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ABSTRACT

An Optimal Split of School Classes*

In many countries, schools have responded to the COVID-19 pandemic by splitting up classes. While the purpose of dividing classes is clearly health-related, the process of doing so poses an interesting question: what is the best way to divide a class so as to maximize the incentive for students to perform better? Using a constructive example, we demonstrate how social-psychological unhappiness can be the basis for an incentive structure that optimally nudges students to improve their performance. The example is based on evidence that students aspire to improve their performance when it lags behind that of other students with whom they naturally compare themselves. For a given set of m students, we quantify unhappiness by the index of relative deprivation, which measures the extent to which a student lags behind other students in the set who are doing better than him. We examine how to divide the set into an exogenously predetermined number of subsets in order to maximize aggregate relative deprivation, so that the incentive for the students to study harder because of unfavorable comparison with other students is at its strongest. We show that the solution to this problem depends only on the students' ordinally-measured levels of performance, independent of the performance of comparators. In addition, we find that when m is an even number, there are multiple optimal divisions, whereas when m is an odd number, there is only one optimal division.

JEL Classification: D01, D02, D23, D61, D90, L22, M11, M52

Keywords: social-psychological preferences, distaste for trailing behind others, unhappiness as measured by relative deprivation, pressure to perform better, superior performance of comparators, assignment of students to subclasses, optimum incentive to improve performance

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1. Introduction

The idea developed in this paper is inspired by two observations: a specific and a general one.

First, in many countries, schools have responded to the COVID-19 pandemic by splitting classes. This is to reduce class size so as to ensure that students can keep distance from each other and study while at desks that are far enough apart. Typically, classes are split into two.[1] Countries do this in different ways. For example, according to one protocol (Austria) one group of students attends school half of the week, Monday to Wednesday, the other group, Thursday to Friday, and then, in the following week, the groups swap. In another case (Denmark) schooling for all students is held daily, but is divided into shifts. Students in one of the newly formed "subclass" are completely separated from students in the other newly formed "subclass." The split is strict. Given that the pandemic is still present, a public health measure that was envisaged as temporary has now become a fixture. As we have learned when we placed calls to principals and Ministries of Education in couple of countries (Austria and Denmark), the manner in which students from a given class are assigned to subclasses is rudimentary, for example, alphabetically. In fact, the educational authorities' division protocol appears to be a random split.

Second, there is widespread recognition that comparisons with others elicit substantial behavioral responses. Several studies have shown that people are motivated to perform better when their achievements lag behind those of their comparators. There is ample empirical evidence that the presence of better-performing comparators motivates workers to make more effort and students to perform better. Examples in the case of workers are studies by Falk and Ichino, 2006; Mas and Moretti, 2009; Bandiera et al., 2010; and Cohn et al., 2014. Examples in the case of students, which is of particular interest to us in the context of the current paper, are studies by Sacerdote, 2001; Hanushek et al., 2003; Azmat and Iriberri, 2010; Bursztyn and Jensen 2015; and Garlick, 2018.

When workers, students, consumers, and people in general assess, review, and evaluate what they have in the way of accomplishments (output, productivity, qualifications), grades, and assets (income, consumption, wealth, health status) - henceforth AGA - people prefer to have high absolute AGA than low absolute AGA, and high relative AGA than low

[1] "Classes should be split in half so that they can attend on alternate days." Lead article, *The Economist* magazine, May 2, 2020.

relative AGA. The incorporation of a dimension of relativity implies that AGA are valued in relation to the AGA of others with whom people naturally compare themselves (the "reference group" or the "comparison group").

The extent of the unhappiness experienced by people when they lag behind others can be quantified by means of a social-psychological index: the measure of relative deprivation (presented below).[2]

In this paper we design an optimal protocol for transforming the feeling of relative deprivation that people, in our case students, seek to redress into a performance incentive. A procedure for converting unhappiness, as measured by a social-psychological index, into an instrument for boosting the effort exerted to achieve a gain in AGA has not been presented before. Specifically, we use a constructive example to demonstrate how student's desire to alleviate unhappiness can be harnessed to design a rule of division of school classes that maximizes the aggregate incentive for students to improve their performance.

2. Optimal division

Imagine that a class of four students needs to be divided into two classes; that an exogenously imposed constraint is that the classes should be of equal size (this is so as to equalize the study environments and ensure that no class can accommodate more than two students); and that all four students need to be schooled. How to distribute students $n4$, $n3$, $n2$, and $n1$ between the two classes so that the incentives to study harder are maximized? The numbers $n4 > n3 > n2 > n1$ represent levels of performance that are independent of the performances of comparators, namely how each student performs in isolation from the pressure of the comparators' performances. The differences between these levels are $n2 - n1 = a$, $n3 - n2 = b$, and $n4 - n3 = c$, where $a, b, c > 0$.[3]

Using aggregate relative deprivation, *ARD*, as a measure of the "pressure" to improve performance in class, we seek to maximize the sum of the levels of *ARD* of the two classes. As shown in the Appendix, the relative deprivation of an individual is defined as the sum of the excesses of the variable in question divided by the size of the population. Taking test score

[2] In the Appendix we describe how we construct the index of relative deprivation that we use in this paper.

[3] Similarly: in a supermarket, there are two exits at the two ends of the shop, each with two cash desks, and there are four cashiers on the payroll. The earnings of a cashier are determined, in part, by the number of grocery items processed. Cashiers observe each other at the same exit, but not across both exits.

as an example of such a variable, and considering, also as an example, say, test score vector $y = (1,2,3,4,5)$, the relative deprivation, *RD*, of a student whose test score is 3 is $\frac{1}{5}\left[(4-3)+(5-3)\right] = \frac{3}{5}$. By a similar calculation we get that the *RD* of a student whose test score is 1 is higher at 2, and that the *RD* of the student whose test score is 5 is nil. The *ARD* of a group of students is the sum of the levels of *RD* of the students who form the group; in the case of test score vector $y = (1,2,3,4,5)$ this sum is 4.

Claim 1. When there are four students, $n4 > n3 > n2 > n1$, the assignment options that yield the maximum aggregate "pressure" to improve performance are $\{\{n4,n2\},\{n3,n1\}\}$ and $\{\{n4,n1\},\{n3,n2\}\}$. These assignments yield the same sum of the levels of *ARD*. The assignment option $\{\{n4,n3\},\{n2,n1\}\}$ is not optimal.

Proof. We naturally assume that the ordering of the classes is immaterial, namely we treat $\{\{n4,n3\},\{n2,n1\}\}$ and $\{\{n2,n1\},\{n4,n3\}\}$ as the same option. Thus, there are three possible divisions. Relative deprivation in a class of two is half of the difference between the levels of performance. In division $\{\{n4,n2\},\{n3,n1\}\}$, the sum of the levels of *ARD* is $\frac{1}{2}[(b+c)+(a+b)] = \frac{1}{2}(a+2b+c)$. In division $\{\{n4,n1\},\{n3,n2\}\}$, the sum of the levels of *ARD* is $\frac{1}{2}[(a+b+c)+b] = \frac{1}{2}(a+2b+c)$. And in division $\{\{n4,n3\},\{n2,n1\}\}$, the sum of the levels of *ARD* is $\frac{1}{2}(a+c)$, which is less than $\frac{1}{2}(a+2b+c)$. Q.E.D.

Remark 1. If we assume that the incentive to study harder increases with the difference between the students in their free-from-comparison levels of performance, then the division $\{\{n4,n3\},\{n2,n1\}\}$ does not maximize the incentive to study harder because under divisions $\{\{n4,n2\},\{n3,n1\}\}$ and $\{\{n4,n1\},\{n3,n2\}\}$ the sum of the differences is larger than under division $\{\{n4,n3\},\{n2,n1\}\}$.

Remark 2. From Claim 1 we already see that the optimal divisions of $n4 > n3 > n2 > n1$ into the two subsets $\{\{n4,n2\},\{n3,n1\}\}$ and $\{\{n4,n1\},\{n3,n2\}\}$ will be the same regardless of whether, let us say, $\{n1, n2, n3, n4\} = \{1, 2, 3, 4\}$, $\{n1, n2, n3, n4\} = \{1, 2, 4, 10\}$, or $\{n1, n2, n3, n4\} = \{1, 7, 9, 10\}$. Namely, for positive *a*, *b*, and *c* that can be the same as each other or that can differ from each other, what matters for obtaining an optimal division is the hierarchical order rather than the cardinal values of the performances that are independent of the performance of comparators.

The protocol guiding partition aimed at maximizing the aggregate "pressure" to improve performance applies to the case of any number of students to be assigned to two classes, rather than just to the case of four students to be assigned to two classes. Suppose that there are five students $n5 > n4 > n3 > n2 > n1$ where, in addition to the differences listed above, $n5 - n4 = d$. Obviously, in this case classes are to be of equal size but for one. The assignment option that yields the maximum aggregate "pressure" to improve performance is $\{\{n5, n3, n1\}, \{n4, n2\}\}$. This option yields a combined maximum *ARD* of $\frac{1}{3}[(a+b)+(a+b+c+d)+(c+d)]+\frac{1}{2}(b+c)$. As can be ascertained easily, any reshuffling of this division will yield a lower combined *ARD*. For example, the division $\{\{n5, n4, n1\}, \{n3, n2\}\}$ yields $\frac{1}{3}[(a+b+c)+(a+b+c+d)+d]+\frac{1}{2}b$ which (because $\frac{1}{2}b < \frac{1}{2}(b+c)$) is less than $\frac{1}{3}[(a+b)+(a+b+c+d)+(c+d)]+\frac{1}{2}(b+c)$.

Remark 3. The numerical illustration $n4 > n3 > n2 > n1$ points to a somewhat different procedure for obtaining the same result. We can divide the set of four students into two mutually exclusive and jointly exhaustive subsets, where each subset consists of two adjacent "neighbors." In the case of the numerical illustration $n4 > n3 > n2 > n1$ these two subsets will be $A_1 = \{n1, n2\}$ and $A_2 = \{n3, n4\}$. We then set up the two classes so that in every two-student class we have one student from subset A_1, and one student from subset A_2. Similarly, we can consider a case in which six students $n6 > n5 > n4 > n3 > n2 > n1$ are to be assigned to two classes of three students each. We form three subsets: $A_1 = \{n1, n2\}$, $A_2 = \{n3, n4\}$, and $A_3 = \{n5, n6\}$. From each subset we pick one student. Replication of the proof protocol of Claim 1 yields the following four optimal divisions:

$\{\{n6,n4,n2\},\{n5,n3,n1\}\}$
$\{\{n5,n4,n2\},\{n6,n3,n1\}\}$
$\{\{n6,n3,n2\},\{n5,n4,n1\}\}$
$\{\{n6,n4,n1\},\{n5,n3,n2\}\}$.

Needless to add, the sum of the levels of *ARD* in each of these divisions is the same.

Remark 4. A mirror image of the procedure described in the preceding remark is that allocation to divisions such that each one of them consists of adjacent "neighbors" cannot be optimal. As a matter of fact, an allocation that consists of divisions such that each of them is composed of "pure" sequences yields the *minimal* combined *ARD* and, thus, it can never be optimal. In the case of numerical illustration $n4 > n3 > n2 > n1$, the pair of divisions $\{\{n4,n3\},\{n2,n1\}\}$ yields the lowest combined *ARD*, and in the case of numerical illustration $n5 > n4 > n3 > n2 > n1$, the lowest combined *ARD* is yielded by the pair of divisions $\{\{n5,n4,n3\},\{n2,n1\}\}$ or, for that matter, by the pair of divisions $\{\{n5,n4\},\{n3,n2,n1\}\}$.

We have already seen how to obtain optimal divisions. What remains to be determined is the number of optimal divisions. As hinted by the preceding three numerical illustrations ($n4 > n3 > n2 > n1$, $n5 > n4 > n3 > n2 > n1$, and $n6 > n5 > n4 > n3 > n2 > n1$), this number depends on whether the number of individuals to be assigned, *m*, is even or odd: when *m* is even, the number of optimal divisions is $2^{\frac{m}{2}-1}$ (so that in the case of $n4 > n3 > n2 > n1$ this number is $2^{\frac{4}{2}-1} = 2$, and in the case of $n6 > n5 > n4 > n3 > n2 > n1$ this number is $2^{\frac{m}{2}-1} = 2^{\frac{6}{2}-1} = 4$); when *m* is odd, there is only one optimal solution (so that in the case of $n5 > n4 > n3 > n2 > n1$ this number is 1.) Formal proof of these results, which is tedious, is omitted here (it is available on request). However, the result that when *m* is even the number of optimal divisions is $2^{\frac{m}{2}-1}$ can be elicited from the protocol described in Remark 3. When *m* is even, we partition the *m* students into $\frac{m}{2}$ subsets $A_1 = \{n1, n2\}$, $A_2 = \{n3, n4\}, \ldots, A_{\frac{m}{2}} = \{n(m-1), nm\}$. From each subset we pick one student, which yields $2^{\frac{m}{2}}$ possible combinations. Because, as already mentioned in the proof of

Remark 1, the ordering of the classes does not matter, we need to divide the number $2^{\frac{m}{2}}$ by two, which results in $2^{\frac{m}{2}-1}$.

That when m is odd there is only one optimal division can be explained as follows.

We already noted that when there are five students to be assigned, there is only one optimal division: $\{\{n4,n2\},\{n5,n3,n1\}\}$. The same applies to the case of seven students, where the single optimal division is $\{\{n7,n5,n3,n1\},\{n6,n4,n2\}\}$ and where, once again, any reshuffling will yield a lower combined *ARD*. Why is what is possible in the even case not possible in the odd case? To respond, we look first and again at $n4 > n3 > n2 > n1$. Here, the reason why $\{\{n4,n1\},\{n3,n2\}\}$ is "just as good as" $\{\{n4,n2\},\{n3,n1\}\}$ is that the exchange leaves the combined *ARD* constant: the increased incentive when moving from $\{n4,n2\}$ to $\{n4,n1\}$, namely a gain by one rank, so to speak, is exactly offset by the decreased incentive when moving from $\{n3,n1\}$ to $\{n3,n2\}$, namely a loss by one rank, so to speak. When m is odd, an equivalent offsetting is not possible: a gain from an increased incentive coincides with a bigger loss from a decreased incentive; the odd case does not afford the symmetry rendered by the even case. In the $n5 > n4 > n3 > n2 > n1$ example, upon reassignment from $\{\{n4,n2\},\{n5,n3,n1\}\}$ to $\{\{n4,n1\},\{n5,n3,n2\}\}$, the gain of one rank, so to speak, in the distancing between the two students in the two-student class coincides with a loss of two ranks, so to speak, in the distancing between the students in the three-student class.

From the preceding claim and remarks it follows that the membership of the subsets in an optimal division of a class of n students remains as is if the class size were to increase, for example to $n+2$ students, and that in an optimal division, the top two performers are never assigned to the same subset.

3. Discussion

It is part of human nature to compare oneself with others. The social space in which people study, work, and live, is a comparison space: people value what they have both in an absolute sense and in a relative sense. This is well known and requires no further discussion. Less clear are two main issues that pertain to "type," and to the "conditions" that determine type. The "type" issue is whether sensing relative deprivation translates into an incentive for constructive engagement, yielding an outcome that improves the wellbeing of individuals, or whether it penalizes the wellbeing of individuals. In this paper we presented and analyzed a setting that belongs to the first type. This is not to deny the prevalence of settings of the second type. An example of the latter type - an optimal allocation of patients to hospital rooms - is presented below. The “conditions” issue is under what circumstances the response to relative deprivation is of one type or of the other. The main purpose of this section is to formulate general criteria that help to ascertain or predict whether relative deprivation is of one or the other type.

3.1 Relative deprivation as a penalizing stress factor: An example of optimal allocation of patients to hospital rooms

Let there be four individuals who suffer from the same illness, but with different degrees of severity: individual 1 is the most seriously ill, individual 4 is the least ill. The individuals require hospitalization. Given the scarcity of rooms, the plan is to place all four individuals in one room. It is well recognized that the individuals will be medically stressed, and that individuals 1, 2, and 3 will additionally experience social-psychological stress from comparing the gravity of their illness with that of the individuals / individual who are / is not as severely ill as they are. It then becomes known that the hospital can actually place the individuals in two rooms rather than in one room. There will be no (direct) medical effect from distributing the individuals between two rooms rather than one room. However, because the comparison group will differ, the extent of social-psychological stress will differ, assuming that the hospital room is the comparison environment. How to distribute the four individuals between the two rooms so that aggregate social-psychological stress is minimized?

As before, let the relative deprivation in a group of two be half of the difference between the levels of gravity of the illness of the two. In division {{4,2},{3,1}} as well as in division {{4,1},{3,2}} the sum of the levels of relative deprivation is two. In division {{4,3},{2,1}}, the sum of the levels of relative deprivation is one. Thus, a division of {1,2,3,4} into the two subsets of {4,3} and {2,1} minimizes the group’s aggregate social-psychological stress.

3.2 General criteria that help ascertain whether relative deprivation is of one type or of another

How can we tell when relative deprivation is likely to feature as an incentive for improvement, and when will it give rise to penalizing stress? Several criteria come to mind.

Criterion 1. Consider a setting in which the level of performance of one student is 10, that of each of four others is 11, namely the distribution is {10,11,11,11,11}. Imagine, alternatively, a setting in which the level of performance of one student is 10, and the level of performance of another student is 14, namely the distribution is {10,14}. In the two settings, the sum of the excesses as seen by 10 is the same at four. When will 10 be more likely to make an effort to close the gap, and when will he be more likely to despair? We could reason as follows: seeing that many other comparators perform better but not by much brings closing the gap more within reach than when one other comparator performs much better. Thus, a tentative inference would be that *the spread or distribution* of a given sum of gaps in performance can affect the response to sensing relative deprivation.[4]

Criterion 2. The search for conditions under which a particular response to sensing relative deprivation is to be expected can be helped by unraveling the underlying source. When a gap arises because of merit, and especially if the student who senses relative deprivation believes that the gap could be reduced by productive engagement, then relative deprivation will not result in despair. If the way in which student A is rewarded by the head teacher is better than the way in which student B is rewarded by the head teacher, and the reason for this is that student A produced four quality essays whereas student B have produced only two, and if more intensive study effort by student B will yield a desirable change, then we will witness a very different response by student B to his relative deprivation than if the gap in the head teacher's treatment is due to sheer favoritism. Thus, a tentative inference would be that the reason for the gap matters: *relative deprivation arising from another student having a better grade because that other student deserves a better grade invites toleration, while relative deprivation arising from another student having a better grade not because that student deserves having a better grade invites resentment*.

[4] Note that the relative deprivation of 10 in distribution {10,11,11,11,11}, which is $\frac{4}{5}$, is lower than the relative deprivation of 10 in distribution {10,14}, which is 2.

Criterion 3. From the preceding discussion we can gain the following insight. The likelihood that relative deprivation will result in debilitating stress is inversely related to the likelihood that relative deprivation will be controlled by a manageable effort to study harder. *Effort is the "enemy" of penalizing stress. When not too demanding, it will crowd out stress.*

Criterion 4. A standard approach in economics is to establish a connection between variables by inspecting a change at the margin. As an example, we can draw on income. Suppose that in a given income distribution the income of an individual positioned to the right of the reference individual increases by a certain amount. The same weight need not be attached to a change in the incomes of individuals who are placed at different distances from the reference individual. The reference individual might be more disturbed when an already relatively rich individual in his comparison group acquires additional income than by an equal increase in income of a less rich individual in his comparison group. Put a little differently, the reference individual may be tolerant of an income gain by someone on a similar income rung, but intolerant when someone already significantly richer than himself becomes even richer. This tolerance / intolerance dichotomy could arise from a basic notion of fairness: when looking to the right, the reference individual considers relatively poor "neighbors" more deserving of an income rise than relatively rich "neighbors." But it could also arise from a dichotomy between a capacity to expend effort to narrow a gap when it is small, and frustration at being exposed to an unbridgeable gap.[5] In sum: *a given amount, say of income, obtained by someone close in an individual's comparison group can prompt an effort to narrow the gap, whereas when obtained by someone farther away in an individual's comparison group, it can cause penalizing stress.*

Criterion 5. Evidence that attributes very bad health outcomes to relative deprivation suggests that the experience of relative deprivation alone cannot predict the type of outcome that relative deprivation entails. It appears that for a bad outcome to arise, relative deprivation has to be intersected with a particular psychological state of mind. Perhaps it is appropriate to refer to this state as dysphoria. If this insight is taken a step forward, then it could shed new light on reported associations in the US and in South Korea between relative deprivation and the incidence of suicides.[6] After all, there are many more individuals in both countries that

[5] Consult Stark et al. (2017).

[6] Using data on deaths by suicide in the US so as to identify the importance of interpersonal comparisons and "relative status," Daly et al. (2013) found compelling evidence that individuals care not only about their own

experience relative deprivation, and many more who experience higher levels of relative deprivation than the levels of relative deprivation experienced by people who commit suicide. Thus, returning to our setting of school classes, a criterion that helps ascertain the type of relative deprivation is that *when relative deprivation is intersected with dysphoria, then relative deprivation is of the type that penalizes the wellbeing of students rather than induce them to study harder*.

4. Conclusion

A divide between educational psychology and economics can result in a loss of productivity-enhancing insights that could be obtained by merging perspectives from the two disciplines.

We have studied how to divide a set of students into subsets so as to maximally influence their performance when pressure is exerted on them by the performance of comparators. For given set sizes of four or more, we identified the divisions that maximize aggregate pressure. We showed that the solution depends only on the ordinally-measured performances that are independent of the performance of comparators. In addition, we identified the number of optimal divisions; this number depends on whether the number of individuals to be assigned is even or odd.

Our analysis is based on several implicit assumptions, and thus has its limitations. For example, we assume that when students are assigned into groups, they have no better alternative options.

Some literature maintains that comparisons are made with worse off individuals (less successful students), and not - as we have assumed - with better off individuals (more successful students). Although we believe that the weight of the evidence supports our stance, we note that studies (such as Boyce et al., 2010) which looked at both effects found strong

income but also about the income of others in their local area: Daly et al. showed that individual suicide risk rises with others' income. This finding was obtained using two separate and independent data sets, suggesting that it is not the product of a particular sample design of either data set. The finding is robust to alternative specifications and cannot be explained by geographical variation in suicide classification, cost of living, or access to emergency medical care. The finding is consistent with the idea that relative deprivation, rather than a person's own absolute income, matters for wellbeing, and that the stress it causes can be severe enough to make people take their own life. In a study of suicide risk in South Korea, Pak and Choung (2020, p. 1) conclude as follows: "[O]ur findings suggest that relative deprivation in income is independently associated with higher odds of suicidal ideation and suicide planning or attempt over and above the effect of absolute income and material living conditions. Narrowing the income gap between individuals would be an effective policy response to a suicide epidemic in South Korea."

support that comparisons with better off individuals are substantially more important than comparisons with worse off individuals. To the extent that comparisons could be both ways while those with the better off individuals (more successful students) dominate, our setting in this regard is a “limit” case.

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Appendix: Construction of the index of relative deprivation

Several recent insightful studies in social psychology (for example, Callan et al., 2011; Smith et al., 2012) document how sensing relative deprivation, *RD*, impacts negatively on personal wellbeing, but these studies do not provide a calibrating procedure; a sign is not a magnitude. For the purpose of constructing a measure, a natural starting point is the work of Runciman (1966), who argued that an individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others with whom he naturally compares himself possess that good. Runciman (1966, p. 19) writes as follows: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel deprived," thus implying that the deprivation from not having, say, income y is an increasing function of the fraction of people in the individual's reference group who have y. To aid intuition and for the sake of concreteness, we resort to income-based comparisons, namely an individual feels relatively deprived when others in his reference group earn more than he does. It is assumed here implicitly that the earnings of others are publicly known. Alternatively, we can think of consumption, which might be more publicly visible than income, although these two variables can reasonably be assumed to be strongly positively correlated.

As an illustration of the relationship between the fraction of people possessing income y and the deprivation of an individual lacking y, consider a population (reference group) of six individuals with incomes $\{1,2,6,6,6,8\}$. Imagine a furniture store that in three distinct departments sells chairs, armchairs, and sofas. An income of 2 allows you to buy a chair. To be able to buy an armchair, you need an income that is a little bit higher than 2. To buy any sofa, you need an income that is a little bit higher than 6. Thus, when you go to the store and your income is 2, what are you "deprived of?" The answer is "of armchairs" and "of sofas." Mathematically, this deprivation can be represented by $P(Y>2)(6-2)+P(Y>6)(8-6)$, where $P(Y>y_i)$ stands for the fraction of those in the population whose income is higher than y_i, for $y_i = 2,6$. The reason for this representation is that when you have an income of 2, you cannot afford anything in the department that sells armchairs, and you cannot afford anything in the department that sells sofas. Because not all those who are to your right in the ascendingly ordered income distribution can afford to buy a sofa, yet they can all afford to buy armchairs, a breakdown into the two (weighted) terms $P(Y>2)(6-2)$ and

$P(Y > 6)(8-6)$ is needed. This way, we get to the very essence of the measure of *RD* presented in the main text of the paper: we take into account the fraction of the reference group (population) who possess some good which you do not, and we weigh this fraction by the "excess value" of that good. Because income enables an individual to afford the consumption of certain goods, we refer to comparisons based on income.

Formally, let $y = (y_1,..., y_m)$ be the vector of incomes in population *N* of size *n* with relative incidences $p(y) = \left(p(y_1),..., p(y_m)\right)$, where $m \leq n$ is the number of distinct income levels in *y*, where *n* and *m* are natural numbers. The *RD* of an individual earning y_i is defined as the weighted sum of the excesses of incomes higher than y_i such that each excess is weighted by its relative incidence, namely

$$RD_N(y_i) \equiv \sum_{y_k > y_i} p(y_k)(y_k - y_i)\ . \tag{1}$$

In the example given above with income distribution {1,2,6,6,6,8}, we have that the vector of incomes is $y = (1,2,6,8)$, and that the corresponding relative incidences are $p(y) = (1/6,1/6,3/6,1/6)$. Therefore, the *RD* of the individual earning 2 is $\sum_{y_k > y_i} p(y_k)(y_k - y_i) = p(6)(6-2) + p(8)(8-2) = \frac{3}{6}\cdot 4 + \frac{1}{6}\cdot 6 = 3$. By similar calculations, we have that the *RD* of the individual earning 1 is higher at $3\frac{5}{6}$, and that the *RD* of each of the individuals earning 6 is lower at $\frac{1}{3}$.

We expand the vector y to include incomes with their possible respective repetitions, that is, we include each y_i as many times as its incidence dictates, and we assume that the incomes are ordered, that is, $y = (y_1,..., y_n)$ such that $y_1 \leq y_2 \leq ... \leq y_n$. In this case, the relative incidence of each y_i, $p(y_i)$, is $1/n$, and (1), defined for $i = 1,..., n-1$, becomes

$$RD_N(y_i) \equiv \frac{1}{n}\sum_{k=i+1}^{n}\left(y_k - y_i\right). \tag{1'}$$

Looking at incomes in a large population, we can model the distribution of incomes as a random variable *Y* over the domain $[0,\infty)$ with a cumulative distribution function *F*. We can then express the *RD* of an individual earning y_i as

$$RD_N\left(y_i\right)=\left[1-F(y_i)\right]\cdot E\left(Y-y_i\,|Y>y_i\right). \tag{2}$$

To obtain this expression, starting from (1), we proceed in the following manner:

$$\begin{aligned} RD_N(y_i) &\equiv \sum_{y_k>y_i} p(y_k)(y_k-y_i) \\ &= \sum_{y_k>y_i} p(y_k)y_k - y_i \sum_{y_k>y_i} p(y_k) \\ &= [1-F(y_i)]\sum_{y_k>y_i}\frac{p(y_k)y_k}{[1-F(y_i)]} - y_i[1-F(y_i)] \\ &= [1-F(y_i)]E(Y\,|Y>y_i)-[1-F(y_i)]y_i \\ &= [1-F(y_i)]E(Y-y_i\,|Y>y_i). \end{aligned}$$

The representation in (2) states that the *RD* of an individual whose income is y_i is equal to the product of two terms: $1-F\left(y_i\right)$, which is the fraction of those individuals in the population of n individuals whose incomes are higher than y_i, and $E(Y-y_i\,|Y>y_i)$, which is the mean excess income.

The formula in (2) is quite revealing because it casts *RD* in a richer light than the ordinal measure of rank or, for that matter, even the ordinal measure of status, which have been studied intensively in sociology and beyond. The formula informs us that when the income of individual A is, say, 10, and that of individual B is, say, 16, the *RD* of individual A is higher than when the income of individual B is 15, even though, in both cases, the rank of individual A in the income hierarchy is second. The formula also informs us that more *RD* is sensed by an individual whose income is 10 when the income of another is 14 (*RD* is 2) than when the income of each of four others is 11 (*RD* is $\frac{4}{5}$), even though the excess income in both cases is 4. This property aligns nicely with intuition: it is more painful (more stress is experienced) when the income of half of the population in question is 40 percent higher than when the income of $\frac{4}{5}$ of the population is 10 percent higher. In addition, the formula in (2) reveals that even though *RD* is sensed by looking to the right of the income distribution, it is impacted by events taking place on the left of the income distribution. For example, an exit from the population of a low-income individual increases the *RD* of higher-income individuals (other than the richest) because the weight that the latter attach to the difference between the incomes of individuals "richer" than themselves and their own income rises.

Similar reasoning can explain the demand for positional goods (Hirsch, 1976). The standard explanation is that this demand arises from the unique value of positional goods in elevating the social status of their owners ("These goods [are] sought after because they compare favorably with others in their class." Frank, 1985, p. 7). The distaste for relative deprivation offers another explanation: by acquiring a positional good, an individual shields himself from being leapfrogged by others which, if that were to happen, would expose him to *RD*. Seen this way, a positional good is a form of insurance against experiencing *RD*.

Comment. In Stark et al. (2017), the employment of a set of axioms enables us to introduce a new class of generalized measures of relative deprivation, RD*p*, based on a preference relationship defined on the set of vectors of incomes. The class takes the form of a power mean of order p. A characteristic of the class is that it is capable of accommodating both a decreasing weight (the case of $p > 1$), and an increasing weight (the case of $p \in (0,1)$) accorded to given changes in the incomes of the individuals whose incomes are higher than the income of the reference individual. The incentive for introducing the class arose from acknowledgement of the possibility that the weights that an individual assigns to the incomes of individuals whose incomes are higher than his could depend on the proximity in the income hierarchy of those incomes to his income. The *RD* index (1') is a special case of the RD*p* class when *p* is equal to one, namely when a given change in income, say an increase, of a higher-income individual affects the reference individual equally, regardless of whom to his right receives the increase. In a way, the class is a generalization of the index of relative deprivation presented in the main text of the paper in that for any positive value of the proximity-sensitive parameter *p* different from one, the class exhibits sensitivity to the proximity of changes in the incomes of individuals whose incomes are higher than the income of the reference individual.

Appendix references

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