Optimal incentive mix:
The dual role of promotions and layoffs in firms

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Abstract

Firms offer highly complex contracts to their employees. These contracts contain a mix of various incentives such as fixed wages, bonuses, promise of promotion and threat of firing. This paper aims at explaining the reason why this incentive mix arises. The theoretical model proposed is a job assignment model with heterogeneous employees. In this model the firm is concerned about job assignment because the overall productivity of the firm depends on the quality of the employees and their allocation to jobs. In particular, the purposes of promotions and layoffs become twofold. First, they create incentives for the employees. Second, they work as sorting devices where the firm can adjust the composition of the workforce at any given hierarchical level to maximize profits. These sorting and selection considerations lead to the optimal incentive mix observed in firms. A set of the model's broader predictions is stated explicitly and tested on the personnel records from a large pharmaceutical company. The model's conjectures are shown to be consistent with the data.

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1 Motivation

Detailed information on the compensation systems used in firms is not easily obtained. Nevertheless the evidence that firms use a rich set of incentives, such as fixed wages, bonuses, layoffs and promotions, is accumulating, see Medoff and Abraham (1980, 1981), Baker, Gibbs and Holmström (1994a,b), Gibbs (1995) and Lazear (1992, 2000). Even though economic theory has shown how and why certain incentives should be used, it has not yet explicitly focused on why a mix of incentives arises. This paper aims at building a theoretical model in which the incentive mix observed in firms can be seen as an optimal contract. Furthermore, to avoid the Gibbons and Waldman (1999a,b) critique that many models may be able to explain a single empirical finding, but they often fail to explain a broader pattern of empirical evidence, we explicitly derive a large set of the model's conjectures. These conjectures are tested empirically using the personnel records from a large pharmaceutical company.

Previous studies analyzing the use of incentives in firms follow one of three main streams. First, the performance pay literature, which originates from Mirrlees (1974, 1976) and Holmström (1979, 1982), explains why pay should be linked to output. According to this theory, wages should include variable elements (such as bonuses or piece rate) to reward employee effort. Second, the efficiency wage literature started off by Shapiro and Stiglitz (1984) emphasizes the threat of firing as a motivational factor. This effect arises if employment involves rent (due to higher than market clearing wages) then the fear of loosing the job provides incentives. Thus, employees might provide first-best effort even absent variable pay elements. Finally, tournament theory, initiated by Lazear and Rosen (1981), puts emphasis on competition between workers. If the number of workers rewarded, fired or promoted is pre-set, then workers have an incentive to exert effort.[1]

The relative merits of different incentives are analyzed by MacLeod and Malcolmson (1998). They contrast efficiency wage and performance pay incentives. Under the efficiency wage system, firms pay rents ex-ante and provide incentives by firing shirking workers. Under the performance pay regime, firms pay bonuses ex-post, conditional on effort. The model shows that in general efficiency wage incentives are more expensive. Yet, efficiency wage might arise because it gives compensation up-front. If the firm cannot commit to pay bonuses, then efficiency wages might be the only way to motivate employees. The results, however, do not answer the question why an optimal mix can arise. In the MacLeod and Malcolmson (1998) setup either efficiency wage or performance pay prevails.

Our basic model, which is a traditional principal-agent model with hidden effort, is set in an infinitely repeated context. It is shown, that the optimal behavior of the firm is to motivate the employees through bonus payments and promotions, but it should not fire employees. Thus, the incentive mix observed in firms does not arise. The reason is that firing is socially wasteful, i.e. firing is costly for the firm (recruiting and training of new employees) and for the worker (job search) and in equilibrium the firm shoulders all these costs. These findings are consistent with MacLeod and

[1] From our perspective, however, tournaments employ the very same incentives as suggested by the other two theories. Hence, it will be disregarded in the discussion below. An important point, however, is that tournaments are more efficient than simple performance pay, if there are common shocks and employees are risk-averse. In this case, the tournament regime decreases inefficient risk-bearing and improves welfare. However, tournament games give incentives to sabotage and undermine team effort as shown in Lazear (1989). In industries where team work is important (and sabotage is potentially costly), tournament games can be counterproductive. Thus, when modelling incentives in such industries, competition among employees should be modelled explicitly.

Malcolmson (1998) and absent relational contracting performance pay incentives indeed prevail.

The basic model is extended to accommodate employee heterogeneity. The profit maximizing firm is now concerned about job assignment because the overall productivity of the firm depends on the quality of the employees and their allocation to jobs. In particular, the purposes of promotions and layoffs become twofold. First, they create incentives for the employees as in the basic model. Second, they work as sorting devices where the firm can adjust the composition of the workforce at any given hierarchical level to maximize profits. These sorting and selection considerations lead to the optimal incentive mix observed in firms.

A novel feature of the model is the use of the optimal steady state contract to characterize within firm dynamics. Most importantly, we examine the effect of the selection process on the employees' careers in the firm. In doing so, we can identify sorting and selection effects and link these to employee tenure and rank. As Guasch and Weiss (1980) suggest, we show that selection constitutes an interesting alternative to on-the-job human capital acquisition to explain the effects of tenure on earnings. Furthermore, the firm's selection process has implications for the employees' probability of being promoted, fired and receiving bonus payments which leads to conjectures about the model's broader predictions.

In the empirical part of the paper the model's conjectures are tested formally. First, it is confirmed that the firm engages in sorting. The firm seems to promote employees of high ability rather than those of low ability. Second, it is established that the firm has a positive selection; i.e. individuals with higher rank and higher tenure are more likely to be of high ability. Also, the model's predictions about earnings dynamics, the likelihood of receiving bonus payments, being promoted and being laid off are shown to be consistent with the data.

In the next section, the basic model is outlined. Section 3 extends this model to accommodate employee heterogeneity and address the consequences of sorting and selection for the employees' career evolution. Furthermore, the conjectures of the theoretical model are stated here. The data are presented in section 4 and the close relation between the empirical contract and the theoretical model is emphasized. In section 5, the model's conjectures are tested empirically. A detailed discussion of the model's limitations and suggestions for future research takes place in section 6. Finally, section 7 summarizes and concludes.

2 Basic model

The basic model entails the contracting relationship between a risk-neutral firm and a unit volume of risk-neutral employees. The firm and the employees form a principal-agent relationship. The firm maximizes steady-state expected profit, while the employees maximize their expected period utility.[2]

The model is set in an infinitely repeated set-up. The timing of events within each period is as outlined below:

1. The period starts.

[2] Focusing on steady state profits avoids exploring various convergence paths and computing net present values along these paths. The assumption that workers maximize period utility means that the firm's long-term contracting does not entail deferred compensation or ex-ante rents.

2. The firm offers a contract.

3. The employees inside the firm accept or decline the contract.

4. Potential employees outside of the firm decide on the contract.

5. The employees decide about the effort level.

6. Output is realized and observed by all.

7. Bonuses are paid, employees are promoted or fired.

8. Some employees leave for exogenous reasons.

9. The period starts all over again.

The employees produce probabilistic output which is normalized to 0 (low) and C (high). The probability that the output is C, is θ and naturally $\theta \in [0,1]$. The employee, once accepted the job, can influence the probability of high output by exerting effort. The utility cost of effort exertion is e, and effort increases the probability of success by δ. We assume that $\theta + \delta < 1$.

The utility of the employee's alternative job option is denoted by $\bar{U}$. Furthermore, the additional utility of a promotion is U_P, and the utility cost from being fired is U_F (note that both parameters are positive). The utility loss from firing reflects that if an employee is laid off, then he must search for a new job which is costly as there are frictions on the labor market. On the other hand, assuming that there are utility gains from promotions implies that promotion into higher ranks entails some rent. In the appendix, we provide a detailed motivation for using this closed form representation of managerial rent. Yet, for the sake of tractability we do not derive it formally in the model.

The firm sets a stationary contract through the following four parameters: $\{w, b, \pi_P, \pi_F\}$. First, a fixed wage (w) is offered to all individuals who accept the job. The remaining three parameters are conditioned on performance. It is assumed that bonus payments (b) are paid to well-performing agents, that the firm only considers promoting employees with high observable output (high performance) and only considers firing employees with low observed output (low performance).[3] The conditional promotion probability and the conditional firing probability are denoted by π_P and π_F, respectively. The parameters are (realistically) constrained as follows: $w, b \geq 0$; $\pi_F, \pi_P \in [0,1]$.

Turnover is costly as the firm has replacement and training costs. These costs are summarized in the turnover cost parameter K. There are two sources to turnover in the firm. First, the firm can influence turnover by laying off workers. Second, some workers leave the firm for natural reasons like retirement or family relocation. The volume of firm-initiated separations is $(1 - \theta - \delta)\pi_F$ and the exogenous natural turnover is denoted by $g \in [0,1]$.

The employees desire promotions as they entail rent but the number of employees that can be promoted is constrained by the number of vacant positions at the management level. That is, the volume of promotions $(\theta + \delta)\pi_P$ is constrained by the volume of exogenous job openings at the managerial level, $g' \in [0,1]$.[4] Moreover, since the managerial level is smaller than the non-management level the

[3] In the basic model, these assumptions can be made without the loss of generality.

[4] g' captures both firm- and employee-initiated separations at the management level.

hierarchical size difference ($H > 1$) should also be taken into account. The above argumentation is summarized in the promotion constraint (F).

$$(\theta + \delta)\pi_P \leq \frac{g'}{H} \tag{1}$$

For simplicity the contracting problem at the management level is not modeled explicitly. Managers consist of formerly promoted non-management employees and individuals hired from the external labor market directly into management. These employees contribute to the firm's profit with a fixed value M.[5] We assume that $C < M$, that is, managerial output is more important than employee output. This follows from the fact that managers affect the output of multiple non-management employees.

In order to keep the analysis tractable, it is assumed that the firm has all the bargaining power. Furthermore, in order to rule out indeterminacies two tie-breaking rules are imposed. First, indifferent players act such that the other player is better off. This can be interpreted as goodwill. Second, if the employees and the firm are equally well off with different contracts, the firm chooses the contract which implies the higher fixed wage. This embodies a weak form of commitment concern and a very weak form of employee risk-aversion.

Finally, we formally define our interest, the optimal incentive mix.

Definition 1 (Optimal incentive mix) *The optimal incentive mix arises when the firm strictly prefers to set all the parameters $\{w, b, \pi_P, \pi_F\}$ to non-zero values.*

2.1 Contracting problem

The contracting problem is summarized in the following equations:

$$\max_{w,b,\pi_F,\pi_P} \Pi(w, b, \pi_F, \pi_P) = \max_{w,b,\pi_F,\pi_P} M/H + (\theta+\delta)C - w - (\theta+\delta)b - \left[(1-\theta-\delta)\pi_F + g + \frac{g'}{H}\right] K \tag{2}$$

subject to

$$\begin{aligned}
(IC) \quad & w - e + (\theta+\delta)[b + \pi_P U_P] - (1-\theta-\delta)\pi_F U_F &&\geq && w + \theta[b + \pi_P U_P] - (1-\theta)\pi_F U_F \\
(IR) \quad & w - e + (\theta+\delta)[b + \pi_P U_P] - (1-\theta-\delta)\pi_F U_F &&\geq && \bar{U} \\
(F) \quad & (\theta+\delta)\pi_P &&\leq && \frac{g'}{H} \\
\text{nonnegativity} \quad & 0 &&\leq && w, b \\
\text{probability} \quad & 0 &&\leq && \pi_F, \pi_P \leq 1
\end{aligned}$$

Intuitively, the program above states that the firm maximizes profit. The IC constraint implies that the employee accepting the contract exerts first-best effort and the IR constraint states that the employee is better off accepting the contract. The other constraints follow from the assumptions directly.

[5] Later, with job assignment the firm can adjust managerial output by increasing the quality of the promoted workers. Thus, managerial output will play an important role once employee heterogeneity is accounted for.

2.2 Solving the basic model

The model is first solved by assuming that the three incentive constraints (IR, IC and F) are binding. This implies:

$$b = \frac{e}{\delta} - \pi_F U_F - \pi_P U_P \quad (3)$$

$$w = e - (\theta + \delta)[b + \pi_P U_P] + (1 - \theta - \delta)\pi_F U_F + \bar{U} \quad (4)$$

$$\pi_P = \frac{g'}{H(\theta + \delta)} \quad (5)$$

As there are four parameters to set the contract, choosing a single parameter (let's say the firing parameter) determines the contract through these three equations. Consequently, the original four-parameter incentive problem is reduced to a single parameter optimization. Expressing the profit as a function of π_F, the first order condition reveals that the objective function is maximized when the firing rate is minimized.[6] The intuition is that firing is more costly than bonuses to motivate workers. Firing creates two social costs: cost of hiring and utility loss of firing. In equilibrium, these two costs are born by the firm. Lemma (1) summarizes the result:

Lemma 1 *If the IR, IC and F constraints are binding, then incentives are given through bonuses and promotions, but not through firing. Thus, no optimal incentive mix arises.*

Moreover, even if the constraints are not binding, the optimal incentive mix does not arise. The intuition for the result follows simply from profit maximization. If the no slack condition on promotion probability (5) is violated in a profit-maximizing context, then promotion in itself provides sufficient incentives and both firing and bonuses are set to zero. If the condition on bonuses (3) is satisfied with slack, then by definition the bonus must be zero (else it could be reduced). Finally, if the condition on wage (4) is satisfied with a slack, then the wage must be zero (else again it could be profitably reduced).

Remark 1 *If the optimal solution involves incentive slack (at least one of the IR, IC and F constraints is not binding), then it is optimal for the firm to set either bonuses or fixed wages equal to zero. Thus, even with incentive slack no optimal incentive mix can arise.*

Finally, a few words about rents in the model. There is no ex-ante rent by assumption (and thus the individual rationality constraint (IR) is binding). However, the fact that firing causes disutilities (U_F) shows that interim there is rent. The interpretation is, that even though the worker initially is indifferent between the firm's offer and other offers, going back to the labor market and continue searching is costly. Thus, interim there is rent from retaining the job and the firm can exploit this feature to motivate the worker. Lemma (1) states, however, that motivating through layoffs is more costly than motivating through bonus payments.

3 Job assignment model

In this section, the basic model is extended to accommodate employee heterogeneity. The profit-maximizing firm is now concerned about job assignment, because the quality of the workforce and

[6] The proof is provided in the appendix.

the allocation of employees to jobs (non-management vs. management jobs) are important for the firm's overall productivity. This implies that the firm has an additional motivation (besides incentive purposes) to promote and layoff employees as the two devices can be used to adjust the quality of the workforce at any given hierarchical level.

The job assignment model is used for two purposes. First, it is shown that sorting and selection considerations lead to the optimal incentive mix. Second, the model is utilized to characterize the career path of the individual employee. This proves to be useful for testing the model's predictions.

3.1 Optimal incentive mix

Potential employees are heterogenous. There are good employees (G) with high ability and bad ones (B) with low ability. The good employees are more likely to produce a high output than bad employees, i.e. $\theta_G > \theta_B$, but the ability of the employee cannot be observed by the firm.

The firm's external labor market consists of a proportion μ of high-ability individuals. Since the firm can use promotions and layoffs to sort and select the employees, the composition of the employees in the firm's internal labor market can differ from the composition in the external labor market. For this reason, the proportion of high-ability types in the beginning of the period at the non-management level is denoted by μ_F and the proportion of high-ability types at the managerial level is μ_F^M.

An assumption of strict history independence is imposed. This simplifying assumption means that the firm conditions its decisions only on the current period. Furthermore, it implies that all information about the ability of the employees is conveyed within the firm i.e. other firms are unable to discriminate between the two types of employees based on actions taking place in the firm. Most importantly, outside offers are not conditioned on past achievements such as bonuses. The incentive scheme offered by other firms, however, might imply different utilities for the two different types. Thus, a good employee's outside option will be denoted by $\bar{U}_G$, and a bad employee's utility is $\bar{U}_B$ where $\bar{U}_G \geq \bar{U}_B$.

Under employee heterogeneity the firm still prefers to promote well-performing employees and to layoff individuals with low performance. The reason is twofold. First, promoting well-performing employees and laying off badly performing ones is useful for incentive purposes. Thus, conditional on promoting the firm prefers to promote well-performing employees for incentive reasons. Similarly, conditional on firing, the firm prefers to fire badly performing employees. Second, besides incentives selection considerations also strengthen the promotion and layoff policy. For promotions, as managerial output is relatively more important than non-management output ($C < M$), better quality employees are more desirable for promotion. Thus, the firm prefers to promote high-performance individuals as they are more likely to have high ability. Finally, conditioning layoffs on low performance makes sure that the fired individual is relatively more likely to be of low ability which improves the overall quality of the workforce and hence on productivity.

3.1.1 The period problem

In each period, the same contracting problem is repeated. If a separating equilibrium can be enforced, then the problem is essentially the same as in the homogeneous case.[7] However, if the firm is unable to write a contract that will make the potential employees self-select ex-ante, a pooling equilibrium arises. A sufficient condition for the existence of a pooling equilibrium is stated in Remark (2).

Remark 2 *Separation is impossible ex-ante and the pooling equilibrium prevails, if the bad worker's outside option is sufficiently low (lower than $\bar{U}_B^*$ specified in the appendix is sufficient).*

The intuition behind the result follows from the fact that low-ability individuals are more likely to have low performance than high-ability individuals. Hence the firm can discourage low-ability individuals to apply for the job by punishing low performance. These disincentives can be imposed to a degree where high-ability individuals are indifferent between the contract offered by the firm and other contracts available in the market. However, if low-ability individuals have sufficiently worse outside options than high-ability individuals, then the disincentives the firm can impose might not be strong enough to elicit self selection of good types. One implication of Remark (2) is, that the individual rationality constraint of the bad employees is always satisfied in a pooling equilibrium.

Consequently, the contracting problem of the pooling equilibrium can be summarized as follows[8]

$$\begin{aligned}\max_{w,b,\pi_F,\pi_P} \Pi(w,b,\pi_F,\pi_P) = \max_{w,b,\pi_F,\pi_P} & \frac{M\mu_F^M}{H} + \mu_F(\theta_G+\delta)(C-b) \\ &+(1-\mu_F)(\theta_B+\delta)(C-b) - w - \mu_F(1-\theta_G-\delta)\pi_F K \\ &-(1-\mu_F)(1-\theta_B-\delta)\pi_F K - (g+\frac{g'}{H})K\end{aligned} \tag{6}$$

subject to

$$\begin{array}{lrcl}
(IC_G) & w-e+(\theta_G+\delta)[b+\pi_P U_P]-(1-\theta_G-\delta)\pi_F U_F & \geq & w+\theta_G[b+\pi_P U_P]-(1-\theta_G)\pi_F U_F \\
(IC_B) & w-e+(\theta_B+\delta)[b+\pi_P U_P]-(1-\theta_B-\delta)\pi_F U_F & \geq & w+\theta_B[b+\pi_P U_P]-(1-\theta_B)\pi_F U_F \\
(IR_G) & w-e+(\theta_G+\delta)[b+\pi_P U_P]-(1-\theta_G-\delta)\pi_F U_F & \geq & \bar{U}_G \\
(F) & (\mu_F\theta_G+(1-\mu_F)\theta_B+\delta)\pi_P & \leq & \frac{g'}{H} \\
\text{nonnegativity} & 0 & \leq & b \\
\text{pooling condition} & \bar{U}_B & \leq & \bar{U}_B^* \\
\text{probability} & 0 & \leq & \pi_F,\pi_P \leq 1
\end{array}$$

The first thing to note is that the two IC constraints can be simplified to a single equation[9]

[7] If ex-ante employee self-selection is possible, then the extension to heterogenous employees is trivial and leads to the basic model. Then a separating equilibrium is obtained by adding an additional constraint to the optimization problem which will discourage lower ability workers to apply for the job.

[8] Note that although the profit depends on μ_F, it is taken for granted since the firm cannot control the composition of the workers within the period.

[9] Which is exactly the same as the IC constraint in the basic model (after some simplification).

$$(IC) \qquad \delta[b + \pi_P U_P] + \delta \pi_F U_F \geq e.$$

Hence, if the IC and IR constraints are binding, then the following equalities will hold

$$\begin{aligned} b &= \frac{e}{\delta} - \pi_F U_F - \pi_P U_P \\ w &= e - (\theta_G + \delta)[b + \pi_P U_P] + (1 - \theta_G - \delta)\pi_F U_F + \bar{U}_G. \end{aligned}$$

It is not trivial, however, that the F constraint is binding. Incentive considerations support promotions from inside, but internal promotions decrease the quality of the workforce pool at the non-management level. This has the implication that the pool from which the firm can promote in the future becomes increasingly worse and at the same time it reduces the average productivity of the non-management employees. Consequently, one can not assume in general that the F constraint holds without slack which is a significant deviation from the basic model.

Nevertheless, it is possible to show conditions, under which the firm prefers to promote mostly or exclusively from inside to the managerial level. Using the terminology from Doeringer and Piore (1971), this corresponds to identifying conditions under which the firm has ports of entry. In this context, it can be shown that if the managerial rent (U_P) is sufficiently high, then incentive considerations outweigh quality concerns.

Remark 3 *The firm has ports of entry and the F constraint binds, if the profit is increasing in the promotion probability*

$$\frac{\partial \Pi}{\partial \pi_P} > 0.$$

A sufficient condition for the above is that the managerial rent (U_P) from promotion is high enough.

3.1.2 Steady state equilibrium

The one-period problem alone does not solve the infinitely repeated problem (quite differently from the homogenous case). The problem is that in the one-period game the equilibrium employee composition is undetermined. In order to solve this endogenously, the steady state equilibrium is explored. That is, if the firm applies a stationary firing and promotion policy, then the workforce composition will converge to a steady state value. This steady state value is determined in Lemma (2).

Lemma 2 *In steady state the equilibrium proportion of good workers (μ_F^*) in the non-management level depends on the firing and promotion probabilities:*

$$\mu_F^* = \mu \frac{(1 - \theta_B - \delta)\pi_F + g}{(1 - \theta_G - \delta)\pi_F + \mu(\theta_G - \theta_B)\pi_F + (\theta_G + \delta)\pi_P + g} \tag{7}$$

Moreover, the proportion of good quality workers at the managerial level can be determined in steady state equilibrium.

$$\mu_F^{H*} = \frac{\pi_P \mu_F^*(\theta_G + \delta) + \overbrace{\left(\frac{g'}{H(\mu_F^*\theta_G + (1 - \mu_F^*)\theta_B + \delta)} - \pi_P\right)\mu}^{external\ hiring\ (F\ slack)}}{\pi_P(\delta + \mu_F^*(\theta_G - \theta_B) + \theta_B) + \frac{g'}{H(\mu_F^*\theta_G + (1-\mu_F^*)\theta_B + \delta)} - \pi_P} \tag{8}$$

Lemma (2) allows for some comparative statistics. It follows from equation (7) that increasing internal promotions decrease the equilibrium quality of non-management employees. Increasing firing, however, has the opposite effect. Equation (8) shows the role of internal promotions and external hiring. For instance, the better the pool of internal non-management employees the worse is the effect of external hiring on managerial quality.

3.1.3 Simulating the optimal incentive mix

With these results at hand the equilibrium steady state profit levels can be simulated as a function of the firing parameter and the promotion parameter. Note that direct substitution back to (6) is not possible as μ_F^* and π_P are jointly determined. Nevertheless, it is straightforward to simulate the behavior of the profit function, see Figure (1).[10] It is obvious from the left-hand side graph that the optimal policy for the firm is to promote as many individuals as possible, i.e. $\pi_P^* > 0$. The right hand side graph illustrates the profit as a function of π_F conditional on the highest possible promotion rate. Clearly, the optimal level of firing is non-zero, i.e. $\pi_F^* > 0$. Given π_P^* and π_F^*, it follows that $w^*, b^* > 0$. Hence, it is optimal for the firm to offer a contract that involves fixed wages, bonuses, promotions and firing. This is the optimal incentive mix observed in firms. Lemma (3) summarizes the result of the simulation.

Lemma 3 *In the job assignment model an optimal incentive mix of fixed wages, bonuses, firing and promotion can arise in equilibrium.*

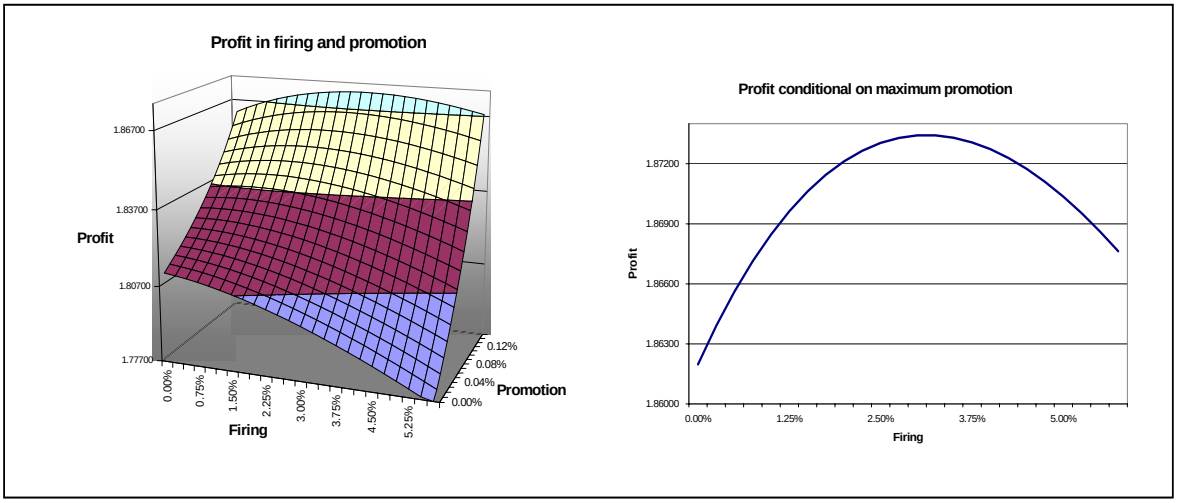

Figure 1: Optimal incentive mix in case of sorting.

[10] The parameters used: $\mu = .4$, $\theta_G = .5$, $\theta_B = .2$, $\delta = .2$, $C = 9$, $H = 25$, $K = 3$, $e = .85$, $g = 6\%$, $g' = 2\%$, $U_F = 1$, $U_P = 1$, $\bar{U}_G = 2$, $M = 25$. In equilibrium the incentive mix is: $\pi_F = 3.25\%$, $\pi_P = .15\%$, $w = .9075$, $b = 4.2185$ (note: values are rounded)

3.2 Characterizing employee careers

In the above discussion, it is established that in a firm where the optimal contract involves paying an incentive mix, composition and allocation of the workforce is important for profit maximization. We used Lemma (2) to establish the relation between the firm's choice of firing and promotion probabilities and the steady state composition of the workforce. Lemma (2), however, is silent about the consequences of these policies for individual workers. This section investigates these implications in a firm with ports of entry.[11]

In equilibrium it is optimal for the firm to promote and layoff a proportion of the workforce each period. Because these decisions are based on employee performance, they have different consequences for high- and low-ability employees. In particular, for the high-ability worker the probability of being fired in any given period is $(1-\theta_G-\delta)\pi_F$ and for the low-ability worker the probability is $(1-\theta_B-\delta)\pi_F$. Hence, it is more likely that the bad worker looses his job. By the same logic, good workers are more likely to be promoted as $(\theta_G+\delta)\pi_P > (\theta_B+\delta)\pi_P$. This indicates that there is a close relation between the promotion and layoff parameters and the firm's selection scheme.

This allows for investigating the effects of the optimal contract on selection in rank and tenure. Selection on tenure is always positive, as well-performing agents, who are more likely to be of good quality than the average employee, are promoted.[12] The intuition is also straightforward for selection on tenure. If promotion is sufficiently high compared to firing, then good-ability employees are promoted faster out of the non-management rank than bad-quality ones are weeded out by firing. Thus, selection on tenure is negative. Similarly, selection on tenure can be positive, if firing is sufficiently high compared to promotions. These results are formalized in Lemma (4).

Lemma 4 *With ports of entry: The selection process in the firm is positive in rank (conditional on tenure). Selection on tenure (conditional on rank) is negative if*

$$\pi_P > \pi_F$$

and positive if

$$\pi_P < \pi_F.$$

Previous studies have analyzed the selection mechanism, Medoff and Abraham (1980, 1981), Lazear (1992), Gibbs (1995), and Gibbons and Waldman (1999b). For instance, based on a series of empirical findings Medoff and Abraham (1980, 1981) write *"... the negative within-grade-level correlation which we suspect exists between experience and ability"*. Lazear (1992) continues this discussion arguing that *"Individuals who remain on the job longer do worse than those who are promoted out early. Wages actually decline with job tenure, probably reflecting the fall in the average worker's quality with length of time in the job"*. In the context of our model, it is clear that if only selection due to promotions was at work, the quality of the workforce that is passed over for promotion will decline with tenure. The optimal contract, however, shows that the firm has incentives to lay off a proportion of the low-performing workers each period. That this behavior affects the selection process is a point that is often

[11] The firm studied in this paper makes use of a high degree of internal managerial hirings (i.e. we observe ports of entry). This motivates the above focus.

[12] Remember, we still assume that the F constraint is binding.

missed in the literature. Gibbs (1995) comes closest to this point and concludes that "*... employees are continuously selected out through promotions, demotions, or exits. Because of these selection effects, ability of the group should decline with tenure...*".

In the context of the model, it is clear that negative selection on tenure is a possibility, but not the rule. If the firing probability exceeds the promotion probability, the selection scheme in tenure shifts from negative to positive. As it will turn out in the empirical analysis conducted below, our firm has a positive selection on tenure (conditional on rank) which provides a counterexample to the earlier conclusions drawn about negative within rank employee selection.

Bonus payments can be used to test for selection in the firm. A basic assumption of the model is that the firm rewards high performance with bonuses. Thus, the firm's selection scheme has direct consequences for the way bonuses are paid to the individuals. For instance, in the positive selection case the probability of being of high ability is increasing in tenure and rank, and thus the probability of receiving bonuses is also increasing in tenure and rank. If the firm has a negative selection, the probability of receiving bonuses would only increase in rank. These results are summarized in Corollary (1).

Corollary 1 *The probability of receiving bonus payments is increasing in rank (conditional on tenure). Furthermore, it is increasing in tenure (conditional on rank) if and only if the firm has a positive selection on tenure.*

Selection also affects earnings in tenure. The absence of learning and human capital acquisition in the model and the fact that the individual's external option ($\bar{U}_G$) is time-unvarying imply that the fixed wage in the contract is constant. Yet, the combination of an increasing likelihood of receiving bonuses and the flat wage-tenure profile leads to an increasing earnings-tenure profile.[13] Corollary (2) summarizes the result.

Corollary 2 *If the firm has a positive selection on tenure, then the within rank expected earnings-tenure profile is increasing.*

3.2.1 The firm's choice of selection regime

Corollaries (1) and (2) show that employee sorting is important not only for the observed incentive mix to be an optimal contract, but also because the selection regime predicts the outcome of other key variables in the firm. This makes it interesting to understand what components in the underlying market structure will lead to a particular selection regime. In this context, it is important to recognize that it is the relative magnitude of the firing and promotion parameters that determines the selection scheme. This is the message from Lemma (4). In general, it is more likely that the firm will have positive selection if firing is relatively common and promotions are relatively rare.

First, the quality difference between the high- and low-ability individuals ($\theta_G - \theta_B$) affects the firm's choice of selection scheme. The reason is that the quality difference determines the firm's incentive to sort and select the employees, i.e. a large difference will make it profitable for the firm to fire more

[13] The flat wage-tenure profile is an obvious drawback of the pure job assignment model. For this reason, potential extentions to integrative models where the job assignment model is combined with on-the-job human capital aquisition and learning are discussed below.

workers each period as the quality composition of the workforce has a stronger effect on profits. Thus, large quality differences make positive sorting more likely.

Second, the social costs of firing (U_F and K) have a negative effect on the firing rate. Hence, it reduces the likelihood of a positive selection. This follows from the fact that the firm shoulders all the costs of turnover in equilibrium. A reduction of these costs, however, implies that the firm can rely more on firing both for sorting and incentive purposes. Bringing the argument to the extreme, i.e. a situation where firing is socially costless ($U_F = K = 0$), the efficiency advantage of the performance pay solution actually vanishes. Thus, in this situation firing would be an equally efficient device to motivate employees and in addition it will contribute to sorting. Consequently, firing would dominate bonus payments.

Also, the level of exogenous turnover affects the policy of the firm. When the exogenous turnover at the managerial level (g') goes up, the promotion probability increases due to the binding F constraint. This will make negative sorting more likely. A different argument applies to an increase in the exogenous turnover at the non-management level (g). When non-management employees are more likely to leave for exogenous reasons, it increases the firm's costs to improve on the equilibrium employee composition (μ_F^*). Since firing is less cost-efficient, it will then be reduced, which increases the likelihood of negative selection.

This discussion indicates that employee quality issues and variation in the cost of firing or the level of exogenous turnover are important determinants for the firm's choice of selection scheme. Variation in these parameters across time, sectors and economies will be useful predictors for the type of selection that will be observed in firms.

3.3 Empirical predictions

Besides producing the optimal incentive mix the model provides additional empirical predictions. These predictions are outlined in the conjectures below and will be tested empirically in the following sections. The conjectures are derived directly from the basic assumptions of the model and its lemmas, so they are not proven formally again.

Conjecture 1 (Sorting) *Sorting is a necessary condition for an incentive mix of fixed wages, bonuses, firing and promotions to be an optimal contract.*

Conjecture 2 (Selection) *With ports of entry: The firm has a positive selection on rank (conditional on tenure). Furthermore, it has a positive selection on tenure (conditional on rank) if and only if $\pi_F > \pi_P$. Else, it has a negative selection on tenure.*

Conjecture 3 (Bonus) *The probability of receiving a bonus depends positively on the employees' ability. Thus, the likelihood of receiving a bonus increases in rank. Furthermore, the probability of receiving a bonus is increasing in tenure if and only if the firm has a positive selection on tenure.*

Conjecture 4 (Earnings) *The within rank earnings profile (fixed wages plus bonus) is increasing if and only if the firm has a positive selection on tenure. Else it is declining.*

Conjecture 5 (Promotion) *The likelihood of being promoted depends positively on the employees' ability. Thus, the probability of being promoted increases in tenure if and only if the firm has a positive selection on tenure.*

Conjecture 6 (Firing) *The likelihood of being fired depends negatively on the employees' ability. Thus, the probability of being fired declines in tenure if and only if the firm has a positive selection on tenure.*

4 Data

Four years of monthly personnel records from the main production site of an international pharmaceutical company are used in the empirical analysis.[14] Average full-time employment in the firm over the period 1997 to 2000 is 5055 persons.[15] These workers are distributed across four hierarchical levels ranging from CEO to non-management, see Figure 2. The share of management workers in the firm is 4.75 percent on average over the four years.

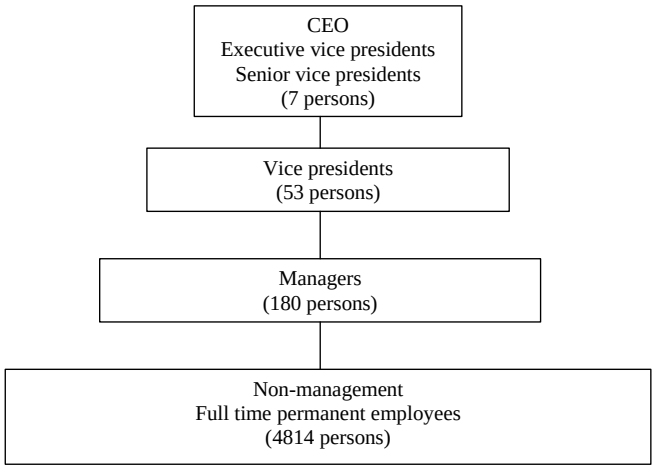

Figure 2: The hierarchy of the firm, 1997 to 2000.

The analysis below uses only those individuals who participate in the performance pay system, i.e. the employees who besides their base salary can get a bonus given sufficiently high performance evaluations. The distinction between strictly fixed paid employees and employees having bonus options can be made by looking at the worker's job category. This exercise reveals that the group of fixed paid employees constitutes 36.96 percent of the workforce and that it mainly consists of production workers.[16]

[14] The study is conducted on monthly observations in contrast to yearly observations. The main motivation for doing this is that the timing of events within a year proves to be important. In the regression results presented below, a "correction" for the use of high-frequency data is made by using clustered standard errors.

[15] The analysis is focused on permanent full-time employment which corresponds to 92.66 percent of all individuals employed on the production site.

[16] Workers choose between a fixed wage and performance-based contracts ex-ante, and hence the analysis is performed conditional on this initial sorting.

The characteristics of the employees calculated from the monthly employee-based observations are presented in the second and third columns of Figure 3. On average, the employees included in the analysis, i.e. those who participate in the performance pay system (column 3), have 8.90 years of tenure. The same group of individuals consists of 60.60 percent women and the average age is 39.51 years.

The level of education in the firm is high. In fact, 21.9 percent of the employees have at least a Master Degree and 11.7 percent have a degree that corresponds to a Bachelor Degree in duration. Only 12.5 percent of the labor force is unskilled.[17] There are three reasons why this structure is observed. First, the firm is operating in an industry where product development is essential for survival. Hence a large proportion of the workforce is engaged in research and development. Second, production is highly automated suggesting that low-skilled labor has been substituted for capital. Finally, in order to comply with the regulations from the Food and Drug Administration (FDA) extensive quality control programmes are implemented which require skilled labor.

	Mean (standard deviations)	
	All employees Observations = 241,951	Employees participating in the performance pay system Observations = 147,558
Age	39.746 (9.125)	39.505 (8.541)
Gender (women = 1)	0.566	0.606
Unskilled	0.125	0.035
Skilled worker	0.266	0.159
Short theoretical education	0.190	0.280
Bachelor degree	0.117	0.174
Masters or PhD degree	0.219	0.352
Tenure less than 2 years	0.126	0.119
Tenure 2 to 5 years	0.303	0.289
Tenure 6 to 10 years	0.273	0.257
Tenure above 10 years	0.299	0.334

Figure 3: Descriptive statistics.

4.1 The empirical contract and the theoretical assumptions

According to the firm's official remuneration principles, it offers wages to the employee such that: "*The base salary is a competitive pay for job function, responsibilities and competencies*". The data provide evidence for the fact that this principle is taken seriously. For instance, a preliminary analysis of the fixed wage component reveals that a set of individual characteristics and information about the

[17] Information on education is missing for 8.21 percent of the employees. However, for the group of employees receiving performance pay only 2.84 percent have missing information on education.

employee's job category predicts wages precisely $\left(R^2 = 0.852\right)$.

The close relation between credentials and wages indicates that the wage is virtually independent of employee performance. Instead, individuals are rewarded for performance through a bonus system. The allocation of bonuses fulfills the criteria that: *"The principles, criteria and targets that will lead to bonus payments should be known by the relevant employee subgroup"*. The size of the bonus pool varies across the different employee subgroups and constitutes 2.5 to 4 percent of the wage sum for non-management workers and 8 percent for non-executive managers.[18] The bonus system is widespread and according to the data 24 percent of workers in non-management receive bonus payments in a given year. The numbers are closer to 75 and 80 percent for managers and vice presidents, respectively.

The institutional settings impose no restriction on whom to fire and the firm seems to use firing frequently. The firm's yearly separation rate is 10.96 percent. The turnover is costly for the firm, but nevertheless 12.64 percent of all separations are initiated by the firm through layoffs. The separation rate for the employees participating in the performance pay system is 6.64 percent. Of these 19.46 (!) percent are initiated by the firm. Thus, firing is a significant component of the observed incentive mix.

There are two motivations for laying off workers. First, the firm's official wage strategy is to: *"Offer attractive salary and employment conditions"* in order to *"attract, develop and retain qualified employees"*. Given the "attractive" wages, the firm can use the threat of a layoff to motivate the workers.[19] Second, the layoffs serve as a sorting device where a proportion of the employees with low performers are forced to separate from the firm in each period. Hence in a pooling equilibrium where both high- and low-ability workers are employed by the firm, layoffs can be used to control the worker composition.

Finally, the firm has ports of entry. This claim is based on the observation that 98.22 percent of the employees are hired into the lowest hierarchical level. One implication of ports of entry is that management vacancies are filled with incumbent employees. Naturally this policy serves both sorting and incentive purposes. The wage premium (unconditional on human capital) associated with a promotion from non-management and into lower and middle management are 52.46 percent and 85.65 percent, respectively.

In the presentation of the firm, it becomes clear that the contract offered to the workers is highly complex in nature. First, the firm pays fixed wages to all workers who accept the contract unconditional on performance. Second, 63 percent of the employees can be rewarded for performance through a bonus system. Third, even though turnover is costly for the firm, a significant part of separations is firm-initiated i.e. layoffs. Finally, the firm has ports of entry which implies that promotions to higher level jobs take place (mainly) from the pool of incumbent employees. In sum, the data describes a contract that contains four incentive parameters: w, b, π_P, and π_F.

[18] The subsequent analysis is based on non-management employees, managers and vice presidents. The executive management is omitted due to lack of data.

[19] All workers in the firm have defined contribution pension plans, hence a layoff will not change the value of the current pension account. In other firms, where workers have defined benefit pension plans, the costs of a layoff in terms of lost pension may be significant. This effect could potentially create large incentives in other firms.

5 Empirical analysis

The conjectures stated above will be tested empirically in this section. A first goal is to establish that the firm is sorting its employees. According to the job assignment model, this is a necessary condition for the observed incentive mix to be an optimal contract. Secondly, the firm's selection scheme is identified to be positive in rank and tenure. This information provides predictions about other key variables in the firm such as wage growth, bonus payments, firing and promotions. For this reason, a test of the model's broader predictions will be conducted in the final part of the analysis.

Conjecture (1) stated that employee sorting is a necessary condition for the incentive mix to be an optimal contract. Preliminary evidence for sorting is found in the description of the empirical contract where it is established that layoffs and promotions are common in the firm, i.e. $\pi_P, \pi_F > 0$. The presence of layoffs and promotions are necessary conditions for sorting, but not sufficient in the sense that random decisions would produce no sorting. Thus, a first test for the presence of sorting is to investigate whether the firm makes random decisions or whether it bases its decisions on information about the employee's expected ability (such as revealed performance). The measure of performance that will be used in the analysis is bonus payments.[20]

To establish that the firm is sorting a multinomial logit model is estimated. In the model the individual is facing the three options: stay within rank, promotion and layoff.[21] The point estimates of a multinomial logit are difficult to interpret, and hence the results of the model are evaluated using its predictions.[22] The effect of a bonus payment on the transition probabilities for the average sample member is presented in row four of Figure 4. A person who receives a bonus payment (i.e. who had high performance) has a 1.361 per mille chance of being promoted in a given month while the layoff probability is as low as 0.297 per mille. In contrast, a person who did not receive a bonus payment (low performance) has almost no chance of being promoted and faces a 2.704 per mille risk of being laid off from the firm in any given month.[23] This is clear evidence for the fact that the firing and promotion decisions are based on employee performance meaning that the firm is consciously sorting its employees.

Conjecture (2) gave conditions when sorting is positive or negative. They can be tested from the empirical evidence presented in Figure 4. First, the results reveal that high-performance employees are relatively more likely to be promoted. Since high-ability individuals are more likely to have a high performance, this finding provides evidence for a selection scheme that is positive on rank. Second, the conditional layoff probability ($\pi_F = 2.704$ per mille) exceeds the conditional promotion probability ($\pi_P = 1.361$ per mille). According to Lemma (4), this implies a positive selection on tenure.

Moreover, the positive selection might be even stronger than shown by these numbers due to a bias in the layoff data. A bias arises in the case where the firm is signalling to the worker that the employment relation will end in the near future. This signal will make the worker look for alternative

[20] Bonuses paid to the individual during the preceeding 12 months are used as an indicator for performance. We interpret bonus payments as signals of high performance, i.e. if the individual recieved a bonus payment in the period, he must have had high performance.

[21] It should be noted that the individuals who separate for natural causes such as retirement or death and the individuals who leave the firm for a new job have been deleted from the sample.

[22] The full regression results of the multinomial logit can be seen in the appendix.

[23] The unconditional yearly promotion probability in the firm is 1.33 percent. In comparison, Lazear (1992) observes a yearly promotion probability close to two percent.

	Destinations		
	Promotion	**Layoff**	**Stay within rank**
Probability of average sample member being in category [sample means in brackets]	1.111 ‰ [1.111 ‰]	1.130 ‰ [1.130 ‰]	99.759 % [99.759 %]
Probability of average sample member being in category given bonus is received.	1.361 ‰	0.297 ‰	
Probability of average sample member being in category given no bonus is received.	0.631 ‰	2.704 ‰	
Change in probability due to bonus	0.730 ‰	- 2.407 ‰	

Note: The predictions are based on a regression with 97,925 observations.

Figure 4: Predicted monthly transitions.

employment which may be obtained before the firm terminates the match. We have reason to believe that this procedure is common. The implication is that the layoffs observed in the firm only constitute a lower bound on the separations that in reality are layoffs.[24]

Conjecture (3) focuses on bonus payments. Since the likelihood of receiving bonuses depends on employee ability, the selection scheme used by the firm and the probability of receiving bonus are closely linked (see Corollary 1). One consequence of the positive selection identified in the firm is that more able workers in general will have longer tenure and will be assigned to higher rank jobs. Hence, with positive selection employees who have longer job tenure and who are located at higher ranks (conditional on tenure) will be more likely to receive bonus payments. This observation provides a first test for the model's broader predictions.

Figure 5 presents logit regressions for the probability of having received a bonus payment during the preceding year. In the first model, tenure and rank are included as explanatory variables together with dummies for job category and time. As expected, both tenure and rank have significantly positive effects on the probability of receiving bonuses. Furthermore, the results are robust to the inclusion of information on demography (age-splines and gender), see model 2.

In model 3, information on education is added. The positive relation between educational attainment and ability and the increased production capacity of educated workers implies, that workers with higher levels of schooling will be more likely to receive bonus payments, see Becker (1964). This result is confirmed by the data. Important for the analysis, however, is to note that the positive effects of tenure and rank on the probability of receiving bonuses survive in model 3. This indicates that ability is signalled only partially through education. Thus, sorting and selection considerations are important even though the level of formal schooling is observed.[25]

[24] For the sake of completeness we remark here, that selection might get weakened by employee-initiated turnover. High-ability employees, who are underappreciated, might leave the firm for better offers. This, however, does not seem to be a serious problem, as it would require outside firms to have systematically better information than the current employer. It seems, that employee-initiated endogenous turnover (if any) is about terminating a bad match, rather than about ability.

[25] The tenure effect identified in Figure (5) could in principle be driven by the fact that managerial employees, who in general have longer tenure, are more likely to recieve bonuses. For this reason, model (3) has been estimated for the

	(1) Logit model	(2) Logit model	(3) Logit model	(4) Fixed-effects Logit model
Constant	-0.975	-1.098	-1.524	-
	(0.056)	(0.072)	(0.162)	
Tenure less that 2 years	-	-	-	-
Tenure 2 to 5 years	0.705	0.698	0.696	0.001
	(0.058)	(0.058)	(0.059)	(0.098)
Tenure 6 to 10 years	0.755	0.713	0.717	-0.100
	(0.058)	(0.059)	(0.059)	(0.119)
Tenure 10 to 15 years	0.821	0.767	0.763	-0.082
	(0.060)	(0.061)	(0.062)	(0.132)
Tenure 15 to 20 years	0.765	0.712	0.720	0.185
	(0.078)	(0.079)	(0.079)	(0.188)
Tenure 20 to 25 years	0.833	0.787	0.800	0.216
	(0.110)	(0.111)	(0.110)	(0.286)
Tenure above 25 years	0.911	0.893	0.900	2.554
	(0.122)	(0.134)	(0.134)	(1.069)
Non-management	-	-	-	-
Managers	0.680	0.662	0.690	0.892
	(0.151)	(0.152)	(0.150)	(0.377)
Vice presidents	1.620	1.596	1.588	1.747
	(0.329)	(0.329)	(0.328)	(0.898)
Unskilled			-	
Skilled worker			0.358	
			(0.133)	
Short theoretical education			0.394	
			(0.133)	
Bachelor degree			0.292	
			(0.137)	
Masters or PhD degree			0.480	
			(0.153)	
Demographic variables	NO	YES	YES	YES
Job category	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES
Log likelihood	-8018	-7996	-7986	-2496
# observations	13704	13704	13704	13704

Note: Standard errors are clustered with respect to individuals in models 1 to 3.

Figure 5: Logit regression for bonus payments.

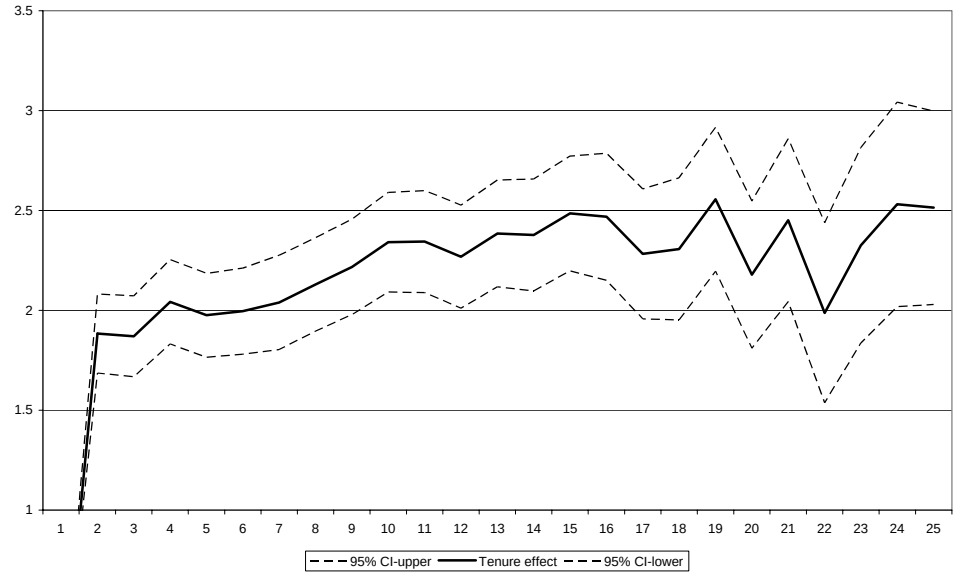

Figure 6: Tenure effect on the likelihood of receiving bonus payments.

An additional test for the relation between tenure and bonus is conducted by estimating a model similar to model (3) in Figure 5 with the tenure-splines substituted for yearly tenure dummies (detailed results are not shown).[26] Figure 6 illustrates the cross-sectional effect of tenure on the likelihood of receiving bonus for the first 25 years of tenure. In accordance with the predictions from positive sorting, the effect is increasing. Thus, the steady increase in the likelihood of receiving bonuses during the first many years of employment reflects that sorting and selection are integrated parts of the firm's policy.

Finally, the cross sectional relation between the probability of getting a bonus and tenure could arise if bonus is paid for tenure and not for performance as we argue in the model. For this reason a fixed-effects model where the individual's ability is "conditioned out" is estimated, model 4 in Figure 5. In this model the tenure effect is insignificant reflecting that ability and not tenure determines the probability of receiving bonus payments. This finding allows us to conclude that the increasing cross-sectional bonus-tenure profile observed in the data is driven by positive selection.

Conjecture (4) describes the within rank earnings-tenure profile. The prediction from the job assignment model is that positive selection will lead to an increasing earnings profile. Several tests of this hypothesis are presented in Figure 7. In the first model, it is established that earnings are increasing in both tenure and rank. These results are robust to the inclusion of information on demography (age and gender) and education, see model 2. Even though these findings support the hypothesis of an increasing within rank earnings-tenure profile, they constitute the Achilles heel of the job assignment model. The reason is that the shape of the earnings profile is motivated only through the increased likelihood of receiving bonus payments in the model. The data, however, shows that not only the likelihood of bonus payments, but also the fixed wage component increases. In the next section, it will

non-management employees only. The results show that the tenure effect is robust.

[26] The maximum level of tenure in the firm is 44 years but less than 3.25 percent of the employees have a tenure above 25 years causing large standard errors on the point estimates of the tenure dummies exceeding the 25th year.

be discussed how the job assignment model can be changed or extended to accommodate these fixed wage dynamics. For now these dynamics are disregarded.

	(1)	(2)
Constant	10.367	10.191
	(0.005)	(0.012)
Tenure less than 2 years	-	-
Tenure 2 to 5 years	0.051	0.040
	(0.005)	(0.004)
Tenure 5 to 10 years	0.128	0.094
	(0.005)	(0.005)
Tenure above 10 years	0.187	0.139
	(0.005)	(0.005)
Non-management	-	-
Managers	0.008	0.014
	(0.008)	(0.008)
Vice presidents	0.315	0.312
	(0.016)	(0.016)
Unskilled		-
Skilled worker		0.055
		(0.010)
Short theoretical education		0.062
		(0.010)
Bachelor degree		0.076
		(0.010)
Masters or PhD degree		0.116
		(0.011)
Demographic variables	NO	YES
Job category	YES	YES
Time dummies	YES	YES
R-squared	0.721	0.749
# observations	147,558	147,558

Note: Standard errors are clustered with respect to individuals.

Figure 7: Mincer earnings regressions.

Conjectures (5) and (6) state the model's predictions on promotions and layoffs. The positive selection identified in the firm implies that non-management employees with longer tenure are of higher average ability than more recently hired individuals. Above it is argued that this will lead to higher average performance of the tenured employees and consequently to a higher probability of promotion. Since the tenure effect on promotions is working through the performance variable, taking the employees performance into account would remove the tenure effect allowing only performance to have a significant effect on promotions. A similar argument goes for the layoff probability. The expected higher average performance of the high-tenured individuals will at first create a negative relation between layoffs and tenure. However, conditional on performance tenure will become insignificant.

These hypotheses are tested empirically using logit models for the probability of being promoted and laid off. The results are presented in Figure 8. A first observation is that promotions and layoffs are very difficult to predict due to absent information on performance. In model 1 where the probability of a promotion is estimated unconditional of employee performance, the expected tenure effect is absent. Instead there is a weak indication that skills are important for the promotion process since individuals with a masters or PhD degree have a marginally higher probability of being promoted than unskilled employees. This result, however, is washed out when conditioning on performance (model 2) indicating that education is important for the promotion process, but only through it's positive effect on the likelihood of getting bonuses. This leads to the conclusion that the firm only promotes workers

who have shown persistent high performance and that the direct effect of tenure on promotions is too weak to be identified in the present firm.

	Promotion regression (1)	**Promotion regression (2)**	**Layoff regression (3)**	**Layoff regression (4)**
Constant	-8.183	-8.603	-8.078	-8.085
	(1.080)	(1.080)	(0.909)	(0.915)
Bonus payment		0.771		-2.223
		(0.247)		(0.259)
Tenure 0 to 2 years	-	-	-	-
Tenure 2 to 5 years	0.307	0.154	0.925	1.211
	(0.400)	(0.398)	(0.741)	(0.742)
Tenure 5 to 10 years	0.215	0.037	1.103	1.456
	(0.425)	(0.421)	(0.736)	(0.739)
Tenure above 10 years	0.042	-0.171	1.267	1.651
	(0.455)	(0.452)	(0.739)	(0.741)
Unskilled	-	-	-	-
Skilled worker	-0.923	-0.925	-1.397	-1.131
	(1.228)	(1.226)	(0.502)	(0.502)
Short theoretical education	-	-	-0.487	-0.589
	(-)	(-)	(0.406)	(0.407)
Bachelor degree	1.216	1.087	0.448	0.798
	(1.039)	(1.041)	(0.393)	(0.390)
Masters or PhD degree	2.146	1.959	-0.307	0.342
	(1.020)	(1.023)	(0.394)	(0.340)
Demographic variables	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES
Log likelihood	-757.57	-752.11	-843.48	-793.20
# observations	67,743	67,743	97,925	97,925

Note: Standard errors are clustered with respect to individuals. In the period studied individuals with a short theoretical education are not promoted. These regressions consider only transitions out of non-management.

Figure 8: Layoff and promotion regressions.

The first layoff regression (model 3) in Figure 8 shows a pattern similar to the promotion regression namely that layoffs are difficult to predict due to absent information on performance. In model 4, it is reestablished that information on bonus payments has a highly significant effect on the decision on who to lay off. Also, there is an indication that layoffs are rare for newly hired individuals, i.e. employees with tenure less than two years. For the remaining employees (tenure above two years), the tenure effect is insignificant.

An additional point that arises from the results is that some employee subgroups (defined by education) are more likely to be laid off than others. This result seems to stay even with taking performance into account. This empirical result, however, may simply be an artefact of missing layoff information. The indicator of layoffs observed in the data only reflects a lower bound on the real extent of layoffs (as discussed above). In particular, the empirical result observed will emerge if the firm is more likely to signal that an employment match is about to end to some employee subgroups relative to others. In this particular case, it is explained if skilled workers (typically production workers) are more likely to get this information than white-collar workers. This can be due to increased sensitivity of information handled by more educated employees. Given the available data, we can only speculate about these matters.

In sum, the empirical evidence presented above provides strong evidence for the model's conjectures. First, using the description of the empirical contract and the multinomial logit regression it is

established that the firm is sorting such that high-performing employees are more likely to be promoted and less likely to be fired. Furthermore, these results show that the firm is using a selection scheme which is positive in both tenure and rank. Using the information on sorting and selection, the model provides prediction about other key variables. The broader predictions related to bonus payments, earnings growth, promotions and layoffs are not rejected by the data.

6 Discussion

In the seminal work by Gibbons and Waldman (1999b), they argue that: *"Any single (empirical) fact may be consistent with a variety of theories, so one way to choose among theories is by evaluating the extent to which each is consistent with a broad pattern of (empirical) evidence"*. This paper has attempted to do exactly that.

As to the initial research question of how the incentive mix observed in firms can be seen as an optimal contract, the paper focuses on the job assignment model. Within this context, it is argued that the firm's sorting and selection concerns provide the answer. This is a contribution in a theoretical sense, but it may not be the only answer to the question. For this reason, we conduct an empirical analysis where evidence for sorting and selection is identified and where the model's broader predictions are tested. In general, there is a close fit between the model's conjectures and the empirical evidence.

However, in one respect, namely in its ability to explain fixed wage dynamics, the model's conjectures seem not to be supported by the data. This suggests that the job assignment model should be extended by other building blocks in a larger integrative model, as in Gibbons and Waldman (1999a, b). In the following, we discuss the empirical findings which an integrative model has to accommodate. Also, we explore possible theoretical explanations and provide suggestions for future research.

Figure 9 presents a set of Mincer wage equations.[27] In contrast to the standard case, these regressions include information on the firm's hierarchy. Thus, the results obtained reflect within rank wage dynamics. Model 1 shows that the wage is increasing in the levels of education, tenure and rank. Model 2 explores the effect of a bonus payment on current wages. The point estimate shows that a bonus payment last period (which can also be interpreted as high performance) increases current wages by 2.61 percent. An extension of this analysis is conducted in model 3 where information of up to 3 years of lagged bonus payments is included. The remarkable result reveals that lagged bonus payments are highly significant. Furthermore, they have similar effects on current wages. Hence it is not the timing, but instead the event of bonus that seems to be important.

To investigate this issue further, model 4 estimates the effect of the last two years' bonus payments on the wage together with an interaction effect capturing the consequences for current wages of receiving bonuses in both years.[28] The interaction term in this regression is insignificant reflecting that bonuses are increasing wages every time they are observed. In other words, the bonus effect is not deflated or magnified when the employee is observed to have a persistent high performance. These empirical observations are not easily explained by the job assignment model, but a variety of other theoretical

[27] In contrast to the earnings regression shown in Figure 7, these regressions are based only on the fixed wage component, i.e. the measure is total earnings minus bonus payments.

[28] The focus on the last two years' bonus payments is only for expositional reasons and could easily be extended to include all three years and their interaction terms.

explanations provide possible explanations. Here, on-the-job human capital acquisition, symmetric learning and asymmetric learning with probabilistic outside offers are discussed.

	(1)	(2)	(3)	(4)
Constant	10.178	10.201	10.272	10.252
	(0.011)	(0.012)	(0.013)	(0.012)
Bonus payment last year		0.026	0.019	0.024
		(0.002)	(0.003)	(0.003)
Bonus payment two			0.017	0.026
years ago			(0.003)	(0.004)
Bonus payment three			0.020	
years ago			(0.004)	
Bonus payment last year				-0.004
and two years ago				(0.005)
Tenure less than 2 years	-	-		
Tenure 2 to 5 years	0.030	0.029	-	-
	(0.004)	(0.005)		
Tenure 5 to 10 years	0.085	0.081	0.048	0.052
	(0.004)	(0.005)	(0.004)	(0.004
Tenure above 10 years	0.128	0.126	0.093	0.098
	(0.005)	(0.007)	(0.004)	(0.004)
Unskilled	-	-	-	-
Skilled worker	0.052	0.050	0.036	0.048
	(0.010)	(0.009)	(0.010)	(0.010)
Short theoretical	0.061	0.062	0.044	0.057
education	(0.010)	(0.010)	(0.011)	(0.010)
Bachelor degree	0.074	0.074	0.063	0.072
	(0.010)	(0.010)	(0.011)	(0.010)
Masters or PhD degree	0.112	0.108	0.101	0.103
	(0.011)	(0.011)	(0.012)	(0.011)
Non-management	-	-	-	-
Managers	0.001	0.006	0.009	0.010
	(0.007)	(0.007)	(0.008)	(0.008)
Vice presidents	0.284	0.296	0.308	0.305
	(0.015)	(0.015	(0.015)	(0.015)
Demographic variables	YES	YES	YES	YES
Job category	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES
R-squared	0.852	0.867	0.882	0.874
# observations	147,558	105,940	32,530	67,247

Note: Standard errors are clustered with respect to individuals in models 1 to 4. The lagged variables included in the regressions determine the number of observations used in the estimation.

Figure 9: Mincer wage equations.

A first potential explanation for the fixed wage dynamics is on-the-job human capital acquisition. Gibbons and Waldman (1999a) show how this model in conjunction with other building block models (job assignment and learning) can be used to explain a large set of empirical evidence. The Gibbons and Waldman model distinguishes between the employees' innate ability which can be high or low and effective ability which is a function of the employees' innate ability and labor market experience. An assumption on symmetric information and steady changes in the employees' effective ability caused by continuous growth in labor market experience leads to fixed wage dynamics. Thus, on-the-job human capital acquisition implies a simple upward sloping fixed wage scheme for all workers. On-the-job human capital acquisition, however, cannot explain without a stretch the significant and positive

coefficient on bonus payments.

A second explanation is offered by symmetric learning. By considering learning the strict history independence assumption of the job assignment model is relaxed. The assumption implied that all information about the employees' ability is "forgotten" after each period. Realistically, however, the firm can learn about what employees are the more likely to be good through the performance history or signals. Moreover, under symmetric learning outside firms can be expected to condition offers on the very same signals. In the context of the model, signals will be education, bonus payments, survival in the firm and promotions.

Relaxing the assumption on history independence and introducing learning into the model has a profound impact on the contract offered. Let us assume that the firm and its competitors are concerned about the employees' perceived type ($\hat{\mu}$), i.e. the expected probability that the worker is of good quality given observable signals. The perceived type can be deduced by Bayesian updating. Formally,

$$\hat{\mu} = \hat{\mu}\left[E, b_0, b_1, b_2..., b_j, tenure = j\right] \tag{9}$$

Where E is the level of education and $b_0, b_1, b_2...$ are past bonus payments.

In order to keep the discussion focused, we assume that only the fixed wage component can respond to variation in the perceived type, i.e. the incentive parameters in the contract become: $\{w(\hat{\mu}), b, \pi_P, \pi_F\}$. Hence, the individual rationality constraint (IR_G) can be written as

$$(IR_G) \qquad w(\hat{\mu}) - e + (\theta_G + \delta)[b + \pi_P U_P] - (1 - \theta_G - \delta)\pi_F U_F = \bar{U}_G(\hat{\mu}).$$

Trivially, better workers have better outside options; thus $\bar{U}_G(\hat{\mu})$ is increasing in perceived type. From the individual rationality constraint it follows that better perceived workers also receive higher wages in the firm. Formally, if these variables are differentiable:

$$\frac{\partial \bar{U}_G(\hat{\mu})}{\partial \hat{\mu}} = \frac{\partial w(\hat{\mu})}{\partial \hat{\mu}} > 0 \tag{10}$$

The third explanation is based on asymmetric learning with probabilistic outside offers. If outside firms cannot observe bonus payments within the firm, the firm still might want to increase the fixed wage of those employees who are more likely to be of high ability. Let us suppose, that outside firms are uncertain with respect to the workers' quality and give imprecise, probabilistic offers. Thus, sometimes good employees leave the firm for alternative jobs. Yet, if the firm learns about the type of the worker, it will increase the wage of those employees, who are likely to be of high ability. The reason to do is that the firm can reduce the probability of high quality employees leaving the firm. Thus, probabilistic outside offers can also lead to wage increases in perceived type as conjectured in equation (10) above. The advantage of the probabilistic outside offer explanation is, that it does not necessarily require that the firm and outside firms would have the same information about employees.[29]

[29] Asymmetric information could in principle lead to strategic rewards, that is the current employer may give bonuses and promote strategically. The reason is that bonuses and promotions are signals to the market about the employees' quality. Waldman (1985) argues that these concerns may lead to inefficiencies. Without commitment problems, however, firms could commit to promote honestly and the inefficiency disappears. As in equilibrium employees foresee commitment problems, the firm would shoulder the cost of these inefficiencies in equilibrium. As the present exposition does not consider commitment problems, strategic rewards are ignored.

In sum, both symmetric and asymmetric learning and to a lesser degree on-the-job human capital acquisition provide explanations for how signals and learning can be used to create fixed wage dynamics. These theories imply, that our job assignment model can be extended to capture wage dynamics. Yet, to step further, more research is needed on firm-level data in order to evaluate the relative merits of the above theories precisely . At this stage, we can only say that wage dynamics can be made consistent with the model.

7 Conclusion

Firms are known to offer highly complex contracts to their employees. In this paper, we study how incentives are used in practice and provide a theoretical motivation for when the incentive mix observed in firms can be seen as an optimal contract.

A basic model with homogenous employees and the four most often used incentive parameters (fixed wages, bonuses, layoffs and promotions) is presented. This model confirms the findings in MacLeod and Malcolmson (1998) that the firm uses performance pay, if it can commit to bonus payments. Thus, the basic model does not exhibit a mix of incentives.

The basic model is extended to accommodate employee heterogeneity. In this job assignment model, the profit-maximizing firm adjusts the equilibrium composition of workforce through sorting and selection. Sorting and selection give a dual role for layoffs and promotions. First, they create incentives for the employees as in the basic model. Second, they can be used to adjust the composition of the workforce at any hierarchical level to maximize profits. It is the firm's sorting and selection concerns that provide the critical result: The optimal incentive mix.

The job assignment model is also used to make predictions about the career development of employees in firms. In particular, aspects of wage and earnings dynamics, bonus payments, promotions and layoffs are addressed. These conjectures are tested empirically using the personnel records from a large pharmaceutical company, and a close relation between the model's predictions and the data is found.

The main contribution of this paper is to refocus formal research on on-the-job sorting - by examining the role of firing. We show that selection on tenure depends on the specificities of the sorting mechanism applied by the firm. In particular, we show that a positive selection in tenure (conditional on rank) is possible. Moreover, our empirical data from a major pharmaceutical firm gives an example for a positive selection. This result questions the conventional wisdom on the necessity of negative selection in tenure.

The paper also opens up new avenues for future research. First, the new focus on on-the-job sorting needs more empirical and theoretical support. Second, the model could be used as part of a larger integrative model as suggested in Gibbons and Waldman (1999a,b). In this context, exploring the interaction between on-the-job sorting and other building blocks could prove to be useful.

8 Appendix

8.1 Proofs

8.1.1 Basic model

Proof of Lemma (1). Notice first, that the profit is a linear function of the firing probability:

$$(\theta+\delta)C - \overbrace{e+(\theta+\delta)\left[b+\frac{g'}{H(\theta+\delta)}U_P\right]}^{w} - (1-\theta-\delta)\pi_F U_F - \bar{U}$$
$$-(\theta+\delta)b - \left[(1-\theta-\delta)\pi_F + g + \frac{g'}{H}\right]K,$$

and that the bonus (b) is cancelling out of the equation. Then the first order condition in terms of the firing parameter can be written as:

$$-(1-\theta-\delta)U_F - (1-\theta-\delta)K < 0$$

Thus, firing is minimized in equilibrium. The result corresponds to our intuition, that the costs of motivating through firing increase in

- the cost of hiring and training (K)
- the utility loss of firing (U_F)
- the likelihood when firing has to be used ($1-\theta-\delta$). ■

Proof of Remark (1). The proof follows directly from the intuition provided prior to the remark. Slack means either zero bonus and firing for promotion slack (equation 5) or zero bonus for bonus slack (equation 3) or zero wage for wage slack (equation 4). Thus, no optimal incentive mix can arise. ■

8.1.2 Job assignment model

Proof of Remark (2). The pooling equilibrium arises if the firm cannot force the workers to self-select ex-ante. That is, the bad type employee will accept the contract, even if the firm sets the parameters of the contract, such that they create the strongest possible incentives. For the sufficient condition consider the following: The strongest incentives trivially include zero fixed wages and unit firing probability. Then the promotion probability is determined through the feasibility (F) constraint of problem (6), and the bonus payment is determined through the individual rationality constraint of the good type in the same problem. Thus:

$$\begin{aligned}\pi_P^* &= \frac{g'}{H(\mu_F\theta_G + (1-\mu_F)\theta_B + \delta)} \\ b^* &= \frac{\bar{U}_G + e + (1-\theta_G-\delta)U_F}{\theta_G+\delta} - \frac{g'}{H(\theta_G+\delta)}U_P\end{aligned}$$

Consequently, a sufficient condition for the pooling equilibrium is as follows:

$$-e + (\theta_B+\delta)[b^* + \pi_P^* U_P] - (1-\theta_B-\delta)U_F = \bar{U}_B^* \geq \bar{U}_B$$

■

Proof of Remark (3). The proof is straightforward. If the profit derivative with respect to π_P is positive, then naturally the upper bound is reached. Furthermore, note that

$$\begin{aligned}\frac{\partial b}{\partial \pi_P} &= -U_P, \\ \frac{\partial w}{\partial \pi_P} &= 0,\end{aligned}$$

and also that $\frac{\partial \mu_F^*}{\partial \pi_P}$ and μ_F^{H*} are independent of U_P. Thus, increasing U_P sufficiently will guarantee that

$$\frac{\partial \Pi}{\partial \pi_P} > 0.$$

■

Proof of Lemma (2). Given a fixed firing probability, the percentage of good workers employed by the firm can be determined. The measure of good employees promoted to managerial ranks is given as:

$$\mu_F^*(\theta_G + \delta)\pi_{P.}$$

Furthermore, the measure of good employees fired is

$$\mu_F^*(1 - \theta_G - \delta)\pi_F,$$

and the measure of good employees exogenously retiring is

$$\mu_F^* g.$$

The steady state condition is that the composition of the workforce does not change any further. This can be captured as the measure of good employees leaving the non-management rank in any period equals to the measure of good employees entering the non-management rank. It is summarized in the following equation:

$$\overbrace{\mu[\underbrace{(1-\mu_F^*)(1-\theta_B-\delta)\pi_F + \mu_F^*(1-\theta_G-\delta)\pi_F}_{\text{measure of all fired}} + g]}^{\text{measure of good type newly hired}}$$
$$= \overbrace{\mu_F^*(1-\theta_G-\delta)\pi_F}^{\text{measure of good type fired}} + \mu_F^* g + \overbrace{\mu_F^*(\theta_G+\delta)\pi_P}^{\text{measure of good type promoted}}$$

The statement on μ_F^* follows directly from here.

For management the steady state composition is given by the following logic. In the steady state only the equilibrium proportion of promoted workers matters, if F binds. All initial differences - if any - are deflated to zero by natural turnover g. The volume of good quality workers promoted is given by $\mu_F^*(\theta_G + \delta)\pi_P$ and the volume of bad quality workers promoted is $(1 - \mu_F^*)(\theta_B + \delta)\pi_P$. Also, if F does not bind, the slack is filled from the outside pool, with average quality μ. Hence, the proportion is given in the lemma. ■

Proof of Lemma (3). It is straightforward to create an example of an interior solution. Take, for instance, the parameter choice used for Figure (1). The parameters used there: $\mu = .4$, $\theta_G = .5$, $\theta_B = .2$, $\delta = .2$, $C = 9$, $H = 25$, $K = 3$, $e = .85$, $g = 6\%$, $g' = 2\%$, $U_F = 1$, $U_P = 1$, $\bar{U}_G = 2$, $M = 25$.

In equilibrium the incentive mix is: $\pi_F = 3.25\%$, $\pi_P = .15\%$, $w = .9075$, $b = 4.2185$ (note: values are rounded).

For the simulation: Expressing profit as the function of the firing (π_F) and the promotion parameter (π_P) is straightforward using the following easing notation:

$$\mu_F^* = \mu \frac{(1-\theta_B-\delta)\pi_F + g}{(1-\theta_G-\delta)\pi_F + \mu(\theta_G-\theta_B)\pi_F + (\theta_G+\delta)\pi_P + g}$$

The expression is:

$$\begin{aligned}
\Pi(\pi_F,\pi_P) = & \frac{\pi_P\mu_F^*(\theta_G+\delta) + \left(\frac{g'}{H(\mu_F^*\theta_G+(1-\mu_F^*)\theta_B+\delta)} - \pi_P\right)\mu}{\pi_P(\delta+\mu_F^*(\theta_G-\theta_B)+\theta_B) + \frac{g'}{H(\mu_F^*\theta_G+(1-\mu_F^*)\theta_B+\delta)} - \pi_P}\frac{M}{H} \\
+ \quad & \mu_F^*(\theta_G+\delta)\left(C - \frac{e}{\delta} + \pi_F U_F + \pi_P U_P\right) \\
& +(1-\mu_F^*)(\theta_B+\delta)\left(C - \frac{e}{\delta} + \pi_F U_F + \pi_P U_P\right) \\
& -e + (\theta_G+\delta)(\frac{e}{\delta} - \pi_F U_F) - (1-\theta_G-\delta)\pi_F U_F - \bar{U}_G \\
& -\mu_F^*(1-\theta_G-\delta)\pi_F K \\
& -(1-\mu_F^*)(1-\theta_B-\delta)\pi_F K \\
& -(g+\frac{g'}{H})K
\end{aligned}$$

This two-variable equation is depicted on Figure (1). The F constraints, however, must be separately checked in the simulation. ■

Proof of Lemma (4). For the first statement consider (and remember that F is binding):

$$\begin{aligned}
\mu_F^* &< \mu_F^{M*} = \frac{\mu_F^*(\theta_G+\delta)}{\mu_F^*(\theta_G+\delta)+(1-\mu_F^*)(\theta_B+\delta)} \\
\mu_F^*(\theta_G+\delta)+(1-\mu_F^*)(\theta_B+\delta) &< (\theta_G+\delta) \\
(1-\mu_F^*)(\theta_B+\delta) &< (1-\mu_F^*)(\theta_G+\delta)
\end{aligned}$$

For the second statement consider the system:

$$\begin{array}{c} \Pr(b_t=1|i) = \theta_i+\delta \\ \Pr(b_t=0|i) = 1-\theta_i-\delta \end{array},$$

$$\begin{array}{c} \Pr(promotion=1|i,b_t=1) = \pi_P \\ \Pr(promotion=1|i,b_t=0) = 0 \end{array},$$

and

$$\begin{array}{c} \Pr(layoff=1|i,b_t=0) = \pi_F \\ \Pr(layoff=1|i,b_t=1) = 0 \end{array}$$

where $i = B, G$. Then,

$$\begin{aligned}
\Pr(tenure &= j, rank=r, b_t=b, b_{t-1}=b, ..., b_{t-j}=b|G) \\
&= k_{(j,r)}\left[(\theta_G+\delta)\pi_P\right]^r\left[1-(\theta_G+\delta)\pi_P-(1-\theta_G-\delta)\pi_F\right]^j \\
&= k_{(j,r)}\left[(\theta_G+\delta)\pi_P\right]^r\left[1-\pi_{S|G}\right]^j,
\end{aligned}$$

$$\begin{aligned}\Pr(tenure &= j, rank = r, b_t = b, b_{t-1} = b, ..., b_{t-j} = b|B) \\ &= k_{(j,r)} \left[(\theta_B + \delta)\,\pi_P\right]^r \left[1 - (\theta_B + \delta)\pi_P - (1 - \theta_B - \delta)\pi_F\right]^j \\ &= k_{(j,r)} \left[(\theta_B + \delta)\,\pi_P\right]^r \left[1 - \pi_{S|B}\right]^j\end{aligned}$$

for $j \geq 0, 0 \leq r \leq j$ where r and j are integers and $k_{(j,r)}$ are coefficients depending on j and r. $\pi_{S|G}$ and $\pi_{S|B}$ are the probabilities of leaving a given rank for the good and bad workers, respectively. Recalling that the initial proportion of good types hired by the firm is μ, then the probability of being of a good type conditional on tenure, rank and past bonus payments becomes

$$\begin{aligned}P_G &= \Pr(G|tenure = j, rank = r, b_t = b, b_{t-1} = b, ..., b_{t-j} = b) \\ &= \frac{\mu k_{(j,r)} \left[(\theta_G + \delta)\,\pi_P\right]^r \left[1 - \pi_{S|G}\right]^j}{\mu k_{(j,r)} \left[(\theta_G + \delta)\,\pi_P\right]^r \left[1 - \pi_{S|G}\right]^j + (1 - \mu) k_{(j,r)} \left[(\theta_B + \delta)\,\pi_P\right]^r \left[1 - \pi_{S|B}\right]^j}\end{aligned}$$

and the probability of being a bad type given tenure, rank and past bonus payments is

$$P_B = \Pr(B|tenure = t, rank = r, b_t = b, b_{t-1} = b, ..., b_{t-j} = b) = 1 - P_G$$

using the fact that good workers are more productive than bad workers, i.e. $\theta_G = \theta_B + \epsilon$ with $\epsilon > 0$, and the notation that $\pi = \pi_P = \pi_F + \gamma$, we can calculate the ratio of the proportion of good workers relative to the proportion of bad workers in the firm at a given tenure and rank

$$\begin{aligned}M_G &= \frac{P_G}{P_B} \\ &= \frac{\mu k_{(j,r)} \left[(\theta_G + \delta)\,\pi_P\right]^r \left[1 - (\theta_G + \delta)\pi_P - (1 - \theta_G - \delta)\pi_F\right]^j}{(1 - \mu) k_{(j,r)} \left[(\theta_B + \delta)\,\pi_P\right]^r \left[1 - (\theta_B + \delta)\pi_P - (1 - \theta_B - \delta)\pi_F\right]^j} \\ &= \frac{\mu \left[(\theta_B + \epsilon + \delta)\,\pi\right]^r \left[1 - (\theta_B + \epsilon + \delta)\pi - (1 - \theta_B - \epsilon - \delta)(\pi - \gamma)\right]^j}{(1 - \mu) \left[(\theta_B + \delta)\,\pi\right]^r \left[1 - (\theta_B + \delta)\pi - (1 - \theta_B - \delta)(\pi - \gamma)\right]^j} \\ &= \frac{\mu \left[(\theta_B + \epsilon + \delta)\,\pi\right]^r \left[(1 - \pi) + (1 - \theta_B - \epsilon - \delta)\gamma\right]^j}{(1 - \mu) \left[(\theta_B + \delta)\,\pi\right]^r \left[(1 - \pi) + (1 - \theta_B - \delta)\gamma\right]^j} \\ &= \frac{\mu}{(1 - \mu)} \left(\frac{(\theta_B + \epsilon + \delta)}{(\theta_B + \delta)}\right)^r \frac{\left[(1 - \pi) + (1 - \theta_B - \epsilon - \delta)\gamma\right]^j}{\left[(1 - \pi) + (1 - \theta_B - \delta)\gamma\right]^j} \\ &= \frac{\mu}{(1 - \mu)} \left(\frac{(\theta_B + \epsilon + \delta)}{(\theta_B + \delta)}\right)^r \left(\frac{\left[(1 - \pi) + (1 - \theta_B - \epsilon - \delta)\gamma\right]}{\left[(1 - \pi) + (1 - \theta_B - \delta)\gamma\right]}\right)^j \\ &= \frac{\mu}{(1 - \mu)} (A)^r (B)^j\end{aligned}$$

When $\gamma = 0 => \pi_P = \pi_F, \epsilon > 0$

$$M_G = \frac{\mu}{(1 - \mu)} \left(\frac{(\theta_B + \epsilon + \delta)}{(\theta_B + \delta)}\right)^r (1)^j$$

hence

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} = 0$$

In the case where $\gamma > 0 => \pi_P > \pi_F$ and $\epsilon > 0$ we have that $A > 1, B < 1$ hence

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} < 0$$

and when $\gamma < 0 => \pi_P < \pi_F$ and $\epsilon > 0$ we have that $A > 1, B > 1$ hence[30]

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} > 0$$

■

Proof of Lemma (1). Corollary (4) has direct implications for the bonus payments since

$$\begin{aligned}
\Pr(bonus) &= P_G(\theta_G + \delta) + P_B(\theta_B + \delta) \\
&= P_G(\theta_G + \delta) + (1 - P_G)(\theta_B + \delta) \\
&= P_G(\theta_G + \delta) + (1 - P_G)(\theta_G - \epsilon + \delta) \\
&= \theta_G + \delta - \epsilon(1 - P_G)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{d\Pr(bonus)}{d(j)} &= \epsilon\frac{dP_G}{d(j)} \\
\frac{d\Pr(bonus)}{d(r)} &= \epsilon\frac{dP_G}{d(r)}.
\end{aligned}$$

Finally, using

$$\frac{dM_G(x)}{dx} = \frac{d}{dx}\frac{P_G(x)}{1 - P_G(x)} > 0 => \frac{dP_G(x)}{dx} > 0,$$

and

$$\frac{dM_G(x)}{dx} = \frac{d}{dx}\frac{P_G(x)}{1 - P_G(x)} < 0 => \frac{dP_G(x)}{dx} < 0$$

the corollary follows. ■

Proof of Corollary (2). Follows directly from Corollary (1). ■

8.2 Managerial rent

The model assumes that managerial rent exists, yet it only handles it in a closed form. It seems necessary to provide reasons for this closed form representation. There are two major explanations and both of them build on the same observation, namely that managers affect the work of many subordinates.

The first approach justifies rents by incentives. Calvo and Wellisz (1979) argue that rent is an increasing function of hierarchical rank. Their model is based on costly supervision, where shirking employees are punished by firing. This punishment, however, is only effective as long as there are rents with respect to the job. As managers affect the work of many subordinates, they are given more rents to ensure no shirking. Thus, the firm might be tempted to offer higher compensation for managers even if the nature of the work is not different and all the workers and managers are identical. In sum, a wage (and rent) ladder might prevail, even absent quality differences.

The second rationale, suggested here, stems from the firm's desire to curb managerial turnover (more than the turnover of the non-management level). The intuition is simple and again it rests on the observation that managers affect the output of many other employees. If a non-management worker leaves, it disrupts his own output. If a manager leaves, the leave disrupts the output of the

[30] In general $A > 0$ for all feasible values of γ if $\epsilon > 0$.

manager and all his subordinates. Consequently, the firm would like to give stronger incentives for managers than for workers to stay. If outside offers are probabilistic, rents can be used to induce loyalty. Thus, in this setting the manager's compensation will include rents (and higher ones than that of the employees).

Finally, managerial rent has empirical support. It is generally understood that employees prefer to be promoted with the ongoing conditions. Thus, managerial work is more desirable, supporting the first, incentive explanation. Also, voluntary managerial turnover is lower than voluntary employee turnover, which lends support to the second explanation. In our data set, for instance, lower level managers and vice presidents have a 9.80 percent and 15.35 percent lower turnover, respectively, than employees.

Concluding, we are comfortable with the closed form modeling of managerial rents. The available empirical and theoretical evidence seems to support its existence.

8.3 Additional regressions

	Layoff	**Promotion**
Constant	-10.912	-8.603
	(1.023)	(1.117)
Bonus payment received	-2.222	0.768
during the preceding year	(0.268)	(0.249)
Tenure less than 2 years	-	-
Tenure 2 to 5 years	1.211	0.155
	(0.736)	(0.388)
Tenure 6 to 10 years	1.456	0.038
	(0.736)	(0.412)
Tenure above 10 years	1.651	-0.170
	(0.734)	(0.432)
Unskilled	-	-
Skilled worker	-1.314	-0.925
	(0.504)	(1.225)
Short theoretical education	-0.589	-
	(0.403)	(-)
Bachelor degree	0.798	1.088
	(0.391)	(1.032)
Masters or PhD degree	0.343	1.959
	(0.402)	(1.009)
Demographic variables	YES	YES
Time dummies	YES	YES
Log likelihood	-1545.22	
# observations	97,925	

Note: Reference category is: "Stay within rank". In the period studied individuals with a short theoretical education are not promoted.

Multinomial logit.

9 References

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