A Multisector Model of Labor Markets in Developing Countries: Implications for poverty reducing policies

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James Albrecht, Catalina Gutierrez, Pierella Paci

Abstract

We develop a multi-sector model of labor markets in developing countries in which workers can be in any of four states i) employed in the formal urban sector, ii) employed in the informal urban sector, iii) unemployed in the urban sector or, iv) employed in agriculture. Workers differ in the productivity of formal sector employment and there are floor wages below which firms can't set wages. Migration between urban and rural areas can take place at a cost. We find that the informal sector consists of both workers that are there because of rationing of formal employment brought about by floor wages, as well as workers which end up in this sector because of choice. We also find that workers with the lowest level of formal sector productivity locate in rural areas, while those with higher productivity migrate to the cities. We use the model to understand the impact of different policies on the allocation of workers among different employment options and on poverty. We then extend the model to take into account inter-linkages between labor markets and growth, in particular we allow for feedback effects of labor markets on growth through learning by doing.

1 Introduction

Improvement in the quality and quantity of employment opportunities is increasingly recognized as one of the main transmission channels between growth and poverty reduction. This is reflected in (i) a generalized concern with 'jobless growth' as the potential cause of the failure of growth to reduce poverty in a number of countries (ii) the related growing debate on how to foster employment intensive growth and, iii) an increasing interest on the effects of labor markets on growth. More recently, emphasis has been shifted to the fact that in low and middle income countries where unemployment is a luxury, poverty is more likely to be associated with low productivity (and low earnings) than lack of employment. In these cases it is the impact of growth on the quality—rather that the quantity—of employment opportunities that matters for poverty reduction.

Despite the obvious importance of labor income and employment in making growth work for the poor, our understanding of the role of labor markets in transmitting the benefits of growth to the poor has remained clouded. Our knowledge on the feedback effect of the impact of labor markets on growth is even more limited, despite the well documented importance of growth for poverty reduction. One of the main constraints we face in remedying this gap in knowledge is the lack of a structural model that links labor markets poverty and growth, which takes into account the specificities of labor markets in developing countries. In particular, labor markets in developing countries (DC's) are better understood under a multi-sector framework, where a 'modern' or 'good jobs' urban sector coexists with a bad jobs or 'traditional' urban sector, and an agricultural sector, both of which offer low earnings opportunities. Often, most of the working population is employed in the last two. Movements of labor force from the low productivty sectors to the high productivity sectors -which is most commonly referred to as structural change in the development literature- is at the core of growth and poverty reduction in DC's. Movement between sectors is often limited and unemployment is a 'luxury', thus being more prevalent among the middle class than among the poor.

With very few exceptions, current models of labor markets are strongly biased towards explaining developed countries labor markets outcomes, where there are only two possible states for active population: being employed or unemployed. In addition workers face no barriers to moving between states and the agricultural sector is treated as any other sector in the economy, all of which are equally productive. Policy analysis has also concentrated on issues not always relevant to developing countries; such as the impact of growth on unemployment, or the effects of unemployment insurance on labor market outcomes. Both are mot issues in developing countries given the nature of unemployment in DC's. Instead, issues such as what is the impact of educational expansions, of increases in productivity in agriculture, of reducing barriers to mobility between rural and urban sectors, of minimum wages, or of public sector pay, and how do labor markets affect countries growth prospects, have been largely unexplored . All these policies are at the forefront of the policy agenda in developing countries.

The lack of a structural model that takes into account the specificities of labor markets in developing countries has hampered our understanding of their role in transmitting the benefits of growth to the poor, as well as our understanding of the impact of different policies on labor market outcomes and its feedback on growth, in low and middle income countries.

This paper is tries to remedy this knowledge gap by developing such a model. We begin by extending the model of Albretch et al. (2007) in which agents differ in their formal sector productivity, to allow for an agricultural sector, urban-rural migration and floor wages. In Albrecht et al., workers choose to be in the informal sector not as a result of rationing of informal jobs, but as a worker's decision determined by his or her level of human capital and potential productivity in the formal sector. This does not mean that workers in the informal sector are as well off as those in the formal sector. It only implies that they would not necessarily be better off in the other sector.

Although this is consistent with some evidence for Latin America countries, there is still some controversy, and rationing of formal jobs is supported in other countries. We show that by introducing minimum wages, we can combine both views of informality, the one that sees it as a result of rationing and the one that sees it as a result of choice. In addition we also consider, rural urban migration, as an essential feature determined labor market outcomes in developing countries.

The first section presents the basic model with urban rural migration. Section 2 extends the model to allow for minimum wages. Some preliminary policy experiments using data from two or three developing countries are performed in section 3. Section 4 extends the model to take into account feedback effects from labor markets to growth, discussing its implications and section 5 concludes.

2 The model with agricultural sector

Workers can be in one of four states i) unemployed in the urban areas and looking for jobs, ii)employed in the urban informal sector, iii) employed in the urban formal sector and iv) employed in the agricultural (rural) sector.

In the formal sector each firm employs only one worker. Workers differ in the maximum productivity in formal-sector jobs. In particular, workers are distributed according to a distribution function$F(y)$, $0 \leq y \leq 1$. The parameter $y$can be loosely interpreted as human capital or the skills of each worker, which can be a combination of education, experience and other unobserved talents. A worker's output in a formal-sector job depends on his or her type. Formal-sector jobs are started at the maximum productivity y. However job-specific shocks (as different from overall economy shocks) arrive at a Poisson rate λ and affect the productivity of a the worker in that particular job. These shock can be caused by structural shifts in demand that change he relative price of the good produced, or by changes in the unit cost of production. They are real shocks associated with shifts in technology or tastes that affect a particular firm. These shocks are *iid* draws from a continuous distribution $G(x)$, where $0 \leq x \leq 1$. There are three possibilities to consider. First, if the realized value of the shock x is sufficiently low, it is in the mutual interest of both the firm and the worker to end the job and the worker will become unemployed. As in the standard MP model, there would be a value of productivity $R(y)$, which will depend on the workers type, below which it is not worth keeping the job. Thus with a probability $G(R(y))$ the shock will end in a termination of the job. Second, if the realized shock is $R(y) \leq x \leq y$, the productivity of the worker will change to x. Therefore, with probability $G(y) - G(R(y))$ the jobs continues after the shock but a at a level of productivity x. Finally, if the realization of the shock is $x > y$ then the job continues at the maximum level of productivity y.That is, with probability $1 - G(y)$ the job continues at maximum productivity. We will assume that in the formal sector at the maximum productivity the output from a job that employes a worker of type y is $A_1 y$, where A_1 is an exogenous technological parameter. When a shock arrives output of worker y will change to $A_1 y'$, where y' will take values between $R(y)$ and y, as described above.

In the informal sector there are no returns to skills, and all workers receive income y_0.Informal sector jobs are also subject to shocks, but in this case all shocks end the job and workers become unemployed again. Shocks to the informal sector arrive at a Poisson rate δ. While employed in the informal sector, workers do not search for formal employment.

We assume that in the rural areas the only activity is agriculture. In the agricultural sector workers receive the average product of labor in the sector. That is, income when employed in agriculture is equal to $y_a = A_a L_a^{\gamma-1}$, with $\gamma < 1$ and $A_0 \geq 1$. This production function just reflects decreasing marginal productivity of labor for a fixed amount of land. While employed in agriculture workers do not search for jobs in the urban economy. In order to search for a job workers need to migrate to urban areas. Migration is costly and workers incur in a cost $M(y)$. Migration can take place either from urban to rural areas or vice-versa, in both cases the cost of migration is the same.

Unemployment is a residual state, anyone not employed in the formal, informal or agricultural sectors is unemployed and looking for a job. While unemployed workers receive a flow of income b, which can be the flow income equivalent to the value of leisure, unemployment benefits or transfers from other family members and friends. We assume that $y_0 > b$. While unemployed, workers find informal sector opportunities at an exogenous Poisson rate α, which they can take-up or not. They also search for formal jobs. We assume that it takes time for workers to find a job and for firms to find suitable candidates to fill the vacancies. We can expect that the higher the number of vacancies relative to job seekers the easier it is to find a job for the unemployed and the harder to fill vacancies. By the same token, very few vacancies and many unemployed workers means its harder to find jobs and easy to fill vacancies. Thus, the rate of arrival of formal job opportunities to the unemployed will depend on both vacancies and unemployment. We take the standard approach that of modeling the number of matches (meetings between an unemployed worker and a firm with a vacancy) by a constant returns to scale production function, in which the inputs are the number of vacancies and of unemployed workers: $mL = m(uL, vL)$, where v is the vacancy rate, u is the unemployment rate and L is the total labor force. This means that, in any unit of time, the probability of an unemployed workers finding a formal employment opportunity (i.e. meeting a potential employer) is $mL/uL = m(v/u, 1)$. If we define $\theta = v/u$, then the rate of arrival of formal employment opportunities to unemployed workers will be $m(\theta)$.Clearly, this rate is increasing in θ, meaning that the higher the number of vacancies the easier it is to find a job. The rate at which firms will meet workers will be $m(\theta)/\theta$[1].

Once workers and firms meet they decide to engage in production whenever it is in the advantage of both parties to do so and they bargain on a wage, which we will denote by $w(y, y)$, with the first term in parenthesis reflecting the workers type and the second the current productivity of the job (all jobs are started at maximum worker productivity). This will depend on the benefits of each in engaging in production and on the outside opportunities. The benefits to the firm will depend on the output of the worker A_1y, on the wage $w(y, y)$ it has to pay him or her and other costs of recruitment. The outside option to the firm, that is the benefits of not hiring the worker and keeping the vancany open, will depend on how costly it is to keep the vacancy open, and how likely it is to find a better worker. The benefits to the unemployed of accepting the job will depend on the wage he receives. His outside option will be give by the income when unemployed, and the likelihood of finding better employment of options if he rejects the job. Once firm and worker are engaged in production, they renegotiate the wage every time a shock arrives and it is worth to keep the match. We will now explain the decisions faced by firms and workers and their optimizing choices.

Aside from the idiosyncratic shocks that hit formal and informal employment, and of which the workers have perfect foresight (i.e. they correctly anticipate the rate of arrival of these shocks), there are general shocks to the economy which are unanticipated. These shocks affect the productivity of each sector, that is they change the value of the technological parameters A_1, A_a, and y_0. they can be again interpreted as changes in tastes or technology that affect the whole economy, rather than a particular firm (job)

2.1 The problem of the worker

Workers can be in four possible states: i) employed in the formal sector, ii) employed in the informal sector, iii) unemployed and iv) employed in the agricultural sector. We will denote the fraction of workers in each state as l_a for the share in agriculture, l_1 for the share in formal employment, l_0 for the share in informal employment and u for the share unemployed. In each state, as conditions change workers take optimizing decisions. When idiosyncratic shocks arrive while they are employed in the formal sector, they decide whether to

[1]The meeting funtion has the standard properties: i) $m(\theta)$ is increasing in θ ii) $m(\theta)/\theta$ is decreasing in θ, iii) $\lim_{\theta\to 0} m(\theta) = 0$ and the $\lim_{\theta\to\infty} m(\theta) = \infty$ and iv) $\lim_{\theta\to 0} m(\theta)/\theta = \infty$ and the $\lim_{\theta\to\infty} m(\theta)/\theta = 0$

continue in the job or end it and go back to the pool of unemployed workers. When they are employed in the formal sector if an idyosincratic shock arrives the jobs is automatically destroyed. So there are not decision to make. If he is unemployed he needs to decide whether to accept formal or informal jobs as offers arrive. In addition to these decisions, workers decide where to locate: in the urban areas where they can find formal and informal opportunities but also be unemployed, or in the rural areas where they can only engage in agricultural production. For any given values of A_1, A_a, and y_0, the distribution of workers between urban and rural areas must reflect and optimizing behavior. Once new shocks to A_1, A_a, and y_0 arrive they need to decide whether to stay where they are, or migrate incurring in the migration cost $M(y)$. The decisions of the worker can be summarized by the value of being in each state. These values can be represented by value functions, and correspond to the discounted expected income of being in each of the states. These values depend on the probability of finding a job (formal or informal) on the arrival rate of idiosyncratic shocks and on a worker's type, all of which are known to the worker. For a worker of type y, we will denote these values as $U(y)$ for the value of being unemployed, $N_1(y, y')$ for the value of being employed at productivity y', $N_0(y)$ for the value of being employed in the informal sector and $N_a(y)$ for the value of being employed in the agricultural sector . The appendix describes in more detail the derivation of these value functions and their rationale.

For a worker of type y , the value of being unemployed $U(y)$, will be given by

$$rU(y) = b + \alpha \max[N_0(y) - U(y), 0] + m(\theta) \max[N_1(y, y) - U(y), 0] \quad (1)$$

The value of being unemployed thus depends on the income flow from unemployment b, and the option value of being unemployed which is juts the likelihood that the worker will receive a formal or informal job offer in the future. With probability α he will find an informal sector opportunity, in which case he will have to choose whether to take it or not. He will take it if the difference between the value of taking it and of continuing to be unemployed is positive if he takes-up the job his net gain will be $N_0(y) - U(y)$, if he rejects it his net gain will be zero . With probability $m(\theta)$ the worker will meet a firm with a vacancy and he will have to decide whether to take it or not. He will take it if the value of being employed is larger than the value of staying unemployed, in which case he will have a net gain of $N_1(y, y) - U(y)$.otherwise he will stay unemployed and have a net gain of zero.

The value of being in an informal sector job $N_0(y)$ will be given by:

$$rN_0 = y_0 + \delta(U(y) - N_0(y)) \quad (2)$$

It reflects the fact that while being employed in the informal sector the worker receives an income flow of y_0 and in the future with probability δ a shock will arrive and the informal job will be destroyed. In which case he will gain the value of unemployment $U(y)$ and loose the value of informal employment $U(y) - N_0(y)$. He will thus have a net gain of $U(y) - N_0(y)$

The value of being in an agricultural job $N_a(y)$ will be given by:

$$rN_a = y_a \tag{3}$$

It just reflects the present discounted value of income from agricultural activities, as we are assuming no idiosyncratic shocks will affect this employment.

Finally, the value for a worker of type y of being in a formal sector job at current productivity y', $N_1(y)$ will be given by:

$$\begin{aligned} rN_1(y,y') &= w(y,y') + \lambda G(R)[U(y) - N_1(y,y')] + \\ &\lambda \int_{R(y)}^{y} [N_1(y,x) - N_1(y,y')]g(x)dx + \lambda[1-G(R)][N_1(y,y) - N_1(y,y')] \end{aligned} \tag{4}$$

It reflects the fact that the worker receives a current flow of income equal to his wage $w(y,y')$, and that in the future several things can happen: i) he can loose the job with probability $\lambda G(R)$ (i.e. the probability of a shock arriving λ times the probability that the new productivity falls bellow the reservation productivity $R(y)$), in which case he will have a net gain of $U(y) - N_1(y,y')$.ii) with probability λ a shock will arrive that is bellow y but does not destroy the match, in which case he will have an expected net gain of $\int_{R(y)}^{y} [N_1(y,x) - N_1(y,y')]g(x)dx$ and, iii) with probability $\lambda[1-G(R)]$ he will receive a shock that will return the productivity of the worker to its maximum potential and he will have a net gain of $N_1(y,y) - N_1(y,y')$.

2.2 Location and migration decisions by workers

At any point in time and for given values of A_1, A_a, and y_0 worker's location choices have to be optimal, which means that best choice for workers both in urban areas or rural areas is not to migrate. Therefore the expected income from staying in the current area of residence has to be bigger than or equal to the expected income from migration minus the migration cost $M(y)$. The expected income from agricultural work is $Na(y)$. The expected income from locating in urban areas will be determined by the value of finding a job in the formal sector $N_1(y,y)$, finding a job in the informal sector $N_0(y)$ and, being unemployed $U(y)$ and the probabilities of being in each state. Let $n_1(y)$, $n_0(y)$

and $u(y)$ denote the fraction of time (or equivalently the probabilities) that an agent of type y spends in employment in the formal sector, employment in the informal sector and unemployment, the the expected value of locating in the cities will be given by:

$$V_c(y) = N_1(y, y) * n_1(y) + N_0(y) * n_0(y) + U(y) * u(y)$$

The probabilities of being in each state are endogenous and will be determined in equilibrium. They will depend on the worker's type. In equilibrium no migration takes places, which means that for all worker types located in rural areas:

$$Na(y) \geq V_c(y) - M(y) \tag{5}$$

and for all worker types located in the cities:

$$V_c(y) \geq Na(y) - M(y) \tag{6}$$

For simplicity we will assume that initially all agents were located in rural areas, and that migration took place until equation (5) held for all y. We will assume that $M(y)$is either independent of y or decreasing in y. This means that there will be a threshold level of y such that 5 will hold with equality, and above which all workers will migrate to the cities. We thus start with a perfectly segregated population will low type agents locating in the rural areas and high type agents locating in urban ones.

2.3 The problem of the firm

Firms post vacancies if it is profitable to do so. Once they meet a potential worker they decide whether to engage in production or instead search for another worker. Once a job is filled and shocks arrive firms have to decide whether to terminate the job or continue at the new productivity level. The problem of the firm can be expressed in terms of the value of holding a vacancy unfilled and the value of having a vacancy filled with a worker of productivity type y. The value to a firm of a job filled with a worker of type y producing at current productivity level y', $J(y, y')$ is the present discounted value of profits. As long as this value is positive it is in the interest of the firm to initiate or maintain the match. Because $J(y, y')$ is increasing in y, there will be a value of $y = R_f(y)$ below which it is not profitable to initiate the match or continue it. $J(y, y')$ will be determined by:

$$
\begin{aligned}
rJ(y,y') &= A_1 y' - w(y,y') + \lambda G(R)[V - J(y,y')] + \\
&\lambda \int\limits_{R_f(y)}^{y} [J(y,x) - J(y,y')]g(x)dx + \lambda[1 - G(R)][N_1(y,y) - J(y,y')]
\end{aligned}
$$

This expression reflects the fact that the current profit from the job is just the output produced by worker y minus the wage paid: $A_1 y' - w(y,y')$ plus the discounted expected flow of future profits. The expected flow of future profits will be determined by the shock process. With probability $\lambda G(R_f(y))$ a shock will arrive which will turn the productivity of the current job to a value bellow $R_f(y)$, and thus it is not worth for the firm to maintain, the match will terminate and the firm will loose the value of the filled job and will now have a vacancy, thus experiencing a net gain of $V - J(y,y')$.The term $\int\limits_{R(y)}^{y} [J(y,x) - J(y,y')]g(x)dx$ reflects the expected net gain of receiving a shock between $R_f(y)$ and y. Finally, With probability $\lambda[1 - G(R_f(y))]$ a shock will arrive that will reset the productivity to its maximum value.

On the other hand the value of a vacancy is defined by the expression:

$$
rV = -c + \frac{m(\theta)}{\theta} E \max[J(y,y) - V, 0)
$$

This expression reflects the expected discounted benefit of having an open vacancy. There is a cost c per period of time of keeping the vacancy open, and that with probability $\frac{m(\theta)}{\theta}$ the firm will meet a worker. the expected benefit from this meeting will depend on whether they engage in production or not. as long as the net gain from doing so $J(y,y) - V$ is positive, the match will take place and the firm will have a net gain of $J(y,y) - V$.Otherwise the firm will keep the vacancy open and have a net gain of 0. the expected value of $J(y,y) - V$ will in depend on the probability of meeting a worker of type y, which will be determined by the distribution of y types in the unemployment pool. This will be dealt with later.

In equilibrium firms will post vacancies until it is not longer profitable to do so. Because we assume free entry in equilibrium the value of having an open vacancy has to be equal to zero. That is, firms will enter into the market until and post jobs until it is no longer profitable to keep entering.

2.4 Wage determination mechanism

We assume that wages are bargained in anon cooperative way between employers and worker. The power of the worker can be expressed by a parameter $\beta < 1$, and the power of the firm will be $(1-\beta)$.As in most of this literature we assume Nash bargaining. This type of bargaining guarantees that the outcome is the best possible for both players in a non cooperative game in which both parties are trying to split a 'surplus' from a match. Here the surplus arises from the fact workers can't find jobs instantly and it takes some time to fill the vacancies so there is an option value of not making the match for both parties. The difference between the option value and the value of making the match generates a 'surplus'. The solution to the Nash bargaining problem is the wage for which the surplus from the match to both firm and worker is maximized:

$$\max_{w(y,y')} [N_1(y,y') - U(y)]^{\beta}[J(y,y) - V]^{1-\beta} \tag{7}$$

using the fact that in equilibrium $V = 0$ it is easy to verify that the solution to this problem is:

$$w(y,y') = \beta A_1 y' - (1-\beta) rU(y)$$

That is the wage will be a weighted average of the output produced by the worker and the value of the outside option to the worker (the value of being unemployed).

2.5 Reservation productivities:

The Nash bargaining problem has an important property which is that workers and firms will always agree on the productivity level at which it is beneficial to both engage in production, that is the reservation productivity for the worker and the firm are the same, that is $R_f(y) = R(y)$ [2]. This means that that the reservation productivity can be obtained either from finding the value of $y = R(y)$ such that the surplus or to the to the worker is zero. This will hold true as long as wages are bargained. When we introduce wage floors, such as minimum wages, bellow we will see that this condition will no longer holds for all worker types.

Setting $J(y,y) = 0$ and subsisting gives:

[2]This fact comes from the FOC of problem 7, taking logs and diferentiting yileds: $(1-\beta)[N_1(y,y') - U(y)] = \beta[J(y,y)\ ]$, thus when $N_1(y,y') - U(y) > 0$, $J(y,y)$ is also bigger than zero.

$$R(y) = rU(y) - \frac{\lambda}{r+\lambda} \int_{R(y)}^{y} [1 - G(x)]dx \tag{8}$$

For any given worker type y, the left hand side is increasing in $R(y)$ and the right hand side is decreasing in $R(y)$, which means a unique solution exists.

2.6 Cut-off productivities and the value of unemployment

By solving for (2) and (4) and substituting in (1) we can find an expression for $rU(y)$ in terms of the reservation productivity and the values of the parameters:

$$rU(y) = b + \alpha \max\left[\frac{y_0 - rU(y)}{r+\delta}, 0\right] + \frac{\beta m(\theta)}{r+\lambda} \max\left[A_1 y - rU(y) + \int_{R(y)}^{y} [1 - G(x)]dx, 0\right] \tag{9}$$

For every worker type equations (8) and (9) represent two equations in the unknowns $R(y)$ and $U(y)$. Each worker when unemployed will face two different decisions: i) to accept or reject a formal employment opportunity when one arrives and, ii) to accept or reject a formal sector opportunity when one arrives. The first decision will be determined by choosing the maximum between $\frac{y_0 - rU(y)}{r+\delta}$ and 0. If $\frac{y_0}{r} \geq U(y)$ the worker will accept formal employment, otherwise he will reject it. The second decision is determined by choosing the maximum between $A_1 y - rU(y) + \int_{R(y)}^{y} [1 - G(x)]dx$ and 0. Workers for which $A_1 y + \int_{R(y)}^{y} [1 - G(x)]dx \geq rU(y)$ will always accept formal employment, workers for which the opposite hold will always reject it. The appendix shows that $rU(y)$ is continuous and increasing in y and that given our assumption that $y_0 > b$ workers can be classified only into three types: i) workers who will always accept formal employment and always reject informal employment, workers who accept both formal and informal, and workers who will always accept informal employment but not formal employment. It also shows that there will be two cut-off values of worker productivity y^* and y^{**}, with $y^* \leq y^{**}$, such that workers of type $y \leq y^*$ will only accept informal employment, workers of type $y^* \leq y \leq y^{**}$ will accept both formal and informal employment and, workers with type $y \geq y^{**}$ will only accept formal employment.

The intuition is straightforward; workers who have low formal sector productivity will have a low value of being employed in the formal sector, for some of them the value is so small that it is not worth to be formally employed because they have higher expected income in the informal sector. These workers will never accept a formal job. For other workers the value of being employed in the formal sector is larger, but not large enough for them to stay unemployed waiting only for a formal match to arrive, instead, as soon as they find an informal opportunity they take it up, but if a formal opportunity arrives they will also take it. For the workers with very high formal sector productivity it is worth to wait unemployed until a formal match arrives because their expected income from a formal match is very high compared with the informal-sector income.

For each of these worker types both reservation productivities and the values of unemployment will take different forms, in particular they will differ on two constant terms. The unemployment values can be expressed as:

$$rU(y) = k_1 + k_2 \left[A_1 y + \frac{\lambda}{r+\lambda} \int\limits_{R(y)}^{y} [1 - G(x)] dx \right]$$

with k_1 given by

$$k_1 = \begin{array}{lll} \frac{b(r+\delta)+\alpha y_0}{r+\delta+\alpha} & for & y \leq y^* \\ \frac{[b(r+\delta)+\alpha y_0](r+\delta)}{(r+\delta+\alpha)(r+\lambda)+(r+\delta)m(\theta)\beta} & for & y^* \leq y \leq y^{**} \\ \frac{b(r+\lambda)}{r+\lambda+m(\theta)\beta} & for & y \geq y^{**} \end{array}$$

and k_2 given by

$$k_2 = \begin{array}{lll} 0 & for & y \leq y^* \\ \frac{(r+\delta)m(\theta)\beta}{(r+\delta+\alpha)(r+\lambda)+(r+\delta)m(\theta)\beta} & for & y^* \leq y \leq y^{**} \\ \frac{m(\theta)\beta}{r+\lambda+m(\theta)\beta} & for & y \geq y^{**} \end{array}$$

Cut-off productivities will be given by

$$y^* = \frac{b\,(r+\delta) + \alpha y_0}{r+\alpha+\delta} - \frac{\lambda}{r+\lambda} \int_{R(y^*)}^{y^*} (1 - G\,(x))\, dx$$

$$y^{**} = (1+\tau) \frac{(y_0 - b)(r+\lambda) + m\,(\theta)\,\beta y_0}{m\,(\theta)\,\beta} + \lambda s - \frac{\lambda}{r+\lambda} \int_{R(y^{**})}^{y^{**}} [1 - G(x)] dx.$$

2.7 The decision to locate in urban or rural areas

Once we have this reduced form expression for expression for $U(y)$ we can find reduced form expressions for $N_1(y, y')$ and $N_0(y)$. It is easy to show that both are non-decreasing in y and as a consecuence the value of being in the urban areas V_c is non-decreasing in y. On the other hand the value of being in the agricultural sector $N_a(y)$ is $y_a = A_a L_a^{\gamma-1}$ with L_a being the equilibrium number of workers in the rural sector. Thus the value of being employed in agriculture will depend on the allocation decision of workers. For the cost of migration $M(y)$, we can consider two alternatives the first is to assume that it is independent of y. A second approach, probably more realistic, would be to assume $M(y)$, is decreasing in the worker's type, meaning that is more skilled workers have a lower cost of migration. This assumption can be broadly interpreted to reflect several facts: first, more educated workers are better able to access information and reduce costs, and second, they can also have higher non labor income which implies that, in relative terms its cheaper for them to migrate.

2.7.1 Migration cost independent of y

To find the equilibrium allocation of workers between urban and rural areas we can rewriting 5 and 6 using the fact $y_a = A_a L_a^{\gamma-1}$, we can then define y^c and

y^r as the values for which:

$$V_c(y^c) + M = A_a \left[F(y^c)\right]^{\gamma-1}$$

and

$$V_c(y^r) - M = A_a \left[F(y^r)\right]^{\gamma-1}$$

Since $\gamma < 1$, the right hand side of the above equations is decreasing in y, while the LHS is increasing in y.Thus the values of y^c and y^r will be unique, moreover $y^c < y^r$. Figure 1 illustrates these values. The flat portion in the expected values of locating in the cities is a result of $rU(y)$ being constant for those agents with $y < y^*$. For other agents this value is increasing in y.

For any initial segregated allocation of workers, the equilibrium allocation after a change in the values of $Vc(y)$, M or A_a, will depend on the location of y^c and y^r relative to the initial allocation. In the begining of time we start with a perfectly segregated allocation of workers, as a result of our assumption that all workers started in the rural sector and those for which it was optimal to migrate, did so. Let the threshold type which was indifferent between migrating or not be denoted by $\widehat{y}$. This means that all agents with income $y > \widehat{y}$ will be located in the cites and the rest in rural areas. Then

Once the values of $Vc(y), M$ or A_a change, the new allocation may entail or not migration (in either way). To see this note that if the initial allocation, is

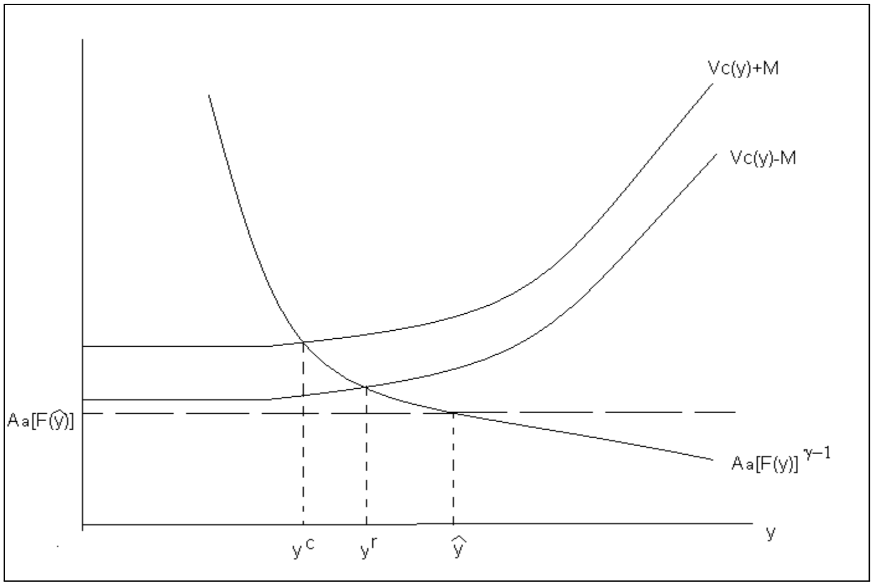

Figure 1: Equilibrium levels of y^c and y^r

such that $y^c < y^r < \widehat{y}$ (as illustrated in figure 1), then for workers with skills $y^r < y < \widehat{y}$ (which are all located in rural areas) it will be optimal to migrate to the cities, rural to urban migration will take place, as this happens the value of being in agriculture rises, until it is no longer optimal for any other worker to migrate and $V_c(y^r) - M = A_a\left[F(y^r)\right]^{\gamma-1}$, In this case the equilibrium number of workers in rural areas will be $F(y^r)$. On the other hand if $y^c \leq \widehat{y} \leq y^r$ then no migration will take place and the equilibrium number of workers in rural areas will be $F(\widehat{y})$. Finally, if $\widehat{y} < y^c < y^r$ then for those workers with skills $\widehat{y} < y < y^c$ (which are all located in the cities) it will be optimal to migrate, urban to rural migration will take place, as this happens the value of being agriculture will decrease until it is no longer for any other worker to migrate, $V_c(y^c) + M = A_a\left[F(y^c)\right]^{\gamma-1}$ and the equilibrium number of workers in rural areas will be $F(y^c)$. In any case the population will be segregate according to their type with lower skill workers locating in rural areas. Thus, for an initial allocation of workers determined by $\widehat{y}$, the equilibrium number of workers in the rural areas will be given by (normalizing the population to 1):

$$L_a = \begin{cases} F(y^c) & for \quad \widehat{y} < y^c < y^r \\ F(\widehat{y}) & for \quad y^c < \widehat{y} < y^r \\ F(y^r) & for \quad y^c < y^r < \widehat{y} \end{cases} \tag{10}$$

And the distribution of worker types in the cities will be the original distribution of types truncated to the left at $F^{-1}(L_a)$. let this distribution be denoted by:

$$f_c(y) = \frac{f(y)}{1 - L_a}$$

Let $\widehat{\widehat{y}} = F^{-1}(L_a)$.The threshold level $\widehat{\widehat{y}}$ can be smaller or bigger than the threshold level y^{**} and/or y^*, depending on the parameter values. If $\widehat{\widehat{y}} < y^*$ then there would be four types of workers: those with $y < \widehat{\widehat{y}}$, would locate in rural areas, those with $\widehat{\widehat{y}} < y < y^*$ will locate in the cities but accept only formal sector jobs, those with $y^* < y < y^{**}$ will locate in the cities and accept both formal and informal sector jobs, and those with $y > y^{**}$ will locate in the cities and accept only formal sector jobs. If $y^* < \widehat{\widehat{y}}, < y^{**}$, then there will be only two types of workers in the cities, those that accept both formal and informal jobs and those that only accept formal jobs. If $\widehat{\widehat{y}}, > y^{**}$ then workers in the cities will only accept formal jobs and there will be no informal sector. (I am not sure if we can find some restrictions that will limit how high can y^r go).

2.7.2 Migration Cost increasing in y

TBF.

2.8 Steady-State Conditions

The model's steady-state conditions allow us to solve for the unemployment rates, $u(y)$, for the various worker types. Let $u(y)$ be the fraction of time a worker of type y spends in unemployment, let $n_0(y)$ be the fraction of time that this worker spends in informal-sector employment, and let $n_1(y)$ be the fraction of time that this worker spends in formal-sector employment. Of course, $u(y) + n_0(y) + n_1(y) = 1$.

Workers of type $y < y^*$ flow back and forth between unemployment and informal-sector employment. There is thus only one steady-state condition for these workers, namely, that flows out of and into unemployment must be equal,

$$\alpha u(y) = \delta(1 - u(y)).$$

For $y < y^*$ we thus have

$$\begin{aligned} u(y) &= \frac{\delta}{\delta + \alpha} \\ n_0(y) &= \frac{\alpha}{\delta + \alpha} \\ n_1(y) &= 0. \end{aligned} \tag{11}$$

There are two steady-state conditions for workers with $y^* \leq y \leq y^{**}$, (i) the flow out of unemployment to the informal sector equals the reverse flow and (ii) the flow out of unemployment into the formal sector equals the reverse flow,

$$\begin{aligned} \alpha u(y) &= \delta n_0(y) \\ m(\theta) u(y) &= \lambda G(R(y))(1 - u(y) - n_0(y)). \end{aligned}$$

Combining these conditions gives

$$\begin{aligned} u(y) &= \frac{\delta \lambda G(R(y))}{\lambda(\delta+\alpha) G(R(y)) + \delta m(\theta)} \\ n_0(y) &= \frac{\alpha \lambda G(R(y))}{\lambda(\delta+\alpha) G(R(y)) + \delta m(\theta)} \\ n_1(y) &= \frac{\delta m(\theta)}{\lambda(\delta+\alpha) G(R(y)) + \delta m(\theta)}. \end{aligned} \tag{12}$$

Finally, for workers with $y > y^{**}$ there is again only one steady-state condition, namely, that the flow from unemployment to the formal sector equals the flow back into unemployment,

$$m(\theta) u(y) = (1 - u(y)) \lambda G(R(y)).$$

This implies

$$\begin{aligned} u(y) &= \frac{\lambda G(R(y))}{\lambda G(R(y)) + m(\theta)} \\ n_0(y) &= 0 \\ n_1(y) &= \frac{m(\theta)}{\lambda G(R(y)) + m(\theta)}. \end{aligned} \tag{13}$$

Total unemployment is obtained by aggregating across the population,

$$u = \int_0^{y^*} u(y) f_c(y) dy + \int_{y^*}^{y^{**}} u(y) f_c(y) dy + \int_{y^{**}}^{1} u(y) f_c(y) dy.$$

Where the only difference from ANV 2007 is that now the distribution of workers in the cities is the population distribution truncated at L_a.

2.9 Equilibrium

We use the free-entry condition to close the model and determine equilibrium labor market tightness. Setting $V = 0$ in equation (**??**) gives

$$c = \frac{m(\theta)}{\theta} E \max[J(y), 0].$$

To determine the expected value of meeting a worker, we need to account for the fact that the density of types among unemployed workers is contaminated. Let $f_u(y)$ denote the density of types among the unemployed. Using Bayes Law,

$$f_u(y) = \frac{u(y)f_c(y)}{u}.$$

The free-entry condition can thus be rewritten as

$$c = \frac{m(\theta)}{\theta} \int_{y^*}^{1} J(y) \frac{u(y)}{u} f_c(y) dy.$$

After substitution for $J(y)$, this becomes

$$c = \frac{m(\theta)}{\theta} \int_{y^*}^{1} (1-\beta) \left(\frac{y - R(y)}{r + \lambda} \right) \frac{u(y)}{u} f_c(y) dy. \tag{14}$$

Equation (14), of course, only makes sense if its right-hand side is positive. Since $J(y^*) = 0$ and $J(y)$ is increasing in y for $y \geq y^*$, a necessary condition for equation (14) to have a solution is $y^* < 1$. From equation (**??**), a simple sufficient condition for $y^* < 1$ is

A *steady-state equilibrium with formal-sector employment* is a labor market tightness θ, together with a reservation productivity function $R(y)$, unemployment rates $u(y)$, and cutoff values y^* and y^{**} and $\widehat{\widehat{y}}$ such that

(i) the value of maintaining a vacancy is zero

(ii) matches are consummated and dissolved if and only if it is in the mutual interest of

the worker and firm to do so

(iii) the steady-state conditions hold

(iv) formal-sector matches are not worthwhile for workers of type $y < y^*$

(v) informal-sector matches are not worthwhile for workers of type $y > y^{**}$

(vi) there is no migration and workers with income $y < \widehat{\widehat{y}}$ are located in rural areas, while agents with income $y \geq \widehat{\widehat{y}}$ are located in the cities.

3 Extension of the model to account for floor wages

In progress

4 Policy experiments

Main policy levers that the model should provide input to:

- TFP growth in the different sectors.

- Capital accumulation/changes in the cost of capital (to be determined)
- The effect of skill/educational expansions
- Labor market legislations changes, in particular minimum wages, and non wage labor costs (payroll taxes and severance pay)
- Lowering of costs to urban/rural mobility
- Labor market information and/or matching efficiency
- Scope for efficiency improvements (i.e. scope for policy interventions) and efficiency equity tradeoffs
- Clearly show which are the determinants of 'vacancy creation'

5 Inter-linkages between employment and growth

In this section we Incorporate the above model in a dynamic growth setting to extend it in the following directions :

1) Incorporate creative destruction and capitalization effects: Several papers have studied the effect of growth on unemployment using search models. Pissarides (2000, Ch. 3) and Aghion and Howitt (1998) have extensive treatments of the problem. In these papers, growth is the results of technological change as in the neoclassical Solow model. Two types of technological change are analyzed; in the first case technology is not embodied in capital, so that all jobs benefit from innovation. In this case, higher growth reduces unemployment due to the 'capitalization effect': As firms expect to benefit from future technological advances then investors are encouraged to create more vacancies and plants. In the case where new technology is incorporated in capital and jobs need to be destroyed in order to create more advanced plants of production, higher growth increases unemployment. This direct creative destruction effect will increase the flow of workers into unemployment, increasing steady state unemployment. In addition to this effect the rapid obsolescence of plants reduces the payoff to firm's investments. By discouraging the creation of new plants, which are the source of new job openings, it tends to reduce the job finding rate and increase unemployment. This is what Aghion and Howitt call indirect creative destruction effect. Both types of innovation can coexist. For example, Mortensen and Pissarides (1998) develop a model in which firms have the choice to either upgrade or close the plant and open a new one. There is a cost for upgrading and there is also a sunken cost of setting a new plant. They show that, there is an optimal size of the cost of upgrading beyond which it is always more profitable to destroy the plants, in this case the creative destruction effect takes place and higher productivity growth raises unemployment. Instead, for sufficiently low updating costs the capitalization effect takes place and growth reduces unemployment. They show that when there is a match specific idiosyncratic productivty, then for a given updating cost, it will always be better for firms with low idiosyncratic productivity to destroy the match and firms with high idiosyncratic productivty to update the technology. The fact that workers have different productivities in the formal sector will mean that growth in the

formal sector may generate updating of jobs for the more skilled thus increasing employment for this group and plant closing for the most unskilled. It will therefore generate the same effect as skilled biased technical change even when technological change is skill neutral. The impact on the structure of employment among sectors will need to be assessed.

2) Allow for feedback effects of labor markets to growth: Labor markets can also feedback on growth. Current literature on endogenous growth has described tow main channels through which labor market regulation and labor market structure can affect growth. The first assumes growth is the product of innovation brought about by research and development, and the second introduces learning by doing as primary source of growth. We believe this second channel to be relevant for developing countries. Learning by doing takes place for employed workers only, and a higher learning-by-doing rate might be expected from the formal or modern sector of the economy, when compared to agriculture or informal employment. When workers are unemployed or employed in low learning-by-doing activities, the rate of growth of the economy can be lower than the rate of growth of an economy where workers are employed in high-learning by doing activities. Therefore labor regulation and the structure of the labor market will determine the growth rate of an economy by affecting the allocation of workers between low-learning-by-doing activities and high-learning by doing activities.

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