Analysis of Two Higher Education Funding Systems under Uncertainty*

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Abstract

We present a theoretical model to analyze the effects of two higher education loan schemes on individual schooling decisions, in a world where graduate earnings are stochastic. The paper is structured in two parts. The first part is static, we assume a single lifetime shock on graduate incomes and we compare the individual expected utilities under a mortgage loan (ML) and an income contingent loan (ICL). We find that when agents are risk-neutral they always strictly prefer an ICL. When they are risk-averse, for increasing uncertainty the expected utility under an ICL system is higher. The second part is dynamic, we assume a graduate income following a geometric Brownian motion. Applying the Euler-Maruyama method to generate the incomes, we develop a numerical iterative solution to compute the average utilities under the two funding schemes. We find that an ICL scheme is preferred to a ML scheme when the initial level of the income is low and the uncertainty on future wages is increasing. However, if the individuals are very risk-averse and receive a low initial wage, the differences between the two funding systems decline and a ML becomes the most advantageous system.

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1 Introduction

An under-researched question in the literature is whether investing in education is risky. An individual making schooling decisions is likely to be only imperfectly aware of her abilities, the probability of success, the job and the earnings that may be obtained after completing an education.

The Weiss (1974) lifecycle model assumed completely imperfect capital markets. He studies the risk adjusted average rate of return to schooling, which is the subjective discount rate at which the individual would be indifferent between acquiring a certain level of education and having no education at all. Weiss finds that the risk adjusted average returns to education sharply decrease as the risk aversion increases. Weiss'model has been extended by Hause (1974) and Levahari and Weiss (1974). Particularly interesting is the work of Olson,White and Shefrin (1979). They assume that consumption equals income in each period after schooling and educated individuals get a random stream of income that varies according to the level of education achieved. Olson, White and Shefrin allow borrowing to finance education, and in particular they consider a mortgage loan that is paid back only after the completion of schooling. They find that the estimated real returns of college are large, and the estimated risk adjustments for college are small but positive.

The model that we present in this paper follows the traditional literature assuming that the effects of education on earnings are stochastic, however the analysis is developed comparing two higher education financing schemes: a mortgage loan and an income contingent loan. The choice of this two systems is related to the UK Reform of Higher Education, approved in 2004 and that will be effective from 2006. The current UK higher education financing system is based on an up-front fee fixed across universities and courses. Only students whose family income exceeds a given amount pay the fee in full, the others are exempted. The Higher Education Reform increases the tuition, enlarges the number of students liable and universities can set their fees up to a maximum £3000 p.a. Fees will be covered by a system of subsidized loans. The innovation is the introduction of an income-contingent scheme to repay the loans. Graduates start to pay back only when their incomes are above £15,000 per year and at 9 per cent fixed repayment rate. There is a zero real interest rate and repayments are made through the tax system as a payroll deduction. A similar higher education system is effective in Australia since 1989, the difference is the presence of increasing thresholds of income and increasing repayment rates.

In the recent literature fully integrating risk into the analysis of investment in human capital where borrowing is possible has remained technically problematic. For example, Hogan and Walker (2001) use "real options" theory to model optimal schooling choices, when the returns are stochastic, and they find the opposite results that individuals stay longer in school as risk increases. The crucial difference lies in the ease with which lifelong learning is possible - being able to return to education matters. Pistaferri and Padulla (2001) extend the Olson, White and Shefrin's model to consider two types of risk: employment risk and wage uncertainty, within an imperfect credit market framework. However they do not take into account the education financing systems. Similar work by Hartog and Serrano (2003) analyze the effects of stochastic post schooling earnings on the optimal schooling length, and show a negative effect of risk on investment. In our paper, we use the same framework of Hartog and Serrano but incorporating the possibility that students can take out a loan to finance their education costs.

Our first contribution is a comparative analysis between a mortgage loan and an income-contingent loan, assuming that the graduate incomes are affected by a single lifetime shock. The model is static, the uncertainty affects the level of the income, which remains constant for all the working life. According to the intensity of the variance it is possible to have big variations among the graduate wages. The analysis is developed, initially algebraically, using the latest techniques applied in this literature. Expected utility is derived, under ICL and ML, with CRRA and CARA utility functions, for risk neutral and risk-averse individuals. After the theoretical analysis we undertake some simulations in MATLAB to investigate the sensitivity to parameter values. We find that under risk neutrality the individuals strictly prefer an ICL, and under risk aversion they prefer an ICL for increasing level of uncertainty.

Our second contribution is more technical, we assume that the growth rate of the graduate income follows a geometric Brownian motion. The model is dynamic, the uncertainty in this case affects the income each year during the individual working life. The initial level of the income is not random and it generates many paths that grow following a deterministic trend. However, in a single path the incomes change year by year according to the intensity of the volatility of the Brownian motion.

It is very complex to find an explicit equation for the repayment period under an income contingent loan. Therefore, we develop in MATLAB a numerical iterative solution to make the comparison between the two financing schemes. Using the Euler-Maruyama method, we first solve numerically the stochastic differential equation that produces the random incomes. We generate a stochastic path of incomes along the individual working life, and we work out the repayment period under an income contingent loan using the cumulative sum of the annual repayments. We compute the individual utility as the discounted sum of net incomes during and after the repayment of the debt. A similar procedure is used for a mortgage loan, but with a fixed repayment period. We replicate the process generating a high number of income paths and we compute the average of all the single utilities, under both funding schemes. We, finally, compare the average utilities under the two systems, repeating the simulations for different parameter values. We obtain a database of around 27000 observations of the difference between the average utility under an ICL and under a ML.

We find that the sign of the difference is strongly affected by the initial income. When the individuals start their working life with low incomes an ICL is more beneficial, and the extent of the preference is increasing in the variance. Instead for high initial wages a ML is more advantageous. The trend of the difference between the average utilities depends on the ratio between the deterministic growth rate and the volatility. If the deterministic rate is high, the trend is increasing and therefore an ICL becomes more profitable. The effect of the risk aversion is evident when the individuals have low initial wages, in fact if they feel very risk-averse they find more advantageous a ML.

2 The Individual Decision Problem

Consider the individual at the end of compulsory school deciding whether investing in more education or starting to work. This choice involves two levels of potential income and is represented by a binary variable

$$d = \begin{cases} 0 & \text{if she does not go to college} \\ 1 & \text{if she goes to college} \end{cases}$$

If the individual does not go to college she receives at the beginning of period zero and for all her working life a deterministic income $X < 1$. This is a strong assumption, since the non-graduate income could be also random, but for the purposes of our model the uncertainty affects only the graduate income.
Education is costly and people going to college have zero income during that period. We assume the existence of a simple capital market where individuals can borrow only to finance their fees and living expenses. Upon graduation, as in Hartog and Serrano (2003), income is uncertain because subject to a random shock. For simplicity, the shock has a single lifetime realization, after which the income remains constant at the new level reached. Let y be the shock with $E(y) = 1$ and $Var(y) = \sigma^2$.

In this model individuals cannot insure the wage risk and seek to maximize the expected lifetime utilities. Utility is defined over the individuals' income stream. We assume, as in Olson, White and Shefrin (1979), that consumption, c, is always equal to income for people not going to school and in each period after school, since individuals cannot borrow and lend.

$$d = 0 \implies c = X \quad \text{for ever}$$

$$d = 1 \implies c = \begin{cases} y & \text{for ever after college} \\ 0 & \text{during college} \end{cases}$$

As stated earlier, students take out loans to avoid negative consumption while at school. In general, if persons do not invest in education they have the following expected utility:

$$V(0) = \int_0^\infty e^{-\rho t} u(X)\, dt. \tag{1}$$

People attending college start to repay their loan after s years of school and for T years. Assuming a general repayment schemes, we define R as the general per-period payment. The expected utility is:

$$V(1) = E\left\{\int_{s}^{T+s} e^{-\rho t}u\left(y-R\right)dt + \int_{T+s}^{\infty} e^{-\rho t}u\left(y\right)dt\right\} \tag{2}$$

3 Comparing Mortgage and Income Contingent Loans under Risk-Neutrality

The Government finances the investment in higher education issuing debt that is paid back only with the graduates' repayments. The students take out a loan of fixed size that cover all the costs of the university, that are equal for all the courses and subjects. The loan is repaid according to two financing systems: Mortgage Loan and Income Contingent Loan. The scheme is fully funded and the participation is obligatory, there is no opting out choice. As noted by Olson, White and Shefrin (1979), under a mortgage loan scheme is possible to escape through bankruptcy. However, in our model we assume that all the debt is paid off and there is no default[1].

3.1 Mortgage Loan

The individuals take out a loan equal to C and repay through T equal, fixed and periodical instalments φ, at a certain real interest rate, r. For simplicity, (and UK relevance) $r = 0$. In our model the formula[2] for the instalment that

[1] The case of the students' default is analyzed in another future work.

[2] We assume the instalments are worked out applying a French Amortization method, therefore they include capital and interests. This method is useful because we can obtain single payments that correspond to a fixed percentage of the total cost. If we consider a total loan C, a real interest r and a repayment period of T years, the formula for a single instalment is: $\varphi = \frac{C\,r}{[1-(1+r)^{-T}]}$. Taking the limit and applying L'Hopital's rule: $\lim_{r\to 0}\varphi = \lim_{r\to 0}\frac{C}{T(1+r)^{-T-1}} = \frac{C}{T}$

we use is: $\varphi = C/T$. The repayment period is therefore just

$$T = C/\varphi. \tag{3}$$

3.2 Income Contingent Loan

The individuals borrow an amount equal to the total cost of education, C, and start to pay back their loan after graduation according to level of their income. Under this scheme if the wage is below a minimum threshold no payment is due. If the wage increases, a greater portion of the debt is repaid and all the loan is paid off in less time. Therefore, the main difference with a mortgage loan is that the repayment period, $\tilde{T}$, is **random**. In our model, for simplicity, we assume no initial threshold and the total cost of schooling is given by a fixed percentage (γ) of the random graduate income.

$$C = \gamma \int_{s}^{\tilde{T}+s} y \, dt \tag{4}$$

We solve the integral and work out the repayment period:

$$\tilde{T} = \frac{C}{\gamma y} \quad . \tag{5}$$

Substituting this parameter in the equation (2) we obtain the expected utility under an income contingent loan.

3.3 Risk Neutrality and Expected Costs

We assume first that individuals are risk neutral, i.e. $u(y) = y$. So we can consider only the costs to compare the two repayment schemes. We work out the present value of the costs, substituting for each scheme the respective repayment period, T and $\tilde{T}$, and discounting to $t = 0$.

Proposition 1. *The utility from ICL is greater than the utility from ML*

$$V(1)_{ICL} > V(1)_{ML}.$$

Proof. See Appendix A.

The first theoretical result found in our analysis is that under an ICL the expected utility is lower than under a ML. This result could sound strange because individuals are risk neutral, but it depends on the expected costs of education that are greater with a ML than an ICL. If we assume a general repayment method R, the present value of the education cost is:

$$PVC = \int_0^T R\, e^{-\rho t} dt = \frac{R}{\rho}\,[1 - e^{-\rho T}]$$

Taking the derivatives of PVC with respect to T, we can easily observe that this function is concave [3]. Consider now a loan with a certain repayment period of 10 years, and another loan with two even probability repayment periods of 5 or 15 years. The concavity property implies that the expected present value of the cost of an uncertain repayment is lower than the present value of the expected cost of a certain repayment:

$$E(PVC) = \frac{1}{2}\, PVC(5) + \frac{1}{2}\, PVC(15) < PVC(10).$$

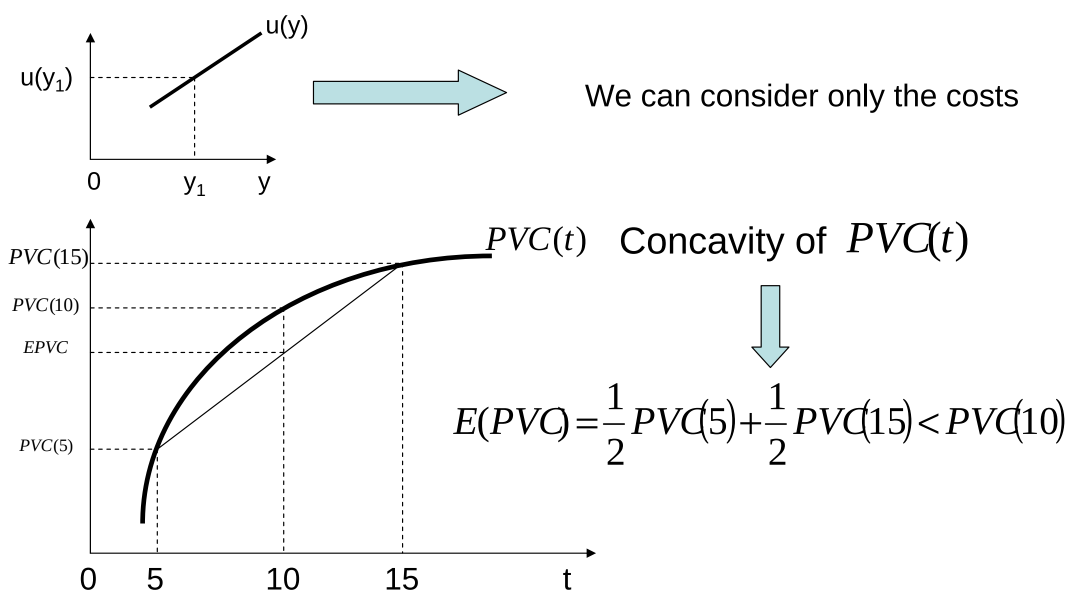
See Figure 1.

4 Comparing Mortgage and Income Contingent Loans under Risk Aversion

In this Section we consider individuals who are risk averse and we work out their expected utility (represented by the equation (2)), under a mortgage loan and an income contingent loan system. We consider the assumptions stated in Section 2 and we develop the analysis using two types of utility function: constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). We omit the majority of calculations that are showed in more detail in the Appendices B and C.

[3] $\frac{\vartheta^2 PVC}{\vartheta T^2} = -R\rho\, e^{-\rho T} < 0$ for all T.

Figure 1: Expected Cost and Risk Neutrality



4.1 Expected Utility with a Mortgage Loan

Under a mortgage loan, the expected utility is obtained substituting $R = \varphi$ in equation (2):

$$V_{ML} = \int_{s}^{T+s} e^{-\rho t} E\left[u\left(y - \varphi\right)\right] dt + \int_{T+s}^{\infty} e^{-\rho t} E\left[u\left(y\right)\right] dt. \tag{6}$$

To get a closer-form solution for V_{ML}, we use a second order Taylor expansion around the mean $E[y - \varphi] = 1 - \varphi$ [4] for the utility during the repayment period, and around $E[y] = 1$ for the utility after the repayment period:

$$E[u(y - \varphi)] \simeq u\left(1 - \varphi\right) + \frac{1}{2} u^{''}\left(1 - \varphi\right) \sigma^2. \tag{7}$$

$$E[u(y)] \simeq u(1) + \frac{1}{2} u^{''}(1) \sigma^2. \tag{8}$$

We develop our analysis using both a CARA and CRRA utility function:

$$u(y) = -\frac{1}{a}\, e^{-ay} \quad \text{CARA}$$

and

$$u(y) = \frac{y^b}{b} \quad CRRA.$$

where a is the risk aversion parameter and $b = 1 - a$.

After simplifying [5], we get:

$$V_{ML_{CARA}} = \frac{e^{-\frac{\rho C + a\varphi + as + s\rho\varphi}{\varphi}}}{2a\rho} \left[-2 + a^2\sigma^2 + e^{a\varphi}\left(2 + a^2\sigma^2\right) - e^{\frac{\rho C}{\varphi} + a\varphi}\left(2 + a^2\sigma^2\right)\right]. \tag{9}$$

And if we use a CRRA utility function the expected utility is:

$$\begin{aligned} V_{ML_{CRRA}} = \frac{e^{-\rho s}}{\rho} \{ & \left(1 - e^{-\frac{\rho C}{\varphi}}\right) \left[\frac{(1-\varphi)^b}{b} + \frac{1}{2}(b-1)(1-\varphi)^{b-2}\sigma^2\right] \\ & + e^{-\frac{\rho C}{\varphi}} \left[\frac{1}{b} + \frac{1}{2}(b-1)\sigma^2\right] \}. \end{aligned} \tag{10}$$

[4] See Pistaferri and Padula (2001) and Hartog and Serrano (2003).
[5] see Appendix B for the proof.

4.2 Expected Utility with the Income Contingent Loan

Under an income contingent loan we do not know how long people take to repay their education debt, therefore in the general equation of the expected utility the random income appears twice. First in the integral's bounds as random repayment period, second as argument of the utility function.

$$V_{ICL} = E\left\{ \int_{s}^{\frac{C}{\gamma y}+s} e^{-\rho t} u\left[y\left(1-\gamma\right)\right] dt + \int_{\frac{C}{\gamma y}+s}^{\infty} e^{-\rho t} u\left(y\right) dt \right\} \tag{11}$$

Solving the integral we get the following equation:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} E\left\{ \left[1 - e^{-\frac{\rho C}{\gamma y}}\right] u\left[y\left(1-\gamma\right)\right] + \left[e^{-\frac{\rho C}{\gamma y}}\right] u\left(y\right) \right\}. \tag{12}$$

To simplify the calculations we define all the expression included in the expected value operator as $g(y)$. This trick allows us to apply a second order Taylor expansion of $E[g(y)]$ around the mean $E\left[y\right] = 1$. Then, the equation (12) becomes:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} \left[g(1) + g''(1)\frac{\sigma^2}{2} \right]. \tag{13}$$

The remaining procedure consists of calculating the value of $g(1)$ and $g''(1)$, in general and with CARA and CRRA utility functions in particular Finally, we substitute the expressions found in equation (13), and we obtain the following results. If we use a CARA utility function, we get after simplifying:

$$\begin{aligned} EU_{ICL_{CARA}} = & \frac{e^{-\frac{a\gamma+(c+\gamma s)\rho}{\gamma}}}{2a\rho}\{-2[1 - e^{a\gamma} + e^{a\gamma+\frac{\rho C}{\gamma}}] \\ & + \frac{1}{\gamma^2}[(-a^2\gamma^2[1 - e^{a\gamma}(\gamma-1)^2 + e^{a\gamma+\frac{\rho C}{\gamma}}(\gamma-1)^2] \\ & + 2\rho C[1 + e^{a\gamma}(\gamma-1)]a\gamma + \rho C(e^{a\gamma}-1)(\rho C - 2\gamma))\sigma^2]\}. \end{aligned} \tag{14}$$

And using a CRRA utility function the expected utility [6] is:

$$\begin{aligned} EU_{ICL_{CRRA}} = & \frac{e^{-\left(s+\frac{C}{\gamma}\right)\rho}}{2b\gamma^2\rho}\{e^{\frac{\rho C}{\gamma}}(1-\gamma)^b\gamma^2[2 + (b-1)b\sigma^2] - [(1-\gamma)^b - 1] \\ & \cdot [2\gamma^2 + ((b-1)b\gamma^2 + 2(b-1)C\gamma\rho + C^2\rho^2)\sigma^2]\}. \end{aligned} \tag{15}$$

[6] The expected utility with an income contingent loan is equal to the expected utility with a mortgage loan if $\varphi = \gamma$ and the variance of the income is zero.

5 Simulations

In this Section we use the equations of the expected utility derived previously and we compare them through simulations. We assign numerical values to each parameter, and see how the variations affect the difference between the expected utility under a mortgage loan and the expected utility under an income contingent loan. The analysis is developed using both a CRRA utility function and a CARA utility function. In particular, for a mortgage loan we use the equations (10) and (9), and for an income contingent loan the equations (15) and (14).

We consider the following vectors of parameters:

$\sigma = [0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.2]$. In our equations σ is the standard deviation of the wage. Therefore, the higher is σ, the higher is the uncertainty around the level of the income. In the UK Labour Force Survey of 2002, the average income of graduate people is around £25000, with a standard deviation $\sigma = 0.64$. Taking into account this value, we assign six values to σ: from the lowest case with no uncertainty to a standard deviation of 120% of the income.

$\varphi = [500 \quad 600 \quad 800 \quad 1000 \quad 1200 \quad 2400]$, we set six possible installments under a ML[7].

$C = [6000 \quad 8000 \quad 10000 \quad 12000]$, we consider six possible costs of the education, assuming a $s = 4$ years full-time degree. Therefore, we obtain the following repayment periods under a ML: $T = [24 \quad 20 \quad 15 \quad 10 \quad 5]$.

$\gamma = [0.05 \quad 0.09 \quad 0.15 \quad 0.25 \quad 0.5]$, we assign 5 values to the rate of repayment under an ICL. The value of 9% is the one chosen in the UK Reform.

$\rho = [0.02 \quad 0.08 \quad 0.15 \quad 0.25 \quad 0.5]$, we assign 5 increasing values to the subjective discount rate.

$a = [0 \quad 0.25 \quad 0.5 \quad 0.75]$, we set 4 increasing values for the risk aversion parameter. In the model we used $b = 1 - a$ for the CRRA utility func-

[7] The values in the program are calibrated assuming ten thousand as unit of measure of the expected income, which is equal to 1 in the model.

tion. These values have been chosen following the literature.

The simulations are performed using MATLAB, and we compute the differences $V_{ICL} - V_{ML}$. We analyze all the possible combinations of the above parameters and we obtain a database of 5760 observations.

We consider the following cases.

- σ and a increasing: $C = £10000$, $\varphi = £1000$, $T_{ML} = 10$ years, $\rho = 8\%$, $\gamma = 9\%$, CRRA utility function. Given a and for σ increasing the difference $V_{ICL} - V_{ML}$ is positive and increasing. This means that the higher is the uncertainty on the income, the most an ICL is preferred. Given σ, for increasing risk aversion $V_{ICL} - V_{ML}$ is increasing. However, if we decrease φ in order to have a repayment period of $T_{ML} = 20$ years, a ML is preferred and it becomes more advantageous for increasing a. (Table 1). If we use a CARA utility function, we observe that $V_{ICL} - V_{ML}$ is positive and almost constant for increasing level of uncertainty. Given σ, for higher level of a $V_{ICL} - V_{ML}$ first decreases and then increases. If the repayment period augments there is no change in the trend and in the sign of $V_{ICL} - V_{ML}$. (Table 2)

- σ and ρ increasing: $C = £10000$, $\varphi = £1000$, $T_{ML} = 10$ years, $a = 0.5$, $\gamma = 9\%$, CRRA utility function. If we keep σ constant, $V_{ICL} - V_{ML}$ is positive and increasing up to $\rho = 8\%$; for higher values of ρ it is decreasing. If we increase the ML repayment period to $T_{ML} = 20$ years, the sign and the trend of $V_{ICL} - V_{ML}$ changes. ML is the preferred systems (except when $\sigma = 1.2$) and the difference $V_{ICL} - V_{ML}$ first decreases, and then when ρ is higher than 8% it starts to increases. We can notice that $V_{ICL} - V_{ML}$ converges to zero when ρ is very high. If we keep ρ constant, the difference $V_{ICL} - V_{ML}$ increases for σ increasing. In particular when $T_{ML} = 20$ years, the sign of $V_{ICL} - V_{ML}$ changes from negative to positive if σ is very high. Therefore, an ICL becomes the most advantageous system. (Table 3). Using a CARA utility function and keeping σ constant the trend of $V_{ICL} - V_{ML}$ is increasing. For low level of ρ a ML is preferred, for higher ρ $V_{ICL} - V_{ML}$ sharply increases and then it remains constant. A higher repayment period under a ML does not affect this behavior. (Table 4)

- σ and φ increasing : $C = £10000$, $\rho = 8\%$, $a = 0.5$, $\gamma = 9\%$, CRRA utility function. Given σ, if φ increases the repayment period under ML decreases and an ICL becomes more beneficial. The trend of $V_{ICL} - V_{ML}$ is increasing, its sign instead is negative for low φ and positive for high φ. Keeping φ constant, if σ is increasing the trend of $V_{ICL} - V_{ML}$ is always increasing, instead its sign depends on the level of φ. (Table 5). When we use a CARA utility function we obtain similar results, however the sign of $V_{ICL} - V_{ML}$ is always positive (Table 6).

- σ and γ increasing: $C = £10000$, $\varphi = £1000$, $T_{ML} = 10$ years, $a = 0.5$, $\rho = 8\%$, CRRA utility function. Given σ, the trend of $V_{ICL} - V_{ML}$ is decreasing as γ increases, since an ICL becomes worse. We observe that for increasing σ the trend is increasing, and its sign is positive for low level of γ and always negative if γ is very high. (Table 5). The results are the same with a CARA utility function.(Table 6)

- φ and γ increasing: $C = £12000$, $\sigma = 0.6$, $a = 0.5$, $\rho = 8\%$, CRRA utility function. Given $\gamma = 5\%$, $V_{ICL} - V_{ML}$ is increasing as φ increases. The sign of $V_{ICL} - V_{ML}$ is negative for low φ: a ML has longer repayment period and it is more beneficial. For high φ, the repayment under a ML is short and an ICL become more advantageous. Given φ, for higher values of γ the trend is decreasing because the utility of an ICL reduces. (Table 5.) With a CARA utility function, the previous results are confirmed, but $V_{ICL} - V_{ML}$ decreases more slowly for increasing φ. (Table 6)

The uncertainty on the level of the income affects strongly the trend of $V_{ICL} - V_{ML}$. In fact, if the standard deviation is increasing $V_{ICL} - V_{ML}$ is always increasing, and an ICL becomes more advantageous. The sign of $V_{ICL} - V_{ML}$ depends on the length of repayment period. The individuals prefer the system with the longest repayment period, since the real interest rate is zero they can exploit the advantages of an implicit subsidy, due to the decreasing present value of the payments. Therefore, when φ is low the repayment period under a ML is long and this system becomes more attractive. The same happens for low levels of the ICL repayment rates, this system gives higher expected utility then it is preferred. The effect of the risk aversion is affected by the presence of the uncertainty, therefore increasing a does not change the trend and the sign of $V_{ICL} - V_{ML}$, it just augments the size of $V_{ICL} - V_{ML}$ in absolute value. So if a system is preferred to the other, the higher risk aversion strengthens this preference. The subjective discount rate instead affects the trend of $V_{ICL} - V_{ML}$, which converges to zero for high values of ρ, therefore discounting a lot reduces the differences between the two systems.

6 Increasing Income

In the second part of our work we change the assumptions on the income. After graduating the individuals receive a wage that is not affected by a single lifetime shock but it increases following a geometric Brownian motion. We first consider the case of a constant growth rate of the income during the working life, then we add a stochastic component and we compute the individual expected utilities of the individuals under the two repayment schemes.

6.1 Constant Growth Rate

We assume that $y(t)$ is the value of £1 of graduate income after time t increasing at a constant rate λ for all the individual working life, ending in period T_{max} . Then $y(t)$ satisfies the ordinary differential equation (ODE)

$$dy(t)/dt = \lambda y(t). \tag{16}$$

The solution of the ODE gives the level of income $y(t) = e^{\lambda t} y_0$, where y_0 is the initial wage after graduating. We consider a logarithmic utility function $\log u = \log y_0 + \lambda t$ and we work out the expected utilities under the two higher education funding systems.

Under a mortgage loan the repayment period is given by equation(3) $T = C/\varphi$. We assume for simplicity that the initial wage y_0 is higher than the instalment φ, in the next section we consider a more general case. Using equation (6) we substitute T and $\log(u)$. Noticing that all the components are deterministic, we obtain the following expression for the expected utility

$$\begin{aligned} EU_{ML} &= \int_s^{(C/\varphi)+s} e^{-\rho t} \left(\log(y_0 - \varphi) + \lambda t\right) dt + \int_{(C/\varphi)+s}^{T_{\max}} e^{-\rho t} \left(\log y_0 + \lambda t\right) dt \\ &= \frac{1}{\rho^2} (e^{-(s+T_{\max})\rho} [e^{\rho T_{\max}} \lambda(1 + s\rho) - e^{\rho s} \lambda(1 + T_{\max}\rho) + \\ &\rho(-e^{\rho s} \log y_0 + e^{\rho(T_{\max} - \frac{C}{\varphi})} (\log y_0 - \log(y_0 - \varphi)) + e^{\rho T_{\max}} \log(y_0 - \varphi))]). \end{aligned} \tag{17}$$

The cost of education with an income contingent loan is given by (4), and under the new assumptions on income it is

$$C = \gamma \int_s^{\widetilde{T}+s} e^{\lambda t} y_0 \, dt. \tag{18}$$

Solving equation(18) for $\widetilde{T}$, we get the repayment period

$$\widetilde{T} = \frac{\log[e^{\lambda s} + \frac{\lambda C}{\gamma y_0}]}{\lambda} - s. \tag{19}$$

The expected utility under an ICL is given by

$$EU_{ICL} = E\left\{ \int_s^{\widetilde{T}} e^{-\rho t} u\left[y\left(1-\gamma\right)\right] dt + \int_{\widetilde{T}}^{T_{\max}} e^{-\rho t} u\left(y\right) dt \right\}. \tag{20}$$

Substituting the expression for $\widetilde{T}$ and the log utility we get

$$\begin{aligned} EU_{ICL} = & \int_s^{\frac{\log[e^{\lambda s}+\frac{\lambda C}{\gamma y_0}]}{\lambda}} e^{-\rho t}\left(\log(y_0(1-\gamma)) + \lambda t\right) dt + \\ & \int_{\frac{\log[e^{\lambda s}+\frac{\lambda C}{\gamma y_0}]}{\lambda}}^{T_{\max}} e^{-\rho t}\left(\log y_0 + \lambda t\right) dt \\ = & \frac{1}{\rho^2}\{e^{-(s+T_{\max})\rho t}(e^{\rho T_{\max}}\lambda(1+s\rho) - e^{\rho s}\lambda(1+T_{\max}\rho)) + \\ & \rho[-e^{\rho s}\log y_0 + e^{-\rho s}\log(-(1-\gamma)y_0) + \\ & (\log y_0 - \log(-(1-\gamma)y_0))(e^{\lambda s} + \frac{\lambda C}{\gamma y_0})^{-\frac{\rho}{\lambda}}]\}. \end{aligned} \tag{21}$$

The two expressions of the expected utilities found in equations (17) and (21) can be used for numerical simulations, in order to see which funding system is more profitable. However our main task in this work is to compare the two schemes under uncertainty. Therefore, we will see later that the constant wage growth is a particular case of the stochastic income growth.

7 Stochastic Income

We assume that the growth rate of the income is affected by a white noise process, formally defined as the derivative of the standard Brownian motion, or standard Wiener process, $W(t)$

$$\epsilon = dW(t)/dt.$$

The derivative does not exist in the usual sense, since the Brownian motion is nowhere differentiable. However this process is used with the convention that its meaning is given by integral representation. If $\sigma(y,t)$ is the intensity of the noise at point y at time t, then it is common agreement that $\int_0^T \sigma(y(t),t)\epsilon(t)dt = \int_0^T \sigma(y(t),t)dW(t)$. In our case, adding a white noise $\sigma\epsilon(t)$ to the constant growth rate of the income in equation (16), we obtain the stochastic differential equation, SDE

$$dy(t)/dt = (\lambda + \sigma\epsilon)y(t). \tag{22}$$

where $\sigma \geq 0$, $\lambda > 0$, are some constants, and $y(0)$ is the deterministic component of the income.
This means that $y(t)$ satisfies

$$dy(t)/y(t) = \lambda dt + \sigma dW(t). \tag{23}$$

This expression can be interpreted heuristically as expressing the relative or percentage increment dy/y in y during an instant of time dt. Then, the expected instantaneous growth rate is λ, and the standard deviation of the instantaneous growth rate is σ.

To solve the SDE we introduce the Itô process $R(t)$ given by
$dR(t) = rdt + \sigma dB(t)$. We rewrite the SDE as

$$dy(t) = y(t)dR(t) \tag{24}$$

this means that $y(t)$ is the stochastic exponential, $\varepsilon(R)$, of $R(t)$. The solution of equation(24) is

$$\begin{aligned} y(t) &= y(0)\varepsilon(R)(t) \\ &= y(0)\exp[R(t) - R(0) - \frac{1}{2}[R,R](t)\,]. \end{aligned} \tag{25}$$

$R(t)$ is easily found to be $R(t) = rt + \sigma B(t)$, $R(0) = 0$, and its quadratic variation $[R,R](t)$ is the quadratic variation of an Itô process and equal to

$$[R,R](t) = \int_0^t \sigma^2 ds = \sigma^2 t.$$

Substituting these expressions in equation(25) we get

$$\begin{aligned} y(t) &= y(0)\exp[\lambda t - \sigma W(t) - \frac{1}{2}\,\sigma^2 t\,] \\ &= y(0)\exp[(\lambda - \frac{1}{2}\,\sigma^2)t + \sigma W(t)\,]. \end{aligned} \tag{26}$$

This process is a geometric brownian motion. Since $W(t)$ is Normally distributed with $E(W(t)) = 0$ and $Var(W(t)) = t$, the transformation function $f(y) = \ln y$ is also Normally distributed with

$$E(\ln y(t)) = \ln y_0 + (\lambda - \frac{1}{2}\,\sigma^2)t \tag{27}$$

and

$$Var(\ln y(t)) = \sigma^2 t. \tag{28}$$

Therefore, $y(t)$ has a lognormal distribution with

$$E(y(t)) = y_0\, e^{\lambda t}$$

$$Var(y(t)) = y_0^2\, e^{2\lambda t}(e^{\sigma^2 t} - 1).$$

8 Expected Utilities with Brownian motion

Our target as in the previous section is to find the individual expected utilities under both a mortgage loan system and an income contingent loan. However this task is not very easy when the income follows a geometric brownian motion. We can find an algebraic solution for the mortgage loan scheme, but not for the income contingent loan; therefore we develop a numerical method and we compare the expected utilities through some simulations.

8.1 Difficulties

Assuming a logarithmic utility function we have the following expression

$$\ln y = \ln(y_0) + (\lambda - \frac{1}{2}\,\sigma^2)t + \sigma W(t).$$

Under a mortgage loan, we can work out the expected utility applying the general equation(10) and using the equation(27) for the expected value of a log income. We obtain

$$
\begin{aligned}
EU_{ML} = &\int_{s}^{(C/\varphi)+s} e^{-\rho t}\left(\log(y_0-\varphi)+(\lambda-\frac{1}{2}\sigma^2)t\right)dt+ \\
&\int_{(C/\varphi)+s}^{T\max} e^{-\rho t}\left(\log y_0+(\lambda-\frac{1}{2}\sigma^2)t\right)dt \\
= &\frac{1}{2\rho^2}(e^{-(s+T_{\max})\rho}[(e^{\rho T_{\max}}(1+s\rho)-e^{\rho s}(1+T_{\max}\rho))(2\lambda-\sigma^2)+ \\
&2\rho(-e^{\rho s}\log y_0+e^{\rho(T_{\max}-\frac{C}{\varphi})}(\log y_0-\log(y_0-\varphi))+ \\
&e^{\rho T_{\max}}\log(y_0-\varphi))]).
\end{aligned}
\tag{29}
$$

Under an income contingent loan the repayment period is not fixed but it depends on the annual income, which is stochastic in our case. In the previous section, with a non stochastic income, we got the repayment period solving for $\widetilde{T}$ the equation (18) of the education cost. In this case the cost is given by the following expression

$$C=\gamma\int_{s}^{\widetilde{T}+s} y_0\exp[(\lambda-\frac{1}{2}\sigma^2)t+\sigma W(t)]dt \tag{30}$$

To obtain $\widetilde{T}$ we should solve the integral of the exponential of a brownian motion, and according to the recent literature on this field it is a very complex task. Therefore to overcome this obstacle we adopt a numerical method.

8.2 Numerical Method

Our objective is to compare the value of the utility under a mortgage loan and an income contingent loan scheme, for a generated path of stochastic income. We developed the method in several steps.

1. We generate a path of annual incomes for an individual working life. Since the problem requires a discrete solution, we prefer to apply the Euler-Maruyama method to the SDE (23), instead of using the close form in equation(26). The Euler-Maruyama method takes the form

$$y_j = y_{j-1}+y_{j-1}\lambda\Delta t+y_{j-1}\sigma(W(\tau_j)-W(\tau_{j-1})). \tag{31}$$

To generate the increments $W(\tau_j) - W(\tau_{j-1})$ we compute discretized Brownian motion paths, where $W(t)$ is specified at discrete t values. As explained in Higham (2001) we first discretize the interval $[0, I]$. We set $dt = I/N$ for some positive integer N, and let W_j denote $W(t_j)$ with $t_j = jdt$. According to the properties of the standard Brownian motion $W(0) = 0$ and

$$W_j = W_{j-1} + dW_j \tag{32}$$

where dW_j is an independent random variable of the form $\sqrt{dt}N(0,1)$. The discretized brownian motion path is a 1-by-N array, where each element is given by the cumulative sum in equation(32). To generate equation(31), we define $\Delta t = I/L$ for some positive integer L, and $\tau_j = \Delta t$. As in Higham (2001) we choose the stepsize Δt for the numerical method to be an integer multiple $R \geq 1$ of the Brownian motion increment dt: $\Delta t = Rdt$. Finally, we get the increment in equation(31) as cumulative sum:

$$W(\tau_j) - W_(\tau_{j-1}) = W(jRdt) - W((j-1)Rdt) = \sum_{h=jR-R+1}^{jR} dW_h. \tag{33}$$

2. Income contingent loan. We work out the yearly repayments as fixed percentage of the stochastic incomes generated. We then built a vector whose elements are the cumulative sum of the repayments, in order to see the amount of loan repaid. To obtain the repayment period, we observe the years in which the cumulative sum of the payments is equal[8] to the cost of education. We work out the individual utility as discounted sum of the net incomes during and after the repayment period, up to the end of the working life. We use a Logarithmic and CRRA utility function.

3. Mortgage loan. We set the fixed repayment period as the ratio between the cost of education and the annual instalment. The individual utility is given by the discounted sum of the net incomes during and after the repayment period. We use a Logarithmic and CRRA utility function.

[8]Since it is almost impossible to get a value equal to the cost, when the repayment is greater than it, we infer with certainty that debt has been paid off.

However, it can happen that annual income is lower than the instalment, in a usual mortgage loan the individual repays in the subsequent years at a higher interest rate. Or in case of default he will have a damage of his credit reputation. In our model to highlight a loss of utility in case of no repayment in one year, we compute the level of the utility for that year as negative percentage[9] of the annual income.

4. In point (2) and (3) we obtain a single value of the utility for individual income path generated in point (1). We generalize our method generating a high number of income paths and for each path we compute a level of utility. We then work out the average utility under both financing scheme and the difference of the average in order to compare the two systems.

5. We let the various parameters change and we repeat steps from (1) to (4), observing the trend of the difference of the average utility under the two funding schemes.

9 Simulations

The numerical method previously explained is implemented through a program built in MATLAB, that allows us to do all the simulations required. We consider a CRRA and a Log utility function and seven vectors of parameters:

$Y_0 = [8000 \quad 15000 \quad 30000]$, we consider three levels of initial income. During a a working period of 40 years these incomes generates different paths according to the volatility of the Brownian motion and the deterministic growth rate.

$\sigma = [0 \quad 0.02 \quad 0.05 \quad 0.9 \quad 0.15]$, we assign 5 values to the standard deviation, in order to have different intensities of the effect of the stochastic shock on income. When $\sigma = 0$ there is no stochastic growth; and e.g. $\sigma = 0.05$ means that the maximum annual variation of the income can be 5%, with respect to the case with $\sigma = 0$.

$\lambda = [0.5 \quad 1 \quad 1.5]$, we assign three values to the deterministic growth rate. Applying these rates to the case with no uncertainty, i.e. $\sigma = 0$, we

[9] We set this percentage equal to the average-low interest rate for a typical mortgage loan e.g. around 5%.

obtain for $\lambda = 0.5$ a total increase of the initial income in 40 years of around 40%, meaning a constant increase of 1% per year. If $\lambda = 1$ the total increase of the income at the end of the working life is 63%, that is 2.4% p.a. Finally, $\lambda = 1.5$ corresponds to 77% increase of the initial income after 40 years, that is 4% p.a.

$a = [0 \quad 0.25 \quad 0.5 \quad 0.75]$, we set 4 increasing values for the risk aversion parameter; in the program we use $b = 1 - a$ as in the CRRA utility function considered in the first part of this work. These values have been chosen following the literature.

$\rho = [0.02 \quad 0.08 \quad 0.15 \quad 0.25 \quad 0.5]$, we assign 5 increasing values to the subjective discount rate.

$\varphi = [500 \quad 600 \quad 800 \quad 1000 \quad 1200 \quad 2400]$, we set 6 possible installments under a mortgage loan and two levels of the cost of education $C = [12000 \quad 10000]$. When $\varphi = £1000$ the program applies a cost of £10000 and the resulting repayment period under ML is 10 years. For the other values of φ the cost applied is always £12000, and we get 5 repayment periods under ML, respectively: $T = [24 \quad 20 \quad 15 \quad 10 \quad 5]$.

$\gamma = [0.05 \quad 0.09 \quad 0.15 \quad 0.25 \quad 0.5]$, we assigned 5 values to the rate of repayment under an ICL. The value of 9% is the one chosen in the UK Reform.

The Brownian motion of equation (32) is produced setting $I = 1$ and $N = 160$ in order to have a small value of dt. Using a random number generator we produce 160 "pseudorandom" numbers from the N(0,1) distribution. The increments of equation (33) are computed setting $R = 4$, in order to have 40 annual incomes. A single value of the utility is given by a unique income path, and to obtain a more precise average utility we generated 1000 income paths. Then, we computed all the possible combinations among the parameters above, and we built a database with 27000 different values of the difference between the average utilities under an ICL and a ML. In particular, 21600 values are with a CRRA utility function, and 5400 with a Log utility. Among all the possible cases generated we identified the most relevant. For simplicity, we define $A_{ICL} - A_{ML}$ the difference between the average utilities under ICL and ML.

1. Risk Aversion Changing . We assume low risk aversion $a = 0.25$, low deterministic growth $\lambda = 1\%$, a repayment period under ML $T_{ML} = 10$ years, $\rho = 8\%$ and $\gamma = 9\%$. We consider σ and Y_0 increasing. If the initial income is low, for increasing σ $A_{ICL} - A_{ML}$ is positive, therefore an ICL is preferred to a ML; however the trend is decreasing. For higher levels of Y_0 $A_{ICL} - A_{ML}$ becomes negative, meaning that a ML is favorite, but the trend is slowly increasing. Keeping the same parameters but increasing the risk aversion to $a = 0.75$, the sign of $A_{ICL} - A_{ML}$ and its trend are the same as before, however the size of $A_{ICL} - A_{ML}$ in absolute value is sharply reduced. For example, for the low level of the income the size of $A_{ICL} - A_{ML}$ reduces of around 99%. (Table 7)

2. Deterministic Growth Rate changing. We assume the same parameters of the previous case with low risk aversion, and we increase λ from 1% per year to 4% per year. For a low Y_0 we observe that $A_{ICL} - A_{ML}$ is positive and the trend increasing for higher σ. Therefore, an ICL becomes more advantageous if the uncertainty is increasing. Comparing this case with the one with low deterministic growth, we notice that a higher λ changes completely the trend of $A_{ICL} - A_{ML}$ if Y_0 is low. Instead with higher levels of the initial income, the effect of higher λ is weak. (Table 7)

3. Discount Rate Changing . We use a CRRA utility function and we assume $a = 0.5$, $\gamma = 9\%$, $T_{ML} = 10$ years, $\lambda = 1\%$ and $Y_0 = £8000$. Given σ, for values of ρ up to 15% $A_{ICL} - A_{ML}$ is positive and increasing, for higher ρ the trend is decreasing. Given ρ if σ increases the trend is decreasing. Using a Log utility function, for higher values of ρ the trend is almost constant, given σ. If we consider a high Y_0, $A_{ICL} - A_{ML}$ is always negative and the trend is first decreasing and then increasing, both using CRRA and Log utility functions. We then keep the same parameters as in the first case but we increase λ to 4%. Given σ, the trend of $A_{ICL} - A_{ML}$ is increasing up to $\rho = 25\%$ and then it decreases. However, given ρ, for increasing σ $A_{ICL} - A_{ML}$ is increasing. (Table 8)

4. Repayment Period ML Changing . We assume $a = 0.5$, $\rho = 8\%$, $\lambda = 1\%$, $\gamma = 9\%$ and a repayment period under ML $T_{ML} = 24$ years. In this case $A_{ICL} - A_{ML}$ is always negative for any level of Y_0. The individuals, having the opportunity to repay in a long period and with

zero real interest rate, prefer a ML to an ICL. The trend of $A_{ICL} - A_{ML}$ is slowly increasing for higher σ. If we reduce the repayment period to $T_{ML} = 5$ years, an ICL is always preferred for low and medium levels of Y_0. $A_{ICL} - A_{ML}$ is negative only if $Y_0 = £30000$, in this case in fact a fixed installment of £2400 is preferred to 9% of the wage that will imply to pay back the in less than 5 years. (Table 9)

5. Repayment Rate under ICL changing. We assume $a = 0.5$, $\rho = 8\%$, $\lambda = 1\%$, $\sigma = 5\%$ and $Y_0 = £8000$. We let γ and ϕ increase. If $\gamma = 5\%$ an ICL is always preferred, for $\gamma = 9\%$ an ICL is more advantageous if $T_{ML} < 15$ years, that is $\phi > 800$. For higher γ a ML is preferred. We increase λ to 4% keeping equal the other parameters. If $T_{ML} = 24$ years a ML is always the most advantageous for any γ. In the other cases we observe a decreasing trend of $A_{ICL} - A_{ML}$ as γ and ϕ are higher. The same results are confirmed if we use a Log utility function. (Table 9)

The increase of the initial income affects strongly the sign of $A_{ICL} - A_{ML}$: keeping constant all the other parameters, for low level of Y_0 $A_{ICL} - A_{ML}$ is positive and the individuals prefer an ICL. Instead if the individuals receive high initial wages $A_{ICL} - A_{ML}$ is negative, and a ML is the most advantageous system. The trend of $A_{ICL} - A_{ML}$ depends on the ratio λ/σ. If λ is low, for increasing level of uncertainty $A_{ICL} - A_{ML}$ is decreasing. Instead, keeping constant all the other parameters, if λ is high, for increasing σ the trend of $A_{ICL} - A_{ML}$ is increasing. The effects of higher λ are more evident when Y_0 is low: an ICL is the most advantageous system when the uncertainty is increasing. When the risk aversion increases the sign of $A_{ICL} - A_{ML}$ and its trend remain unchanged. The risk aversion affects the absolute value of $A_{ICL} - A_{ML}$, which declines sharply when a increases. Also in this case the economic effect is clearer when Y_0 is low: the benefits of an ICL reduce when the risk aversion is higher. For small values of the subjective discount rate, keeping constant all the other parameters, $A_{ICL} - A_{ML}$ increases. When ρ is high the trend of $A_{ICL} - A_{ML}$ is decreasing. Finally, the effect of a reduction of the repayment period under a ML is to increase the average utility under an ICL. It becomes the most advantageous system above all for low and medium level of Y_0. The opposite effect is realized when the repayment rate under an ICL is increased.

10 Conclusion

In this work we exposed a theoretical model of schooling under uncertainty, when graduate earnings are stochastic. Assuming that students can take out a loan to finance their higher education, we compared two repayment schemes: mortgage loan and income contingent loan. The first part of the model is static, since the uncertainty affects only the level of the income, which remains constant all over the working life. The main result found is that for risk neutral individuals the expected costs of education under an income contingent loan are lower than under a mortgage loan scheme. If the individuals are risk averse, an income contingent loan is preferred when the level of uncertainty increasing. The extent of the preference of one system over the other is strongly related to the repayment period. The longest the repayment period is, the most a systems becomes profitable.

The second part of the model is dynamic, we assumed that the growth rate of the income follows a geometric Brownian motion. The uncertainty affects the income each year during the individual working life and not only once. We compared the average utilities under the two financing schemes, developing a numerical iterative method. We find that the level of the initial income strongly affects the preference of one system over the other. If the individuals receive a low initial wage, they prefer an ICL above all for increasing level of uncertainty. Instead, if they are very risk averse, and still getting a low initial wage, a ML becomes more beneficial. The size of the deterministic growth rate is also very important, in fact when this rate is high the individuals find an ICL more advantageous for increasing uncertainty. Finally, reducing the repayment period under a ML makes always an ICL more profitable. One further extension, that we will face in another work, is the case for students to be unable to pay off their loan, and the government can choose to impose a default premium to keep its budget balanced.

A Appendix: Proof Proposition 1

Under risk neutrality equation (2) becomes

$$V(1) = E(\int_s^\infty e^{-\rho t} y \, dt) - E(\int_s^{T+s} e^{-\rho t} R \, dt) \tag{34}$$

So we can compare only the expected costs. Under ML the present value of the cost of size C is:

$$\begin{aligned} PVC_{ML} &= \int_s^{T+s} \varphi \, e^{-\rho t} \, dt \\ &= e^{-\rho s} \, \frac{\varphi}{\rho} \, [1 - e^{-\rho \frac{C}{\varphi}}] \, . \end{aligned} \tag{35}$$

Under ICL the present value of the cost of size C is:

$$\begin{aligned} PVC_{ICL} &= \int_s^{\tilde{T}+s} y \, \gamma \, e^{-\rho t} \, dt \\ &= e^{-\rho s} \, \frac{\gamma \, y}{\rho} \Big[1 - e^{-\rho \frac{C}{\gamma y}} \Big] \quad . \end{aligned} \tag{36}$$

Knowing that $E(y) = 1$, we take the expected value of both the equations above.

$$\begin{aligned} E(PVC_{ML}) &= \frac{\varphi}{\rho} \, [1 - e^{-\rho \frac{C}{\varphi E(y)}} \,] e^{-\rho s} \\ E(PVC_{ICL}) &= E \, \big[\frac{\gamma \, y}{\rho} \, (1 - e^{-\rho \frac{C}{\gamma y}} \,) e^{-\rho s} \, \big] \end{aligned} \tag{37}$$

Assuming that the instalment under a mortgage loan is equal to the repayment rate under an income contingent loan: $\varphi = \gamma$, we can easily observe that the expected values can be written:

$$E(PVC_{ML}) = f[E(y)]$$
$$E(PVC_{ICL}) = E f(y)$$

Since $f(y) = \frac{\gamma \, y}{\rho} \big(1 - e^{-\frac{\rho \, C}{\gamma \, y}} \big) e^{-\rho s}$ is a concave function [10], we obtain that the expected costs under ICL are lower then the expected cost under ML: $E(PVC_{ICL}) < E(PVC_{ML})$. According to equation (34) the expected utility under ICL is higher than the expected utility under ML.

[10] $f''(y) = -\frac{\rho C^2 e^{-\left(s + \frac{C}{\gamma y}\right)\rho}}{\gamma y^3}$. It is reasonable to assume that γ, ρ and C are all greater or equal than zero. Therefore, the second derivative of $f(y)$ is always negative when the shock on income is positive: $f''(y) < 0, \quad \forall \, y > 0$.

B Appendix: Expected Utility with a Mortgage Loan

The Taylor approximation in equation (7) is the following

$$\begin{aligned} E[u(y-\varphi)] &= E\left\{ u\left(1-\varphi\right) + u^{'}\left(1-\varphi\right)\left(y-1\right) + \frac{1}{2}u^{''}\left(1-\varphi\right)\left(y-1\right)^{2} \right\} \\ &= u\left(1-\varphi\right) + u^{'}\left(1-\varphi\right)E(y-1) + \frac{1}{2}u^{''}\left(1-\varphi\right)E(y-1)^{2} \\ &= u\left(1-\varphi\right) + \frac{1}{2}u^{''}\left(1-\varphi\right)\sigma^{2}. \end{aligned} \tag{38}$$

Plugging the equations (7) and (8) in the equation (6), substituting $T = C/\varphi$ and solving the integral, we obtain:

$$\begin{aligned} V_{ML} = &\frac{e^{-\rho s}}{\rho}\left(1-e^{-\frac{\rho C}{\varphi}}\right)\left[u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma_{s}^{2}\right] \\ &+\frac{e^{-\rho s}}{\rho}e^{-\frac{\rho C}{\varphi}}\left[u\left(1\right)+\frac{1}{2}u^{''}\left(1\right)\sigma_{s}^{2}\right]. \end{aligned} \tag{39}$$

Finally, substituting the CARA and CRRA utility functions in equation(39) and simplifying we get equations (9) and (10).

C Appendix: Expected Utility with an Income Contingent Loan

In Section (4.2) we defined a new function $g(y)$ as:

$$g(y) = \left[1-e^{-\frac{\rho C}{\gamma y}}\right]u\left[y\left(1-\gamma\right)\right]+\left[e^{-\frac{\rho C}{\gamma y}}\right]u\left(y\right) \tag{40}$$

We rewrite the equation (12)

$$V_{ICL} = \frac{e^{-\rho s}}{\rho}E[g(y)] \tag{41}$$

and we apply a second order Taylor expansion to $E[g(y)]$, around the mean $E\left[y\right]=1$, then:

$$
\begin{aligned}
E[g(y)] &= E\left\{g(1)+g'(1)(y-1)+g''(1)\frac{(y-1)^2}{2}\right\} \\
&= g(1)+g'(1)E(y-1)+\frac{g''(1)}{2}E\left(y-1\right)^2 \\
&= g(1)+g''(1)\frac{\sigma_s^2}{2}.
\end{aligned}
\tag{42}
$$

The equation (41) becomes

$$
V_{ICL}=\frac{e^{-\rho s}}{\rho}\left[g(1)+g''(1)\frac{\sigma_s^2}{2}\right] \tag{43}
$$

From now on we follow this procedure:

1. we work out the value of g(1), in general and with a CARA and CRRA utility functions;
2. we work out the first derivative and the second derivative of g(y), both in general and with a CARA and CRRA utility functions;
3. we calculate $g'(1)$ and $g''(1)$ using both CARA and CRRA utility function;
4. we substitute the equations of $g(1)$ and $g''(1)$, using a CARA and CRRA utility function, in the equation (43) and we obtain equations (53), (14), (15).

- **Value of g(1)**

  In general,

$$
g(1)=\left[1-e^{-\frac{\rho C}{\gamma}}\right]u\left[(1-\gamma)\right]+\left[e^{-\frac{\rho C}{\gamma}}\right]u\left(1\right) \tag{44}
$$

Using a CARA utility function we have

$$
g(1)_{CARA}=-\frac{1}{a}e^{-\frac{a\gamma+\rho C}{\gamma}}\left[1-e^{a\gamma}+e^{\frac{a\gamma^2+\rho C}{\gamma}}\right] \tag{45}
$$

If we use a CRRA utility function we have

$$g(1)_{CRRA} = \frac{1}{b}[-e^{-\frac{\rho C}{\gamma}}((1-\gamma)^b - 1) + (1-\gamma)^b]. \tag{46}$$

- **Value of g'(y)**

  In general,

$$\begin{aligned} g'(y) = {} & u'[y(1-\gamma)](1-\gamma)\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + u[y(1-\gamma)]\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] \\ & + u'(y)[e^{-\frac{\rho C}{\gamma y}}] + u(y)\left[\frac{\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] \end{aligned} \tag{47}$$

using a CARA utility function:

$$\begin{aligned} g'(y)_{CARA} = {} & (1-\gamma)e^{-ay(1-\gamma)}\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + \left[\frac{\rho C e^{-ay(1-\gamma)-\frac{\rho C}{\gamma y}}}{a\gamma y^2}\right] \\ & - \frac{1}{a}e^{-ay-\frac{\rho C}{\gamma y}}\left(\frac{\rho C}{\gamma y^2} - a\right) \end{aligned} \tag{48}$$

using a CRRA utility function:

$$\begin{aligned} g'(y)_{CRRA} = {} & (y(1-\gamma))^{b-1}(1-\gamma)\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + (y(1-\gamma))^b\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma y^2}\right] \\ & + y^{b-1}[e^{-\frac{\rho C}{\gamma y}}] + \left[\frac{y^{b-2}\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma}\right]. \end{aligned} \tag{49}$$

- **Value of g''(y)**

$$\begin{aligned} g''(y) = {} & \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(2\gamma y - \rho C)}{y^4\gamma^2}u[y(1-\gamma)] + \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(-2\gamma y + \rho C)}{y^4\gamma^2}u(y) \\ & - \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C(1-\gamma)}{y^2\gamma}u'[y(1-\gamma)] + \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C}{y^2\gamma}u'(y) \\ & + \left[1 - e^{-\frac{\rho C}{\gamma y}}\right](1-\gamma)^2u''[y(1-\gamma)] + \left[e^{-\frac{\rho C}{\gamma y}}\right]u''(y). \end{aligned} \tag{50}$$

Now we work out $g''(y)$ using a CARA utility function and in $y = 1$

$$g''(1)_{CARA} = \frac{e^{-\frac{a\gamma+\rho C}{\gamma}}}{a\gamma^2}\{-a^2[1 - e^{a\gamma}(\gamma - 1^2 + e^{a\gamma+\frac{\rho C}{\gamma}}(\gamma - 1)^2]\gamma^2 \\ + 2ac[1 + e^{a\gamma}(\gamma - 1)]\rho\gamma + \rho C(e^{a\gamma} - 1)(\rho C - 2\gamma)\}. \tag{51}$$

Using a CRRA and evaluating in $y = 1$

$$g''(1)_{CRRA} = \frac{1}{b\gamma^2}\{e^{-\frac{\rho C}{\gamma}}[(b-1)b\gamma^2[1 + (e^{\frac{\rho C}{\gamma y}} - 1)(1-\gamma)^b] \\ + 2\rho C(b-1)\gamma(1 - (1-\gamma)^b) + C^2\rho^2(1 - (1-\gamma)^b)]\}. \tag{52}$$

- **Results**

Substituting $g(1)$ and $g''(1)$ in equation (43) we get the general expected utility under an income contingent loan:

$$V_{ICL} = \left[1 - e^{-\frac{\rho C}{\gamma}}\right] u\left[(1-\gamma)\right] + \left[e^{-\frac{\rho C}{\gamma}}\right] u\left(1\right) \\ + [\frac{e^{-\frac{\rho C}{\gamma}}\rho C(2\gamma - \rho c)}{\gamma^2} u[1-\gamma] + \frac{e^{-\frac{\rho C}{\gamma}}\rho C(-2\gamma + \rho c)}{\gamma^2} u(1) \\ - \frac{2e^{-\frac{\rho C}{\gamma}}\rho C(1-\gamma)}{\gamma} u'[1-\gamma] + \frac{2e^{-\frac{\rho C}{\gamma}}\rho C}{\gamma} u'(1) \\ + \left[1 - e^{-\frac{\rho C}{\gamma}}\right](1-\gamma)^2 u''[1-\gamma] + \left[e^{-\frac{\rho C}{\gamma}}\right] u''(1)]\frac{\sigma_s^2}{2}. \tag{53}$$

Substituting in equation (43) the equations for $g(1)$ and $g''(1)$ with CARA and CRRA utility functions, we obtain equations (14)) and (15).

Table 1: $V_{ICL} - V_{ML}$ - CRRA - Uncertainty and Risk Aversion

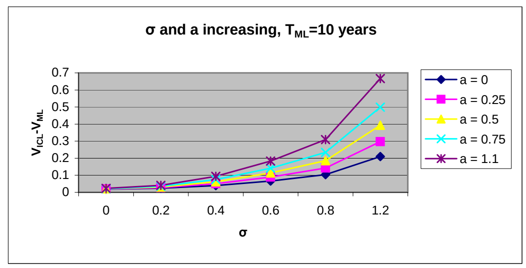
C = £ 10000 φ = £1000 T_{ML} = 10 ρ = 8% γ = 9%

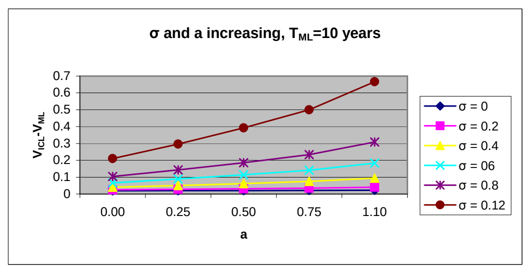
	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.00	0.0188	0.0241	0.04	0.0665	0.1037	0.2098
0.25	0.0197	0.0274	0.0504	0.0889	0.1427	0.2966
0.50	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.75	0.0215	0.0348	0.0746	0.1410	0.2339	0.4993
1.10	0.0229	0.0408	0.0944	0.1838	0.3089	0.6664

σ = 0 σ = 0.2 σ = 0.4 σ = 06 σ = 0.8 σ = 0.12

C = £ 10000 φ = £500 T_{ML} = 20 ρ = 8% γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.00	-0.1189	-0.1136	-0.0976	-0.0711	-0.0339	0.0722
0.25	-0.1222	-0.1156	-0.0959	-0.0630	-0.0170	0.1144
0.50	-0.1256	-0.118	-0.0952	-0.0574	-0.0043	0.1472
0.75	-0.1290	-0.1207	-0.0958	-0.0544	0.0037	0.1696
1.10	-0.1340	-0.1252	-0.0989	-0.0550	0.0064	0.1817





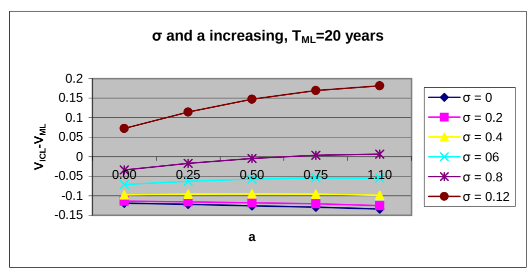
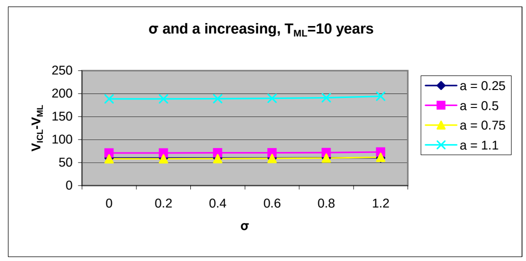


Table 2: $V_{ICL} - V_{ML}$ - CARA - Uncertainty and Risk Aversion

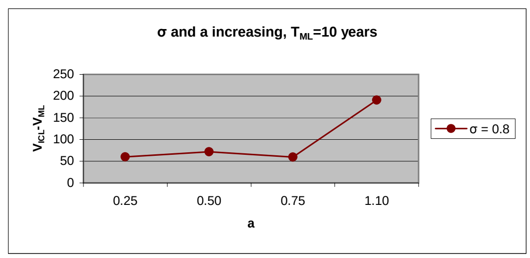
C = £ 10000 φ = £1000 T_{ML} = 10 ρ = 8% γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.25	59.749	59.767	59.823	59.917	60.048	60.421
0.50	71.050	71.098	71.244	71.485	71.824	72.791
0.75	58.260	58.350	58.619	59.068	59.697	61.492
1.10	188.383	188.544	189.028	189.835	190.963	194.189



C = £ 10000 φ = £500 T_{ML} = 20 ρ = 8% γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.25	59.6069	59.6244	59.6771	59.7649	59.8878	60.239
0.50	70.9039	70.9495	71.0863	71.3143	71.6335	72.5456
0.75	58.1099	58.1946	58.449	58.8729	59.4664	61.162
1.10	188.2261	188.3783	188.8348	189.5956	190.6608	193.7043



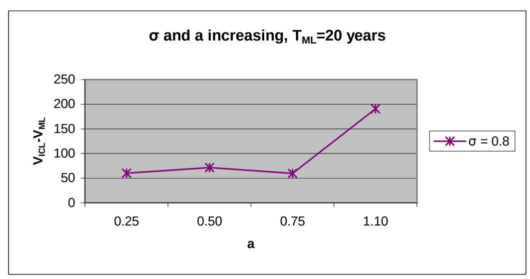
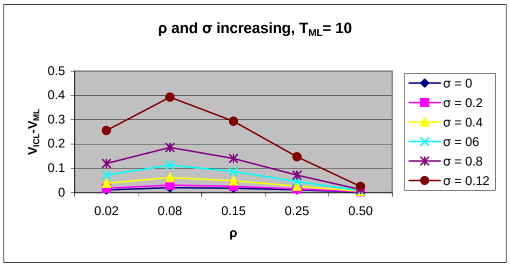


Table 3: $V_{ICL} - V_{ML}$ - CRRA - Uncertainty and Subjective Discount Factor

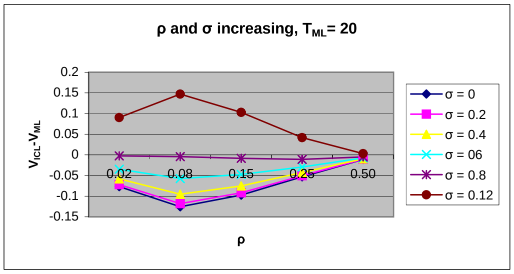
C = £ 10000 φ = £1000 T_{ML} = 10 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	0.0114	0.0182	0.0386	0.0726	0.1201	0.2559
0.08	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.15	0.0183	0.0260	0.0489	0.0872	0.1407	0.2937
0.25	0.0115	0.0153	0.0266	0.0455	0.0720	0.1476
0.50	0.0028	0.0034	0.0053	0.0085	0.0129	0.0256



C = £ 10000 φ = £500 T_{ML} = 20 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-0.0767	-0.072	-0.0581	-0.0349	-0.0025	0.0903
0.08	-0.1256	-0.118	-0.0952	-0.0574	-0.0043	0.1472
0.15	-0.0973	-0.0918	-0.0751	-0.0473	-0.0083	0.1029
0.25	-0.0531	-0.0505	-0.0426	-0.0294	-0.0109	0.0418
0.50	-0.0111	-0.0107	-0.0096	-0.0076	-0.0049	0.0029



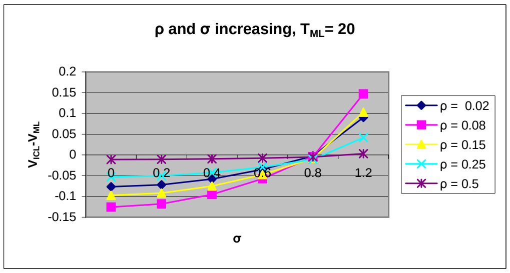
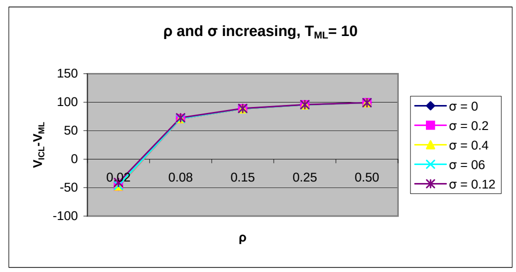


Table 4: $V_{ICL} - V_{ML}$ - CARA - Uncertainty and Subjective Discount Factor

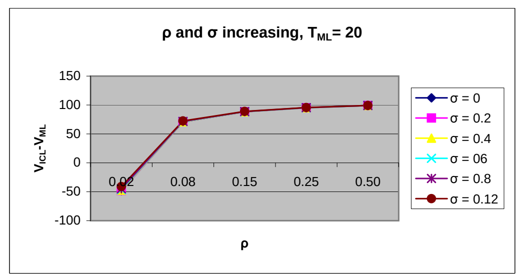
C = £ 10000   φ = £1000 T_{ML} = 10   a = 0.5   γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-47.956	-47.760	-47.170	-46.186	-44.809	-40.876
0.08	71.050	71.098	71.244	71.485	71.824	72.791
0.15	88.370	88.393	88.461	88.574	88.732	89.183
0.25	95.334	95.343	95.373	95.421	95.489	95.684
0.50	99.143	99.145	99.150	99.159	99.171	99.207



C = £ 10000   φ = £500 T_{ML} = 20   a = 0.5   γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-48.0444	-47.8499	-47.2663	-46.2937	-44.932	-41.0415
0.08	70.9039	70.9495	71.0863	71.3143	71.6335	72.5456
0.15	88.2546	88.2751	88.3366	88.4391	88.5826	88.9926
0.25	95.2689	95.2775	95.3033	95.3463	95.4064	95.5783
0.50	99.129	99.1305	99.1351	99.1427	99.1535	99.1841



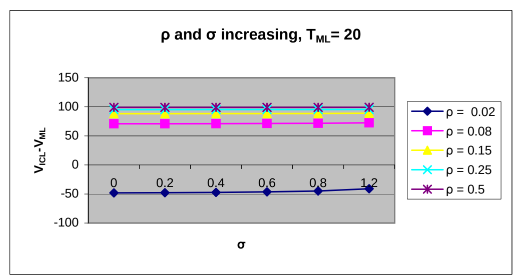
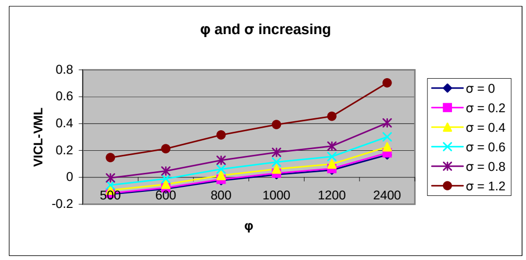


Table 5: $V_{ICL} - V_{ML}$ - CRRA - ICL and ML Parameters

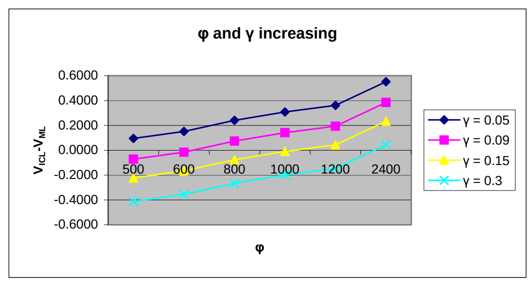
C = £ 10000 a = 0.5 ρ = 8% γ = 9%

	σ					
φ	0	0.2	0.4	0.6	0.8	1.2
500	-0.1256	-0.1180	-0.0952	-0.0574	-0.0043	0.1472
600	-0.0852	-0.0769	-0.0520	-0.0106	0.0474	0.2131
800	-0.0238	-0.0144	0.0139	0.0610	0.1270	0.3156
1000	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
1200	0.0545	0.0656	0.0990	0.1546	0.2324	0.4548
2400	0.1674	0.1823	0.2270	0.3014	0.4056	0.7034



C = £ 12000 a = 0.5 σ = 0.6 ρ = 8%

	γ			
φ	0.05	0.09	0.15	0.3
500	0.0948	-0.0720	-0.2226	-0.4118
600	0.1516	-0.0153	-0.1658	-0.3550
800	0.2409	0.0740	-0.0765	-0.2658
1000	0.3080	0.1412	-0.0094	-0.1986
1200	0.3609	0.1940	0.0435	-0.1458
2400	0.5514	0.3845	0.2340	0.0447



C = £ 10000 φ = £ 1000 T_{ML} = 10 a = 0.5 ρ = 8%

	σ					
γ	0	0.2	0.4	0.6	0.8	1.2
0.05	0.1461	0.1583	0.1948	0.2557	0.3409	0.5844
0.09	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.15	-0.0726	-0.0653	-0.0433	-0.0065	0.0449	0.1919
0.30	-0.1811	-0.1779	-0.1685	-0.1528	-0.1307	-0.0678

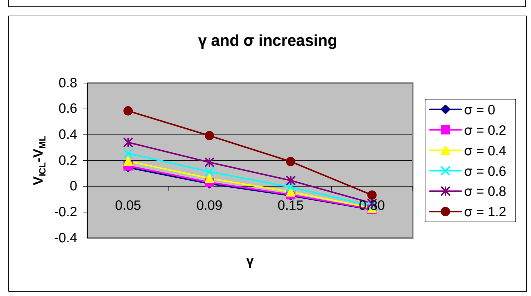
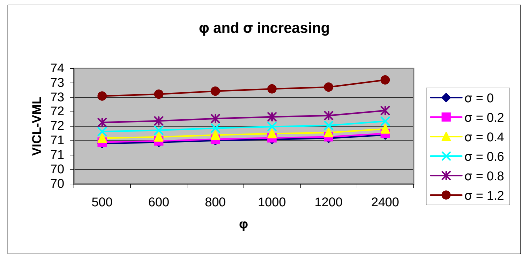


Table 6: $V_{ICL} - V_{ML}$ - CARA - ICL and ML Parameters

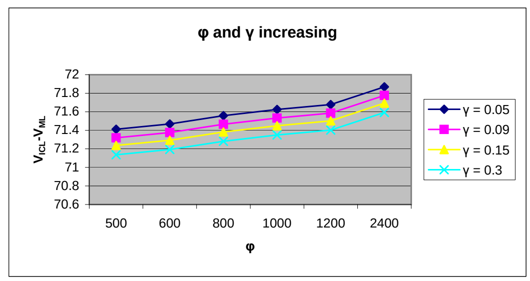
C = £ 10000 a = 0.5 ρ = 8% γ = 9%

	σ					
φ	0	0.2	0.4	0.6	0.8	1.2
500	70.904	70.950	71.086	71.314	71.634	72.546
600	70.944	70.991	71.130	71.361	71.685	72.611
800	71.006	71.053	71.195	71.433	71.765	72.714
1000	71.050	71.098	71.244	71.485	71.824	72.791
1200	71.084	71.133	71.281	71.526	71.870	72.853
2400	71.197	71.250	71.409	71.673	72.044	73.102



C = £ 12000 a = 0.5 σ = 0.6 ρ = 8%

	γ			
φ	0.05	0.09	0.15	0.3
500	71.412	71.3198	71.2366	71.1362
600	71.4687	71.3765	71.2933	71.1929
800	71.558	71.4658	71.3826	71.2822
1000	71.6252	71.533	71.4498	71.3494
1200	71.678	71.5858	71.5026	71.4022
2400	71.8685	71.7763	71.6931	71.5927



C = £ 10000 φ = £ 1000 T_{ML} = 10 a = 0.5 ρ = 8%

	σ					
γ	0	0.2	0.4	0.6	0.8	1.2
0.05	71.126	71.175	71.321	71.564	71.905	72.878
0.09	71.050	71.098	71.244	71.485	71.824	72.791
0.15	70.994	71.041	71.183	71.419	71.749	72.694
0.30	70.931	70.977	71.114	71.342	71.661	72.573

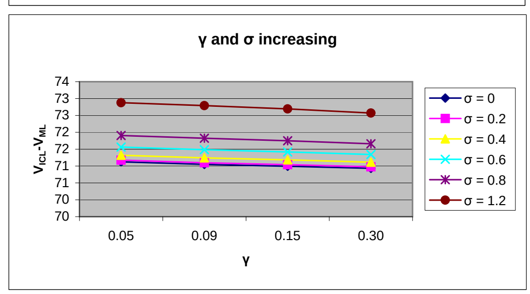
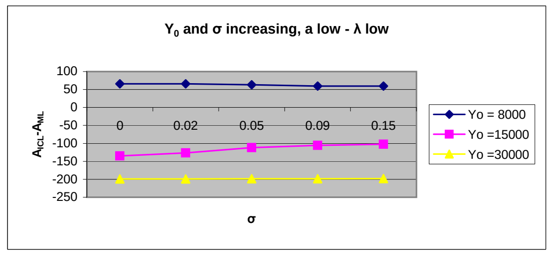


Table 7: $A_{ICL} - A_{ML}$ - Risk Aversion and Deterministic Growth

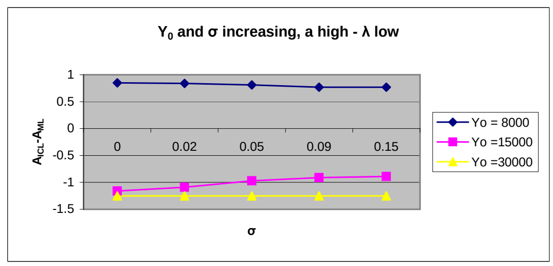
a = 0.25 φ = £1000 ρ = 8% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	65.94	66.06	63.19	59.62	59.41
15000	-134.92	-126.37	-111.66	-105.49	-102.42
30000	-198.84	-198.79	-198.7	-198.55	-198.23



a = 0.75 φ = £1000 ρ = 8% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	0.85	0.84	0.81	0.77	0.77
15000	-1.16	-1.09	-0.97	-0.91	-0.89
30000	-1.25	-1.25	-1.25	-1.25	-1.25



a = 0.25 φ = £1000 ρ = 8% λ = 4%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	7.59	16.22	23.68	26.2	27.48
15000	-128.48	-128.42	-128.29	-128.08	-127.08
30000	-245.57	-245.53	-245.44	-245.29	-244.99

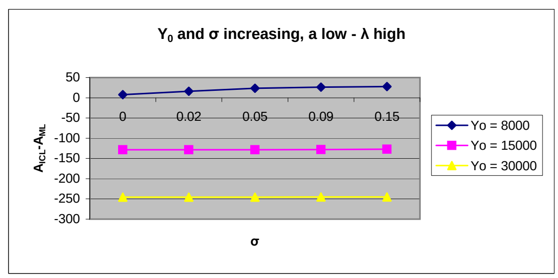
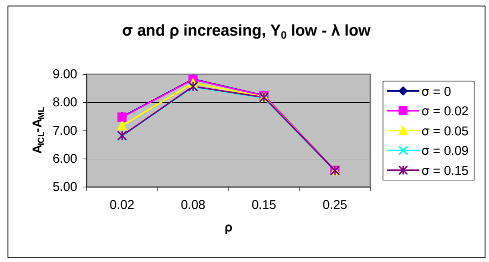


Table 8: $A_{ICL} - A_{ML}$ - Uncertainty and Subjective Discount Factor

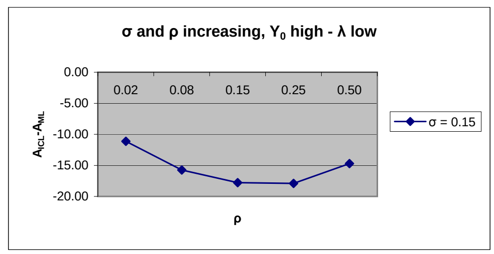
a = 0.5 Y_0 = £ 8000 φ = £1000 λ = 1% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	2.68	2.66	1.99	1.17	1.11
0.08	7.49	7.48	7.18	6.81	6.82
0.15	8.83	8.82	8.69	8.55	8.57
0.25	8.25	8.24	8.20	8.16	8.18
0.50	5.59	5.59	5.58	5.58	5.58



a = 0.5 Y_0 = £ 30000 φ = £1000 λ = 1% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	-11.12	-11.13	-11.13	-11.13	-11.12
0.08	-15.75	-15.76	-15.76	-15.76	-15.75
0.15	-17.79	-17.79	-17.80	-17.80	-17.79
0.25	-17.91	-17.91	-17.91	-17.91	-17.91
0.50	-14.69	-14.69	-14.69	-14.70	-14.70



a = 0.5 Y_0 = £ 8000 φ = £1000 λ = 4% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	-4.32	-2.70	-1.32	-0.89	-0.64
0.08	1.63	2.45	3.14	3.36	3.50
0.15	4.33	4.71	5.03	5.14	5.21
0.25	5.19	5.33	5.45	5.48	5.52
0.50	4.24	4.25	4.26	4.27	4.27

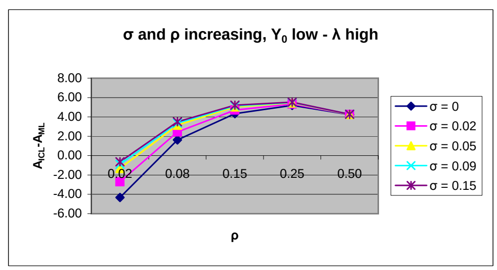


Table 9: $A_{ICL} - A_{ML}$ - ICL and ML Parameters

a = 0.5 φ = £ 500 T_{ML} = 24 ρ = 8% γ = 9% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	-19.65	-18.91	-18.58	-18.50	-18.40
15000	-31.50	-31.50	-31.51	-31.36	-30.96
30000	-35.97	-35.98	-35.98	-35.98	-35.86

a = 0.5 φ = £ 2400 T_{ML} =5 ρ = 8% γ = 9% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	38.08	38.82	39.15	39.22	39.30
15000	7.61	7.60	7.60	7.74	8.12
30000	-9.40	-9.40	-9.41	-9.41	-9.30

a = 0.5 σ =5% ρ = 8% Y_0 = 8000 λ = 1%

	γ			
φ	0.05	0.09	0.15	0.3
500	4.64	-18.58	-40.07	-62.29
600	11.93	-11.29	-32.78	-55.00
800	23.42	0.20	-21.29	-43.51
1000	26.19	7.18	-8.22	-39.75
1200	38.81	15.59	-5.90	-28.12
2400	62.37	39.15	17.66	-4.57

a = 0.5 σ =5% ρ = 8% Y_0 = 8000 λ = 4%

	γ			
φ	0.05	0.09	0.15	0.3
500	-1.61	-22.52	-44.04	-72.18
600	5.52	-15.39	-36.91	-65.06
800	17.02	-3.90	-25.42	-53.56
1000	21.67	3.14	-15.75	-30.60
1200	32.79	11.87	-9.64	-37.79
2400	57.30	36.38	14.87	-13.28

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