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ABSTRACT

Electoral Incentives and the Allocation of Public Funds*

It is widely believed that politicians allocate public resources in ways to maximize political gains. But what is less clear is whether this comes at a cost to welfare; and if so, whether alternative electoral rules can help reduce these costs. In this paper, we address both of these questions by modeling and estimating politicians' decisions to allocate public funds. We use data from Brazil's federal legislature, which grants each federal legislator a budget to fund public projects in his state. We find that 26 percent of the public funds are distorted relative to a social planner's allocation. We then use the model to simulate several potential policies reforms to the electoral system, including adopting approval voting and implementing term limits. We find that an approval voting system reduces the distortions by 7.5 percent. Term limits also reduce distortions, but come at the cost of more corruption, which makes it a welfare-reducing policy.

JEL Classification: H40

Keywords: distributive politics, public goods, corruption, electoral rules, term limits

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1 Introduction

A central function of government is the provision of public goods and services. For instance, in 2014 governments throughout the world spent on average more than 34 percent of their countries' GDP on public goods and services.[1] When allocated and spent efficiently, these expenditures can be important drivers of economic development and key determinants of quality of life. But, as an extensive theoretical literature on distributive politics has argued, politicians care about getting elected and these electoral incentives can often distort how public expenditures, and hence public goods, are allocated. While the existence of these distortions are rarely disputed, there is no empirical evidence on their magnitude or whether they can be reduced, which is what we ultimately care about.

The main contribution of this paper is to fill this gap in the literature. Specifically, we answer two questions: What is the size of the distortions generated by electoral incentives? And, can electoral policies reduce these distortions? We answer them in four steps.

We begin by providing descriptive evidence on the main factors that underlie a politician's decision to allocate public funds using data from Brazil's federal legislature. Consistent with findings in other settings, we observe three main patterns in the data. First, the number of votes a politician receives in a given region is strongly correlated with the amount of public funds he had allocated there during the previous term. Other studies have documented this relationship in other contexts as evidence that voters reward politicians who provide them with transfers (e.g. Levitt and Snyder (1997), Manacorda, Miguel, and Vigorito (2011)). Second, studies have also shown that politicians with electoral incentives target regions with more votes (Cary M. Atlas (1995), Rodden (2002), Knight (2008)). We document a similar pattern, but find that politicians also target regions with few voters, suggesting that non-electoral motives may also affect a politician's allocation decisions. Lastly, we provide evidence that legislators who run for reelection allocate funds differently from those who do not, with the former targeting their funds to regions that have relatively more voters and the latter favoring poorer and less developed areas. This last pattern is consistent with a large literature documenting the importance of reelection incentives on spending and policy choices (e.g. Besley and Case (1995), List and Sturm (2006), Lim (2013)).

We then develop a model sufficiently rich to not only capture these patterns in the data, but to also allow for the evaluation of alternative electoral policies. In so doing, our model highlights the following five key aspects of the allocation decisions of politicians.

[1] This statistic comes from the 2014 Index of Economic Freedom.

First, some politicians may value citizens' welfare more than others. In our model, politicians care about getting elected as well as aggregate welfare within the district, and they do so to varying degrees. This heterogeneity in the degree of altruism of politicians can help explain why they target poorer places that offer few votes.

Second, we explicitly model the decision to run for reelection. Politicians who decide to run are likely to have stronger reelection incentives than those who do not, which can affect how they allocate public funds. By endogenizing this decision, our model can capture the observation that politicians who run for reelection target regions with more voters, whereas those who do not target relatively poorer and less populated areas.

Third, we assume that citizens value public funds and vote for the politicians who they believe will provide them with more resources in the future. With this feature of the model, we can explain why in the data residents are more likely to vote for the incumbents who had provided them with the most resources.

Fourth, it is common in politics for the actions of one politician to affect the decisions of other politicians. While this tends to be the case in general, it is particularly relevant in settings where incumbents belong to and compete in the same electoral district. Because citizens can only vote for one candidate, an incumbent may not want to allocate public funds to places that he believes other incumbents are targeting. Our model explicitly accounts for the existence of this type of strategic interactions.

Finally, as exposed in recent media accounts in Brazil, the funds politicians allocate may not translate fully into aggregate welfare gains. This could be because politicians steal some of the funds. Or, the funds are designated for public goods that may not be entirely non-rival, in the sense that one person's use of the good may reduce the welfare derived by other residents in the region. Our model allows for both of these possibilities when mapping public funds into aggregate welfare.

Given these features, we estimate our model by Simulated Method of Moments (SMM) using data on the universe of public funds allocated by each federal legislator in Brazil from 1996-2013. Brazil's federal legislature provides an ideal setting to estimate a model of how politicians allocate public funds across regions. Each year the Brazilian Constitution grants each federal legislator a budget of BRL$1.5 million (US$750,000) to fund public projects in the state where the legislator is elected.[2] This constitutional provision allows us to investigate what factors

[2]The study of Brazil's budgetary amendments has a long tradition in the comparative politics literature. Since Ames (1995) classic study, there have been a number of empirical studies investigating both the allocation of these budget appropriations, as well as their electoral returns (e.g. Samuels (2003); Pereira and Renno (2003)). The

influence the politicians' allocation decisions without worrying about the endogeneity of who has access to these funds, which is an important concern in other contexts, such as in the U.S. Congress. Moreover, these budgetary appropriations, which are commonly earmarked for large-scale development projects, have important welfare considerations.

Based on our estimates, we find that allocation patterns can be explained by two types of politicians: an egoistic type, who cares almost exclusively about electoral incentives; and an altruistic type, who also values the welfare implications of their decisions. The candidate pool comprises 60 percent of egoistic types.

Our estimates also suggest that legislators who decide not to run for reelection engage in 15 percent more corruption than deputies who have reelection concerns. This finding is consistent with a large literature on term limits, and in particular with Ferraz and Finan (2011) who find that mayors in Brazil with reelection incentives misappropriate 27 percent fewer resources than mayors without reelection incentives. In addition to the fraction of resources diverted, we also estimate the degree of rivalry of the public goods funded by the transfers. For every dollar a municipality receives, slightly less than one cent of it represents a pure public good. Given the average population size of municipalities in the data, our estimate implies that legislators are funding goods that are approximately halfway between a pure public good and a private one.

Consistent with the descriptive evidence, we find that voters are quite responsive to public expenditures, and this responsiveness varies across regions. For instance, if an incumbent transfers all of his resources from the region with the smallest political gains to the one with the largest political gains his probability of getting reelected increases by 52 percentage points. Incumbency advantage also matters in our model. Incumbent politicians have a probability of being elected that is 36 percentage points higher than politicians that are not in power, all else equal.

Given our model estimates, we then proceed to answer our first question: What is the size of the distortions generated by electoral incentives? We find that 26 percent of public funds are distorted relative to a social planner's allocation. Egoistic politicians who target regions with more votes at the cost of poorer and more productive places are responsible for 74 percent of these distortions. Had the pool of politicians only consisted of altruistic incumbents, the distortions would reduce to 16 percent. These findings highlight not only the importance of

most recent example is Firpo, Ponczek, and Sanfelice (2012). The authors show that politicians tend to reward municipalities that supported them in the previous election, and that among the legislators who seek reelection, voters reciprocate by voting for the candidates who have brought more resources to their localities. Our study complements and extends this body of work. Many of these results, which we replicate in our reduced-form analysis, help to motivate our model and its underlying assumptions.

unobserved heterogeneity in the allocation of public funds, but also the need to account for it when trying to understand allocation decisions.

Lastly, we use counterfactual simulations to answer the second question: Can electoral policies reduce these distortions? To address this question, we simulate three possible policy reforms. First, we consider the effects of adopting approval voting rules, which allow residents to vote for more than one candidate. We find that approval voting has the desired effect of reducing distortions, but the size of the decline is limited. For example, if Brazil's government adopted a system that allowed residents to vote for 8 candidates, the distortions in the allocation of public funds would decrease by 7.5 percent.

Although this represents an important improvement in aggregate welfare, there are two countervailing forces that limit the efficacy of approval voting. On the one hand, under approval voting the incentive to target regions with the most voters is not as strong. Thus, as politicians shift their resources to other more productive municipalities, the distortions decrease. On the other hand, approval voting lowers incumbency advantage, therefore making elections more competitive. As we allow residents to select an increasing number of candidates, the probability that a challenger is elected rises substantially. For instance, going from the current system in which residents can vote for only one candidate to an 8-person voting system increases the probability that a challenger wins by 29 percent. But as elections become more competitive, the incentive to target regions with more voters become stronger and, if these places are less productive, distortions increase.

The second policy experiment considers an exogenous increase in the number of challengers. Political contestation is widely considered the hallmark of a healthy political system. And while we do not necessarily dispute this claim, our findings suggest that policies aimed at increasing candidate entry will only increase the distortions in the allocation of public funds. Given our previous finding, this is hardly a surprise. As political competition increases, political considerations become more salient.

The final policy we consider is the introduction of term limits. Brazil currently allows legislators to be elected an indefinite number of times, but several countries have argued for, and in some cases implemented, term limits as a way to improve representation and reduce politician pandering. In our model the advantage of this policy is that electoral incentives would no longer influence the way public funds are allocated. The disadvantage is that legislators who do not run divert significantly more of the funds than those who still face reelection incentives. The ability to determine which of these two effects dominate is an important contribution of our model. When we compare the results of our model to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions do decrease by almost 40 percent (from 26 to 16 percent). However, because of the increase in corruption, welfare as a whole actually goes down by 2 percent, suggesting that this policy is not welfare improving.

Overall, our findings contribute to two broad strands of the literature. First, our study relates to an extensive literature in both economics and political science that investigates the causes and consequences of distributive politics. As Golden and Min (2013) report in an excellent and comprehensive review of this vast literature, there have been numerous studies documenting the importance of electoral incentives in the allocation of public goods and services, often at the expense of aggregate welfare. And yet despite this attention, our study is, to our knowledge, the first to quantify the welfare consequences of electoral incentives, and to show how different electoral rules can help reduce potential deviations from a social planner's allocation.

Our focus on electoral rules naturally relates to a more specific literature within distributive politics that examines the importance of electoral rules on public goods provision.[3] The focus of this literature has been mostly theoretical. For example, in his seminal study, Myerson (1993) shows how different electoral systems can cause candidates to target minorities of voters even when voters are identical. Lizzeri and Persico (2001) extend the Myerson model to also include public goods. They show that, compared to a system of proportional representation, a winner-take-all system will result in the under-provision of public goods even when they are highly desirable. Persson and Tabellini (2005) show that majoritarian systems generate less public goods provision than systems with proportional representation.

We complement this literature in various ways. Most of the models in this literature have focused exclusively on the electoral motives of politicians. In our model, we allow politicians to also care heterogeneously about their district's welfare. This feature of the model is important for explaining why politicians target regions with few votes. Moreover, these models also tend to only consider the allocation decisions of politicians who have decided to run. Here, we model the decision to allocate public funds jointly with the decision to run. This extension allows us to capture the fact that the allocation decisions of politicians with reelection incentives differ from those who forgo reelection.

The empirical literature has relied mostly on reduced-form evidence at the country or state level. For example Milesi-Ferretti, Perotti, and Rostagno (2002) and Persson and Tabellini

[3] See Persson and Tabellini (2000) for a general review of the literature. The implications of different vote-counting schemes for candidate behavior is also reviewed in a 1995 JEP issue, see for example Levin and Nalebuff (1995).

(2005) examine the effects of different political institutions, including electoral systems, on outcomes such as economic policy, government spending, and budget deficits. They show that the type of electoral system affects both the size and type of federal spending. Besley and Case (2003) examine how differences in political institutions across the U.S. states, such as term limits, can affect various policy outcomes. However, one limitation of relying on cross-country or cross-state variation is the inability to control for unobserved heterogeneity, which we show is an important driver of allocation decisions. One notable exception is the paper by Beath et al. (2014), who use experimental variation in Afghanistan to show how electoral rules can affect political selection.

In contrast to the reduced-form literature, this paper uses a structural approach to understand the effects of electoral institutions on the allocation of public funds. In this regard, our paper is methodologically similar to Stromberg (2008).[4] In that paper, he structurally estimates how U.S. presidential candidates allocate their campaign resources across states. He finds that had the U.S. adopted a popular vote system, as opposed to the Electoral College system, the allocation of campaign spending would be much less concentrated.

The paper proceeds as follows. The next section describes Brazil's Federal legislature and provides some institutional background. Section 3 describes the data we use to estimate the model and presents the reduced-form evidence that motivates the study. The model is then presented in Section 4, followed by a discussion of our estimation approach in Section 5. In Section 6, we discuss the identification of our model's parameters. Section 7 presents both our estimation results, as well as our policy simulations. Section 8 concludes the paper.

2 Background: Brazil's Federal Legislature

Brazil's federal legislature provides an ideal laboratory to study the allocation of public funds. In this section, we provide some basic background on Brazil's political system, and highlight some of the institutional features that facilitate our analysis.

Brazil's federal legislature, also referred to as Chamber of Deputies (we will use the terms "Deputy" and "Legislator" interchangeably), consists of 513 seats allocated across 26 states according to population size. Each state represents a multi-member voting district, where candidates can receive votes from residents of any of its municipalities. As opposed to a single-

[4]More broadly, our paper is related methodologically to a nascent literature in political economy that structurally estimates the behavior of politicians. Examples include among others: Diermeier, Keane, and Merlo (2005), Da Silveira (2012), Aruoba, Drazen, and Vlaicu (2015), and Kang (2015).

member district, incumbents not only face competition from new potential challengers, but also from the other incumbents. Given this institutional feature, it is important for us to allow for the possibility of strategic interactions.

Nationwide elections for the legislature are held on a four-year cycle and incumbents can be elected an unlimited number of times. Despite the lack of any term limit, legislative careerism is relatively absent in Brazil. For instance, on average only 75 percent of deputies run for reelection. Candidates are elected based on the D'hondt open-list proportional representation method, which determines how the available seats are allocated. Specifically, seats are allocated to parties based on the total number of votes their candidates receive. Then given the number of seats a party has earned, candidates within the party are elected based on their vote total. Voting in Brazil is mandatory, and although the electorate can vote for the political party, this option is rarely exercised as elections tend to be highly individualized. It is also not unusual for several elected officials to change parties during their electoral terms. In the 49^{th} parliamentary session, for example, 55 percent of deputies switched parties during their term. With such a low degree of party loyalty, both from the standpoint of the politician as well as the electorate, objectives of the party are unlikely to play an important role in the allocation of public funds. As a result, our empirical analysis does not focus on party politics but instead on the individual behavior of federal deputies.

Another advantage of our setting is that the primary responsibility of federal deputies is to allocate public funds and, unlike in other settings, access to these funds is exogenous. Brazil's legislature is comparatively weak and seldom legislates on issues of national concern (Ames 1995). As a Federal Deputy from Ceará stated in the Brazilian newspaper *Folha de São Paulo* on February 21, 1988: "A political career in Brazil is closely connected to success in bringing home material benefits . . . Especially in the poorest regions, communities judge their deputies on what they bring home"; or Federal Deputy Joaquim Haickel expressed: "The primary function of a deputy is getting resources; legislating comes second." (Mainwaring 2002). To facilitate these objectives, federal deputies have had the right to submit pork-barrel amendments to the budget since 1988. Before 1996, members of Congress were not limited in the number of budgetary proposal and between 1992 and 1995 averaged close to 137 per year per member. But starting from 1996, the year we begin our analysis, Brazil's constitution has allowed each member of Congress discretion on how to allocate a fixed budget of BRL$1.5 million per year.

3 Summary Statistics and Descriptive Evidence

In this section, we present some summary statistics and descriptive evidence that will help develop and estimate our structural model.

3.1 Data sources

To provide the descriptive evidence, we assemble an extensive database of political and municipal characteristics for the period 1996 to 2013 and for all states in Brazil. The data used for this study combines administrative data from three sources.

To investigate budgetary allocations, we collected budgetary amendment data from the Federal Chamber of Deputies. Each budgetary amendment issued from 1996 to 2013 records the author's name, and the amount, type, and location of the public investment. These data are then merged with election data from the Tribunal Superior Eleitoral (TSE). These data contain vote total for each candidate by municipality, along with various individual characteristics, including gender, education, occupation, and party affiliation. We use this information to construct our primary measure of political support – municipal vote share – as well as various other measures of electoral performance and competition, such as the candidate's rank and vote total. Our final data source is from the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística (IBGE)). The 2000 and 2010 population censuses provide several socioeconomic and demographic characteristics such as poverty rates, income inequality, population density, share of the population that is urban, and share of the population that is literate.

3.2 Summary Statistics

Table A.1 in the on-line appendix presents some basic information on the budgetary amendments issued by federal deputies.[5] For convenience, the results are based on the 50^{th} legislature, which issued budgetary amendments from 1996-1999 and faced the possibility of reelection in 1998. Although for the reduced-form analysis we restrict the sample to a single term, the patterns we present here are similar to those found in Firpo, Ponczek, and Sanfelice (2012) who also consider additional terms.

[5]The sample also contains the budget allocation of 135 non-elected deputies who were originally voted as alternates, but later replaced elected deputies who were unable to fulfill their responsibilities. Inclusion of these deputies does not affect our estimation results.

On average, each year a deputy proposes 15 budgetary amendments that are approved, with an approximate value of $1.3 million reais.[6] Both the number and the amount decline slightly over the electoral cycle, although differences across years are not large. The geographic variation in the distribution of these public works is considerable (see Figure A.1 in the on-line appendix). More than 10 percent of municipalities did not receive a single public work during the 1996-1999 term, with the median municipality only receiving BRL$280,000 in budgetary amendments. In contrast, the top one percent of municipalities receive BRL$10,000,000.

Table A.2 in the on-line appendix displays summary statistics by state corresponding to the 1998 election. On average, 68 percent of deputies from the 1996-1999 term, who had issued a budgetary amendment, ran for reelection in 1998. Of those, 65 percent were reelected. These averages however, mask considerable variation across states. In some states reelection rates are above 80 percent, while in others they are even below 50 percent. Elected federal deputies average anywhere between 5 to 21 times the number of votes that non-elected deputies receive. Federal deputies are predominately male (90 percent), and only 60 percent of them have a college degree.

3.3 Descriptive Evidence

Several features of the data guide our modeling choices. Our most basic assumption is that voters care about the public funds they received from politicians, and reward the ones who supply them. This assumption is based on the strong association observed in Panel (a) of Figure 1 between the number of votes a deputy receives in a particular municipality and the amount of funds he provided to the municipality during the term. This relationship appears linear, which is consistent with politicians targeting larger projects to more populated municipalities. Consistent with this explanation, we do see some concavity when we instead plot on the y-axis the share of votes received in the municipality (see Panle (b) of Figure 1). Larger projects may do little to increase one's vote share within a municipality, but can increase one's vote total if targeted to a more populated municipality. In Panel (c) of Figure 1, we indeed see that larger projects are targeted to more populated places.

In Table A.3 in the on-line appendix, we explore the robustness of these correlations to unobserved deputy and municipal characteristics. Because each state is a multi-member district,

[6] As mentioned previously, federal deputies were allowed up to 20 outlays totaling up to 1.5 million per year. The limits are in general reached. However in our analysis, we only consider outlays targeted to a municipality and exclude the ones that are designed to benefit either the state or the country as a whole.

we can estimate these correlations controlling for the fixed characteristics of the deputy (e.g. valence, education levels) as well as the fixed characteristics of the municipality (e.g. poverty levels, party affiliation). To do so, has little impact on the estimated relationship between electoral performance and public spending in the municipality.

Another way to visualize the data, which will motivate how we model the voting rule, is to first measure how deputies rank within a municipality in terms of the amount of funds they had allocated there. We then examine how this ranking relates to their vote share within the municipality. In Figure 2, we describe this relationship by ploting the coefficients from a regression of the share of votes within a municipality on a set of indicators that measure a deputy's ranking in the municipality. The regression controls for municipal fixed-effects. Deputies ranked first receive vote shares that are 27 percentage points higher than those ranked above 22 (the excluded category). The electoral returns to coming in second fall considerably, as second-place finishers only have a 7 percentage point advantage. The vote shares of deputies ranked third and above are statistically indistinguishable from the excluded category. These results suggest that when casting their ballots, voters prefer the candidate who provides them with the most public funds.

Thus far, the evidence suggests that voters do care and reward politicians who supply them with more public funds, and that politicians in turn target more resources to places with more voters. But this does not necessarily imply that electoral returns are the only motives when deciding where to allocate these public works. Politicians may target more populated places because their funds are needed more in those regions. To provide more persuasive evidence on the relevance of electoral incentives, we compare the allocations of public funds by incumbents who chose to participate in the elections with the allocations of those who did not. In Panel (a) of Figure 3, we plot the distribution of public funds by poverty levels for the two groups of deputies. Panel (b) plots a similar figure using the municipality's human development index, which is a commonly used composite index of a country's (or in our case a municipality's) life expectancy, educational attainment, and income level. As both graphs indicate, incumbents who do not run for reelection are, on average, much more likely to target poorer and less developed municipalities, relative to those with electoral concerns.

While the differences reported in Figure 3 are suggestive of electoral incentives, together with Panel (c) of Figure 1 they also document that deputies allocate a significant fraction of their funds to poorer regions with fewer votes. Thus, electoral incentives may not be the only motive behind the deputies' decisions. Politicians appears to transfer part of their budget based on the needs of a municipality.

4 Model

In this section, we model the politicians' allocation decisions with three goals in mind. First, the model should be sufficiently rich so as to perform interesting policy evaluations. Second, it should be able to generate the empirical patterns documented in the previous section, namely (i) deputies allocate resources to municipalities with both high and low political returns; (ii) deputies who choose not to run for reelection allocate their resources differently from those who do run; and (iii) residents are more likely to vote for the deputy who supplied them with the most public funds during the term. Third, the model should incorporate strategic interaction among politicians to take into account that the actions of an incumbent are likely to affect the allocation decisions of other incumbents.

Given these objectives, consider an economy in which, in term t, J deputies make two choices. First, they decide how to allocate a fixed amount of resources $\bar{Q}$ among M municipalities. Second, they choose whether to run for reelection at the end of the term. Each municipality is populated by N^m individuals who choose for which politicians to vote at the end of the term. Let $q^{j,m}$ denote the amount of resources that deputy j allocates to municipality m; $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ denote the collection of allocations chosen by deputy j; $q = \left\{q^1, \ldots, q^J\right\}$ denote the allocations of all deputies; and $q^{-j} = \left\{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\right\}$ denote the allocations of all deputies except j. Finally, let $Q^m = \sum_{j=1}^{J} q^{j,m}$ be the total amount of funds received by municipality m.

Before we can characterize the optimal decisions of deputies, we have to specify the preferences of deputies and voters, the electoral rule, and how each deputy interacts with his political rivals. This will be the subject of the next three subsections.

4.1 Preferences

Voters' Preferences. Voters in municipality m have preferences over the total amount of resources allocated to the municipality by the J deputies, Q^m. Their preferences also depend on a set of municipality characteristics X^m and a variable K^m which accounts for every other factor that affects the residents' welfare. We characterize an individual's preferences with the following welfare function:

$$w_m\left(Q^m, X^m, K^m\right),$$

where w_m is increasing in Q^m. The total welfare of municipality m can then be calculated by multiplying w_m by the number of people living in the municipality:

$$W_m(Q^m, X^m, K^m) = N^m w_m(Q^m, X^m, K^m).$$

Note that the welfare function can vary across municipalities. With this heterogeneity, we can account for any potential productivity differences across regions in the use of public funds. Without this feature, we would not be able to distinguish between an environment in which politicians target regions with more voters for political reasons versus one in which politicians target more productive places that happen to have more voters.

The amount of public funds a municipality receives may not necessarily translate fully into welfare gains, for at least two reasons. First, deputies may divert a fraction of the funds, and this amount may depend on whether or not they run for reelection. Second, the public good produced with the allocated funds may not be fully non-rival, in that the use of the good by one individual may reduce the welfare derived by other residents of the region. Our welfare function can account for both of these possibilities by redefining Q^m as the actual value of the public good that reaches municipality m after taking into account the diversion of resources and rivalry concerns.

Formally, we allow for diversion by including in the model the parameters $\phi^{'}_{R}$ and $\phi^{'}_{NR}$, where $1 - \phi^{'}_{R}$ and $1 - \phi^{'}_{NR}$ measure the fraction of resources stolen by deputies who run and do not run for reelection, respectively. These parameters allow for the possibility that deputies who run for reelection may have different, and generally weaker, incentives to divert funds. To obtain the actual amount received by municipality m, we multiply the nominal amount of funds transferred by deputy j to municipality m by $\phi^{'}_{R}$, if he chooses to run for reelection, and by $\phi^{'}_{NR}$, if he chooses not to run.

We add rivalry in the model by introducing the parameter $\phi^{'}$, which measures the severity of the reduction in welfare due to the use of the public good by other residents. The parameter $\phi^{'}$ takes a value equal to 1 if deputies fund the production of a good that is non-rival and a value smaller than 1 if there is some degree of rivalry. To obtain the amount of pure public good enjoyed by municipality m, Q^m, we then multiply the actual amount of funds received by municipality m by the parameter $\phi^{'}$. The variable Q^m is therefore defined as follows:

$$Q^m = \phi^{'} \left[\sum_{j=1}^{J_R} \phi^{'}_{R} q^{j,m} + \sum_{j=1}^{J_{NR}} \phi^{'}_{NR} q^{j,m} \right], \tag{1}$$

where J_{R} is the number of deputies who choose to run for reelection and J_{NR} is the corresponding number for deputies who do not participate in the election.

At most two of the three parameters in Equation (1) can be identified. To see this, we can rewrite Q^m as follows:

$$Q^m = \phi^{'} \phi^{'}_{R} \left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \frac{\phi^{'}_{NR}}{\phi^{'}_{R}} q^{j,m} \right] = \phi \left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \phi_{NR} q^{j,m} \right]. \tag{2}$$

From the last definition of Q^m, we can see that only ϕ and ϕ_{NR} can be identified, which measure the degree of rivalry relative to the amount actually transferred by incumbents who run and the fraction of funds not diverted by incumbents who do not run relative to those who do.

Deputies' Preferences. Deputies have different preferences depending on their decision to run for reelection. If they choose to run, their utility function comprises four parts. First, these deputies derive utility from the opportunity of being in power. We will refer to this component as the egoistic motive. Second, they derive utility from the welfare of the people living in their district. We will refer to this part of the deputies' preferences as the altruistic motive. The last two components of the utility function are a utility cost of running for reelection and a preference shock.

Specifically, let v_p^j be deputy j's utility from being in power in the next term, v_{np}^j the utility if he is not in power in the next term, $\bar{\bar{C}}_R$ the cost of running for reelection, $\bar{\epsilon}^j$ the preference shock, and α^j the weight that deputy j assigns to the altruistic motive. Lastly, denote by p^j the probability that deputy j wins the election at the end of the term, which will be derived in the next subsection. Then, for a particular allocation of resources by all deputies $q = \{q^1, \ldots, q^J\}$, incumbent j's utility can be written in the following form:

$$\bar{U}_R^j(q) = p^j(q)\, v_p^j + \left(1 - p^j(q)\right) v_{np}^j + \alpha^j \sum_{m=1}^{M} W_m\left(Q^m, X^m, K^m\right) - \bar{\bar{C}}_R + \bar{\epsilon}^j\left(q^j\right).$$

The first part of the utility function, $p^j(q)\, v_p^j + (1 - p^j(q))\, v_{np}^j$, measures the expected utility of running for reelection and represents the egoistic motive of politician j. Provided that $v_p^j \geq v_{np}^j$, which is the only case in which the deputy will choose to compete in the next election, with this part we can account for the fact that politicians tend to allocate more resources to municipalities with higher electoral returns. The second part of the utility, $\alpha^j \sum_m W_m$, describes the altruistic motive of a politician and it allows us to explain why politicians might transfer part of their funds to municipalities with limited political returns.

To allow for sufficient heterogeneity, we will assume that the preference shock $\bar{\epsilon}^j$ comprises two parts. The first part depends on the allocation chosen by deputy j, which we will denote by $\bar{\varepsilon}^j(q)$. The second part, which we will denote by $\bar{\nu}_R$, does not vary by allocation, but is specific to the decision of running for reelection.

We can rewrite deputy j's utility in the following form:

$$\bar{U}_R^j(q) = p^j(q)\left(v_p^j - v_{np}^j\right) + \alpha^j \sum_{m=1}^{M} W_m - \bar{C}_R + \bar{\varepsilon}^j\left(q^j\right) + \bar{\nu}_R,$$

where $\bar{C}_R = \bar{\bar{C}}_R - v_{np}^j$. Since v_p^j and v_{np}^j do not vary with the allocation chosen by deputy j, we can divide the politician's utility by $\alpha^j + v_p^j - v_{np}^j$ and obtain

$$U_R^j(q) = (1 - \beta_j)\, p^j(q) + \beta^j \sum_{m=1}^{M} W_m - C_R + \varepsilon^j\left(q^j\right) + \nu_R.$$

From this alternative formulation of the politician's utility, we can see clearly the tradeoff deputies face when choosing how to allocate their budget across municipalities. They can allocate their resources either to increase their probability of remaining in power or to increase the welfare of the people residing in a region. How much a deputy is willing to trade off between these two considerations depends on the parameter $\beta^j = \frac{\alpha^j}{\alpha^j + v_p^j - v_{np}^j}$. We interpret β^j as the degree of altruism of deputy j, and we will refer to it as deputy j's type.

If deputy j decides not to run for reelection, his utility function is identical to the utility of an incumbent who chooses to run except that the probability of winning and the cost of running are now equal to zero. It therefore takes the following form:

$$\bar{U}_{NR}^j(q) = v_{np}^j + \alpha^j \sum_{m=1}^{M} W_m\left(Q^m, X^m, K^m\right) + \bar{\varepsilon}^j\left(q^j\right) + \bar{\nu}_{NR}.$$

If we divide the utility by the same value used for a deputy who participates in the election, $\alpha^j + v_p^j - v_{np}^j$, we have a utility function that depends on the type β^j:

$$\bar{U}_{NR}^j(q) = \bar{v}_{np}^j + \beta^j \sum_{m=1}^{M} W_m\left(Q^m, X^m, K^m\right) + \varepsilon^j\left(q^j\right) + \nu_{NR}.$$

We can see from this utility function that political incentives do not play a role, and only welfare considerations affect the allocation decisions of deputies who do not run. This feature of the model allows us to generate the observed pattern that deputies who choose not to run are more likely to allocate resources to poorer municipalities with fewer votes. The fact that these incumbents care only about welfare does not imply, however, that they will maximize aggregate municipal welfare. They can always divert part of their funds.

Note that diverted funds only enter a deputy's utility through the welfare function. This simplification is without loss of generality. Because the fraction diverted does not vary across regions, the amount stolen is the constant fraction $\left(1-\phi_R^{'}\right)\bar{Q}$ for deputies who run and $\left(1-\phi_{NR}^{'}\right)\bar{Q}$ for deputies who do not. Therefore, the diversion of resources only affects the deputies' decisions through W_m.

4.2 Residents' Voting Decisions and Deputies' Strategic Interactions

The probability that politician j wins the election depends both on the number of available seats, S, and his total number of votes. Residents vote based on two factors – the amount of future resources they expect to receive from a given politician, and a voting preference shock, $\xi^{i,j,m}$. How voters form expectations over the level of future public funds depends on whether the politicians is an incumbent or a challenger. For incumbents, voters form expectations using the following three variables: the amount of funds the deputy transferred to their municipality during the current term; the amount of funds the other incumbents transferred to the municipality; and the deputy's characteristics X^j.[7] Specifically, the amount $q^{j,m\,'}$ a voter in municipality m expects to receive from deputy j in the next term takes the following form:

$$E\left(q^{j,m\,'}\left|q^{j,m},q^{-j,m},X^j\right.\right)=f\left(q^{j,m},q^{-j,m},X^j\right).$$

Voters must also form expectations over the amount of public funds they will receive from the J_C challengers participating in the elections. Because challengers were not in power during the previous term, voters cannot condition their expectations on previous allocations. Instead, we assume that voters believe that, if a challenger is elected, he will select one of the feasible allocations with a probability that corresponds to the probability with which incumbents choose that allocation in the current term. This assumption guarantees consistency of the deputies'

[7] The data support this assumption. Conditional on legislator fixed-effects, a region that received public funds in the previous term is 45.8 (robust standard error = 0.011) percentage points more likely to receive public resources in the next term.

choices across terms.

We can now formalize the voter's decision. Let $J_E = J_R + J_C$ denote the number of politicians taking part in the elections. Then, individual i in municipality m votes for politician j, if the following condition is satisfied:

$$j = \text{argmax}_{j \in J} \left\{ f\left(q^{1,m}, q^{-1,m}, X^1\right) + \xi^{i,1,m}, \ldots, f\left(q^{J_E,m}, q^{-J_E,m}, X^{J_E}\right) + \xi^{i,J_E,m} \right\}. \tag{3}$$

Note that this voting decision is consistent with residents' preferences. Because voters' welfare increases with the amount of public funds received, conditional on the shock it is optimal for the district residents to vote for the politician who is expected to transfer the largest amount of resources. With this voting rule, we can rationalize the relationship observed in the data between the ranking of a deputy within a municipality in terms of the amount of public funds provided and his share of votes.

Using Equation (3) we can determine the total number of votes received by politician j in the district. Let d_j^i be a variable equal to 1 if resident i plans to vote for candidate j and 0 otherwise and denote by $N = \sum_{m=1}^{M} N^m$ the number of voters in the district. Lastly, let θ_j a district-level voting shock that determines the share of residents planning to vote for politician j who abstain from voting, with $0 \leq \theta_j \leq 1$. This shock can be interpreted as news about the candidate arriving just before the election. The total number of votes politician j receives in the district can then be computed as follows:

$$nv(j) = (1 - \theta_j) \sum_{i=1}^{N} d_j^i.$$

The probability that deputy j wins an election with S seats is:

$$P_{win}(j) = P\left(nv(j) > nv(k) \text{ for all } k \text{ except at most } S - 1\right).$$

In our model the utility and therefore the choices of deputy j clearly depend on the decisions of all other deputies. To deal with these strategic interactions, we make two assumptions. First, deputies make simultaneous decisions. Second, deputies do not know the type of the other legislators, where the type represents the degree of altruism β. Deputies know, however, the distribution function $\pi(\beta)$ from which the types are independently drawn.

Given these assumptions, when deputy j chooses his optimal allocation and whether to run, he does not know and cannot calculate the optimal decisions of the other incumbents. Deputy j

can only compute, given the opponents' characteristics, the probability that his political rivals will choose whether to run and one of the possible allocations. Let d_R^j be deputy j's decision to compete in the next election. We will denote by $\sigma\left(q^h, d_R^h \left| X^h\right.\right)$ the probability that deputy j assigns to incumbent h choosing whether to run and allocation q^h given characteristics X^h. Under the assumption that types are drawn independently from the same distribution, the probability that j's rivals choose the sequence of choices $q^{-j} = \left\{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\right\}$ and the sequence of decisions to participate $d_R^{-j} = \left\{d_R^1, \ldots, d_R^{j-1}, d_R^{j+1}, \ldots, d_R^J\right\}$ can then be written in the following form

$$\sigma_{-j}(q^{-j}, d_R^{-j}|X^{-j}) = \Pi_{h \neq j}\sigma\left(q^h, d_R^h \left| X^h\right.\right),$$

where $X^{-j} = \left\{X^1, \ldots, X^{j-1}, X^{j+1}, \ldots, X^J\right\}$.

4.3 Deputies' Optimal Decisions

Now that the politicians' and residents' preferences, the voting decisions, and the strategic interactions have all been specified, we can describe the deputies' choices. We will do this in two steps. Conditional on the decision to run for reelection, we first discuss how deputies choose the optimal allocation of resources. We then determine whether it is optimal for them to run.

Consider first the case in which it is optimal for deputy j to participate in the election. Conditional on running, he chooses the allocation that maximizes the expected value of his utility, where the expectation is taken over the decisions of his political rivals. Specifically, deputy j selects the allocation $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ that solves the following problem:

$$V_R^j\left(X^m, X^{-j}, \beta^j\right) = \max_{q^j} \int \left[\left(1-\beta^j\right) p^j\left(q\right) + \beta^j \sum_{m=1}^{M} W_m\right] d\sigma_{-j}(q^{-j}, d_R^{-j}|X^{-j}) - C_R + \varepsilon^j\left(q^j\right) + \nu_R$$

$$s.t. \sum_{m=1}^{M} q^{j,m} \leq \bar{Q}, \tag{4}$$

where $V_R^j\left(X^m, X^{-j}, \beta^j\right)$ is the value of competing in the election.

Consider now the case in which it is optimal for deputy j not to run for reelection. In this case, his utility does not depend on the political incentives, and the optimal allocation solves the following problem:

$$V_{NR}^{i}\left(X^{m},X^{-j},\beta^{j}\right)=\max_{q^{j}}\int\left[\bar{v}_{np}^{j}+\beta^{j}\sum_{m=1}^{M}W_{m}\right]d\sigma_{-j}(q^{-j},d_{R}^{-j}|X^{-j})+\varepsilon^{j}\left(q^{j}\right)+\nu_{NR}$$

$$s.t.\sum_{m=1}^{M}q^{j,m}\leq\bar{Q}, \tag{5}$$

where $V_{NR}^{j}\left(X^{m},X^{-j},\beta^{j}\right)$ denotes the value of not running.

We can now determine whether deputy j will compete in the election. He will if

$$V_{R}^{j}\left(X^{m},X^{-j},\beta^{j}\right)\geq V_{NR}^{j}\left(X^{m},X^{-j},\beta^{j}\right).$$

We conclude the description of the model by outlining the timing of the game played by the politicians and by providing a definition of its equilibrium. It is straightforward to describe the timing. First, nature reveals β^{j} to the politicians. The deputies' preference shocks are then realized, and the politicians simultaneously decide how much to transfer to each municipality subject to their resource constraint and whether to run for reelection. Lastly, the voting preference and district-level shocks are realized and residents cast their vote. The equilibrium that characterizes our model is a Bayesian-Nash equilibrium, which can be defined as follows.

Definition 1 *Allocations $q^{1*},\ldots,q^{J*}$ and the deputies' decisions to run for reelection $d_{R}^{1*},\ldots,d_{R}^{J*}$ are a Bayesian-Nash equilibrium if, for each deputy j, conditional on q^{-j*} and d_{R}^{-j*}, the decisions q^{j*} and d_{R}^{j*} maximize deputy j's expected utility.*

We will show that an equilibrium exists in the next section, after we introduce the assumptions required for estimating the model.

4.4 The Social Planner's Problem

Before we discuss how we estimate our model, it is useful to define the benchmark we will use to measure the size of the distortions. We define our benchmark as the allocation $Q^{sp}=$

$\left\{Q^1, \ldots, Q^M\right\}$ of aggregate funds, $J \times \bar{Q}$, that maximizes aggregate welfare, i.e.

$$Q^{sp} = \underset{Q^1,\ldots,Q^M}{\arg\max} \sum_{m=1}^{M} W_m$$
$$s.t. \sum_{m}^{M} Q^m \leq J \times \bar{Q}.$$

We will then use deviations from this benchmark in the allocation of public funds as our measure of distortions.

5 Econometric Implementation

In this section, we discuss functional form assumptions and the estimation method. To estimate the proposed model, we need to make functional form assumptions for the probability that deputy j wins an election, the welfare function, and the distribution of the shocks.

Probability of Winning an Election. For the probability of winning an election, we make the following assumptions. First, we assume that the expected allocation function $f\left(q^{j,m}, q^{-j,m}, X^j\right)$ is linear in its arguments and independent of $q^{-j,m}$. While these assumptions might seem strong, it turns out that they are not overly restrictive. We have explored more flexible functional forms, and have allowed $f(\cdot)$ to be a function of the maximum amount of resources allocated by deputy j's opponents. However, once we control for $q^{j,m}$, these other terms have no significant effect on the results. Second, we allow the constant in $f\left(q^{j,m}, q^{-j,m}, X^j\right)$ to vary between incumbents and challengers, but not within incumbents and challengers. The difference between the two constant terms can be interpreted as a measure of the incumbency effect. Third, we allow the coefficient on the current allocation $q^{j,m}$ to vary across municipalities. Consequently, conditional on $q^{j,m}$, voters will have different expectations about future allocations depending on which municipality they live in. These assumptions imply the following form for $f\left(q^{j,m}, q^{-j,m}, X^j\right)$:

$$f\left(q^{j,m}, q^{-j,m}, X^j\right) = \gamma_{0,j} + \gamma_{1,m} q^{j,m}.$$

Finally, we assume that the voting preference shocks $\xi^{i,j,m}$ are drawn from a type I extreme-value distribution and the district-level voting shocks θ_j from a uniform distribution with support $[0, \sigma_\theta]$, with $\sigma_\theta \leq 1$.

Given these assumptions, we can compute the probability that a deputy wins an election in a way that approximates the electoral system used in Brazil. At the time incumbents make decisions, they do not know which challengers will enter the electoral race or the realization of the district-level shocks θ_j. They therefore choose the optimal allocation and whether to participate in the election using the expected probabilities of winning the election that correspond to different choices, where the expectation is taken over the probability distribution of challengers' allocations and district-level shocks.[8]

The expected probabilities of winning corresponding to particular choices by incumbents is calculated as follows. For a particular set of deputies' choices q and d_R, we draw a set of district-level shocks and a set of allocations voters and incumbents believe will be selected in the subsequent term by challengers. Using Equation (3) and the assumption that the voting preference shocks are distributed according to an extreme value distribution, we then compute the total number of residents who plan to vote for each candidate. The district-level shocks are then used to determine the actual number of votes received by each politician. The candidates ranked within the top S vote totals win the election for this particular set of challengers' choices and district-level shocks, where S is the number of available seats. We then draw a new set of challengers' allocations and district-level shocks and re-compute who wins the elections. These steps are repeated 1000 times to calculate the conditional expected probabilities of winning the election. This process must be repeated for each set of possible deputies' choices.

Welfare Function. We specify the welfare function to meet several objectives. It should be concave to account for the decreasing returns to public funds. It should be flexible enough to accommodate the effects of other goods and services that are not financed by the deputies (e.g. projects funded by other public offices, public goods generated by private organizations, private projects, and private consumption). Finally, it should allow for the possibility that the funds are more productive and valued more in some municipalities than in others, thus generating higher welfare in those municipalities.

Based on these objectives, we assume the following welfare function:

$$w_m = \rho_m \log \left(y^m + Q^m \right),$$

[8] We can determine the incumbents' optimal decisions using the deputies' expected probabilities of winning the election because their utility is linear in those probabilities. Without linearity, we would have to compute the expectation over challengers and shocks of their entire utility function.

where y_m is per-capita income of municipality m and Q^m is the actual amount of non-rival public good received by municipality m as defined in Equation (2). With the logarithmic specification we can allow for decreasing returns. The municipality per-capita income y_m enables us to account for the existence of other goods and services that may affect the individual welfare and for the possibility that their value varies by municipality. Finally, with the coefficient ρ_m we can allow for two types of heterogeneity across municipalities. First, we can introduce productivity differences across municipalities. Second, we can account for possible differences in the weights a deputy assigns to the welfare of different municipalities. Notice that we can only identify the joint effect of productivity differences and differences in weights. This identification issue is a general result and does not depend on the functional form we have assumed. Indeed, to separately identify the effect of productivity differences from the effect of differences in preferences, one would need to observe data on productivity of projects located in different municipalities, which is not available for the regions we consider.

Preference Shocks. Along with the voting preference shocks and the district-level shocks, which were discussed earlier, our model is characterized by the two preference shocks we introduced when describing the deputies' preferences. The first shock ν_R is independent of the allocation chosen by the deputy and only affects the decision to run. It is drawn from a normal distribution with mean 0 and variance σ_ν. The second shock $\varepsilon^j(q^j)$ depends on the allocation selected by the deputy. To add flexibility to the model we allow the allocation-specific shock to have a different distribution depending on whether the deputy chooses to compete in the next election. Specifically, we assume that the shock is drawn from a normal distribution with mean 0, but with variance $\sigma_{\varepsilon,R}$ if the deputy chooses to run for reelection and with variance $\sigma_{\varepsilon,NR}$ if he decides not to run.

Model Estimation. Together with our functional form assumptions, we impose the following assumptions to make the estimation computationally tractable. First, we assume that the number of types is discrete. In the estimation, we experimented with two and three types and the data are consistent with two types of deputies: egoistic types (low β) and altruistic types (high β). Second, we discretize the provision of public goods into four choices. Specifically, a deputy can choose to give 0 percent, 33.33 percent, 66.66 percent, or 100 percent of the budget to a given municipality subject to the constraint that the allocations must add up to BRL$1.5 million. Third, as it is standard in the estimation of games, we will assume that only one equilibrium is observed in the data (Draganska et al. 2008).

Lastly, we estimate the model for the state of Roraima. This state, which is located in the northwestern part of Brazil, comprises 15 municipalities, which we aggregate into 4 macro-regions. We do this for two reasons. First, public projects assigned to one municipality are likely to benefit the surrounding municipalities as well. Typical examples include hospitals and schools. The aggregation of municipalities into macro-regions allows us to mitigate the effects of these spillovers. The second reason is that computationally the estimation becomes manageable. Each term Roraima elects 8 deputies to the Chamber. With four regions and four possible choices for each region, each of the 8 deputies can select among 20 feasible allocations. To construct the deputy's expected utility for one of the allocations he may choose, we have to consider all possible combinations that can be selected by the deputy's rivals. This implies that to solve the deputy's problem we have to consider $20^8 = 2.56e^{10}$ combinations. Even with the use of Message Passing Interface (MPI), which allows us to use simultaneously multiple processors in the estimation, and the aggregation of municipalities into macro regions, our model is computationally demanding. Without the aggregation, the estimation would be extremely difficult.

Now that we have outlined all the assumptions of the model, we are in position to prove that a Bayesian-Nash Equilibrium exists.

Proposition 1 *The model estimated in this paper has a Bayesian-Nash equilibrium in mixed strategies. Moreover, for every $\epsilon > 0$, the model is consistent with a pure-strategy ϵ-equilibrium.*

Proof. In the Appendix. ■

Given our assumptions, we estimate 18 parameters, which we separate into five sets. The first set includes the three parameters related to the politician's type: β_L, β_H, and π. The second set characterizes the voting decisions. This includes the voting function parameters γ_0, $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, $\gamma_{1,4}$, and the district-level shock parameter σ_θ. The third set consists of the productivity parameters ρ_1, ρ_2, ρ_3, and ρ_4. Since each deputy's allocation to the different localities must add up to BRL$1.5 million, we can only identify three out of the four parameters. We therefore normalize their sum to 1 and estimate three of them. The fourth set is composed of the two parameters that determine the actual amount of non-rival public good enjoyed by a municipality: ϕ and ϕ_{NR}. The last set includes the cost of competing in the elections ν, and the variances of the preference shocks σ_ν, $\sigma_{\varepsilon,R}$, and $\sigma_{\varepsilon,NR}$.

We estimate these parameters by simulated method of moments (SMM) using data on allocation choices, decisions to run for reelection, and electoral outcomes. Specifically, for one set of parameters we consider an initial set of beliefs and simulate the deputies' decisions. Given the deputies' decisions, we compute the beliefs generated by the model and compare them with the initial beliefs. If the distance between the two sets of beliefs is large, we re-simulate the model using as initial beliefs the beliefs generated by the model. When the distance between the initial and the simulated beliefs is sufficiently small, we compute the simulated moments used in the estimation and compare them with the corresponding data moments. We compute the standard errors using the asymptotic distribution of the estimated parameters.

6 Identification Discussion and Moments Selection

In this section, we discuss the identification of the model's parameters. The model is sufficiently complex that a mathematical proof of their identification is not possible. Instead, we provide a heuristic argument for the variation we use to identify each one of them.

Productivity Parameters

Identification. Four productivity parameters characterize the welfare function: ρ_1, ρ_2, ρ_3, and ρ_4. As discussed in the previous section, we normalize their sum to one and estimate only ρ_2, ρ_3, and ρ_4. To identify these parameters, we exploit the fact that, in our model, deputies who do not run for reelection allocate resources based only on welfare considerations. Under this assumption, the allocation decisions of these deputies identify the productivity parameters. Specifically, we use the difference between their average allocation to region 1 and their average allocation to region i to identify ρ_i, for $i = 2, 3, 4$.

In principle, incumbents who do not run may still have other electoral motives that affect their allocation decisions. In practice, for the state of Roraima, these motives are limited. Among the deputies who do not run for reelection, 65 percent have remained out of politics. The electoral motives for these deputies are likely to be nonexistent, or at best minimal. Of the remaining 35 percent, 85 percent sought an elected office in the capital city, such as vice mayor or vice governor, and 15 percent ran for the State Legislator or for a federal seat in the Senate. These politicians do have electoral motives and, given their career choices, have an incentive to target region 1, because it has a majority of the voters and is where the capital city resides. In these cases, we will estimate a lower bounds for the distortions.

Another possibility to consider is that deputies who choose not to run for reelection allocate more resources to their hometown for personal reasons, such as funding projects that will benefit themselves and their neighbors. The deputies from Roraima all come from region 1. Thus, if these hometown motives are present, our estimates of the distortions will again represent lower bounds.

Moments. To estimate the welfare parameters ρ_2, ρ_3, and ρ_4, we follow the previous discussion and use as moments the average share of resources allocated to regions 1, 2, and 3 by incumbents who choose not to run for office. The corresponding share for region 4 cannot be used because it is equal to one minus the sum of the shares for the other regions.

Altruism Parameters

Identification. Given the productivity parameters, we can then identify the altruism parameters β_L, β_H, and π using the difference in allocations between incumbents who compete in the elections and those who do not. To see why, first consider the case in which there is only one type of deputy. The difference in allocations between those who run and those who do not identifies the parameter β. If there is no difference, the degree of altruism will be identified to be 1. If deputies who compete in the election transfer a larger fraction of resources to municipalities with higher political gains, β will be less than 1 and will approach 0 as this difference increases.

With two types, the model can account for situations in which there are two distinct groups of deputies who run for reelection. The first allocates a larger fraction of their funds to regions with high political gains when compared to deputies who do not run. The second group also targets their funds to regions with high political gains, but to a lesser extent, again when compared to deputies who do not participate.

Moments. In addition to the three moments used to identify the welfare parameters, we also use the average share of resources allocated to regions 1, 2, and 3 by incumbents who decide to run for reelection.

Voting Function Parameters

Identification. To identify the incumbency effect γ_0, we compare the average probability of getting elected between incumbents and challengers. For the parameters that measure the effect of public funds on the number of votes $\gamma_{1,m}$, $m = 1, \ldots, 4$, we use the following sources of variation: the probability that an incumbent wins the elections conditional on transferring a large share of funds to region m and the share of resources allocated to region m by deputies who were reelected and by deputies who lost reelection. Conditional on the number of voters in region m, a high probability of winning if a large budget share is transferred to that locality indicates that funds allocated there translates into a high number of votes. Thus, $\gamma_{1,m}$ must be large. Similarly, a large difference between the share of resources allocated to region m by deputies who were reelected and deputies who lost reelection indicates that in that locality public funds transform easily into votes and, hence, $\gamma_{1,m}$ must be large. Finally, to identify the support of the district-specific shock σ_θ, we compare the probability of winning the election if a large budget share is allocated to the region with most voters with the corresponding probability for the region with the smallest number of voters. If the distance is small, the probability of winning the election is determined predominantly by district-specific shocks. If the distance is large, the probability is determined mostly by deputies' transfers and σ_θ must be small.

Moments. To estimate the voting function parameters we add to the set of moments (i) the difference in the average probability of election between incumbents and challengers, (ii) the difference in the average share of resources allocated to region m between politicians who won reelection and those who lost, for $m = 1, \ldots, 4$, and (iii) the average probability with which an incumbent is reelected conditional on transferring to municipality m at least 2/3 of their resources for $m = 1, \ldots, 4$.

Rivalry and Diversion of Resource Parameters

Identification. To identify the rivalry parameter ϕ, we use the correlation between the difference in per-capita GDP between two municipalities and the corresponding difference in the amount of funds received. To understand why this variation identifies this parameter, consider two regions with different per-capita GDP. Deputies without electoral incentives but some degree of altruism will transfer public funds to the region with lower per-capita GDP until its residents' marginal welfare is equal to the marginal welfare of the residents' of the region with higher per-capita GDP. The same argument holds for deputies with electoral incentive and some degree of altruism, except that these deputies will only narrow the distance between the marginal welfare of the two regions without equating it, where the distance depends on the relevance of the electoral incentives and the degree of altruism. Now consider the case in which ϕ is low, and hence there is a high degree of rivalry in Q_m. In this environment, conditional on the productivity parameters, the transfers to the region with lower per-capita GDP have to be significantly larger to reduce the difference between the regions' marginal welfare. If instead, ϕ is high, a smaller difference in transfers is sufficient to generate the required outcome. We can therefore pin down the parameter ϕ using the described variation in per-capita GDP and allocated funds.

The identification of the diversion parameter ϕ_{NR} requires a similar type of variation for incumbents who do not run. If these deputies divert more funds than incumbents who run, a small difference in per-capita GDP requires bigger differences in funds transferred to the two regions to equate marginal welfare. Therefore, we can identify ϕ_{NR} using the correlation between differences in per-capita GDP and differences in the amount transferred by deputies who forgo reelection.

It is important to remark that the only requirement for the described variation to provide identification of ϕ and ϕ_{NR} is the concavity of the welfare function. The functional form we have selected helps us reduce the number of parameters accounting for rivalry and diversion to two. But the same argument applies to more general concave functions.

Moments. Following the previous discussion, we use two moments to estimate ϕ and ϕ_{NR}. To construct the first moment, for deputies who run, we first compute the difference between the public funds allocated to the region with the highest per-capita GDP, region 1, and the public funds allocated to any other region. We then calculate the difference in per-capita GDP between region 1 and any other region. Finally, we compute the ratio of the two differences described above and take the average, where the average is taken over regions and terms. We therefore use as our first moment $E\left[\frac{Q_R^{n,k} - Q_R^{n,highGDP}}{y^{highGDP} - y^k}\right]$, where $Q_R^{n,m}$ is the amount of funds allocated to region m according to the data. The second moment is constructed using the same procedure for deputies who do not run, which produces $E\left[\frac{Q_{NR}^{n,k} - Q_{NR}^{n,highGDP}}{y^{highGDP} - y^k}\right]$.

Cost of Running Parameter and Variance Parameters

Identification. The last four parameters of the model are the cost of running, ν, the variance of the shocks to the decision to run σ_ν, and the variances of the preference shocks $\sigma_{\varepsilon,R}$ and $\sigma_{\varepsilon,NR}$. We identify the cost of running using the fraction of deputies who choose to run for reelection. We identify σ_ν using the difference in the probability of running between deputies who transferred most of their resources to the region with the highest political gains and deputies who allocated most of their funds to the region with the lowest political gains. If σ_ν is low, the described difference should be large. To understand why, notice that only incumbents for whom the benefits of being reelected outweigh the cost of participation will chose to run in the election. This group is composed of deputies with a probability of being reelected that is sufficiently large: if the probability is low, it is not worth paying the cost of participation. With a low σ_ν, the probability of winning will be sufficiently high predominantly for deputies who allocated funds to the region with high political gains. The difference between our two variables will therefore be large. Using the same line of reasoning, if σ_ν is high, the decision to run is mostly based on individual shocks. As a consequence, the described difference will be small. Finally, the variances of the preference shocks $\sigma_{\varepsilon,R}$ and $\sigma_{\varepsilon,NR}$ can be identified using the variance of the allocations chosen by deputies who ran for reelection and the variance for those who did not, respectively.

Moments. Following the previous discussion, the cost of running parameter and the variance of the shock to the decision to run are estimated by using the fraction of incumbents who choose to participate in the election and the probability of running conditional on transferring to municipality m at least $2/3$ of the budget for $m = 1, 2, 3$. The variances of the preference shocks are estimated by using the following two moments: the variance across regions of the allocation of resources conditional on running and the same variance conditional on not running.

7 Results

In this section, we present the estimates of our structural model and our policy simulations. As mentioned above, the model is estimated for the state of Roraima. We focus on this state because it is representative of a group of states that are poorer and less populated. For example, in Roraima 56 percent of households live below the poverty line, and average per-capita income is only R$133 per month. For these states it is particularly important to understand how resources are allocated and how to reduce the effect of political distortions since it could lead to large and positive effects on individual welfare.

Of the four macro regions that compose the state, region 1, which contains the capital city, is the wealthiest and most populated. It has a population of 80,293 inhabitants and its GDP per capita is BRL$5,833. In contrast, the other three regions are much less populated and much poorer. Region 2 is the least populated with 9,658 inhabitants, followed by region 4 (10,495) and region 3 (10,820). These regions have a similar ranking in terms of per-capita GDP.

7.1 Parameter Estimates

Productivity Parameters: As we discussed in Section 6, the allocation decisions of deputies who chose to forgo reelection identify the productivity parameters $\rho_1, \ldots, \rho_4$. To get a sense for how this variation identifies these parameters, in Panel (a) of Figure 4 we plot the average allocations by incumbents who did not run (denoted by the darker bars). We see that region 1 receives only 10% of the available funds, regions 2 and 3 receive more than twice that amount, and region 4 receives the largest proportion of funds at 40%. Our model will estimate productivity parameters to match this pattern in the data, which is precisely what we see in Table 1. Region 1 has the lowest productivity level at 0.03, regions 2 and 3 have similar estimated productivity parameters at 0.260 and 0.238, and region 4 has by far the highest estimated productivity with a coefficient equal to 0.472. The lighter bars in Panel (a) of Figure 4 and the first three rows of Table 2 show that, with these parameter estimates, our model can match almost perfectly the observed allocation pattern for deputies who forgo reelection. The simulated data deviates from the actual data by at most 1.2 percentage points.

To interpret the magnitude of these welfare parameters, we compute the change in welfare after reallocating one dollar from the lowest productivity region (region 1) to one of the other three regions. The welfare effect of reallocating one dollar is 3.05 times larger for region 4 than for region 2 and 4.3 times larger for region 4 than for region 3.

Altruism Parameters: The altruism parameters are identified by the difference in allocations between incumbents who ran versus those who did not. The darker bars in Panel (b) of Figure 4 depict the average allocations of incumbents who ran for reelection. The allocations display a clear U-shaped pattern: region 1 receives the largest fraction of resources, followed by region 4 which receives 2% less than region 1. Region 2 receives 24% of the funds, and region 3 receives the least with only 17% of the funds. To match this pattern in the data, the model needs two types of deputies with different degrees of altruism. We need one who cares almost exclusively about reelection incentives to explain the large fraction allocated to region 1, and a second, who cares both about altruism and electoral incentives, to explain why deputies who run for reelection transfer such a large fraction of resources to region 4, which has limited political gains but high productivity. Consistent with this argument, we estimate $\beta = 0.038$ for the egoistic type, and a much higher β of 0.173 for the altruistic type. Based on these estimates, an altruistic type is willing to substitute 1 vote for 44.3 dollars of welfare, whereas an egoistic type would require a welfare improvement of 232 dollars in order to trade off a vote.

To get a better sense for how this unobserved heterogeneity affects the allocation of public funds, in Figure 5 we compute the simulated share of funds allocated to the various regions distinguishing between the two types of politicians. The differences across politicians are quite stark: egoistic deputies allocate more than 36 percent of their funds to region 1, where the electoral incentives are the strongest. Altruistic deputies instead target region 4 relatively more, where the productivity of an extra dollar is the highest. They also give a substantial share of their funds to regions 2 and 3, which are also significantly more productive than region 1. The combination of these two types of politicians allows the model to explain the U-shaped pattern.

The proportion of types in the pool of politicians is estimated by matching the difference in allocations at the two ends of the U-shaped distribution. If the difference is large and positive then the model requires a larger fraction of egoistic deputies to rationalize the data. If, instead, the difference is large and negative, the model needs a large fraction of altruistic types. In the data the difference is positive but small, with region 1 receiving only 2% more funds than region 4. We therefore need a fraction of altruistic types that is slightly lower than 0.5, and we estimate $\pi = 0.404$.

With these parameter estimates we match the average allocations of deputies who compete in the elections well. Table 2 indicates that the largest difference between the simulated and actual data is quite small at 2.3 percentage points for region 1.

Voting Parameters: The incumbency parameter γ_0 is identified by the difference in the average probability of winning an election between incumbents and challengers. Table 2 reports that incumbents have on average a 51 percentage point advantage over challengers. Our model matches this difference well with $\gamma_0 = 0.032$. Based on this estimated parameter, we can compute the incumbency advantage by calculating the average probability that a deputy wins reelection over all possible allocations, and compare it to the same probability when $\gamma_0 = 0$. The incumbency effect is substantial. The average probability of winning the elections increases from 27.7 percent without incumbency advantage to 63.2 percent with the advantage, an increase of 35.5 percentage points.

Two sets of moments identify the parameters that measure the effects of public funds on vote shares, $\gamma_{1,1}, \ldots, \gamma_{1,4}$. The first set comprises the probabilities that an incumbent wins the election conditional on transferring at least 2/3 of his budget to a given region. The second set consists of the differences in the fraction of resources allocated to a given region between an incumbent who won reelection versus one who ran but did not win. We report these moments in Table 2. Region 1 has the highest probability of winning conditional on allocating at least 2/3 of one's budget, followed by region 2, region 4, and region 3. To see how these moments affect our parameter estimates, consider the coefficient $\gamma_{3,1}$. In the data, the incumbents who allocate 2/3 of their budget to region 3 win only 50 percent of the time. Given that a deputy can achieve that probability of winning simply by virtue of his incumbency advantage, the model will want to estimate $\gamma_{1,3}$ close to zero. This is precisely what we get, with $\gamma_{1,3} = 0.00001$ and statistically indistinguishable from zero. The low estimate for $\gamma_{1,3}$ is also consistent with the small difference in fraction of resources allocated to region 3 between incumbents who won and lost the elections.

The argument for the identification of the parameters $\gamma_{1,1}, \gamma_{1,2}$, and $\gamma_{1,4}$ is similar once we account for the number of voters in the regions. As we documented above, region 1 has the highest number of voters, so the model does not need a large coefficient for region 1 ($\gamma_{1,1} = 0.035$) in order to explain the high electoral return to allocating there. In contrast, region 2 has only 1/8 of the population of region 1. And yet, if a deputy allocates 2/3 of his budget to that region, he wins with a probability of 63.4 percent, which is significantly higher than the probability generated by the incumbency advantage. Similarly, the difference between winners and losers in the fraction of the budget allocated to region 2 is just 1/3 of the difference for region 1 – quite smaller than the difference in population size. In order to match these moments, our model requires $\gamma_{1,2}$ to be three times the size of $\gamma_{1,1}$ (0.103). A similar logic applies to region 4, and given its data moments, we estimate $\gamma_{1,4} = 0.030$.

A direct interpretation of the parameters $\gamma_{1,1}, \ldots, \gamma_{1,4}$ is difficult for two reasons. First, because of the budget constraint, a deputy cannot increase the amount of transfers he allocates to one region without reducing transfers to the other regions. Second, the reallocation of resources from one region to another triggers general equilibrium responses by other deputies. To provide an economic interpretation of these parameters, we therefore calculate how much the probability of getting reelected changes as an incumbent shifts his resources out of one region and into another. Based on our estimates, Region 1 is the most attractive region in terms of electoral returns: if a deputy were to shift all of his resources from Region 3 (the least attractive) to Region 1, his probability of reelection would increase by 52 percentage points. By comparison, transferring those resources to Region 2 or Region 4 would increase the likelihood of reelection by 18 and 7 percentage points, respectively.

The last parameter affecting the voting decisions is the upper bound of the district-level shock, σ_θ. As we discussed, we use the difference between two probabilities to identify this parameter: the probability of winning conditional on allocating at least 2/3 of one's budget to region 1 and the probability of winning conditional on allocating at least 2/3 of one's budget to region 3. In the data, this difference is quite large and equal to 37.5 percentage points, suggesting that the allocations have large effects on voters' decisions and that the model does not need large district-level shocks to explain the observed patterns. Consistent with this observation, we estimated $\sigma_\theta = 0.066$, which implies that politicians lose at most 6.6% of the votes because of the arrival of district-level shocks.

With our estimated voting parameters we can match extremely well the probability a deputy wins the election conditional on transferring at least 2/3 of his funds to a region. According to Table 2, the largest difference between the simulated and data moments is only 0.9 percentage points for region 2. With regard to the differences between the allocations of deputies who win and those who do not, we can match the ranking, but are slightly off on the levels.

Rivalry and Diversion Parameters: We identify the diversion parameter ϕ using the correlation between the following two variables for deputies who run for reelection: the difference in per-capita GDP between region 1 and any other region, and the corresponding difference in allocated funds. In the data, this moment is equal to -0.190, indicating that incumbents who choose to run for reelection transfer more funds to the region with the highest per-capita GDP. This is to be expected since these deputies care about being elected and the region with the highest per-capita GDP also has the most votes. At the same time, this number is substantially smaller than what it would be absent welfare considerations. To match this relatively small number, our model requires a $\phi = 0.008$, suggesting that for each dollar that is spent in a municipality slightly less than one cent of it represents a pure public good. This estimate implies that the allocated resources fund projects that are not pure public goods, but still have a high degree of non-rivalry. To see this note that the average population across municipalities in the state of Roraima is 27,817. For the funded projects to be a private good, the rivalry parameter ϕ would therefore have to be equal to 0.00004. With the estimated ϕ, we match the data moment well: the simulated moment is -0.171 – just below the level observed in the data.

To identify the diversion parameter ϕ_{NR}, we use the same correlation used to identify ϕ, but for deputies who forgo reelection. In the data, the corresponding moment is equal to 0.158. Given that these incumbents have no electoral incentives, a positive moment indicates that the region with the highest per-capita GDP receives fewer funds from them than poorer regions. The moment is also not large, suggesting that the model needs a relatively small fraction of diverted funds to explain the data. We estimate a $\phi_{NR} = 0.852$, which implies that incumbents who forgo reelection divert 15% more funds than those who run. Overall, the estimated ϕ_{NR} highlights an important tradeoff: although deputies who do not run place more weight on welfare considerations, they also divert more resources. We will explore this tradeoff further when we consider a policy of a one-term limit.

Cost of Running: We estimate the cost of running parameter and the variances as a fraction of the value of running to simplify their interpretation. The cost of running parameter is identified using the fraction of deputies who run for reelection. In the data, 71% of incumbents run for reelection. To match this moment our model needs a relatively small utility cost of running, which is estimated to be 0.019, or about 2% of the total utility value of running. With this estimated parameter we can match well the data moment: our simulated fraction of incumbents who choose to run is 70%.

As a specification test of our model, Figure 6 examines how our model matches other moments of the data not used in the estimation. In Panel (a), we plot the probability an incumbent runs for office conditional on allocating 1/3 of his budget to a particular region. Similarly in Panel (b), we plot the probability an incumbent is reelected conditional on allocating 1/3 of his budget to a particular region. For the probability of running, the model matches these additional moments extremely well. For the probability of winning, however, the model consistently overestimates the probability of winning in all regions except 1. Given the incumbency advantage that deputies enjoy, the model cannot reduce the probability of winning in regions 2-4 without further lowering the probability of winning in region 1, which the model underestimates relative to the data. Although the model does not match the levels well, it is able to capture the ordering across regions.

7.2 Political Distortions

Having estimated our model, we can answer the first question of the paper: To what extent do political incentives affect the allocation of public funds? To address this question, we compare the distribution of public funds generated by the model to the social planner's allocation, which is defined in Section (4.4) as the allocation that maximizes the total welfare of the state based on our estimated welfare functions. Note that this allocation differs from the allocations of deputies who do not run for two important reasons: unlike the social planner's allocation, both stealing and the beliefs over the actions of the other incumbents affect the decisions of deputies who decide not to run.

Figure 7 plots the allocation of public funds relative to the social planner's allocation. Electoral incentives distort 26% of the public funds, with all of the distortions occurring towards region 1, and away from the less populated and poorer regions. Given our estimates, aggregate welfare is therefore maximized by allocating relatively more funds to regions 2, 3, and 4. Political distortions arise because politicians cannot afford to ignore region 1, which is the region with the most votes. This of course raises the question as to whether changes to the electoral rules can help reduce these distortions, which is the second aim of the paper.

7.3 Policy Evaluations

In this section, we investigate how different electoral rules affect the distribution of public funds. We focus on two potential changes to the electoral rules: adoption of rank-score voting and term limits. In addition to these two specific policy changes, we also simulate the effects of an increase in the number of challengers. For each policy simulation, it is necessary to recalculate the set of beliefs deputies have so that they are consistent under the new environment. We do this using fixed-point iterations.

Rank-Score Voting

Rank-score voting is an important class of voting rules that allows voters to rank all or a subset of the candidates. Specifically, suppose J candidates compete in the elections. Rank-score voting is defined by a vector of scores $(s_1, \ldots, s_J)$, with $s_1 \geq \ldots \geq s_J$, that assigns points to politicians according to a voter's ranking: s_1 points are assigned to the candidate ranked at the top by the voter, s_2 points are assigned to the second-ranked candidate, and so forth. The S seats in the election are won by the politicians with the S highest total number of points. Rank-score voting is believed to be a more expressive form of voting and to encourage residents to vote sincerely. Here, we evaluate whether it also has the benefit of reducing the distortions created by electoral incentives.

Under Brazil's current system residents can only vote for a single candidate, which corresponds to the scoring rule $(1, 0, \ldots, 0)$ and is commonly referred to as plurality voting. We will compare it to the following set of N-person scoring rules, which is commonly referred to as approval voting: $(1, 1, 0, \ldots, 0)$, $(1, 1, 1, 1, 0, \ldots, 0)$, $(1, 1, 1, 1, 1, 1, 0, \ldots, 0)$, and $(1, 1, 1, 1, 1, 1, 1, 1, 0, \ldots, 0)$.

Figure 8 presents the results from simulating the 2-person, 4-person, 6-person, and 8-person scoring rules, along with the original results (the base case).[9] As before, we plot the allocation of public funds across the four regions generated by the N-person scoring rules relative to the

[9]For each simulation, in addition to finding a fix point, we also need to compute an exploded logit in order to determine the total number of points each candidate receives. As we increase the number of candidates a voter can list, the computation becomes increasingly more expensive. Thus, due to these computational constraints, we linearly extrapolate the results for the 8-person rule.

social planner's allocation. Based on these simulations, we find that the deviations from the social planner decrease as we allow voters to rank increasingly more candidates. For instance, if the government was to adopt a 6-person rule, our model indicates that the distortions in the allocation of public funds will decrease by 5.1 percent. The adoption of an 8-person rule would reduce the distortions even further to 7.5 percent.

Although these reductions are sizable, it does raise the question as to why rank scoring does not reduce the distortions even further, or perhaps even eliminate them. The reason is straightforward: there are two countervailing forces at play. On the one hand, this policy provides an incentive for politicians to reallocate their funds away from region 1. Because of the winner-take-all nature of the original voting rule, politicians have a strong political incentive to target region 1, where a majority of the voters reside. With score voting, as politicians receive points even if voters do not rank them at the top, the necessity to come in first in region 1 diminishes, allowing politicians to allocate more funds to other regions.

On the other hand, this policy makes elections more competitive, which can be seen in Panel (a) of Figure 9. As we increase the number of candidates voters can rank, the probability that a challenger wins a seat increases substantially. Under the current system, the probability that a challenger wins an election is only 14 percent. But going from a 1-person scoring rule to an 8-person scoring rule increases the probability that a challenger wins by 29 percent. As the election becomes more competitive, incumbents want to target region 1 more, which increases distortions. This second effect limits the efficacy of rank scoring to further reduce distortions in the allocation of public funds.

The results depicted in Panel (a) of Figure 9 illustrate another important effect of rank-score voting. In our model, voters believe that challengers will distribute public funds similarly to the current incumbents. Because of this consistency, the major difference between incumbents and challengers in the probability of winning the elections is the incumbency advantage. The fact that challengers win more often as residents can rank an increasing number of candidates, indicates that rank-score voting can be effective in reducing incumbency advantage.

Candidate Entry

For the same reason why rank score voting cannot completely eliminate distortions, policies aimed at increasing candidate entry will not be effective. In Panel (b) of Figure 9, we plot the results of a series of policy simulations in which we double the number of challengers in combination with rank-score voting. Independent of the number of candidates that can be ranked, the distortions in the allocation of public funds are larger when the number of challengers has been doubled. As the figure also depicts, the difference in distortion levels increases as we move from a 1-person scoring rule to an 8-person scoring rule. The effects of these simulations are consistent with the effects from the previous policy. As elections become more competitive due to the increase in the number of challengers, incumbents have a greater incentive to target region 1, thus increasing distortions. This incentive becomes even stronger as we allow voters to rank more candidates, since it increases the probability that a challenger gets elected.

Term Limits

The final policy that we consider is to limit incumbents to a single term. Brazil currently allows deputies to be elected an indefinite number of times, but several countries have argued for, and in some cases implemented, term limits as a way to improve representation. It is also a policy that the Chambers of Deputy recently voted in favor of for Brazil's executive branches.[10]

In our model the advantage of the policy is that electoral incentives would no longer influence the way public funds are allocated. The disadvantage is that deputies who do not run divert significantly more of the funds than deputies who still face reelection incentives. The ability to determine which of these two effects dominates is an important contribution of our model. When we compare the results of our model to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions decrease from 26 to 16 percent. However, because of the increase in corruption, welfare as a whole actually goes down by 2 percent, indicating that this policy is not welfare improving. Observe that, even without electoral incentives, the distortions do not go to zero because incumbents form beliefs about their rivals' allocations.

8 Conclusions

A central question in distributive politics is how politicians target public funds. In this paper, we present a novel approach to the empirical analysis of how politicians allocate public funds in an environment in which other politicians are behaving strategically. This approach allows us, among other things, to compute the extent to which political incentives distort the transfer of these public funds away from the social planner's allocation, and to investigate the effects of electoral rules and political competition in mitigating these distortions.

[10]On 5/27/2015, the Chamber of Deputies voted to eliminate reelection for the office of President, Governor, and Mayor. For more information see: http://noticias.uol.com.br/politica/ultimas-noticias/2015/05/27/camara-vota-o-fim-da-reeleicao.htm

Using data from Brazil's federal legislature, we find that at least 26 percent of the value of the budget amendments that legislators can allocate are distorted relative to the social planner's allocation. These distortions are driven by the behavior of non-altruistic politicians, who put little weight on the welfare of the municipalities, and thus target regions that have more voters, but are less productive. We also explore the effects of approval voting on the allocation of public funds. We find that while approval voting does reduce the distortions associated with political incentives, the effects can be muted because of two countervailing forces. Although approval voting reduces the incentives politicians have to target regions with more voters, it also reduces their incumbency advantage, which in turn exacerbates the electoral incentive effects.

Overall our study highlights the importance of political institutions for the allocation of public expenditures, and in particular the type of distortion that can arise when the incentives between the politicians and a social planner are not aligned. But how much we can change institutions to align incentives is still an open question, and in this case our results suggest that perhaps more emphasis should be placed on attracting better types of politicians (Dal Bó, Finan, and Rossi (2013); Ferraz and Finan (2009)).

Although our model fits the data well, it is quite parsimonious and can be extended and generalized in several directions that represent exciting possibilities of future research. One possible extension would be to make the game dynamic. As Diermeier, Keane, and Merlo (2005) correctly emphasize, politicians are forward-looking agents whose career choices are dynamic in nature. Although our model captures this behavior to some degree in the decision to run for reelection, it would be interesting to model these decisions more explicitly, such as the decision to enter higher offices. Another extension would be to add political parties into the model. While we do not think that this is an important feature of Brazilian politics, one could potentially exploit a nested structure to extend the model in this direction for other countries. Other directions of future research will ultimately depend on the collection of new data. For instance, with data on campaign spending, one could easily extend our model to examine whether budgetary amendment complement or substitute campaigning. One could then investigate the impact of campaigning financing laws on not only electoral performance but also public funds allocation.

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9 Tables and Figures

Table 1: Parameter Estimates

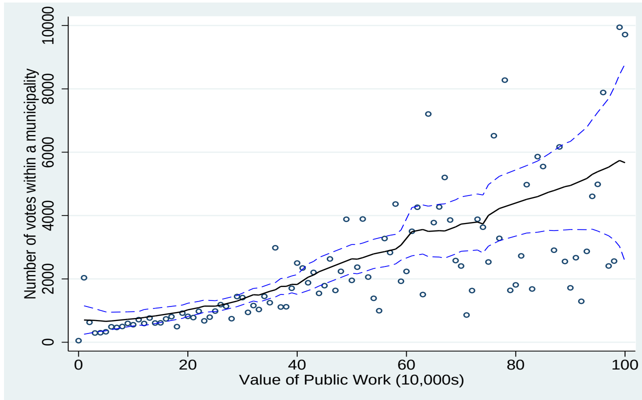
	Parameter	Estimate	Std Err
Welfare			
Productivity in region 1	ρ_1	0.030	0.006
Productivity in region 2	ρ_2	0.268	0.003
Productivity in region 3	ρ_3	0.230	0.003
Productivity in region 4	ρ_4	0.472	0.004
Altruism			
Egoistic type	β_H	0.038	0.001
Altruistic type	β_L	0.173	0.008
Proportion of altruistic types	π	0.404	0.006
Voting Decisions			
Incumbency advantage	γ_0	0.032	0.000
Effects of public funds on vote shares, region 1	$\gamma_{1,1}$	0.035	0.001
Effects of public funds on vote shares, region 2	$\gamma_{1,2}$	0.103	0.004
Effects of public funds on vote shares, region 3	$\gamma_{1,3}$	0.00001	0.0001
Effects of public funds on vote shares, region 4	$\gamma_{1,4}$	0.030	0.002
Degree of rivalry	ϕ	0.008	0.001
Diversion of funds if not running	ϕ_{NR}	0.852	0.057
Upper bound of support of district-level shock	σ_θ	0.066	0.0008
Decision to Run and Shocks			
Cost of running	ν_L	0.019	0.001
Variance of shocks on decision to run	σ_ν	0.010	0.0005
Variance of preference shocks if running	$\sigma_{\varepsilon,R}$	0.016	0.0007
Variance of preference shocks if not running	$\sigma_{\varepsilon,NR}$	0.015	0.0002

Notes: This table presents the model's parameter estimates. The standard errors are computed using the asymptotic distribution of the estimated parameters.

Table 2: Moments Used in the Estimation

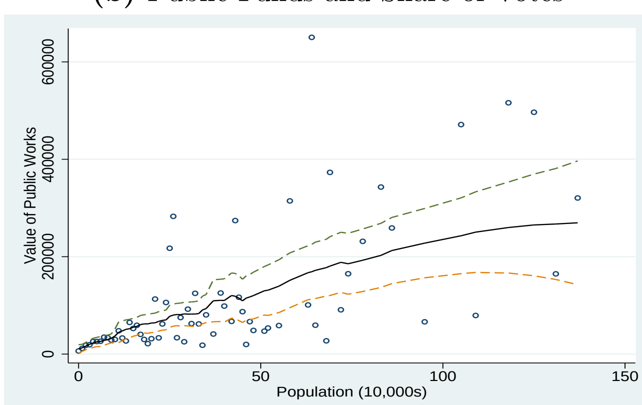
Moments	Model (1)	Data (2)
Welfare		
Avg. share allocated if not running, region 1	0.116	0.104
Avg. share allocated if not running, region 2	0.229	0.229
Avg. share allocated if not running, region 3	0.261	0.250
Altruism		
Avg. share allocated if running, region 1	0.323	0.300
Avg. share allocated if running, region 2	0.233	0.242
Avg. share allocated if running, region 3	0.155	0.167
Voting Function		
Pr(Winning) for incumbents - Pr(Winning) for challengers	0.492	0.508
Pr(Winning) if at least 2/3 of budget given to region 1	0.854	0.857
Pr(Winning) if at least 2/3 of budget given to region 2	0.634	0.625
Pr(Winning) if at least 2/3 of budget given to region 3	0.499	0.500
Pr(Winning) if at least 2/3 of budget given to region 4	0.543	0.546
Difference in avg. share allocated: winners versus losers, region 1	0.197	0.200
Difference in avg. share allocated: winners versus losers, region 2	0.071	0.008
Difference in avg. share allocated: winners versus losers, region 3	0.022	-0.033
Difference in avg. share allocated: winners versus losers, region 4	0.050	-0.025
Covariance bet. difference in allocations and GDP if running	-0.171	-0.190
Covariance bet. difference in allocations and GDP if not running	0.122	0.158
Cost of Running and Variances		
Probability of running	0.701	0.714
Probability of running if at least 2/3 given to region 1	0.921	0.875
Probability of running if at least 2/3 given to region 2	0.708	0.727
Probability of running if at least 2/3 given to region 3	0.504	0.500
Variance of allocation shocks if running	0.004	0.003
Variance of allocation shocks if not running	0.010	0.012

Notes: This table presents the moments used to estimate the model's parameter. Column 1 reports simulated moments based on 5,000 simulations. Column 2 reports the data moments.



(a) Public Funds and Number of Votes

(b) Public Funds and Share of Votes



(c) Public Funds and Population Size

Figure 1: Public Expenditures and Outcomes

Notes: This figure depicts the unadjusted relationship between the amount of public funds a deputy allocated to the municipality during the 1996-1999 term and the number of votes (panel A) or share of votes (panel B) he received from the municipality during the subsequent election. Panel C depicts the unadjusted relationship between population size and the amount of public funds a deputy allocated to the municipality during the 1996-1999 term. The solid line was computed using lowess. The dashed lines are the corresponding 95 percent confidence intervals. Each dot represents the mean of the dependent variable computed based on equally-sized bins.

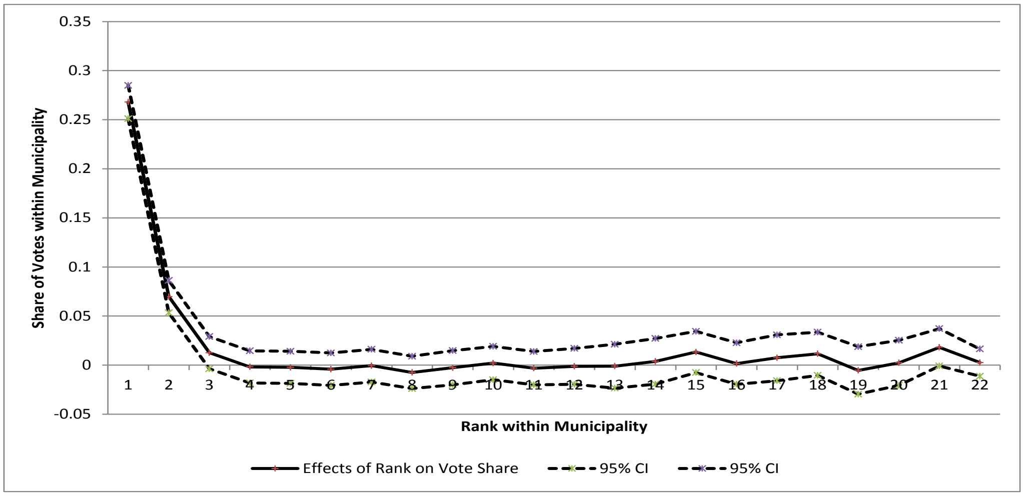
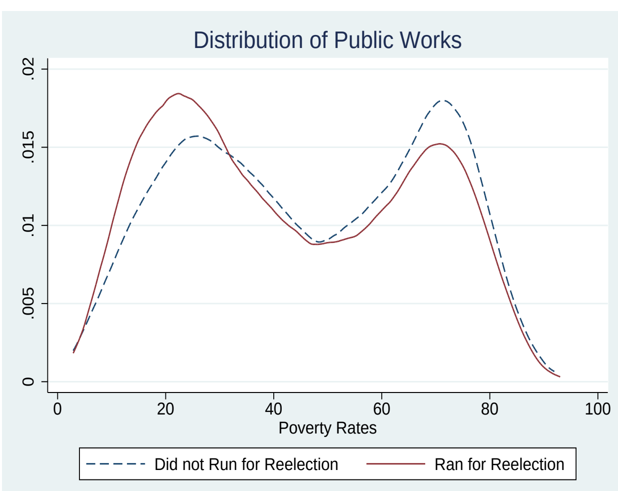
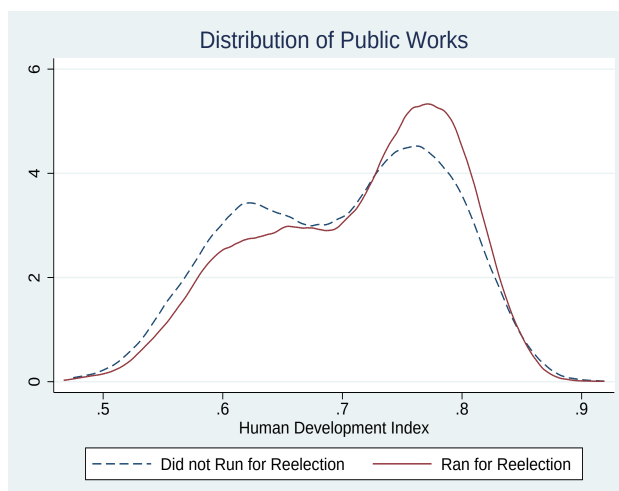


Figure 2: Vote Share and Rank by Amount of Public Funds

Notes: This figure plots coefficient estimates from a regression that regressed the share of votes an incumbent received in a particular municipality on a set of dummies indicating the incumbent's rank within the municipality. An incumbent's rank was determined by the amount of public funds he allocated to that municipality. Incumbents ranked above 22 are the excluded category. The dashed lines correspond to the 95 percent confidence interval of each estimated coefficient.



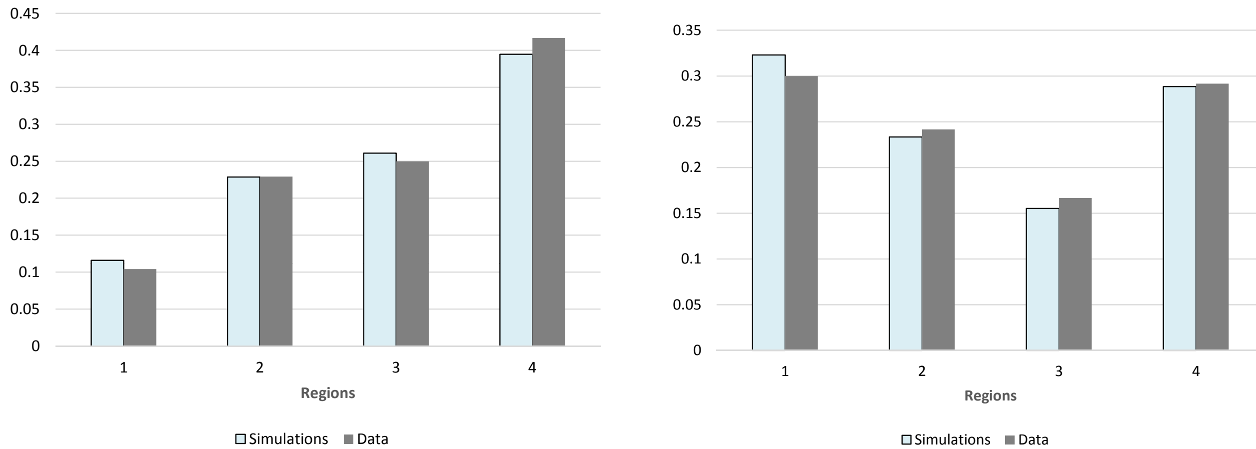
(a)



(b)

Figure 3: Distribution of Public Funds by Welfare Levels

Notes: The Figure in Panel (a) depicts kernel density plots of the allocation public funds by poverty level of the municipality. The Figure in Panel (b) depicts kernel density plots of the allocation public funds by the municipality's Human Development Index. These density plots are estimated separately for incumbents who ran for reelection and those that did not. These figures were computed based on a sample of 5,550 municipalities.



(a) Did not run for reelection

(b) Ran for reelection

Figure 4: Comparison Between Model's Prediction and Actual Allocations

Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. In Panel (a), we compute the allocations for incumbents who did not run for reelection. In Panel (b), we compute the allocations for incumbents who ran reelection. The share of public funds is computed by region and averaged over 5,000 simulations.

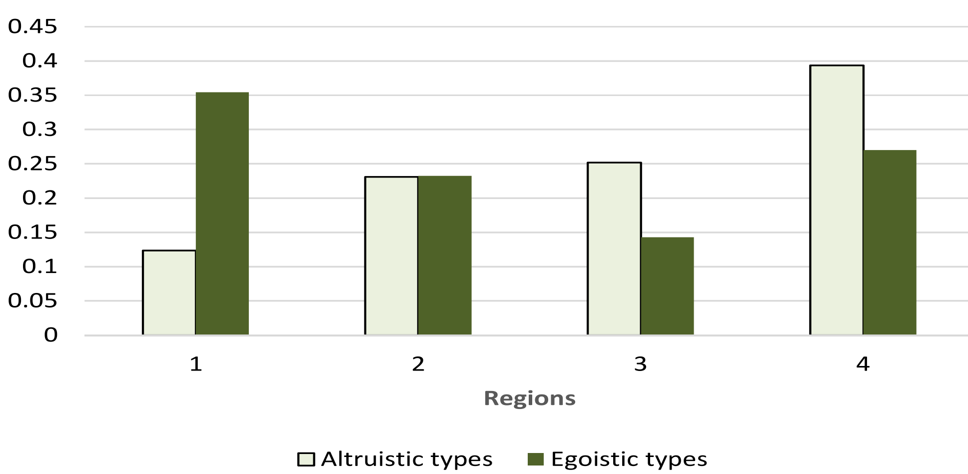
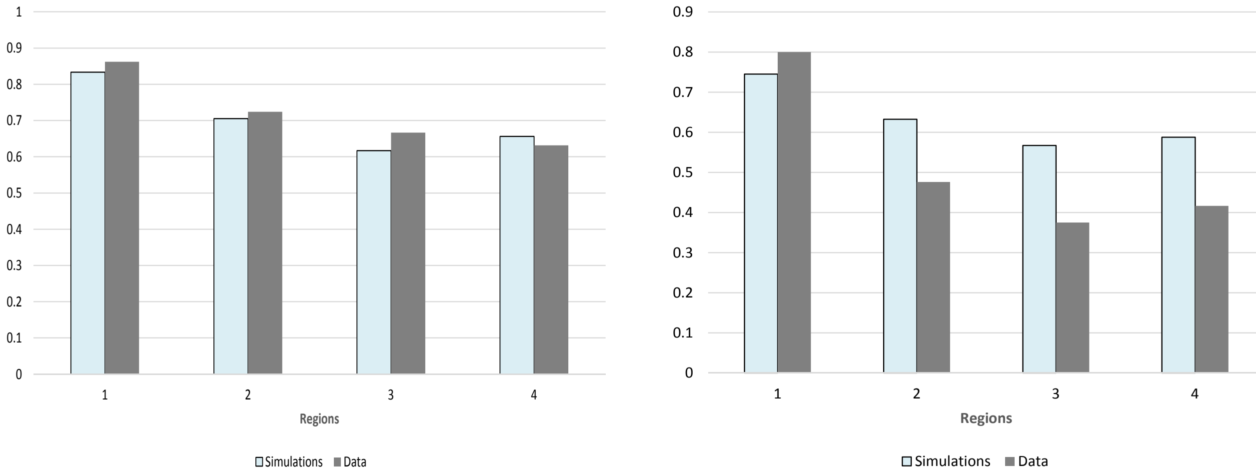


Figure 5: Distribution of Allocations By Politician Type

Notes: This figure plots the allocation of public funds by politician type. The share of public funds is computed by region and averaged over 5,000 simulations.



(a) $Pr(\text{Running}|\text{Allocation} > 1/3)$ **(b)** $Pr(\text{Winning}|\text{Allocation} > 1/3)$

Figure 6: Comparison Between Model's Prediction and Actual Allocations - Moments not Matched

Notes: Panel (a) plots the probability of running conditional on allocating more than 1/3 of one's budget to a particular region, as predicted by the model and in the data. Panel (b) plots the probability of winning conditional on allocating more than 1/3 of one's budget to a particular region, as predicted by the model and in the data.

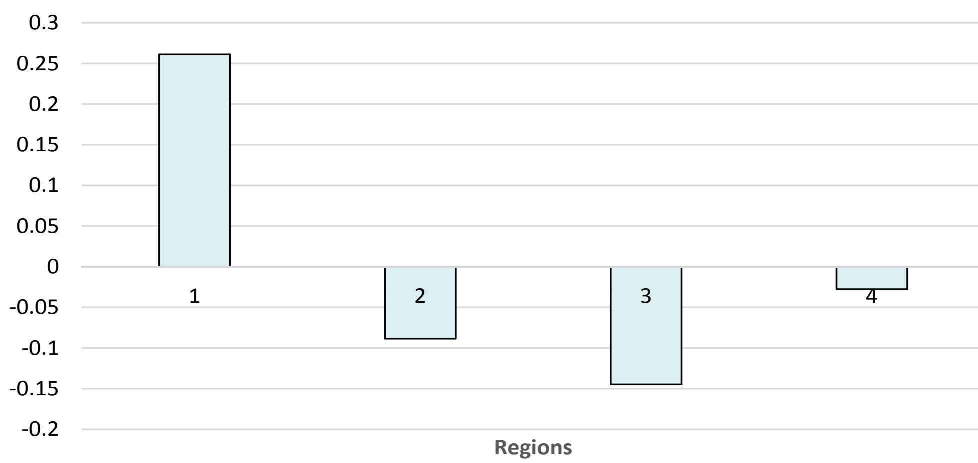


Figure 7: Deviation from the Social Planner

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations.

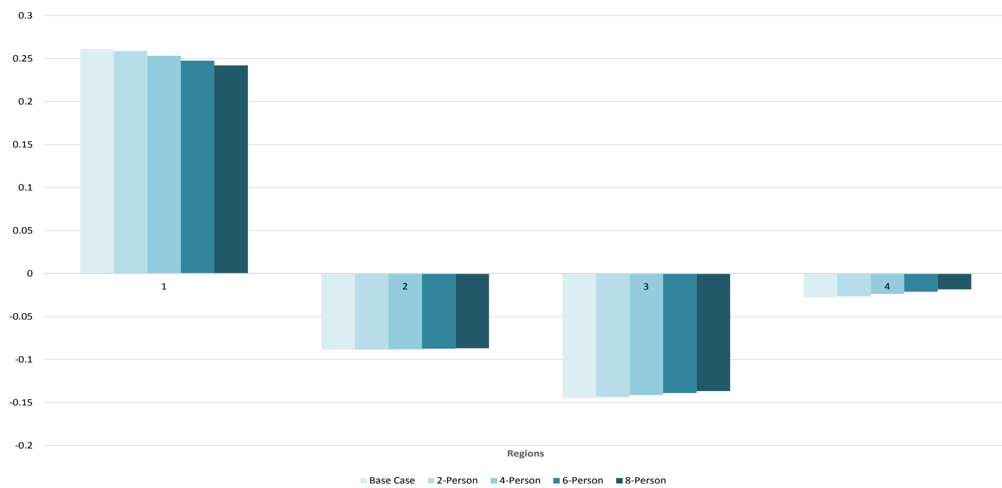
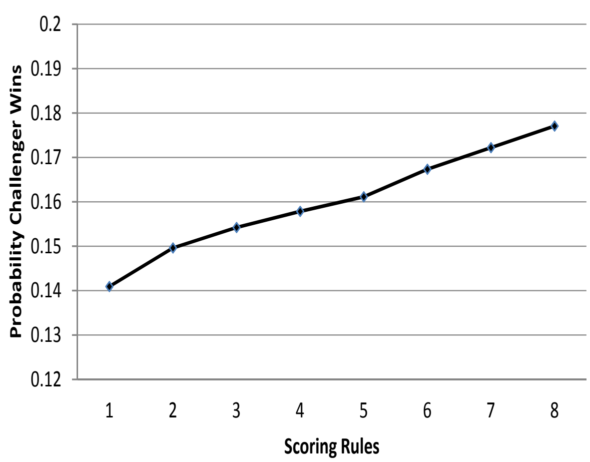
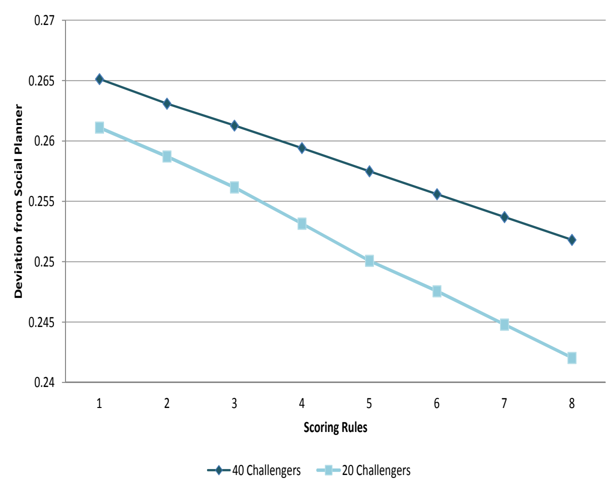


Figure 8: Policy Simulations: N-Person Electoral Rules

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations. The 2-person policy refers to the scoring rule: $(1, 1, 0, \ldots, 0)$. The 4-person policy refers to the scoring rule: $(1, 1, 1, 1, 0, \ldots, 0)$. The 6-person policy refers to the scoring rule: $(1, 1, 1, 1, 1, 1, 0, \ldots, 0)$, The 8-person policy refers to the scoring rule: $(1, 1, 1, 1, 1, 1, 1, 1, 0, \ldots, 0)$, etc. The base case policy refers to our original results.



(a) Probability a Challenger Wins



(b) Challenger Competition

Figure 9: Policy Simulations

Notes: See note in Figure 8.

On-line Appendix

A Appendix: Tables and Figures

Table A.1: Budgetary Amendments

Year	Total	Average per Deputy	Average Amount per Deputy	Standard Deviation
1996	3859	16.82	1,370,380	380,295.4
1997	4955	16.72	1,335,346	200,642.7
1998	6106	14.35	1,300,812	194,080.3
1999	5275	13.28	1,266,827	229,759.3
Total	20195	15.12	1,313,702	253048.100

Notes: These data are available online at: http://www2.camara.leg.br/

Table A.2: Summary Statistics the 1998 Elections for Federal Deputies

	Number of legislators	Number of Candidates	Number of Parties	Reelection Rates	Share ran for reelection	Average Votes Among Non-elected	Average Votes Among Elected	Share male	Share college degree
Acre	8	51	13	0.33	0.40	2,095	11,185	0.82	0.47
Alagoas	9	52	20	0.54	0.86	5,549	49,648	0.88	0.71
Amazonas	8	48	15	0.58	0.57	6,538	62,900	0.90	0.54
Amapá	8	63	17	0.80	0.38	1,690	8,180	0.90	0.52
Bahia	39	118	23	0.77	0.76	8,293	76,461	0.96	0.65
Ceará	22	99	20	0.64	0.72	7,810	79,824	0.90	0.58
Espírito Santo	10	72	17	0.64	0.56	8,325	55,960	0.86	0.58
Goias	17	89	20	0.62	0.69	6,470	68,159	0.88	0.56
Maranhão	18	79	20	0.64	0.81	5,346	59,167	0.92	0.51
Minas Gerais	53	352	26	0.83	0.64	7,333	80,671	0.92	0.55
Mato Grosso do Sul	8	53	20	0.62	0.50	6,837	49,380	0.87	0.68
Mato Grosso	8	48	15	0.58	0.71	5,919	58,555	0.85	0.58
Pará	17	102	16	0.68	0.60	7,600	51,443	0.93	0.52
Paraíba	12	55	14	0.56	0.67	9,043	56,390	0.89	0.84
Pernambuco	25	110	23	0.69	0.64	8,024	71,696	0.89	0.65
Piauí	10	62	16	0.54	0.86	3,223	67,905	0.84	0.58
Paraná	30	195	24	0.70	0.69	8,997	70,444	0.95	0.58
Rio de Janeiro	46	434	29	0.75	0.59	5,600	78,009	0.88	0.57
Rio Grande do Norte	8	47	15	0.64	0.71	6,783	81,350	0.89	0.68
Rondônia e	8	59	13	0.54	0.71	4,187	24,294	0.90	0.42
Roraima	8	38	14	0.80	0.75	1,228	9,305	0.92	0.63
Rio Grande do Sul	31	194	20	0.67	0.62	11,934	74,606	0.92	0.68
Santa Catarina	16	104	17	0.40	0.60	12,724	62,713	0.85	0.63
Sergipe	8	50	23	0.45	0.80	3,392	46,154	0.96	0.42
São Paulo	70	656	29	0.77	0.59	8,885	105,326	0.90	0.60
Tocantins	8	44	10	0.67	0.75	3,878	30,028	0.70	0.61
Brazil	513	284	23	0.68	0.65	7,437	70,709	0.90	0.59

Notes: Data are available: http://www.tse.jus.br/

Table A.3: Relationship between Electoral Performance and Allocation of Public Funds

Dependent variable	Number of Votes			Vote Share		
	(1)	(2)	(3)	(4)	(5)	(6)
Public Funds ($100,000s)	677.491	679.28		0.028	0.028	
	[50.198]	[49.705]		[0.001]	[0.001]	
Rank within the municipality			-452.763			-0.027
			[35.468]			[0.001]
Municipal intercepts	N	Y	Y	N	Y	Y
Deputy intercepts	N	Y	Y	N	Y	Y
Observations	154,139	154,139	154,139	154,139	154,139	154,139
R-squared	0.47	0.47	0.36	0.2	0.16	0.12

Notes: Column 1 reports the unadjusted correlation between the amount of public funds a deputy allocated to the municipality during the 1996-1999 term and the number of votes he received from the municipality during the subsequent election. Column 2 reports the same relationship as in Column 1 but adjusts for both deputy and municipal fixed-effects. Column 3 reports the relationship between the number of votes a deputy received and his ranking in the municipality with respect to the amount of public goods he provided. Columns 4-6 replicate the regressions in columns 1-3 but use the deputy's vote share in the municipality as the dependent variable. The estimation has been restricted to only those incumbents that ran for reelection. Robust standard errors in brackets.

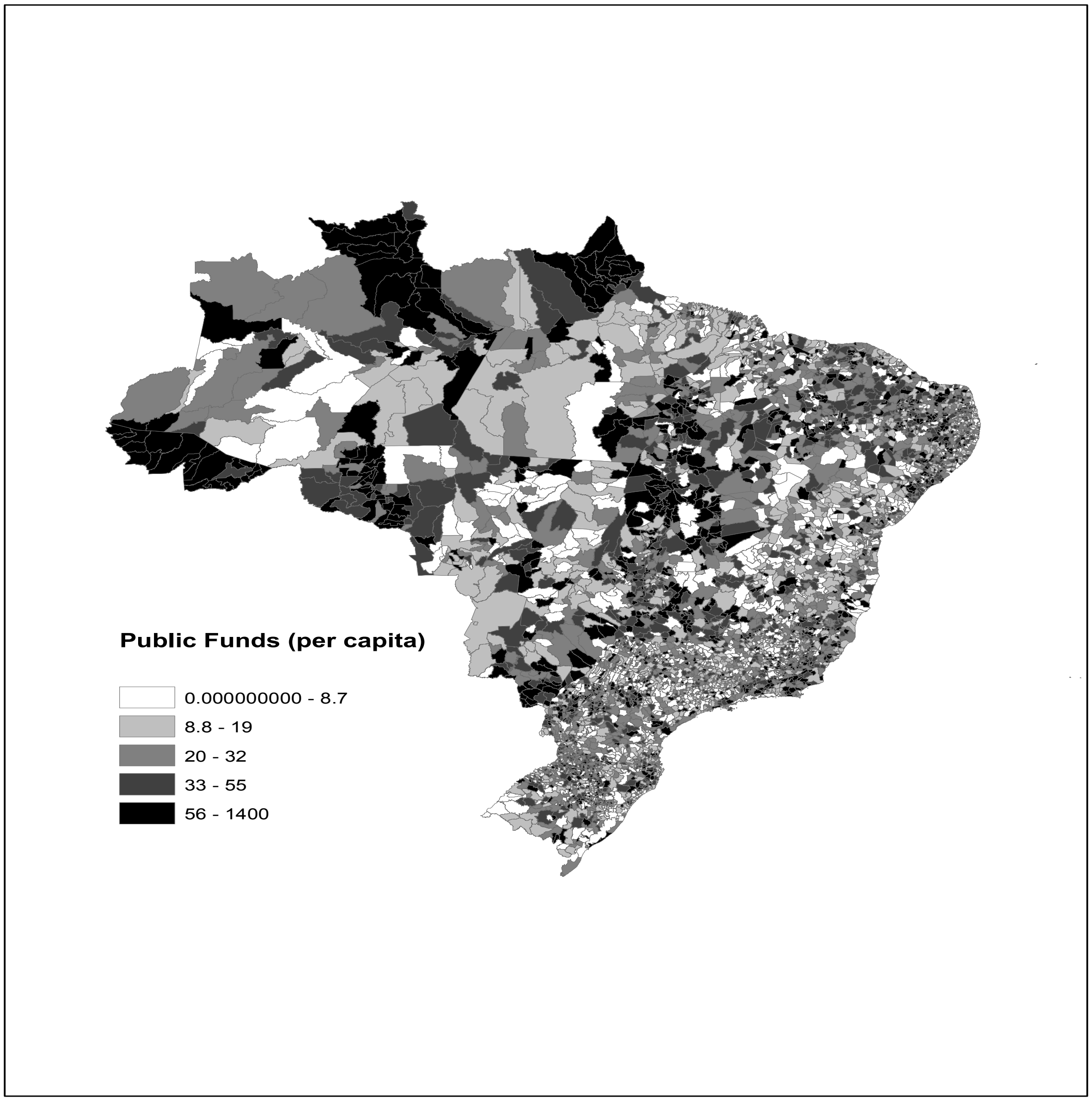


Figure A.1: Distribution of Budgetary Amendments Per Capita

Notes: The map depicts the distribution of public funds per capita during the 1996-1999 term by municipality.

B Appendix: Proof of Proposition 1

The proof is based on the existence results established in Milgrom and Weber (1985). To use their results is helpful to rewrite our model in the following way. Let

$$U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = \left(1-\beta^j\right) p^j\left(q, d_R\right) + \beta^j \sum_{m=1}^{M} W_m$$

and

$$U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = \bar{v}_{np}^j + \beta^j \sum_{m=1}^{M} W_m.$$

Then, deputy j chooses the optimal allocation and whether to run according to the following problem:

$$\max_{d_R^j}\left\{\max_{q^j}\int\left[U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right), \max_{q^j}\int U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\right]\sigma\left(d\beta^{-j}\right)\right\}$$

The problem can alternatively be written in the following form:

$$\max_{q^j, d_R^j} d_R^j \int U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right) + \left(1-d_R^j\right)\int U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right).$$

Or equivalently,

$$\max_{q^j, d_R^j}\int d_R^j U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) + \left(1-d_R^j\right) U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) d\sigma\left(\beta^{-j}\right).$$

We can therefore redefine the utility of deputy j as

$$U^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) = d_R^j U_R^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right) + \left(1-d_R^j\right) U_{NR}^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right).$$

Deputy j's problem can therefore be written as follows:

$$\max_{q^j, d_R^j}\int U^j\left(q^j, d_R^j, q^{-j}, d_R^{-j}; \beta^j, \beta^{-j}\right)\sigma\left(d\beta^{-j}\right).$$

We can now define a pure-strategy and a mixed-strategy Bayesian Nash equilibrium for this setting.

Definition 2 *A pure-strategy Bayesian Nash equilibrium is a vector of strategies* $s = \left(s^1, \ldots, s^J\right)$ *such that for every* $j \in J$*:*

$$s^j\left(\beta^j\right) = \underset{q^j, d_R^j}{\arg\max} \int U^j\left(q^j, d_R^j, s^{-j}\left(\beta^{-j}\right); \beta^j, \beta^{-j}\right) \sigma\left(d\beta^{-j}\right).$$

To define a mixed-strategy Bayesian Nash equilibrium, for every type β^j, let $m^j\left(s^j; \beta^j\right)$ be a probability measure over the strategy space S^j, and M^j player j's set of such mixed strategies. Then, we can extend the deputy j's utility to the set of mixed strategies by an expected utility calculation:

$$U^j\left(m^j, m^{-j}; \beta^j, \beta^{-j}\right) = \int_{S_1} \cdots \int_{S_J} U^j\left(s^j, s^{-j}; \beta^j, \beta^{-j}\right) m^1\left(ds^1; \beta^1\right) \ldots m^J\left(ds^J; \beta^J\right).$$

We can now introduce the mixed extension of the initial game in pure strategy $G = \left(S^j, U^j\right)_{j=1}^J$ as $\bar{G} = \left(M^j, U^j\right)_{j=1}^J$.

Definition 3 *A mixed-strategy* m^* *is a mixed-strategy Bayesian Nash equilibrium of the initial game* G *if* m^* *is a pure-strategy Bayesian Nash equilibrium of the extended game* $\bar{G}$*.*

Theorem 1, Propostion 1, and Proposition 3 in Milgrom and Weber (1985) establish that a game of incomplete information of the type considered in this paper has a mixed-strategy Bayesian Nash equilibrium if two conditions are satisfied: (i) the set of actions available to each player S^j is finite and (ii) the types of the players, $\beta^1, \ldots, \beta^J$, are drawn from independent distributions. In the model we estimate, each player has a finite set of actions since she can choose among four possible allocations of resources. Moreover, by assumption, types are drawn independently from the same distribution. Hence, a mixed-strategy Bayesian Nash equilibrium exists.

Let a pure-strategy ϵ-equilibrium is defined as a vector of strategies $m = \left(m^1, \ldots, m^J\right)$ such that for every player j and every alternative pure strategy $m^{j'}$,

$$\int U^j \left(m^j, m^{-j}; \beta^j, \beta^{-j}\right) \sigma \left(d\beta^{-j}\right) + \epsilon \geq \int U^j \left(m^{j'}, m^{-j}; \beta^j, \beta^{-j}\right) \sigma \left(d\beta^{-j}\right).$$

A Corollary to Theorem 1 in Milgrom and Weber (1985) establishes that a pure-strategy ϵ-equilibrium exists in a game of incomplete information if, in addition to the two conditions required for the existence of a mixed-strategy Bayesian Nash equilibrium, the players' beliefs over types are atomless. In the model we estimate, we do not need to make specific assumptions on the beliefs over types. Moreover, in the data we only observe the actions played by the deputies. Hence, we cannot make inference over the distribution of types. Our estimated model is therefore consistent with distributions over types that are atomless and distributions that are not atomless. It is therefore also consistent with a pure-strategy ϵ-equilibrium.