# Pensions and Strategic Prevention of Unemployment

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#### Abstract

If unionized workers gain an insider position through past affiliation with their firm, young workers entering the labor market will bear the resulting insider-outsider unemployment burden. The adoption of a pay-as-you-go pension scheme provides an incentive for the insider union to prevent unemployment by lowering wages. The reason is a higher social security tax base in the future induced by higher productivity of (formerly) young workers through on-the-job learning.

This paper presents a stylized overlapping generations model implementing the above arguments. By deriving the steady state equilibria it is shown that unemployment can be partially or fully abolished, provided that labor demand is sufficiently elastic with respect to wages. The conditions for the equilibria to be subgame perfect are also given.

**Keywords:** insider-outsider unemployment, learning by doing, payas-you-go pension system

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#### 1 Introduction

The topic of this paper is the interaction of insider-outsider unemployment (see Lindbeck and Snower, 1986) and a pension system. The underlying idea is quite simple and intuitive: If insiders receive a reward from outsiders, they will have an incentive to "let outsiders in" and thus to forgo some of their insider rents. A pay-as-you-go pension system can provide such a transfer if some assumptions hold: The insider-outsider status must be reflected by the age of the worker, or more precisely, her tenure. The idea that there is a correlation between being young and being an outsider is not new, see for example Pissarides (1989).

Furthermore, if the social security contribution rate is roughly constant, the pension system is only able to provide the reward from (former) outsiders to insiders if the future labor income tax base is broadened. In this paper the mechanism is a productivity enhancement of (former) outsiders due to training-on-the-job (or learning-by-doing) effects. Expressed differently, insiders invest in the human capital of the young (outsider) generation by letting them work. To do this, they have to lower wages which they normally would not do. But they reap the returns of their investment during retirement in the form of higher pensions. Obviously, insiders will implement this "investment project" only if its present value is positive. This paper derives sufficient conditions that insiders will actually let all outsiders work in the context of a suitably simplified overlapping generations (OLG) model.

Apart from obvious connections to the insider-outsider literature and OLG models, this paper is related to Konrad (1995) and Cremer et al. (1992).

In the former article the investment in the young generation takes the form of physical and (the formation of) human capital. The contribution on intra-family transfers by Cremer et al. (1992) is roughly comparable with respect to the schooling aspect, but (apart from other features) models the effect of the pension system differently. There the pension scheme does not provide an individual strategic incentive because of free-rider effects; instead its influence is indirect via a shifted income position in another strategic game about bequests.

While the increase of human capital is also necessary in the present model, the treatment of unemployment is new and complements the existing literature. The explicit capital transfer or provision of education is replaced by a learning-by-doing mechanism through which higher employment leads to higher human capital levels. In reality, of course, both strategic (as well as non-strategic) channels could coexist, for instance if formal education, physical capital, and on-the-job-learning are (partly) complements.

Also, note that the phenomenon called "unemployment" in the present

Figure 1: The OLG structure

t t+1 t+2 t+3 t+4

pot. active pot. active retired

pot. active pot. active retired

pot. active pot. active retired

(etc.)

**Note**: Individuals enter the labor market immediately after birth, and they are potentially active (pot. active) for two periods, after which they retire.

paper need not imply that a part of the labor force is completely unused. Instead, adopting a dual labor market framework, "employment" could be interpreted as employment in the "good jobs" sector, whereas "unemployed" workers would have to conduct tasks in the "bad jobs" sector, where they cannot acquire any skills.

The remainder of this paper includes a description of the model (section 2), a brief characterization of the equilibrium path in the absence of a pension system (section 3), an analysis of the situation with such a system (section 4), and finally concluding comments (section 5).

# 2 The setup

The economy consists of an infinite sequence of overlapping generations that live for three periods. In each period a new generation of individuals is born and immediately enters the labor market. Its members attempt to supply labor for two periods, followed by one period of retirement. For simplicity, there is no population growth, so the cohort size is constant and is normalized to 1. In each period there are three generations alive; young workers (outsiders), old workers (insiders or outsiders, see below), and retirees. A sketch of the structure of this OLG economy is given in figure 1.

Effective labor units are homogeneous, where the effective labor supply of each individual is given by l or h, with 0 < l < h. We assume on-the-job learning, so a newly employed is only capable of supplying l for the next period, whereas after one period of experience a worker's potential rises to h for the remaining active life span. In the rest of this paper, h is normalized,

h = 1. The learning-by-doing aspect explains the necessity of a three-period OLG model with two (potentially) active periods.

Output is produced according to a function with neoclassical properties such as decreasing returns to effective labor inputs,  $f[A_t]$ , f' > 0, f'' < 0, where  $A_t$  is effective employment in period t. Note that we use square brackets for the argument of this function throughout this paper. For technical reasons we also assume that f'' is a monotonous function. Effective employment consists of both types of workers, whose labor supply is identified by subscripts "low" and "high":

$$A_{t} = l \left( A_{low,t}^{1} + A_{low,t}^{2} \right) + A_{high,t}^{2}, \tag{1}$$

where superscripts help to identify which generation supplies the respective amount of labor: 1 stands for workers in their first active period (=young), 2 for old workers. By assumption there cannot exist young high-productivity workers in this model.

For simplicity, it is assumed that the centralized insider union is in a monopoly position and sets wages accordingly. If there are no insiders in the economy, the labor market outcome is competitive with wages and employment at the level compatible with full employment. Note that it is assumed that there is no wage discrimination between (employed) effective labor units. Employment levels are set by the (otherwise passive) firm and are therefore determined by the "right-to-manage" condition, simply equating marginal productivity to marginal costs. This merely means that the firm implements the value of its labor demand function given the wage rate  $W_t$  set by the union or determined by competition:

$$f'[A_t] = W_t \tag{2}$$

If wages are set by an insider union at the beginning of a period, we assume that jobs are allocated serving the tenured workers (new insiders) first.<sup>2</sup> However, some insiders may be fired if the wage level exceeds the productivity of the marginal insider. As we also assume insider-biased "solidarity" (or alternatively high risk aversion among insiders), the union will set wages such that all insiders remain employed. With this assumption, the event tree describing a worker's possible *curriculum vitae* looks as shown in figure 2. The formalization of this restriction is given by:

 $<sup>^1\</sup>mathrm{Bargaining}$  models would complicate the analysis, but would yield little additional insight.

<sup>&</sup>lt;sup>2</sup>This can be motivated by firing costs.

Figure 2: Possible CVs of an individual employed retired with h (=1)become insider employed retired with lemployed enter as with loutsider retired unemployed unemployed

$$A_{high,t}^2 = A_{low,t-1}^1 \tag{3}$$

Of course, in the absence of strategic considerations this means that the insider union will set wages at the productivity level of the marginal insider, which will cause all outsiders to remain or become unemployed.

Finally, it is useful to introduce the outsider employment quota  $q_t, \forall t: 0 \leq q_t \leq 1$ , as the central variable describing the evolution of the model. It is assumed that firms do not discriminate between young and old outsiders, randomly drawing the needed workers from the outsider supply pool. With the employment quota the various employed labor quantities can be described as follows; note that these are not effective units. First, the amount of employed young outsiders in t, who by definition have low productivity, is given by (recall that the cohort size is unity):

$$A_{low,t}^1 = q_t \tag{4}$$

Because of (3) this is also the amount of insiders in t + 1, who are high productivity workers. Next, all workers who were unemployed in their first active period  $(1 - q_{t-1})$  remain outsiders. The amount of employed old outsiders in t is therefore

$$A_{low,t}^2 = q_t(1 - q_{t-1}). (5)$$

Plugging these substitutions into the right-to-manage condition (2) yields

$$W_t = f' \left[ lq_t(2 - q_{t-1}) + q_{t-1} \right]. \tag{6}$$

# 3 Equilibrium path without a pension system

It is quite easy to describe the resulting equilibrium sequence of employment quotas in this model without further assumptions. Imagine an initial situation in t with full employment, when the insider union is founded. At the beginning of the next period t+1, the insiders will increase wages up to the point of equating them with the marginal insider's productivity. The new generation of currently young workers will remain unemployed, hence  $q_{t+1}=0$ . When the insiders retire at the beginning of t+2, there will be no insiders anymore, and the labor market will clear, so  $q_{t+2}=1$ . The situation in t+2 is identical to the one in t, so the cycle starts again. The unique equilibrium path is oscillating forever between complete outsider unemployment and full employment.

# 4 The model with a pension system

Consider a pension system where labor income is taxed at the constant rate b.<sup>3</sup> The revenue is equally distributed among retirees. Apart from that, workers have free access to a highly stylized capital market with constant interest rate r.

The relevant strategic decision is made by the insiders who set wages taking into account the effect of this decision on the labor market equilibrium and future consequences, especially for productivity levels of younger workers. However, the stress in this paper is on purely egoistic motives, without any intergenerational altruism or solidarity. The utility of insiders in period t can be indexed by the following expression:

<sup>&</sup>lt;sup>3</sup>It is also possible to analyze other systems, e.g. where pension payments are linked to current or past labor income. Then there may be additional incentives for insiders to lower wages, as more employed workers represent a broader tax base, at least if labor demand is sufficiently elastic. On the other hand, if pensions are linked to one's own past income, there is a counteracting incentive for insiders to increase their wages in order to also reap higher pensions in the future. But the assumption of a constant social security tax rate seems reasonable as a first approximation, because societies seem to tolerate contribution rate variations only to certain extent (especially upwards). So even if formally the pension system specifies a link to labor income in the short run, legislation will be changed if the implied contribution rate hits some threshold. This can currently be observed in Germany.

$$V_t = (1 - b)W_t + \frac{1}{1 + r}bW_{t+1}A_{t+1}$$
(7)

The first term is simply labor income net of social security taxes for an individual insider's effective labor supply h = 1. The second component is the transfer received from the pension system, which is the equally distributed (among the retired generation, with size normalized to one) discounted  $(\frac{1}{1+r})$  share (b) of the future aggregate labor income. Applying the above equalities (1), (3), (4), (5), and (6), the problem of the insiders is given by:

$$\max_{q_t} V_t = (1 - b) f' \left[ lq_t (2 - q_{t-1}) + q_{t-1} \right] + \frac{b}{1 + r} \left\{ f' \left[ lq_{t+1} (2 - q_t) + q_t \right] \left( lq_{t+1} (2 - q_t) + q_t \right) \right\},$$
(8)

where the range of the choice variable is limited by construction:

$$0 \le q_t \le 1. \tag{9}$$

From (8) it is clear that the insiders' utility also depends on the actions of next period's insiders who set  $q_{t+1}$ . This is a serious complication of the analysis, because it gives rise to an infinite interdependence, potentially making today's optimal choice susceptible to actions of generations in the distant future. Providing an accurate description of all possible equilibrium paths is therefore not straightforward, if it is possible at all.

Instead we will pursue a more modest approach by applying OLG game theory in the spirit of Salant (1991) or Kandori (1992), analyzing Nash and subgame perfect equilibria defined below. Furthermore, we will only consider steady state equilibria, which implies that our analysis only covers symmetric solutions where all players choose the same action in equilibrium.

To this end, we adopt Salant's (1991) notion of a strategy which in the context of the present model yields the following definition. (We restrict ourselves to pure strategies.)

**Definition 1** A strategy for the insider union of period t is a rule which determines the union's choice of  $q_t$  as a function of the history of the economy up to t, i.e. as a function of the finite sequence  $\{q_i\}_{i=0}^t$ . This function must be defined everywhere, i.e. every conceivable historical path of the economy must be assigned a corresponding choice for  $q_t$ .

After this clarification, the definition of an equilibrium is standard:

**Definition 2** An OLG Nash equilibrium is a set of strategies for each player (each period's insider union) which specify individually optimal actions given the set of the strategies of other players.

As mentioned before, we will only derive all possible steady state equilibrium paths, because cyclical or irregular developments seem too difficult to analyze. Furthermore, we will derive these paths based on very special strategy sets, not even coming close to discussing all possible strategies leading to these steady state equilibria.

### 4.1 The full employment equilibrium

As a first and most obvious candidate, consider the following potential equilibrium: Let the strategy of each period's insider union be the choice  $q_t = 1$  for all possible histories of the economy, thus implementing full employment. This is an almost trivial strategy because it is actually independent of the history of the game; it therefore represents a very special case of definition 1. The important property, however, is that it does not depend on the future of the game, which will only become known ex post. We obtain the following result.

**Proposition 1** The strategy set  $\forall (t \geq t_0) : q_t = 1$  (where  $t_0$  is the time of the introduction of the pension system) represents an OLG Nash equilibrium if labor demand is sufficiently elastic. In this equilibrium unemployment is completely abolished. More precisely, the following set of conditions is sufficient for the result to hold, where the argument  $l(2-q_t)+q_t$  will be abbreviated by  $\alpha$ . The inverse own price elasticity of labor demand

$$\varepsilon = \frac{f''[\alpha]}{f'[\alpha]} \alpha \le 0 \tag{10}$$

is not necessarily constant, but must satisfy

$$\varepsilon > -1$$
 (11)

over the entire range of the argument. Additionally, we need the following supplements:

$$\frac{\varepsilon}{1+\varepsilon} \ge -\frac{b}{1-b} \frac{2}{1+r} (1-l) \tag{12}$$

$$\forall q_t : f''' \cdot \left( \frac{1 + (l-1)q_t}{(2 - q_t)l} - 1 \right) \ge 0 \tag{13}$$

Proof Obviously, we have to check whether for the insider union of an arbitrary period t it is optimal to choose  $q_t = 1$ , if all other unions choose this full employment quota. As only the immediately preceding and following periods are relevant for the utility of the union of period t, this means establishing the optimality of  $q_t = 1$  given  $q_{t-1} = q_{t+1} = 1$ . As was already described above, there will be full employment every other period in the economy without a pension system. Thus the strategic decision problem encompasses that of the first insider generation facing the pension system right after its introduction. It is only important that in (8)  $q_{t-1} = q_{t+1} = 1$  can be used. The utility becomes

$$V_t|_{q_{t-1}=q_{t+1}=1} = (1-b)f'[lq_t+1] + \frac{b}{1+r}\{f'[\alpha]\alpha\}.$$
 (14)

It is immediately clear that without a pension system (b=0) the optimal  $q_t$  is zero, as it enters the only remaining term as a positive argument of a decreasing function. The same result emerges in the absence of on-the-job-learning (l=1): The second term becomes an irrelevant constant (recall the abbreviation  $\alpha = l(2-q_t) + q_t$ ), and in the first term again a higher  $q_t$  diminishes insiders' utility. For other cases, the given set of conditions ensures that the utility of insiders is increasing in  $q_t$ , making the corner solution  $q_t = 1$  the optimal choice. To see this, differentiate (14) with respect to  $q_t$  and divide by  $f'[\alpha]$ :

$$(1-b)\frac{f''[lq_t+1]}{f'[\alpha]}l + \frac{b}{1+r}(1-l)\left\{\frac{f''[\alpha]}{f'[\alpha]}\alpha + 1\right\}$$
(15)

Using the definition for  $\varepsilon$  to substitute the first part of the term in braces, it is clear that  $\varepsilon > -1$  is necessary for the whole expression to be positive for all  $q_t$ . Furthermore, note that we can write

$$\frac{f''[lq_t+1]}{f'[\alpha]} = \frac{f''[\alpha]}{f'[\alpha]} + \Delta_1,\tag{16}$$

where  $\Delta_1$  denotes an approximation error, defined as

$$\Delta_1 = \frac{f''[lq_t + 1] - f''[\alpha]}{f'[\alpha]}.$$
(17)

It is useful to determine in which cases this error is nonnegative. If we assume a monotonous function f'', a comparison of the arguments of the functions in the numerator of  $\Delta_1$  yields the following possibilities:

$$\left(\forall q_t : f''' = 0 \lor \frac{1 + (l-1)q_t}{(2 - q_t)l} - 1 = 0\right) \Rightarrow \Delta_1 = 0 \tag{18}$$

$$\left(\forall q_t : f''' \stackrel{\geq}{\leq} 0 \land \frac{1 + (l - 1)q_t}{(2 - q_t)l} - 1 \stackrel{\geq}{\leq} 0\right) \Rightarrow \Delta_1 \geq 0 \tag{19}$$

Combining these cases yields condition (13). Returning to the derivative of insiders' utility, we can now write the condition of it being nonnegative as

$$(1-b)\left(\frac{\varepsilon}{\alpha} + \Delta_1\right)l + \frac{b}{1+r}(1-l)(1+\varepsilon) \ge 0. \tag{20}$$

A few rearrangements lead to:

$$\frac{\varepsilon + \Delta_1 \alpha}{1 + \varepsilon} \ge -\frac{b}{1 - b} \frac{1}{1 + r} \frac{(1 - l)^2}{l} \left( \frac{2l}{1 - l} + q_t \right) \tag{21}$$

To arrive at the conditions as stated in the proposition, note the following properties: First, the critical value for  $\frac{\varepsilon + \Delta_1 \alpha}{1+\varepsilon}$  depends on  $q_t$ . This is admissible but inconvenient. It can easily be seen that the maximum of the critical value is attained for  $q_t = 0$ , wherefore we choose that value for the sufficient condition. Second, if  $\Delta_1 \geq 0$ , then  $\Delta_1 \alpha \geq 0$ , and hence

$$\Delta_1 \ge 0 \Rightarrow \frac{\varepsilon + \Delta_1 \alpha}{1 + \varepsilon} \ge \frac{\varepsilon}{1 + \varepsilon}.$$
(22)

This means that under the given assumption for  $\Delta_1$ , if  $\frac{\varepsilon}{1+\varepsilon}$  exceeds the right hand side of (21), then the really relevant condition will also hold.

Interpreting the result verbally, unemployment is only prevented if the on-the-job-learning effect is strong enough to enhance future effective employment to such an extent that it outweighs the necessary decrease of contemporaneous and future wages. Whether this holds depends on the shape of

the production function f in relation to the influence of the parameters b, r, l. The stated condition (12) is more likely to hold if the contribution rate b is higher, or if the productivity enhancement through on-the-job-learning 1-lis higher, or if the discount rate r is lower. This result confirms the intuition with respect to the effects of these parameters (as there is no possibility to substitute labor inputs in this stylized model). Note that the condition (13) concerning the third derivative of the production function includes as a special case f''' = 0, implying a quadratic production function and a linear labor demand schedule. Also, note that if the training-on-the-job effect is strong (l is low),  $\frac{1+(l-1)q_t}{(2-q_t)l}-1$  is likely to be greater than zero, such that f''' would have to be nonnegative, implying a convex labor demand curve. (Strictly speaking, this is not necessary for the equilibrium property, but necessary for this particular sufficient condition to hold.) However, in the special case of l = 0.5 (where learning on the job exactly doubles a worker's productivity), the sign of f''' is completely irrelevant for the sufficiency of the condition set.

### 4.2 Equilibria with partial outsider unemployment

The previous result represents the corner solution of the optimization problem of insiders. Next, consider the possibility of an interior steady state equilibrium with  $q_t$  inside (0; 1]. (Of course, a steady state with  $q_t = 0$  is impossible, because as described before, there will then be no insiders in the next period and the labor market will clear with  $q_{t+1} = 1$ .) The following result is found.

**Proposition 2** The strategy set  $\forall (t \geq t_0) : q_t = \overline{q}$  (where again  $t_0$  is the time of the introduction of the pension system, and  $\overline{q} \in (0;1]$ ) represents an OLG Nash equilibrium if the following set of conditions hold (under the additional assumption that a local maximum is also a global one):

$$\frac{\varepsilon}{1+\varepsilon} = -\frac{b}{1-b} \frac{1-l\overline{q}}{1+r} \overline{q} \left( 1 + \frac{1}{l(2-\overline{q})} \right)$$
 (23)

where  $\varepsilon$  is now evaluated at  $l\overline{q}(2-\overline{q})+\overline{q}$ . Note that  $\varepsilon>-1$  again is necessary.

$$f''\left[l\overline{q}\left(2-\overline{q}\right)+\overline{q}\right]\left(2-\overline{q}\right)=f''\left[1+l\overline{q}\right] \tag{24}$$

Note that f''' < 0 is necessary here.

*Proof* The proof is somewhat tedious but straightforward and will only be briefly described. The first order condition is defined by

$$\frac{dV_t|_{q_{t-1}=q_{t+1}=\overline{q}}}{dq_t}|_{q_t=\overline{q}} = 0, \tag{25}$$

which yields the first of the stated conditions. Note that it is not guaranteed that for all possible constellations of  $\varepsilon$ , b, r, l there is a permissible  $\overline{q}$  which satisfies this condition.

For the first generation after the introduction of the pension system the situation is a little different because those insiders face  $q_{t-1} = 1$ . As  $\overline{q}$  is already fixed by the steady state condition, it must also be true that

$$\frac{dV_t|_{q_{t-1}=1,q_{t+1}=\overline{q}}}{dq_t}|_{q_t=\overline{q}} = 0. {26}$$

A comparison of these two points of the derivatives leads to the second stated condition, which is a strong requirement, but possible. As  $1+l\overline{q}>l\overline{q}$  ( $2-\overline{q}$ ) +  $\overline{q}$  and f''<0, this can only hold if f'''<0. It turns out that this latter property of f is sufficient for the second order conditions for the two cases

$$\frac{d^2V_t|_{q_{t-1}=q_{t+1}=\overline{q}}}{dq_t^2}|_{q_t=\overline{q}} < 0, \frac{d^2V_t|_{q_{t-1}=1,q_{t+1}=\overline{q}}}{dq_t^2}|_{q_t=\overline{q}} < 0$$
 (27)

to hold. As mentioned in the proposition, this of course only guarantees the existence of a local maximum. The coincidence with a global maximum in these cases is simply assumed.  $\blacksquare$ 

This steady state result with interior solutions is less attractive, because it imposes strong requirements on the shape of the production function. However, the solutions cannot be ruled out.

# 4.3 Subgame perfection

A Nash equilibrium is given by a strategy set that represents best answers to the actions of other players, if everybody adheres to the equilibrium strategies. A standard refinement of the equilibrium concept is the notion of subgame perfection, where it is required that equilibrium strategies also include best answers to non-equilibrium actions, where different "branches" of the game tree are reached. In the context of the present model this leads to the following definition.

**Definition 3** An OLG subgame perfect equilibrium is a set of strategies for each period's player that represents a Nash equilibrium in each possible subgame, no matter whether the subgame is attained in equilibrium or not. In the present model, the subgames are given by all possible evolutions of the economy measured by the sequence  $\{q_t\}_{t=t_0}^{\infty}$  (where  $t_0$  again is the period of the introduction of the pension system), combined with all possible subgame starting periods  $t=t_0...\infty$ .

This sounds more complicated than it actually is, because in our model the only historical realization that matters is  $q_{t-1}$ . If we discard the whole game involving the first generation after the introduction of the pension system as trivial, the remaining subgames are isomorphic: Because of the infinite existence of the economy there is no difference between varying subgame starting periods, as there will always be an infinite number of periods ahead. Therefore, a check of subgame perfection only involves analyzing whether a given strategy set provides a Nash equilibrium for arbitrary values of  $q_{t-1}$ . It should be noted that the above definition is possible because of the definition of strategies dependent only on the history of the game as in Salant (1991) and Kandori (1992).

Applying this check to full employment strategy set considered in proposition 1 with a corner solution we find the following result.

**Proposition 3** The strategy set analyzed in proposition 1,  $\forall (t \geq t_0) : q_t = 1$ , represents an OLG subgame perfect equilibrium if the following set of sufficient conditions hold.

$$\varepsilon > -1$$
 (28)

$$\forall (q_t, q_{t-1}) : f''' \cdot \{lq_t (2 - q_{t-1}) + q_{t-1} - l(2 - q_t) - q_t\} \ge 0$$
 (29)

$$\frac{\varepsilon}{1+\varepsilon} \ge -\frac{b}{1+b} \frac{1}{1+r} (1-l) \tag{30}$$

Proof The proof is similar to the one of Nash equilibrium. The sufficient conditions ensure that  $V_t|_{q_{t+1}=1}$  is increasing in  $q_t$ , and this again makes the corner solution  $q_t = 1$  optimal. Differentiating  $V_t|_{q_{t+1}=1}$  w.r.t.  $q_t$  and dividing by  $f'[\alpha]$  yields:

$$(1-b)\frac{f''[lq_t(2-q_{t-1})+q_{t-1}]}{f'[\alpha]}(2-q_{t-1})l + \frac{b}{1+r}(1-l)(\varepsilon+1)$$
(31)

Again, we approximate the first fraction:

$$\frac{f''[lq_t(2-q_{t-1})+q_{t-1}]}{f'[\alpha]} = \frac{f''[\alpha]}{f'[\alpha]} + \Delta_2$$
 (32)

$$\Delta_2 = \frac{f''[lq_t(2 - q_{t-1}) + q_{t-1}] - f''[\alpha]}{f'[\alpha]}$$
(33)

Again, we impose some sufficient conditions (not reported) such that  $\Delta_2 \geq 0$ , which lead to (29) in analogy to the proof of proposition 1. Some calculations lead to

$$\frac{\varepsilon + \Delta_2[\alpha]}{1 + \varepsilon} \ge -\frac{b}{1 + b} \frac{1}{1 + r} \frac{1}{2 - q_{t-1}} \left(\frac{2l}{1 - l} + q_t\right) \frac{\left(1 - l\right)^2}{l} \tag{34}$$

As before, if the approximation error  $\Delta_2$  is nonnegative, we can ignore it, and it can easily be seen that the right hand side reaches a maximum for  $q_t = q_{t-1} = 0$ , wherefore we choose those values for the sufficient condition.

Note that the third condition is stronger than before. The absence of the factor 2 increases the right hand side (decreases its absolute value) such that  $\varepsilon$  must also be greater than before (its absolute value must be lower) to satisfy the new condition.

We still need to clarify the possibility of subgame perfection in the case of steady state equilibria with interior solutions,  $\overline{q} \in (0; 1]$ .

**Proposition 4** The strategy set analyzed in proposition 2,  $\forall (t \geq t_0) : q_t = \overline{q}$ , represents a subgame perfect equilibrium if in addition to the conditions given before it also holds that

$$\forall q_{t-1} : f'' [2l\overline{q} + (1 - l\overline{q}) q_{t-1}] (2 - q_{t-1}) = f'' [1 + l\overline{q}].$$
 (35)

*Proof* As in the proof of proposition 2, the steps are straightforward and are only briefly described. As mentioned before, the check for optimality given deviations from the equilibrium strategies amounts to letting the past

realization vary and ascertain that optimality still applies. Therefore the check refers to all possible values of  $q_{t-1}$ . This is a generalization of the condition needed for the first generation, where  $q_{t-1} = 1$ . The first order condition is given by

$$\frac{dV_t|_{q_{t+1}=\overline{q}}}{dq_t}|_{q_t=\overline{q}} = 0. (36)$$

Comparing the terms in the resulting equation that vary with  $q_{t-1}$  to the analogous terms in the conditions needed for the existence of Nash equilibria, we arrive at the stated additional condition. Then the strategy of playing  $q_t = \overline{q}$  is optimal for all past developments. (As before, it can be seen that f''' < 0 is required and ensures that the second order conditions hold. Again, it is assumed that local maxima are also global maxima.)

This latter result has very strong requirements and will only apply to a very narrow class of production functions. Thus any Nash OLG steady state equilibria with employment quotas between zero and one are very unlikely to be also subgame perfect.

## 5 Conclusions

This paper presented a highly stylized OLG model combining insider-outsider unemployment theory with a generational conflict due to the earlier arrival of older workers on the labor market, which gave them the insider power. It was also assumed that workers' productivity is enhanced through learning-by-doing. Then it could be shown that a pay-as-you-go (defined contribution) pension system can induce insiders to prevent unemployment, because the pension system lets them participate in the increased earnings potential of (former) outsiders. However, this effect outweighs the necessary reduction of wages only if labor demand is sufficiently elastic.

For reasons of tractability we assumed a monopoly union, but the general thrust of the argument applies to other wage bargaining constellations as well, as long as the insider union has some power in determining wages. On the other hand, it should be noted that an essential assumption is the feature that the decision of a centralized insider union affects the entire labor force. Thus we described effects arising in a corporatist economy, otherwise the argument would be weakened by free-rider incentives.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup>An entirely different interpretation was pointed out by Hans Friederiszick: The analysis could also relate to a smaller economic entity. The owner of a small firm could be

At first sight it may seem possible to apply the argument of this model to a funded pension system as well: Higher employment levels could raise the marginal productivity of capital and therefore lead to higher interest rates, which could provide the necessary incentive for insiders who are accumulating funds for their retirement. However, the major caveat is that this effect would only work in a closed economy, because otherwise the analyzed country would be a price taker with respect to interest rates in the world capital market.

On the other hand, the central argument in this paper should not be interpreted as the belief that a pay-as-you-go pension scheme is good and funded systems are bad. For instance, capital was not even included as an input factor in the production function. Also, demographic problems that presently plague social security arrangements in industrial countries were assumed away. The aim of the model was simply to focus on a specific strategic incentive for insider unions which has hitherto not been analyzed in the literature.

The main prediction of this model is that among corporatist economies which face insider-outsider problems, those with intergenerational transfer systems of the type assumed in this paper will have –ceteris paribus—lower unemployment. In this model insiders were rewarded for not causing unemployment through higher pensions. But note that other elements of the welfare state could in principle serve a similar purpose: for instance, unemployment benefits are financed by insiders' taxes. However, whereas pension systems usually have their own budget, it is probably more difficult to link lower unemployment to lower taxes at some point in time. In general, the whole issue is somewhat related to the popular argument by Calmfors and Driffill (1988), claiming that centralized institutions take their possibly adverse (side) effects on economic performance better into account than sector-wide institutional arrangements.<sup>5</sup>

From an empirical perspective, there are unfortunately serious obstacles to assessing the relevance of the theoretical implications of the present model. For a comparison we would need corporatist economies with insider problems, but without a traditional pension system. These are very hard to find, and this complication is aggravated by the fact that for statistical reasons we ideally would also need more than just one. For the time being, we leave this question open for future exploration.

induced to hire somebody who could become his successor, even if the worker were not profitable. The acquisition of firm-specific human capital on the part of the worker would increase his reservation utility of taking over the firm. Hence the owner could charge a higher price, rewarding him for the earlier implicit employment "subsidy".

<sup>&</sup>lt;sup>5</sup>Their more specific "hump-shape" hypothesis has not withstood careful empirical analysis, however; see OECD (1997).

We do not claim, however, that the effect described in this model accounts for the virtual omnipresence of pay-as-you-go systems in insider-plagued economies. These systems with intergenerational transfers are typically installed in times of imminent urgency, i.e. when older generations face poverty because the economy's capital stock has been wiped out, be it because of wars or after natural catastrophes. It is clear that the social policy component of pension systems is much more pronounced than any of the rare efficiency reasons one can find.

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