DISCUSSION PAPER SERIES

IZA DP No. 3850

# A New Model of Wage Determination and Wage Inequality

Guillermina Jasso

November 2008

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

# A New Model of Wage Determination and Wage Inequality

**Guillermina Jasso**
*New York University*
*and IZA*

Discussion Paper No. 3850
November 2008

IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

IZA Discussion Paper No. 3850
November 2008

# ABSTRACT

## A New Model of Wage Determination and Wage Inequality*

This paper proposes a new model of wage determination and wage inequality. In this model, wage-setters set workers' wages; they do so either directly, as when individuals vote in a salary committee, or indirectly, as when political parties, via the myriad of social, economic, fiscal, and other policies, generate wages. The recommendations made by wage-setters (or arising from their policies) form a distribution, and all the wage-setter-specific distributions are combined into a single final wage distribution. There may be any number of wage-setters; some wage-setters count more than others; and the wage-setters may differ among themselves on both the wage distribution and the amounts recommended for particular workers. We use probability theory to derive initial results, including both distribution-independent and distribution-specific results. Fortuitously, elements of the model correspond to basic democratic principles. Thus, the model yields implications for the effects of democracy on wage inequality. These include: (1) The effects of the number of wage-setters and their power depend on the configuration of agreements and disagreements; (2) Independence of mind reduces wage inequality, and dissent does so even more; (3) When leaders of democratic nations seek to forge an economic consensus, they are unwittingly inducing greater economic inequality; (4) Arguments for independent thinking will be more vigorous in small societies than in large societies; (5) Given a fixed distributional form for wages and two political parties which either ignore or oppose each other's distributional ideas, the closer the party split to 50-50, the lower the wage inequality; and (6) Under certain conditions the wage distribution within wage-setting context will be normal, but the normality will be obscured, as cross-context mixtures will display a wide variety of shapes.

JEL Classification: C02, C16, D31, D6, J31

Keywords: wage-setter, power, consensus, independence of mind, dissent, form of government, probability distributions, shifted exponential distribution, shifted general Erlang distribution, shifted mirror-exponential distribution, Gini coefficient

Corresponding author:

Guillermina Jasso
Department of Sociology
New York University
295 Lafayette Street, 4th Floor
New York, NY 10012-9605
USA
E-mail: gj1@nyu.edu

* This research was supported in part by the U.S. National Science Foundation under Grant No. SBR-9321019 and partly carried out while the author was a Fellow at the Center for Advanced Study in the Behavioral Sciences in 1999-2000. Early versions of portions of this paper were presented at the annual meeting of the American Sociological Association, New York, August 1996, at the Conference in Honor of James S. Coleman, Reimers Stiftung, Bad Homburg, October 1996, and at the annual meeting of the Public Choice Society, San Francisco, March 1997. I am grateful to participants at those meetings and especially to Samuel Kotz, the anonymous referees, and the Editor for many valuable comments and suggestions. I also gratefully acknowledge the intellectual and financial support of New York University. This paper is forthcoming in *Rationality and Society*.

## 1. INTRODUCTION

This paper proposes a new model of wage determination and wage inequality. Wages and the wage distribution touch virtually every aspect of human experience, and are linked to fundamental features of individual and society, from health and happiness to status and the sense of justice to crime and revolution. The search for knowledge about wage processes and social and economic inequality has produced several rich literatures, including classic philosophical inquiries (e.g., Plato, Laws, Book V; Aristotle, Politics, Book II; Aquinas (Summa Theologica II-I, Q. 105, art. 2); Rousseau 1755 [1952]); religious narratives (e.g., Leviticus 19, 25; Matthew 19, 20), sermons (e.g., St. John Chrysostom (386-407 [1860]), St. Antoninus), and scholarly contributions in the social sciences and statistics (e.g., Jencks et al. 1972; Champernowne and Cowell 1998; Kleiber and Kotz 2003).[1]

The new wage model began as an attempt to provide an account of the wage process that for at least some contexts would be more true to life than extant models. In the world of the model, wage-setters set workers' wages; they do so either directly, as when individuals vote in a salary committee, or indirectly, as when political parties, via the myriad of social, economic, fiscal, and other policies, generate wages. The recommendations made by wage-setters (or arising from their policies) form a distribution, and all the wage-setter-specific distributions are combined into a single final wage distribution. The model has three key features: (1) the number of wage-setters may vary; (2) some wage-setters may count more than others; and (3) wage-setters may disagree with each other on both the wage distribution and the amounts

[1] For example, Plato's (Laws, Book V) intuition that social relations and societies suffer when the largest income exceeds five times the smallest income, Aristotle's (Politics, Book II) observation that inheritance laws and laws on procreation generate economic inequality, John Chrysostom's (Homily 66.3, "On the Gospel of Matthew") effort to estimate the proportions poor and rich in the city of Antioch, Gregory the Great's (Moralia in Job, xxi) insight, "Where there is no sin, there is no inequality,"and Aquinas' (Summa Theologica II-I, Q. 105, art. 2) spirited defense of the injunction in Leviticus 19 to pay workers promptly have helped shape social thought and social science. Moreover, there is a strong sense that inequality is a bad which needs to be "corrected", as noted by Aristotle (Politics, Book II, Chapter 9). Leviticus 25 provides a way to correct the bad, establishing a jubilee year every fiftieth year -- when inequalities are erased and the ancestral allotments are restored -- and Jesus imagines a more radical upheaval where inequality is preserved but "the last will be first, and the first last" (Matthew 19:30).

recommended for particular workers. The model has a simple mathematical structure, and analysis of the model yields interesting and novel implications for the effects of the three features -- the number of wage-setters, their power, and their agreements and disagreements – on wage inequality and the shape of the wage distribution. The paper reports both distribution-independent results and results based on classical probability distributions.

As work with the model progressed, it quickly became apparent that the three features of the model coincide with three basic democratic principles: (1) that as many people as possible should share in government; (2) that they should count equally; and (3) that people are free to hold whatever views they want. Thus, the simple wage model turns out to yield implications for the effects of democracy – indeed, of the form of government – on wage inequality.[2]

Along the way we encounter classic themes that have fired the imagination across human history – the benevolent dictator, the sinless world, the promise and peril of political parties, the contributions and cost of consensus, the value of dissent, the special beneficent possibility of duarchy, and the equally beneficent possibility of mixed government.[3]

Closer to the daily life of social science, the new wage model revisits the classic question of the shape of the income distribution – whose roots go at least as far back as Chrysostom (386-407 [1860]) and which has captivated a diversity of commentators (Lebergott 1959).

---

[2] The democratic principles, too, have a rich history, better known under the rubric of forms of government (going back at least as far as Plato's Republic and Aristotle's Politics). For example, Thomas Aquinas (Summa Theologica, II-I, Q. 105, art. 1), commenting on law and invoking Aristotle (Politics, Book II), notes: "[A]ll should take some share in the government, for this form of constitution ensures peace among the people, commends itself to all, and is guarded by all."

[3] For example, discussion of the benevolent dictator goes back to Plato. Augustine (City of God, Book V, Ch 24) echoes Plato, and thus begins a great tradition of ruler-saints – like Louis IX and the Empress Pulcheria. [In the complex interplay of sociobehavioral mechanisms, it is not without interest that poor saints outnumber rich saints and scholar-saints outnumber ruler-saints.] Similarly, duarchy, or rule by two (also called biarchy, diarchy, dyarchy, and duumvirate), goes back to antiquity. Duarchy was for the most part unsuccessful, perhaps due to the "envy of equals" (Lewis 1852:II,79-80; see also Plutarch's (c. 46-120 [1952]) Pyrrhus and Lysander and Gibbon (1776 [1952], Vol I, Ch 6), cited by Lewis). A notable exception involves Pulcheria, who, besides reigning alone, also did a stint of duarchy. Finally, the idea of a mixed government – combining elements of monarchy, oligarchy, democracy – has found appeal throughout the ages (see, for example, Thomas Aquinas, Summa Theologica, II-I, Q. 105, art. 1).

Specifically, we return to the pioneering work of Ammon (1899) and Pigou (1924), who sought to learn the reasons why income was not normally distributed, and we derive conditions under which the wage distribution will exhibit normality.

Is the new wage model plausible? As elaborated below, in many real-world contexts, wages and wage schedules are set by special committees and boards, sometimes with help from special compensation consultants.

Moreover, empirical research on distributive justice demonstrates not only the pervasive human impulse to form ideas about the just wage but also the profound individualism of ordinary people who disagree with each other on all aspects of the wage distribution – bringing to life the Hatfield principle, "Equity is in the eye of the beholder" (Walster, Berscheid, and Walster 1976:4). If people's wage-setting activities are informed by their ideas of justice, then it is not unreasonable to expect recommended wages to reflect some of the diversity that permeates ideas of justice.

The work reported in this paper builds on and contributes to several disciplines and subdisciplines, especially sociology, economics, political science, and statistics. For example, it contributes to political science and economics by advancing knowledge about democracy, political parties, and wage inequality. Within sociology, it contributes not only to social stratification and political sociology but also to economic sociology. As well, the paper contributes to the study of probability distributions, especially the exponential, Erlang, general Erlang, and the recently-introduced mirror-exponential (Jasso and Kotz 2007), reporting some new formulas and introducing a new subfamily called the shifted mirror-exponential.[4]

While the model, like all models, provides only a simplified abstraction of a complex reality, we believe that it holds some promise for a successful social science undertaking.

---

[4] To illustrate, the new wage model expands economic tools and concomitantly underscores a basic tenet of economic sociology – viz., that full understanding of economic phenomena which are embedded in social contexts (such as wages) requires ideas and insights from across the social sciences (Smelser [1963] 1976; Hicks 1963; Sørensen 1977, 1979; Ben-Porath 1980; Granovetter 1981, 1985, 1988; Granovetter in Swedberg 1990:112; Swedberg and Granovetter 1992, 2001; Smelser and Swedberg 1994, 2005; Dobbin 2004, 2005; Zelizer 2007).

Although this paper presents only initial results, we believe that it will be possible to obtain many more results by further application of probability theory and also by application of other tools.

Almost all the implications reported in this paper are conditional and nuanced. But one is strikingly free of conditions: dissent is universally superior to independence in reducing inequality, and independence is superior to agreement. The democratic principle guards the freedom to agree or disagree. But if reducing inequality is the goal, then independent-mindedness trumps agreement, and opposite-mindedness trumps independent-mindedness. Being at loggerheads has no peer.

The paper is organized as follows: Section 2 presents the basic model. Section 3 sets forth the basic mathematization and basic formulas. Section 4 reports the model's distribution-independent implications, and Section 5 follows with analysis of shifted subfamilies of the exponential, Erlang, general Erlang, and mirror-exponential distributions. A short note concludes the paper.

## 2. BASIC ELEMENTS OF THE NEW WAGE MODEL: WORKERS AND WAGE SETTERS, RECOMMENDED WAGE DISTRIBUTIONS AND WAGE DECISIONS, POWER AND CONSENSUS

### 2.1. Workers, Wage-Setters, and the Recommended Wage

To fix ideas, we begin with a situation familiar to many readers. Every year, during the late spring or early summer, professors are notified of their salary for the following academic year. The new salary usually consists of the current salary plus a salary increase which is intended to reflect such factors as research productivity, teaching excellence, etc., constrained by the salary budget allocated to the department. In preparation for the salary decision, professors submit a written list of their publications and other contributions during the previous year (or, at some institutions, during the previous two or three years, to offset the temporal unevenness of the publication stream and other contributions). The individuals who review these materials and settle on the recommendations to be made to higher university authorities are, at some institutions, a special salary committee of the departmental faculty or the standing "executive" committee; at other institutions, salary recommendations are made solely by the departmental chair, and at still others, by the Dean.[5]

In the situation just described, and in many other workplace settings, there are thus two kinds of actors: workers and the wage-setters who make recommendations for the workers' wages. In general, each wage-setter recommends a wage amount, denoted $x$, for each worker. Indexing wage-setters by $i$ ($i = 1,..., N$) and workers by $j$ ($j = 1,..., J$), the wage recommended by the $i$th wage-setter for the $j$th worker is written $\boldsymbol{x}_{ij}$. This is the wage-setter-specific/worker-specific wage (Table 1, panel A).

-- Table 1 about here –

A convenient way to collect all the wage recommendations made by all the wage-setters for all the workers is by means of a matrix. Let each row of the matrix represent a wage-setter, and let each column of the matrix represent a worker. Thus, each element of the matrix is the wage recommended by the wage-setter occupying the row for the worker occupying the column. Table 1 (panel B) presents the wage matrix, denoted $\boldsymbol{X}$.

Accordingly, each row of the matrix gives rise to a wage-setter-specific distribution. Following the usual notational conventions, the wage-setter-specific distribution is denoted $X$. This paper focuses on the $\boldsymbol{X}_i$ distributions, of which the number is $N$, one for each wage-setter.[6]

In the special case of a single wage-setter (who may be called a "dictator" because he or she alone sets the salary distribution), the wage matrix collapses to a vector, as shown in Table 1

---

[5] The history of professorial salaries and merit increases is insightfully discussed in Clark (2006).

[6] The wage matrix gives rise to (up to) three kinds of distributions. Here we are concerned with only one of these distributions, the wage-setter-specific distribution of wage recommendations for all the workers, that is, the row-specific distributions. The column-specific distributions – the set of wage recommendations for a single worker – are very much of interest in other problems, such as in the study of reputation. Moreover, in some wage situations, every person is both a worker and a wage-setter and makes wage recommendations for all workers, self included. In such case, the matrix is square and gives rise to a third kind of distribution, namely, the distribution of wages recommended by self for self (found on the principal diagonal of the matrix).

(panel C), and there is only one *X* distribution.

Recapitulating, the fundamental elements of the new wage model are:

1. the fundamental actors – worker and wage-setter
2. the fundamental quantity – the wage
3. the fundamental matrix – the wage matrix
4. the fundamental distribution – the wage-setter-specific recommended wage distribution.

The model accommodates many other special features, some of which will be developed below, but the foregoing constitute the fundamental elements, paralleling the fundamental elements in other topical domains (Jasso 2004).

## 2.2. Recommended Wage Distributions and Wage Decisions

### 2.2.1. The Wage-Setter-Specific Recommended Wage Distribution

The wage-setter-specific recommended wage distribution embodies two kinds of considerations, one concerning the overall form and shape of the distribution – "what should the distribution look like" -- and the other concerning the recommendation for each worker – "who should get what". These have been discussed in at least two literatures, the literature on income distribution and the literature on distributive justice.

Chipman and Moore (1980:402) distinguished between two meanings of distribution: (1) the distribution of the reward amounts received by particular individuals (which may be called the "proper-name" distribution); and (2) the "anonymous" frequency distribution of reward amounts. They pointed out that the English language unfortunately has a single term for both meanings, in contrast to the French (répartition and distribution, respectively).

In the study of justice, Brickman, Folger, Goode, and Schul (1981) introduced the distinction between principles of microjustice -- which pertain to allocations to particular individuals -- and principles of macrojustice – which pertain to the overall form of the distribution. Jasso (1983) elaborated the distinction and established the mathematical relations between principles of microjustice and principles of macrojustice.

This paper highlights two macro and two micro features which will form the basis for micro and macro consensus, developed below. The two macro features, not surprisingly, are the arithmetic mean and the dispersion. For example, in some situations there is a budget restriction, such that all the wage-setters' recommended-wage distributions must have the same mean:

$$E(X_1) = E(X_2) = \ldots = E(X_N) . \qquad \textbf{(1)}$$

In other situations, however, wage-setters may recommend not only the workers' wages but also the salary budget (presumably not without consequence for non-salary items in the budget, such as advertising faculty searches or lodging visiting speakers).

The two micro features highlighted are the absolute amount recommended for each worker and the worker's relative rank in the recommended wage distributions.

### 2.2.2. Combining the Wage Recommendations into a Wage Decision

Whenever there is more than one wage-setter, the wage-setters' recommended wage amounts for particular workers are combined into a single final wage, denoted *y*, and, concomitantly, the recommended wage distributions $X_i$ are combined into a single final wage distribution, denoted *Y*. We shall, for convenience, refer to these as the wage decision, although, of course, in many real-world contexts this decision is not binding but rather constitutes a recommendation to a higher authority (e.g., the Provost).

Combining the individual wage-setters' recommended wage amounts for a particular worker may be thought of as generating a weighted average. Ignoring subscripts for the worker, this final wage may be written:

$$y = w_1 x_1 + w_2 x_2 + \ldots + w_N x_N , \qquad \textbf{(2)}$$

where there are *N* wage-setters and the weights are nonnegative and sum to one. Formally, $w_i \geq 0$ and $\Sigma w_i = 1$.

Concomitantly, combining the individual wage-setters' recommended wage distributions into a single wage distribution may be thought of as generating a weighted average of the *X* distributions:

$$Y = w_1 X_1 + w_2 X_2 + \ldots + w_N X_N . \quad \textbf{(3)}$$

It is immediately evident that the weights represent the wage-setters' power, as will be developed below.

### 2.3. Power and Consensus

Whenever the number of wage-setters exceeds one, two new considerations come into play: power and consensus. These are basic features of the wage-determination process developed here, and they have far-reaching implications.

#### 2.3.1. Power of the Wage-Setters: Weights and the Weight Matrix

Power refers to the weight placed on each wage-setter's recommendation when all the recommendations are combined into the final wage distribution. As noted above, the pooling of recommendations may be thought of as a process by which a weighted mean is calculated. Suppose, for example, that the first of three wage-setters recommends a salary of $67,000 for a particular professor, while the second and third wage-setters recommend for that same professor, respectively, a salary of $72,000 and $77,000. If all three wage-setters have equal power, the salary decision will be the unweighted (i.e., equally-weighted) mean, or $72,000. On the other hand, if the first wage-setter is very powerful, his/her view may count for 80% of the result, with the two other wage-setters each counting for 10%; in this case, the salary decision will be $68,500. Indeed, if the first wage-setter has absolute power, the salary decision will be $67,000.

A wage-setter's power is represented by the weight $w$ associated with his/her wage recommendation. We attach subscripts to the weight to denote the identity of the wage-setter. Thus, $w_i$ is the weight attached to $X_i$, the wage-recommendation distribution of the $i$th wage-setter.

It may happen that the weights differ not only by wage-setter but also by worker. For example, in an academic department, theorists may count more heavily in the remuneration of theorists, and ethnographers may count more heavily in the remuneration of ethnographers, etc. In such case, a second subscript is used to distinguish workers, exactly as with the wage. Thus, the wage recommended by the $i$th wage-setter for the $j$th worker is, as above, $x_{ij}$, and the attached weight is $w_{ij}$ (Table 2, panel A).

– Table 2 about here –

Accordingly, the formula for the *j*th worker's wage is given by:

$$y_j = \sum_{i=1}^{N} x_{ij} w_{ij} . \quad \textbf{(4)}$$

As with the wage, a convenient way to collect all the weights attached to the wage recommendations made by all the wage-setters for all the workers is by means of a matrix. Let each row of the weight matrix represent a wage-setter, and let each column of the matrix represent a worker. Thus, each element of the matrix is the weight attached to the wage recommendation made by the wage-setter occupying the row for the worker occupying the column. Table 2 (panel B) presents the weight matrix, denoted $\boldsymbol{W}$.

The weight matrix, together with its cells and its regions, provides a wealth of information. Entries of zero and one reveal that a wage-setter has no power or absolute power, respectively, with respect to the worker occupying the column. The quantity $1/N$, viz., the weight when all wage-setters have equal say, operates as a key benchmark in distinguishing between the relatively more or less powerful among the wage-setters. For example, if one of the weights in a row is one or if none go below $1/N$, the wage-setter occupying that row is on the high end of the power spectrum. Meanwhile, the weights in a column, which must sum to one, reveal the circle of the powerful over the worker occupying the column.

Two special cases deserve mention. First, in the special case of a dictatorship, the weight matrix collapses to a row vector, as shown in Table 2 (panel C), and all the entries are ones. Second, in the special case where weights do not differ across worker, the weight matrix collapses to a column vector, as shown in Table 2 (panel D). In this second case, the power configuration is universal, covering all workers, and, depending on the content of the weights, we may say that there is a universal dictatorship, or a universal triumvirate, or a universal democracy. The implications to be derived in Sections 4 and 5 are for this case of universal weights.

When weights do differ across worker, however, it is useful to characterize the worker-specific power configuration. Thus, if one wage-setter has full power in setting a particular worker's wage, we may say that there is dictatorship specific to that worker; if three wage-setters have equal power in setting a particular worker's wage, we may say that there is a triumvirate specific to that worker.

The weight matrix also leads to straightforward representation of each wage-setter's power distribution – comprising all the weights in the row occupied by a particular wage-setter -- and by parameters of that distribution, such as the mean,

$$E(Power_i) = \frac{1}{J}\sum_{j=1}^{J} w_{ij}, \quad \mathbf{(5)}$$

and variance. Note that wage-setters' power distributions may differ sharply in one or more parameters. For example, suppose that one wage-setter has full power over the wage for some workers and zero power for others; another wage-setter has $1/(N-1)$ weight in all recommended wages except those where the first wage-setter has exclusive say. It is possible for the average power of the two wage-setters to be the same, but the measures of dispersion may differ.

In sum, the conception of power in this model comprises three sets of power relations. First, the wage-setter has power over the worker (whenever the weight $w_{ij}$ is nonzero), so that each wage-setter has power over several workers and, concomitantly, each worker is under the power of several wage-setters. Second, each wage-setter is involved in a hierarchy of power relations with the other wage-setters (if the weights are not all equal). Third, this hierarchy of power relations among wage-setters can be multidimensional, if the power configuration differs across target workers.

### 2.3.2. Micro and Macro Consensus: Association Among the Recommended Wage Distributions and Whether They Are Identical or Different

The wage-setters may agree or disagree with each other concerning the micro and macro features – or, equivalently, the proper-name and anonymous distributions. In particular, they may agree or disagree with each other concerning the four features highlighted above: (1) the mean of the overall distribution; (2) the dispersion in the distribution; (3) the workers' relative ranks; and (4) the workers' recommended wage amounts.

At one polar extreme, two wage-setters may have exactly the same views with respect to both micro and macro features, and their recommended wage distributions are identical. Alternatively, their recommended wage distributions may differ in one or several ways. They may differ not only in absolute wages but also in rank-ordering. For example, two professors may view the contributions of a peer in diametrically opposite terms, the one proposing that the third should be the highest-paid member of the department, the second arguing that the third is overrated and should be the lowest-paid full professor.

All the patterns of agreements and disagreements are logically possible except two: If the worker-specific wage amounts are the same across two or more wage-setters, then (1) the worker-specific relative ranks must be the same, and (2) the anonymous distributions must also be the same.

To illustrate, look at two rows of the wage matrix in Table 1. If two rows are identical, then both the proper-name distributions and the anonymous distributions are identical. But suppose that two cell entries are interchanged in one row. Now the two proper-name distributions are no longer identical, although the anonymous distributions remain identical.

To prepare for analysis of important special cases, we focus on two dimensions of consensus. The first – <u>macro consensus</u> – considers whether the anonymous distributions are identical or different. Following standard practice, henceforth we refer to the anonymous distributions simply as "distributions". The second dimension – <u>micro consensus</u> – captures basic aspects of the association between the proper-name distributions, namely, whether two or more of the recommended wage distributions are independent and, if dependent, whether positively or negatively associated.

In this initial analysis of the new wage model we work mainly with three polar types of association: (1) independence; (2) perfect positive association; and (3) perfect negative association. Thus, the proper-name or micro dimension of consensus can range from perfect agreement to perfect disagreement, crossing an intermediate region of random association.

To characterize micro consensus, it is natural to begin with the statistical notions of independence and dependence. Following Stuart and Ord (1987:28), the term "independence" is used to mean "complete independence", viz., in a set of $N$ variates all the marginal distributions of all orders are independent. Intuitively, independence indicates mutual obliviousness. Smith's wage recommendations are irrelevant to Jones' wage recommendations, and Jones' wage recommendations are irrelevant to Smith.

Continuing, dependence is of two kinds, positive and negative, corresponding to agreement and disagreement. To characterize the two extremes we use the notions of perfect positive association and perfect negative association. Perfect positive association denotes the case in which each worker has the same relative rank on all the wage-setters' recommended wage distributions. Perfect negative association of two recommended wage distributions denotes the case in which the workers' rank ordering in one distribution is exactly the reverse of the rank ordering in the other; thus, one ranking is the conjugate ranking of the other (Kotz, Johnson, and Read 1982:145).[7]

Note that while two independent wage-setters are oblivious of each other, two dependent wage-setters may be very much mindful of each other, agreeing with each other in the positive-association case and disagreeing with each other in the negative-association case.

Combining now the two dimensions of consensus, we obtain a typology of six polar types. A version of the typology for the case of two wage-setters is presented in Table 3.

– Table 3 about here –

To fix ideas, Table 4 illustrates the six polar types for the simple case of two wage-setters and four workers. The top left cell represents wage-setters who are like-minded with respect to both micro and macro features. In the bottom left cell, the wage-setters are like-minded with

[7] An example of perfect negative association is the Biblical idea that "the last will be first, and the first last"(Matthew 19:30).

respect to the worker ranking but not with respect to the wage amounts or the overall distribution. The top middle cell represents wage-setters who are independent-minded with respect to micro features but like-minded with respect to the anonymous distribution. In the bottom middle cell, the wage-setters are independent-minded about micro features and they have different views concerning the overall distribution. The top right cell represents wage-setters who are exactly opposite-minded with respect to both wage amounts and relative ranks but who are like-minded about the overall distribution. Finally, the bottom right cell portrays wage-setters who are exactly opposite-minded with respect to both wage amounts and relative ranks and who have different views concerning the overall distribution.[8]

– Table 4 about here –

The conditions of independence and of perfect positive association can be extended immediately to the case of *N* distributions. The condition of perfect negative association, however, requires special treatment, and for that we introduce the idea of a faction – which will be substantively important as well -- and the principle of organized subsets proposed by Berger, Fisek, Norman, and Zelditch (1977:126-127).

In many situations it is possible to discern subsets of wage-recommendation variates which are internally identical but which are independent of each other. As a simple example, suppose that there are two wage-setters and their wage-recommendation variates are independent (as in the middle column of Table 4). Now suppose that two new wage-setters are brought in, and each joins with one of the two continuing wage-setters, agreeing exactly with him/her. In this case, each of the two pairs has internal perfect positive association, but the two pairs are independent. To characterize this type of situation, we may say that there are *M* independent <u>factions</u>, each faction generating a recommended wage distribution, denoted $\boldsymbol{X}_m$, and each faction composed of *n* individuals, where $\Sigma n_m = N$.

Of course, two factions need not be independent; they may also be perfectly negatively

---

[8] In this simple example, all six distributions have the same mean – 10. Thus, the wage-setters who espouse different distributions nonetheless agree on the distribution's average.

associated. To characterize this situation, we use the principle of organized subsets introduced by Berger et al. (1977:126-127), applying it as follows: (i) there are *N* wage-setters, arranged in two factions; (ii) within faction, all the recommended wage distributions are perfectly positively associated; and (iii) the two factions are perfectly negatively associated with each other.

The weights associated with each faction represent the proportion of all the wage-setters in each of the two factions (possibly reflecting as well differential wage-setter power).

### 2.4. Remarks on the Basic Wage-Setter Model

In the model, wage-setters set workers' wages, the wage-setters may differ in power, and they may be oblivious to, or agree or disagree with, each other. It has not been necessary to introduce supply and demand considerations, though such considerations may at times influence the activities of the wage-setters (especially when a worker has received an outside offer), and, indeed, one of the interpretations of the particular probability distribution used in Section 5 below relies on supply and demand. Nonetheless, the situation can still be understood as wage-setters making up their minds. As discussed in economic sociology, the model we have sketched, although like all models a simplification, appears to adequately describe the wage determination process in an academic department and to do so without invoking markets. As the economists Boyes and Happel (1989:39) note, "Academics (even economists) are not used to thinking of allocations within their institutions in terms of a market system."

Of course, the model is applicable to many situations besides that of professors. Some firms employ professional wage-setters -- the compensation consultants. There may be collective bargaining, and a myriad of special types of bargaining, such as national bargaining, company bargaining, and plant bargaining (Chamberlain 1951; Dunlop 1958). A firm's board of directors may decide compensation for the chief executive officer, as studied by O'Reilly, Main, and Crystal (1988). There may be governmental councils that decide the pay structure for one or more sectors of the economy, as in the Wages Councils and Wages Boards in the United Kingdom, or statutorily-empowered individuals who decide the pay structure for a political jurisdiction, as the justice of the peace in English counties under Queen Elizabeth I (Sells 1939;

Tolles 1964; Elliott 1991). In all these cases, it seems natural to model the basic features of the wage-determination process in terms of *N* wage-setters, each generating a recommended wage distribution, together with a process for combining the *N* recommended wage distributions, the $X_i$, into one final wage distribution, *Y*.[9]

### 2.5. Political Parties as Wage-Setters

Consider a society with one or more political parties. Associated with each political party is a set of policies, and this set of policies gives rise to a potential wage distribution for the population (Jasso 1989:260-261). Suppose that the final wage distribution *Y* is a weighted average of the potential wage distributions generated by the parties, with the weights representing each party's relative power (proportion of the electorate, say, or seats in a legislative body). This situation may be modeled exactly as the faction situation introduced above. In the special case of two political parties, the party split is represented by the proportions in the two parties (in the electorate or in the legislature), denoted *p* and (1-*p*).

* * *

Table 5 summarizes the substantive elements of the model and the corresponding mathematical or statistical representation. In the general case, there can be any number of wage-setters; their weights may be equal or unequal; the recommended wage distributions may be identical or different; and the association between them may be positive, independent, or negative. There is one overarching question: What are the effects of these features on inequality in the final wage distribution *Y* ? Below we analyze some special cases and obtain initial results.

– Table 5 about here –

## 3. MATHEMATICAL AND STATISTICAL BUILDING-BLOCKS

This section assembles the basic building-blocks to be used in the subsequent sections.

---

[9] Compensation consultants have been much in the news recently, as focus on large executive compensation packages extends to the consultants who design them (Morgenson 2008); moreover, in the economic turmoil of 2008 the American public and legislators seek a hand in executive compensation.

### 3.1. Preliminary Considerations

#### 3.1.1. Constraints on Conditions of Micro Consensus

We have introduced micro consensus, and in Sections 4 and 5 will assess the effects of micro agreement, independence, and micro disagreement on inequality in the final wage distribution. However, not all situations are amenable to all three types of association between the recommended wage distributions. Here we examine briefly some constraints.

At the outset we note that it is always possible to have perfect positive association. Whatever an existing recommended wage distribution, it can be replicated exactly (as in the case of identical/perfect-positively-associated $X$ distributions, illustrated in the top left cell of Table 4); similarly, its rank-ordering can be replicated exactly (as in the case of different/perfect-positively-associated $X$ distributions, illustrated in the bottom left cell of Table 4).

Consider now the case of perfect negative association (illustrated in the right column of Table 4). It is obvious that perfect negative association can occur only in situations with two wage-setters, for if a third wage-setter is in perfect negative association with one of two existing wage-setters, he or she must perforce be in perfect positive association with the other. Thus, perfect negative association is a condition of duarchies and duumvirates – and two-party systems. It is an important special case, and we will obtain results applicable both to the composite factions introduced by Berger et al. (1977), discussed in Section 2.3, and to two-party systems.

Similarly, independence is not always possible. To illustrate, if (1) there are two wage-setters and (2) the $X$ distributions are identical, then in order to satisfy independence the number of workers must be a square (4, 9, 16, etc.) or the multiple of a square (for example, 8, 12, etc., when the square is 4; 18, 27, etc., when the square is 9; and so on). By induction, it can be further shown that, continuing with identical $X$ distributions, if there are three wage-setters, the number of workers must be a cube (8, 27, 64, etc.) or the multiple of a cube. The general formula for this class of problems is $c$ to the $N$, where $c$ is the number of distinct $x$ values and $N$ is, as before, the number of wage-setters. To illustrate with an example provided by an anonymous referee, if the number of wage-setters is two and there are two distinct values of $x$, then independence requires that the number of workers be 4, 8, 12, etc. (as in the top cell in the middle column of Table 4). This implies that in small groups the number of workers can be such as to render independence impossible.

#### 3.1.2. Representing Inequality

A major objective is to assess the effects of the main elements of the model – the number of wage-setters, their power, and their agreements and disagreements with respect to micro and macro aspects of the wage distribution – on the magnitude of inequality in the final wage distribution. As is well-known, there are many measures of inequality, and, notwithstanding commonalities in important subsets of socioeconomic situations (such as the set of two-parameter probability distributions discussed in Jasso and Kotz 2008), in the general case different inequality measures may generate different inequality-orderings of distributions. Thus, it would be desirable to analyze the wage model using a basic set of inequality measures.[10]

Here we follow a two-pronged approach. In Section 4, which focuses on distribution-free results, we rely on the variance, which is not dimensionless. Accordingly, mathematical results based on the variance can be given an inequality interpretation only if the means are equal. Thus, variance-based results will apply directly to cases of identical distributions (the top rows of Tables 3 and 4), to cases where the distributions are different but have equal means (a subset of the bottom rows of Tables 3 and 4), and to contrasts within a given set of distributions. In Section 5, which focuses on modeling distributions, we rely on the Gini coefficient.

### 3.2. Basic Formulas

Table 6 reports basic formulas for the case of two wage-setters. The table follows the general format of Tables 3 and 4, showing the six main cases generated by the three types of micro consensus and the two types of macro consensus. In addition, Table 6 distinguishes, within each of the six main cells, between the case in which the two wage-setters have equal

---

[10] The four inequality measures analyzed by Jasso and Kotz (2008) would be a reasonable starter set: the coefficient of variation, the Gini coefficient, Atkinson's measure defined as the ratio of the geometric mean to the arithmetic mean, and Theil's MLD.

weights and the case of unequal weights. The general formula appears in panel A, and the special-case formulas in panel B.[11]

– Table 6 about here –

As shown, the general formula for *Var(Y)* is a special weighted sum of the two *X* variances and the covariance between the two *X* distributions. The formulas for the special cases in panel B are derived from the general formula in panel A. The formulas for the cases in which the two *X* distributions are independent (in the middle column) are well-known; the others are either in the literature or easily derived.

Table 7 reports the corresponding formulas for the more general case of *N* wage-setters. As in Table 6, the general formula appears in panel A and the special-case formulas in panel B. Note that the righthand column of Table 7 (panel B) pertains to negative association rather than to perfect negative association, given that, as discussed above, perfect negative association occurs in the case of two wage-setters. Moreover, the interpretation of negative association is that the sum of the weighted covariances is negative – that is, negative weighted covariances dominate positive weighted covariances. Again, as with Table 6, the formulas are either well-known or are easily derived.

– Table 7 about here –

Most of the theorems to be presented in Section 4 will require for their proof the formulas in Tables 6 and 7.

### 3.3. The Effects of Introducing One New Wage-Setter

Besides relying on Tables 6 and 7 to assess the effects of the number of wage-setters and other features on wage inequality, it is illuminating to consider little illustrations. In this section we examine the case where one new wage-setter is introduced into a group of wage-setters. Returning to the example of the faculty salary committee, suppose that there are three independent wage-setters, and the committee is enlarged to four members. The incoming wage-

[11] Here, and in the rest of the paper, the weights are universal; that is, each wage-setter has the same weight for all workers (as in Table 2, panel D, and discussed in Section 2.3.1).

setter may develop an independent recommended wage distribution or, alternatively, may become a partisan of one of the three original wage-setters. Suppose that the rules are that each wage-setter has an equal vote. How do the two alternatives differ in their effect on wage inequality? To answer this question, we derive and contrast formulas expressing the variance of *Y* before and after addition of the new wage-setter, under the assumption that the *X* distributions are identical with finite variance.

Table 8 reports the formulas for the *Y* variance, together with the change in the variance from Time 1 to Time 2 and the proportional increase or decrease. In Table 8, *N* denotes the number of wage-setters prior to the coming of the new wage-setter. For completeness, we include the case where the new wage-setter has no power -- for example, a committee may include one person who can enter the discussion but cannot vote; this situation is labeled (1). The situation where the new wage-setter has equal power and develops an independent recommended wage distribution is labeled (2), and the case of the partisan new wage-setter is labeled (3). As shown, introducing an independent wage-setter reduces wage inequality; the variance of *Y* declines by a factor of $\{1/[N(N+1)]\}$, or a proportionate decrease of $[1/(N+1)]$. When the new wage-setter becomes a partisan of one of the original wage-setters, however, wage inequality increases; it increases by the factor $\{(N-1)/[N(N+1)^2]\}$, or a proportionate increase of $[(N-1)/(N+1)^2]$.

– Table 8 about here –

## 4. IMPLICATIONS OF THE WAGE MODEL: DISTRIBUTION-INDEPENDENT RESULTS

### 4.1. Theorems on Inequality and Democracy

Three principles of democracy are fundamental. First, the electorate – the governed who give consent – should include as much of the population as possible. Second, members of the electorate should have equal votes. Third, members of the electorate should be free to hold and express their own opinions on all matters, and thus to agree or disagree with other members of the body politic.

In earlier times, going back to antiquity, ideas about democracy and experiments with democratic principles included only a small fraction of the governed, perhaps only men or only free men or only property owners. But the vision has steadily grown, so that increasingly the electorate spans all adult citizens free of disabling cognitive impairments.[12] With respect to the second principle, it is widely held that voters should count equally, as in "One person, one vote."[13] And with respect to the third principle, the right to hold and express a variety of opinions has come to be seen as not only upholding the dignity and worth of every person but also generating superior public policies, as ideas from every corner are examined, contrasted, revised and refined (in a process not too different from scientific work).

The question addressed in this section is: How do these three fundamental principles of democracy affect wage inequality? To address the effect of the first principle, we examine the effects of $N$ the number of wage-setters on wage inequality. To address the effect of the second principle, we examine the effects of the pattern of weights on wage inequality (where, as described in Section 2, the weights are nonnegative and sum to unity). To address the effect of the third principle, we examine the effects on wage inequality of type of micro consensus – viz., like-mindedness, independent-mindedness, and opposite-mindedness concerning the wages each worker should receive. If democracy reduces inequality, then (1) the effect of $N$ should be positive, namely, the larger the number of wage-setters, the lower the inequality, and (2) inequality should be lower when the wage-setters' weights are equal than when they are unequal. It is not obvious what the effects of type of micro consensus should be, given that the principle allows freedom to agree or disagree.

**THEOREM 1 (Wage Inequality and the Number of Wage-**

[12] Classic debates focused on slavery, race, and gender. Current controversies focus, inter alia, on voting rights of citizens in prison and of noncitizen residents.

[13] Not all procedures in democratic countries conform to this rule. Consider, for example, the U.S. Senate, in which equal representation is of each state rather than each person.

**Setters)**: *The effect on wage inequality of the number of wage-setters depends on the configuration of micro consensus, macro consensus, and wage-setter power; increasing the number of wage-setters can increase or decrease inequality or have no effect. Specifically:*

**THEOREM 1.1 (No Effect of the Number of Wage-Setters in the Case of Perfect Micro and Macro Like-Mindedness)**: *Given N wage-setters whose recommended wage distributions are identical, have finite variances, and are perfectly positively associated, inequality in the final wage distribution is exactly the same as in the recommended wage distributions. Formally:*

$$\text{Ineq}_Y = \text{Ineq}_X, \quad \textbf{(6)}$$

*where the subscripts indicate the recommended and final wage distributions.*

**THEOREM 1.2 (Effect of the Number of Wage-Setters in the Case of Equally-Weighted, Independent-Minded Wage-Setters)**: *Given N equally-weighted wage-setters whose recommended wage distributions have equal means and finite variances and are independent, inequality is a decreasing function of the number of wage-setters*.

**THEOREM 1.3 (Effect of Introducing an Equally-Weighted Wage-Setter into a Set of Independent-Minded, Equally-Weighted Wage-Setters)**: *Given N equally-weighted wage-setters whose recommended wage distributions have equal means and equal finite variances and are independent, increasing the number of equally-empowered actors can have opposite effects -- can either increase or decrease wage inequality -- conditional on the* *independence of mind of the incoming wage-setter.*

**Proof of Theorem 1.1**: We construct a simple proof based on the quantile function (QF):

$$
\begin{aligned}
Q_Y(\alpha) &= \sum_{i=1}^{N} w_i \left[Q_X(\alpha)\right] \\
&= \left[Q_X(\alpha)\right] \sum_{i=1}^{N} w_i \\
&= \left[Q_X(\alpha)\right].
\end{aligned}
\qquad \textbf{(7)}
$$

As shown, given that the distributions are identical and the sum of weights must be unity, the QF of the final wage distribution is the same as the QF of the recommended wage distributions. Finally, if *X* and *Y* have the same QF, they must have the same variance.

$$\text{If } Q_Y(\alpha) = Q_X(\alpha), \text{ then } Var(Y) = Var(X). \qquad \textbf{(8)}$$

For an alternate proof, look at the formulas for *Var(Y)* in the top two rows of the left column in Tables 6 and 7. The formulas do not include *N*, and thus it is obvious that in these cases *N* has no effect on *Var(Y)*.

**Proof of Theorem 1.2**: It is obvious from inspection of the formulas for *Var(Y)* in the equal-weights/independent cases in Tables 6 and 7 that the first partial derivative of *Var(Y)* with respect to *N* is negative.

**Proof of Theorem 1.3**: This result follows from the formulas presented in Table 8 (panels B and C).

□

**Remarks on Theorem 1**. Theorem 1.1 provides a benchmark, so to speak, against which to gauge inequality reduction. Note that neither the number of wage-setters nor their weights affect inequality in this case. Note also that the effects shown in Table 8 and used in the proof of Theorem 1.3 are conditional on the number of wage-setters, leading to a theorem on population size presented in Section 4.2 below.

**THEOREM 2 (Wage Inequality and Wage-Setter Power)**: *The effect on wage inequality of equal or unequal weights among the* *wage-setters depends on the configuration of micro consensus and macro consensus; equal power can increase or decrease inequality or have no effect. Specifically:*

**THEOREM 2.1 (Effect of Wage-Setter Power in the Case of Like-Minded Wage-Setters Whose Recommended Wage Distributions Are Identical with Finite Variances)**: *Given N wage-setters whose recommended wage distributions are identical, have finite variances, and are perfectly positively associated, inequality is the same whether the wage-setters have equal or unequal power*.

**THEOREM 2.2 (Effect of Wage-Setter Power in the Case of Independent-Minded Wage-Setters Whose Recommended Wage Distributions Have Equal Finite Variances)**: *Given N wage-setters whose recommended wage distributions have equal finite variances and are independent, inequality is lower when the wage-setters have equal power than when they are unequal*.

**THEOREM 2.3 (Effect of Wage-Setter Power in the Case of Independent-Minded Wage-Setters Whose Recommended Wage Distributions Have Equal Means and Different Finite Variances)**: *Given N wage-setters whose recommended wage distributions have equal means and different finite variances and are independent, unequal weights can achieve a lower wage inequality than equal weights, provided that the weights vary inversely with the variances of the recommended wage distributions*.

**THEOREM 2.4 (Effect of Wage-Setter Power in the Case of Two Opposite-Minded Wage-Setters Whose Recommended**

**Wage Distributions Are Identical with Finite Variances)**:

*Given two wage-setters whose recommended wage distributions are identical, have finite variances, and are perfectly negatively associated, inequality is lower when the wage-setters have equal power than when they are unequal.*

**Proof of Theorem 2.1**: The proof is the same as for Theorem 1.1. Whether the weights are equal or unequal, they still sum to unity.

**Proof of Theorem 2.2**: The formulas in the top two rows in the middle column of Tables 6 and 7 indicate that the weights operate as an attenuation factor on the variance of *Y*. Looking at Table 7, in the case of equal weights, the attenuation factor is $1/N$; in the case of unequal weights, the factor is the sum of the squared weights. The sum of the squares of the weights is minimized when the weights are equal, so that the attenuation factor is smaller in the equal-weights case than in the unequal-weights case. Hence, *Var(Y)* is smaller in the equal-weights case than in the unequal-weights case.

**Proof of Theorem 2.3**: The proof is based on the optimal weights result in Kotz, Johnson, and Read (1985). The variance of a weighted average, where the component distributions have equal means, is minimized, with respect to the weights, when each weight, $w_i^*$, is the following decreasing function of the corresponding variance:

$$w_i^* = \frac{[Var(X_i)]^{-1}}{\sum_{i=1}^{N} [Var(X_i)]^{-1}} . \qquad \textbf{(9)}$$

Intuitively, if larger weights are associated with smaller *X* variances, then unequal weights lead to a smaller *Y* variance than do equal weights. Put differently, wage inequality declines if the proponents of low-inequality wage schemes are more powerful than the proponents of high-inequality wage schemes.[14]

[14] An alternate proof of Theorem 2.2 can be constructed by replacing the unequal variances in the proof of Theorem 2.3 with equal variances.

**Proof of Theorem 2.4**: Look at the top two rows of the right column in Table 6. Each formula has two terms, and both terms include the weights. In the lefthand term, *Var(X)* is multiplied by the sum of the squared weights. The sum of the squared weights (where the weights are nonnegative and sum to unity) is minimized when the weights are equal. Hence, the lefthand term is smaller when the weights are equal. Meanwhile, the righthand term includes the product of the weights as a multiplier of the covariance. In this case of perfect negative association, the covariance is negative, and thus the righthand term is negative. The product of the weights is maximized when the weights are equal. Hence, the righthand term is larger (but of negative sign) when the weights are equal. Therefore, via the operation of both terms, *Var(Y)* is minimized when the weights are equal.

□

**Remarks on Theorem 2**. We shall encounter again the evocative theme of Theorems 2.2 and 2.4 – the closer the weights to equality, the lower the wage inequality. Meanwhile, Theorem 2 strengthens the foundation for analyzing the part played by the principle of equal power in the operation of wage inequality. Whether equal power increases or decreases wage inequality or has no effect depends on the configuration of types of micro and macro consensus.

**THEOREM 3 (Wage Inequality and Micro Consensus)**: *Given N wage-setters whose recommended wage distributions have finite variances, inequality in the final wage distribution is lower when the wage-setters are independent-minded than when they are like-minded and lower still when they are opposite-minded. Formally:*

$$\text{Ineq}^{pos} > \text{Ineq}^{ind} > \text{Ineq}^{neg}, \qquad \textbf{(10)}$$

*where the superscripts indicate the type of micro consensus, that is, whether the recommended wage distributions are positively associated, independent, or negatively associated.*

**Proof**: The variance of a weighted sum is written (Table 7, panel A):

$$Var(Y) = \sum_{i=1}^{N} w_i^2 \left[ Var(X_i) \right] + \sum_{i,k}^{N(N-1)/2} 2 w_i w_k Cov(X_i, X_k) \, , \tag{11}$$

where the covariance may be written:

$$Cov(X_i, X_k) = E(X_i X_k) - [E(X_i)][E(X_k)] \, . \tag{12}$$

When the $X_i$ distributions are mutually independent – the case portrayed in the middle columns of Tables 3, 4, 6, and 7 – the covariance equals zero and the rightmost term in the formula for *Var(Y)* drops out. When the $X_i$ distributions are positively associated, the covariance term is positive and *Var(Y)* will exceed that in the independent case; and when the $X_i$ distributions are negatively associated (in the sense that the sum of the weighted covariances is negative, as discussed in Section 3), the covariance term is negative and *Var(Y)* will be smaller than in the independent case.

□

Accordingly, within each row of Tables 3, 4, 6, and 7, inequality declines from left to right. Note that Theorem 3 holds whether the weights are equal or unequal and, of course, whether the recommended wage distributions are identical or different. Note also that the relation in expression (10) holds for any magnitude of positive or negative association (including the two perfect extreme types highlighted in much of this paper).

Thus, Theorem 3 provides a strong and pristine result, without the contingencies of Theorems 1 and 2. Independence of mind reduces wage inequality, and dissent does so even more.

### 4.2. Theorem on Population Size

**THEOREM 4 (Democracy and Demography)**: *Given N equally-weighted wage-setters whose recommended wage distributions have equal means and equal finite variances and are independent, the reduction in wage inequality that occurs as a result of introducing an independent wage-setter grows smaller as N increases; and, similarly, the increase in wage inequality that* *occurs as a result of introducing a partisan wage-setter also grows smaller as N increases.*

**Proof**: It is obvious from the formulas in Table 8 (panel C) that the decrease in wage inequality in case (2) and the increase in wage inequality in case (3) are both decreasing functions of *N*.[15]

□

Thus, when the group of wage-setters is very large, the effects of introducing a new wage-setter are small, whether the new wage-setter is independent or a partisan of one of the original wage-setters. This result suggests that arguments for independent thinking will be more vigorous in small societies than in large societies.[16]

### 4.3. Theorems on Consensus and Inequality

As discussed in Section 2.3, the case where there are *M* factions is formally equivalent -- in its effects on wage inequality -- to the case where there are $N=M$ wage-setters. Thus, it would appear that even if each faction has thousands of wage-setters, their presence does not contribute to the reduction of wage inequality. To make their voices "count," they would have to develop an independent or opposite mind and leave the faction. Of course, the power configuration, in the form of the $w_i$ and the $w_m$ , also plays a part.

The equivalence of the wage-setter case and the faction case is expressed formally:

**THEOREM 5 (Identical Effects of *C* Independent Wage-Setters and *C* Independent Wage-Setting Factions)**: *The variance of Y is the same under two distinct systems: (1) a system where there are C independent wage-setters and no one else has any power, and (2) a system where there are C independent*

[15] Theorem 4 could be considered a corollary to Theorem 1.3. It is reported separately both so as not to disrupt the flow of Theorems 1-3 and because population size is important in its own right.

[16] Note that the population size in the two key settings associated with the rise of the democratic spirit -- Greek city-states and American colonial towns -- was relatively small.

*factions, and the recommended wage distributions and weights of the factions replicate the recommended wage distributions and weights of the C wage-setters.*

**Proof**: This is a basic feature of the model's construction.

□

To illustrate, consider a triumvirate in which each wage-setter has weight equal to 1/3. Suppose that the society switches to a democratic regime, so that thousands of workers become wage-setters. If the new wage-setters align themselves in equal numbers with each member of the triumvirate, so that there are now three independent factions, each with a single recommended wage distribution, then the variance of $Y$ remains unchanged.

Similarly, if the original triumvirate has weights of ½, 1/4, and 1/4 and the new wage-setters align themselves in those proportions -- half with the leader whose weight was ½, one-quarter each with the other two leaders -- then, again, the variance of $Y$ remains unchanged.

Thus, there is no reduction in wage inequality to accompany increases in the number of partisan wage-setters, as there is when the number of independent wage-setters increases (Theorem 1.2). This has striking real-world implications. The hope that increasing the number of voters will reduce economic inequality can be dashed, absent independence of mind or a degree of dissent.

The case of full agreement and dictatorship merits its own theorem:

**THEOREM 6 (Equivalence of Dictatorship and Full Micro/Macro Agreement)**: *In terms of wage inequality, a society in which everyone agrees on the recommended wage distribution and on the wage amounts for particular workers is equivalent to a society governed by a dictator.*

**Proof**: It is evident from Tables 6 and 7 that a society with $N$ wage-setters whose recommended wage distributions are identical and perfectly positively associated leads to exactly the same $Var(Y)$ as a society with a single wage-setter. In both cases, $Var(Y)$ equals $Var(X)$.

□

### 4.4. Gregory the Great and the Sinless World

When would wage inequality equal zero? Gregory the Great ([540-604] 1844) observes in his famous principle (Moralia in Job, xxi), "Where there is no sin, there is no inequality." While Gregory did not argue for the converse, we take logical license and pay tribute to Gregory the Great, calling a world without inequality a world without sin:

> **THEOREM 7 (Sinless World)**: *The final wage distribution is Equal – that is, has zero inequality – when (1) the number of wage-setters is two, (2) they are perfectly opposite-minded, (3) their recommended wage distributions have a correlation of -1, and (4) either (4a) the two recommended wage distributions are identical and equally-weighted, or (4b) the two recommended wage distributions are different and unequally-weighted and the ratio of their weights is the inverse of the ratio of their standard deviations.*

**Proof**: The proof is based on simple algebraic manipulation of the formulas in Table 6 and the optimal weights result in Kotz, Johnson, and Read (1985). For example, to prove the part including condition 4a, it is straightforward to show that the formula for *Var(Y)* in the case of two identical recommended wage distributions with unequal weights reduces to:

$$Var(Y) \ = \ [Var(X)]\,[w_1 - w_2]^2 \ . \qquad \mathbf{(13)}$$

Accordingly, *Var(Y)* equals zero when the weights are equal.

□

Of course, as will be seen in Theorem 13, a benevolent dictator can also produce a wage distribution with zero inequality.

### 4.5. Theorems on Political Parties

George Washington did not like political parties. He feared that they would destroy the Union, writing urgently in his Farewell Address (1796): “Let me . . . warn you in the most solemn manner against the baneful effects of the spirit of party generally." We might ask about the fate of wage inequality in societies with political parties whose policies generate the wage distribution. As noted earlier, the wage model developed in this paper applies to political parties in the case where final outcomes can be represented as a weighted average of the policies of all the parties. This occurs in explicit proportional-representation systems, and, given party alignments and the ebb and flow of party dominance, may occur as well in systems where, in principle, the dominant party could set policy alone.

All the theorems presented above thus apply to political parties. For example, Theorem 1.2 may be re-expressed as follows:

**THEOREM 8 (Effect of the Number of Parties in the Case of Independent-Minded Parties of Equal Relative Size)**: *Given N parties of equal relative size and with associated potential wage distributions which have equal means and finite variances and are independent, inequality is a decreasing function of the number of parties.*

Similarly, Theorem 3 may be re-expressed as follows:

**THEOREM 9 (Inequality and Micro Agreement)**: *Given N political parties with associated potential wage distributions of finite variance, inequality in the final wage distribution is lower when the parties are independent-minded than when they are like-minded and lower still when they are opposite-minded. Formally:*

$$\text{Ineq}^{\text{pos}} > \text{Ineq}^{\text{ind}} > \text{Ineq}^{\text{neg}}, \quad \textbf{(14)}$$

*where the superscripts indicate the type of micro consensus, that is, whether the associated potential wage distributions are positively associated, independent, or negatively associated.*

Of course, political parties, in contrast to individuals, are not particularly prone to be like-minded, and thus the practical import of Theorem 9 is the superiority of dissent versus independence in achieving inequality reduction. Note that Theorems 8 and 9 may be useful in analyzing multi-party traditions in Europe and around the world.

In the remainder of this section, we focus on two-party systems.

**THEOREM 10 (Two-Party Systems, Dissent, and Inequality)**: *Given a two-party system, holding constant party relative size, wage inequality is minimized when the two parties advocate policies that are directly opposite each other.*

**Proof**: The proof follows from Theorem 9 and the proof of Theorem 3 on which Theorem 9 is based.

□

**THEOREM 11 (Two-Party Systems, Party Relative Size, and Inequality)**: *Given a two-party system and associated potential wage distributions that are either (1) independent with equal finite variances or (2) identical with finite variances and perfectly negatively associated, wage inequality is minimized when the two parties are of equal relative size.*

**Proof**: The proof follows from the proofs of Theorems 2.2 and 2.4, on which Theorem 11 is based.

□

Theorem 7 on Gregory the Great and the sinless world leads directly to a theorem on the Workers' Paradise:

**THEOREM 12 (Two-Party Systems and the Workers' Paradise)**: *The final wage distribution in a two-party system is Equal – that is, has zero inequality – when (1) the two parties are perfectly opposite-minded, (2) their associated potential wage distributions have a correlation of -1, and (3) either (3a) the wage distributions are identical and the two parties have equal power,* *or (3a) the wage distributions are different and the two parties have unequal strength and the ratio of their relative size is the inverse of the ratio of the distributions' standard deviations.*

**Proof**: The proof follows from the proof of Theorem 7, on which Theorem 12 is based.

### 4.6. Theorem on Dictatorship

It is obvious by now that there are conditions under which that fabled figure – the benevolent dictator -- can minimize inequality, echoing centuries of social thought since Plato (Republic; Laws, Book IV). Formally:

**THEOREM 13 (Benevolent Dictator)**: *Rule by a single wage-setter minimizes wage inequality if and only if either (1) that wage-setter's recommended wage distribution is Equal, or (2) that wage-setter's recommended wage distribution has the lowest inequality and there is no pair of potential wage-setters generating an Equal wage distribution (i.e., satisfying Theorem 7).*

**Proof**: A proof can be constructed based on the results in Tables 6 and 7.

□

### 4.7. Theorems on the Shape of the Wage Distribution

What determines the shape of the wage distribution? This classic question, whose roots stretch at least as far back as John Chrysostom's (386-407 [1860]) estimation of the proportions rich, poor, and in the middle in the city of Antioch, has fired the imagination in every age (Ammon 1899; Pigou 1924; Lebergott 1959; Kleiber and Kotz 2003). The wage model analyzed in this paper yields a pertinent result:

**THEOREM 14 (Number of Wage-Setters and Normality of the Wage Distribution)**: *As the number of independent wage-setters increases (where the component recommended distributions have finite variances), the shape of the wage distribution tends to normality.*

**Proof**: The proof relies on the version of the central limit theorem owed to Liapunov (1900, 1901), as strengthened by Lindeberg and Feller, where the component independent variates may have different distributions, provided that they have finite variances (Stuart and Ord 1987; Wolfson 1985).

□

Thus, according to the wage-determination account outlined in this paper, if the wage distribution is not normal, it must be because the number of wage-setters is small or because the wage-setters are not independent.

Note, however, that our model is consistent with a situation in which wage-setting may occur in separate groups. For example, in a university, each department may conduct its separate wage-setting activity. Similarly, wage-setting may be confined to occupations, to firms, to sectors, or to political jurisdictions. Thus, Theorem 14 is confined to the entity within which wage-setting occurs.

Suppose that within each wage-setting entity the wage distribution is normal. What then would be the shape of the overall wage distribution? This is a problem in pooling distributions, and the shape of the overall distribution would depend on the number, mean, and variance of the component distributions and the share of the total population in each. In general, the density function of a mixture of a finite number of normals is easy to write, but the parameters are not always easy to derive. Moreover, the mixture distribution may exhibit a wide variety of shapes, including the normal, and it may be unimodal or multimodal.[17] Thus, a non-normal wage distribution is not inconsistent with normal wage distributions within wage-setting entities. Formally:

**THEOREM 15 (Multiple Groups and Non-Normality of the Wage Distribution)**: *If wage-setting occurs in separate groups,*

[17] Famous examples of mixtures of normal distributions include Galton's normal mixture of normals (sketched by Edgeworth) and Karl Pearson's bimodal mixture of two normals. For formulas and graphs, see Everitt (1985) and Stigler (1986:312).

*the within-group wage distributions may be normal but the overall wage distribution nonnormal.*

**Proof**: The proof is based on standard statistical results, as sketched above.

□

**Remark on Theorem 15**. If the separate groups within which wage-setting occurs are characterized by different configuration of the relevant factors – number of wage-setters, configuration of wage-setter power, micro and macro consensus – then it is easy to imagine that the wage distribution may, or may not, be normal within wage-setting context and that the ensuing cross-context mixtures produce, as Jasso and Kotz (2007:321) put it, "a dazzling diversity of shapes". Wage distributions will be symmetrical and asymmetrical, unimodal and multimodal, narrow and wide, short and tall. There will be Mexican hats and fedoras, bowler hats and top hats.

## 5. IMPLICATIONS OF THE WAGE MODEL: RESULTS BASED ON CONTINUOUS PROBABILITY DISTRIBUTIONS

As shown above, the number of wage-setters, their agreements and disagreements, and their relative power combine to produce the inequality in the wage distribution. For example, given independent-minded and equally-weighted wage-setters whose recommended wage distributions have finite variances, as the number of wage-setters increases, the inequality in the final wage distribution decreases (Theorem 1.2) and the distribution tends to normality (Theorem 14). And dissent is the tool par excellence for reducing inequality, followed by independence of mind (Theorem 3). We now explore the wage model using continuous univariate probability distributions. In this initial exploration, the situation is simple and the distribution highly tractable, but more elaborate situations can be analyzed in the future.

### 5.1. Setup

To begin, and as above, let there be $N$ wage-setters. Let the wage distributions recommended by the $N$ wage-setters be identical – that is, there is macro agreement – and with finite variances. The first task is to select a modeling distribution for the wage-setter-specific distribution. The main requirement for the modeling distribution is that it be defined on the positive support, ideally with a positive infimum (so as to represent a situation with a minimum wage). The modeling distribution need not be any of the modeling distributions commonly used to represent the income distribution (see, for example, the distributions discussed by Dagum (1985) and by Kleiber and Kotz (2003)), for, as discussed above and in Jasso and Kotz (2007:321), the income distribution routinely modeled may be a mixture of many wage distributions, each specific to a wage-setting context.

Modeling distribution for each wage-setter's recommended wage distribution – the shifted exponential. To model the wage-setter-specific recommended wage distributions, we choose the exponential with location parameter $a$ equal to the positive infimum and with scale parameter set at unity (Johnson, Kotz, and Balakrishnan 1994:494). The probability density function (PDF) for this shifted exponential is given by:

$$f(x) = \exp[-(x-a)] , \quad x > a > 0 . \tag{15}$$

The exponential has several advantages. First, it has been extensively studied, so that its properties and relations to other variates are well established (see, for example, Galambos and Kotz (1978), Johnson, Kotz, and Balakrishnan (1994:494-572), and the references cited therein). Second, it is highly tractable. Third, in the study of heavy tails, the exponential emerges, along with the gammas, as intermediate between the light-tailed distributions and the heavy-tailed distributions, by several criteria for classifying distributions. As Bryson (1985:598) puts it, "the exponential and gamma families seem to occupy a middle ground". Fourth, the exponential provides a natural model for situations in which supply and demand considerations are important and wage increases at an increasing rate with relative rank, as noted by Jasso and Kotz (2007:320). Fifth, the exponential also provides a natural way to represent work units in which the number of workers is smaller, the greater the skill or experience.[18] The exponential thus

[18] Military units are a good example. For example, in the officer grades, the number of slots decreases with rank; thus, there are fewer generals than colonels, fewer colonels than

seems an ideal modeling distribution for this initial exploration.

Next, we derive the distributions for some pertinent special cases that arise when the wage-setters' recommended wage distributions are shifted exponentials. As above, the wage-setters may be equally or unequally weighted, and their recommended wage distributions may be related in three main ways – they may be perfectly positively associated, independent, or perfectly negatively associated.

### 5.2. The Case of Perfectly-Positively-Associated Shifted Exponentials

In the case of perfect positive association, the final wage distribution – the weighted sum of the *N* shifted exponentials – remains unchanged. It is the same original shifted exponential whose PDF is given in expression (15). Whether the weights are equal or unequal does not matter. This result is well-known and has been widely used (e.g., in derivation of multi-good status distributions (Jasso 2001; Jasso and Kotz 2007)). However, if a proof is desired, a simple proof along the lines of the proof for Theorem 1.1 can be constructed.

### 5.3. The Case of Independent Shifted Exponentials

#### 5.3.1. The Case of Independent/Equally-Weighted Shifted Exponentials

As is well known, the exponential is the special case of a gamma arising when the gamma's shape parameter equals unity. By the special "reproductive property," the sum of independent and identically distributed (iid) exponentials is a gamma with shape parameter equal to the number of variates in the sum (Johnson, Kotz, and Balakrishnan 1994:340).[19] Thus, the

---

lieutenant colonels, fewer lieutenant colonels than majors, etc. (Rostker, Thie, Lacy, Kawata, and Purnell 1993). Moreover, across entire military units, the number of enlisted is always greater than the number of officers – producing the E/O ratio which, for example, in the United States during World War II stood at ten. Recent decreases in the E/O ratio have spurred debates concerning the "officer bloat" and the possibility that modern warfare requires lower E/O ratios. We note that academic departments with more full professors than assistant professors (or even than more assistant and associate professors combined) refer to themselves rather more generously as "topheavy".

[19] In fact, as Johnson, Kotz, and Balakrishnan (1994:340) note, the reproductive property holds even if the gammas in the sum have different shape parameters. Given selection of the exponential as the modeling distribution for each wage-setter's recommended wage distribution, this analysis does not make use of that property. Future research, however, might explore a situation in which the wage-setters' distributions can be modeled by gammas with different shape

PDF of the sum of these iid shifted exponentials is given by:

$$f(x) = \frac{(x-a)^{N-1} \exp[-(x-a)]}{\Gamma(N)}, \quad x > a > 0, \tag{16}$$

where, as before, $a$ denotes the infimum, and the shape parameter (usually denoted by some other letter) is here denoted by $N$, to indicate its interpretation in the wage model. When the shape parameter is an integer, the distribution is known as the Erlang. In this case, given that the number of wage-setters $N$ (i.e., the number of exponentials) must be an integer, the distribution is an Erlang.

Of course, we require the average of the $N$ variates, rather than their sum, and so we carry out a change-of-variable to obtain the PDF of the final wage distribution $Y$:

$$g(y) = \frac{N^N (y-a)^{N-1} \exp[-N(y-a)]}{\Gamma(N)}, \quad y > a > 0. \tag{17}$$

Inspection of the formula for the PDF reveals that the distribution is also Erlang, with a new scale parameter, formerly unity but now equal to the reciprocal of $N$, embodied in the two new occurrences of $N$ (the initial factor $N$ and the multiplier $N$ in the argument of the exponential function, both in the numerator). Cognizant of the location parameter, we call it the shifted Erlang.[20]

Accordingly, because the distribution arising from the average of $N$ independent and identical exponentials is Erlang, we already know many of its properties. First, the mean is equal to the sum of 1 (the mean of the standard exponential) and the infimum $a$. The mode and median are each a sum of $a$ and a monotonic function of $N$. The variance, which is of particular interest, is a monotonic function of $N$ – viz., $1/N$. Finally, the formula for the Gini coefficient turns out to be a simple modification of a well-known formula for the unshifted Erlang and gamma

---

parameters.

[20] The power $N$ to which the initial factor in the numerator of the PDF of $Y$ in (17) is raised is not a new occurrence of $N$, but it is invisible in equation (16) because the scale factor in the PDF in (16) is equal to unity.

distributions (see, for example, McDonald and Jensen (1979:856), Johnson, Kotz, and Balakrishnan (1994:341), Kleiber and Kotz (2003:164), and Jasso and Kotz (2007)):

$$G = \left[ \frac{\Gamma(N + 1/2)}{\sqrt{\pi}\,\Gamma(N + 1)} \right] \left( \frac{1}{a+1} \right) , \tag{18}$$

where $N$ denotes the number of wage-setters, $a$ denotes the minimum wage, and the modification consists of attaching the simple righthand factor to adjust for the shift in the origin. This is only the first of many formulas which include the shift factor $(1/(a+1))$, which, being a proper fraction, serves to attenuate the quantity.

The first partial derivatives of the variance and the Gini coefficient with respect to $N$ are negative, indicating that, as $N$ increases, both the variance and the Gini coefficient decrease. Moreover, their left limits are both zero. Thus, in this case of equally-weighted independent shifted exponentials, as expected from Theorem 1.2, both the variance and the Gini coefficient approach zero as $N \rightarrow \infty$.

Table 9 reports the PDF, variance, and Gini coefficient in the shifted Erlang arising from identical, independently distributed, and equally-weighted shifted exponentials for $N$ the number of wage-setters from 1 to 10. As shown, both indicators of inequality diminish quickly. The variance, which is 1 in the exponential, is cut in half with the addition of a single new independent wage-setter, and by 5 wage-setters it registers .2. Ignoring the shift factor $(1/(a+1))$, the Gini coefficient, which is .5 in the unshifted exponential, declines to .375 when $N$ equals 2; by 5 wage-setters it is approximately .246, and by 10 wage-setters, it has declined to .176. Of course, taking into account the shift factor reduces all these values of the Gini coefficient still more.

– Table 9 about here –

To provide visual illustration and to solve for the mode and median, we set the parameter $a$ to .25. Table 10 reports the median and mode and, for contrast, the mean, for values of $N$ from 1 to 10. Figure 1 depicts the PDF of the ten Erlangs. It is visually obvious that as the number of wage-setters increases, the distribution grows progressively more concentrated.

– Table 10 about here –

– Figure 1 about here –

Table 10 also shows that the mean, median, and mode display the pattern characteristic of right-skewed gamma distributions (Groeneveld and Meeden 1977; MacGillivray 1985) – namely, the mean is greater than the median, and the median is greater than the mode.

Finally, it is also evident from Figure 1 that as the number of wage-setters increases, the variate tends more and more to the normality expected of the gamma family (Johnson, Kotz, and Balakrishnan 1994:340) and expected as well from the central limit theorem in Theorem 14. Yet, however expected, it is striking how quickly the Erlang takes on the shape of the normal. Thus, it is not surprising that the phrase "domain of attraction" has come to be used to refer to the set of distributions that tend to a particular other distribution. The Erlang, together with all distributions with finite variance, "is in the domain of attraction to the normal," as Stout (1985:417) evocatively puts it.

#### 5.3.2. <u>The Case of Independent/Unequally-Weighted Shifted Exponentials</u>

When the iid exponentials are unequally weighted, the ensuing distribution is known as a <u>general Erlang</u> or a <u>general gamma</u>. The formula for the PDF in the unshifted case is reported by Johnson and Kotz (1970:222) and, more recently, by Johnson, Kotz, and Balakrishnan (1994:552). The PDF for the shifted general Erlang is only a minor modification, namely:

$$g(y) \;=\; \sum_{i=1}^{N} \left( \prod_{i \neq j} \left(w_i - w_j\right)^{-1} \right) w_i^{N-2} \exp[-N(y-a)/w_i]\,, \quad y > a > 0\,, \quad w_i \neq w_j\,. \qquad \textbf{(19)}$$

Using this formula as a starting point, we derived the formula for the special case of two wage-setters with any combination of unequal weights. Because in this case, the two weights may be thought of as the party split $p$, where $p$ represents one party's proportion of the population (or legislature) and $p$ and (1-$p$) sum to one, we express the formula for the PDF in terms of $p$:

$$g(y) \;=\; \left( \frac{1}{1-2p} \right) \left[ \exp\left( -\frac{y-a}{1-p} \right) \;-\; \exp\left( -\frac{y-a}{p} \right) \right], \quad y > a > 0\,, \quad 0 < p < .5\,. \qquad \textbf{(20)}$$

Figure 2 depicts the PDF of four shifted general Erlangs corresponding to two parties with values of the party split $p$ of .1, .2, .3, and .4. Figure 2 also includes the PDF of the shifted Erlang corresponding to two equally-weighted wage-setters (already shown in Figure 1).

– Figure 2 about here –

Working with expression (20), we obtained the formula for the Gini coefficient in this special two-$N$/independent/unequally-weighted shifted general Erlang, which turns out to be beautiful and spartan:

$$\left(\frac{p^2 - p + 1}{2}\right)\left(\frac{1}{a+1}\right) . \qquad \textbf{(21)}$$

Recall that the Gini coefficient for the independent/equally-weighted Erlang case at $N = 2$ is .375 times the shift factor (Table 9). Using formula (21), it is straightforward to establish that the limits of the Gini coefficient as $p$ goes from zero to .5 are .5 and .375 (again in each instance multiplied by the shift factor). Taking the derivative of (21) with respect to $p$ shows that inequality is at a minimum when the population (or legislature) is evenly divided between the two parties. This result, expected from Theorem 11, is not without practical importance. Commentators concerned about such exact splitting of the electorate may take comfort in its effects on wage inequality.

### 5.4. <u>The Case of Perfectly-Negatively-Associated Shifted Exponentials</u>

#### 5.4.1. <u>The Case of Perfectly-Negatively-Associated/Equally-Weighted Shifted Exponentials</u>

A pair of equally-weighted shifted exponentials that are perfectly negatively associated gives rise to a shifted version of the ring(2)-exponential variate obtained by Jasso (2001) and analyzed by Jasso and Kotz (2007). As would be expected from the original ring(2)-exponential, an important feature of this variate is that its lower extreme value shifts upward – in this case to ($a$ + ln(2)) or approximately $a$ + .693. Concomitantly, inequality declines perceptibly. Ignoring the shift factor, the Gini coefficient, which registers .5 in the perfectly-positively-associated case and .375 in the case of two independent wage-setters, declines to (ln(2) - ½) or approximately .193 in the negative-association case. As expected from Theorem 10, dissent dramatically reduces inequality.

### 5.4.2. The Case of Perfectly-Negatively-Associated/Unequally-Weighted Shifted Exponentials

To define the case of $N$ negatively-associated recommended wage distributions, we follow Jasso and Kotz (2007), who adopt the principle of organized subsets proposed by Berger et al. (1977:126-127), and apply the principle as follows: (i) the $N$ wage-setters are arranged in two factions; (ii) within each faction, the wage-setters are perfectly positively associated; and (iii) the two factions are perfectly negatively associated. This case gives rise to a shifted version of the mirror-exponential variate introduced by Jasso and Kotz (2007). As in the original mirror-exponential, there is an expression for the quantile function but none for the PDF or the CDF. The formula for the QF is given by:

$$Q(\alpha) = a - (p - p^2)\ln\{\alpha + [(p - p^2)(1 - \alpha)^2]\} - (1 - p)^2\ln[(1 - p)(1 - \alpha)] - p^2\ln[p(1 - \alpha)]\,, \quad \textbf{(22)}$$

which differs from the QF for the original unshifted mirror-exponential only by the addition of the initial righthandside factor $a$ (the infimum, representing minimum wage).

The Gini coefficient is given by:

$$\left[\frac{-p^3\ln(p) - (1 - p)^3\ln(1 - p)}{p(1 - p)} - \frac{1}{2}\right]\left(\frac{1}{a+1}\right). \quad \textbf{(23)}$$

This differs from the formula for the original unshifted mirror-exponential only in the shift factor $(1/(a+1))$.

### 5.5. Effects of Like-/Independent-/Opposite-Mindedness: Contrasting the Shifted Exponential, Shifted Erlang, Shifted General Erlang, and Shifted Mirror-Exponential

Table 11 collects the formulas for the major quantities associated with the three variates representing the final wage distribution in the case of two equally-weighted wage-setters – the shifted exponential in the positive-association case, the shifted Erlang in the independent case, and the shifted ring(2)-exponential in the negative-association case. Figure 3 depicts the PDFs of the three variates (the shifted exponential also appears in Figure 1, and the shifted Erlang in both Figures 1 and 2).

– Table 11 about here –

– Figure 3 about here –

It is evident from Table 11 and Figure 3 that inequality declines as we progress from like-mindedness to independent-mindedness to opposite-mindedness, illustrating Theorems 3, 9, and 10. For example, as already noted, the Gini coefficient (Table 11, bottom row) declines, ignoring the shift factor, from .5 to .375 to .193. As an instrument for reducing inequality, being at loggerheads appears to have no peer.

Table 12 summarizes the variates obtained for all six cases – including the three unequal-weights cases as well as the equal-weights cases described in Table 11 and depicted in Figure 3.

– Table 12 about here –

To examine the case of unequal weights – that is, the case in which the party split departs from .5 – we report in Table 13 the Gini coefficient in the final wage distribution for the shifted exponential, the shifted general Erlang, and the shifted mirror-exponential for party splits from .05 to .5. First, as expected from Theorems 3, 9, and 10, inequality declines from left to right across the table, going from like-mindedness to independent-mindedness to opposite-mindedness. Second, as expected from Theorem 11, in the independent and negative-association cases, the Gini coefficient declines as the two parties become more evenly divided. Figure 4 depicts both the consensus effect and the party split effect.

– Table 13 about here –

– Figure 4 about here –

## 6. CONCLUDING NOTE

This paper proposes a new model of wage determination and wage inequality. In this model, wage-setters set workers' wages, and there are three key features: (1) the number of wage-setters may vary; (2) some wage-setters may count more than others; and (3) wage-setters may disagree with each other on both the wage distribution and the amounts recommended for particular workers.

As shown above, the number of wage-setters, their agreements and disagreements, and their relative power combine to produce the inequality in the wage distribution. For example, given independent-minded and equally-weighted wage-setters whose recommended wage distributions have finite variances, as the number of wage-setters increases, the inequality in the final wage distribution decreases (Theorem 1.2) and the distribution tends to normality (Theorem 14). An important result, both for its generality and its substance, is that dissent is the preeminent tool for reducing inequality, followed by independence of mind (Theorem 3). Thus, when leaders of democratic nations seek to forge an economic consensus they are unwittingly inducing greater economic inequality.

New avenues for research include both theoretical and empirical work. Theoretically, it will be possible to derive many further implications for special cases, including: (1) wage-setting situations where all recommended wage distributions must preserve the current rank-ordering; (2) the case where the variates are independently and identically distributed, as in Section 5, but drawn from a family other than the exponential; and (3) the case where the variates are independent but not identical, in particular, drawn from two different families, as in the case recently investigated by Nadarajah and Kotz (2005). Finally, it will be useful to explore mixture distributions for modeling the wage distribution in large populations which incorporate several wage-setting entities.

Empirically, the model can be applied and tested in a variety of ways. These include a new emphasis on the behavior of wage-setters. While most studies of wage attainment focus on the characteristics of workers, the new model suggests that it might be worthwhile to focus on the characteristics of wage-setters. Moreover, not only are the characteristics of wage-setters potentially important but so also is their network of social relations, in particular, the processes that lead to agreement and disagreement among them. Further, the structural features of the wage-setting situation, especially the decision rule that gives some wage-setters more power than others, might profit from scrutiny.

Future research might also use the wage model to examine historical experience with benevolent dictators, duarchies, utopian experiments, and the totalitarian forms to which they sometimes lead.

Another way to empirically assess the wage model is to examine data on ideas of just earnings, in particular (1) data collected by the International Social Science Programme (ISSP), and (2) data collected via factorial survey methods. The ISSP has asked respondents, via the Inequality Modules fielded in 1987, 1992, and 1999, to provide the just earnings for sets of 9-11 occupations (for a list of the occupational titles, see Jasso 2007:229). Factorial surveys of just earnings have been carried out since Jasso and Rossi (1977). These studies yield matrices that assemble the earnings regarded as just for a set of fictitious workers by a set of respondents, and enable estimation of respondent-specific just earnings functions and just earnings distributions and their parameters (Jasso 1994; Jasso 2006a:379-407; Osberg and Smeeding 2006). Published fragments of just earnings matrices include those reported in Jasso (2006a:389-392) and Jasso and Meyersson (2007:133).

As a quick prelude to future work, we cast into wage-model matrix form one of the factorial survey data decks analyzed in Jasso (2006b) – where 23 respondents each form a just earnings amount for each of 20 fictitious workers, generating 23 recommended wage distributions and 253 covariances. These data reveal the pervasive individualism enshrined in Hatfield's (Walster et al. 1976:4) principle. Of the 253 covariances, 50 are negative (19.8 percent). Moreover, the final wage distribution (i.e., where each worker's wage is the average of the 23 recommended wage amounts) has a smaller variance than all but two of the respondent-specific recommended wage distributions. Thus, if these respondents' ideas of just earnings became their recommended wage distributions, then, as expected from Theorem 3, their disagreements would attenuate final wage inequality.

Turning to the larger question of democracy and inequality, since Plato and Aristotle, it has been a useful and appealing exercise to rank-order the forms of government according to their potential for increasing the common good. In that spirit, we provide the rank-ordering implied by the new wage model from the special vantage point of minimizing inequality. To be sure, the common good means much more than inequality reduction. Yet, as noted above, since antiquity, inequality has aroused suspicions and the impulse to "correct" it (Aristotle, Politics, Book II, Chapter 9).

Here then is the inequality-minimizing rank-ordering implied by the new wage model:

1. Perfect Equality (Gregory the Great's Sinless World). Perfect equality is possible in the following regimes:

1.1. The Benevolent Dictator Regime. Provided that the wage distribution recommended by the benevolent dictator is an Equal distribution. [Theorem 13]

1.2. A Duarchy. Provided that there is perfect dissent and the two recommended wage distributions have a correlation of -1, and either of the following conditions is satisfied

1.2.1. The two recommended wage distributions are identical and the two wage-setters have equal power, or

1.2.2. The two recommended wage distributions are different and the two wage-setters have unequal power and the ratio of their weights in the inverse of the ratio of their standard deviations. [Theorems 7 and 12]

2. Inequality Minimization. Inequality is minimized, but not completely eradicated, in the following regimes:

2.1. The Benevolent Dictator Regime. Provided that the wage distribution recommended by the benevolent dictator has the lowest inequality of all recommended wage distributions. [Theorem 13]

2.2. Democracy. Provided that the democracy has a fixed income distribution (in the anonymous sense) and the wage-setters exhibit independence of mind. [Theorem 1.2]

2.3. Regime with Unequally-Empowered Wage-Setters. Provided that inequality in the recommended wage distributions varies inversely with power. [Theorem 2.3]

Finally, note that the top contenders, with the exception of the Benevolent Dictator, combine elements of different forms of government, consistent with the classic appeal of mixed government (Thomas Aquinas, Summa Theologica, II-I, Q. 105, art. 1) .

# REFERENCES


Ammon, Otto. 1899. "Some Social Applications of the Doctrine of Probability." Journal of Political Economy 7:204-237. Translated by Carlos C. Closson.

Aristotle. [384-322 BC] 1952. The Works of Aristotle. 2 volumes. Translated by W. D. Ross. Chicago: Britannica Press.

Augustine, Saint. ([413-426] 1952). The City of God. Translated by Marcus Dods. Chicago: Britannica.

Ben-Porath, Yoram. 1980. "The F-Connection: Families, Friends, and Firms and the Organization of Exchange." Population and Development Review 6:1-30.

Berger, Joseph, M. Hamit Fisek, Robert Z. Norman, and Morris Zelditch, Jr. 1977. Status Characteristics and Social Interaction. New York: Elsevier.

Blau, Peter M. 1977. Inequality and Heterogeneity: A Primitive Theory of Social Structure. New York: Free Press.

Boyes, William J., and Stephen K. Happel. 1989. "Auctions as an Allocation Mechanism in Academia: The Case of Faculty Offices." Journal of Economic Perspectives 3:37-40.

Brickman, Philip, Robert Folger, Erica Goode, and Yaacov Schul. 1981. "Micro and Macro Justice." Pp. 173-202 in M. J. Lerner and S. C. Lerner (eds.), The Justice Motive in Social Behavior. New York, NY: Plenum.

Bryson, M. C. 1985. "Heavy-Tailed Distributions." Pp. 598-601 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 3. New York: Wiley.

Chamberlain, N. W. 1951. Collective Bargaining. New York: McGraw-Hill.

Champernowne, David G., and Frank A. Cowell. 1998. Economic Inequality and Income Distribution. Cambridge, UK: Cambridge University Press.

Chipman, John S., and James C. Moore. 1980. Real National Income with Homothetic Preferences and a Fixed Distribution of Income. Econometrica 48:401-422.

Clark, William. 2006. Academic Charisma and the Origins of the Research University.

Chicago, IL: University of Chicago Press.

Dagum, Camilo. 1985. "Income Distribution Models." Pp. 27-34 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 4. New York: Wiley.

Dobbin, Frank. 2004. "The Sociological View of the Economy." Pp. 1-46 in F. Dobbin (ed.), The New Economic Sociology: A Reader. Princeton, NJ. Princeton University Press.

______. 2005. "Comparative and Historical Approaches to Economic Sociology." Pp. 26-48 in N. J. Smelser and R. Swedberg (eds.), The Handbook of Economic Sociology, Second Edition. Princeton, NJ. Princeton University Press.

Dunlop, John T. 1958. Industrial Relations Systems. New York: Holt.

Elliott, Robert F. 1991. Labor Economics: A Comparative Text. London: McGraw-Hill.

Everitt, B. S. 1985. "Mixture Distributions." Pp. 559-569 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 5. New York: Wiley.

Galambos, Janos, and Samuel Kotz. 1978. Characterizations of Probability Distributions, Lecture Notes in Mathematics, 675. Berlin, Germany: Springer-Verlag.

Gibbon, Edward. [1776-1789] 1952. The Decline and Fall of the Roman Empire. Six volumes. Chicago: Britannica.

Granovetter, Mark. 1981. "Toward a Sociological Theory of Income Differences." Pp. 11-47 in Ivar Berg (ed.), Sociological Perspectives on Labor Markets. New York: Academic.

______. 1985. "Economic Action and Social Structure: The Problem of Embeddedness." American Journal of Sociology 91:481-510.

______. 1988. "The Sociological and Economic Approaches to Labor Market Analysis: A Social Structural View." Pp. 187-216 in George Farkas and Paula England (eds.), Industries, Firms, and Jobs: Sociological and Economic Approaches. New York: Plenum Press.

Gregory the Great, St. c. 579-604 [1849]. Patrologiae Cursus Completus. Series Latina, Volumes 75-79. Edited by J. P. Migne.

Groeneveld, Richard A., and Glen Meeden. 1977. "The Mode, Median, and Mean Inequality." The American Statistician 31:120-121.

Hicks, John. 1963. The Theory of Wages. New York: St Martin's Press.

Jasso, Guillermina. 1983. "Fairness of Individual Rewards And Fairness of the Reward Distribution: Specifying the Inconsistency between the Micro and Macro Principles of Justice." Social Psychology Quarterly 46:185-199.

______. 1989. "Self-Interest, Distributive Justice, and the Income Distribution: A Theoretical Fragment Based on St. Anselm's Postulate." Social Justice Research 3:251-276.

______. 1994. "Assessing Individual and Group Differences in the Sense of Justice: Framework and Application to Gender Differences in Judgments of the Justice of Earnings." Social Science Research 23:368-406.

______. 2001. "Studying Status: An Integrated Framework." American Sociological Review 66:96-124.

______. 2004. "The Tripartite Structure of Social Science Analysis." Sociological Theory 22:401-431.

______. 2006a. "Factorial Survey Methods for Studying Beliefs and Judgments." Sociological Methods and Research 34:334-423.

______. 2006b. "Homans and the Study of Justice." Pp. 203-227 in A. Javier Treviño (ed.), George C. Homans: History, Theory, and Method. Boulder, Colorado: Paradigm Press.

______, and Peter H. Rossi. 1977. "Distributive Justice and Earned Income." American Sociological Review 42:639-651.

______, and Samuel Kotz. 2007. "A New Continuous Distribution and Two New Families of Distributions Based on the Exponential." Statistica Neerlandica 61:305-328. Manuscript available as IZA Discussion Paper No. 2598 at www.iza.org.

______ and ______. 2008. "Two Types of Inequality: Inequality Between Persons and Inequality Between Subgroups." Sociological Methods and Research 37:31-74.

Manuscript available as IZA Discussion Paper No. 2749 at www.iza.org.

_______ and Eva M. Meyersson Milgrom. 2008. "Distributive Justice and CEO Compensation." *Acta Sociologica* 51:123-143. Manuscript available as IZA Discussion Paper No. 3236 at www.iza.org. Published article at http://asj.sagepub.com/ .

Jencks, Christopher, Marshall Smith, Henry Acland, Mary Jo Bane, David Cohen, Herbert Gintis, Barbara Heyns, and Stephan Michelson. 1972. Inequality: A Reassessment of the Effect of Family and Schooling in America. New York, NY: Basic Books.

John Chrysostom, St. c. 386-407 [1860]. Patrologiae Cursus Completus. Series Graeca. Volumes 47-64. Edited by J. P. Migne.

Johnson, Norman L., and Samuel Kotz. 1970. Continuous Univariate Distributions, Volume 1. New York, NY: Wiley.

Johnson, Norman L., Samuel Kotz, and N. Balakrishnan. 1994. Continuous Univariate Distributions, Volume 1. Second Edition. New York, NY: Wiley.

Kleiber, Christian, and Samuel Kotz. 2003. Statistical Size Distributions in Economics and Actuarial Sciences. Hoboken, NJ: Wiley.

Kotz, Samuel, Norman L. Johnson, and Campbell B. Read. 1982. "Conjugate Ranking." P. 145 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.). Encyclopedia of Statistical Sciences, Volume 2. New York: Wiley.

Kotz, Samuel, Norman L. Johnson, and Campbell B. Read. 1985. "Tukey's Inequality for Optimal Weights." Pp. 361-362 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 9. New York: Wiley.

Lebergott, Stanley. 1959. "The Shape of the Income Distribution." American Economic Review 49:328-347.

Lewis, George Cornewall. 1852. A Treatise on the Methods of Observation and Reasoning in Politics. Two Volumes. London: Parker.

Liapunov, Alexander M. 1900. "Sur Une Proposition de la Théorie des Probabilités." Bulletin Academie Imperiale Sciences St. Petersburg 13:359.

______. 1901. "Nouvelle Forme du Théorème sur la Limite de Probabilité." Memoire Academie Imperiale Sciences St. Petersburg 12:1.

MacGillivray, H. L. 1985. "Mean, Median, Mode, and Skewness." Pp. 364-367 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 5. New York: Wiley.

McDonald, James B., and Bartell C. Jensen. 1979. "An Analysis of Some Properties of Alternative Measures of Income Inequality Based on the Gamma Distribution Function." Journal of the American Statistical Association 74:856-860.

Morgenson, Gretchen. 2008. "How Big a Payday for the Pay Consultants? New York Times, 22 June 2008.

Nadarajah, Saralees, and Samuel Kotz. 2005. "On the Linear Combination of Exponential and Gamma Random Variables." Entropy 7:161-171.

O'Reilly, Charles, Brian Main, and Graef Crystal. 1988. "CEO Compensation as Tournament and Social Comparison: A Tale of Two Theories." Administrative Science Quarterly 33:257-274.

Osberg, Lars, and Timothy Smeeding. 2006. "'Fair' Inequality? Attitudes Toward Pay Differentials: The United States in Comparative Perspective." American Sociological Review 71:450-473.

Pigou, Arthur C. 1924. The Economics of Welfare. Second Edition. London: Macmillan.

Plato. 4th century B.C. [1952]. The Dialogues of Plato. Translated by Benjamin Jowett. Chicago: Britannica.

Plutarch. c. 46-120 [1952]. Lives of the Noble Grecians and Romans. The Dryden Translation. Chicago: Britannica.

Rostker, Bernard, Harry Thie, James Lacy, Jennifer Kawata, and Susanna Purnell. 1993. The Defense Officer Personnel Management Act of 1980: A Retrospective Assessment. R-4246-FMP. Santa Monica, CA: RAND.

Rousseau, Jean Jacques. 1755 [1952]. "A Discourse on the Origin of Inequality." Pages 323-

366 in Rousseau. Translated by G. D. H. Cole. Chicago: Britannica.

Sells, Dorothy M. 1939. British Wages Boards: A Study in Industrial Democracy. Washington, DC: Brookings Institution.

Smelser, Neil J. 1963 [1976]. The Sociology of Economic Life. Second edition. Englewood Cliffs, NJ: Prentice-Hall.

______, and Richard Swedberg. 1994. "The Sociological Perspective on the Economy." Pp. 3-26 in N. J. Smelser and R. Swedberg (eds.), The Handbook of Economic Sociology. Princeton, NJ. Princeton University Press.

______, and Richard Swedberg. 2005. "Introducing Economic Sociology." Pp. 3-25 in N. J. Smelser and R. Swedberg (eds.), The Handbook of Economic Sociology, Second Edition. Princeton, NJ. Princeton University Press.

Sørensen, Aage B. 1977. "The Structure of Inequality and the Process of Attainment." American Sociological Review 42:965-978.

______. 1979. "A Model and a Metric for the Analysis of the Intragenerational Status Attainment Process." American Journal of Sociology 85:361-384.

Stigler, Stephen M. 1986. The History of Statistics: The Measurement of Uncertainty before 1900. Cambridge, Massachusetts: Harvard University Press.

Stout, W. 1985. "Domain of Attraction." Pp. 416-418 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 2. New York: Wiley.

Stuart, Alan, and J. Keith Ord. 1987. Kendall's Advanced Theory of Statistics. Volume 1: Distribution Theory. Fifth Edition. New York: Oxford University Press.

Swedberg, Richard. 1990. Economics and Sociology: Redefining Their Boundaries: Conversations with Economists and Sociologists. Princeton, New Jersey: Princeton University Press.

______, and Mark Granovetter. 1992. "Introduction". Pp. 1-26 in Mark Granovetter and Richard Swedberg (eds.), The Sociology of Economic Life. Boulder, Colorado:

Westview Press.

______, and ______. 2001. "Introduction to Second Edition". Pp. 1-30 in Mark Granovetter and Richard Swedberg (eds.), The Sociology of Economic Life, Second Edition. Boulder, Colorado: Westview Press.

Thomas Aquinas, Saint. ([1267-1273] 1952). Summa Theologica. Two volumes. Translated by the Fathers of the English Dominican Province. Revised by Daniel J. Sullivan. Chicago: Britannica.

Tolles, N. Arnold. 1964. Origins of Modern Wage Theories. Englewood Cliffs, New Jersey: Prentice-Hall.

Walster, Elaine, Ellen Berscheid, and G. William Walster. 1976. "New Directions in Equity Research." Pp. 1-42 in Leonard Berkowitz and Elaine Walster (eds.), Equity Theory: Toward a General Theory of Social Interaction. New York: Academic.

Washington, George. 1796 [1935]. George Washington's Farewell Address. Edited by Victor H. Paltsits. New York: New York Public Library.

Wolfson, D. 1985. "Lindeberg-Feller Theorem." Pp. 1-2 in Samuel Kotz, Norman L. Johnson, and Campbell B. Read (eds.), Encyclopedia of Statistical Sciences, Volume 5. New York: Wiley.

Zelizer, Viviana A. 2007. "Pasts and Futures of Economic Sociology." American Behavioral Scientist 50:1056-1069.

*Note*: Articles from the Encyclopedia of Statistical Sciences also appear in the second edition of that work, published in 2005, as well as in the online edition of 2006.

**Table 1. The Wage Matrix: *N* Wage-Setters and *J* Workers**

**A. The Wage-Setter-Specific/Worker-Specific Wage**

$$x_{ij}$$

where $x$ denotes the recommended wage, and the wage-setters are indexed by $i$ ($i = 1,\ldots, N$) and the workers by $j$ ($j = 1,\ldots, J$).

**B. Wage Matrix**

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1J} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2J} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & x_{N3} & \cdots & x_{NJ} \end{bmatrix}$$

**C. The Case of a Single Wage-Setter**

If there is only one wage-setter, the wage matrix collapses to a vector:

$$\boldsymbol{x}_{.j} = [x_{.1} \;\; x_{.2} \;\; x_{.3} \;\; \cdots \;\; x_{.J}]$$

**Table 2. The Weight Matrix: *N* Wage-Setters and *J* Workers**

**A. The Wage-Setter-Specific/Worker-Specific Weight**

$$w_{ij}$$

where $w$ denotes the weight, the wage-setters are indexed by $i$ ($i = 1,\ldots, N$) and the workers by $j$ ($j = 1,\ldots, J$), and the weights are nonnegative and for each worker sum to one. That is, $w_i \geq 0$ and $\Sigma w_i = 1$.

**B. Weight Matrix**

$$\boldsymbol{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \cdots & w_{1J} \\ w_{21} & w_{22} & w_{23} & \cdots & w_{2J} \\ w_{31} & w_{32} & w_{33} & \cdots & w_{3J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & w_{N3} & \cdots & w_{NJ} \end{bmatrix}$$

**C. The Case of a Wage-Setter with Absolute Power**

If one wage-setter has absolute power, the weight matrix collapses to a vector:

$$\boldsymbol{w}_{.j} = [w_{.1} \;\; w_{.2} \;\; w_{.3} \;\; \ldots \;\; w_{.J}] = [1 \;\; 1 \;\; 1 \;\; \ldots \;\; 1]$$

**D. The Case of a Single Weighting Scheme for All Workers**

If weights do not differ by worker, the weight matrix collapses to a vector:

$$\boldsymbol{w}_{i.} = \begin{bmatrix} w_{1.} \\ w_{2.} \\ w_{3.} \\ \vdots \\ w_{N.} \end{bmatrix}$$

**Table 3. Types of Consensus in the Case of Two Wage-Setters, by Whether the Recommended Wage Distributions Are Identical or Different and the Association Between the Two Distributions**

| Macro Consensus | Micro Consensus: Association Between $X_1$ and $X_2$ | | |
|---|---|---|---|
| | **Perfect Positive** | **Independent** | **Perfect Negative** |
| $X_1$ and $X_2$ Identical | Macro Agreement<br>Micro Agreement | Macro Agreement<br>Micro Independence | Macro Agreement<br>Micro Disagreement |
| $X_1$ and $X_2$ Different | Macro Disagreement<br>Micro Agreement | Macro Disagreement<br>Micro Independence | Macro Disagreement<br>Micro Disagreement |

*Notes*: Extension to $N$ distributions is straightforward except for the two cases of perfect negative association. As described in the text, the $N$-variate perfect-negative-association case is specified as follows: (i) there are $N$ wage-setters, arranged in two factions; (ii) within each faction, all the recommended wage distributions are perfectly positively associated; and (iii) the two factions are perfectly negatively associated with each other.

**Table 4. Six Wage Matrices: Two Wage-Setters and Four Workers**

| Macro Consensus | Micro Consensus: Association Between $X_1$ and $X_2$ | | |
|---|---|---|---|
| | Perfect Positive | Independent | Perfect Negative |
| $X_1$ and $X_2$ Identical | $\begin{bmatrix} 7 & 8 & 10 & 15 \\ 7 & 8 & 10 & 15 \end{bmatrix}$ | $\begin{bmatrix} 8 & 8 & 12 & 12 \\ 8 & 12 & 8 & 12 \end{bmatrix}$ | $\begin{bmatrix} 7 & 8 & 10 & 15 \\ 15 & 10 & 8 & 7 \end{bmatrix}$ |
| $X_1$ and $X_2$ Different | $\begin{bmatrix} 8 & 9.5 & 10.5 & 12 \\ 8.5 & 9 & 11 & 11.5 \end{bmatrix}$ | $\begin{bmatrix} 8 & 8 & 12 & 12 \\ 10 & 12 & 10 & 12 \end{bmatrix}$ | $\begin{bmatrix} 8 & 9.5 & 10.5 & 12 \\ 11.5 & 11 & 9 & 8.5 \end{bmatrix}$ |

**Table 5. Basic Structure of the Wage Determination Model**

| Elements of the Wage Model | Mathematical Representation |
|---|---|
| wage-setters' recommendations | variates $X_i$ |
| number of wage-setters | number $N$ |
| wage-setter power | variate weights $w_{ij}$ |
| macro consensus | identical/different variates |
| micro consensus | association between variates |

**Table 6. Formulas for the Variance of the Final Salary Distribution in the Case of Two Wage-Setters**

| Macro Consensus/ Wage-Setter Power | Micro Consensus: Association Between the Recommended Wage Distributions $X_1$ and $X_2$ — Perfect Positive | Independent | Perfect Negative |
|---|---|---|---|
| A. General Formula for the Variance of the Weighted Sum of Two Recommended Wage Distributions | | | |
| | | $Var(Y) = w_1^2 Var(X_1) + w_2^2 Var(X_2) + 2w_1 w_2 Cov(X_1,X_2)$ | |
| B. Specific Formulas for Main Special Cases | | | |
| $X_1$ and $X_2$ Identical Equal Weights | $Var(X)$ | $\frac{Var(X)}{2}$ | $\frac{Var(X)}{2} + \frac{Cov(X_1,X_2)}{2}$ |
| $X_1$ and $X_2$ Identical Unequal Weights | $Var(X)$ | $[Var(X)][w_1^2 + w_2^2]$ | $[Var(X)][w_1^2 + w_2^2] + 2w_1 w_2 Cov(X_1,X_2)$ |
| $X_1$ and $X_2$ Different Equal Weights | $\frac{E[Var(X_i)]}{2} + \frac{Cov(X_1,X_2)}{2}$ | $\frac{E[Var(X_i)]}{2}$ | $\frac{E[Var(X_i)]}{2} + \frac{Cov(X_1,X_2)}{2}$ |
| $X_1$ and $X_2$ Different Unequal Weights | Formula in A | $w_1^2 Var(X_1) + w_2^2 Var(X_2)$ | Formula in A |

*Note*: The covariance term is positive in the positive-association cases and negative in the negative-association cases.

**Table 7. Formulas for the Variance of the Final Salary Distribution in the Case of *N* Wage-Setters**

| Macro Consensus/ Wage-Setter Power | Micro Consensus: Association Between the Recommended Wage Distributions $X_i$ | | |
|---|---|---|---|
| | **Perfect Positive** | **Independent** | **Negative** |
| | A. General Formula for the Variance of the Weighted Sum of the *N* Recommended Wage Distributions | | |
| | $Var(Y) = \sum_{i=1}^{N} w_i^2 \left[Var(X_i)\right] + 2 \sum_{i=1}^{N} \sum_{k>i}^{N} w_i w_k Cov(X_i, X_k)$ | | |
| | B. Specific Formulas for Main Special Cases | | |
| $X_i$ Identical Equal Weights | $Var(X)$ | $\frac{Var(X)}{N}$ | $\frac{Var(X)}{N} + \frac{2}{N^2} \sum_{i=1}^{N} \sum_{k>i}^{N} Cov(X_i, X_k)$ |
| $X_i$ Identical Unequal Weights | $Var(X)$ | $[Var(X)] \sum_{i=1}^{N} w_i^2$ | $[Var(X)] \sum_{i=1}^{N} w_i^2 + 2 \sum_{i=1}^{N} \sum_{k>i}^{N} w_i w_k Cov(X_i, X_k)$ |
| $X_i$ Different Equal Weights | $\frac{E\left[Var(X_i)\right]}{N} + \frac{2}{N^2} \sum_{i=1}^{N} \sum_{k>i}^{N} Cov(X_i, X_k)$ | $\frac{E\left[Var(X_i)\right]}{N}$ | $\frac{E\left[Var(X_i)\right]}{N} + \frac{2}{N^2} \sum_{i=1}^{N} \sum_{k>i}^{N} Cov(X_i, X_k)$ |
| $X_i$ Different Unequal Weights | Formula in A | $\sum_{i=1}^{N} w_i^2 \left[Var(X_i)\right]$ | Formula in A |

*Notes*: The covariance term is positive (negative) in the positive(negative)-association cases. Perfect negative association occurs only when *N* equals 2.

**Table 8. The Effects on Wage Inequality of Introducing One New Wage-Setter into a Group of *N* Independent-Minded and Equally-Weighted Wage-Setters, where the Recommended Wage Distributions Are Identical with Finite Variance**

| Time 1 | Time 2 | | |
|---|---|---|---|
| | (1)<br>New Wage-Setter Has No Power | (2)<br>New Wage-Setter Independent, with Equal Power<br>$\left(\frac{1}{N+1}\right)$ | (3)<br>New Wage-Setter a Partisan, with Equal Power<br>$\left(\frac{1}{N+1}\right)$ |
| **A. Formulas for *Var(Y)*** | | | |
| $\frac{Var(X)}{N}$ | $\frac{Var(X)}{N}$ | $\frac{Var(X)}{N+1}$ | $\frac{(N+3)Var(X)}{(N+1)^2}$ |
| **B. Change in *Var(Y)* from Time 1 to Time 2** | | | |
| | 0 | $\frac{Var(X)}{N(N+1)} > 0$ | $-\frac{(N-1)Var(X)}{N(N+1)^2} < 0$ |
| | No Change | Inequality decreases | Inequality increases |
| **C. Size of Proportional Increase/Decrease in *Var(Y)*** | | | |
| | | $\frac{1}{N+1}$ | $-\frac{N-1}{(N+1)^2}$ |

*Notes*: The number of wage-setters at Time 1 is equal to *N*. The recommended wage distributions of all wage-setters have the same finite variance, *Var(X)*.

**Table 9. PDF, Variance, and Gini Coefficient in the Shifted Erlang Distribution Arising When the Wage-Setters Are Equally-Weighted and Their Recommended Wage Distributions Are Shifted Exponentials Identically and Independently Distributed, by Number of Wage-Setters**

| Number of Wage-Setters | PDF | Variance | Gini Coefficient |
|---|---|---|---|
| **A. General Formulas** | | | |
| $N$ | $\frac{N^N (y-a)^{N-1} \exp[-N(y-a)]}{\Gamma(N)}$ | $\frac{1}{N}$ | $\left[\frac{\Gamma(N+1/2)}{\sqrt{\pi}\,\Gamma(N+1)}\right]\left(\frac{1}{a+1}\right)$ |
| **B. Formulas for the PDF and Values of the Variance and Gini Coefficient** | | | |
| 1 | $\exp[-(y-a)]$ | 1 | $.5\left(\frac{1}{a+1}\right)$ |
| 2 | $4\,(y-a)\exp[-2(y-a)]$ | .5 | $.375\left(\frac{1}{a+1}\right)$ |
| 3 | $\frac{27\,(y-a)^2 \exp[-3(y-a)]}{2}$ | .333 | $.3125\left(\frac{1}{a+1}\right)$ |
| 4 | $\frac{128\,(y-a)^3 \exp[-4(y-a)]}{3}$ | .25 | $.273\left(\frac{1}{a+1}\right)$ |
| 5 | $\frac{3125\,(y-a)^4 \exp[-5(y-a)]}{24}$ | .2 | $.246\left(\frac{1}{a+1}\right)$ |
| 6 | $\frac{1944\,(y-a)^5 \exp[-6(y-a)]}{5}$ | .167 | $.226\left(\frac{1}{a+1}\right)$ |
| 7 | $\frac{823543\,(y-a)^6 \exp[-7(y-a)]}{720}$ | .143 | $.209\left(\frac{1}{a+1}\right)$ |
| 8 | $\frac{1048576\,(y-a)^7 \exp[-8(y-a)]}{315}$ | .125 | $.196\left(\frac{1}{a+1}\right)$ |
| 9 | $\frac{43046721\,(y-a)^8 \exp[-9(y-a)]}{4480}$ | .111 | $.185\left(\frac{1}{a+1}\right)$ |
| 10 | $\frac{15625000\,(y-a)^9 \exp[-10(y-a)]}{567}$ | .1 | $.176\left(\frac{1}{a+1}\right)$ |

*Notes*: The parameter $a$ represents the minimum income.

**Table 10. Mean, Median, and Mode in the Shifted Erlang Distribution Arising When the Wage-Setters Are Equally-Weighted and Their Recommended Wage Distributions Are Shifted Exponentials Identically and Independently Distributed, by Number of Wage-Setters**

| Number of Wage-Setters | Mean | Median | Mode |
|---|---|---|---|
| **A. General Formulas** | | | |
| $N$ | $a + 1$ | $a + \frac{N-1+\ln(2)}{N}$ | $a + \frac{N-1}{N}$ |
| **B. Special Case where $a$ = .25** | | | |
| 1 | 1.25 | .943 | .25 |
| 2 | 1.25 | 1.097 | .75 |
| 3 | 1.25 | 1.148 | .917 |
| 4 | 1.25 | 1.173 | 1 |
| 5 | 1.25 | 1.189 | 1.05 |
| 6 | 1.25 | 1.199 | 1.083 |
| 7 | 1.25 | 1.206 | 1.107 |
| 8 | 1.25 | 1.212 | 1.125 |
| 9 | 1.25 | 1.216 | 1.139 |
| 10 | 1.25 | 1.219 | 1.15 |

*Notes*: The parameter $a$ represents the minimum income.

**Table 11. Associated Functions, Major Parameters, and Other Properties of the Shifted Exponential, the Shifted Erlang, and the Shifted Ring(2)-Exponential Distributions Which Arise in Wage Analysis: Equally-Weighted Case**

| Feature/Property | One Wage-Setter | Two Wage-Setters, Equally Weighted | | |
|---|---|---|---|---|
| | | Positively Associated | Independent | Negatively Associated |
| Variate | Shifted Exponential | Shifted Exponential | Shifted Erlang | Shifted Ring(2)-Exponential |
| Support | $x > a$ | $x > a$ | $x > a$ | $x > a + \ln(2)$ |
| PDF | $e^{-(x-a)}$ | $e^{-(x-a)}$ | $4(x-a)e^{-2(x-a)}$ | $\frac{4e^{-(x-a)}}{\sqrt{e^{2(x-a)}-4}}$ |
| CDF | $1-e^{-(x-a)}$ | $1-e^{-(x-a)}$ | $1-\{e^{-2(x-a)}[2(x-a)+1]\}$ | $\sqrt{1-\frac{4}{e^{2(x-a)}}}$ |
| QF | $a+\ln\left(\frac{1}{1-\alpha}\right)$ | $a+\ln\left(\frac{1}{1-\alpha}\right)$ | --- | $a+\ln\left(\frac{2}{\sqrt{1-\alpha^2}}\right)$ |
| Mean | $a+1$ | $a+1$ | $a+1$ | $a+1$ |
| Variance | 1 | 1 | .5 | .178 |
| Median | $a+\ln(2)$ | $a+\ln(2)$ | $a+\left(\frac{\ln(2)+1}{2}\right)$ | $a+\ln\left(\frac{4}{\sqrt{3}}\right)$ |
| Mode | $a$ | $a$ | $a+\left(\frac{1}{2}\right)$ | $a+\ln(2)$ |
| Skewness | 2 | 2 | $\sqrt{2}$ | 2.63 |
| Kurtosis | 9 | 9 | 6 | 13.1 |
| Gini coefficient | $\left(\frac{1}{2}\right)\left(\frac{1}{a+1}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{a+1}\right)$ | $\left(\frac{3}{8}\right)\left(\frac{1}{a+1}\right)$ | $\frac{\ln(2)-1/2}{a+1}$ |

*Notes*: The parameter $a$ represents the minimum income. When $a$ equals zero, the shifted distributions reduce to the standard exponential, the Erlang, and the original ring(2)-exponential derived by Jasso (2001) in the study of status and analyzed by Jasso and Kotz (2007).

**Table 12. Final Wage Distributions in Two-Party Society, with Recommended Wage Distributions Identical Shifted Exponentials, by Type of Micro Consensus and Whether the Two Parties Are Equal or Unequal in Size**

| Party Split | Micro Consensus: Association between Two Parties' Wage Distributions | | |
|---|---|---|---|
| | Perfect Positive | Independent | Perfect Negative |
| Equal | Shifted Exponential | Shifted Erlang | Shifted Mirror-Exponential |
| Unequal | Shifted Exponential | Shifted General Erlang | Shifted Mirror-Exponential |

*Notes*: In the equal party split, $p = .5$. The variate in the equal-split/perfect-negative-association case is the shifted version of the ring(2)-exponential derived by Jasso (2001) in the study of status and analyzed by Jasso and Kotz (2007).

**Table 13. Gini Coefficient in Two-Party Society, with Recommended Wage Distributions Identical Shifted Exponentials, by Type of Micro Consensus and Party Split**

| Party Split | Micro Consensus: Association between Two Parties' Wage Distributions | | |
|---|---|---|---|
| | **Perfect Positive** | **Independent** | **Perfect Negative** |
| Variate | Shifted Exponential | Shifted General Erlang | Shifted Mirror-Exponential |
| .05 | $.5\left(\frac{1}{a+1}\right)$ | $.476\left(\frac{1}{a+1}\right)$ | $.434\left(\frac{1}{a+1}\right)$ |
| .1 | $.5\left(\frac{1}{a+1}\right)$ | $.455\left(\frac{1}{a+1}\right)$ | $.379\left(\frac{1}{a+1}\right)$ |
| .15 | $.5\left(\frac{1}{a+1}\right)$ | $.436\left(\frac{1}{a+1}\right)$ | $.333\left(\frac{1}{a+1}\right)$ |
| .2 | $.5\left(\frac{1}{a+1}\right)$ | $.42\left(\frac{1}{a+1}\right)$ | $.295\left(\frac{1}{a+1}\right)$ |
| .25 | $.5\left(\frac{1}{a+1}\right)$ | $.406\left(\frac{1}{a+1}\right)$ | $.263\left(\frac{1}{a+1}\right)$ |
| .3 | $.5\left(\frac{1}{a+1}\right)$ | $.395\left(\frac{1}{a+1}\right)$ | $.237\left(\frac{1}{a+1}\right)$ |
| .35 | $.5\left(\frac{1}{a+1}\right)$ | $.386\left(\frac{1}{a+1}\right)$ | $.218\left(\frac{1}{a+1}\right)$ |
| .4 | $.5\left(\frac{1}{a+1}\right)$ | $.38\left(\frac{1}{a+1}\right)$ | $.204\left(\frac{1}{a+1}\right)$ |
| .45 | $.5\left(\frac{1}{a+1}\right)$ | $.376\left(\frac{1}{a+1}\right)$ | $.196\left(\frac{1}{a+1}\right)$ |
| .5 | $.5\left(\frac{1}{a+1}\right)$ | $.375\left(\frac{1}{a+1}\right)$ | $.193\left(\frac{1}{a+1}\right)$ |

*Note*. The variates arising in the perfect positive, independent, and perfect negative cases are the shifted exponential, shifted general Erlang, and shifted mirror-exponential, respectively.

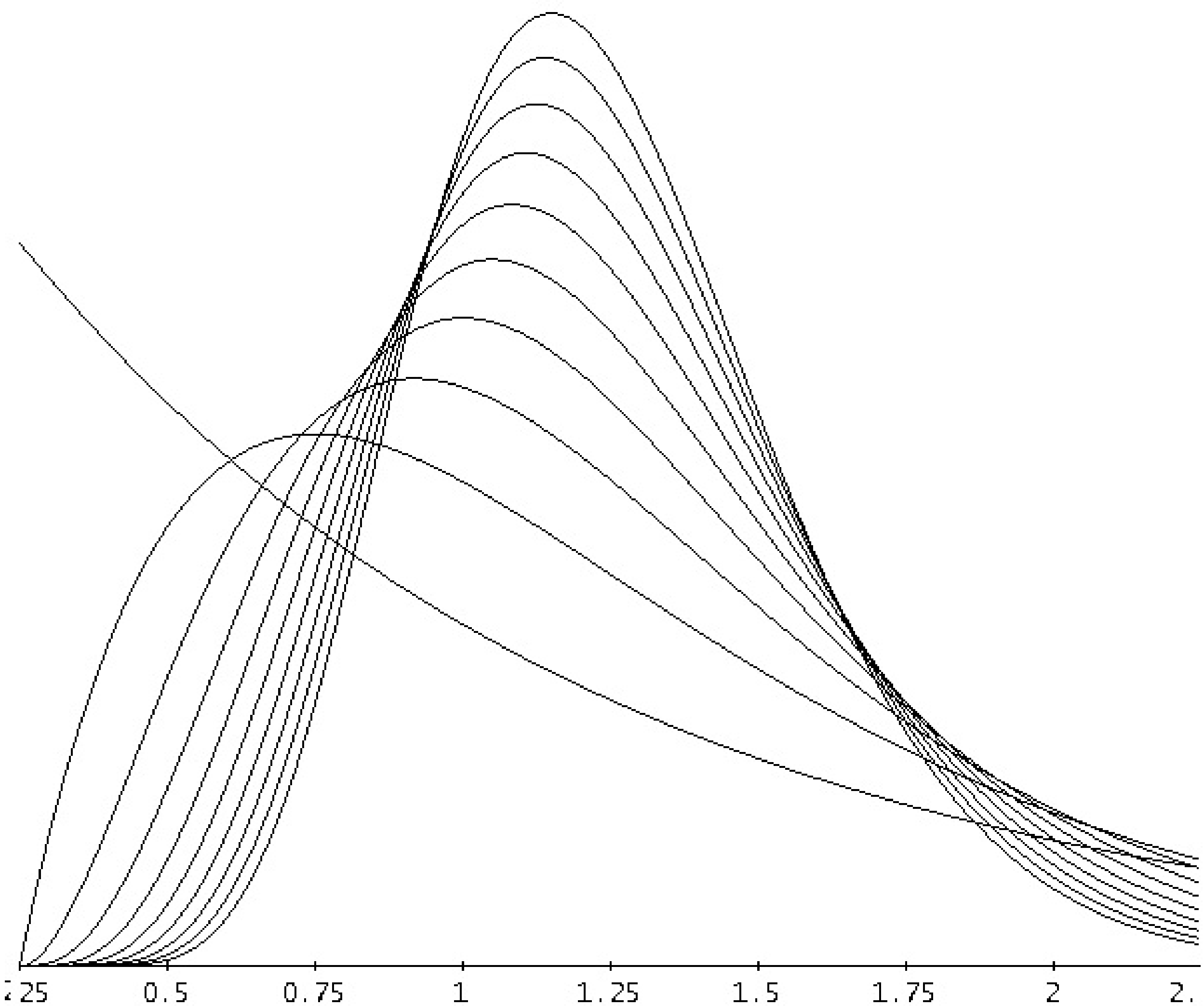


Figure 1. Graphs of the Probability Density Function of the Wage Distribution in a Society with Equally Powerful and Independent-Minded Wage-Setters, for Number of Wage-Setters from 1 to 10. The variate is the shifted Erlang distribution which arises when the wage-setters are equally weighted and their recommended distributions are independently and identically distributed as shifted exponentials. The graphs are for shifted Erlangs arising from shifted exponentials with a mean of 1.25 and a minimum of .25. Looking at the middle region of the plot (approximately 1.25 on the horizontal axis), the graphs line up from bottom to top corresponding to number of wage-setters from 1 to 10. As the number of wage-setters increases, the distribution gets more concentrated about its mean of 1.25 and also more symmetric. When the number of wage-setters is one, the shifted Erlang reduces to the shifted exponential.

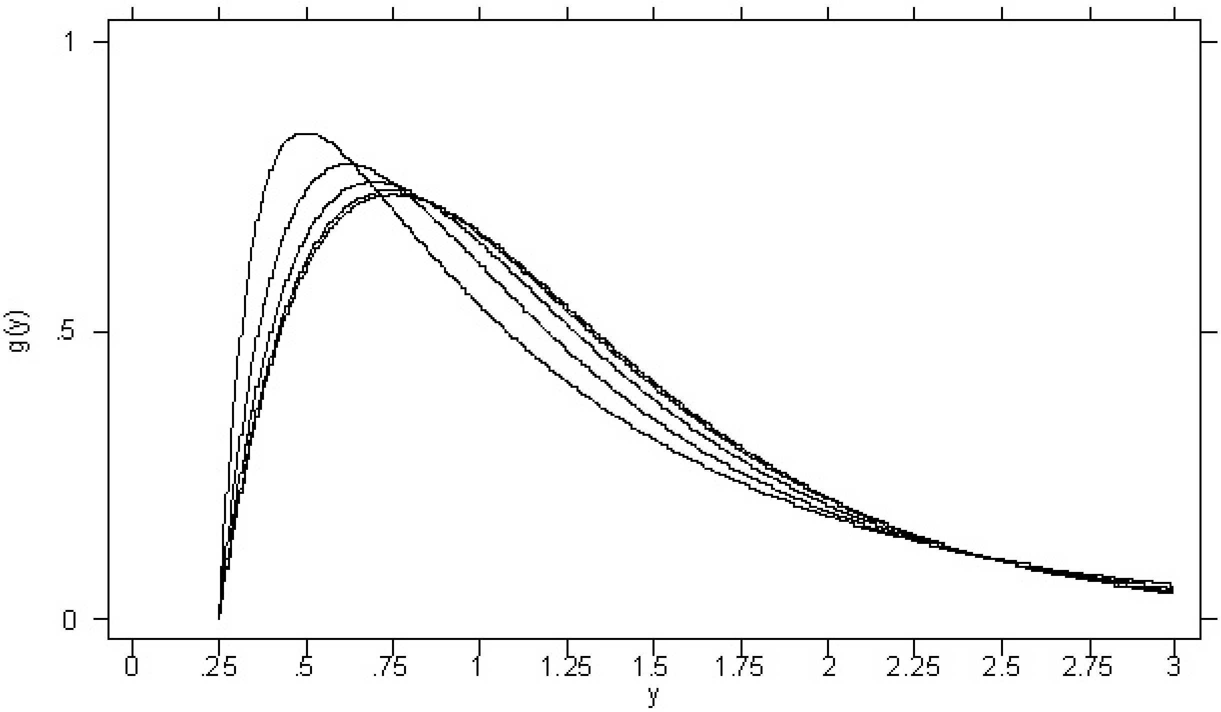


Figure 2. Graphs of the Probability Density Function of the Wage Distribution in a Society with Two Unequally Powerful Independent-Minded Wage-Setters. From top to bottom at the mode, the five variates correspond to party split $p$ = .1, .2, .3, .4, and .5. The top four are shifted general Erlang variates, and the bottom variate (with two equally powerful wage-setters) is the shifted Erlang also depicted in Figure 1. The graphs are for variates arising from shifted exponential variates with a minimum of .25 and a mean of 1.25.

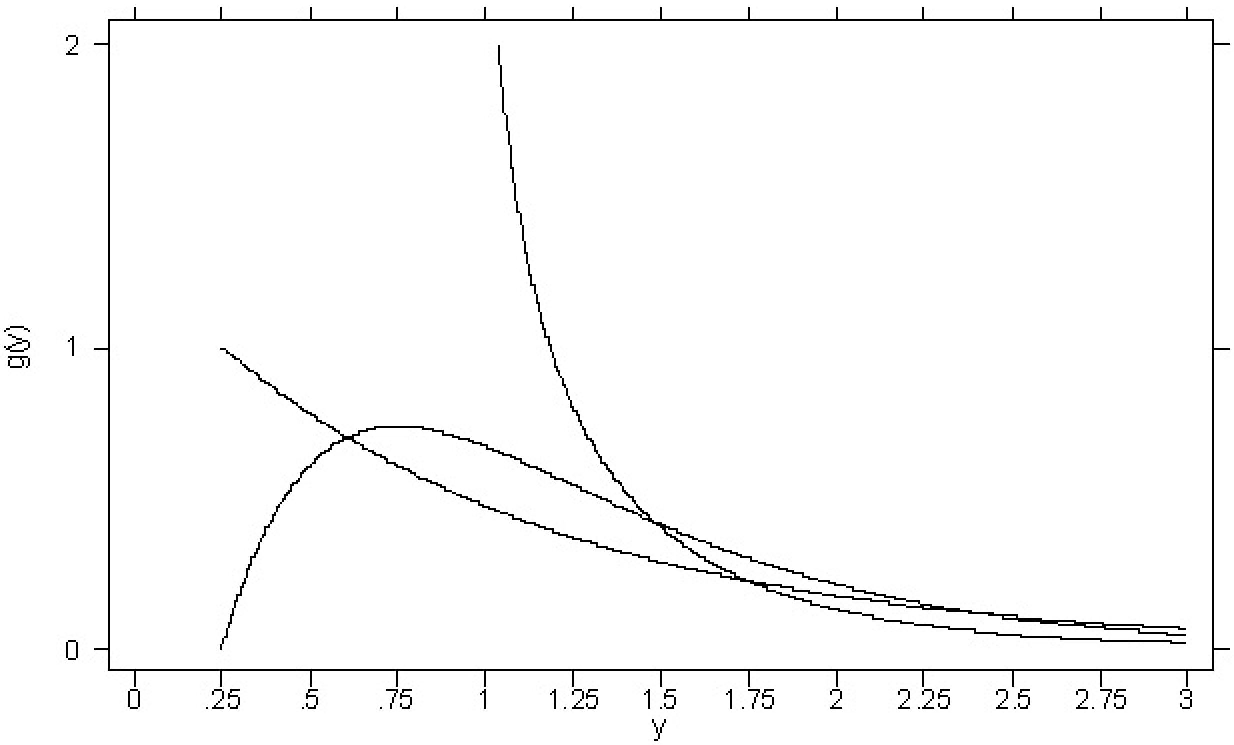


Figure 3. Graphs of the Probability Density Function of the Wage Distribution in a Society with Two Equally Powerful Wage-Setters. The three variates are the shifted exponential, shifted Erlang, and shifted ring(2)-exponential which arise when the wage-setters are like-minded, independent-minded, and opposite-minded, respectively. The graphs are for variates arising from shifted exponential variates with a minimum of .25 and a mean of 1.25. At the mean, the graphs line up from bottom to top corresponding to the shifted exponential, shifted Erlang, and shifted ring(2)-exponential. The shifted exponential is also depicted in Figure 1, and the shifted Erlang in both Figures 1 and 2.

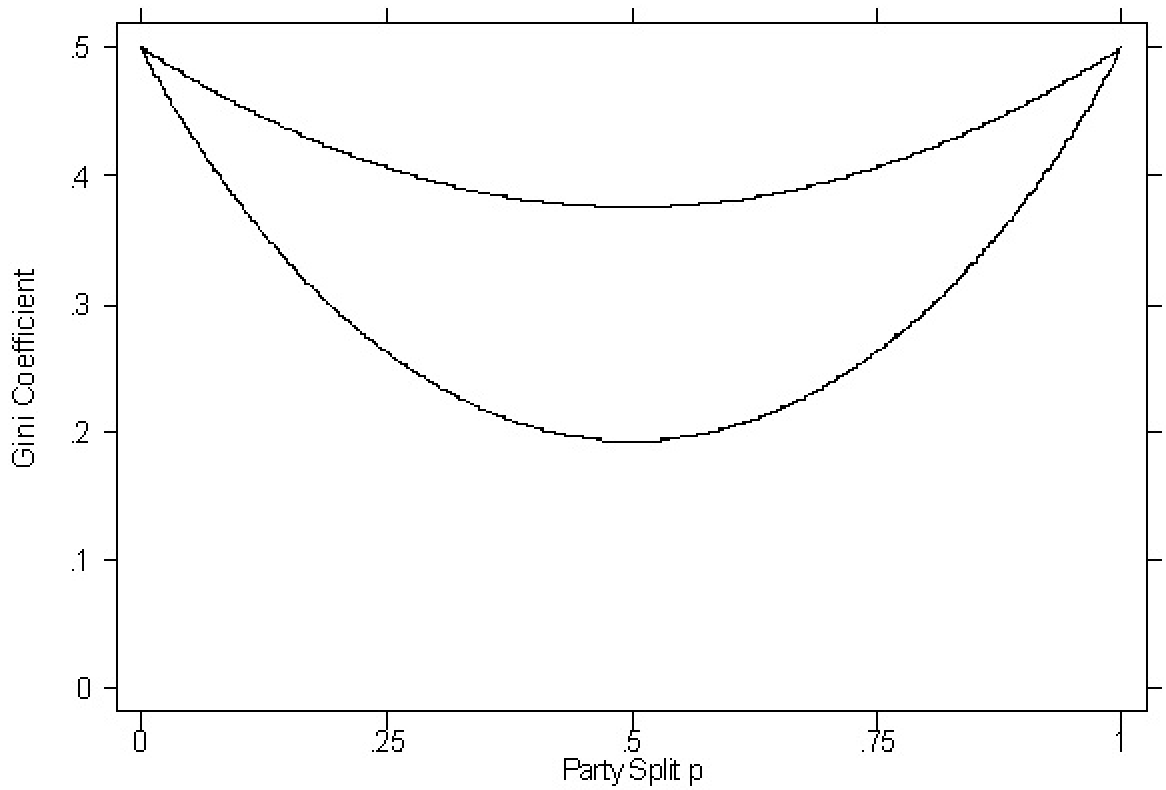


Figure 4. Gini Coefficient in Two-Party Society, by Party Split *p*. Graphs of the Gini coefficient in the shifted general Erlang (upper graph) and the shifted mirror-exponential (lower graph) arise from independent and negatively-associated wage distributions, respectively. In both situations, the Gini coefficient is at its minimum when the two parties are equally-sized. For every party split, the Gini coefficient is lower when the two parties are oppositely-minded than when they are independent-minded. Graphed values do not include the shift factor (see Table 13).