

Firm Training and Wage Rigidity

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Abstract

Based on overwhelming evidence suggesting that wages are in practice downwards rigid, we impose the restriction that wages of the current period are not allowed to be smaller than in the preceding period, whereas otherwise wages are negotiated freely via standard Nash-bargaining.

Thereby we are able to analyze the consequences of downwards wage rigidity on the determination of wages, on the degree of wage-compression and on firm's training investments. It turns out, that wage rigidity will increase wage-compression. But contrary to the work of Acemoglu this is not sufficient to increase firms' training investments. The reason lies in the endogeneity of separations.

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1 Introduction

Wage-rigidity is receiving more and more attention, especially in the RBC- and the empirical literature. It can be seen as a powerful device to bring in line the variability of vacancies and unemployment in RBC-models with the empirical facts - models without rigidity usually produce unemployment-fluctuations which are way too low, because wages are adjusting too fast to macroeconomic shocks. Concerning the empirical literature evidence on wage-rigidity comes from three different brands: econometrics, surveys and experiments. Broadly speaking, all of them agree that there is a considerable amount of wage-rigidity, especially when it comes to downwards movements.

This work is building on the idea that wage-rigidity will have effects on the wage-structure of an economy, potentially creating or increasing wage-compression.¹ According to the influential work of Acemoglu² wage-compression will induce firms to invest in the human capital of their workers because they can reap some of the returns to training. This is in contrast to the traditional theory on human capital based on Becker (1962). Becker argued that a firm will never pay for a worker's general human capital³ because this kind of training would increase wages one-to-one. But at the same time workers would invest themselves in their human capital and they would do so efficiently because they are the only beneficiaries. This is another difference to the models of Acemoglu where training-investments are inefficiently low due to various externalities.⁴

As will be discussed in the following section, the evidence on firm training and wage compression is in rare unison: firms are paying even for general training and wages are indeed compressed. But there is a field in which the evidence is not so clear: minimum wages. As discussed in Acemoglu and Pischke (2002) and described in more detail in

¹A compressed wage-structure means that when the productivity of a worker is improved, the wage of that worker will increase by less than productivity.

²See for instance, Acemoglu (1997) and Acemoglu/Pischke (1999a).

³General human capital - as opposed to specific human capital - can be used without any restrictions in other firms.

⁴On future employers or on the workers.

the section on empirical literature below, the evidence pointing to a positive relationship between minimum wages and firm training⁵ is rather poor. This has lead Acemoglu and Pischke (2002) to adjust their models to bring back in line theory and evidence. In this paper we are trying to give an alternative explanation for a possibly negative relationship between inflexible wages and firm training.

In most aspects, our model is very similar to the models of Acemoglu, for instance there are two periods, only firms can invest in training, the first period is the training period etc. However, there are two important differences: the way in which wages are determined and the endogeneity of separations. We are sticking to the usual assumption of Nash-bargaining but add the restriction of nominal wage-rigidity. We are able to show that wage-rigidity can indeed lead to a higher degree of wage-compression by altering the way wages are determined. Nevertheless, contrary to the models of Acemoglu this is not sufficient to improve firm-training. This result is created by the effect of rigidity on separations which become more frequent, as is confirmed by the empirical evidence. In Acemoglu's models separations take place at an exogenous rate and therefore this effect is ruled out.

The remainder of the paper is organized as follows. In the proceeding chapter the empirical literature on both, wage-rigidity and firm training will be discussed. Then we illustrate our own wage mechanism and compare it to other rules used in the theoretical literature. In section five we will present the general framework of the model before we outline a benchmark model in which wages are determined without any restriction. Chapter six discusses the model with rigid wages and chapter seven concludes.

2 Empirical Literature

Baker, Gibs and Holmstrom (1994) are using records of all management employees of a medium-sized US firm in a service industry over the period of 1969 to 1988. Excluding

⁵As suggested by the their models because minimum wages increase wage compression.

foreign employees⁶ they arrive at a sample of 62.957 observations and try to determine the firms' wage policy. They report that nominal wage-cuts are extremely rare: overall they observe less than 200 cases which is less than 0.32 % of all observations. On the other hand, zero nominal changes can be quite frequent, reaching up to more than 15% of observations in the year 1977. Real wage cuts are much more frequent than nominal wage cuts: for instance 15% of workers of the 1975 cohort are receiving a lower real wage in 1985 than their starting wage. Not very surprisingly, real wage cuts are especially frequent in years of high inflation: in the high-inflation years around 1980 40% of workers had to suffer declines in their real wages.

Fehr and Götte (2000) do a similar exercise for two Swiss firms (large and medium-sized) in the service industry.⁷ In the large firm only 1.7 % of 35.779 observations are wage cuts. These are even less frequent in midium sized firm: 0.4 % of 20.236 observations. Fehr and Götte are not only using this firm-data but as well cross-sectional data from the Swiss Labor Force Survey and Social Insurance Files. Controlling for measurement error, they find that at most 5 % of workers are receiving wage cuts, while for more than 50% of workers nominal rigidity is preventing wage cuts. This wage rigidity does not vanish during periods of low GDP-growth.

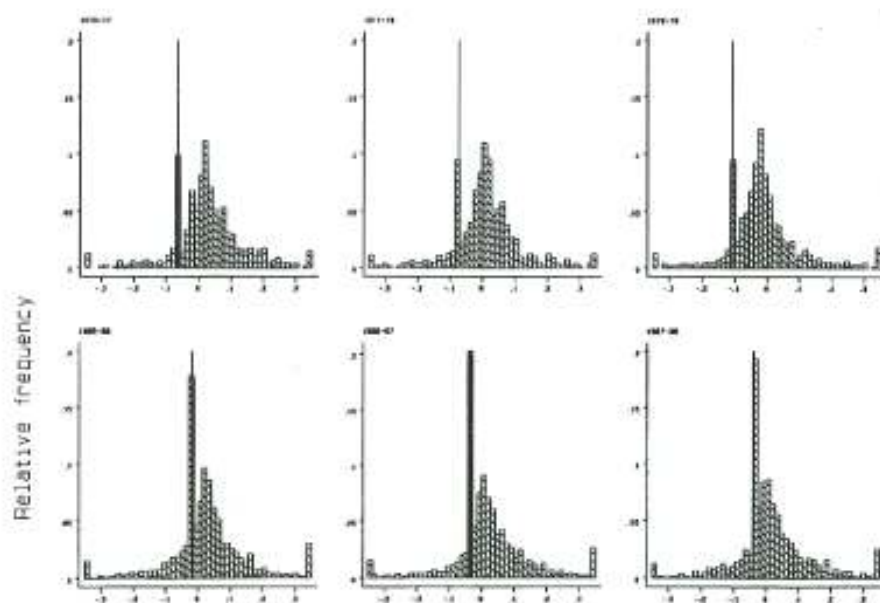
Card and Hyslop (1997) are using the US Panel Study of Income Dynamics. Figure (1) is illustrating their results for the years of relatively high inflation 1976-1979 and the years of low inflation 1985-1988. Again there is sharp peak at zero nominal wage changes and nominal wage cuts are quite rare while real wage cuts are more frequent. The comparison of the periods of high and low inflation makes clear that nominal-wage rigidity is especially important during times of low inflation: Zero nominal wage changes are even more frequent. Combined with a low rate of inflation this preventing real wage cuts as well.

Evidence of cross-sectional studies is not so clear as it might seem: Parkin (2001)

⁶Due to limited comparability.

⁷See Beissinger/Knoppik (2001) for a study on Germany.

Figure 1: Log real wage changes



surveys ten panel studies and finds that between ten and twenty percent of wage changes are negative. However, reporting errors seem to be quite important in these studies, leading to overstatements of the frequency of wage cuts.

A more detailed review of the econometric literature on wage rigidities can be found in Malcomson (1999) or Howitt (2002).

A different kind of evidence comes from Bewley (1999 and 2002) who was asking US managers directly why they behave the way they do. He finds an unusual deal of consensus that the most important factor inhibiting wage cuts is the fear that this might be interpreted as a hostile act and that this would lead to lower morale in the workforce, thus decreasing effort and productivity. For the same reason, firms do not replace workers by unemployed who would be willing to work for less. On the other hand, managers are less reluctant to fire workers during a recession to improve productivity and profits. Although this will clearly lower the morale of the fired worker, she is no longer in the firm to spread the bad morale to other workers. A similar study has been done by Agell and

Benmmarker (2003) for Sweden. They find as well that the morale of the work force is an important obstacle to nominal wage cuts. Besides worker morale, the legal framework and institutions (unions) are important factors.⁸

Finally, a third piece of evidence on wage rigidity comes from experimental studies on reciprocal behavior. For instance, Fehr and Falk (1999) find that firms are not willing to hire underbidders and that workers who accept lower wages also provide lower effort if effort cannot be contracted. This is clearly confirming the views of managers as reported by Bewley. However, as soon as effort can be contracted, underbidders are no longer refused.⁹

Lets turn to the relationship between firm training and wages that has been the subject of numerous studies. Loewenstein/Spletzer (1998 and 1999) find for the US that firms frequently pay for general training, that general training usually increases productivity by more than wages and that training is translated into higher wages if it was provided by a previous and not the current employer. This clearly suggests that wages are compressed and that training is general to a large degree - otherwise future employers would not pay higher wages. At the same time it can be neglected that workers are paying indirectly for the training through receiving lower starting wages. Similar results are derived by Barron et al (1999) for the US, by Booth/Bryan (2002) for the UK and by Gerfin (2003a and 2003b) for Switzerland.

A more direct test of the Acemoglu and Pischke model described in the introduction is provided by Bassanini/Brunello (2003). Using the European Community Household Panel (ECHP) they find a positive relationship between wage-compression and training.

To my knowledge there are no empirical studies directly relating the incidence of wage-rigidity and firm training. However, when it comes to minimum wages¹⁰ the evidence is

⁸See as well Agell/Lundborg (1995).

⁹For other experiments on reciprocal behavior see for instance Camerer/Thaler (1995) or Falk, Gächter and Kovacs (1998).

¹⁰Which will in part work similar as wage-rigidities because both are preventing the wage from decreasing.

not so clear: Acemoglu/Pischke (2002) find no evidence that minimum wages reduce training but as well little evidence that they increase training. Therefore they create a new model in which workers are willing to accept wage cuts in order to finance part of the training. This might be prevented by the minimum wage and lead to a decrease in training. Thereby, we have two countervailing effects on training investments: on the one hand the compression on the wages structure is increased, but on the other hand workers cannot finance training through lower wages. Our own work might as well shed some light on this discussion: we are showing that things might change considerably if one is allowing for endogenous separations.

3 Wage-setting

Since the wage-setting mechanism is the main feature of our model, this section discusses it in detail in and compares it to other approaches in the literature. We assume that wages are determined according to Nash-bargaining¹¹ but with the restriction that wages cannot decline from one period to the other, so that the wage of the prior period is the minimum for the current period. Besides that, the wage of the prior period has no influence on the outcome of the bargaining, thus that in fact the wage of the prior period is acting as an outside option. This can be motivated by arguing - in line with the empirical literature discussed above - that a decrease in the wage will be interpreted by the worker as a hostile act leading to the provision of zero effort. The firm foreseeing this "option" will never pay a lower wage than the one it has paid in a previous period. This is clearly in line with the evidence of surveys and experiments discussed above.

Strand (2003) has modelled this idea more explicitly and put it into the framework of an efficiency-wage model developing a synthesis between wage-bargaining ala Nash and efficiency wages. The firm sets the required effort in order to maximize its profits. Then wages are negotiated according to Nash-bargaining. However, if this would lead

¹¹See for instance Shaked/Sutton (1984).

to a wage, which is too low to assure that workers do not shirk, the firm will pay at least the non-shirking wage. In that way the efficiency wage acts as a minimum wage to the Nash-bargaining. In our model the wage of the last period plays the role of this efficiency minimum-wage - whenever the worker is receiving less than in the past the firm will expect her to shirk and therefore it will never lower the wage. In order to keep the model simple¹² we omit this micro-foundation and simply assume, that the previous wage acts as a minimum.

Especially in the RBC-literature there can be found many other approaches to model wage rigidity. In this literature rigidity is used to amplify fluctuations of unemployment and vacancies over the business cycle because flexible wages usually adjust too quickly to shocks. Consequently, the variability of unemployment and vacancies is too low to match the empirical facts. The most simple method was chosen by Shimer (2004) who simply assumes that the wage is totally constant. By using this extreme rule he can demonstrate that a rigid wage can amplify the fluctuations of unemployment and vacancies sufficiently to better match the empirical facts. Hall (2003) also sets the wage constant but allows renegotiations in case a threat point of the two parties is violated, thus avoiding inefficient separations. Krause/Lubik (2003) stick to Nash-bargaining but with the modification that the actual wage is a weighted average of the Nash-wage and a wage norm. Another frequently used approach is the wage-staggering introduced by Taylor (1979) according to which wages are fixed for some periods until they can be renegotiated.

Danthine/Kurmann (2004a) try to incorporate rigid wages in an efficiency-wage model. However, this approach is rather ad hoc since it is simply assumed that the previous wage reduces the effort of the worker whereas the current wage improves it. Consequently, firms are more reluctant to lower wages because this will reduce effort. More funded appears the approach in Danthine/Kurmann (2004b). The authors stress that wage rigidity can be constructed in efficiency-wage models quite easily by moving the wage reference from external to internal. Usually the wage reference is assumed to be external¹³ and wages will

¹²Actually the focus is on firm training and not on wage bargaining.

¹³For instance related to average earnings of the work force or unemployment benefits.

respond quickly to macroeconomic shocks. In contrast, Danthine and Kurmann suggest the firm's earnings per worker as wage reference. They demonstrate that this is sufficient to create a considerable amount of wage rigidity.

One common drawback of all the approaches discussed above is, that they are creating rigidity in both directions: upwards and downwards. This is clearly ad odds with the empirical literature showing that rigidity is mainly restricting downwards movements. It is an advantage of the approach used in this paper that it is only preventing downwards adjustments of wages without affecting upwards movements. Additionally, it is able to create the large mass of zero-changes observed by the econometric literature. Another drawback of the efficiency wage literature is, that it is contradicting the evidence found by Bewley (2002). He argues that monetary incentives are only important when it comes to wage-cuts. If the management decides to increase the wage of its employees, they will improve their effort only temporarily. By the time they will get used to the higher level of wages and perceive that they have earned it. Consequently their effort will go back to normal. To the contrary, wage cuts have permanent effects on the morale of the workforce and therefore they are avoided. Our model is clearly better able to take account of that fact.

Another motivation to stick to Nash-bargainig is that it is most commonly used in the literature on firm training and specifically in the most influential work by Acemoglu and Pischke. Thereby we are able to allow direct comparisons between our model and the related literature.

We believe that Nash-bargainig is predominant in the training literature not without good reason. It seems quite arbitrary to model training in an efficiency-wage model: how should this training affect the effort of the worker. If one believes that the worker will interpret training as sign of kindness and react by providing more effort this might lead to wages declining with human-capital which is clearly at odds with empirical findings. To my knowledge the only work directly incorporating firm training and efficiency wages is Katsimi (2003). In this model training will bind the worker and the firm more closely

together: the worker knows that the firm will be more reluctant to fire her in case of recessions. This is giving more credibility to the threat of firing as punishment to shirking. Consequently, a lower wage will be sufficient to fulfill the shirking condition. Thus this model is creating a counterfactual negative relationship between training and wages.

4 General structure of the Model

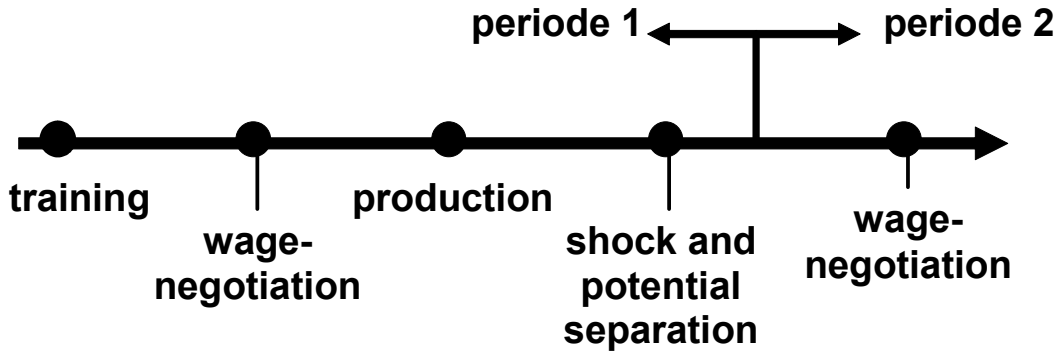
In the background of the model lies the idea of a labor-market described by a standard matching function,¹⁴ such that unemployed workers searching for a new job are facing a certain probability to be successful that is dependent on the tightness of the labor market. Tightness is defined as the relation of searching workers and posted vacancies, so that an increase in labor market tightness implies a better chance to find a job for each worker. Of course the opposite is true for firms which are posting a vacancy: the higher the tightness of the market, the lower the chances to find a worker. However, it is not necessary to explicitly model the labor market to draw meaningful conclusions. To keep things as simple as possible I will therefore not describe in detail the value of a vacancy and the value of unemployment.

Thus we are concentrating at the pair of a firm and a worker who have already met. The firm is employing only one worker. After two periods the relationship will be terminated and both parties return to the labor market. I could as well assume that the worker will die after this second period. However, since this does not change the results in any way but does only complicate the wage functions, I will not do so.

At the beginning of the first period, the firm has the possibility to train the worker, which will improve her productivity y instantaneously. In discrete-time models featuring firm-training it is usually assumed that the first period is the training period and productivity is improved only in the second period. However, in order to give a meaningful interpretation to the idea of wage rigidity, it is necessary that two wage-negotiations take

¹⁴See Pissarides (2000).

Figure 2: Time path



place after the training-decision. Otherwise the wage rigidity would just be equivalent to a minimum wage and its effect on wage compression via altering the way wages are determined could not be analyzed. Wages are negotiated after the training decision but still at the beginning of the first period, i.e. before production takes place.¹⁵¹⁶

At the end of period one the pair is hit by an idiosyncratic shock π with distribution $f(\pi)$ which is added to the productivity of the worker so that the output of period two equals $y + \pi$. Thus we are able to endogenize the separation decision: the match will only be destroyed in the case of shocks that are bad enough. At the beginning of period two both parties can search for a new job in case of a separation. If they are staying together, the wage will be newly negotiated, but now with the restriction that the wage cannot be below the wage of the prior period. The timing of important events is illustrated in figure (2).

To better illustrate the effects of wage rigidity we will first describe a benchmark model, in which wages are negotiated without any restriction.

¹⁵The timing of wage-negotiations is common in the training-literature - see for instance Acemoglu/Shimer (1999).

¹⁶We will assume that the output of the worker is high enough to assure positive wages.

5 Benchmark

5.1 Value functions

The model described in this section is very similar to the models of Acemoglu and Pischke.¹⁷ However, there is a major difference with respect to separations: Acemoglu usually assumes an exogenous rate of separation, whereas we are endogenizing separations via stochastic, idiosyncratic productivity-shocks:¹⁸ whenever the shock lies below a certain threshold the worker and the firm will separate and return to the labor market. The advantage of this approach is that training will influence the probability of separations. It does not seem very likely that all workers have the seem risk of losing their job, no matter how well trained they are.

The value of a firm with an employee shall be described by $J(y)$ while a vacancy is denoted by V . The value of a worker occupying a job is $W(y)$ and the value of an unemployed worker is $U(h)$, where h denotes the amount of human capital of that worker. The value of unemployment is dependent on human capital if training is assumed to be general. I do not consider any other forms of human capital such as for instance education.

The value of a filled job at the beginning of the first period is:¹⁹

$$J(y_t) = y_t(h) - w_t^b - c(h) + \rho \int_{y_q}^{\max} J(y_{t+1})f(y_{t+1})dy_{t+1} - \rho \int_{\min}^{y_q} Vf(y_{t+1})dy_{t+1} \quad (1)$$

The output of the worker $y_t(h)$ is a positive function of her human capital. The cost of training $c(h)$ is assumed to be increasing as well. Either the output of training has to be growing at a declining rate or the cost of the training has to be increasing at an enhancing rate to assure an interior solution.

¹⁷See for instance Acemoglu/Pischke (1999a).

¹⁸See for instance Pissarides (2000).

¹⁹The notation is very much in line with Pissarides (2000) or the Appendix in Acemoglu/Pischke (1999a).

The first three terms are showing revenues and expenditures of the current period (output minus wages and training costs), while the integrals are giving the expected value of the firm next period, which has to be discounted by the factor ρ . y_q is the threshold-productivity:²⁰ if output turns out be lower than this threshold, the partnership will be terminated and the firm will get the (constant) value of a vacancy (second integral). If output is above y_q , the match will continue. In this case the firm-value is $J(y_{t+1})$ which is dependent on the actual realization of the shock y_{t+1} . All these cases have to be weighted with their respective probabilities and added up over the domain of the probability distribution [min, max]. The threshold y_q is defined by:

$$J(y_q) = V \tag{2}$$

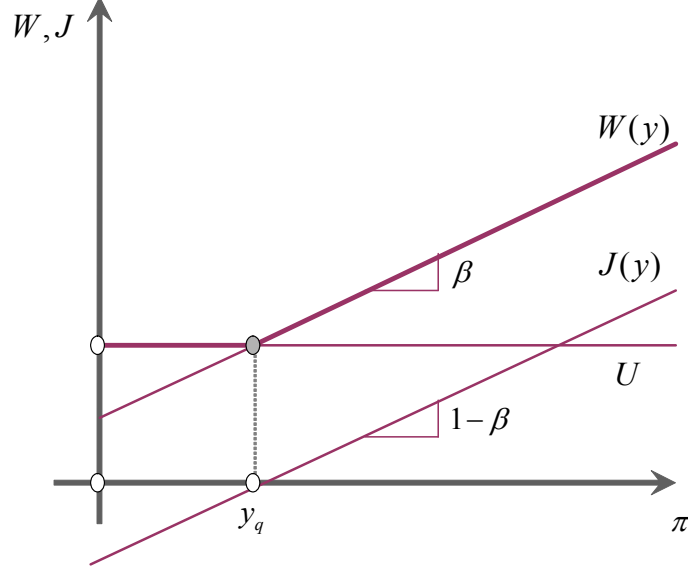
or alternatively

$$W(y_q) = U(h)$$

so that both parties are indifferent between continuing the partnership (in which case they would get $J(y_{t+1})$ resp. $W(y_{t+1})$) and terminating it (in that case they would get V resp. $U(h)$). For any shock lower than y_q , both parties will agree to separate because their values at the outside labor-market are higher. For the remainder of the paper I will assume free entry of firms, so that the value of a vacancy will be zero at any time - if it were positive, new firms would enter the market, lowering the probability of all firms to find a worker and thereby driving down the value of a worker. To the contrary, if the value of a vacancy were negative, some firms would exit the market, the chances of the remaining firms to find a worker go up and thereby the value of a vacancy until it has reached its equilibrium level zero - only then there will be no incentives for further adjustment.

²⁰If I have called the separation-threshold y_q , referring to quits, because in the benchmark model both parties agree to separate. This will be different in the rigidity model and thus there I refer to the separation as a firing and will use y_F to denote the threshold.

Figure 3: Value functions of the second Period - Benchmark



The value of an employed worker in period one is very similar to the value of a firm:

$$W(y_t) = w_t^b + \rho \int_{y_q}^{\max} W(y_{t+1}) f(y_{t+1}) dy_{t+1} - \rho \int_{\min}^{y_q} U(h) f(y_{t+1}) dy_{t+1} \quad (3)$$

The income of the current period is just equal to the wage, since the costs of the training are paid by the firm. The integrals are again illustrating the expected value of the worker in the second period: if the output lies above y_q the match will continue and the worker is getting value $W(y_{t+1})$, if output lies below the threshold-productivity she will quit and have the value of an unemployed worker $U(h)$.

Since the match will terminate with certainty after the second period the second-period value functions are just equal to the respective incomes during that period plus the respective values at the labor market:

$$J(y_{t+1}) = y_{t+1} - w_{t+1}^b + \rho V = y_{t+1} - w_{t+1}^b$$

$$W(y_{t+1}) = w_{t+1}^b + \rho U(h)$$

Figure (3) shows the value functions of workers and firms for period two in dependence of the productivity shock and the quitting threshold. The slope of the value functions of the match is equal to $1 - \beta$ resp. β because the threatpoint of both parties is independent of the idiosyncratic shock. Thus the outcome of the shock will be shared according to the respective bargaining powers of the parties. Due to the same reason is the value of unemployment characterized by a horizontal line. The worker will prefer the state with the higher value. Therefore she will choose unemployment for any shock lower than y_q . In the graph this is illustrated by the thick line. The firm has the alternative between $J(y_{t+1})$ and the value of a vacancy which is equal to zero. Of course, whenever the value of the job lies below zero, the firm will prefer to terminate the relationship. As can be seen in the picture and was explained further above, the firm and the worker will agree on whether to stay together or whether to separate: The worker's value function intersects the value of unemployment at the same value of productivity at which the job's value function turns negative.

5.2 Wages

As was mentioned above, wages are determined by Nash-bargaining,²¹ according to which the surplus of the match over the threat-points²² of both parties is shared corresponding to their bargaining strength. From the perspective of the worker this means that her surplus over the threat-point ($W - U$) has to be equal to the rent of the whole match ($W + J - U - V$) multiplied with her bargaining-power β :

$$W - U = \beta(W + J - U) \tag{4}$$

²¹See for instance Shaked and Sutton (1984) for a game-theoretic foundation or Pissarides (2000) for an application to the matching framework.

²²The threatpoint or fall-back position in a bargain is the value a party would get in case of a breakdown of negotiations. As is standard, I assume that both parties are able to turn back to the labor-market instantaneously.

The bargained wage will assure that the surplus of the match is shared according to this rule. For both periods this will in the following wage formula:²³

$$w_t^b = (1 - \rho) U(h) + \beta (y_t(h) - (1 - \rho) U(h)) \quad (5)$$

This is a standard result: the worker will get at least the value according to her threat-point²⁴ plus a share β of the surplus over that threat-point.

Using the value and wage functions we find that the quitting-threshold is given by the sum of both adjusted threat-points:

$$y_q = (1 - \rho) U(h)$$

Thus the two parties will agree to separate whenever the output of the second period lies below the value of unemployment. In this case the negotiated wage will be so low that it is more profitable for the worker to look for another job.²⁵ But still the wage is so high that it lies above the output of the worker and the firm is making losses. Therefore both, the firm and the worker are better in the case of a separation.

5.3 Wage compression

The degree of wage compression can be determined by taking the derivative of wages with respect to productivity resp. firm-training:

$$\frac{\partial w_t^b}{\partial h} = \beta \frac{\partial y_t}{\partial h} + (1 - \beta) (1 - \rho) \frac{\partial U(h)}{\partial h} < \frac{\partial y_t}{\partial h} \quad (6)$$

²³Here we can see the advantage of assuming that workers do not die after the second period. Otherwise, we would have a different wage-rule for both periods and both wages would be more complicated since $(1 - \rho) U(h)$ would have to be replaced by $U_t(h) - \rho U_{t+1}(h)$.

²⁴Where the threat-point has to be adjusted due to discounting.

²⁵Remember that the shock was idiosyncratic, so that the output of the worker in an alternative firm is not affect.

As can be seen in the equation above and is proved in the Appendix, wages are reacting to training less than output. In other words, the wage structure is compressed. According to Acemoglu and Pischke (1999a) this is sufficient and necessary to induce firm-sponsored training.

Equation 6 is illustrating how the wage is reacting to changes in productivity. It might seem surprising that this equation is not so simple. The reason is that the wage is not only affected by the output but also by the alternatives of both parties.

The first term is the direct effect of training on wages: since the worker is always getting a share β of the value of production it will get as well a share β of the training's value. But there is an additional, more indirect effect of training on wages which is working via the bargaining position of the worker. If training is assumed to be general, it will increase the value of unemployment to the worker since she will earn a higher wage if she is finding a new job. This will improve the threat-point of the worker and thereby her bargaining position. In consequence, the wage will be higher. This effect is captured by the second term of the equation.

5.4 Training

Before wages are negotiated the firm will decide privately about the amount of training. The optimal decision is found by taking the derivative of the first-period value function with respect to training and setting it equal to zero:

$$\frac{\partial J(y_t)}{\partial h} = \frac{\partial y_t}{\partial h} - \frac{\partial w_t^b}{\partial h} - c'(h) + \rho \int_{y_q}^{\infty} \frac{\partial J(y_{t+1})}{\partial h} f(y_{t+1}) dy_{t+1} - \rho \frac{\partial y_q}{\partial h} J_{t+1}(y_q) f(y_q) = 0$$

The first term is the marginal cost of additional training, the second and the third term illustrate the effect on current profits: the output increases but at the same time the wage will increase as well. The fourth term is showing the effect of training on the firm-value next period while the last term is illustrating the change of the quitting-threshold. However, this last term will drop out since the value of the firm at this threshold is zero

by the definition of the threshold (see equation ??).²⁶

By using the value functions and the wage rule as defined above and rearranging we arrive at the following - more meaningful - equation:

$$c'(h^b) = \left[\frac{\partial y_t}{\partial h} - \beta \frac{\partial y_t}{\partial h} - (1 - \beta)(1 - \rho) \frac{\partial U(h^b)}{\partial h} \right] [1 + (1 - F(y_q))] \quad (7)$$

where we have marginal costs on the left hand side and marginal revenues on the right hand side. The terms inside the first square brackets are showing the marginal revenue per period while the second square brackets are giving the expected number of cases in which the firm and the worker will stay together. The first 1 is standing for the first period: there cannot be a separation during that period so the match will survive with probability one. However, it will continue into period two only if the realization of the shock lies above the threshold y_q . $F(\pi)$ is the cumulative distribution function of $f(\pi)$. Therefore $F(y_q)$ is giving the probability of a separation and $1 - F(y_q)$ the probability of survival.

Lets get back to the terms in the first square brackets on the right hand side of equation 7: Overall this is illustrating the effect of training on the value of the firm each period (given that the match continues), which is the increase of the output of the worker net of the increases in wages as defined in equation 6. By inspection of equations 7 and 6 it becomes clear, that the firm will invest in the worker's human capital if and only if the wage-structure is compressed. Otherwise, the term inside the first square brackets - and thereby the marginal revenues to training - would be zero (or even negative). Considering this point we are able to confirm the results of Acemoglu and Pischke.

²⁶It should be noted that this last result is NOT due to our assumption that the value of a vacancy is equal to zero. If it were not zero, there would be an additional term in the value function, as illustrated in equation 1.

6 Rigidity model

6.1 Value functions

As already mentioned above, the rigidity model differs from the benchmark only with respect to the wage-negotiations of the second period. In principle, these are the same with the only restriction that the wage is not allowed to drop from the first period to the second. As we will see later, the restriction to wage-negotiations in the second period has consequences for the wages of the first period as well, although these are still freely negotiated. To account for the possibility that rigidity becomes binding, we have to add another state to the description of the second period, so that we can distinguish between situations in which the wage of the previous period is restricting the negotiations of the last period and situations in which it is not. To make things clear we add a superscript r to denote value functions describing the restricted case and a superscript u to denote the reverse case:

$$J^u(y_{t+1}) = y_{t+1} - w_{t+1}^r \quad (8)$$

$$J^r(y_{t+1}, w_t) = y_{t+1} - w_t^r \quad (9)$$

Again the value of the second period for the firm are straight forward, it's just production minus wages. The unrestricted value function $J^u(y_{t+1})$ is exactly the same as in the benchmark model, whereas the restricted value function $J^r(y_{t+1}, w_t)$ has another state-variable which is the wage of the previous period. The values of workers are straight forward as well:

$$W^u(y_{t+1}) = w_{t+1}^r + \rho U \quad (10)$$

$$W^r(w_t) = w_t^r + \rho U \quad (11)$$

It should be noted that, in contrast to the firm value, the restricted value function of a worker will no longer depend on the output of the match - the worker will receive the same wage in any case - of course, only so long as the match is not destroyed.

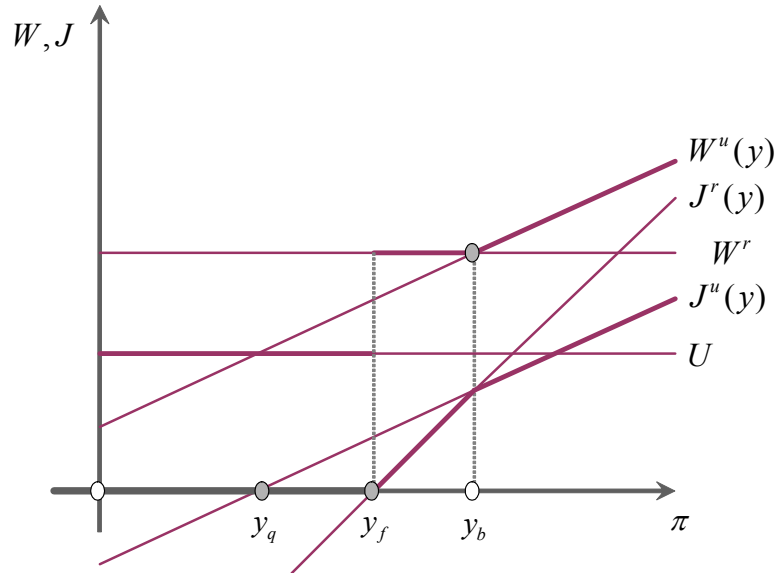
Clearly this distinction between two different value functions for the second period has consequences as well for the values of the first period:

$$\begin{aligned}
J(y_t) = & y_t(h) - w_t^r - c(h) + \rho \int_{y_b}^{\max} J^u(y_{t+1})f(y_{t+1})dy_{t+1} + \\
& \rho \int_{y_f}^{y_b} J^r(y_{t+1}, w_t)f(y_{t+1})dy_{t+1} - \rho \int_{\min}^{y_f} Vf(y_{t+1})dy_{t+1} \quad (12)
\end{aligned}$$

$$\begin{aligned}
W(y_t) = & w_t^r + \rho \int_{y_b}^{\max} W^u(y_{t+1})f(y_{t+1})dy_{t+1} + \\
& \rho \int_{y_f}^{y_b} W^r(w_t)f(y_{t+1})dy_{t+1} - \rho \int_{\min}^{y_f} U(h)f(y_{t+1})dy_{t+1} \quad (13)
\end{aligned}$$

The interpretation of the value-functions is analogous to the benchmark-model: Again the terms without an integral are giving the earnings of the first period, while all the integrals taken together make out the expected value of the second period. The distinction between the cases where the first-period wage is restricting and where it is not, necessitates another threshold discriminating between these two cases. This threshold is called y_b here, to denote whether the wage is binding or not. So whenever the (partly) random y_{t+1} lies above this threshold, the freely negotiated wage of the second period will be higher than the wage of the first period and the restriction will not be binding. In this case both parties are receiving the unrestricted value of period two, $J^u(y_{t+1})$ resp. $W^u(y_{t+1})$. Whenever output is below y_b the negotiated wage would lie below the wage of the previous period so that wages would have to be cut down. However, the management does fear the bad effects of wage-cuts on the morale of the work-force and is therefore preferring to keep the

Figure 4: Value functions of the second Period - Rigidity-model



wage constant. Thus in this case the firm and the worker will get the restricted values of period two $J^r(y_{t+1}, w_t^r)$ resp. $W^r(w_t^r)$.

As in the benchmark there is a separation-threshold (y_f), such that the match will be terminated for lower shocks. Just as in the benchmark-model in such a case the parties are receiving the values V and $U(h)$. The thresholds and their relations to one another will be discussed in more detail further below.

Again the value functions and thresholds can be illustrated graphically (see figure (4)). The unrestricted value functions have the same slopes as the value functions in the benchmark model (see figure (3)): $1 - \beta$ resp. β . The restricted value function of the worker is horizontal: Since in these cases the worker gets a fixed wage, the value will be independent of actual productivity. In turn, the restricted value function of the firm will have slope one: The wage is fixed and therefore any increase in output will lead to an increase in firm value one to one.

Again the thick line is indicating the actual value of the worker resp. the firm for all levels of the shock. The worker will prefer the larger of the unrestricted and the restricted

value so whenever output lies below the threshold y_b the wage-restriction will become binding. For the firm it's just the other way around: Due to the fear of bad morale it will always get the lower alternative. However, the firm has the possibility to fire the worker and it will do so whenever the value of the firm becomes negative - this implies the second kink of the thick line at the threshold y_f , which lies clearly above y_q . From the worker's perspective this means a jump from the horizontal line W^r to the value of unemployment, which is horizontal as well. It is clear that - in contrast to the benchmark - the worker will always prefer to stay employed. For shocks between y_f and y_q the worker would even be willing to accept a wage cut in order to stay employed. However, the firm will not accept this because it fears that the worker will provide no effort in such a case.

6.2 Wages

Since the wages are determined for the first time at the beginning of the first period there is no previous-period wage that could become a restriction. Consequently, wage negotiations are unrestricted. Nevertheless, wage rigidity will play a role in these negotiations since the prospect of a binding restriction alone is enough to alter the outcome of the bargain. It is this modification that gives rise to enhanced wage compression, as will be shown further below.

The wage is again found by plugging in the value functions into the sharing rule of Nash-bargaining (equation 4):²⁷

$$w_t^r = (1 - \rho)U + \beta (y_t - (1 - \rho)U) + \frac{\beta \rho \int_{y_f - y_t}^{y_b - y_t} \pi_t f(\pi_t) d\pi_t}{1 + \rho F(y_b) - \rho F(y_f)} \quad (14)$$

Compared to the wage of the benchmark model (see equation 5) there is one additional term (the last term).²⁸ Besides that, the wage outcomes are equivalent. But what is this

²⁷See the Appendix for a proof.

²⁸It should be noted, that the value of the integral is negative since both of the boundaries of the integral are negative.

additional term? It is the compensation of the firm for the possibility that it might have to pay a wage "too high", i.e. not according to the unrestricted bargaining rule. In the nominator we can find the deviation of the output of the second period from the output of the current period, in all those cases that the wage restriction is binding but the worker not fired. Remember that the output of the second period is equal to $y_t + \pi_t$, while the output of the first period is just y_t - thus π_t is the deviation from one period to the other. This deviation is irrelevant for all those cases that the wage is freely negotiated, because in these cases the wage is adjusted accordingly. But for all those states that the wage would have to be cut and this is hindered by wage rigidity, the adjustment is not possible. This is benefitting the worker because she will get a wage higher than she would get otherwise (under the condition of free bargaining) but hurting the firm. These possibilities are foreseen by both parties and reflected in the value-functions. Thus because the worker will have an advantage over the firm in the second period and both parties are foreseeing this, the worker has to compensate the firm by accepting a lower first-period wage, compared to the benchmark. In this sense, wage rigidity can be interpreted as an insurance against wage cuts. The firm is providing the insurance to the worker and is paying a wage that is at least as high as the wage of the current period. The difference between the first-period wage in the benchmark and the rigidity model is the insurance premium that the worker is paying to the firm.

Remains to explain the term in the denominator, which is equal to one plus the probability that the wage is binding in the second period. Thus the denominator is giving the expected number of cases, that the currently negotiated wage will be paid. To interpret this, it is useful to rearrange the wage-equation by multiplying both sides with the denominator:²⁹

$$(1 + F(y_b) - F(y_f))w_t = \beta \left[(1 + \rho F(y_b) - \rho F(y_f))y_t + \rho \int_{y_f - y_t}^{y_b - y_t} \pi_t f(\pi_t) d\pi_t \right]$$

Now we can see on the left hand side of the equation the wage payments of the firm for all those cases that the currently negotiated wage has to be paid. And on the right hand

²⁹To save notation I have left out the terms related to the threat-point of the worker.

side inside the square brackets we can see the expected output in these cases: the output y_t plus the expected deviations from this output. According to Nash-bargaining, the worker should get a share according to her bargaining strength β and therefore this term has to be multiplied by β . Summarizing it can be said, that the bargaining of the first period is assuring that overall, both parties are compensated according to their respective bargaining-strengths. If one party is expected to have in the future an advantage over the other party, the Nash-bargain of the presence is assuring that the profiting party is compensating the aggrieved party.

Wage negotiations of the second period are the same as in the benchmark model. Since the match will end after the second period for sure, wage-rigidity will no longer play any role. Thus the wage is:

$$w_t^r = \text{Max}[(1 - \rho)U(h) + \beta(y_t(h) - (1 - \rho)U(h)), w_{t-1}^r]$$

either the freely negotiated wage as given in equation 5 or the wage of the previous period.

It can be shown that the wage structure of the rigidity model is more compressed than in the benchmark:³⁰

$$\frac{\partial w_t^r}{\partial h} < \frac{\partial w_t^b}{\partial h}$$

Thus an increase in firm training will have less effects on the wages of employees. According to the predictions of Acemoglu and Pischke (1999a) this should lead to higher firm-training. We will try to answer this question further below but first we will discuss the thresholds in more detail.

6.3 Discussion of thresholds

The binding-threshold (separating the states where the wage rigidity is relevant and where it is not) can be defined in three equivalent ways:

³⁰For a proof see in the Appendix.

$$W^u(y_b) = W^r(w_t) \tag{15}$$

$$J^u(y_b) = J^r(y_b, w_t^r)$$

$$w_t^r(y_t) = w_{t+1}^r$$

where $w_t^r(y_t)$ is denoting the freely bargained wage. Thus, as the last equation illustrates, at the quitting-threshold the bargained wage is just equal to the wage of the previous period. Therefore the worker is indifferent between the old wage and the freely negotiated wage and, consequently, both the restricted and the unrestricted values are equal to each other. Alternatively, the threshold could be defined by using firm-value functions.

The separation threshold of the rigidity-model y_f is not equal to the separation threshold of the benchmark model y_q due to the inflexibility of wages. The letter f is used to denote the firing which will take place if output is lower than this threshold. It is referred to as a firing because in such a case the worker would prefer to keep up the relationship since she will always get the same wage $w_{t+1} = w_t$ and therefore never has any interest to terminate the relationship. Nevertheless, for the firm a termination is more profitable and it will therefore fire the worker. This is in contrast to the benchmark-model where both parties would agree to separate if output were below y_q . It might be criticized that this will lead to inefficient separations: both parties might be better off, if they would agree on a lower wage. However, this is exactly what the evidence of surveys is telling us: managers are preferring layoffs to wage cuts because it moves the problem of bad morale outside of the firm. In other words, the firm does not favor the wage cut because it anticipates that this will induce the worker to shirk and this would be even worse than a separation. In that sense, it cannot be spoken of inefficient separations although it might seem so.

Although the separation-thresholds are different, they are defined in a very similar way:

$$J^r(y_f, w_t^r) = V \tag{16}$$

Just as in the case of the quitting-threshold, the firing-threshold is found by setting the value of the firm equal to its threat-point, the value of a vacancy. If the output is lower than y_f , then to the firm the value of keeping up the match will be lower than terminating it and thus it will fire the worker. Of course, if the output is low enough that a separation will occur, the output will be so low that the wage rigidity is binding (if the parties were not separating). Therefore we have to use the restricted value function J^r . The use of the restricted value-function is explaining the difference between y_f and y_q . In the rigidity model the firm is not able to lower the wage from one period to the other, whereas in the benchmark model the wage can go down to zero. As long as the wage in the first period was not negative, it follows that for low (bad) shocks the wage of the second period in the rigidity model has to be higher than in the benchmark model.³¹ Due to this higher wage, the firm will be less reluctant to fire the worker and consequently separations are more frequent. Summarizing we can state the following about the order of thresholds:

$$y_b > y_f > y_q \implies F(y_b) > F(y_f) > F(y_q)$$

Consequently we can distinguish four different intervals for the second period. From \max to y_b wages and values will be the same in both models. From y_b to y_f the wage-rigidity will become relevant so that the wage in the benchmark is lower. From y_f to y_q a worker will be fired in the rigidity model but not in the benchmark model. Below y_q workers in both models get unemployed.

By plugging in the value and wage functions into the definitions of the thresholds we can easily find that:

$$y_b = y_t + \frac{\rho \int_{y_f - y_t}^{y_b - y_t} \pi_t f(\pi_t) d\pi_t}{1 + \rho F(y_b) - \rho F(y_f)} \tag{17}$$

³¹This is true for all states below the binding-threshold.

$$y_f = w_t^r \quad (18)$$

These equations can be interpreted as follows: Because the wage adjustment³² is no longer necessary in the second period (the relationship will be terminated afterwards), the bargained wage of the second period will be principally higher for equal productivities. Therefore the output of the worker has to fall by the value of that adjustment-term in order to make rigidity binding. More straight forward is the interpretation of the second threshold: Since the worker is getting w_t^r for sure if the output is below the binding-threshold y_b , the firm will get the residual of the output over that wage. This residual will turn negative as soon as the output lies below the wage and then the firm will fire the worker.

6.4 Training

Again the optimal amount of training is found by setting the first-order derivative of the value function equal to zero:

$$\begin{aligned} \frac{\partial J(y_t)}{\partial h} &= -c'(h) + \frac{\partial y_t}{\partial h} - \frac{\partial w_t^r}{\partial h} + \rho \int_{y_b}^{\infty} \frac{\partial J^u(y_{t+1})}{\partial h} f(y_{t+1}) dy_{t+1} + \rho \int_{y_f}^{y_b} \frac{\partial J^r(y_{t+1}, w_t)}{\partial h} f(y_{t+1}) dy_{t+1} \\ &\quad - \rho \frac{\partial y_b}{\partial h} J_{t+1}^u(y_q) f(y_q) + \rho \frac{\partial y_b}{\partial h} J_{t+1}^r(y_q, w_t) f(y_q) - \rho \frac{\partial y_f}{\partial h} J_{t+1}^r(y_f, w_t) f(y_f) \\ &= -c'(h) + \frac{\partial y_t}{\partial h} - \frac{\partial w_t^r}{\partial h} + \rho \int_{y_b}^{\infty} \frac{\partial J^u(y_{t+1})}{\partial h} f(y_{t+1}) dy_{t+1} + \rho \int_{y_f}^{y_b} \frac{\partial J^r(y_{t+1}, w_t)}{\partial h} f(y_{t+1}) dy_{t+1} = 0^{33} \end{aligned}$$

After plugging in the definition of values and wages as given in equations 14 and rearranging we arrive at the following equation which is very similar to the benchmark:

$$c'(h^r) = \left[\frac{\partial y_t}{\partial h} - \beta \frac{\partial y_t}{\partial h} - (1 - \beta)(1 - \rho) \frac{\partial U(h^r)}{\partial h} \right] [1 + (1 - F(y_f))] \quad (19)$$

³²The additional term in equation 14.

³³Note that $J_{t+1}^u(y_q) = J_{t+1}^r(y_q, w_t)$ and $J_{t+1}^r(y_f, w_t) = 0$ by definition of the thresholds.

Actually the only difference between the optimality condition of the benchmark given in equation 7 and the equation above is the separation probability inside the second square-brackets at the right hand side. The reason for this surprising result lies in the flexibility of wage-bargaining in the first period - as already discussed in the section above, it will assure that the value of the whole match is shared according to the respective bargaining powers of the parties. Mathematically the additional term in the wage of the first period will assure that the effects of wage rigidity on the values will just cancel out.

Consequently, the only difference between the two models when it comes to firm training, is the difference in separation rates. A higher separation rate will make it less likely that the firm can get a return on its investment by paying wages below productivity in the second period. Of course the firm can still get some return on the training during the first period, but a higher risk of termination in the second period will lower the profitability of the second period. As already discussed in the section above, this separation probability is higher in the rigidity model compared to the benchmark. It follows that training in the rigidity model will be lower, not higher as predicted by the models of Acemoglu and Pischke:

$$y_f > y_q \implies P(y_f) > P(y_q) \implies h^r < h^b$$

The difference in results is due to the endogeneity of separations. In the models of Acemoglu and Pischke separations occur at an exogenous rate. Thus training can have no effect on the probability of separations. This is not very plausible, given that training is improving the output of a worker in any state of the world. Instead a worker with higher productivity should be more able to overcome bad times. The concept of exogenous separation-rates seems even more problematic in the context of minimum wages or wage rigidity. Both phenomena are restricting the flexibility of the firm in a severe way: they do not allow the firm to cut wages below a certain level. It appears only natural that firms will react by being less reluctant to fire workers - and indeed this is confirmed by empirical surveys like the ones of Bewley (1999) or Agell and Lundborg (1995). Therefore it appears even more important to allow for endogenous separations in the context of such

restrictions. It might be interesting to explore the potential of endogenizing separations in models of minimum wages to bring back in line the theoretical results with empirical findings.

7 Conclusion

By endogenizing the separation decision we were able to show that higher wage compression does not necessarily lead to more training investments as implied by the models of Acemoglu and Pischke who assume that jobs are destroyed at an exogenous rate. Their assumption implies that all workers face the same risk of losing their job no matter how productive they are. This is not only implausible but as well at odds with the empirical literature on firm training that is pointing towards a negative relationship between a worker's training and her turnover-rate.³⁴ By assuming that the productivity of the match is hit by an idiosyncratic shock we are able to endogenize the separation decision so that workers are only fired if the shock lies below a certain threshold. The higher the human capital of the worker the lower lies this separation-threshold implying a higher risk of getting unemployed for untrained workers in comparison to trained (or better trained) workers.

Wage rigidity is modelled by implying the restriction that wages of the current period are not allowed to be smaller than wages in the preceding period, whereas otherwise wages are negotiated freely via standard Nash-bargaining. Worker and firm - foreseeing this - will negotiate a lower starting wage than in an unrestricted world. In principle the firm is offering the worker an insurance against wage-cuts and the lower starting wage is the insurance premium.

We were able to show that this wage rule will lead to an increase in wage compression compared to a benchmark model with unrestricted Nash-bargaining. However, at the same time the rigid wage will lead to higher turnover rate since firms are not allowed

³⁴See for instance Lynch (1991).

to lower the wage. This is again in line with the empirical literature suggesting that managers prefer to lay off workers to wage cuts, because they fear the adverse effects on morale.³⁵ Due to this increase in the probability of separations firm training will be lower in the rigidity model.

This is especially interesting with regard to the empirical literature on the effect of minimum wages on firm training. As noted by Acemoglu/Pischke (2002) the results of these empirical studies are rather mixed. Applying the endogeneity of separations to a model of minimum wages will likely lead to similar results as pointed out in this paper. This might be an interesting field for further research.

8 Literature

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³⁵See Bewley (2002).

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9 Appendix A: wage compression in the benchmark

Wages were not compressed in the benchmark only if U would equal $\frac{y}{1-\rho}$. In this extreme case the threat-point of the worker is so high, that she will always get a wage equal to

her output. The derivative of the wage with respect to productivity as given in equation 6 would then be:

$$\frac{\partial w_t^b}{\partial h} = \beta \frac{\partial y_t}{\partial h} + (1 - \beta) \frac{\partial y_t}{\partial h} = \frac{\partial y_t}{\partial h}$$

Wages react one to one to changes in productivity and thus the wage structure is not compressed.

Values of U higher than $\frac{y}{1-\rho}$ will make no sense, since in that case, the alternatives of the worker are unambiguously better, the pair would separate immediately at the beginning of the first period and no training would take place. Of course in practice the threat-point of the worker will be much lower. It is implausible that the worker would get a wage above productivity in another firm (which would be implied by $U = \frac{y}{1-\rho}$ since the denominator is smaller than zero). Additionally, the worker will not find a new job with certainty but will stay unemployed with a certain probability. This will further decrease the value of unemployment.

All this factors will diminish the sensitivity of the worker's threat-point to productivity and thus create wage compression.

10 Appendix B: wage-rule of the rigidity model

We are using the same rule to determine the wage as in the benchmark, equation 4:

$$W - U = \beta(W + J - U)$$

By plugging in the value functions for J and W as defined in equations 12 and 13 we get:³⁶

$$U = w_t + \rho \int_{y_b}^{\max} W^u(y_{t+1}) f(y_{t+1}) dy_{t+1} + \rho \int_{y_f}^{y_b} W^r(w_t) f(y_{t+1}) dy_{t+1} + \rho \int_{\min}^{y_f} U(h) f(y_{t+1}) dy_{t+1} - \beta [y_t(h) + \rho \int_{y_b}^{\max} J^u(y_{t+1}) f(y_{t+1}) dy_{t+1} + \rho \int_{y_f}^{y_b} J^r(y_{t+1}, w_t) f(y_{t+1}) dy_{t+1} +$$

³⁶Since the training cost is already sunk, it does not appear in the wage negotiations.

$$\begin{aligned}
U] = & \rho \int_{y_b}^{\max} W^u(y_{t+1})f(y_{t+1})dy_{t+1} + \rho \int_{y_f}^{y_b} W^r(w_t)f(y_{t+1})dy_{t+1} + \rho \int_{\min}^{y_f} U(h)f(y_{t+1})dy_{t+1} - \\
& \beta[y_t(h) + \rho \int_{y_b}^{\max} [J^u(y_{t+1}) + W^u(y_{t+1})]f(y_{t+1})dy_{t+1} + \rho \int_{y_f}^{y_b} [J^r(y_{t+1}, w_t) + W^r(w_t)]f(y_{t+1})dy_{t+1} + \\
& \rho \int_{\min}^{y_f} U(h)f(y_{t+1})dy_{t+1} - U]
\end{aligned}$$

Using the fact that the wage-rule equation 4 is valid in the second period as well, the unrestricted value-functions will cancel out (with the exception of the value of unemployment). Plugging in the equations 9 and 11 for the remaining value-functions of the second period we get:

$$\begin{aligned}
U = & w_t + \rho \int_{y_b}^{\max} U(h)f(y_{t+1})dy_{t+1} + \rho \int_{y_f}^{y_b} [w_t + \rho U(h)]f(y_{t+1})dy_{t+1} + \rho \int_{\min}^{y_f} U(h)f(y_{t+1})dy_{t+1} - \\
& \beta[y_t(h) + \rho \int_{y_b}^{\max} U(h)f(y_{t+1})dy_{t+1} + \rho \int_{y_f}^{y_b} [y_{t+1} + \rho U(h)]f(y_{t+1})dy_{t+1} + \\
& \rho \int_{\min}^{y_f} U(h)f(y_{t+1})dy_{t+1} - U]
\end{aligned}$$

By merging the terms with U the equation simplifies to:

$$\begin{aligned}
w_t + \rho \int_{y_f}^{y_b} [w_t + (\rho - 1)U(h)]f(y_{t+1})dy_{t+1} + (\rho - 1)U = \\
\beta[y_t(h) + \rho \int_{y_f}^{y_b} [y_{t+1} + (\rho - 1)U(h)]f(y_{t+1})dy_{t+1} + (\rho - 1)U]
\end{aligned}$$

Now use the definition of $y_{t+1} = y_t + \pi$:

$$\begin{aligned}
w_t + \rho \int_{y_f}^{y_b} [w_t + (\rho - 1)U(h)]f(y_{t+1})dy_{t+1} + (\rho - 1)U = \\
\beta[y_t(h) + \rho \int_{y_f}^{y_b} [y_t + \pi + (\rho - 1)U(h)]f(\pi)dy_{t+1} + (\rho - 1)U]
\end{aligned}$$

The only term in this equation that is random is the π on the right hand side. All the other terms are constant and can therefore be taken out of the integral:

$$\begin{aligned}
w_t + (\rho - 1)U + \rho[F(y_b) - F(y_f)][w_t + (\rho - 1)U(h)] + (\rho - 1)U = \\
\beta[y_t(h) + (\rho - 1)U + \rho[F(y_b) - F(y_f)][y_t + (\rho - 1)U(h)] + \rho \int_{y_f}^{y_b} \pi f(\pi)dy_{t+1}]
\end{aligned}$$

By joining the terms and bringing all the U to the right hand side we get:

$$w_t[1 + \rho[F(y_b) - F(y_f)]] =$$

$$(1-\rho)U[1+\rho[F(y_b)-F(y_f)]]+\beta[(y_t(h)-(1-\rho)U)[1+\rho[F(y_b)-F(y_f)]]+\rho \int_{y_f}^{y_b} \pi f(\pi) dy_{t+1}]$$

Finally, we arrive at the wage given in equation 14 by dividing through in square brackets on the left hand side:

$$w_t = (1 - \rho)U + \beta[(y_t(h) - (1 - \rho)U)] + \frac{\beta \rho \int_{y_f}^{y_b} \pi f(\pi) dy_{t+1}}{1 + \rho[F(y_b) - F(y_f)]}$$

11 Appendix C: wage compression in the rigidity model

To see whether the wage-compression of the benchmark or the rigidity model is higher it is sufficient to look at the extra term in equation 14 giving the wage of the rigidity model, since the remaining terms (output and the value of unemployment) will react equally in both models. The wage structure of the rigidity model will be more compressed if this term is decreasing with output and vice versa. First of all we define the extra-term in the rigidity wage as Λ and use the assumption of uniformly distributed productivity shocks to get:

$$\begin{aligned} \Lambda &= \frac{\rho \int_{y_f - y_t}^{y_b - y_t} \pi_t f(\pi_t) d\pi_t}{1 + \rho F(y_b) - \rho F(y_f)} = \frac{\rho \int_{y_f - y_t}^{y_b - y_t} \pi_t \frac{1}{\max - \min} d\pi_t}{1 + \frac{\rho(y_b - y_f)}{\max - \min}} = \frac{\rho \frac{(y_b - y_t)^2 - (y_f - y_t)^2}{2(\max - \min)}}{\frac{\max - \min + \rho(y_b - y_f)}{\max - \min}} \\ &= \frac{\rho(y_b - y_t)^2 - (y_f - y_t)^2}{2[\max - \min + \rho(y_b - y_f)]} = \rho \frac{y_b^2 - y_f^2 - 2y(y_b - y_f)}{2[\max - \min + \rho(y_b - y_f)]} \end{aligned}$$

By noting that $y_b^2 - y_f^2 = (y_b + y_f)(y_b - y_f)$ this equation simplifies to:

$$\Lambda = \rho \frac{(y_b + y_f - 2y)(y_b - y_f)}{2[\max - \min + \rho(y_b - y_f)]}$$

Now the derivative of Λ with respect to productivity can be written as:

$$\begin{aligned} \frac{\partial \Lambda}{\partial y} &= \rho \frac{\frac{\partial (y_b + y_f - 2y)}{\partial y} (y_b - y_f) 2[\max - \min + \rho(y_b - y_f)]}{4[\max - \min + \rho(y_b - y_f)]^2} + \\ &+ \rho \frac{\frac{\partial (y_b - y_f)}{\partial y} (y_b + y_f - 2y) 2[\max - \min + \rho(y_b - y_f)] - \frac{\partial (y_b - y_f)}{\partial y} 2\rho(y_b + y_f - 2y)(y_b - y_f)}{4[\max - \min + \rho(y_b - y_f)]^2} = \end{aligned}$$

$$\rho \frac{\frac{\partial(y_b+y_f-2y)}{\partial y}(y_b-y_f)2[\max-\min+\rho(y_b-y_f)]}{4[\max-\min+\rho(y_b-y_f)]^2} + \rho \frac{\frac{\partial(y_b-y_f)}{\partial y}(y_b+y_f-2y)2[\max-\min]}{4[\max-\min+\rho(y_b-y_f)]^2} \quad (20)$$

It turns out to be useful to not further split up the derivatives of the sum and the difference of the thresholds. For convenience let me repeat the definitions of these thresholds as given in equations 17 and 18:

$$y_b = y_t + \frac{\rho \int_{y_f-y_t}^{y_b-y_t} \pi_t f(\pi_t) d\pi_t}{1+\rho F(y_b)-\rho F(y_f)}$$

$$y_f = \beta y_t + (1-\beta)(1-\rho)U + \beta \frac{\rho \int_{y_f-y_t}^{y_b-y_t} \pi_t f(\pi_t) d\pi_t}{1+\rho F(y_b)-\rho F(y_f)}$$

Consequently the difference between the two thresholds is:

$$y_b - y_f = (1-\beta)(y_t + \frac{\rho \int_{y_f-y_t}^{y_b-y_t} \pi_t f(\pi_t) d\pi_t}{1+\rho F(y_b)-\rho F(y_f)} - (1-\rho)U) \quad (21)$$

while the sum of the two thresholds is given by:

$$y_b + y_f = (1+\beta)(y_t + \frac{\rho \int_{y_f-y_t}^{y_b-y_t} \pi_t f(\pi_t) d\pi_t}{1+\rho F(y_b)-\rho F(y_f)}) + (1-\beta)(1-\rho)U \quad (22)$$

By taking the derivatives of equations 21 and 22 with respect to productivity y , plugging them both in in equation 20 and bringing all terms with Λ to the left hand side we get:

$$\begin{aligned} & \frac{\partial \Lambda}{\partial y} \left(1 - \frac{(1+\beta)\rho(y_b-y_f)}{2[\max-\min+\rho(y_b-y_f)]} + \rho \frac{-(1-\beta)(y_b+y_f-2y)2[\max-\min]}{4[\max-\min+\rho(y_b-y_f)]^2} \right) \quad (23) \\ = & \rho \frac{[1+\beta+(1-\beta)(1-\rho)\frac{\partial U}{\partial y}-2](y_b-y_f)2[\max-\min+\rho(y_b-y_f)]}{4[\max-\min+\rho(y_b-y_f)]^2} + \\ & + \rho \frac{(1-\beta)(1-(1-\rho)\frac{\partial U}{\partial y})(y_b+y_f-2y)2[\max-\min]}{4[\max-\min+\rho(y_b-y_f)]^2} \end{aligned}$$

This equation looks rather complicated. However, the only thing of relevance are the signs of the nominators. The term in brackets on the left hand side of the equation is

clearly positive since the second term inside the brackets is smaller than one while the third term is positive (since $y_b + y_f - 2y < 0$ as can be seen from equation 22). Consequently, Λ will have the same sign as the right hand side of the equation above. Thus the problem boils down to the determination of the signs of $[1 + \beta + (1 - \beta)(1 - \rho)\frac{\partial U}{\partial y} - 2]$ (taken out of the first term) and of $(1 - (1 - \rho)\frac{\partial U}{\partial y})$ (taken out of the second term) - the signs of all the other terms are obvious.

To do so we need to look more closely at the threat-point of the worker: As discussed in Appendix A U will most likely be lower than $\frac{y}{1-\rho}$. Nevertheless it is useful discuss this extreme case to clarify the relationship of the thresholds.

Given that $U = \frac{y}{1-\rho}$, the difference of the thresholds will simplify to:

$$y_b - y_f = (1 - \beta) \frac{\rho \int_{y_f - y_t}^{y_b - y_t} \pi_t f(\pi_t) d\pi_t}{1 + \rho F(y_b) - \rho F(y_f)}$$

The only solution to this equation is obviously $y_b = y_f = y_t$, since a positive difference between the two thresholds would imply a negative value for the integral - which is a contradiction - and vice versa. It follows that the wages in the benchmark and the rigidity model are exactly equal to each other. Moreover, all the thresholds will be equal as well, so that the worker will quit, whenever her output decreases due to a negative shock - in that case she can earn more on the labor market no matter whether wages are principally downwards rigid. Since the wage rigidity can never be binding there is no need to compensate the firm and so wages in both models will be equal.

However, as discussed in Appendix A, whenever U is smaller than $\frac{y}{1-\rho}$, the threat point of the worker will be less sensitive to changes in productivity than output. Then it is obvious that the following is true:

$$1 + \beta + (1 - \beta)(1 - \rho)\frac{\partial U}{\partial y} - 2 < 0$$

$$1 - (1 - \rho)\frac{\partial U}{\partial y} > 0$$

It follows that the right hand side of equation 23 is unambiguously negative. Thus the extra term in the wage of the rigidity model Λ will become more and more negative as productivity increases and the difference between wages in the benchmark and the rigidity

model will become larger. In other words, the wage-structure is more compressed when wages are downwards rigid, whenever $U < \frac{y}{1-\rho}$.