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Saumik Paul

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ABSTRACT

A Decline in Labor's Share with Capital Accumulation and Complementary Factor Inputs: An Application of the Morishima Elasticity of Substitution*

The role of capital accumulation as a driver of the labor income share requires capital and labor to be substitutes, which appears paradoxical in a world predominantly characterized by complementarity between capital and labor. This paper argues that the composition of skills in the labor force and an identification of the elasticity parameters between capital and different skills of labor can reconcile the opposing views. Using a framework with capital-skill complementarity and variable substitution elasticities, the Morishima elasticity of substitution is applied to identify the elasticity parameters at different skill levels and derive the necessary condition for capital accumulation to coexist with a declining labor income share when capital and labor are complements. Empirical evidence supports this proposition.

JEL Classification: E21, E22, E25

Keywords: labor income share, production function, Morishima elasticity of substitution

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Introduction

In recent years, a large body of research has documented a global decline in the labor income share (LIS, here on)[1]. The burgeoning literature offers several explanations and one of them is the role of capital accumulation (Karabarbounis and Neiman, 2014, Piketty, 2014). A 25% decline in the relative price of capital (investment), according to Karabarbounis and Neiman (2014), causes a 2.5 percentage point decline in the global LIS. The authors show that a drop in the relative price of investment produces an increase in the capital-labor ratio, and this mechanism forces the LIS to decrease when capital and labor in the production technology are substitutes. Both Karabarbounis and Neiman (2014) and Piketty (2014)[2] estimate the elasticity of substitution between capital and labor to be greater than 1, which is at odds with an extant literature[3] that predominantly documents the elasticity of substitution between capital and labor to be less than 1, or complementary components.

This paper validates the capital accumulation mechanism in a world predominantly characterized by complementarity between capital and labor. I show that in the presence of changing skill composition of the labor force and the variation in the elasticity of substitution between capital and labor across skill levels, a decline in the relative price of capital can lower the LIS when capital and aggregate labor are complements. Karabarbounis and Neiman (2014) estimate the elasticity of substitution parameter for different skill levels to be around 1.25. However, their empirical strategy does not identify the differences in substitutability between capital and labor at different levels of skill, and for this reason, the skill composition of the labor force does not alter their main results[4]. Also, other studies[5] that model capital-skill complementarity using a nested-CES production technology do not address the identification of elasticity parameters and their implications for the purported link between capital accumulation and the LIS.

To assess the quantitative significance of the elasticity parameters, I use a production framework with the variable elasticity of substitution (VES) technology that allows for substitution elasticities to vary with the capital-labor ratio at different levels of skills. I apply the Morishima elasticity of substitution (MES) and its comparative statics properties to (1) identify the substitution elasticities between capital and different skills of labor, and (2) measure the responsiveness of labor income share to

[1] Blanchard and Giavazzi (2003), Gollin (2002), Elsby, Hobijn, and Sahin (2013), Piketty and Zucman (2014), Piketty (2014), Karabarbounis and Neiman (2014), among other studies.
[2] Piketty (2014) argues for a channel of capital accumulation through growth in aggregate savings.
[3] Leon-Ledesma, McAdam, and Willman (2010), Oberfield and Raval (2014), and Chirinko and Mallick (2017), among others.
[4] Karabarbounis and Neiman (2014) acknowledge this limitation in footnote 25 (page 98). However, the authors conjecture that the skill-composition may have played some role in the declining labor income share.
[5] Arpaia, Perez and Pichelmann (2009); Elsby, Hobijn, and Sahin (2013).

changes in the price of capital in the presence of capital-skill complementarity. The MES extends the two-input Hicksian elasticity of substitution to a multi-input case (Blackorby and Russel, 1989). Based on the properties of the MES, I derive the necessary and sufficient conditions for a decline in the LIS caused by a fall in the relative price of capital. I show that if the necessary condition holds, capital accumulation can lead to a decline in the labor income share when capital and labor are complements.

The first step in this process is to consider a production framework with three input factors in capital, skilled-labor and unskilled-labor. Let σ_{Agg} denote the elasticity of substitution between capital and aggregate labor, and σ and ρ stand for the elasticity of substitution between capital and unskilled-labor and capital and skilled-labor, respectively. An assumption of $\sigma > \rho$ implies capital-skill complementarity (Krusell et al., 2000) and $\sigma_{Agg} > 1$ is the sufficient condition[6] for a decline in the LIS resulting from a fall in the relative price of capital (Karabarbounis and Neiman, 2014; Piketty, 2014). In a three-factor production function, the sufficient condition with capital-skill complementarity can be written as $\sigma > \rho > 1$. I identify and estimate the substitution parameters (σ and ρ) using the MES, and based on its properties, derive the necessary condition for the aforementioned mechanism to work as $\sigma > 1 > \rho$ and $|1-\rho| < |1-\sigma|$. The necessary condition is a less strict condition than the sufficient condition, and it allows for capital and skilled-labor to be complements when capital and unskilled-labor are substitutes in parallel with capital-skill complementarity.

Depending on the aggregation rule that combines σ and ρ into σ_{Agg}, capital and aggregate labor can be complementary (i.e., $\sigma_{Agg} < 1$) when the necessary condition $\sigma > 1 > \rho$ and $|1-\rho| < |1-\sigma|$ is met. Thus, a decline in the relative price of capital can cause a decline in the aggregate LIS when a decrease in the LIS for the unskilled labor force outweighs an increase in the LIS for the skilled labor. I consider a three-factor production framework that assumes (1) the variable elasticity of substitution between capital, skilled- and unskilled-labor and (2) capital-skill complementarity. Using this framework, I derive the expressions for the MES parameters, and then the necessary condition. Empirical estimates using data on the US for the period from 1977 to 2005 support this proposition. The necessary condition holds in 25% of the cases. Assuming σ_{Agg} as a weighted arithmetic mean of σ and ρ (Jones, 1965; Oberfield and Raval, 2014), I graphically show the feasible set of values for the weights that satisfy the condition for complementarity between capital and aggregate labor, i.e., $\sigma_{Agg} < 1$. Atkinson (2009) advocates an application

[6]A non-unitary elasticity of substitution (σ) between capital and labor plays a crucial role in explaining movements in the labor income share. The role of σ in analyzing the factor income shares has been noted since the seminal work of Hicks (1932) and Robinson (1933). Following the Hicksian partial elasticity of substitution, Elsby, Hobijn, and Sahin (2013) demonstrate the relationship between labor income share (L_S) and σ as $dlnL_S = -(1-L_S)\frac{\sigma-1}{\sigma}dln(\frac{K}{L})$, suggesting a drop in L_S when σ > 1. With σ = 1, factor income shares remain constant.

of the MES to gain a richer set of possible distributional outcomes when labor market is characterized by heterogeneous skills in a production function with more than two inputs. This appears to be the first study that applies the MES to understand the forces behind a decile in the LIS.

This paper combines and contributes to a rich set of literatures. First and foremost, this study contributes to the growing literature on the drivers of the LIS. A large body of research has documented a global decline in the LIS and offers several explanations (Elsby, Hobijn, and Sahin, 2013; Piketty, 2014). The assumption of a non-unitary elasticity of substitution (σ) between capital and labor plays a crucial role in explaining the changes in the LIS. The "accumulation view" assumes capital and labor to be gross substitutes ($\sigma_{Agg} > 1$), whereas studies have predominantly estimated σ_{Agg} to be less than one (Oberfield and Raval, 2014; Chirinko and Mallick, 2017). This paper reconciles the opposing viewpoints by showing that capital accumulation and a decline in the labor income share can coexist in the presence of complementarity between capital and labor.

Rognlie (2015) and Grossman et al. (2017), are two recent works that corroborate this study. Rognlie (2015) supports the "scarcity view", which assumes an increase in capital share due to the relative scarcity of some forms of capital in place of the "accumulation view". When considering a multisector model with different types of capital, he distinguishes between the ${\sigma_{Agg}}^{NET}$ (the relationship between net capital-output ratio and net rental rate of capital) and ${\sigma_{Agg}}^{GROSS}$ (the standard definition) by highlighting the role of depreciation. He demonstrates that ${\sigma_{Agg}}^{NET} < {\sigma_{Agg}}^{GROSS}$, ${\sigma_{Agg}}^{GROSS} > 1$ does not necessarily imply ${\sigma_{Agg}}^{NET} > 1$; as a result, following the "scarcity view," a decline in the LIS with a higher capital-output ratio can be attained with ${\sigma_{Agg}}^{NET} < 1$. In another paper, Grossman et al. (2017) uses human capital accumulation in a standard neoclassical growth framework, and defines the three elasticity parameters, $\sigma_{HC,PC}$ (between human capital and physical capital) as $\sigma_{HC,RL}$ (between human capital and raw labor) and $\sigma_{L,PC}$ (between total labor and physical capital). They show that if $\sigma_{HC,PC} < \sigma_{HC,RL}$, and $\sigma_{L,PC} < 1$, then a constant level of schooling would mean that movement in the share of labor as a proportion of national income and the rate of labor productivity growth would be positively correlated across steady states. This way, a decrease in labor productivity growth would correspond to a drop in the LIS. Grossman et al shows that a decline in the LIS is feasible with $\sigma_{Agg} < 1$ if labor productivity growth slows down[7].

[7] To demonstrate the mechanism, Grossman et al. (2017) considered a drop in the interest rate relative to the growth rate of wages, which prompts individuals to achieve a higher level of human capital for any steady-state level of technology and the size of capital stock. Since, $\sigma_{HC,PC} < \sigma_{HC,RL}$ (i.e., human capital is more complementary to physical capital than raw labor) this generates a shift in the relative factor demand in favor of a rise in the capital income share.

The economic rationale behind the use of capital-skill complementarity in explaining LIS trends comes from well-documented trends showing an increase in the supply and relative wages of skilled labor over time. The literature on the skill-biased technical change (SBTC) argues that an increase in the demand for skills is a potential driver of this upward trend (Acemoglu and Autor, 2011). The availability of cheaper capital equipment could also increase the demand for skilled labor with or without SBTC (Krusell, Ohanian, Rios-Lull, and Violante, 2000). More recently, Buera, Kaboski, and Rogerson (2015) argue for a systematic reallocation of value-added shares toward high-skill intensive sectors, which they term the skill-biased structural change (SBSC), to explain an increase in the supply of skills.

I follow the works of Blackorby and Russel (1989) and Anderson and Moroney (1993) to estimate ρ and σ. Blackorby and Russel (1989) show that the MES parameters can be approximated by the differences in own-price and cross-price elasticities. Moreover, MES directly links the changes in relative factor input prices to changes in the LIS. However, the MES is symmetric and constant only if the technology has either the implicit CES structure, the explicit Cobb-Douglas structure, or if there are only two inputs in the production function (Blackorby and Russell, 1981; Kuga, 1979 and Murota, 1977). In a three-factor production function, the MES becomes asymmetric and non-constant, which suggests for a framework that allows the elasticity of substitution to vary (Arrow, 1961; Miyagiwa and Papageorgiou, 2007). I work with a three-factor production function with the variable elasticity of substitution technology (Lu and Fletcher, 1968) that allows capital-skill complementarity and derive the necessary and the sufficient conditions for a decline in the LIS resulting from a decline in the relative price of capital.

Finally, this paper relates to the literature on estimating the aggregate elasticity of substitution using disaggregated (sectoral or plant level) data (Oberfield and Raval, 2014; Herrendorf, Herrington, and Valentinyi, 2015). There is no consensus on the economic environment that the aggregate elasticity of substitution is derived from (Chirinko, 2008; Leon-Ledesma, McAdam, and Willman, 2010, Miyagiwa and Papageorgiou, 2007). In an early and influential work, Jones (1965) using a multi-sectoral framework showed that the aggregate elasticity of factor substitution can be expressed as the weighted arithmetic mean of sectoral elasticity parameters. Oberfield and Raval (2014) consider both the elasticity of substitution between factor inputs within a plant and the reallocation of factor inputs across plants to estimate σ_{Agg}, and their estimates suggest that $\sigma_{Agg} < 1$.

The rest of the paper is organized as follows. I begin section 2 with a brief discussion of capital-skill complementarity and the identification problem associated with the estimation of the elasticity of substitution. After this, I introduce the concept of the MES and an application of the MES to identify the elasticity of substitution parameters. This sets the stage for analyzing the characteristics of the MES. Using the properties of the MES, I construct the necessary and the sufficient condition for a decline in the price of capital to cause a fall in the labor income share. Section 3 shows the derivation of these two conditions based on the MES in a production framework with the variable elasticity of substitution technology. In section 4, I discuss the empirical results. Section 5 concludes.

2. Theoretical Constructs

2.1 Capital-skill Complementarity in a nested-CES Framework

Consider labor is heterogeneous in skills and the elasticity of substitution between capital and labor differs across different skill levels (Grilliches, 1969; Krusell, Ohanian, Rios-Lull, and Violante, 2000 [KORV, here on]). With the three inputs of capital (K), skilled labor (S), and unskilled labor (U), the CES production function can be nested in three ways: $Y = f[(K,S)U]$, $Y = f[(K,U)S]$, and $Y = f[(S,U)K]$ (nested-inputs are within the first bracket). Since $Y = [(S,U)K]$ boils down to a standard 2-factor CES production, the other two functions can be used to examine the link between capital-skill complementarity and the labor share of income (Karabarbounis and Neiman, 2014). I write $Y = f[(K,S)U]$ as

(1) $$Y = f[(K,S)U] = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset)S^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + (1-\theta)U^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

θ and $\emptyset$ denote distribution parameters; σ denotes the elasticity of substitution between K and U (similarly, between U and S); ρ denotes the elasticity of substitution between K and S. Equation (1) shows capital-skill complementarity when $\sigma > \rho$. The condition $\sigma > \rho$ implies that substitutibility between capital and unskilled-labor is higher than the substitutability between capital and skilled-labor. Using the nested-CES production function (Equation1), Karabarbounis and Neiman (2014) derived the following equation to estimate the elasticity of factor substitution:

(2) $$\frac{L_S}{1-L_S}\widehat{L_S} = \alpha + (\sigma-1)\hat{\delta} + \beta\widehat{\left(\frac{U}{K}\right)} + \varepsilon^{8}.$$

Equation (2) suggests that a positive relationship between trends in the LIS and changes in the price of capital goods ($\hat{\delta}$) relative to skilled labor is only possible when $\sigma > 1$, i.e. capital and labor are substitutes. If this condition holds, a fall in the price

[8] L_S denotes the labor income share; $\widehat{L_S}$ denotes changes in the labor income share; $\hat{\delta}$ denotes changes in the relative price of investment goods (capital); $\widehat{\left(\frac{S}{K}\right)}$ denotes changes in the ratio between skilled labor and capital; α and β are regression parameters (constants), and ε denotes the idiosyncratic error term.

of capital good is associated with a lower labor income share, which summarizes the accumulation view of the labor income share. If capital is more substitutable with unskilled labor than skilled labor (Grilliches, 1969; Berman, Bound, and Grilliches, 1994), then a drop in the relative price of capital results in a larger drop in the share of income for the unskilled labor than for the skilled labor[9].

I denote the aggregate elasticity of substitution between capital and labor as σ_{Agg}, which shows the degree of substitutability between capital and aggregate labor, composed of the skilled and the unskilled types. Following equation 2, the accumulation view of the labor income share requires $\sigma_{Agg} > 1$. An estimate of the σ_{Agg} using capital and the aggregate labor can be approximated by a weighted average of the elasticities of substitution between capital and labor across different skill groups. However, a direct estimation of σ_{Agg} masks the role of differential capital-skill substitutability unless σ and ρ are identified. I discuss the identification problem in greater detail in the following section.

2.2. The Identification Problem

In a production framework with more than two inputs, an identification problem in the estimation of the elasticity parameters can be caused by simultaneous changes in the factor prices. Since σ and ρ ($\rho \neq \sigma$ following capital-skill complementarity) are determined simultaneously, equation 2 cannot identify the value of ρ (the elasticity of substitution between nested inputs) independent of σ. In other words, if the estimate of β is a function of σ and ρ, then equation 2 alone cannot solve the value of both elasticities of substitution[10]. Karabarbounis and Neiman (2014) estimate σ using skilled labor by replacing $\widehat{\left(\frac{U}{K}\right)}$ with $\widehat{\left(\frac{S}{K}\right)}$ and found almost identical values (around 1.25) in both cases. this outcome is likely to be driven by the identification problem.

Some studies address this identification problem by suggesting alternative aggregation methods. Oberfield and Raval (2014) use a nested-CES structure similar to Equation (1) with capital, labor, and materials instead of two types of labor. They estimate σ_{Agg} as a convex combination of the elasticity of demand, the elasticity of substitution between materials, and the capital-labor bundle. This provides a novel way of estimating σ_{Agg}, but the same identification problem persists since they combine capital and labor and effectively reduce the production structure from a three-input to a two-input case. Similarly, Elsby, Hobijn, and Sahin (2013) also

[9] This is also related to the large literature on skill-biased technical change (SBTC). See Grilliches (1969), Acemoglu (2002), Autor, Levy, and Murnane (2003), Caselli (1999), among others.
[10] Karabarbounis and Neiman (2014) acknowledged this limitation in their paper (footnote 25).

contend that σ_{Agg} becomes a weighted average of ρ and σ[11]. Arpaia, Perez, and Pichelmann (2009) acknowledge the differences in the elasticity of substitution between the two kinds of labor and capital. However, none of these papers attempt to identify the differences between ρ and σ, and find their implications for the LIS trends.

In a multi-input nested production structure (equation 1), the identification of both types of elasticities of substitution is crucial because they govern the links between changes in the relative factor income shares and changes in the relative factor prices (Blackorby and Russel, 1989; Anderson and Moroney, 1993). For example, using the nested-CES production function in Equation (1), the condition $\sigma > \rho$ implies that capital is more substitutable with unskilled labor than with skilled labor and the differential capital-skill substitutability can contribute to the LIS trend in various ways, including changes in the skill-premium through technological progress (discussed at length in KORV)[12]. Similarly, Diamond et al. (1978) caution that elasticities could only be identified when factor price movements are independent of the bias of technological changes.

In the following section, I discuss an alternative framework to identify and estimate both types of elasticities of substitution (ρ and σ) using the concept of the Morishima Elasticity of Substitution, which holds the prices of other factor inputs constant and adjusts the measure of the elasticity of substitution accordingly.

2.3. The Morishima Elasticity of Substitution (MES)

For a multi-input production structure, Morishima (1967) introduced a measure of factor substitutability, which later Blackorby and Russell (1975) termed as the Morishima Elasticity of Substitution (MES). The MES is essentially a two-factor one-price elasticity of substitution (TOES), which measures the percentage change in the input ratio between two inputs resulting from a one percent change in the price of one input (Chambers, 1988, page 96). In a multi-input framework, let Y denote the output quantity, $\bar{P}$ as the output price, x as input quantity vector and p as input price vector for inputs $i = 1(1)k$. Then an expression of the MES in the context of a cost minimization problem (Chambers, 1988; Blackorby and Russell, 1989) can be written as

[11] Elsby, Hobijn, and Sahin (2013) demonstrate the relationship between labor income share (L_S) and capital-skill complementarity as $dlnL_S = -(1 - L_S)[\{\varphi\rho + (1-\varphi)\sigma\}dln\left(\frac{K}{L}\right) + \{\frac{S}{1-S}(1-\varphi)\rho - \varphi\sigma\}dln(S)]$.

[12] KORV demonstrated capital-skill complementarity as the key feature of technology. They provided empirical evidence based on a theoretical framework that hypothesized elasticity of substitution was higher between capital and unskilled labor than between capital and skilled labor.

(3) $$MES_{ij}(Y,p) = \frac{\partial log\frac{x_i(Y,p)}{x_j(Y,p)}}{\partial log\frac{p_j}{p_i}}$$ [13]

The expression for MES in equation (3) shows changes in the cost-minimizing optimal input ratio resulting from a percentage change in the price ratio induced by a change in p_i. The MES holds prices of other factor inputs constant and adjusts the measure of the elasticity of substitution accordingly[14]. The factor inputs x_i and x_j are Morishima complements if $MES_{ij}(Y,p) < 1$ and x_i and x_j are Morishima substitutes if $MES_{ij}(Y,p) > 1$.

It is evident from Equation (3) that $MES_{ij} \neq MES_{ji}$, that MES is asymmetric as the value, and that the sign of MES differs between the price changes of input x_i and changes in the price of input x_j. Blackorby and Russell (1989) claim that the MES is a natural multi-input generalization of the Hicksian two-input elasticity of substitution. These authors also showed that MES is a sufficient statistic to quantitatively and qualitatively assess the effects of changes in the price ratio on relative factor input shares. As Equation (4) shows, the MES provides a direct link between the factor prices and the ratio of factor input uses.

(4) $$\frac{\partial log\frac{p_i x_i(Y,p)}{p_j x_j(Y,p)}}{\partial log\frac{p_i}{p_j}} = 1 - MES_{ij}(Y,p) = 1 - \frac{\partial log\frac{x_i(Y,p)}{x_j(Y,p)}}{\partial log\frac{p_j}{p_i}}.$$

This property of MES makes it an ideal choice to study changes to factor income shares resulting from changes in relative factor prices.

2.4. Identification of Substitution Parameters using the MES

Moving on, I discuss the expressions for MES (similar in Equation 3) in the context of a three-input nested-CES structure. I rewrite the CES production structure in Equation (1) as a two-stage function consisting of two sub-processes, or nests, as follows:

(5) $$Y = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset)S^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + \quad (1-\theta)U^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = N_1(K,S) + N_2(U).$$

[13] The MES can be expressed as a function of its own price and the cross-price elasticities of two inputs in the following way $MES_{ij}(Y,p) = \frac{\partial log x_j(Y,p)}{\partial log p_i(Y,p)} - \frac{\partial log x_i(Y,p)}{\partial log p_i(Y,p)}$.

[14] As originally suggested by Pigou (1934), one way to address this issue is to hold output and other input factors, except for one of the two in the ratio, constant.

From Equation (5), ρ denotes the intra-nest elasticity of substitution between K, and S and σ denote the inter-nest elasticity of substitution between K and U. The sub-processes N_1 (with inputs K and S) and N_2 (with just input U) are mutually exclusive and exhaustive. Following Anderson and Moroney (1993), I write the expressions for ρ and σ as

(6) $$\rho = MES_{KS} = \frac{dlogS}{dlogp_K} - \frac{dlogK}{dlogp_K}, \quad K, S \in N_1$$

(7) $$\sigma = MES_{KU} = \theta \left[\frac{\partial logN_2}{\partial logp_{N_1}} - \frac{\partial logN_1}{\partial logp_{N_1}}\right] - \frac{\partial logK}{\partial logp_K}, \quad K \in N_1;\ U \in N_2.$$

In Equation (7), the expression $\frac{\partial logN_2}{\partial logP_{N_1}} - \frac{\partial logN_1}{\partial logP_{N_1}}$ refers to the MES between N_1 and N_2. Given the nested-CES production structure in equation 1, ρ can be estimated either through MES_{KS} or MES_{SK}, but $MES_{KS} \neq MES_{SK}$ due to asymetric properties of the MES. Similarly, the inter-nest MES is asymmetric ($MES_{KU} \neq MES_{UK}$), the value of σ will therefore change if we let the price of unskilled labor change instead of the price of capital (Blackorby and Russel, 1989; Anderson and Moroney, 1993).

Denote $p_K = r$ (rate of interest), $P_S = w_S$ (skilled wages), and $P_U = w_U$ (unskilled wages). From Equations (4) and (5), the expression for the effects of changes in the rate of interest relative to changes in wages for the skilled-labor on changes in the share of skilled-labor income relative to capital income becomes:

(8) $$\frac{dlog\frac{W_S S}{r K}}{dlog\frac{W_S}{r}} = 1 - MES_{SK}(= \rho) = 1 - \frac{dlogK}{dlogW_S} + \frac{dlogS}{dlogW_S}$$

Equation (8) indicates that a drop in the relative price of capital relative to wages for skilled-labor (an increase in the ratio $\frac{W_S}{r}$) leads to a decline in the skilled-LIS (i.e., $\frac{dlog\frac{W_S S}{r K}}{dlog\frac{W_S}{r}} < 0$) if $MES_{SK} > 1$. In other words, the skilled-LIS declines due to the availability of cheaper capital when capital and skilled-labor (intra-nest inputs) are gross substitutes.

(9) $$\frac{dlog\frac{W_U U}{r K}}{dlog\frac{W_U}{r}} = 1 - MES_{UK}(= \sigma) = 1 - \theta \left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] + \frac{dlogK}{dlogP_K}$$

Similarly, Equation (9) shows that a drop in the relative price of capital (an increase in the ratio $\frac{W_U}{r}$) leads to a drop in the unskilled-LIS (i.e., $\frac{dlog\frac{W_U U}{r K}}{dlog\frac{W_U}{r}} < 0$) if $MES_{UK} > 1$ or if capital and unskilled-labor (inter-nest inputs) are gross substitutes.

2.5. Capital Accumulation and Decline in the LIS when $\sigma_{Agg} < 1$

The MES expressions in Equations (6) and (7) suggests that $\rho \neq \sigma$. If the elasticity of substitution between capital and skilled labor (intra-nest) is different from the elasticity of substitution between capital and unskilled labor (inter-nest), then the size of the gap between the values of ρ and σ has a direct bearing on the aggregate LIS changes. If both ρ and σ are greater than 1, i.e., capital is gross substitutes with both skilled and unskilled labor, then from Equations (8) and (9), it is sufficient to have a declining LIS with cheaper capital. I provide a more formal discussion of the sufficient and the necessary condition in Proposition 1.

Proposition 1: If $\sigma > \rho$, then (i) the sufficient and (ii) the necessary conditions for a decline in the aggregate LIS with a drop in the relative price of capital are (i) $\sigma > \rho > 1$ and (ii) $\sigma > 1 > \rho$ and $|1-\rho| < |1-\sigma|$, respectively.

I provide a rough sketch of the proof. At equilibrium (i.e., when marginal products equal factor prices) a simple expression for the aggregate LIS (L_S) can be written as

$$(10) \qquad L_S = \frac{W_S S + W_U U}{y}, \text{ or } \frac{L_S}{1-L_S} = \frac{W_S S}{r K} + \frac{W_U U}{r K}$$

Taking log and differentiating Equation (10) with respect to the log of input-price ratios, we get

$$(11) \qquad \frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{\overline{W}}{r})} = \frac{dlog\frac{W_S S}{r K}}{dlog\frac{W_S}{r}} + \frac{dlog\frac{W_U U}{r K}}{dlog\frac{W_U}{r}},$$

$$\text{or, } \frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{\overline{W}}{r})} = 1 - \rho + 1 - \sigma = 2 - \rho - \sigma.$$

In Equation (11), $\overline{W}$ represents weighted average wage in the labor market. If L_S declines as a result of a fall in r relative to $\overline{W}$, then the sum of the signs of the terms on the right-hand side of Equation (11) must be negative. Since $\sigma > \rho$, the condition $\rho > 1$ ensures that both elasticities are greater than 1. Thus, it is straightforward from Equation (11) that the sign of the sum of the right-hand side terms of Equation (11) become negative when $\rho > 1$. Equation (11) is similar to the regression model (equation 2) used by Karabarbounis and Neiman (2014). They estimated the substitution elasticities of capital for both skilled and unskilled labor to be greater than 1.

However, it can also be shown that $\sigma > \rho > 1$ is not the necessary condition for a decline in the aggregate LIS. If $\sigma > \rho$ and $\sigma > 1$, then ρ can be less than unity, as long as the following inequality holds: $\left|\frac{dlog\frac{W_S S}{r K}}{dlog\frac{W_S}{r}}\right| < \left|\frac{dlog\frac{W_U U}{r K}}{dlog\frac{W_U}{r}}\right|$ or $|1-\rho| < |1-\sigma|$. Thus, a less strict condition with $\sigma > 1 > \rho$, can also produce a decline in the aggregate LIS. This proves the necessary condition for a decline in the labor share from a drop in the relative price of capital. This condition implies that a decline of the LIS alongside a drop in the relative price of capital occurs when the loss of income share due to a decrease in the unskilled labor force outweighs the income gain due to an increase in the skilled labor force.

The expression for the responsiveness of the ratio of factor shares to the ratio of factor prices (equation 11) can be empirically estimated using four possible combinations of the MES following its asymmetric property: σ can be approximated by either MES_{UK} or MES_{KU}, and similarly ρ can be estimated by either MES_{SK} or MES_{KS}. It results in a set of four combinations (MES_{SK}, MES_{UK}), (MES_{SK}, MES_{KU}), (MES_{KS}, MES_{UK}) and (MES_{KS}, MES_{KU}) to assess the empirical validity of the comparative statics in Equation (11). I elaborate on this point in section 4.

Proposition 2: If the necessary condition is met, and then $\sigma_{Agg} < 1$ is feasible subject to the aggregation rule that combines σ and ρ into σ_{Agg}.

This proposition implies that it is feasible to have an estimate of the aggregate elasticity of substitution (σ_{Agg}) to be less than unity (capital and labor are complements) when the necessary condition $\sigma > 1 > \rho$ holds. To proceed, I need to make some plausible assumptions about the functional relationship between σ_{Agg}, σ, and ρ, i.e., $\sigma_{Agg} = \phi(\sigma, \rho)$ where ϕ represents the aggregation rule. Oberfield and Raval (2014) derive a closed-form expression for σ_{Agg}, where the aggregate elasticity of substitution between capital and labor can be expressed as a weighted average of the sectoral (industry-level) elasticity of substitution parameters. Following Oberfield and Raval (2014), I write

(12) $\sigma_{Agg} = (1 - \aleph)\varepsilon + \aleph\theta$

where $\aleph$ represents a heterogeneity index, which takes a value of zero if the capital intensity is the same across sectors (or industries). ε is the sectoral level elasticity of the substitution parameter and θ represents the elasticity of demand. Using a multisectoral model, Rognlie (2015) derived an analytical solution to σ_{Agg} as a function of five gross elasticities of substitution $(\sigma_Z, \sigma_F, \sigma_{G_1}, \sigma_{G_2}, \sigma_H)$[15]. He further

[15] σ_Z is the elasticity of demand for housing services; σ_F is the elasticity of substitution between real estate and other services; σ_{G_1} is the elasticity of substitution between structures and land in non-housing sectors; σ_{G_2} is the elasticity of substitution between structures and land in the housing sector; and finally, σ_H represents the elasticity of substitution between equipment and labor.

highlighted the role of net elasticity of substitution (${\sigma_{Agg}}^{NET}$), which shows changes in the real capital to net output ratio and the net rental rate of capital. Following his "scarcity view," a decline in the LIS with a higher capital-output ratio can be attained using ${\sigma_{Agg}}^{NET} < 1$.

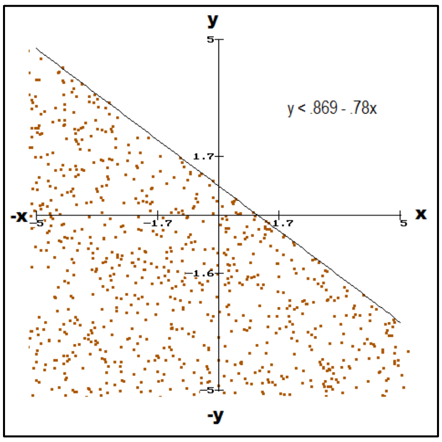
The primary goal here is to show that there exists a set of weights for σ and ρ, for which $\sigma_{Agg} < 1$ under the condition $\sigma > 1 > \rho$. There is no consensus on the economic environment the elasticity considered in. Following Jones (1965) and Oberfield and Raval (2014), I consider the aggregate elasticity of factor substitution as the weighted arithmetic mean of elasticity parameters across different skills. Equation (13) shows the aggregate elasticity of substitution between capital and labor as a linear weighted average of σ and ρ (with y and x as weights)

(13) $\sigma_{Agg} = y\sigma + x\rho.$

Imposing the condition for aggregate complementarity between capital and labor ($\sigma > \rho$), I consider a numerical example with the values of $\sigma = 1.15$ and $\rho = .9$[16]. Using these values, I can show $\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog\left(\frac{\overline{W}}{r}\right)} < 0$ since $|1 - .9| < |1 - 1.15|$. Therefore, this numerical example supports both capital-skill complementarity and a decline in the labor income share. I now impose the condition for complementarity between capital and aggregate labor, $\sigma_{Agg} < 1$. Applying this condition to equation (13), it turns into an inequality (equation 14):

(14) $1 > 1.15y + .9x$

Figure 1 A feasible range of weights for $\sigma_{Agg} < 1$



Note: This graph shows a numerical example. It shows the feasible range of value for an equation showing inequality.

[16] These parametric values are in line with the existing literature (Karabarbounis and Neiman, 2014; Rognile, 2015).

Source: Author.

As a final step, I consider a graphical illustration of the relationship between σ_{Agg}, σ, and ρ. In Figure 1, I show the set of permutations of x and y that satisfy Equation (14). The feasible range of numerical values that satisfy Equation (14) are plotted in Figure 1. Any point (combination of weights, x and y) to the left of the diagonal line (the dotted region) implies that the weighted average (σ_{Agg}) of σ (weight measured on the y axis) and ρ (weight measured on the x axis) must be less than the unity for the given values of $\sigma = 1.15$ and $\rho = .9$. Following the hypothetical example, the dotted region in Figure 1 suggests complementarity between capital and labor ($\sigma_{Agg} <$ 1) for a feasible set of values of σ and ρ, which corresponds with a decline in the LIS. While it shows that an estimate of σ_{Agg} can be less than unity, the validity of the condition $\sigma > 1 > \rho$ remains an empirical question. I next derive the necessary condition using a theoretical framework.

3. The MES in a Variable Elasticity of Substitution Production Framework

In this section, I derive the expression for the MES using a production function with three inputs: capital, skilled-labor and unskilled-labor. The CES production technology has mostly been adopted[17] to derive the relationship between factor income shares and the elasticity parameter of factor substitution (σ). There are two main reasons why I chose to work with a variable elasticity of substitution (VES here on) framework instead and derive the expressions for the MES using the VES framework.

First, in a standard two-input model if an increase in the capital-labor ratio causes σ to vary, then the elasticity of substitution between factor inputs varies along the isoquant. Karagiannis, Palivos, and Papageorgiou (2005) consider σ as a linear function of the capital-labor ratio and show that factor income shares do vary with the capital-labor ratio. The choice between the CES and the VES is subject to empirical validity[18]. In a recent paper, Paul (2019) shows that the VES specification of technology is preferred to that of the CES, as σ varies with the factor income shares in 14 out of the 23 Japanese industries for the period from 1970 to 2012.

Second, as discussed in section 2.3, the MES can be expressed as a difference between the own price and the cross-price compensated elasticities of two inputs, in

[17] Bentolila and Saint-Paul (2003); Elsby, Hobijn, and Sahin (2013); Karabarbounis and Neiman (2014)

[18] Sato and Hoffman, 1968; Kazi, 1980; Revankar, 1971) reject CES model specifications in favor of the VES model. Lovell (1968), Tsang and Yeung (1976) and Zellner and Ryu (1998) provide evidence that in certain sectors the CES model provides a better fit to the data compared to the VES model.

the following way: $MES_{ij}(Y,p) = \frac{\partial log x_j(Y,p)}{\partial log p_i(Y,p)} - \frac{\partial log x_i(Y,p)}{\partial log p_i(Y,p)}$. This suggests that $MES_{ij} \neq MES_{ji}$, as the value and the sign of the MES differs between the price changes of input x_i and changes in the price of input x_j. Blackorby and Russell (1981), Kuga (1979) and Murota (1977) provide detailed analysis on the characterization of the MES, and show that the MES is symmetric and constant only if the technology has the implicit CES structure, or the explicit Cobb-Douglas structure, or there are only two inputs in the production function. Thus, in a three-factor production function, the MES becomes asymmetric and non-constant. It also allows the MES to vary across different levels of output if production function is not tied up with the CES technology.

I work with the version of the VES production function provided by Lu and Fletcher (1968), and then extend it to a three-factor production function to incorporate capital-skill complementarity. Lu and Fletcher (1968) derive a VES model based on a log-linear form of the relationship between value-added per unit of labor ($\frac{Y}{L}$), a constant term (β_0), the wage rate, the capital-labor ratio and an error term (ε). Equation (15) shows a similar log-linear model of valued-added per unit of labor with skilled-labor (S) and unskilled-labor (U) and w_S and w_U are the wage rates in the respective sectors. The total labor employed is composed of skilled-labor and unskilled-labor, $L = S + U$, and $\frac{K}{S}$ and $\frac{K}{U}$ are the capital to skilled labor and capital to unskilled labor ratios, respectively.

$$(15) \quad log\frac{Y}{L} = \beta_0 + \alpha_1 log w_S + \alpha_2 log w_U + \beta_1 log\frac{K}{S} + \beta_2 log\frac{K}{U} + \varepsilon$$

Equation (15) serves as the basis for a class of production functions with variable elasticity of factor substitutions assuming $\beta_1 \neq 0$ and $\beta_2 \neq 0$. The non-zero values of these parameters imply a direct link between output per unit of labor and capital to labor ratios, for both skilled and unskilled labor. I use a three-factor version of the Lu and Fletcher (1968) VES technology model as shown in equation (16) where labor input from each sector is multiplied by an additional factor, the capital per unit of labor in the corresponding sector. The aggregate elasticity of factor substitution (termed ε_{Agg}) using the production function in equation (16) can be written as a function of β_1, β_2 and ε. Non-zero values of the parameters β_1 and β_2 add variability to the elasticity of factor substitution, whereas ε represents the constant part of the aggregate elasticity of factor substitution.

The condition $\beta_1 < \beta_2$ indicates capital-skill complementarity (similar to the condition $\sigma > \rho$ in a nested-CES production function in Equation 1), which implies that capital is more substitutable with the unskilled labor than with the skilled labor. In addition, the functional form in equation (15) also avoids sequential production process (or nesting) as done in the CES framework to implement the capital-skill complementarity condition. This makes the empirical estimation of the MES straightforward.

(15) $$Y = \left[(1-\theta_1-\theta_2)K^{\frac{\varepsilon-1}{\varepsilon}} + \theta_1\left(\frac{K}{S}\right)^{\frac{\beta_1}{\varepsilon}} S^{\frac{\varepsilon-1}{\varepsilon}} + \theta_2\left(\frac{K}{U}\right)^{\frac{\beta_2}{\varepsilon}} U^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Assuming perfectly competitive labor and product markets, the total cost (C) function can be written as $C = rK + w_S S + w_U U$. Following simple steps of algebraic calculations from the first order conditions of the cost minimization problem, I can write an expression for the ratio of the wages of skilled-labor to unskilled-labor as a function of capital, cost-sharing parameters, skilled and unskilled labor and the elasticity parameters.

(16) $$\frac{w_S}{w_U} = \frac{\theta_1}{\theta_2}\left(\frac{\varepsilon-\beta_1-1}{\varepsilon-\beta_2-1}\right)K^{\frac{\beta_1-\beta_2}{\varepsilon}}\left[\frac{U^{\frac{1+\beta_2}{\varepsilon}}}{S^{\frac{1+\beta_1}{\varepsilon}}}\right].$$

Equation (16) can be rewritten as a compensated demand function for capital, as follows:

(17) $$K = \left[\frac{\theta_2}{\theta_1}\left(\frac{\varepsilon-\beta_2-1}{\varepsilon-\beta_1-1}\right)\frac{S^{\frac{1+\beta_1}{\varepsilon}}}{U^{\frac{1+\beta_2}{\varepsilon}}}\right]^{\frac{\varepsilon}{\beta_1-\beta_2}}\left[\frac{w_S}{w_U}\right]^{\frac{\varepsilon}{\beta_1-\beta_2}}$$

Taking partial derivatives of $logK$ with repsect to $logw_S$ and $logw_U$, I derive the cross-price elasticities as follows:

(18) $\frac{\partial logK}{\partial logw_S} = \frac{\varepsilon}{(\beta_1-\beta_2)}$ and $\frac{\partial logK}{\partial logw_U} = \frac{\varepsilon}{(\beta_2-\beta_1)}$.

From the first order conditions, a compensated demand function for skilled-labor can be written as

(19) $$S = \left[\frac{\theta_1}{\theta_2}\left(\frac{\varepsilon-\beta_1-1}{\varepsilon-\beta_2-1}\right)\right]^{\frac{\varepsilon}{1+\beta_1}}\left[\frac{w_U}{w_S}\right]^{\frac{\varepsilon}{1+\beta_1}} K^{\frac{\beta_1-\beta_2}{1+\beta_1}} U^{\frac{1+\beta_2}{1+\beta_1}}.$$

I derive the own-price elasticity for skilled-labor by taking partial derivative of $logS$ with respect to $logw_S$

(20) $$\frac{\partial logS}{\partial logw_S} = \frac{\varepsilon}{(1+\beta_1)}.$$

Similarly, from the set of first order conditions, a compensated demand function for unskilled-labor can be written as

(21) $$U=\left[\frac{\theta_2}{\theta_1}\left(\frac{\varepsilon-\beta_2-1}{\varepsilon-\beta_1-1}\right)\right]^{\frac{\varepsilon}{1+\beta_2}}\left[\frac{w_U}{w_S}\right]^{\frac{\varepsilon}{1+\beta_2}}K^{\frac{\beta_2-\beta_1}{1+\beta_2}}U^{\frac{1+\beta_1}{1+\beta_2}}.$$

I derive the own-price elasticity of unskilled-labor by taking partial derivative of $logU$ with respect to $logw_U$, and get

(22) $$\frac{\partial logU}{\partial logw_U}=\frac{\varepsilon}{(1+\beta_2)}.$$

Plugging the expressions from equations (18), (20) and (22) into equations (6) and (7), I can write the following expressions for the MES_{SK} and the MES_{UK}:

(23) $$MES_{SK}=\frac{\partial logK}{\partial logw_S}-\frac{\partial logS}{\partial logw_S}=\varepsilon\left[\frac{1+2\beta_1-\beta_2}{(\beta_1-\beta_2)(1+\beta_1)}\right]$$

(24) $$MES_{UK}=\frac{\partial logK}{\partial logw_U}-\frac{\partial logU}{\partial logw_U}=\varepsilon\left[\frac{1+2\beta_2-\beta_1}{(\beta_2-\beta_1)(1+\beta_2)}\right].$$

To note, since I use the VES model instead of a nested CES framework, the expression for MES_{UK} does not include any inter-nest substitution parameters, unlike the case in equation 7. Combining equations (11), (23) and (24), I get an expression for the changes in the factor income share ratio as a result of changes in the factor price ratio[19]:

(25) $$\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{W}{r})}=2-\frac{\varepsilon}{(\beta_1-\beta_2)}\left[\frac{1+2\beta_1-\beta_2}{(1+\beta_1)}-\frac{1+2\beta_2-\beta_1}{(1+\beta_2)}\right]$$

The derivative in equation (25) suggests that a drop in the price of capital leads to a lower labor income share if $\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{W}{r})}<0$. In other words, the accumulation view of the decline in the labor income share holds when the condition in equation (26) is satisfied.

(26) $$\frac{\varepsilon}{(\beta_1-\beta_2)}\left[\frac{1+2\beta_1-\beta_2}{(1+\beta_1)}-\frac{1+2\beta_2-\beta_1}{(1+\beta_2)}\right]>2$$

As a final step, I derive the values of β_1 and β_2 using the same numerical example discussed in section 2.4 with $\sigma=MES_{UK}=1.15$ and $\rho=MES_{SK}=.9$. Since there are three unknowns (ε,β_1 and β_2) to solve from two equations, (23) and (24), the solutions for β_1 and β_2 contain the unknown constant part of the substitution parameter, ε. Making an additional assumption that $\varepsilon=1$, gives a set of two equations with two variables, β_1 and β_2.

[19] Since $\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{W}{r})}=\frac{dlog\frac{W_S S}{rK}}{dlog\frac{W_S}{r}}+\frac{dlog\frac{W_U U}{rK}}{dlog\frac{W_U}{r}}=1-MES_{SK}+1-MES_{UK}$

(27) $\frac{1+2\beta_1-\beta_2}{(\beta_1-\beta_2)(1+\beta_1)} = .9$

(28) $\frac{1+2\beta_2-\beta_1}{(\beta_2-\beta_1)(1+\beta_2)} = 1.15$

Solving equations (27) and (28), I get the values of β_1 and β_2 at approximately[20] -.64 and .076, respectively, which satisfies the condition for capital-skill complementarity, $\beta_1 < \beta_2$.

The theoretical exercise serves two purposes. First, it derives the expressions for the MES using a production framework with the VES technology. Second, using a numerical example it supports the purported relationship between a decline in the LIS and capital accumulation when capital and labor are complements in a production framework that combines the VES technology and capital-skill complementarity. I provide empirical evidence in the following section.

4. Empirical Evidence

This section provides empirical evidence to three propositions. First, whether the condition $\sigma > 1 > \rho$ is empirically supported. Second, if a drop in the relative price of capital forces labor income share to decline, i.e., $2 - \sigma - \rho < 0$ (or $|1-\rho| < |1-\sigma|$ when $\sigma > 1 > \rho$ is met. Together, these two conditions contitute the necessary condition. Finally, if there is complementarity between capital and aggregate labor when the first two propositions hold. I begin with a brief description of the empirical model and then describe the data. A discussion of the main results concludes this section.

4.1. Empirical model specification

To estimate the MES, I use a translog variable output function (Kohli, 1982; Diewert and Morrison, 1982; Sharma, 2002) analogous to the theoretical model discussed in the previous section. I adopt the empirical model specification of Sharma (2002). Sharma (2002) derives the MES for a variable profit function. Based on the estimates of the MES, he concludes that an increase in the price of US imports results in substitution into capital and labor services. A translog variable output function with constant returns to scale can be written as

(29) $lnY = \beta_0 + \sum_i \beta_i ln\, p_i + \sum_i \sum_j \beta_{ij} ln\, p_i \, ln\, p_j, \quad i,j = K,S$ and U

[20] The calculation involves imaginary roots. For this reason, the solutions show only approximated values of β_1 and β_2.

where $\sum_i \beta_i = 1$, $\beta_{ij} = \beta_{ji}$, $\sum_i \beta_{ij} = 0$, and $\sum_j \beta_{ij} = 0$. Differentiating the expression for variable output in Equation 29 with respect to factor prices, and applying the Hotelling's Lemma, profit maximizing factor shares can be expressed as a function of factor prices, as follows:

(30) $$i^{Sh} = \beta_i + \sum_i \beta_{ij} ln\, p_j$$

In Equation 30, i^{Sh} represents the factor income share of factor i $(= K, S$ and $U)$ and p_j is the price of the j^{th} factor. Adding a stochastic term, it turns into a set of three-equation econometric model, where $p_K = r$, $p_S = W_S$, and $p_U = W_U$ (Equation 31). A standard assumption of the stochastic terms with zero means and homoscedastity for each equation is maintained. Each of these factor shares equations can be separately estimated, however the contemporaneous stochastic terms associated with the dependent variables could be correlated since $\sum_i i^{Sh} = 1$, and $\sum_i \varepsilon_i = 0$. These conditions also produce a singular covariance matrix in the stochastic terms. Thus, I drop one of the equations and apply seemingly unrelated regression (Zellner 1962) to jointly estimate a set of two regression equations.

(31) $$\begin{cases} K^{Sh} = \beta_K + \beta_{KK} ln\, r + \beta_{KS} lnW_S + \beta_{KU} lnW_U + \varepsilon_K \\ S^{Sh} = \beta_S + \beta_{SK} ln\, r + \beta_{SS} lnW_S + \beta_{SU} lnW_U + \varepsilon_S \\ U^{Sh} = \beta_U + \beta_{UK} ln\, r + \beta_{US} lnW_S + \beta_{UU} lnW_U + \varepsilon_U \end{cases}$$

For the translog production function, the own-price (η_{ii}) and cross-price (η_{ij}) elasticities of demand can be calculated using the expressions in Equation 32. I then compute the MES following $MES_{ij} = \eta_{ji} - \eta_{ii}$.

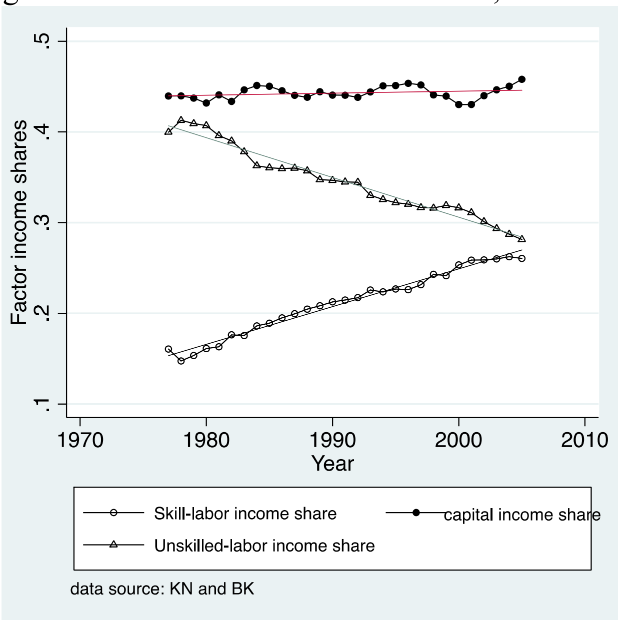
(32) $$\eta_{ij} = \begin{cases} \frac{\hat{\beta}_{ij}}{i^{Sh}} + j^{Sh}, & i \neq j \\ \frac{\hat{\beta}_{ij}}{i^{Sh}} + i^{Sh} - 1, & i = j \end{cases}$$

4.2. Data

To estimate the empirical model (Equation 31), I require data on factor income shares and factor prices. Since data on the labor income share by skills are not readily available, I create a new dataset by combining data available from Karabarbounis and Neiman (2014) (KN, hereon), Burea, Kaboski and Rogerson (2015) (BKR, hereon), the Penn World Tables (PWT, Mark 7.1) and the World Development Indicators (WDI) from the World Bank. Constrained by the availability of data, the empirical analysis is restricted to the US for the period from 1977 to 2005.

KN measure labor income share by combining national income and product accounts data across various sources, including data collected from country-specific national accounting sources on the Internet, the OECD, the UN, and physical books. Their baseline analysis focuses on the LIS within the corporate sector. The authors claim that the trend in corporate LIS is less susceptible to measurement problems (Gollin, 2002) in comparison with the aggregate LIS. I primarily use the aggregate LIS data as it closely fits the capital-skill complementarity argument developed in this paper. The aggregate LIS is simply measured as compensation of employees divided by GDP. The second set of the aggregate LIS data comes from a combined OECD-UN database. The second dataset does not include the country-specific information that KN collected from other sources. As KN documents at length, the price of capital (investment) relative to consumption goods also declined alongside a global decline in the labor income share. I use data the relative price of capital from two sources, PWT and WDI, and merge it with BKR containing data on wages by different skill level, to compute the factor prices for capital, skilled- and unskilled-labor.

Figure 2 Factor income shares in the US, 1977-2005



Note: Authors' calculation based on a combined data set using data sets from Karabarbounis and Neiman (2014) and Buera, Kaborski and Rogerson (2015).

To compute data on skilled and unskilled labor income share, I follow the definition of BKR on high-skill share of labor income in high-skill and low-skill sectors. The high-skill sectoral shares are generated by adding up the percentage of labor compensation across all age and gender for high-skill from four industries (financial intermediation, real estate and business services, education, and health and social work) available in the EUKLEMS database. Also, the sector shares are calculated as compensation-weighted averages of the industry-specific shares. I multiply the aggregate LIS data with sectoral shares of the high-skill labor for each sector and sum them up to generate the aggregate income shares for both types of skills. Figure 2 plots the LIS trends. Both capital income share and the LIS for skilled workers show an upward trend, whereas the unskilled workers predominantly drive a decline in the aggregate LIS. This is consistent with capital-skill complementarity and a declining cost of capital.

4.3. Empirical Findings

Table 1 reports the empirical results. To provide further robustness, I estimate the baseline model (Equation 31) using four different combinations of data, which is indicated in the column headings. To avoid a non-singular matrix with the stochastic terms, the coefficients of the factor prices are separately estimated from two sets of joint-regressions models: (1) with capital income share and skilled labor income share as dependent variables, and (2) with capital income share and unskilled labor income share as dependent variables. The Breusch-Pagan statistic outcomes suggest that stochastic terms are significantly correlated between the capital share and the unskilled-labor share regressions when they are jointly estimated. The own-price demand elasticities, calculated based on the estimated parameters, range between -.15 and -.65. The cross-price elasticities due to changes in the price of capital are positive, suggesting both relationships, between skilled-labor and capital, and unskilled-labor and capital, as substitutes. However, the magnitude of the effect of changes in the capital price on the stock of unskilled-labor is three-times higher than that for skilled-labor. This again broadly supports the notion of capital-skill complementarity.

Table 1 Regression outcomes on the substitutability of factor inputs

		1	2	3	4
		KN-BK-PWT	KN-BK-WDI	UN/OECD-BK-PWT	UN/OECD-BK-WDI
Estimated coefficients of log factor prices	$\hat{\beta}_{KK}$	0.061	0.043	-0.020	-0.037
	$\hat{\beta}_{KS}$	0.028	0.041	0.086	0.090*
	$\hat{\beta}_{KU}$	-0.121	-0.075	-0.001	0.022
	$\hat{\beta}_{SK}$	-0.186**	-0.164**	-0.164**	-0.143*
	$\hat{\beta}_{SS}$	0.093	0.064	0.077	0.051
	$\hat{\beta}_{SU}$	-0.062	-0.149	-0.107	-0.187
	$\hat{\beta}_{UK}$	0.125	0.121	0.184*	0.180*
	$\hat{\beta}_{US}$	-0.121	-0.105	-0.164**	-0.142*
	$\hat{\beta}_{UU}$	0.183	0.224	0.108	0.166
Correlation $(\varepsilon_K, \varepsilon_S)$		-0.139	-0.171	-0.136	-0.159
Breusch-Pagan statistic		0.562	0.844	0.537	0.731
Correlation $(\varepsilon_K, \varepsilon_U)$		-0.672	-0.652	-0.603	-0.580
Breusch-Pagan statistic		13.104***	12.333***	10.559***	9.739***

Source: Author

Note: Empirical outcomes are performed using a combined dataset comprising of Karabarbounis and Neiman (2014) and Buera, Kaborski and Rogerson (2015). Each regression model has 28 observations. The standard errors are bootstrapped with 200 reps. I use the sureg command in STATA, and apply the small-sample properties. It shifts the test statistics from chi-squared and z statistics to F statistics and t statistics, respectively. Breusch-Pagan statistic shows the degree of correlation between the stochastic terms. The four models reflect different combinations of the data sources as indicated in the column headings. *** , ** and * imply statistical significance at 1% level, 5% level, and at 10% level, respectively.

In panel A (Table 2), I report the computed values of the MES. The MES is calculated using the cross-price and the own-price demand elasticities. Both MES_{UK} and MES_{KU} are candidate measures for σ, whereas ρ can be approximated by either MES_{SK} or MES_{KS}. The empirical support for the asymmetric property of the MES is strong, as $MES_{KU} \neq MES_{UK}$ and $MES_{KS} \neq MES_{SK}$. The estimated MES follow an order of hierarchy $MES_{KU} > MES_{SK} > MES_{UK} > MES_{KS}$, which is consistent across all the models. The estimated value of MES_{KU} suggests that capital and unskilled-labor are Morishima substitutes, i.e., $\sigma > 1$, whereas the rest of the MES measures suggest Morishima complementarity between factor inputs. This leaves us with two possible combinations (MES_{SK}, MES_{KU}) and (MES_{KS}, MES_{KU}) from each model, with a total of eight out of 16 possible cases that satisfy the first part of the necessary condition ($\sigma > 1 > \rho$). Panel B of Table 2 shows the comparative statics outcomes. The condition $2 - \sigma - \rho < 0$ is met in four (MES_{SK}, MES_{KU}; MES_{SK}, MES_{KU}; MES_{SK}, MES_{KU} and MES_{KS}, MES_{KU}) out of eight cases suggesting a drop in the labor income share with a decline in the relative price of capital. Taken together, four out of the 16 cases show a drop in the relative price of capital forces the aggregate labor income share to decline when the necessary condition is met.

Table 2 The MES and the comparative statics outcomes

	1	2	3	4
	KN-BK-PWT	KN-BK-WDI	UN/OECD-BK-PWT	UN/OECD-BK-WDI
A. Morishima elasticity of substitutions				
MES_{SK}	0.635	0.799	0.840	0.971
MES_{UK}	0.200	0.193	0.692	0.580
MES_{KS}	-0.004	0.137	0.284	0.421
MES_{KU}	1.214	1.245	1.568	1.595
B. Comparative statics outcomes for the sample that satisfies the condition: $\sigma > 1 > \rho$				
$2 - MES_{SK} - MES_{KU}$	> 0	< 0	< 0	< 0
$2 - MES_{KS} - MES_{KU}$	> 0	> 0	> 0	< 0

Source: Author.

Note: Panel A shows the Morishima elasticity of substitutions using estimated parameters for four models. Panel B shows the comparative statics outcomes for the sample with combination of the MES that meets the necessary condition: $\sigma > 1 > \rho$ and $2 - \sigma - \rho < 0$ (or $|1 - \rho| < |1 - \sigma|$). If $2 - \sigma - \rho < 0$, then it implies a drop in the labor income share with a decline in the relative price of capital.

As a final step, I replicate the graphical illustration in Figure 1. Figure 2 shows the relationship between σ_{Agg}, σ, and ρ using the MES estimates for four cases that satisfy $\sigma > 1 > \rho$ and $2 - MES_{SK} - MES_{KU} < 0$ (the necessary condition). Assuming that σ_{Agg} can be approximated by a weighted linear combination of σ and ρ (i.e., $\sigma_{Agg} = y\sigma + x\rho$, where y and x are weights), and imposing the condition for complementarity between capital and aggregate labor, $\sigma_{Agg} = y\sigma + x\rho < 1$, I get the following inequalities form the four cases:

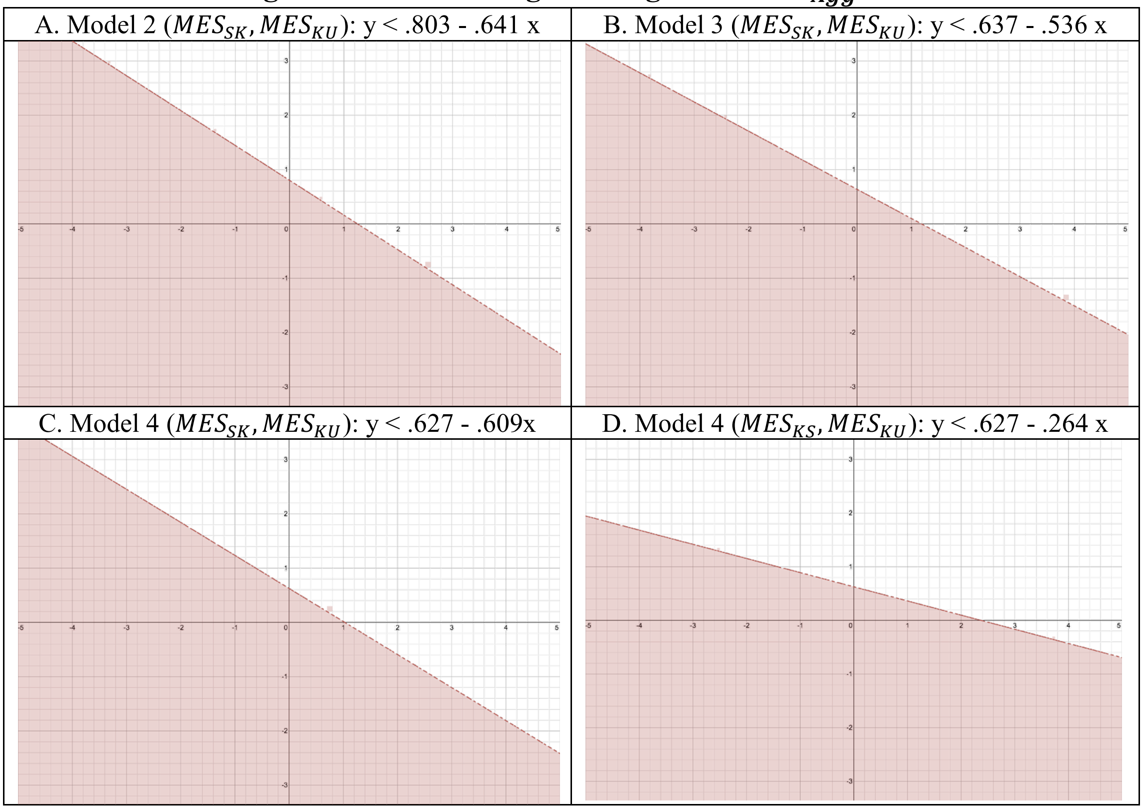
(A) y < .803 - .641 x, derived from $1.245\,y + .799\,x < 1$ (Model 2; MES_{SK}, MES_{KU}),

(B) y < .637 - .536 x, derived from $1.568\,y + .840\,x < 1$ (Model 3; MES_{SK}, MES_{KU}),

(C) y < .627 - .609 x, derived from $1.595\,y + .971\,x < 1$ (Model 4; MES_{SK}, MES_{KU}), and

(D) y < .627 - .264 x, derived from $1.595\,y + .421\,x < 1$ (Model 4; MES_{KS}, MES_{KU}).

Figure 2 Feasible range of weights when σ_{Agg} < 1



A. Model 2 (MES_{SK}, MES_{KU}): y < .803 - .641 x

B. Model 3 (MES_{SK}, MES_{KU}): y < .637 - .536 x

C. Model 4 (MES_{SK}, MES_{KU}): y < .627 - .609x

D. Model 4 (MES_{KS}, MES_{KU}): y < .627 - .264 x

Source: Author.

Note: This graph shows a numerical example. It shows the feasible range of value for an equation showing inequality. The x-axis measures the weight of ρ and the y-axis measures the weight of σ.

The shaded regions in each of the graphs in Figure 2 indicate the feasible set of values for different combinations of weights satisfying σ_{Agg} < 1. The empirical exercise in this section provides support to the theoretical propositions developed in the earlier sections. The required conditions for a decline in the aggregate LIS with a decline in the relative price of capital is met in four out of 16 cases. The validity of $\sigma_{Agg} < 1$ remains an empirical question, which is subject to the aggregation principle that combines σ_{Agg} from elasticity parameters between capital and labor across skills. There is no consensus on the economic environment the elasticity considered in (Chirinko, 2008; Leon-Ledesma, McAdam, and Willman, 2010; Miyagiwa and Papageorgiou, 2007). I use linear weights following Jones (1965) and Oberfield and Raval (2014). The possibilities of non-linear weights, or a more complicated functional relationship between σ_{Agg}, σ, and ρ remains an open question and requires robust empirical evidence to support it.

5. Conclusion

Economists have always been concerned with the functional distribution of income. As emphasized by both Atkinson (2009) and Glyn (2009), the study of factor income shares play an important role in understanding the relationship between national income and personal income, the relationship between wage inequality and wealth inequality, and how they link to overall income inequality and concerns for fairness in different sources of income. In recent years, a large body of research has documented a global decline in the LIS. The burgeoning literature offers several explanations, including the role of capital accumulation (Karabarbounis and Neiman, 2014, Piketty, 2014) behind a global decline in the LIS that requires capital and labor to be substitutes at the aggregate level. This appears paradoxical in a world that is predominantly characterized by complementarity between capital and labor.

This paper reconciles these opposite factions. I show that the composition of skills in the labor force, and identification of the elasticity parameters between capital and different skills can influence the relationship between capital accumulation and the LIS. Using a framework with capital-skill complementarity and variable substitution elasticities, I apply the Morishima elasticity of substitution to identify the elasticity parameters at different skill levels, and based on its properties, I derive the necessary condition that allows capital accumulation to coexist with a declining labor income share when capital and labor are complements. By applying the U.S.annual data from 1977 to 2005, I find support for the necessary condition in four out of 16 cases. The validity of $\sigma_{Agg} < 1$ remains an empirical question, which is subject to the aggregation principle that combines σ_{Agg} from elasticity parameters between capital and labor across skills. This requires further theoretical development on the aggregation principle with robust empirical support. I leave this task for future studies.

Finally, the necessary condition refines our understanding of the drivers of a declining LIS. In the presence of the skill composition of the labor market, a faction of the labor market can alone produce a decline in the aggregate LIS. The relevance of capital-skill complementarity for the labor share of income can also be drawn using a two-stage production structure (Goldin and Katz, 1996). In the first stage, skilled workers adopt new technologies and efficiently use capital, thus showing high capital-skill complementarity. In the second stage, unskilled workers continue the mechanical process of machine maintenance indicating a relatively low level of capital-skill complementarity. Such practices are common across both developing and developed countries and provide an important link between capital-skill substitutability and factor income shares.

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