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Allocative Efficiency of Non-Farm Enterprises in Agricultural Households

Evidence from Malawi

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ABSTRACT

Allocative Efficiency of Non-Farm Enterprises in Agricultural Households

Evidence from Malawi*

According to standard economic theory, households will equate marginal revenue products of inputs across different activities within the household. We test this prediction using data on agricultural plots and non-farm enterprises in Malawi. Specifically, we test whether the marginal product of labor is equal across agricultural and non-agricultural production within a household. To the best of our knowledge, this is the first such test using non-farm data. We are able to control for many household characteristics using household fixed effects and we find the marginal product of labor is consistently higher in non-farm production than agricultural production. However, when focusing the analysis on households that operate both types of enterprises in the same year, we are unable to reject the null hypothesis of allocative efficiency.

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1 Introduction

According to standard economic theory, households will equate marginal revenue products of inputs across different activities within the household. However, this result relies on restrictive assumptions, which may not hold for agricultural households in developing countries. As such, whether households do equate marginal products across productive activities is an empirical question. Moreover, the answer to this question is not only interesting in its own right, but can shed light on some of the underlying market conditions which characterize many agricultural households.

Under traditional economic assumptions, profit maximization results in the equality of marginal revenue product[1] of inputs across productive activities; if allocated in any other way, it would be possible to increase profits by reallocating inputs. However, a number of issues arise that may cause deviations from the standard first-order conditions (FOCs) of MRPL equality. Specifically, lumpiness of inputs—the inability to marginally change input allocation—and differing risk profiles are two such issues (Barrett et al., 2008; Stiglitz, 1974).[2] Furthermore, as Udry (1996) points out, non-cooperative household bargaining models can have equilibria in which allocative efficiency does not hold.

Using the Living Standards Measurement Study (LSMS) from Malawi, we test the assumption of equality of marginal products across household activities. We examine whether the marginal product of labor is equal across agricultural and non-agricultural production within a household. We are able to control for time-invariant household characteristics using household fixed effects and we find the marginal product of labor is consistently higher in non-farm production than agricultural production. However, the prediction of equality of marginal products applies most directly to households that operate both types of activities at the same time. When we focus our analysis on these households, we are unable to reject the null hypothesis of allocative efficiency. To the best of our knowledge, this is the first such finding using non-farm data and testing for allocative efficiency across such enterprises within a household.

[1] Throughout this paper, we use the terms marginal product of labor and revenue marginal product of labor interchangeably, as does much of the literature. Both terms generally refer to the latter of the two.

[2] As Barrett et al. (2008) point out, it is common in the literature to refer to any deviation from the textbook FOCs as allocative inefficiency. As such, when we refer to allocative inefficiency, we are referring to such deviations.

This result is tentative, as it may be the case that our production function is mis-specified or that our data do not provide enough power for the test. But the reported result does suggest that Malawian households do not allocate their labor across agricultural and non-farm activities inefficiently. This finding provides support for the labor supply literature using shadow wages of household members as a measure of the opportunity cost of time within a household.

This results stands in contrast to allocative inefficiency found by Udry (1996). He analyzed whether farming households allocated resources across plots efficiently, and found evidence that they did not. In this paper, we study whether farming households allocate resources efficiently across farm and non-farm activities, and find tentative evidence that households allocate their resources efficiently.

This finding also suggests that Malawian households could not increase their income by reallocating their labor across farm and non-farm activities.

The rest of this paper is organized as follows. In the next section, we discuss the related literature. We then turn to the theory that informs our study as well as our methodology in Section 3. In Section 4, we discuss the data, which are from the Malawi LSMS. We present results of our analyses in Section 5, and conclude with some final comments in Section 6.

2 Literature

This paper relates to several separate—but related—veins of economic literature. First, the marginal revenue product of labor—or shadow wage—is integral to more recent research on labor supply. Specifically, the shadow wage can be used as a valid measure of the opportunity cost of time within the household, as it is a more robust measure of this cost than the market wage. Second, allocative efficiency is related to the intrahousehold bargaining literature. While many models of the household predict a Pareto efficient allocation of labor—and thus equality of MRPL—non-cooperative models do not necessarily make this prediction.

The estimation of marginal products is integral to research on labor supply. In addition to the prediction of MRPL equality across household activities, early labor supply work—since at least Gronau (1977)—that aimed to estimate structural labor supply parameters for agricultural households in developing countries used the market wage rate as a measure of the marginal revenue product of labor within the household, even for individuals that did not sell labor on the market. Indeed, if input and output markets are complete, hired labor is a perfect substitute for family labor, and individuals do not have location-based work preferences, then the market wage is a valid proxy for the individual-specific shadow wage.[3] However, many economists have rightly remarked that these assumptions can be overly restrictive in most developing countries, where market failures may be commonplace (Berg, 2013; Karlan et al., 2012).

Jacoby (1993) and Skoufias (1994) were the first to note that the marginal revenue product of labor on the farm is a more robust estimate of the shadow wage of household members under incomplete markets than is the prevailing market wage. Since then, many papers have relied on this method to estimate structural labor supply parameters, including Abdulai and Regmi (2000), Barrett et al. (2008), and Seshan (2014), to name but a few. Despite these advances, many practical problems remained. In an attempt to remedy one major barrier, Barrett et al (2008) developed a robust way of estimating household-specific shadow wages even under conditions of allocative inefficiency.[4] However, as Skoufias (1994) first pointed out, this method can result in biased estimates. Specifically, the method does not include non-agricultural household outputs in shadow income calculations. Given the ubiquity of non-farm household labor in most developing countries, the bias could theoretically be significant.

Le (2009) offers one possible solution. Le suggests using shadow wages from agricultural production functions to impute the missing household outputs. While Jacoby and Skoufias both omit household outputs while constructing the shadow income, Le derives a way to calculate household output using agricultural marginal products. This is a significant improvement over ignoring household output altogether. However, this relies on one important assumption: that the standard first order conditions hold. That is, that the marginal product of labor is identical across different

[3]Transaction costs and risk can also affect the market/shadow wage equality. These assumptions imply that the commonly cited $MPL = w$ equality is not a necessary condition for individual "rationality" under incomplete markets.

[4]They define allocative inefficiency as simply a rejection of the common $MPL = w$ condition. Again, this does not necessarily imply irrationality.

production activities within the household.

Advances in economic modeling have resulted in predictions regarding the conditions under which the assumption of MRPL equality will not hold. For example, differing risk profiles or lumpiness of inputs across productive activities can result in allocative inefficiency (Barrett et al., 2008). If risk profiles differ across activities, households optimize by equalizing their expected marginal utilities across activities. Assuming risk aversion, this implies that households allocate more resources to the more certain activity, resulting in a lower MRPL for that activity. Lumpiness of inputs can also cause allocative inefficiencies. The textbook first-order condition assumes the ability of a household to marginally reallocate inputs. If, for example, a plot is located far away from the household, there are large fixed travel costs. This implies that a household may not be able to increase labor on the plot only marginally; rather, the household is only able to increase labor by an amount at least as large as the fixed travel cost. This inability to perfectly adjust on the intensive margin can lead to allocative inefficiencies within the household and drive a wedge between marginal products of different activities.

When allocative efficiency fails, another possible explanation can be found in the large body of literature on intrahousehold bargaining. Many economic models of the household assume that the household acts as a single individual or that household decisions result in Pareto-efficient outcomes(Chiappori, 1988; Manser and Brown, 1980; McElroy and Horney, 1981). These so-called unitary, cooperative, or collective models assume household decisions are Pareto optimal and, thus, that marginal products are equated across productive activities within the household. However, non-cooperative models (Balasubramanian, 2013; Lundberg and Pollak, 1993; Strauss and Thomas, 1995) make no such assumption, and under these models marginal products may not be equated across activities.

3 Theoretical Background and Methodology

In this section, we discuss the theoretical background and methodology that informs our study. Much of the theoretical exposition below is a simplified model based on Barrett et al. (2008), with non-farm production included. In our sample, relatively few households hire in labor, and we therefore do not separate out hired labor for the model. We assume a two-person household[5] maximizes:

$$U(X, H^i)$$

subject to

$$\begin{aligned} p_X X \leq & p_f F(L_F^i, Z; K_F, A) - p_Z Z_F + p_n N(L_N^i; K_N) + w^i L_M^i + Y, \\ T^i \geq & H^i + L_F^i + L_N^i + L_M^i, \end{aligned}$$

where p_x is the price of the aggregate consumption good, X; i indexes household members, $i \in M, F$; H^i is leisure; p_F is the price of farm output, which is given by the production function $F(\cdot)$, which in turn satisfies the common assumptions of differentiability and concavity; L_F^i is agricultural labor; Z is other agricultural variable inputs; K_F is a vector of exogenous characteristics affecting agricultural output, including soil quality and weather; A is land, which is considered to be semi-fixed; p_n is the price of the non-farm enterprise output, which is given by the production function $N(\cdot)$, which also satisfies the common assumptions; L_N^i is non-farm enterprise labor; K_N is exogenous characteristics affecting non-farm output; w^i is the wage for individual i, who works L_M^i hours on the market; Y is exogenous income; and T^i is total time endowment.

At an interior solution—that is, for households that operate both types of enterprises—and for individuals that work on the market (assuming away risk and transaction costs for simplicity), both constraints will bind, and the first order conditions (FOCs) are:

$$p_f \frac{\partial F}{\partial L_F^i} = p_n \frac{\partial N}{\partial L_N^i} = \frac{\frac{\partial U}{\partial H^i}}{\frac{\partial U}{\partial X}} = w^i \tag{1}$$

This is the standard treatment in the literature: at the optimum, the marginal revenue product of inputs is equated across all activities and is equal to the wage rate, assuming the individual is active

[5]The model easily generalizes to larger households.

on the market. However, if an individual does not work on the market, then the above equalities will no longer include the wage rate, but rather the shadow wage,

$$w^{i*} = w^i + \frac{\lambda_B}{\lambda_T}, \tag{2}$$

where λ_B is the Lagrange multiplier on the budget constraint and λ_T is the multiplier on the time constraint. This again assumes the household operates at least one non-farm and one agricultural enterprise. In this scenario, the marginal productivities across activities within the household are still equated under the most common assumptions. It is this prediction of equal marginal products that we test in this paper.

This result need not hold in the presence of risk. Here we discuss two types of risk that affect input allocation: output risk and price risk. Output risk is the risk associated with uncertainty in production. While farmers, for example, make agricultural decisions—planting, applying fertilizer, allocating labor, etc.—based on what they expect to happen, idiosyncratic shocks can cause actual outcomes to deviate from expected outcomes. As such, under these circumstances the above model needs to be amended; the household will maximize expected utility and the expectation will be taken over output.

Output risk results in a familiar outcome: the marginal revenue product of labor in the safer activity will be lower than the marginal revenue product of labor in the riskier activity. Let us suppose a household operates both a plot and a non-farm enterprise. Moreover, assume agriculture is riskier than the non-farm activity. In this case, the household will allocate higher levels of inputs—relative to the case with certainty—to the safer activity, non-farm production. The result is that the marginal revenue product of labor in the non-farm activity is lower than the marginal revenue product of labor in agriculture, even if agriculture offers higher expected returns.

Price risk, on the other hand, results in a different prediction. Barrett (1996) showed that price risk has different results depending on whether a household is a net buyer or a net seller of a crop. Specifically, he showed that net buyers will tend to over-allocate—relative to the case with certainty—productive inputs under risk while a net seller will under-allocate inputs. In our context, suppose the agricultural household grows maize on its plot. Moreover, assume the household is a net buyer of maize, as is common in Malawi. In this case, the household will allocate more labor to maize production, resulting in a lower MRPL in agriculture than in non-farm production. Therefore in this scenario, two different types of risk would result in different predictions.

The basic steps involved in testing MRPL equality are as follows, and are similar to those in Linde-Rahr (2005), who studied allocative efficiency across plots planted with different crops. First, we estimate production functions for both agricultural and non-farm enterprises. Second, we compute the marginal revenue products of labor across different activities within the household. With these MPL estimates, we can then test the main research question; whether shadow wages—or marginal revenue products of labor—are equal across activities within the household.

In order to estimate the production functions, we first use a Cobb-Douglas specification as in Barrett et al. (2010).[6] In the first set of results, we estimate production functions for agricultural and non-farm enterprises separately. Specifically, we estimate:

$$\ln Q_{iht} = \alpha_h + \sum_j \beta_j \ln \gamma_{jiht} + \delta C_{iht} + R_{rt} + \eta_{mt} + \rho_{ht} + \varepsilon_{iht}, \tag{3}$$

where Q_{iht} is value of output for enterprise i in household h in wave t, γ_{jiht} are inputs j, C_{iht} is a vector of controls that may affect output and which differ depending on whether the enterprise is agricultural or not, and ε_{iht} is a conditional mean-zero error term. We also include a household-specific intercept, α_h, in order to allow for fixed effects. Moreover, R_{rt} controls for region/wave fixed effects and η_{mt} is a set of dummy variables indicating the month of interview in each wave. We include the latter due to the fact that agricultural production is seasonal. Finally, ρ_{ht} is an asset index constructed individually for each household in each wave. The asset index is constructed by including dummy variables indicating whether a household owns a number of different assets in a factor analysis with a single factor. We include the asset index in case assets owned by the household are correlated with any unobserved non-labor inputs. In non-farm regressions, we include also industry fixed-effects. In agricultural regressions, we restrict attention to plots planted with

[6] We also include estimates using a translog specification. We discuss this more below.

only maize, pigeon peas, tobacco, groundnuts, soya, or any combination of these crops.[7]

After constructing estimates for production functions, we then calculate the marginal revenue product of labor for each input. We calculate the marginal products for the first specification with the following formula:

$$\frac{\partial Q}{\partial L} = \frac{\hat{Q}_{iht}}{L_{iht}} \beta_L, \tag{4}$$

where β_L is the coefficient on labor, $\hat{Q}_{iht}$ is predicted output for enterprise i, and L_{iht} is labor used. This formula is used for both types of enterprises. We use predicted output,$\hat{Q}_{iht}$, instead of actual output as predicted output is our best estimate for what the person is trying to maximize. We assume the difference between actual output and predicted output are idiosyncratic shocks that the household can not predict, and therefore should not affect their labor allocation decisions. That said, we check our results using actual output in the appendix.

We also present results using a translog specification in order to examine the robustness of our findings. We estimate:

$$\ln Q_{iht} = \alpha_h + \sum_j \beta_j \ln \gamma_{jiht} + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln \gamma_{jiht} \ln \gamma_{kiht} + \delta C_{iht} + R_{rt} + \eta_{mt} + \rho_{ht} + \varepsilon_{iht}, \tag{5}$$

where γ_{jiht} and γ_{kiht} are inputs for enterprise i in household h in wave t, and the other variables are defined as in previous specifications. We construct our MRPL estimates for agricultural plots using:

$$\frac{\partial Q}{\partial L} = \frac{\hat{Q}_{iht}}{L_{iht}} \left[\beta_L + \beta_{LL} log L_{iht} + \beta_{LA} \log A_{iht} + \beta_{LF} \log F_{iht} \right], \tag{6}$$

where β_{LL} is the coefficient on the labor squared term, β_{LA} is the coefficient on the interaction between labor and acreage, and β_{LF} is the coefficient on the interaction between labor and fertilizer

[7] Part of the process of constructing the marginal revenue products involves finding prices for the agricultural output. We construct these prices by taking medians at the lowest geographical level of aggregation with at least 5 number of observations. The crops we choose are four of the more common crops in Malawi, and thus offer a sufficient number of observations with which to create median prices.

use. We calculate the MRPL for non-farm enterprises using:

$$\frac{\partial Q}{\partial L} = \frac{\hat{Q}_{iht}}{L_{iht}} \left[\beta_L + \beta_{LL} log L_{iht} \right]. \tag{7}$$

We make use of household fixed effects to identify the production functions. For households that operate a single type of enterprise in both waves, the identifying assumption is that any unobservables are household specific and time invariant. For households that operate two of the same type of enterprise in only one wave, on the other hand, the identifying assumption is relaxed somewhat, as time invariance is no longer necessary.[8]

Equations 4, 6, and 7 provide our estimates of the marginal revenue product of labor for each production activity. With these estimates, we can then turn to testing whether the marginal product of labor is equated across household activities. Given that a household may have more than one of each type of enterprise, we construct the difference between MRPL's as:

$$MRPL_{ht}^{NF} - MRPL_{ht}^{AG} = \psi_{ht}, \tag{8}$$

where we are computing the difference between each marginal product within the same household before then finding the median of the difference. In practice, we compute each MRPL by taking the mean of all enterprises of that type within a household and wave. We then construct a difference variable, ψ_{ht}, equal to the difference between the household level MRPLs. Therefore, the median of the difference over the whole sample is not simply the difference between the median agricultural MRPL and the median non-farm MRPL in the sample.

When we elaborated the theory above, we noted that MRPL equality holds only for an interior solution; in other words, MRPL equality holds only for households that operate both types of enterprises. Therefore, we only use households that operate both non-farm and agricultural enterprises in the same wave to construct MRPL estimates. In fact, economic theory makes no predictions

[8] Ideally, we would treat households separately in each wave. However, with household fixed effects regressions would be identified only by households that operate two of the same type of enterprise in the same wave. In order to then compare non-farm and agricultural MRPL, we would ideally need households that operate two of *both* types of enterprise in the same wave. Unfortunately, we do not have enough data to do this.

regarding marginal products in different households,[9] nor does it make predictions regarding the equality of marginal products across time. As such, our main interest lies only in those households that operate both types of enterprises simultaneously. In some specifications we use all households to estimate the production functions. Though we still compute marginal revenue products only for those households that operate both types of enterprises in the same wave. In this case the assumption would be that households which operate both types of enterprises in the same wave use the production technology as households that do not operate both types of enterprises.

Restricting the sample in such a way raises some concerns regarding the external validity of our conclusions. If the restricted sample is very different from the entire sample of households in Malawi, then the conclusions may not be true for all households. Specifically, households that operate both agricultural plots or non-farm enterprises in the same wave may have access to different types of production technology (e.g. higher quality plots). In all of our MRPL estimates, we therefore include results calculated from production functions using several different subsets of the population. In general, we start by estimating the production function with the entire sample. We then proceed to estimate production functions with more restricted samples, until arriving at production function estimates for the most restrictive sample, which is the same sample of households we use to construct our MRPL estimates.

Finally, in order to conduct valid inference, we bootstrap the standard errors by running 1,000 replications. Since we are interested in a multi-step estimator, bootstrapping offers a convenient and valid option for estimating standard errors. We use household fixed effects in our preferred specifications, and we therefore set up the bootstrap to draw households across waves. In other words, each bootstrap does not randomly draw plots or enterprises, but rather randomly draws households, and all observations for that household from both waves.

[9] Note that if certain assumptions are met, the marginal product in different households should all be equal to the prevailing market wage.

4 Data

In order to test the null hypothesis of allocative efficiency, we use the Living Standards Measurement Study (LSMS) survey from Malawi. The Malawi LSMS data consist of two waves: one collected from 2010-2011 and the other collected in 2013. The first wave was organized such that 12271 households were surveyed. However, only 3247 of these households were designated as panel households. These panel households were resurveyed again in 2013, but the non-panel households were not. As such, the 2010 wave is much larger than the 2013 wave.

Malawi has some of the lowest rates of non-farm enterprise creation in sub-Saharan Africa, with just 17 percent of rural households operating one, as compared to 30 percent in Ethiopia, 59 percent in Niger, 51 percent in Nigeria, 41 percent in Tanzania, and 45 percent in Uganda (Nagler and Naudé, 2014, p. 9). However, the Malawi data are the only LSMS sample with the adequate non-farm data in a panel format. While Ethiopia, Tanzania, and Uganda have more than one wave completed, the questions on labor used for non-farm enterprises only allows one to look at number of workers. Malawi, on the other hand, has data on workers, days, and hours. In the Malawi LSMS, there are approximately 800 households that operate two or more plots across both waves—or approximately 2000 individual plots—and 700 households that operate two or more non-farm enterprises across both waves—or approximately 1600 enterprises. In some of our regressions—and all of our marginal revenue product estimates—we also restrict the sample to households that operate both an agricultural and a non-farm enterprise in the same wave; of these households there are 589, with 773 non-farm enterprises and 881 plots among them.

Table 1 presents summary statistics for two of our samples. The first two columns include all households that operate a non-farm enterprise and/or a plot with maize, pigeon peas, groundnuts, soya, or tobacco, either as a monocrop or in any combination of the four. Non-farm statistics are in the top panel while agricultural statistics (one observation per plot) are shown in the bottom panel. The second pair of columns include statistics for only those households that operate both types of enterprises in the same wave. This is the sample from which we calculate all MRPL estimates in this paper.

While agricultural statistics seem to be comparable for both samples, the restricted sample includes households with smaller non-farm enterprises. Revenue is lower in both waves for the restricted sample. The overall size of the enterprises in terms of labor seems to be similar in both samples, although households in the restricted sample seem to hire less labor for non-farm enterprises than households in the entire sample. In agriculture, the restricted sample has lower average revenue in the first wave but higher average revenue in the second wave. Moreover, a higher percentage of households in the restricted sample reported hiring labor in both waves of the survey.

Interestingly, households that operate both non-farm and agricultural enterprises tend to be slightly less educated than all households that operate non-farm enterprises but slightly more educated than all households that operate agricultural plots. Similarly, households that operate both types of enterprises are slightly older than households that operate non-farm enterprises but slightly younger than households that operate agricultural enterprises.

5 Results

We present the primary results in Tables 2-5. Table 2 and Table 3 present results using a Cobb-Douglas production function. Specifically, Table 2 contains production function estimates while Table 3 presents the corresponding MRPL estimates. In Table 2, the results in the first pair of columns are from regressions using enumeration area (EA) fixed effects.[10] Moving to the second pair of columns, we re-estimate the first column with household fixed effects in place of EA fixed effects. In Model 3, we restrict the estimating sample to only those households that operate both types of enterprises in either wave, while in the final pair of columns, we further restrict the estimating sample to only those households that operate both types of enterprises in a single wave. Note that the households used in the last column are identical to the households we use to construct MRPL estimates, while in the previous three columns we use some households in the regressions that we drop to calculate MRPL.

[10] The enumeration area (EA) is the smallest geographical area recognized in the data and was the basis for survey sampling.

The production function results in Table 2 are largely similar across columns, with the exception of an increase in agricultural productivity in the seventh column. As we move from EA fixed effects to household fixed effects, the largest change in coefficient is for fertilizer. The decrease in the magnitude of the coefficient suggests there may be some omitted household-specific inputs in the first specification that are positively correlated with fertilizer, such as farm equipment. The coefficients on labor and land increase in magnitude as we restrict the sample, implying households which operate both types of enterprises are more productive then the whole sample. Not surprisingly, the R^2 increases substantially after the inclusion of household fixed effects; in fact, our production functions explain more over 70 percent of the variation in output after we include household fixed effects.

MRPL estimates are presented in Table 3. In Table 3, each column corresponds to the same pair of columns in Table 2. In all columns, we use the same 3,910 observations to estimate MRPL. These observations pertain to 1,444 households with 2,189 plots and 1,721 non-farm enterprises. In Table 3 we include the median for three separate statistics: agricultural MRPL, non-farm MRPL, and the difference between the two (equation 8). The median values are consistently smaller than the means (not shown), suggesting the overall distribution is skewed. Moreover, the estimates using the mean are less precisely estimated than the estimates using the median. For these reasons, we prefer the median for all three statistics. The estimated MRPL is positive for both non-farm enterprises and agricultural plots in all four models, and the non-farm MRPL is always larger than the agricultural MRPL. Beginning with EA fixed effects in column one, the median difference between non-farm and agricultural MRPL (298 Malawian Kwacha) is positive and highly significant. The statistic in column one suggests that households could reallocate a marginal worker day from agricultural to non-farm work and increase their income by 298 Kwacha.

Moving from EA fixed effects to household fixed effects (column one to two) has a surprisingly small effect on the estimated difference. The difference is still highly significant ($p<0.01$). However, as we move from Model 2 to Model 3, the median difference decreases, although it remains significant. Finally, in the fourth model in which we estimate the production functions using the same sample of households with which we construct the MRPL estimates, the estimated difference in MRPL is still significant. The difference of just 165.5 Kwacha is well less than US$1 at the time of the second wave of the survey. These results suggest that households do not equalize MRPLs across activities, but the difference becomes less significant as we restrict the sample. That is, in Model 4, when the production function is estimated using only the households which operate both types of enterprises in the same wave, the difference in MPRL's across activities is only marginally significant.

It is therefore prudent to examine whether our results are sensitive to choice of production function, and present results from using a translog production function in Table 4 and Table 5. We present results using the same four samples we used in Table 3. As before, the MPRL for non-farm enterprise is always larger than the agricultural MRPL. However, the more flexible functional form produces MRPL's in Table 5 that are smaller than those using the more restrictive Cobb-Douglas production function. The difference in MRPL's in Table 5 are also smaller in 3 of the 4 models. More importantly, in Model 4, the difference is no longer significant. That is, for households which operate both types of enterprises in the same wave, we are unable to reject the null hypothesis of allocative efficiency.

Since our results depend on which sample and specification we use, it is also useful to check how sensitive our results are to outliers. Outliers are not a concern for the second stage of the analysis, as we use medians for the MRPLs, but outliers could be affecting the estimation of the production function. As such, we re-estimate Models 3 and 4 from Table 3 and Table 5 but now drop the top and bottom five percent of revenue from the sample when estimating the production function. We present the Cobb-Douglas results in Table 6 and the translog results in Table 7. Table 6 shows that removing the outliers did not change the Cobb-Douglas results much. However, in Table 7, we see that outliers did significantly impact the translog estimates. The MRPL's reported in Table 7 are lower than before for both agricultural and non-farm production, and the difference between the MRPL's is not significant.

We include one final set of results in the appendix. So far, we have computed MRPL using predicted output rather than actual output. In appendix table Table A1, we present results from the Cobb-Douglas specifications using actual output to construct MRPL. The estimated difference in MRPL between non-farm enterprises and agriculture actually increases and is statistically significant, even in the most restrictive sample.

In appendix figure Figure A1, we present results using percentile confidence intervals. In the results presented so far, we bootstrapped standard errors and used standard errors to perform hypothesis testing. However, another possibility is to take 95% confidence intervals directly from the bootstrap draws. Figure A1 shows these results. These results do not provide any different inference than the results discussed already. Our ability to reject the null hypothesis of allocative efficiency depends on the sample and specification.

Our preferred specification is Model 4 using the translog production function. First, we prefer Model 4 over the other models as it allows the households which operate both types of enterprises in the same wave to use a different production technology than other households. The results presented above do suggest that these households use a different technology as the MRPL's are mostly larger in Model 4 than in the other models, especially for agriculture. Also, we prefer the translog specification over the Cobb-Douglas specification since it is more flexible. So, when using our preferred specification, we are unable to reject the null hypothesis of allocative efficiency.

A limitation to our analysis which restrains us from concluding that Malawian households are allocating their labor efficiently is whether we have properly specified the production function. The results above have shown the importance of the choice of production function. However, the Malawian LSMS data only allow us to use labor as an input into the non-farm enterprise production function. Although, excluding those other inputs would lead to an upward bias on the coefficient for labor. So, the results presented here likely overestimate the productivity of labor for non-farm enterprises, and including more inputs would then work to lower the MRPL and making it even less likely to reject the null hypothesis of allocative efficiency.

While our results are not conclusive, there are two possible explanations for why households may not be trying to equalize MRPL's across enterprises as tested here. Both explanations are at the individual level. First, it may be that the true bargaining model of the household is not a unitary model (Udry, 1996). If different individuals control agricultural and non-farm enterprises, marginal revenue products of labor may not be equated across agricultural and non-farm enterprises. Second, different individuals within the household may have access to different technologies. Specifically, it may be that males or household heads have access to better technologies than others (Udry, 1996).

However, in both of these cases, marginal revenue products would still be equated across activities for the same individual. In order to explore this possibility, we re-estimated the last two columns of Table 3 using individual fixed effects instead of household fixed effects. We present these results in Table 8. While we lose some precision due to the fewer number of observations identifying our estimates, the coefficients are quite similar to previous results. While we are not able to reject the null hypothesis of allocative efficiency within individuals, it does not appear that the entirety of the difference is explained by individual-level factors.

Another possibility relates to risk. If risk differs across activities, then the equality of MRPL need not hold even with a unitary household model. Specifically, the riskier activity will tend to have higher MRPL, while the safer activity with tend to have lower MRPL as households work to equalize expected MRPL. While we are unable to explicitly test this hypothesis, we do present some suggestive evidence. We are interested here in the variability of output. We first ran regressions of output on district and wave fixed effects. We then calculated the standard deviation of the residuals from these regressions, as a measure of output variability after controlling for location and wave. We present these statistics in Table 9. In the top panel, we present standard deviations of output across households. The variation in output is much higher for non-farm enterprises than agricultural plots in both waves. However, this could simply be masking substantial heterogeneity among firms. In the bottom panel, we present standard deviations of output within households across waves. Within households, the standard deviation is also much higher for non-farm enterprises. This is at least suggestive evidence that non-farm production is riskier than agricultural production.

6 Conclusion

Tests for whether households allocate labor such that the marginal product of non-farm enterprise labor equals the marginal product of agricultural labor provide insight into intrahousehold allocative efficiency. While recent research in the labor supply literature may not assume that the wage is a valid measure of the shadow wage of time within the household, this research does tend to assume some type of intrahousehold allocative efficiency apart from the standard $MRPL = w$ textbook condition. The most common assumption is that the marginal product of labor within the household is equated across different household activities. Theoretically, it is not clear that this is indeed the case. In fact, even assuming complete rationality, deviations from the textbook condition are possible, especially when different activities face different risk profiles. Given the inherent riskiness of agricultural production, intrahousehold allocation decisions are likely fraught with risk. Moreover, recent findings also suggest that household decisions are not always Pareto efficient. As such, the assumption that households allocate labor across activities in such a way that the marginal revenue product is equivalent needs to be tested.

In this paper we test for allocative efficiency across non-farm and agricultural enterprises in Malawian households. In all of our specifications, the MRPL for agriculture is larger than the MRPL of non-farm activities. However, in our preferred specification, we cannot reject the null hypothesis of allocative efficiency. That is, for households that operate both agricultural and non-farm enterprises in the same wave, we do not find conclusive evidence that the households are allocating their labor inefficiently.

One note of caution with this result is that our data did not provide detailed information on inputs into the non-farm production process apart from labor. So, it may be the case that our non-farm production function is misspecified. While we can control for industry and household fixed effects, this may not be entirely satisfactory. Therefore, more research is needed on this topic, especially projects with better information on non-farm production.

Table 1: Summary Statistics for Ag/Non-Ag Observations

	Entire Sample		Restricted Sample	
	Wave 1	Wave 2	Wave 1	Wave 2
	Non-Farm Statistics			
Education of household head	6.98	7.74	6.38	7.50
	(4.37)	(4.61)	(4.09)	(4.53)
Age of household head	39.13	40.70	39.98	41.24
	(13.16)	(12.86)	(13.39)	(12.90)
Household size	2.66	3.02	2.79	3.04
	(1.33)	(1.44)	(1.46)	(1.41)
Revenue	97,615	117,454	58,157	97,526
	(635,241)	(461,311)	(456,136)	(396,416)
Family Labor (Days)	19.96	23.18	20.01	23.21
	(14.85)	(18.28)	(15.74)	(19.05)
Hired Labor (Days)	6.52	11.25	4.55	10.01
	(35.18)	(72.36)	(20.97)	(82.42)
Household hired any labor	0.10	0.13	0.07	0.11
	(0.29)	(0.34)	(0.26)	(0.32)
Wage (Kwacha)	209.83	446.57	189.27	436.72
	(95.43)	(262.94)	(86.74)	(272.97)
	Agricultural Statistics			
Education of household head	5.31	6.10	6.21	7.09
	(4.04)	(4.44)	(3.93)	(4.54)
Age of household head	42.97	45.98	40.05	44.34
	(15.82)	(15.20)	(13.42)	(13.41)
Household size	2.60	3.06	2.79	3.12
	(1.26)	(1.44)	(1.46)	(1.37)
Revenue	80,544	62,640	74,641	72,031
	(1,046,984)	(446,891)	(1,017,453)	(466,341)
Family Labor (Days)	39.47	33.66	37.37	29.69
	(38.02)	(31.66)	(37.38)	(30.60)
Hired Labor (Days)	1.91	3.87	2.10	4.97
	(6.63)	(7.49)	(4.83)	(9.32)
Household hired any labor	0.07	0.29	0.11	0.39
	(0.26)	(0.45)	(0.31)	(0.49)
Acres	2.00	2.03	1.99	1.95
	(1.03)	(4.48)	(0.82)	(0.86)
Fertilizer (kg)	18.06	14.88	21.53	13.73
	(32.88)	(30.78)	(35.90)	(29.24)
Wage (Kwacha)	299.48	490.54	302.27	498.49
	(136.67)	(169.16)	(137.87)	(194.84)
Non-farm observations	8,160	2,068	1,546	728
Ag observations	2,795	1,332	1,185	566

Wave 1 data were collected in 2010-2011. Wave 2 data were collected in 2013. Agricultural observations are single plots planted with any combination of maize, pigeon peas, tobacco, groundnuts and soya. Non-farm observations are single enterprises. The restricted sample includes only households that operated both types of enterprises in the same wave. This is the sample used to construct all of the MRPL estimates in this paper. Standard deviations are in parentheses.

Table 2: Production Function Estimates - Cobb-Douglas

	Model 1		Model 2		Model 3		Model 4	
	Ag Plots	NFEs	Ag Plots	NFEs	Ag Plots	NFEs	Ag Plots	NFEs
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log labor	0.0913***	0.341***	0.0970***	0.360***	0.0816	0.334***	0.172**	0.345***
	(0.0162)	(0.0284)	(0.0336)	(0.0451)	(0.0559)	(0.0558)	(0.0705)	(0.0777)
Log acres	0.710***		0.729***		0.866***		1.032***	
	(0.0674)		(0.0814)		(0.168)		(0.196)	
Log fertilizer	0.0941***		0.0696***		0.0605**		0.0594*	
	(0.00935)		(0.0153)		(0.0291)		(0.0336)	
Asset index	0.224***	0.523***	0.0661	0.176	0.0349	0.139	-0.247	0.235
	(0.0220)	(0.0356)	(0.0936)	(0.109)	(0.137)	(0.117)	(0.181)	(0.186)
Controls								
Enumeration Area FE	Yes	Yes	No	No	No	No	No	No
Household FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Wave/Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month of Interview FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Plot controls	Yes	No	Yes	No	Yes	No	Yes	No
Industry FE	No	Yes	No	Yes	No	Yes	No	Yes
N	10,290	4,124	10,297	4,125	2,869	2,141	2,221	1,749
R^2	0.378	0.553	0.731	0.898	0.716	0.888	0.823	0.925

Standard errors are in parentheses. Models 1 and 2 (columns 1-4) use all observations. Model 3 (columns 5-6) use all households that operate both types of enterprises in either wave. The final model (columns 7-8) uses all households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Marginal Revenue Product of Labor (Median) - Cobb-Douglas

	Model 1 (1)	Model 2 (2)	Model 3 (3)	Model 4 (4)
Ag MRPL	67.19***	59.49**	72.94	184.6***
	(12.96)	(28.83)	(57.39)	(57.40)
NFE MRPL	397.2***	457.5***	311.3***	412.9***
	(37.57)	(88.88)	(77.90)	(77.90)
Diff (NFE-Ag)	297.7***	296.8***	223.6**	165.5*
	(38.24)	(93.87)	(95.12)	(95.15)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. The model numbers correspond to the production function estimates displayed in Table 2. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Production Function Estimates - Translog

	Model 1		Model 2		Model 3		Model 4	
	Ag Plots (1)	NFEs (2)	Ag Plots (3)	NFEs (4)	Ag Plots (5)	NFEs (6)	Ag Plots (7)	NFEs (8)
Log labor (L)	0.0688	-0.107	0.0638	0.198	0.167	0.145	0.0597	0.222
	(0.0553)	(0.0817)	(0.160)	(0.277)	(0.252)	(0.306)	(0.316)	(0.696)
Log acres (A)	1.674***		1.965***		1.984***		2.006**	
	(0.167)		(0.451)		(0.733)		(0.935)	
Log fertilizer (F)	0.282***		0.183		0.171		0.261	
	(0.0467)		(0.133)		(0.227)		(0.274)	
L×L	0.0459**	0.188***	0.0771	0.0575	0.0642	0.0697	0.110	0.0447
	(0.0187)	(0.0290)	(0.0584)	(0.0933)	(0.0884)	(0.109)	(0.122)	(0.244)
A×A	-0.424***		-0.363*		-0.144		-0.245	
	(0.0732)		(0.195)		(0.528)		(0.723)	
F×F	-0.0776***		-0.0273		-0.0520		-0.105	
	(0.0164)		(0.0511)		(0.0840)		(0.0955)	
L×A	-0.182***		-0.299**		-0.375*		-0.300	
	(0.0467)		(0.123)		(0.200)		(0.299)	
L×F	-0.00921		-0.0264		-0.0307		-0.0196	
	(0.00813)		(0.0213)		(0.0368)		(0.0481)	
A×F	0.0293		0.0499		0.147		0.143	
	(0.0277)		(0.0583)		(0.107)		(0.136)	
Asset index	0.218***	0.494***	0.0609	0.175	0.0241	0.137	-0.238	0.238
	(0.0194)	(0.0305)	(0.152)	(0.236)	(0.211)	(0.251)	(0.314)	(0.482)
Controls								
Enumeration Area FE	Yes	Yes	No	No	No	No	No	No
Household FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Wave/Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month of Interview FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Plot controls	Yes	No	Yes	No	Yes	No	Yes	No
Industry FE	No	Yes	No	Yes	No	Yes	No	Yes
N	10,297	4,125	10,297	4,125	2,869	2,141	2,221	1,749
R^2	0.382	0.517	0.746	0.898	0.720	0.889	0.826	0.925

Standard errors are in parentheses. Models 1 and 2 (columns 1-4) use all observations. Model 3 (columns 5-6) use all households that operate both types of enterprises in either wave. The final model (columns 7-8) uses all households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Marginal Revenue Product of Labor (Median) - Translog

	Model 1	Model 2	Model 3	Model 4
	(1)	(2)	(3)	(4)
Ag MRPL	53.18***	24.32	64.01	145.4**
	(12.23)	(22.04)	(45.79)	(65.89)
NFE MRPL	320.7***	366.7***	230.7***	313.6***
	(41.54)	(75.59)	(71.01)	(103.1)
Diff (NFE-Ag)	252.0***	332.5***	172.9**	106.9
	(43.68)	(88.64)	(82.47)	(101.7)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. The model numbers correspond to the production function estimates displayed in Table 4. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: MRPL - Cobb-Douglas Trimmed Outliers

	Model 3	Model 4
	(1)	(2)
Ag MRPL	73.56	188.2***
	(57.57)	(57.62)
NFE MRPL	310.0***	417.0***
	(77.44)	(77.47)
Diff (NFE-Ag)	246.5**	176.1*
	(95.83)	(95.82)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. Model 3 restricts estimation of the production function to only households that operate both types of enterprises in either wave, while model 4 restricts estimation of the production function to only households that operate both types of enterprises in the same wave. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: MRPL - Translog Trimmed Outliers

	Model 3 (1)	Model 4 (2)
Ag MRPL	98.60**	95.79**
	(40.57)	(47.03)
NFE MRPL	220.5***	230.6**
	(61.04)	(97.71)
Diff (NFE-Ag)	82.21	97.47
	(67.44)	(100.7)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. Model 3 restricts estimation of the production function to only households that operate both types of enterprises in either wave, while model 4 restricts estimation of the production function to only households that operate both types of enterprises in the same wave. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: MRPL - Cobb-Douglas Individual FE

	Model 3 (1)	Model 4 (2)
Ag MRPL	94.26	183.5**
	(67.09)	(81.76)
NFE MRPL	276.6***	351.6**
	(98.54)	(171.3)
Diff (NFE-Ag)	110.5	118.0
	(86.53)	(148.7)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. Model 3 restricts estimation of the production function to only households that operate both types of enterprises in either wave, while model 4 restricts estimation of the production function to only households that operate both types of enterprises in the same wave. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Risk Profiles

	Standard Deviation
	(1)
	Across HH
Ag - Wave 1	83,627
Ag - Wave 2	75,352
NFE - Wave 1	105,769
NFE - Wave 2	86,526
	Within HH
Ag	13,406
NFE	43,199

Standard deviations are from the residuals of regressions of output on district and wave fixed effects.

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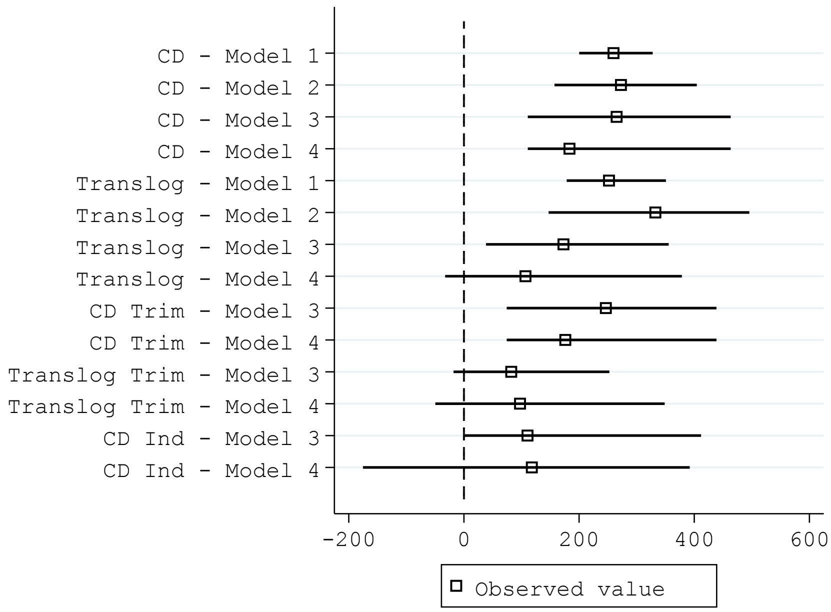
Appendix A

Figure A1: Percentile Confidence Intervals



The upper and lower values for the confidence intervals (CIs) are constructed by taking the corresponding percentiles from the bootstrap samples of 1,000 draws. Since the CIs in this plot correspond to $\alpha = 0.05$, the corresponding lower and upper values are $\alpha/2 = 0.025$ and $1 - \alpha/2 = 0.975$, or the 2.5th and 97.5th percentile. In the table, the models are defined as follows: CD Models 1 through 4 correspond to the four Cobb-Douglas specifications in Table 3, Translog Models 1- 4 correspond to the translog specifications in Table 5, CD Trim Models 3 and 4 correspond to the two models in Table 6, Translog Trim Models 3 and 4 correspond to Table 7 and CD Ind Models 3 and 4 correspond to the individual fixed effects specifications in Table 8. The hollow squares are the difference estimates used in the main body of the paper.

Table A1: MRPL - Actual Output

	Model 3 (1)	Model 4 (2)
Ag MRPL	65.26	137.4***
	(46.38)	(46.38)
NFE MRPL	383.6***	396.1***
	(62.80)	(62.80)
Diff (NFE-Ag)	265.3***	183.3**
	(88.30)	(88.30)

Standard errors are in parentheses. Standard errors are constructed by bootstrapping the procedure 1000 times. Model 3 restricts estimation of the production function to only households that operate both types of enterprises in either wave, while model 4 restricts estimation of the production function to only households that operate both types of enterprises in the same wave. All MRPL estimates are constructed for households that operate both types of enterprises in the same wave. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Appendix B

Derivation of Concavity of Translog Production Function

Let Q = F(K,L), the trans-log form is then:

$$\begin{aligned} lnQ = \alpha_0 + \alpha_1 lnL + \frac{1}{2}\alpha_2 lnK + \frac{1}{2}\alpha_{11}(lnL)^2 + \frac{1}{2}\alpha_{22}(lnK)^2 \\ + \frac{1}{2}\alpha_{12}(lnL \times lnK) + \frac{1}{2}\alpha_{21}(lnK \times lnL) + \varepsilon, \end{aligned} \tag{B1}$$

but this is actually the same as:

$$lnQ = \alpha_0 + \alpha_1 lnL + \alpha_2 lnK + \frac{1}{2}\alpha_{11}(lnL)^2 + \frac{1}{2}\alpha_{22}(lnK)^2 + \alpha_{12}(lnL \times lnK) + \varepsilon. \tag{B2}$$

Implicitly differentiating with respect to L gives us:

$$\frac{1}{Q}\frac{\partial Q}{\partial L} = \frac{\alpha_1}{L} + \frac{\alpha_{11} lnL}{L} + \frac{\alpha_{12} lnK}{L}. \tag{B3}$$

And finally:

$$\frac{\partial Q}{\partial L} = \frac{Q}{L}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right), \tag{B4}$$

which is the formula we use to construct our marginal product values in this paper.

Implicitly differentiating again:

$$\frac{\partial^2 Q}{\partial L^2} = \frac{\frac{\partial Q}{\partial L}L - Q}{L^2}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right) + \alpha_{11}\frac{Q}{L^2}. \tag{B5}$$

But we know that $\frac{\partial Q}{\partial L} = \frac{Q}{L}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right)$, so:

$$\frac{\partial^2 Q}{\partial L^2} = \frac{\left(\frac{Q}{L}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right)\right) L - Q}{L^2}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right) + \alpha_{11}\frac{Q}{L^2} \tag{B6}$$

$$= \frac{\left(Q\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right)\right) - Q}{L^2}\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right) + \alpha_{11}\frac{Q}{L^2} \tag{B7}$$

$$= \frac{Q}{L^2}\left[\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right)^2 - \left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right) + \alpha_{11}\right] \tag{B8}$$

Therefore, the derivative of the marginal product for the translog production function is:

$$\frac{\partial^2 Q}{\partial L^2} = \frac{\partial MRPL}{\partial L} = \frac{Q}{L^2}\left[\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right)^2 - \left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnK\right) + \alpha_{11}\right] \tag{B9}$$

For agricultural plots, this means:

$$\frac{\partial MRPL}{\partial L} = \frac{Q}{L^2}\left[\left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnA + \alpha_{13} lnF\right)^2 - \left(\alpha_1 + \alpha_{11} lnL + \alpha_{12} lnA + \alpha_{13} lnF\right) + \alpha_{11}\right] \tag{B10}$$

For non-farm enterprises (where labor is the only productive input), we similarly have:

$$\frac{dMRPL}{dL} = \frac{Q}{L^2}\left[\left(\alpha_1 + \alpha_2 \ln L\right)^2 - \left(\alpha_1 + \alpha_2 \ln L\right) + \alpha_2\right] \tag{B11}$$

When determining the sign, $\frac{Q}{L^2}$ is always positive. Therefore, the sign of the derivative of MRPL depends on the sign of the expression in brackets in both equations A10 and A11.