A Model of Informal Sector Labor Markets

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Abstract: When labor standards signal where desirable jobs are, they alter the job-matching process, with the result that both aggregate output and employment can increase. Our model distinguishes formal-sector from informal-sector firms by their compliance with labor standards. Workers prefer formal-sector jobs, and take informal-sector jobs only to tide themselves over while searching in the formal sector. To gain access to more workers, more productive employers join the formal sector. We show that not only can the labor standards that give rise to this sector increase aggregate output: suppressing the informal sector, even if it could be done perfectly and at no cost, can lead to lower aggregate output and employment than allowing the informal sector to exist.

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I. Introduction

In many models of informal-sector formation, the informal sector results from, and is the source of, undesirable economic distortions.[1] For example, James Rauch (1991) shows how the informal sector arises in a neoclassical model when some firms choose to remain small to enjoy legal exemption from a mandated minimum wage policy that distorts resources away from first best allocations. A larger informal sector is associated with a more restrictive minimum wage, and greater economic distortions. Tito Boeri and Pietro Garibaldi (2002) show how the informal sector can result from excessive taxation. The government loses tax revenues when firms become informal in order to evade taxes, which forces even higher tax burdens on the tax-paying formal firms. Norman V. Loayza (1996) shows how tax evasion by informal firms inhibits long-term economic growth by reducing productive government expenditures. Two presumptions run through this literature. One is that formal firms would prefer to be free of regulation or taxation from the government. The other is that social welfare would be higher if the regulations on these firms were removed or relaxed.

A small and growing literature questions these presumptions, by showing that formal firms may voluntarily choose formality so as to receive some positive benefit, and that the informal sector has some intrinsic value to the economy.[2] Alec R. Levenson and William F. Maloney (1998) develop a model in which allowing informal operation encourages the start up of new business. Over time, the most productive of these new

[1] Friedrich Schneider and Dominik H. Enste (2000) provide a comprehensive survey of this literature.

[2] A third strand of literature presumes some positive benefit to firms or to workers that choose to operate in the formal sector, but does not allow that permitting the informal sector to exist has some positive social benefit. Examples from this literature include Douglas Marcouiller and Leslie Young (1995); Sylvain Dessy and Stéphane Pallage (2003); Marco Fugazza and Jean-François Jacques (2003); and Rossana Galli and David Kucera (2003).

firms grow and the least productive shut down. Growing firms evolve into formal firms as the benefits of formality - - such as access to various legal protections for increasingly complex business relationships - - become more important to them. Yoshiaki Azuma and Herschel Grossman (2002) develop a model in which allowing an informal sector to opt out of paying taxes may increase government revenues and aggregate output. In their model, there is a distribution of productivity across firms and each firm has strictly private information about its own productivity. A high tax coupled with the provision of government services only to firms that choose to pay can be Pareto superior to a lower tax forcibly levied on all firms.

Our paper expands this literature by showing that when labor regulation is coupled with *voluntary* compliance that allows firms to self-select without penalty into the formal and informal sectors, this can lead to Pareto-improving market outcomes. We work with a simple search model. It is a known feature of search (e.g., Kenneth A. Swinnerton, 1996; Gerard J. van der Berg, 2003) and other models giving firms some degree of labor-market monopsony power (e.g., George Stigler, 1946; V. Bhaskar and Theodore To, 1999), that the imposition of universally enforced labor-cost-increasing regulation can have beneficial effects on aggregate welfare. What has yet to be clarified is the fact that there are instances when allowing firms to choose—without penalty—about whether or not to comply with this regulation can yield an even greater benefit.

In our model, firms differ in productivity, and there are diminishing returns to labor. Random search in the labor market by homogeneous workers implies that in the absence of any labor standards the most productive firms face chronic labor shortages. The government sets labor standards and publicizes the identity of firms that choose to comply with the standards. The government does nothing to penalize firms that do not comply. This combination of labor standards and an "enforcement policy" of identifying the firms that comply can raise aggregate output and employment through two channels. First, the most productive firms will choose to comply, because doing so relieves the labor shortage they face. Once they have been identified as formal firms, workers will look for jobs with them first. This will increase the average productivity of the workforce, as a greater number of workers will go to the more productive firms. Second, the enforcement policy allows less productive firms, which do not face labor shortages, to opt out of compliance. For these firms, the increased labor costs associated with compliance with labor standards would have the typical neoclassical effect of reducing employment and output. These firms provide "fall-back" jobs for searchers who have yet to find formal-sector jobs. Allowing the terms of this employment to be less desirable than at formal-sector firms is better from an aggregate standpoint than regulating the informal sector away by enforcing 100% compliance with labor standards.

In the next section we describe the behavior of workers in our model. Section III discusses labor standards and their impact on the search process. Section IV addresses firms' choice whether or not to comply with labor standards and operate in the formal sector. Section V evaluates the effects of the standard-with-voluntary-compliance policy on aggregate output and employment, and provides examples that compare it both to a policy of no regulation and to one of universally enforced regulation. Section VI concludes. Proofs of all formally stated propositions are in the appendix.

II. Workers

There are k homogeneous workers in the economy, each of whom faces a constant probability of death, $\tau \in (0,1]$, in every period. New workers enter the labor force at rate τ, so that flows out of the labor force due to death are exactly matched by new flows into the labor force. In every period, there will be τk new entrants searching for work for the very first time.

Every worker has a utility function of the form

$$u = x + (m - \ell)v, \tag{1}$$

where m is an endowment of units of time and ℓ is time supplied to a firm as labor. We assume that by performing a home-based activity, any worker can reach a subsistence level of utility per unit of time, which we denote by v. x is a consumption good that is produced outside a worker's home. Workers pay for x from the income they earn working for firms. We normalize the price of x to 1. An implication of equation (1) is that any worker who has a firm-based job that pays a wage greater than v (the marginal product of time spent in home-based production), will want to devote all time to working for the firm. We assume that indifference between home-based and firm-based work is resolved in favor of firm-based work, if workers can find such work.

III. Labor Standards and the Search Process

Remuneration to a worker in the formal sector is administratively regulated by a set of labor standards so that each unit of time spent in formal-sector employment yields the worker a utility value of *at least* $w^* > v$. w^* could, for example, result from a minimum wage policy that says that all workers must be paid at least a wage (denominated in terms of the consumption good x) of w^* for each unit of time devoted to formal-sector work. An equivalent outcome could result from a set of health and safety standards that ensures that each unit of time spent in formal-sector work is less onerous (more enjoyable) than any unit of time spent in home-based production, i.e., each unit of time rather than having a base value to a worker of v has a base value of at least w^*. Since many combinations of labor standards can be conceived to have equivalent utility values, we can think of w^* as the outcome of a set of standards. In what follows, we interpret w^* as some set of labor standards. A formal sector firm is defined as one that adopts labor standards, i.e, that "pays" $w_F \geq w^*$. There is no regulation imposed on the informal sector, so informal sector firms pay $w_I < w^*$.

Government "enforcement" of labor standards amounts simply to identifying formal sector firms by making known their compliance with the standards; there is no punishment for firms that choose not to comply, and operate in the informal sector. We do not model the methods of publicity explicitly, but note that these may include, for example, outright listings of formal or desirable-to-work-for firms, access for formal firms only to a public employment and other job-matching services, or recognition of formal-sector organizations such as a trade union. We also note that the government may not need to serve as publicist; for example, NGOs using labeling schemes or making known firms that practice "social corporate responsibility" can also provide the sort of enforcement we have in mind. We assume that these enforcement activities can be carried on without cost, although allowing for non-zero cost and financing by some positive registration fee levied on firms that wish to be known as formal does not change the nature of our results. Our key assumption is simply that workers can distinguish between formal and informal sector firms when carrying out their search for jobs.[3]

Even though searchers distinguish the set of formal-sector firms from the set of informal ones, they do not know--without searching--which firms will offer them jobs or on what precise terms. Since $w_F > w_I$, there are two sequential episodes of random search at the start of every period. In the first, new entrants to the labor market and workers who have not yet found employment in the formal sector randomly apply for employment to a firm in the formal sector.[4] Any worker who receives an offer accepts it. Formal-sector workers never search again, and remain with the same firm until they die.

In the second episode of search, formal-sector applicants who do not receive a job offer interpret the rejection as a signal that there are no immediate job opportunities in the formal sector, and turn to the informal sector to support themselves while they await the next opportunity to apply to the formal sector for employment. Applicants to informal-sector firms who do not receive offers spend the period "unemployed" in the sense that they do not work for a firm. At the beginning of the next period, workers who spent the previous one unemployed or at informal sector firms join new labor market entrants in search, which repeats the cycle just explained.

IV. Firms

Individual firms are atomistic in the sense that they cannot affect marginally the flow of searchers to their doors (after they identify themselves as formal or informal).

[3] The sorting of firms into the formal and informal sector, and the job offer probabilities associated with each firm type, are equilibrium outcomes determined in Section V. For now, we simply assume that both types of firms exist.

[4] The same idea could be modeled in a matching framework with Nash bargaining or in a model of directed search, by assuming that the government's identification of formal firms facilitates matching in this sector: that is, by assuming that being formal and observing labor standards will speed the rate at which matches are made for the firm relative to what it would be if it chose informality.

From the point of view of searchers, a formal-sector firm can distinguish itself from all informal-sector firms by adopting labor standards, but it has no way to distinguish itself from other formal-sector firms. Since it must pay at least w* per unit of labor hired to signal that it is formal, and since beyond that it can do nothing more to affect the flow of applicants to its door, a formal firm pays no more, i.e., $w_F = w^*$. Any firm that chooses the informal sector, i.e., chooses not to comply with labor standards, knows that any searcher it meets will turn down any offer of less than v, but that offering more than v will not affect its marginal flow of searchers. So informal-sector firms offer $w_I = v$. We now discuss how and why firms sort into the formal and informal sectors.

Normalize to unity the number of firms in the economy. The production function for a firm is

$$\lambda f(\ell), \tag{2}$$

with $f(0) = 0$, $f'(\ell) > 0$, $f''(\ell) < 0$, $\lim_{\ell \to \infty} f'(\ell) = 0$, and $\lim_{\ell \to 0} f'(\ell) = \infty$. λ is an index of firm productivity, and ℓ is labor input. Firms are heterogeneous in λ, which is distributed on $[0, \bar{\lambda}]$ according to the distribution function $A(\lambda)$ with associated density $a(\lambda)$. We assume $a(\lambda)$ is continuous. A firm may operate only in one sector, and chooses the sector that brings it higher profits.

The quantity of labor demanded by a formal sector firm is implicitly defined by $\lambda f'(\ell) = w^*$. We use the notation $\ell^d(w^*/\lambda)$ to stand for this demand for labor. In the informal sector, each firm has labor demand $\ell^d(v/\lambda)$. Note that $\partial \ell^d(w/\lambda)/\partial\lambda > 0$: higher-productivity firms demand more labor, at any given wage rate.

Since searchers within sectors are allocated randomly to firms, a firm can be *labor-supply constrained* within a sector if its demand for labor at the going wage rate is greater than the per-firm supply in that sector. If this constraint could be relaxed, the firm's employment level and profits would both be higher. Given the assumptions of the model, the constraint cannot be relaxed within a sector. But the search process sends all searchers first to formal-sector firms, which means that formal-sector firms have "first dibs" on workers and thus a larger labor supply. So a firm that would be labor supply constrained in the informal sector may find it more profitable to operate in the formal sector, in spite of the higher unit labor costs, because its formal-sector status brings more workers. Letting ℓ^j *(j = I, F)* denote per-firm labor supply in each sector, we have Proposition 1.

Proposition 1: If a formal sector exists, then $\ell^F > \ell^I$.

Corollary 1: A firm never enters the formal sector if $\ell^I \geq \ell^d(v/\lambda)$.

Corollary 2: Formal sector firms are larger than informal sector firms.

From Corollary 1 we see that all firms with values of λ *above* some cut-off level, which we will call λ_1, are supply constrained in the informal sector. λ_1 is the productivity index of the firm where labor demand at the informal sector wage just equals informal-sector labor supply:

$$\lambda_1 = v / f'(\ell^I)\,. \tag{3}$$

Firms in the formal sector will have $\lambda > \lambda_1$. Let us denote by $\lambda_2 > \lambda_1$ the highest productivity index for any firm in the informal sector. The aggregate measure of informal sector firms is $A(\lambda_2)$; $1 - A(\lambda_2)$ firms are formal.

It may happen that in equilibrium, all firms will prefer informality (i.e., that λ_2 = $\bar{\lambda}$). In order to accommodate this possibility, we define λ_{IF} as the productivity level that would be needed for a firm to be *indifferent* between two sectors (i.e., to earn the same profit in either sector). This level of productivity could be outside the support of the distribution of λ. Thus,

$$\lambda_2 = \min(\lambda_{IF}, \bar{\lambda}) \tag{4}$$

To write out the equal-profits condition that defines λ_{IF}, we note that a firm having $\lambda = \lambda_{IF}$ would *not* be supply constrained in the formal sector. If it were, then all formal firms would be supply constrained. They would each hire every searcher they meet, leaving no labor for the informal sector. Consequently, *all* firms would prefer the formal sector, where profits would be positive, to the informal sector, where profits would equal zero, and λ_{IF} would have to equal zero. But this is not possible. So long as labor supply is positive in the formal sector and firms with values of λ at or near zero exist, there will be firms with very low labor demand that will not be supply constrained. We conclude that a firm that would be indifferent between the two sectors would be supply constrained in the informal sector and on its demand curve in the formal sector. λ_{IF} is therefore defined by

$$\lambda_{IF} f(\ell^I) - v\ell^I = \lambda_{IF} f(\ell^d(w^*/\lambda_{IF})) - w^*\ell^d(w^*/\lambda_{IF}). \tag{5}$$

We next derive ℓ^F and ℓ^I, using the logic from Albrecht and Axell (1984). Let q be the probability that a formal-sector job applicant receives a job offer. q is determined in equilibrium, but for now we take it as a parameter. The flow of workers to a formal-sector firm at any search date consists of its share of new entrants into the labor force at that date, $\tau k/[1 - A(\lambda_2)]$; its share of the new-entrants from the previous period who did not get formal-sector jobs and did not die, $(1-q)(1-\tau)\tau k/[1 - A(\lambda_2)]$; its share of the stillliving searchers who first entered two periods in the past, $(1-q)^2(1-\tau)^2\tau k/[1-A(\lambda_2)]$;

and so forth. The total flow equals $\dfrac{\sum_{j=0}^{\infty}(1-\tau)^j(1-q)^j\tau k}{1-A(\lambda_2)}=\dfrac{\tau k}{[1-A(\lambda_2)][1-(1-q)(1-\tau)]}$.

If a firm were to offer jobs to all of the workers who applied, then its potential labor supply would be equal to this flow plus survivors from the total flows from previous periods. Adding these up gives us the potential labor supply to a firm in the formal sector:

$$\ell^F(q,\lambda_2)=\frac{k}{[1-A(\lambda_2)][1-(1-q)(1-\tau)]}. \tag{6}$$

The informal sector provides employment to workers who are unable to secure jobs in the formal sector. Since workers in the informal sector do not wish to work there forever, we do not aggregate all surviving workers who failed to secure employment in the formal sector. Potential labor supply to a firm in the informal sector thus equals its period flow of applicants:

$$\ell^I(q,\lambda_2)=\frac{(1-q)\tau k}{A(\lambda_2)[1-(1-q)(1-\tau])}. \tag{7}$$

V. Equilibrium

We close the model by determining *q*, the probability that a searcher receives a formal-sector job offer, and *p*, the probability of an informal-sector offer.

We begin with *q*. Denote by λ_3 the highest productivity level of a formal firm that is able to satisfy its demand for labor in the formal sector. Firms having $\lambda>\lambda_3$ have such high demand for labor that they are supply-constrained (even in the formal sector), while firms having $\lambda \le \lambda_3$ satisfy their labor demand in the formal sector. For given λ_2 and *q*, λ_3 is defined by

$$\lambda_3 = \begin{cases} \overline{\lambda}, & w^*/f'(\ell^F) \ge \overline{\lambda} \\ w^*/f'(\ell^F) & \text{if } \lambda_2 < w^*/f'(\ell^F) < \overline{\lambda}, \\ \lambda_2, & w^*/f'(\ell^F) \le \lambda_2 \end{cases} \tag{8}$$

The first line of equation (8) describes a situation where, for given λ_2 and *q*, no formal firms are supply constrained: $w^* > \overline{\lambda} f'(\ell^F)$. The second line describes a situation in which the formal sector includes both supply-constrained firms and firms that satisfy their labor demand. The third line describes a situation in which, for given λ_2 and *q,* all formal firms are supply constrained: $w^* < \overline{\lambda} f'(\ell^F)$.

Because of each worker's constant death risk (τ), a formal-sector firm has, in steady state, $\tau\ell^F$ job-openings if it is labor-supply constrained, and $\tau\ell^d(w^*/\lambda)$ if it is not. The steady-state flow of searchers to each formal sector firm is $\tau\ell^F$. Thus, a searcher who contacts a supply-constrained firm (having $\lambda > \lambda_3$) receives an offer with probability $\tau\ell^F/\tau\ell^F = 1$. If a searcher contacts an unconstrained firm (having $\lambda_2 < \lambda < \lambda_3$), the conditional offer probability is only $\tau\ell^d(w^*/\lambda)/\tau\ell^F < 1$, as the firm's flow of job openings, $\tau\ell^d$, is less than its flow of applicants, $\tau\ell^F$. The unconditional probability of receiving an offer from *some* formal-sector firm is just the weighted average of the individual formal-sector firms' offer probabilities:

$$q = \frac{\int_{\lambda_2}^{\lambda_3} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda)d\lambda}{1 - A(\lambda_2)} + \frac{\int_{\lambda_3}^{\overline{\lambda}} a(\lambda)d\lambda}{1 - A(\lambda_2)}. \tag{9}$$

Note that if $\lambda_3 = \overline{\lambda}$, then the last term in equation (9) vanishes, since all formal firms are then on their labor demand curves. If all formal-sector firms are supply constrained, i.e., $\lambda_3 = \lambda_2$, then the first term on the right-hand side of equation (9) vanishes, and $q = 1$.

To complete the model, we follow the derivation of the equation for q to define the probability of a job offer in the informal sector, p:

$$p = \frac{\int_0^{\lambda_1} \frac{\ell^d(v,\lambda)}{\ell^I(q,\lambda_2)} a(\lambda) d\lambda}{A(\lambda_2)} + \frac{A(\lambda_2) - A(\lambda_1)}{A(\lambda_2)} \tag{10}$$

Since turnover occurs in every period in the informal sector (as workers quit to search for "better" formal jobs), the probability of a job offer from an informal firm (p) has no effect on the probability of a formal offer (q) or on the productivity level of the marginal entrant into the formal sector (λ_2). Equation (10) therefore determines p recursively, for given values of the model's other endogenous variables.

Equilibrium values of the other endogenous variables in the model, $\{\ell^I, \ell^F, q, \lambda_1, \lambda_{IF,} \lambda_2, \lambda_3\}$, are determined by equations (3) – (9). To characterize equilibrium further, we simplify by first noting from equations (6) and (7) that ℓ^F and ℓ^I are functions of q and λ_2 but not of any other endogenous variable. Substitute $\ell^I(q,\lambda_2)$ into equation (3) and $\ell^F(q,\lambda_2)$ into equation (8), so that both λ_1 and λ_3 are also functions of only q and λ_2. We denote these functions by $\lambda_1(q,\lambda_2)$ and $\lambda_3(q,\lambda_2)$. Finally, we substitute $\ell^I(q,\lambda_{IF})$ for ℓ^I in equation (5).

We are left with three equations in the unknowns, $\{q, \lambda_{IF}, \lambda_2\}$:

$$\lambda_{IF} f(\ell^I(q,\lambda_{IF})) - v\ell^I(q,\lambda_{IF}) = \lambda_{IF} f\left(\ell^d(w^*/\lambda_{IF})\right) - w^*\ell^d(w^*/\lambda_{IF}) \qquad (11A)$$

$$\lambda_2 = \min(\lambda_{IF}, \overline{\lambda}) \qquad (11B)$$

$$q = \frac{\displaystyle\int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda)d\lambda}{1-A(\lambda_2)} + \frac{\displaystyle\int_{\lambda_3(q,\lambda_2)}^{\overline{\lambda}} a(\lambda)d\lambda}{1-A(\lambda_2)} \qquad (12)$$

We can represent equilibrium solutions for q and λ_2 graphically, and we provide three examples in Figures 1 through 3. We will discuss the distinctive features of each Figure shortly. First, we identify the common features of each case, and explain why at least one stable equilibrium always exists.

Equations (11A) and (11B) define λ_2, the highest productivity index in the informal sector, for any $q \in [0,1]$. If $q = 1$, then all searchers receive offers in the formal sector: none are left over to apply to the informal sector ($\ell^I = 0$). Every firm with $\lambda > 0$ will therefore locate in the formal sector, making $\lambda_2 = \lambda_{IF} = 0$. Decreases in q increase informal labor supply, and increase λ_2. As q becomes progressively smaller, either of two things may happen. One, which is illustrated in Figures 1 and 3, is that λ_2 rises quickly enough to cause all firms become informal ($\lambda_2 = \overline{\lambda}$) at some $q > 0$. In this case, equations (11) give rise to a negatively sloped curve in (q, λ_2)-space, which ranges from the point (1,0) on the horizontal axes, moves up and to the left, and eventually becomes horizontal, at $\lambda_2 = \overline{\lambda}$. The second possibility, illustrated in Figure 2, is that for the entire range of q, $\lambda_2 \leq \overline{\lambda}$. In this case, equations (11) imply a negatively sloped curve with no horizontal portion.

Equation (12) implicitly defines, for any $\lambda_2 \in [0, \bar{\lambda}]$, the function $q(\lambda_2)$, giving the probability of a job offer from a formal firm. The derivative, $q'(\lambda_2)$, can be shown to equal the sum of a positive and a negative term.[5] The positive term arises because as λ_2 increases, i.e., as the formal sector becomes smaller, firms with relatively high labor demand remain in the formal sector, which tends to increase *q*. On the other hand, as λ_2 increases, per-firm labor supply in the formal sector goes up, tending to relax labor-supply constraints and to reduce *q*. For values of λ_2 close to zero, the first effect dominates -- $q'(0) > 0$ -- because the low-λ firms removed from the informal sector when λ_2 rises had been employing relatively few workers, and the availability of these few workers to the relatively large measure of remaining formal-sector firms has little effect on per-firm formal-sector labor supply.

For values of λ_2 close to $\lambda_2 = \bar{\lambda}$ the second effect dominates. When there are few formal sector firms to begin with, dropping a few firms from the informal sector provides a large increase in the potential labor supply for the firms that remain in the formal sector. It can be shown that $\lim_{\lambda_2 \to \bar{\lambda}} q'(\lambda_2) < 0$. Thus, equation (12) does not define a monotonic relationship between *q* and λ_2, but one that starts out positive for low values of λ_2 and is negative by the time λ_2 approaches $\bar{\lambda}$.

q(0) is less than 1: $\ell^F(q,0) > 0$ and $\lim_{\lambda \to 0} \ell^d(w^*/\lambda) = 0$, so that $\lambda_3 > 0$ and the first term on the right-hand-side of equation (12) does not vanish. When $\lambda_2 = \bar{\lambda}$, there are no formal sector firms, so $q(\bar{\lambda}) = 0$.

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[5] A formal derivation of the slope of the graph of equation (12) may be found in the proof of Proposition 3.

An equilibrium always exists. Equations (11) and (12) define continuous relationships between q and λ_2. The horizontal intercept of the graph of equations (11) always is to the right of that of the graph of equation (12). As $q \to 0$, there are two possibilities. The first possibility, which is illustrated in Figures 1 and 3, is that the curves have the same vertical intercept. In this case, there is always at least one equilibrium, i.e., one in which all firms are informal (i.e., the point $(0, \bar{\lambda})$). The second possibility, illustrated in Figure 2, is that the vertical intercept of equation (11) is below that for equation (12). In this case, continuity ensures that the two curves must cross.

In Figure 1, there is no formal sector in equilibrium: the equilibrium has $q = 0$ and $\lambda_2 = \bar{\lambda}$. Such an outcome is clearly possible if complying with labor standards is very costly; or, if in the absence of labor standards, labor-supply constraints are not binding. Here are three, not-mutually-exclusive, ways the equilibrium in Figure 1 could occur. First, $a(\lambda)$ could have heavy density near zero and very little near $\bar{\lambda}$. In this case, the economy is heavily populated with relatively unproductive firms that have low labor demand. Second, the population of workers relative to firms could be so large that binding labor-supply constraints on informal sector firms are never much of an issue. Finally, if labor standards are set too high (w^* much larger than v), then all firms will find operating in the informal sector (with lower wages) to be more profitable, and none will comply with labor standards.

In Figure 2, there is a single equilibrium with both a formal and an informal sector. This occurs when k, the measure of workers, is not too large, so that in the absence of labor standards there are supply-constrained firms; and when labor standards are not very onerous (that is, when w^* is "close" to v).

Finally, in Figure 3, there are multiple equilibria, labeled A, B, and C. "A" and "C" are stable. In "A" the equilibrium has only an informal sector, while in "C" both formal and informal sectors exist. The intuition for the possible existence of two stable equilibria is straightforward. If a large enough formal sector exists (equilibrium C), the formal sector soaks up many workers making it more likely that the informal-sector labor-supply constraint binds on any individual firm should it choose informality; therefore, it is more likely to be most profitable to go formal. In equilibrium A, the formal sector soaks up no workers and so the labor-supply constraint from remaining informal is not as severe as in equilibrium A. Thus in the economy depicted in Figure 3, an individual firm's choice to go formal or not is reinforced by heavy incidence of other firms making exactly the same choice. Formality and informality feed on themselves.

VI. Output and Employment Effects of Labor Market Regulation

In this section of the paper, we show that aggregate output and employment can be higher in the equilibrium in this model, in which firms select the sector in which to operate, than in either an equilibrium in which there is no regulation, or an equilibrium in which all firms are compelled to comply with the labor standard. The latter equilibrium (compelled compliance) has served as a benchmark in the literature on the welfare effects of minimum wages.

In our model, regulation comprises two interconnected features: one is the set of labor standards ($w^* > v$) which impose costs on formality. The second is that regulation directs workers towards higher-productivity (formal) firms. In the absence of labor market regulation, the wage would equal v for all workers, and there would be no reason for workers to prefer work at one firm over another. As a result, per-firm labor supply would then be equal to $k/[1-(1-\hat{p})(1-\tau)]$, where $\hat{p}$ is the equilibrium probability of a job offer in the absence of labor standards.

Let us suppose there would be some firms that are supply constrained in the absence of standards.[6] Lower-productivity firms would operate on their labor demand curves, and hire fewer workers than apply for work, while supply-constrained higher-productivity firms would hire every worker who applied. If the government knew individual firm productivities, it might be able to channel the excess supply from the low-productivity firms to the high-productivity firms, thereby raising both aggregate output and employment. Unfortunately, the government is unlikely to observe all firm productivities, and workers have no incentive to present themselves in larger numbers to high-productivity firms, since all firms pay the same wage.

Labor standards, viewed from this perspective, serve as an allocation device. By raising remuneration for workers, they make search at the (known) formal sector more desirable. Workers search there first. Since not all firms will join the formal sector, this aggregate supply of labor is spread out over fewer firms than in the no-regulation case, tending to raise the per-firm labor supply of the firms that choose the formal sector. However, these firms are also the ones most likely to be labor-supply constrained, and therefore, there can be an offsetting drag on per-firm labor supply because the firms that do join the formal-sector are likely to have higher offer probabilities and less likely leave workers in the pool of searchers, i.e., there is an indirect effect of reducing the number of firms that works through q. In equilibrium, the negative indirect effect never dominates, so

[6] Otherwise, all informal firms will satisfy their labor demands and there will be no formal sector.

that firms that join the formal sector face a greater per-firm labor supply than they would in the absence of regulation.

Proposition 2: In equilibrium, $\ell^F = \dfrac{k/\left[1-A(\lambda_2)\right]}{1-(1-q)(1-\tau)} > \dfrac{k}{1-(1-\hat{p})(1-\tau)}$.

Since only firms with high labor productivity and high labor demand will benefit from access to higher labor supply, the labor standard encourages high-productivity firms to sort themselves into the formal sector. This reallocation can raise aggregate output and employment.

Proposition 3: If some firm faces labor shortages in an unregulated labor market, then a set of labor standards exists that raises aggregate output and aggregate employment.

Our finding that aggregate output can increase has the flavor of a result in Swinnerton (1996), who showed that a minimum wage with compliance compelled on all firms could raise output by reallocating labor from lower to higher productivity activities, on average. However, in that model of compelled compliance, it was possible for aggregate employment to fall, if there was a large reduction in employment among firms that were compelled to pay the minimum wage but were not supply-constrained.

In our model, where complete compliance is not presumed, employment rises, unambiguously, with regulation. Search frictions are reduced for firms that choose formality, so only the firms looking to find a way to attract more workers will pay the additional unit labor costs associated with complying with the labor standards. Meanwhile, labor demand at informal firms is not affected by the increased labor costs of formality. Increased labor supply to firms that choose formality, and maintained labor demand at those that choose informality, can increase aggregate employment when voluntary labor standards are set appropriately.

Because of the effect of the voluntary-compliance regulation regime on search frictions, aggregate output can also be higher relative to the case where there is fully compelled compliance with the labor standard. Figure 4 illustrates this with a typical numerical example. Suppose productivity is uniformly distributed on [0,1], with $f(\ell) = 2\ell^{1/2}$; and with $k = 2.5$, $\tau = 0.05$, and $v = 0.25$. We show aggregate output for different levels of w^* and three different equilibrium assumptions. The first is the equilibrium for the two-sector model. The next is the equilibrium when the labor standard is compelled for all firms. Finally, we show aggregate output in a no-regulation equilibrium (here the level of w^* is irrelevant). Figure 5 keeps track of the corresponding employment effects.

In this example, output and employment are higher in the two-sector equilibrium than in either the full compliance equilibrium or the unregulated equilibrium, so long as w^* is not too high. As w^* rises from v, output and employment is higher in the two-sector model than in either of the other two. In the two-sector model, output is maximized at approximately a 44 percent increase in w^* (0.36) over v (0.25). After this point, aggregate output begins to fall in the two-sector model. The reason is that labor standards initiate high turnover in the informal sector; but at wages that are too high, there is low absorption of these workers in the formal sector because few firms operate there. This effect eventually causes output and employment to fall rapidly. The most rapid decline occurs when w^* is so high that even the most productive firm in the formal sector is not supply constrained. At this point, only neoclassical-type labor demand effects are at work as w^* is increased. With ever-stricter labor standards, aggregate output and employment can fall below not only their levels in the compelled compliance case, but also below levels in an unregulated equilibrium.

VII. Conclusion

Typically the informal sector is portrayed as an undesirable side effect of wrong-headed government regulation. In our model, it is a desirable side effect of regulation set to maximize output or employment. There are just two assumptions needed to generate our departure from the typical portrayal. One is that there are frictions that lead workers to search for jobs and that lead to some firms being able to hire fewer workers than they would like. The other is that a government or regulatory organization cannot eliminate these frictions completely, but can address them to some extent by setting labor standards and publicizing the identity of those firms that choose to comply. Neither assumption is heroic; therefore, we suggest good reason to reconsider the pejorative connotations normally associated with the informal sector.

Our results emphasize that to maximize output or employment, regulation should not be enforced by punishment, but rather by positive reinforcement (identification) of voluntary compliance. A natural question then arises as to why most enforcement regimes are largely punitive in nature. The answer may be that our focus on aggregate indicators may not be sufficient to account for distributional concerns that can drive many observed regulatory outcomes. In our model, regulation is the cause of inequality: ex ante identical workers end up earning different wages or enjoying different labor standards only because some are luckier than others in finding formal-sector work. This distributional implication may not be politically acceptable, and therefore, may affect the punitive stance of the final regulatory outcome. The focus of this paper was efficiency concerns, and we assumed implicitly that maximizing traditional indicators of efficiency was the objective of the regulatory body. With some explicit modeling of the political process that actually determines the objective of regulation, future work may introduce distributional concerns into the story.

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Appendix: Proofs of Propositions

Proposition 1: If a formal sector exists, then $\ell^F > \ell^I$.

Proof: First, suppose for some firm in the formal sector with productivity level $\hat{\lambda}$, that $\ell^d(w^*/\hat{\lambda}) > \ell^F$. Then, it has to be the case that $\ell^F > \ell^I$, or else profits for this firm will be greater in the informal sector, and the firm would not be in the formal sector.

Next suppose that for some firm in the formal sector with productivity level $\hat{\lambda}$, that $\ell^d(w^*/\hat{\lambda}) \le \ell^F$. Then, since $\ell^d(v/\hat{\lambda}) > \ell^d(w^*/\hat{\lambda})$, it has to be the case that $\ell^d(v/\hat{\lambda}) > \ell^I$ and $\ell^I < \ell^F$, or else profits will be greater in the informal than in the formal sector, and the firm would not be in the formal sector.

Proposition 2: In equilibrium, $\ell^F = \dfrac{k/[1-A(\lambda_2)]}{1-(1-q)(1-\tau)} > \dfrac{k}{1-(1-\hat{p})(1-\tau)}$

Proof: For convenience, define $\hat{\ell} = \dfrac{k}{1-(1-\hat{p})(1-\tau)}$. We show that for *any* (q,λ_2) pair that satisfies equation (12), $\ell^F \ge \hat{\ell}$. After defining two functions useful to the proof, we do this in two steps. (i) We show that for the smallest possible $\lambda_2 (\lambda_2 = 0)$, $\ell^F \ge \hat{\ell}$. (ii) We show that increasing λ_2 while satisfying equation (12) always leads to increases in ℓ^F .

Define,

$$G(q,\lambda_2) = q - \{\frac{\int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda)d\lambda}{1-A(\lambda_2)} + \frac{1-A(\lambda_3(q,\lambda_2))}{1-A(\lambda_2)}\}. \tag{A1}$$

Note that $G(q,\lambda_2) = 0$ is equivalent to equation (12).

Next define,

$$H(\hat{p}) = \hat{p} - \{\int_0^{v/f'(\hat{\ell})} \frac{\ell^d(v/\lambda)}{\hat{\ell}} a(\lambda)d\lambda + 1 - A(\frac{v}{f'(\hat{\ell})})\} = 0 \tag{A2}$$

Equation (A2) gives the probability of receiving a job offer ($\hat{p}$) when there is no formal sector and no quitting of jobs.

(i) We now show that for the smallest possible $\lambda_2 (\lambda_2 = 0)$, $\ell^F \ge \hat{\ell}$. If $w^* = v$, the point $(q,\lambda_2) = (\hat{p},0)$ is a solution to $G(q,\lambda_2) = 0$, because if we plug these values into equation (A1), then (A1) is identical to equation (A2). In this case, $\ell^F = \hat{\ell}$. If $w^* > v$, then we see from equation (A1) that $G(\hat{p},0) > 0$, because $\lambda_3(\hat{p},0)$ is no smaller for $w^* > v$ than for $w^* = v$; $\ell^d(w^*/\lambda) < \ell^d(v/\lambda)$; and, $\ell^F(\hat{p},0) = \hat{\ell}$. If we hold λ_2=0 for any $w^* > v$, it must be he case that the q that solves $G(q,0) = 0$ is less than $\hat{p}$. We know this because

$$\frac{\partial G}{\partial q} = 1 + \frac{1}{1-A(\lambda_2)} \int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{[\ell^F(q,\lambda_2)]^2} \frac{\partial \ell^F(q,\lambda_2)}{\partial q} a(\lambda) d\lambda$$
$$= 1 - (1-\tau) \int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{k} a(\lambda) d\lambda > 0. \tag{A3}$$

The sign of this expression follows from noticing that the integrand in the second line is less than one because labor demand at any firm that is not labor-supply constrained cannot be large enough to absorb the entire population of workers. Finally, since ℓ^F is decreasing in q (see equation (A5) below) it follows that for any $q < \hat{p}$, $\ell^F(q,0) > \ell^F(\hat{p},0) = \hat{\ell}$.

(ii) We now show that increasing λ_2 while satisfying equation (12) always leads to increases in ℓ^F. To do this, we establish that the derivative

$$\frac{d\ell^F}{d\lambda_2} = \frac{\partial \ell^F}{\partial q}\frac{\partial q}{\partial \lambda_2} + \frac{\partial \ell^F}{\partial \lambda_2} \tag{A4}$$

is positive.

From equation (7) we have,

$$\frac{\partial \ell^F}{\partial q} = \frac{-(1-\tau)\ell^F}{1-(1-q)(1-\tau)} \tag{A5}$$

$$\frac{\partial \ell^F}{\partial \lambda_2} = \frac{a(\lambda_2)}{1-A(\lambda_2)} \ell^F \tag{A6}$$

We derive $\frac{\partial q}{\partial \lambda_2}$ by noting that equation (A1) may be viewed as defining q implicitly as a function of λ_2. From equation (A1) we have,

$$\frac{\partial G}{\partial \lambda_2} = -\frac{a(\lambda_2)}{1-A(\lambda_2)}\left[-\frac{1-A(\lambda_3)}{1-A(\lambda_2)} + \frac{\ell^d(w^*/\lambda_2)}{\ell^F}\right] \tag{A7}$$

By the implicit function theorem: we have $\frac{\partial q}{\partial \lambda_2} = -\frac{\partial G}{\partial \lambda_2}\Big/\frac{\partial G}{\partial q}$. Substituting from equations (A3) and (A5)-(A7) into equation (A4) yields, after some manipulation:

$$\frac{d\ell^F}{d\lambda_2} = \frac{\partial \ell^F}{\partial \lambda_2}\left[\frac{k-(1-\tau)E^F + (1-\tau)(1-A(\lambda_2))\ell^d(w^*/\lambda_2)}{k\frac{\partial G}{\partial q}}\right] > 0. \tag{A8}$$

In equation (A8), E^F is total formal sector employment, i.e.,

$E^F = \int_{\lambda_2}^{\lambda_3} \ell^d (w^* / \lambda) a(\lambda) d\lambda + [1 - A(\lambda_3)] \ell^F (q, \lambda_2)$. The term $k - (1-\tau) E^F$ is equal to the flow of searchers to the formal-sector in the aggregate at the beginning of each period. Increasing λ_2 decreases the stock of formal-sector firms, and the workers released from the marginal firm switching to the informal sector—i.e., $(1-\tau)(1 - A(\lambda_2)) \ell^d (w^* / \lambda_2)$ -- increases the flow of searchers to each remaining formal-sector firm. In the steady state, each remaining formal sector firm's potential labor supply increases.

Proposition 3: If some firm faces labor shortages in an unregulated labor market, then a set of labor standards exists that raises aggregate output and aggregate employment.

Proof: We derive expressions for the change in aggregate output ($\Delta Y(w^*)$) and aggregate employment ($\Delta E(w^*)$), when an economy goes from having no labor market regulation to having *some* labor market regulation. We then show that $\lim_{w^* \to v} \Delta Y(w^*) = \Delta Y(v)$ and $\lim_{w^* \to v} \Delta E(w^*) = \Delta E(v)$ are strictly positive. Existence of these limits establishes that $\Delta Y(w^*)$ and $\Delta E(w^*)$ are continuous at $w^* = v$, so that we know that at least at values of w^* that are slightly greater than v, $\Delta Y(w^*)$ and $\Delta E(w^*)$ are positive.

<u>Aggregate output:</u>

In the absence of any labor market regulation, aggregate output equals

$$\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d (v/\lambda)) a(\lambda) d\lambda + f(\hat{\ell}) \int_{v/f'(\hat{\ell})}^{\bar{\lambda}} a(\lambda) d\lambda \,, \tag{A9}$$

this is the sum of output at firms that satisfy their labor demand and of firms that are labor-supply constrained.

With labor market regulations, aggregate output becomes

$$\int_0^{v/f'(\ell^I)} \lambda f(\ell^d (v/\lambda)) a(\lambda) d\lambda + f(\ell^I) \int_{vf'(\ell^I)}^{\lambda_2} \lambda a(\lambda) d\lambda + \int_{\lambda_2}^{w^*/f'(\ell^F)} \lambda f(\ell^d (w^*/\lambda)) a(\lambda) d\lambda +$$

$$f(\ell^F) \int_{w^*/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda) d\lambda \,. \tag{A10}$$

The *change* in aggregate output (ΔY) equals

$$\Delta Y(w^*) = \int_0^{v/f'(\ell^I)} \lambda f(\ell^d (v/\lambda)) a(\lambda) d\lambda + f(\ell^I) \int_{vf'(\ell^I)}^{\lambda_2} \lambda a(\lambda) d\lambda + \int_{\lambda_2}^{w^*/f'(\ell^F)} \lambda f(\ell^d (w^*/\lambda)) a(\lambda) d\lambda +$$

$$f(\ell^F)\int_{w^*/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda - \left[\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d(v/\lambda))dA(\lambda) + f(\hat{\ell})\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} \lambda a(\lambda)d\lambda\right]. \qquad \text{(A11)}$$

As w^* approaches v, in the limit, $\lambda_2 \to \lambda_1 = v/f'(\ell^I)$, and $\ell^I = \ell^d(v/\lambda_2)$. Making these changes and also setting w^*'s equal to v everywhere gives us:

$$\lim_{w^*\to v} \Delta Y = \int_0^{v/f'(\ell^F)} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\ell^F)\int_{v/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda - \left[\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\hat{\ell})\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} \lambda a(\lambda)d\lambda\right] \qquad \text{(A12)}$$

From proposition 2, we know that $\ell^F > \hat{\ell}$ which in turn implies that $v/f'(\ell^F) > v/f'(\hat{\ell})$; so, the limit above can be rewritten as

$$\lim_{w^*\to v} \Delta Y = \int_{v/f'(\hat{\ell})}^{v/f'(\ell^F)} \lambda\left[f(\ell^d(v/\lambda)) - f(\hat{\ell})\right]a(\lambda)d\lambda + \left[f(\ell^F) - f(\hat{\ell})\right]\int_{v/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda > 0,$$

since both terms in the sum on the r.h.s are strictly positive.

<u>Aggregate Employment.</u>

With no regulation, aggregate employment is

$$\int_0^{v/f'(\hat{\ell})} \ell^d(v/\lambda)a(\lambda)d\lambda + \hat{\ell}\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} a(\lambda)d\lambda. \qquad \text{(A13)}$$

With the regulation in place it equals:

$$\int_0^{v/f'(\ell^I)} \ell^d(v/\lambda)a(\lambda)d\lambda + \ell^I\int_{vf'(\ell^I)}^{\lambda_2} a(\lambda)d\lambda + \int_{\lambda_2}^{w^*/f'(\ell^F)} \ell^d(w^*/\lambda)a(\lambda)d\lambda + \ell^F\int_{w^*/f'(\ell^F)}^{\bar{\lambda}} a(\lambda)d\lambda \qquad \text{(A14)}$$

The change in employment (due to having regulation) equals (A14) minus (A13). The same reasoning as before establishes that:

$$\lim_{w^*\to v} \Delta E = \int_{v/f'(\hat{\ell})}^{v/f'(\ell^F)} \left[\ell^d(v/\lambda) - \hat{\ell}\right]a(\lambda)d\lambda + \left[\ell^F - \hat{\ell}\right]\int_{v/f'(\ell^F)}^{\bar{\lambda}} a(\lambda)d\lambda > 0$$

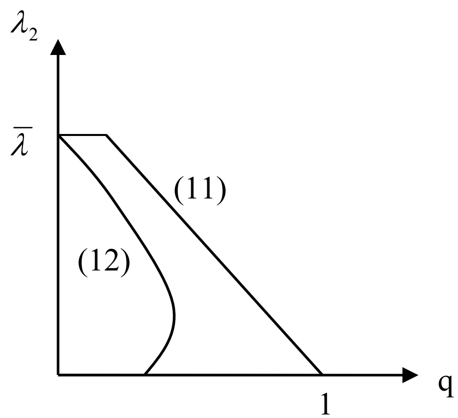


Figure 1: An Equilibrium with No Formal Sector

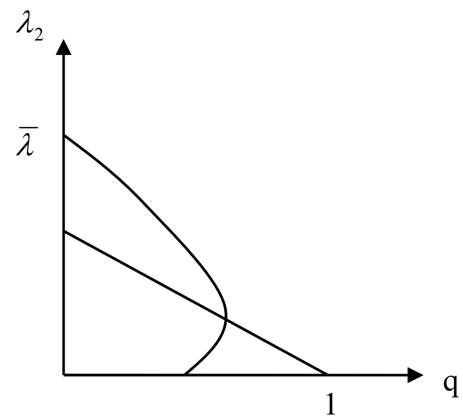


Figure 2: A Unique Equilibrium with a Formal and an Informal Sector

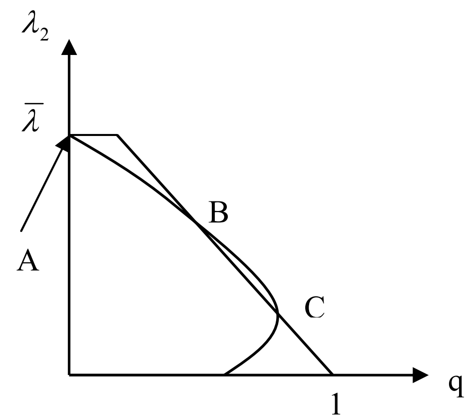
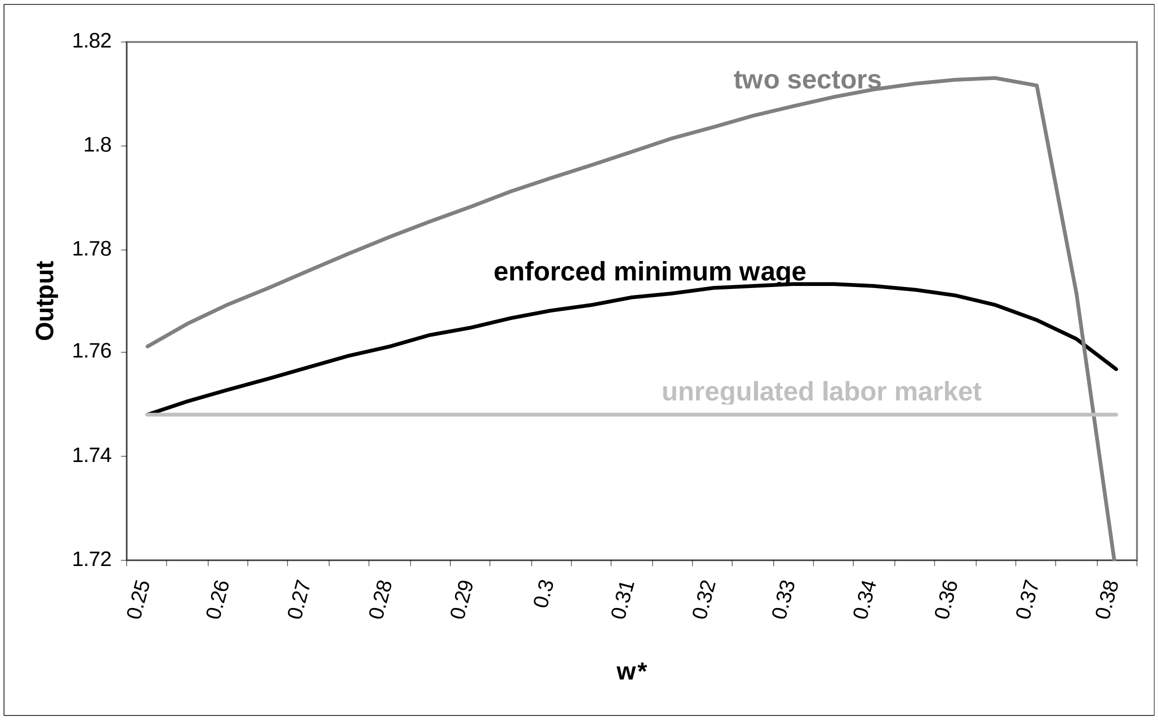


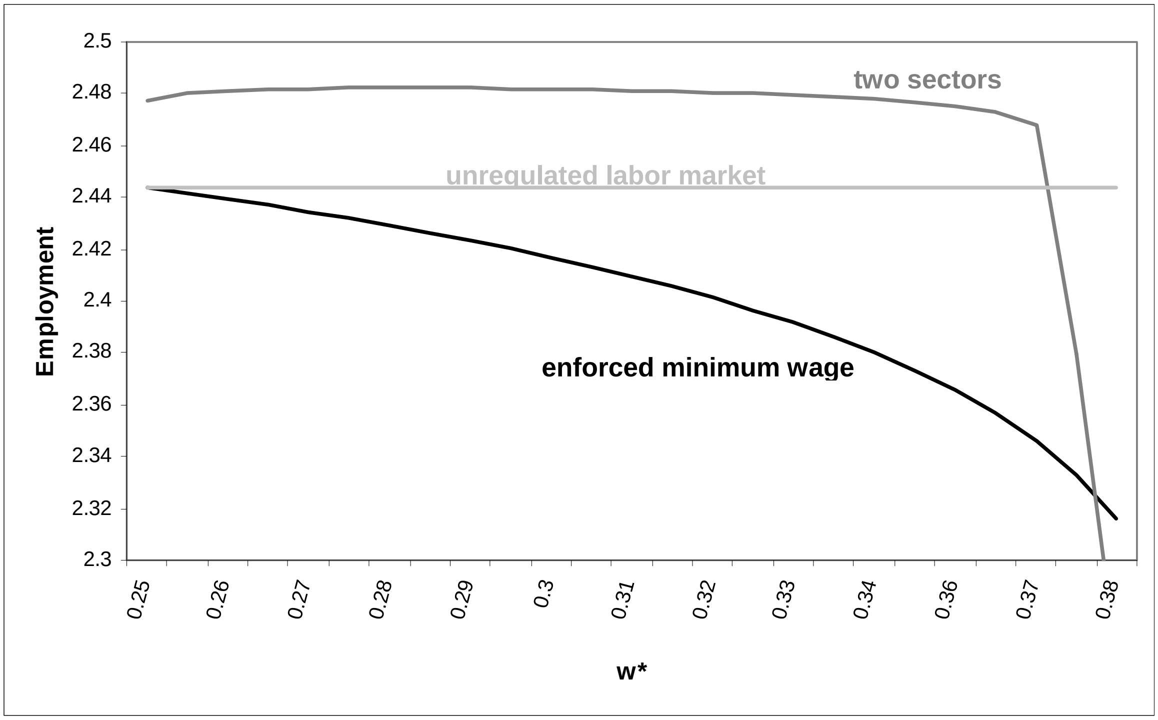
Figure 3: Multiple Equilibria

Figure 4



two sectors
enforced minimum wage
unregulated labor market
Output
1.82
1.8
1.78
1.76
1.74
1.72
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
0.34
0.36
0.37
0.38
w*

Figure 5



two sectors
unregulated labor market
enforced minimum wage
Employment
2.5
2.48
2.46
2.44
2.42
2.4
2.38
2.36
2.34
2.32
2.3
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
0.34
0.36
0.37
0.38
w*