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Risky Human Capital Investment, Income Distribution, and Macroeconomic Dynamics

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ABSTRACT

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This paper analyzes the interaction between intergenerational wealth transmission, human capital investments under uninsurable labor income risk, and economic growth in a small open overlapping-generations economy with heterogeneous agents. It demonstrates how the role of the personal income distribution for an economy's process of development through risky human capital accumulation depends on the shape of the saving function. Consistent with recent empirical evidence, the analysis suggests that the impact of higher inequality on the aggregate human capital stock, and thus, on growth may be positive. This result rests on two features of the model, which both are largely supported by empirical evidence. First, as shown under weak conditions, children's human capital investments are positively affected by parents' income. Second, the marginal propensity to save is increasing in income.

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1   Introduction

This paper analyzes the interaction between intergenerational wealth transmission, human capital investments under uninsurable labor income risk, and economic growth in a small open overlapping-generations economy with heterogeneous agents. It demonstrates how the role of the personal income distribution for an economy's process of development through risky human capital accumulation depends on the shape of the saving function.

The analysis suggests that despite diminishing individual returns to educational investments the impact of higher inequality on the aggregate human capital stock and thus on growth may be positive during the transition to a stationary equilibrium. This result rests on two features of the model, which both are largely supported by empirical evidence. First, children's human capital investments are positively affected by parents' income.[1] For advanced countries, this evidence is surprising at the first glance because credit constraints for human capital investments do not seem to be binding for most individuals (or are, at least, negligible).[2] However, uninsurable labor income risk systematically affects incentives of risk-averse individuals to invest in human capital, an effect which has received surprisingly little attention in the growth literature.[3] For instance, individuals face idiosyncratic and nondiversifiable risk associated with labor demand shocks for specific skills. Moreover, among other sources, they face health and disability risk, uncertainty about the quality of schooling, and risk regarding access to social networks and other social factors which affect individual labor market prospects. The second crucial ingredient of the model which gives rise to a potentially positive relationship between inequality and growth is that the marginal propensity to save is

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[1] Empirical studies have controlled for parents' education, occupation and ability of both children and parents, still finding a positive and substantial effect of parents' income on human capital investments (e.g., Taubman, 1989; Sacerdote, 2002; Plug and Vijverberg, 2003).

[2] Mainly, because government lending for educational purposes largely overcomes this market failure.

[3] See Gould et al. (2001), Bénabou (2002) and Krebs (2003) for notable exceptions, which, however, are concerned with different questions. Gould et al. (2001) are concerned with the evolution of wage inequality and its interaction with technical progress. Bénabou (2002) examines the trade-offs of progressive income taxation for growth and efficiency, whereas Krebs (2003) studies the impact of labor income risk on growth in a framework with ex ante identical agents.

increasing in income (e.g., Menchik and David, 1983; Dynan et al., 2000, 2002). According to the model, adults save in order to bequeath or to make inter vivos gifts, respectively.[4] Intergenerational transfers are optimally allocated to human capital investments and savings for future wealth of the young.[5] With an increasing marginal propensity to save, higher inequality implies that the average reduction in wealth transmission of the rich may outweigh the average increase in wealth transmission of the poor despite diminishing individual returns to educational investments. Consequently, aggregate human capital investment may fall.

The question how the personal income distribution affects an economy's process of development has stimulated both macroeconomic theory and growth empirics in the last decade like almost no other one. Whereas earlier empirical evidence suggests a negative link between inequality and growth (e.g., Alesina and Rodrik, 1994; Persson and Tabillini, 1994; Perotti, 1996), using a new and comprehensive high-quality data set, Deininger and Squire (1998) and Banerjee and Duflo (2003) find practically none, whereas Li and Zou (1998) and Forbes (2000) report a positive relationship. Interestingly, allowing for a non-linear impact of inequality on GDP growth, Barro (2000) finds a negative relationship for developing countries and a positive relationship for more advanced countries.

A positive relationship between inequality and growth in advanced countries is difficult to reconcile with more recent theoretical approaches in the literature on income distribution and macroeconomics.[6] In their pioneering work, Galor and Zeira (1993) show that inequality typically has an adverse effect on the process of development if credit markets are imperfect ("credit-market imperfections approach"). This is because poor individuals cannot borrow sufficiently high amounts to finance an indivisible level of schooling investments.[7] Hence, the credit-market imperfections approach is consis-

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[4]Such a "joy of giving" saving motive has received strong empirical support. See Carroll (2000) for an illuminating discussion of the empirical evidence.

[5]The basic structure of the model builds on Galor and Moav (2004) in a way discussed throughout the paper.

[6]For surveys of this literature, see e.g. Aghion et al. (1999) and Grossmann (2001, ch. 1).

[7]As shown by Bénabou (1996) and Moav (2002), a negative relationship between inequaliy and growth can also be obtained by replacing this non-convexity in the education technology by the

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tent with evidence of a negative relationship between inequality and growth in *developing* countries. In contrast, in the theory proposed in this paper, individuals do not face any borrowing constraints to finance educational investments. Thus, the present model is particularly capable to shed light into the relationship between inequality and growth in *advanced* countries. Both the credit-market imperfections approach and the theory proposed in the present paper may thus be viewed as complementary.[8]

The mechanism underlying a potentially positive effect of inequality on growth proposed by the theory developed in this paper provides an alternative to the classical view, which argues that wealth inequality is positively related to investment-driven growth.[9] The foundation of the classical view by Bourguignon (1981) shows that under an increasing marginal propensity to save, unegalitarian stable equilibria are even Pareto superior to an egalitarian stable equilibrium in the neoclassical growth model of Stiglitz (1969). The reason for this result is that physical capital accumulation, fueled by domestic savings, raises wages such that growth "trickles down" to less wealthy individuals. This mechanism is excluded in the present small open economy framework.

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assumption of diminishing marginal returns to human capital investment. This implies that the aggregate human capital stock increases if educational investment is spread more equally. Another strand of literature deals with the role of imperfect capital markets for the relationship between wealth distribution and entrepreneurship, when project sizes (i.e., required physical capital investments to become entrepreneur) are fixed (e.g., Banerjee and Newman, 1991, 1993; Aghion and Bolton, 1997).

[8]Besides the credit market imperfections approach, it has been argued that high inequality is adversely related to growth because it induces high demand for redistributive taxation in the political process (e.g., Alesina and Rodrik, 1994; Persson and Tabellini, 1994), high fertility (e.g., Perotti, 1996), high social instability (e.g., Venieris and Gupta, 1986; Alesina and Perotti, 1996), low aggregate demand for R&D-intensive products (Zweimüller, 2000), and a low degree of specialization of labor (Fishman and Simhon, 2002). Some other contributions, although suggesting a positive relationship between inequality and growth, are relevant for developing contries only. For instance, if capital markets are imperfect and borrowing constraints are binding even for the rich (i.e., in very poor countries), redistribution to the rich enables more individuals to finance education (e.g., Perotti, 1993; Moav, 2002). Moreover, Galor and Tsiddon (1997) argue that for relatively poor economies, equality in the distribution of human capital may be an impedient to prosperity in the longer run under two conditions: first, the individuals' level of human capital positively depends on the parental level of human capital and, second, technological progress depends on the average level of human capital in the economy.

[9]Galor and Moav (2004) offer a unified approach which combines the classical view and the credit-market imperfections approach. Hypothesizing a co-linear saving function in a closed economy model, they show that inequality and growth are positively related in early stages of development when physical capital accumulation is the prime engine of growth, but are negatively related in mature stages of development when human capital accumulation is the prime engine of growth and borrowing constraints are still binding for the poor.

Rather, the model offers a mechanism which accounts for the crucial role of *human capital* accumulation under idiosyncratic human capital risk in modern societies. It shows that even in a small open economy, in which national savings are unrelated to physical capital investment, the relationship between inequality and an economy's the process of development critically depends on intergenerational wealth transmission and thus on savings behavior.

The paper is organized as follows. Section 2 presents the basic structure of the model. Section 3 analyzes individual education and saving decisions. Section 4 examines the role of inequality for aggregate income dynamics. Section 5 discusses the main ingredients and results of this paper in the light of empirical evidence. The last section concludes. Some technicalities as well as an illustrative example are relegated to an appendix.

2 The Model

Consider a small open overlapping-generations economy with uninsurable risk of educational investments.

2.1 Production of Final Output

In every period, a single homogenous consumption good is produced according to a neoclassical, constant-returns-to-scale production technology. Output at time t, Y_t, is

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t), \quad k_t \equiv K_t/H_t, \tag{1}$$

where K_t and H_t are the amounts of physical capital and human capital employed in period t, the latter being measured in efficiency units. $f(\cdot)$ is a strictly monotonic increasing and strictly concave function which fulfills $\lim_{k \to \infty} f'(k) = 0$ and $\lim_{k \to 0^+} f'(k) = \infty$.

Output is sold to the world market in a perfectly competitive environment, with output price normalized to unity. The rate of return to capital, r_t, is internationally

given and time-invariant, i.e., $r_t = \bar{r}$. Thus, profit maximization of the representative firm in any period t implies that k_t is given by $\bar{r} = f'(k_t)$. Thus, $k_t = (f')^{-1}(\bar{r}) \equiv \bar{k}$. Consequently, the wage rate per efficiency unit of human capital, w_t, reads $w_t = f(\bar{k}) - \bar{k}f'(\bar{k}) \equiv \bar{w}$. Moreover, since $Y_t = H_t f(\bar{k})$ in this economy, Y_t (i.e., the gross domestic product) grows at the same rate as the aggregate human capital stock H_t.[10]

2.2 Individuals and Education Technology

In each period, there is a unit mass of individuals with two-period lives. In the first period, individuals live by their parents and devote their entire time to acquire education. In the second period (adulthood), individuals supply their efficiency units of human capital to the labor market, and allocate their income between consumption and transfers to their offspring (i.e., bequests or inter vivos transfers, respectively). Intergenerational transfers (i.e., savings of adults) are optimally allocated (either by parent or child) between human capital investment and savings of the young for future wealth.[11] Individuals are identical with respect to their preferences and their ability to acquire human capital, but may differ in family wealth. So far, this overlapping-generations structure follows Galor and Moav (2004). However, in contrast to their model, individuals face idiosyncratic human capital risk. Moreover, in order to focus on this aspect and to make the analysis particularly applicable to advanced countries in which credit constraints to finance higher education seem to play a minor role, I assume that individuals can freely borrow for educational purposes, e.g. due to government lending.[12] (In contrast, developing countries do not have this type of institution, and thus are not focus of the present analysis.) However, as a matter of fact, even in advanced countries human capital risk is uninsurable (e.g. Arrow, 1971), which is

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[10]The capital-skill complementarity underlying production function (1) is empirically well supported; see e.g. Goldin and Katz (1998). This technology is common in the literature on income distribution, human capital and growth.

[11]Human capital investments can be thought of both schooling and nonschooling forms of training.

[12]The present model also differs to Galor and Moav (2004) in that our small open economy assumption excludes the feedback mechanism from aggregate savings to factor prices which also underlies the results in Bourguignon (1981). Moreover, in order to study the role of savings behavior for the relationship between inequality and growth, no particular functional form on utility is imposed.

therefore assumed.

An individual i born in period t (a member i of generation t) with investment e_t^i (in units of the consumption good) in education obtains

$$h_{t+1}^i = h(e_t^i, \tilde{a}) \tag{2}$$

efficiency units of human capital. $\tilde{a}$ is a random variable which follows an i.i.d. process and is drawn each period from a (cumulative) distribution function $\Phi(\tilde{a})$ with support $\mathcal{A} = [\underline{a}, \bar{a}] \subset \mathbb{R}$. The random shock realizes only after investment decisions are made, i.e., in the beginning of the second period of life. The function $h(e, a)$ fulfills the following properties.

A1. For all $a \in \mathcal{A}$, $h_e(e,a) > 0$, $h_{ee}(e,a) < 0$, $h_a(e,a) > 0$, $h_{ea}(e,a) > 0$, $\lim_{e \to \infty} h_e(e,a) = 0$ and $\lim_{e \to 0^+} h_e(e,a) = \infty$.

(h_e denotes the first partial derivative of h with respect to e, etc.) $h_{ee} < 0$ implies that expected marginal returns to educational investment are diminishing.[13] Moreover, given that $h_a > 0$ which merely serves as a convention, $h_{ea} > 0$ implies that the variance of earnings increases with human capital investment e.[14] Finally, the latter two conditions in A1 ensure interior solutions with respect to educational investment decisions.[15]

Denote by s_t^i and b_t^i the amount of savings invested in the financial market and the

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[13] This reflects the fact that "human capital is inherently embodied in humans and the existence of physiological constraints subjects its accumulation at the individual level to diminishing returns" (Galor and Moav, 2004, p. ??).

[14] See Levhari and Weiss (1974) for a discussion of this assumption and supporting empirical evidence. For more recent evidence, see Pereira and Martins (2002, 2003). A similar type of risk also underlies the model of Bénabou (2002). There are other notions of labor income risk. For instance, Gould et al. (2001) argue that an increasing variance of sectoral shocks increase educational attainment of workers which differ in ability because general education reduces the costs of moving across sectors.

[15] The modelling strategy to assume that human capital investment is riskier than physical capital investment seems plausible (Krebs, 2003). First, human capital risk is nondiversifiable since embodied in individuals, whereas diversified portfolios of financial capital can be held. Second, many forms of financial assets in advanced countries are indeed almost risk-free (e.g. government bonds).

amount of wealth received by member i of generation t, respectively, i.e., $s_t^i = b_t^i - e_t^i$. Thus, income of member i of generation t as an adult is given by

$$I_{t+1}^i = \bar{w} h_{t+1}^i + \bar{R} s_t^i = \bar{w} h(e_t^i, \tilde{a}) + \bar{R}\left(b_t^i - e_t^i\right) \equiv I(b_t^i, e_t^i, \tilde{a}), \tag{3}$$

where $\bar{R} \equiv 1 + \bar{r}$. Utility U_t^i of member i of generation t is given by a utility function u which is defined over consumption c_{t+1}^i as an adult and transfer b_{t+1}^i to her offspring ("joy of giving"), i.e.,

$$U_t^i = u(c_{t+1}^i, b_{t+1}^i). \tag{4}$$

A2. $u_c > 0$, $u_b > 0$, $u_{cc} < 0$, $u_{cc} u_{bb} - (u_{cb})^2 > 0$, $u_{cb} \geq 0$ and $\lim_{I \to \infty} u_c(I, 0) < \lim_{I \to \infty} u_b(I, 0)$.

Thus, u is strictly monotonic increasing and strictly concave, which, as will become apparent below, implies risk aversion of individuals. Moreover, the latter two relations in A2 imply normality of intergenerational transfers b_{t+1}^i, which is the empirically relevant case.

Finally, assume that there are two groups of dynasties in the initial period $t = 0$. A fraction $\lambda \in (0, 1)$ of ("rich") young individuals in $t = 0$ receives a transfer $b_0^R > 0$ and a fraction $1 - \lambda$ of ("poor") individuals receives a transfer $b_0^P \in [0, b_0^R)$ from their parent. Thus, in the aggregate, an amount $B_0 \equiv \lambda b_0^R + (1 - \lambda) b_0^P$ is initially transferred. Adult individuals possess an aggregate human capital stock H_0 in the initial period, i.e., initial output is $Y_0 = H_0 f(\bar{k})$.[16]

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[16]Introducing endogenous growth, e.g. by assuming that the aggregate human capital stock H_t enters the education technology as positive externality (following Glomm and Ravikumar, 1992, among others), i.e., letting $h_{t+1}^i = h(e_t^i, \tilde{a}, H_t)$, does not alter the main insights of this paper. The main focus lies on the impact of initial inequality (in $t = 0$) on subsequent growth averaged over a longer period (i.e. on $Y_t/Y_0 - 1$, and thus on Y_t, $t \geq 1$), in order to address the empirical literature on inequality and growth (see section 5.1 for a discussion). Introducing endogenous growth through human capital accumulation simply implies that initial inequality also affects the period-by-period growth rate $Y_t/Y_{t-1} - 1$ in a qualitatively similar fashion as Y_t both during transition to a stationary equilibrium and in the long run.

3 Individual Decisions

Note that income $I_{t+1}^i = I(b_t^i, e_t^i, \tilde{a})$ of an adult member i of generation t is a random variable ex ante, but is known in period $t+1$ (i.e., after realization of the shock) when allocating income to consumption and transfer to her offspring. The budget constraint of such an individual in $t+1$ reads $c_{t+1}^i + b_{t+1}^i \leq I_{t+1}^i$. Thus, under the additional constraint $b_{t+1}^i \geq 0$, her optimal transfer in $t+1$ is given by

$$b(I_{t+1}^i) \equiv \underset{b_{t+1}^i \geq 0}{\arg\max}\, u(I_{t+1}^i - b_{t+1}^i, b_{t+1}^i). \tag{5}$$

$b(I)$ is called "saving function" (as b_{t+1}^i equals forgone consumption of an adult) and has the following properties.

Lemma 1. *Under A2. There exists an income level $\underline{I} \geq 0$ such that $b(I_{t+1}^i) > 0$ and $b'(I_{t+1}^i) > 0$ for all $I_{t+1}^i > \underline{I}$.*

Proof. Note that, according to (5), b_{t+1}^i is implicitly given by the first-order condition

$$\Omega(b_{t+1}^i, I_{t+1}^i) \equiv -u_c(I_{t+1}^i - b_{t+1}^i, b_{t+1}^i) + u_b(I_{t+1}^i - b_{t+1}^i, b_{t+1}^i) \leq 0, \tag{6}$$

which holds with equality if $b_{t+1}^i > 0$. According to (6), $\Omega_b = u_{cc} - 2u_{cb} + u_{bb}$ and $\Omega_I = -u_{cc} + u_{cb}$, i.e., A2 implies $\Omega_I > 0$ and $\Omega_b < 0$.[17] First, suppose $b_{t+1}^i = 0$. Since $\Omega_I > 0$, the left-hand side of the inequality in (6) is strictly increasing in I_{t+1}^i. Hence, using $\lim_{I \to \infty} u_c(I, 0) < \lim_{I \to \infty} u_b(I, 0)$ from A2, eventually, $b_{t+1}^i > 0$ if income I_{t+1}^i exceeds some level $\underline{I} \geq 0$. Second, for $b_{t+1}^i > 0$, applying the implicit function theorem reveals

$$b'(I) = -\frac{\Omega_I}{\Omega_b} = \frac{u_{cc} - u_{cb}}{u_{cc} - 2u_{cb} + u_{bb}} > 0 \tag{7}$$

under A2. ∎

From the optimal allocation of income earned as an adult, we can derive the fol-

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[17]To confirm $\Omega_b < 0$, use $u_{cc} < 0$ and $u_{cc}u_{bb} - (u_{cb})^2 > 0$ from A2 to obtain $u_{bb} < (u_{cb})^2/u_{cc}$. Thus, $\Omega_b = u_{cc} - 2u_{cb} + u_{bb} < u_{cc} - 2u_{cb} + (u_{cb})^2/u_{cc} = (u_{cc} - u_{cb})^2/u_{cc} < 0$. Of course, this is nothing else than showing that strict concavity of u implies strict quasiconcavity of u.

lowing properties of indirect life-time utility,

$$v(I_{t+1}^i) \equiv u(I_{t+1}^i - b(I_{t+1}^i), b(I_{t+1}^i)). \tag{8}$$

Lemma 2. *Under A2. $v(I)$ is a strictly monotonic increasing and strictly concave function.*

Proof. First, if $b_{t+1}^i = b(I_{t+1}^i) = 0$, we have $v(I_{t+1}^i) = u(I_{t+1}^i, 0)$; thus, $v'(I) = u_c(I, 0) > 0$ and $v''(I) = u_{cc}(I, 0) < 0$. If $b(I) > 0$, then $v'(I) = u_c(I - b(I), b(I)) > 0$, according to (8), (6) and the envelope theorem. Thus, $v''(I) = u_{cc} + b'(I)(u_{cb} - u_{cc})$. Substituting (7) into the latter expression, we obtain $v''(I) = u_{cc} - (u_{cb} - u_{cc})^2/(u_{cc} - 2u_{cb} + u_{bb})$. Manipulating the latter expression implies $v''(I) = \left[u_{cc}u_{bb} - (u_{cb})^2\right]/\Omega_b$ (recall $\Omega_b = u_{cc} - 2u_{cb} + u_{bb} < 0$). Hence, $v''(I) < 0$, according to the strict concavity of $u(c, b)$ presumed (assumption A2). This concludes the proof. ∎

According to Lemma 2, individuals are risk-averse. Throughout the remainder of the paper, the widely accepted assumption of "decreasing absolute risk aversion" is maintained.

A3. $A(I) \equiv -v''(I)/v'(I)$ is strictly decreasing in I.[18]

An amount of transfers b_t^i received by a member i of generation t is allocated to savings for future wealth, s_t^i (with a safe return $\bar{R}$), and the risky investment in education, e_t^i, to maximize expected life-time utility $E\left[v(I_{t+1}^i)\right] = E\left[v(I(b_t^i, e_t^i, \tilde{a}))\right]$, where E is the expectation operator. Thus, using (3), the optimal human capital investment is given by

$$e(b_t^i) \equiv \arg\max_{e_t^i \geq 0} E\left[v\left(\bar{w}h(e_t^i, \tilde{a}) + \bar{R}\left(b_t^i - e_t^i\right)\right)\right]. \tag{9}$$

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[18]Note that $A'(I) < 0$ if and only if $-v'''(I)/v''(I) > A(I)$, i.e., $-v'(I)$ is "more concave" that $v(I)$. See, e.g., Gollier (2001) for a discussion of the plausibility of this assumption. See also Carroll (2002) for empirical evidence which is consistent with decreasing absolute risk aversion in the context of portfolio decisions in financial markets.

One can then derive the following result.

Proposition 1. (Human capital investment). *Under A1 and A3, the human capital investment is strictly increasing in family wealth, i.e., $e'(b_t^i) > 0$.*

Proof. See appendix A. ∎

Proposition 1 coincides with a result (derived from a two-period model with exogenous wealth) of the pioneering work by Levhari and Weiss (1974). (See also Eaton and Rosen, 1980.) The intuition for the result is the following. Under convention $h_a > 0$, the assumption $h_{ea} > 0$ implies that risk is increasing with the level of investment in human capital. In contrast, investing in physical capital (i.e., financial assets) is risk-free. Hence, if the degree of absolute risk aversion, $A(I)$, is decreasing in income I, such that $A(I_{t+1}^i) = A(I(b_t^i, \cdot, \cdot))$ is decreasing in b_t^i, then individuals with larger b_t^i invest more in risky education.

In fact, $e'(b_t^i) > 0$ is well-supported empirically even in advanced economies in which credit constraints seem to play a negligible role for human capital investments.[19] (See section 5.3 for a brief review of empirical evidence.) There is no theoretical prediction, however, how the marginal propensity to invest in education, $e'(b_t^i)$, changes with b_t^i. Moreover, the rare existing empirical evidence on the sign of $e''(b)$ is rather inconclusive.[20] In order to focus the analysis, I shall thus suppose the following.

A4. The impact of a change in family wealth b_t^i on the magnitude of the marginal propensity to invest in education is negligible, i.e., $|e''(b_t^i)| \approx 0$ for all $b_t^i \in \mathbb{R}_+$.

[19]Note that without uncertainty, i.e., if $\underline{a} = \bar{a} \equiv a$, the optimal schooling investment is given by $Wh_e(e_t^i, a) = R$, according to (9). Thus, under certainty, e_t^i is independent of b_t^i. For instance, this coincides with a result by Galor and Moav (2004) when credit constraints are not binding in their model.

[20]There exist some estimates for the impact of parental income on children's earnings which allow for non-linearity. Whereas Becker and Tomes (1986) suggest that the marginal impact is diminishing, if anything, Behrman and Taubman (1990) find a positive marginal impact. To see how these findings relate to the present model, first, define earnings of a member i of generation t as function of her parent's income (suppressing $\tilde{a}$): $E(I_t^i) \equiv \bar{w}h(e(b(I_t^i)), \cdot)$, where the relationships $e_t^i = e(b_t^i)$ and $b_t^i = b(I_t^i)$ have been used. Thus, $E'(I) = \bar{w}h_e(e(b(I)), \cdot)e'(b(I))b'(I)$ and $E''(I) = \bar{w}\left[h_{ee}e'(b)^2b'(I)^2 + h_e e''(b)b'(I)^2 + h_e e'(b)b''(I)\right]$. Thus, for instance, $E''(I) \approx 0$ if $e''(b) \approx 0$ and the effects driven $h_{ee} < 0$ and $b''(I) > 0$ (which is the empirically relevant case as argued in section 5.2) approximately cancel.

Assumption A4 is not crucial for any of the mechanisms discussed in the subsequent analysis. However, it helps to focus the discussion on the role of diminishing marginal returns to human capital and the effects arising from intergenerational wealth transmission for the inequality-growth relationship.

4 The Role of Inequality for Income Dynamics

This section studies the aggregate behavior of the economy which results from individual decisions analyzed in the preceding section. In particular, it is examined how inequality in initial family wealth, for a given initial aggregate transfer B_0 (and conditional on initial GDP, $Y_0 = H_0 f(\bar{k})$), affects the process of development. For this purpose, it is useful to recall the dynamical system of the considered economy.

A given transfer b_t^i to a member i of generation t is optimally allocated to savings, s_t^i, and education investments, $e_t^i = e(b_t^i)$, in period t (Proposition 1). According to (2), this leads to an individual amount

$$h_{t+1}^i = h(e(b_t^i), \tilde{a}) \equiv \hat{h}(b_t^i, \tilde{a}) \tag{10}$$

of efficiency units of human capital, supplied during adulthood (which is a random variable). Thus, denoting the economy's c.d.f. of family wealth in period t by $\Psi_t(b)$, with support $\mathcal{B}_t \subset \mathbb{R}_+$,[21] the aggregate human capital stock at $t+1$ is given by

$$H_{t+1} = \int_{\mathcal{A}} \int_{\mathcal{B}_t} \hat{h}(b, \tilde{a}) d\Psi_t(b) d\Phi(\tilde{a}). \tag{11}$$

Aggregate income is given by $Y_{t+1} = H_{t+1} f(\bar{k})$.[22]

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[21]Note from the assumptions on initial conditions that $\mathcal{B}_0 = \{b_0^P, b_0^R\}$ and $\Psi_0(b) = 0$ for $0 \le b < b_0^P$, $\Psi_0(b) = 1 - \lambda$ for $b_0^P \le b < b_0^R$, and $\Psi_0(b) = 1$ for $b \ge b_0^R$.

[22]Note that the human capital risk considered in the model is consistent with risk associated with skill specificity in the following sense. Suppose individuals acquire skills which are applicable in a single "industry" only (which may also be interpreted as specific task) and there are ideosyncratic productivity shocks across industries (Wildasin, 2000). To see that this is consistent with the risk considered here, suppose there is a continuum $[0,1]$ of intermediate goods industries, indexed by j. Output $q_t(j)$ in industry j at t is produced with industry-specific human capital $\hat{H}_t(j) \equiv \int_{\mathcal{B}_t} \hat{h}(b, \theta(j)) d\Psi_t(b)$,

According to (3) and (10), given realization a of the random variable $\tilde{a}$ after educational investments are made, income in $t+1$ of an adult individual i reads

$$I_{t+1}^i = \bar{w}h(e(b_t^i), a) + \bar{R}\left(b_t^i - e(b_t^i)\right) = I(b_t^i, e(b_t^i), a) \equiv \hat{I}(b_t^i, a). \tag{12}$$

I_{t+1}^i is then optimally allocated to consumption, c_{t+1}^i, and transfers to the offspring,

$$b_{t+1}^i = b(I_{t+1}^i) = b(\hat{I}(b_t^i, a)) \equiv \hat{b}(b_t^i, a). \tag{13}$$

According to (13), the wealth transfer within each dynasty i follows a discrete time Markov process defined by the difference equation $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$.

Due to the small open economy assumption, there are no feedback effects through factor price changes from aggregate variables to individual behavior. This not only simplifies the analysis but also excludes the mechanism suggested by the classical view: i.e., that higher inequality fosters growth fueled by domestic accumulation of physical capital if the marginal propensity to save is increasing. It will turn out, however, that wealth accumulation nevertheless plays a fundamental role for the inequality-growth relationship by affecting resources available for human capital accumulation at the individual level.

For simplicity, let me restrict attention to the case in which even for the worst realizations of $\tilde{a}$, income of an adult increases (without bound) with the amount of transfer received as child.[23] Formally, this means the following.

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where where $\theta(j)$ is the realization of an i.i.d. shock $\tilde{\theta}$. (Note that because industries are symmetric and b and $\tilde{\theta}$ are independently distributed, skill supply across industries is fully symmetric.) Suppose this production technology simply reads $q_t(j) = \hat{H}_t(j)$. Thus, the the aggregate stock of human capital in period $t+1$, $H_{t+1} = \int_{j \in [0,1]} \hat{H}_t(j)dj = \int_{j \in [0,1]} q_t(j)dj$ can be thought of a composite input of (perfectly substitutable) intermediate goods. Let $\tilde{\theta}$ and $\tilde{a} \in \mathcal{A}$ (still with c.d.f. $\Phi(\tilde{a})$) be related such that $\theta(j) \equiv \Phi^{-1}(j)$ is the realization of the shock in industry $j \in [0,1]$ (i.e., after realization of shocks, industries are ordered such that $j = \Phi(a)$, where a is a realization of $\tilde{a}$). Thus, one can write $H_{t+1} = \int_{j \in [0,1]} \int_{\mathcal{B}_t} \hat{h}(b, \tilde{\theta}(j))d\Psi_t(b)dj = \int_{\mathcal{A}} \int_{\mathcal{B}_t} \hat{h}(b, \tilde{a})d\Psi_t(b)d\Phi(\tilde{a})$, which coincides with (11).

[23]Moreover, it is implicitly assumed throughout the paper that any young individual with zero wealth is able to pay back the loan $\bar{R}e(0)$, which equals such an individual's optimal amount of lending, by her labor income even for the worst realization of the shock $\underline{a}$, i.e., $\hat{I}(0, \underline{a}) = \bar{w}h(e(0), \underline{a}) - \bar{R}e(0) \geq 0$.

A5. For all $a \in \mathcal{A}$, $\hat{I}_b(b, a) > 0$, $b \in \mathbb{R}_+$, and $\lim_{b \to \infty} \hat{I}(b, a) = \infty$.[24]

The next two results characterize the Markov process $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$.

Lemma 3. *Under A2 and A5, for all $a \in \mathcal{A}$. There exists $\underline{b}_a \geq 0$ such that $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$.*

Proof. Recall from Lemma 1 (which is implied by A2) that there exists $\underline{I} \geq 0$ such that $b(I) > 0$ and $b'(I) > 0$ if $I > \underline{I}$. Also recall from (13) that $\hat{b}(b, a) = b(\hat{I}(b, a))$, which implies $\hat{b}_b(b, a) = b'(\hat{I}(b, a))\hat{I}_b(b, a)$. Thus, under A5, for all $a \in \mathcal{A}$, a gradual increase in b eventually must lead to a level of b, denoted by $\underline{b}_a$, such that $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$. This concludes the proof. ∎

For notational simplicity in what follows, suppress the random variable $\tilde{a}$ by defining wealth and income one period after (indicated by subscript '1') the level of wealth started in b_t^i as

$$\tilde{b}_1(b_t^i) \equiv \hat{b}(b_t^i, \tilde{a}), \tag{14}$$

$$\tilde{I}_1(b_t^i) \equiv \hat{I}(b_t^i, \tilde{a}). \tag{15}$$

Lemma 4. *Under A1-A5, for any realization $a \in \mathcal{A}$ of the random shock $\tilde{a}$. (i) If $b''(I) \leq 0$, then $\tilde{b}_1''(b) \leq 0$. (ii) If, by contrast, $b''(I) > 0$, then the sign of $\tilde{b}_1''(b)$ is ambiguous.*[25]

Proof. First, note that (whenever differentiable)

$$\tilde{b}_1'(b) = b'(\tilde{I}_1(b))\tilde{I}_1'(b), \tag{16}$$

---

[24]Using (12), it is easy to check that $e'(b) \leq 1$ is sufficient for $\hat{I}_b(b, a) > 0$ to hold. That is, if a marginal increase in b does not lead to a decline of investment in the financial market, then $\hat{I}_b > 0$. However, although plausible, $e'(b) \leq 1$ is not ensured by the assumptions made so far. One can show, for instance, that $e'(b) < 1$ if, for all $a \in \mathcal{A}$, $-\frac{A'(\hat{I}(b,a))\bar{w}}{A(\hat{I}(b,a))} \geq \frac{h_{e\tilde{a}}(e(b),a)}{h_{\tilde{a}}(e(b),a)h_e(e(b),a)}$, or if $|h_{ee}|$ is sufficiently large.

[25]Note that in the case $b(I) = 0$ for $I \leq \underline{I}$ and $b(I) > 0$ for $I > \underline{I}$, $\underline{I} \geq 0$, the function $b(I)$ is not differentiable at $I = \underline{I}$ (and $b'(I) > 0$ for $I > \underline{I}$ under A2, according to Lemma 1). The subsequent analysis neglects this for simplicity, implicitly stating results for $I \neq \underline{I}$ only.

according to (13)-(15). Hence,

$$\tilde{b}_1''(b) = b''(\tilde{I}_1(b))\tilde{I}_1'(b)^2 + b'(\tilde{I}_1(b))\tilde{I}_1''(b). \tag{17}$$

Moreover, according to (12) and (15), we have

$$\tilde{I}_1'(b) = \left(\bar{w}h_e(e(b), a) - \bar{R}\right)e'(b) + \bar{R}, \tag{18}$$

$$\tilde{I}_1''(b) = \bar{w}h_{ee}(e(b), a)e'(b)^2 + \left(\bar{w}h_e(e(b), a) - \bar{R}\right)e''(b), \tag{19}$$

$a \in \mathcal{A}$. Under A4, we can neglect the second summand on the right-hand side of (19), i.e., $\tilde{I}_1''(b) < 0$, according to $h_{ee} < 0$ and $e'(b) > 0$ (Proposition 1) under A1 and A3. Hence, under A2 (which implies $b'(I) \geq 0$, according to Lemma 1), we have $\tilde{b}_1''(b) \leq 0$ if $b''(I) \leq 0$, according to (17). This confirms part (i). To confirm part (ii), note that, if $b''(I) > 0$, the sign of the first summand on the right-hand side of (17) is strictly positive for all $a \in \mathcal{A}$ under A5. This concludes the proof. ∎

According to Lemma 4, if the saving function $b(I)$ is concave (i.e., if $b''(I) \leq 0$), then the transfer of an adult to her offspring will be concave in the transfer she received as a child (i.e., $\tilde{b}_1''(b) \leq 0$). However, if the marginal propensity to save (for adults) is increasing (i.e., if $b''(I) > 0$), the transfer of an adult to her offspring may be convex in the transfer received herself as a child (i.e., $\tilde{b}_1''(b) > 0$ is possible). The empirical relevance of an increasing marginal propensity to save (and bequeath, respectively) is discussed in section 5.2. Its consequences for the process of development are analyzed in the following.

4.1 Inequality and the Process of Development

Recall that, initially, there are two groups of individuals, rich and poor, and the aggregate initial transfer is $B_0 = \lambda b_0^R + (1 - \lambda)b_0^P$. To study the role of inequality for the growth process, suppose the distribution of initial transfers changes (in a lump-sum

fashion) to

$$\breve{b}_0^R \equiv b_0^R - \varepsilon, \quad \breve{b}_0^P \equiv b_0^P + \varepsilon\lambda/(1-\lambda), \tag{20}$$

i.e., aggregate family wealth, B_0, is held constant. Under restriction $\varepsilon < (1-\lambda)(b_0^R - b_0^P)$, i.e., as long as $\breve{b}_0^P < \breve{b}_0^R$, the economy is said to be more equal, the higher ε. The following analysis derives comparative-static results with respect to changes in ε.[26]

4.1.1 Short Run Impact of Higher Equality

Let $g_{s,t} \equiv (Y_s - Y_t)/Y_t$ be the growth rate of aggregate output (or GDP, respectively) between periods s and t, $s > t \geq 0$. First, consider the impact of an increase in ε on the aggregate human capital stock H_1, which gives us the *short run* effect of higher equality on aggregate (or per capita) income, $Y_1 = H_1 f(\bar{k})$, and thus, on the initial growth rate $g_{1,0} = H_1/H_0 - 1$ of the economy (recall $Y_0 = H_0 f(\bar{k})$). Again, suppressing the random variable for notational simplicity, let

$$\tilde{h}_1(b_t^i) \equiv \hat{h}(b_t^i, \tilde{a}). \tag{21}$$

Under (20), the aggregate human capital stock in period 1, H_1, can be written as

$$\begin{aligned} H_1 &= E\left[\lambda\tilde{h}_1(\breve{b}_0^R) + (1-\lambda)\tilde{h}_1(\breve{b}_0^P)\right] \\ &= E\left[\lambda\tilde{h}_1\left(b_0^R - \varepsilon\right) + (1-\lambda)\tilde{h}_1\left(b_0^P + \varepsilon\lambda/(1-\lambda)\right)\right] \equiv \hat{H}_1(\varepsilon), \end{aligned} \tag{22}$$

according to (11) and the specification of the initial distribution of transfers. From this, we obtain the following result.

Proposition 2. (Impact of higher equality in the short run). *Under A1, A3 and A4, higher equality of initial family wealth is associated with higher aggregate income Y_1 (and thus, faster growth $g_{1,0}$), i.e., $\hat{H}_1'(\varepsilon) > 0$.*

---

[26]Note that income inequality and inequality of family wealth are closely related, according to Lemma 1.

Proof. According to (22), differentiating $\hat{H}_1(\varepsilon)$ with respect to ε yields

$$\hat{H}_1'(\varepsilon) \;=\; \lambda E\left[\tilde{h}_1'(\breve{b}_0^P) - \tilde{h}_1'(\breve{b}_0^R)\right], \text{ where} \tag{23}$$

$$\tilde{h}_1'(b) \;=\; h_e(e(b), a)e'(b), \tag{24}$$

$a \in \mathcal{A}$, according to (10) and (21). Since $\breve{b}_0^P < \breve{b}_0^R$, (23) implies that $\hat{H}_1'(\varepsilon) > 0$ if, for instance, for all $a \in \mathcal{A}$ and for all $b \in \mathbb{R}_{++}$, $\tilde{h}_1''(b) < 0$. Note that

$$\tilde{h}_1''(b) = h_{ee}(e(b), a)e'(b)^2 + h_e(e(b), a)e''(b), \tag{25}$$

according to (24). The first summand on the right-hand side of (25) is negative since $e'(b) > 0$, according to Proposition 1 (which holds under A1 and A3), and $h_{ee} < 0$. Thus, if $e''(b) \leq 0$, or if $e''(b)$ is positive but small in magnitude as supposed in A4, we have $\tilde{h}_1''(b) < 0$. This confirms the result. ∎

Proposition 2 is in line with a standard result in the literature on inequality and growth when growth is driven by human capital investments (e.g., Galor and Zeira, 1993; Bénabou, 1996; Moav, 2002). Intuitively, higher equality fosters growth when the poor have a higher expected marginal return to education. This applies when two conditions are met. First, individual human capital investment is an increasing function of wealth (usually derived from borrowing constraints rather than from uncertainty as in the present paper), which holds since $e'(\cdot) > 0$, according to Proposition 1. Second, marginal returns to education are diminishing, which is reflected by the assumption $h_{ee} < 0$ in A1. Hence, the aggregate human capital stock typically increases in the short run if (initial) wealth, and thus, if (initial) educational investment is spread over the population more equally. However, as will become apparent in the following, due to the effects of intergenerational wealth transmission, this may not hold anymore in the medium run.

4.1.2 Medium Run Impact of Higher Equality

Given family wealth b_t^i in period t, human capital of a member i of generation $t+1$ in period $t+2$ is

$$h_{t+2}^i = \hat{h}(b_{t+1}^i, \tilde{a}) = \tilde{h}_1(b_{t+1}^i) = \tilde{h}_1(\hat{b}(b_t^i, \tilde{a})) = \tilde{h}_1(\tilde{b}_1(b_t^i)) \equiv \tilde{h}_2(b_t^i), \qquad (26)$$

according to (10), (21), (13) and (14). For later use, the following auxiliary results are established.

Lemma 5. *Under A1-A3 and A5, for any realizations of random shocks, $\tilde{h}_2'(\cdot) \geq 0$.*
Proof. First, note that

$$\tilde{h}_2'(b) = \tilde{h}_1'(\tilde{b}_1(b))\tilde{b}_1'(b) = h_e(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))\tilde{b}_1'(b) \qquad (27)$$

according to (10), (21) and (26). Substituting (16) into (27) leads to

$$\tilde{h}_2'(b) = h_e(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))b'(\tilde{I}_1(b))\tilde{I}_1'(b). \qquad (28)$$

Under A2, (whenever differentiable) $b'(\cdot) \geq 0$, according to Lemma 1, and, under A1 and A3, $e'(\cdot) > 0$, according to Proposition 1. Moreover, $\tilde{I}_1'(b) > 0$ under A5. Thus, recalling $h_e > 0$, Lemma 5 follows from (28). ∎

Lemma 6. *Under A1-A5, for any realizations of random shocks. If $b''(I) \leq 0$, then $\tilde{h}_2''(b) \leq 0$. (ii) If, by contrast, $b''(I) > 0$, then the sign of $\tilde{h}_2''(b)$ is ambiguous.*
Proof. Note that (27) implies

$$\tilde{h}_2''(b) = h_{ee}(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))^2\tilde{b}_1'(b)^2 + h_e(e(\tilde{b}_1(b)), \tilde{a})\left[e''(\tilde{b}_1(b))\tilde{b}_1'(b)^2 + e'(\tilde{b}_1(b))\tilde{b}_1''(b)\right]. \qquad (29)$$

The first summand on the right-hand side of (29) is non-positive since $h_{ee} < 0$. Moreover, if $|e''(b)|$ is small in magnitude as supposed in A4, the first term in square brackets of (29) is negligible. Regarding the second term in square brackets, if $b''(I) \leq 0$, then

$\tilde{b}_1''(\cdot) \leq 0$ according to part (i) of Lemma 4. In addition, recall $e'(\cdot) > 0$ from Proposition 1 (which holds under A1 and A3). Thus, if $b''(I) \leq 0$, then the second term in square brackets of (29) is non-positive. Hence, $\tilde{h}_2''(b) \leq 0$ if $b''(I) \leq 0$, confirming part (i) of Lemma 6. However, if $b''(I) > 0$, then we may have $\tilde{b}_1''(b) > 0$, according to part (ii) of Lemma 4. Consequently, we may have $\tilde{h}_2''(b) > 0$ if $b''(I) > 0$, confirming part (ii) of Lemma 6. This concludes the proof. ■

Given $\breve{b}_0^i$, the intergenerational transfer within dynasty i in period 1, after realization $a_0 \in \mathcal{A}$ of the shock at $t = 0$, is given by $\breve{b}_1^i \equiv \hat{b}(\breve{b}_0^i, a_0)$, $i = R, P$. Thus, using (20) and (26), the aggregate human capital stock in period 2 may be written as

$$
\begin{aligned}
H_2 &= E\left[\lambda E\left[\tilde{h}_1(\breve{b}_1^R)\right] + (1 - \lambda)E\left[\tilde{h}_1(\breve{b}_1^P)\right]\right] &\qquad (30)\\
&= E\left[\lambda E\left[\tilde{h}_2\left(b_0^R - \varepsilon\right)\right] + (1 - \lambda)E\left[\tilde{h}_2\left(b_0^P + \varepsilon\lambda/(1 - \lambda)\right)\right]\right] \equiv \hat{H}_2(\varepsilon).
\end{aligned}
$$

This leads to the following result.

Lemma 7. *Under A1-A5, for any realizations of random shocks. If $b''(I) \leq 0$, then $\hat{H}_2'(\varepsilon) \geq 0$. (ii) If, by contrast, $b''(I) > 0$, then we may have $\hat{H}_2'(\varepsilon) < 0$.*

Proof. First, note that (30) implies

$$
\hat{H}_2'(\varepsilon) = \lambda E\left[E\left[\tilde{h}_2'(\breve{b}_0^P) - \tilde{h}_2'(\breve{b}_0^R)\right]\right] \qquad (31)
$$

Since $\breve{b}_0^P < \breve{b}_0^R$, (31) implies that $\hat{H}_2'(\varepsilon) > (=, <)0$ if, for instance, for all realizations $a_0, a_1 \in \mathcal{A}$ at $t = 0, 1$ and for all $b \in \mathbb{R}_+$, $\tilde{h}_2''(b) < (=, >)0$. Finally, use Lemma 6. This confirms Lemma 7. ■

The intuition of Lemma 7 is as follows. On the one hand, a diminishing marginal return to human capital investment leads to a positive impact of higher equality of family wealth on H_2, i.e., the short run effect of higher equality on the aggregate human capital stock which underlies Proposition 2 also applies two periods after a lump-sum redistribution. To see this intuitively, recall that the wealth transfer from parent to

child is an increasing function of parent's income. Consequently, an increase in ε implies that members of generation 0 which belong to a poor dynasty (endowed with b_0^P) are, on average, richer as parents, and thus transfer, on average, more to their offspring (i.e., to members of generation 1). In turn, on average, members of generation 1 from poor families invest more in education whereas the opposite holds for rich dynasties (with family wealth b_0^R in the initial period). Hence, on the other hand, and in contrast to the short run effect of higher equality, the impact of an increase in ε on the aggregate human capital stock in period 2, H_2, also depends on the intergenerational wealth transmission as function of parents' income. As a consequence, the positive effect from $h_{ee} < 0$ on aggregate human capital investment may be dominated: Poor dynasties transmit more wealth, on average, when ε is higher, rich dynasties transmit less. If $b''(I) > 0$, then the average reduction in wealth transmission of the rich may outweigh the average increase in wealth transmission of the poor. Formally, this has been established in Lemma 4, which states that $\tilde{b}_1''(b) > 0$ is possible if $b''(I) > 0$. As implied by Lemma 6, initially richer dynasties may thus sharply reduce their educational investment, on average. Thus, despite diminishing marginal returns to education, reflected by $h_{ee} < 0$, $H_2 = \hat{H}_2(\varepsilon)$ may decrease with ε. Intuitively, the "more convex" $b(I)$ and the "less concave" $h(e, \cdot)$ in e is, the "more likely" is $\hat{H}_2'(\varepsilon) < 0$, which reflects a positive relationship between inequality and growth.

To draw conclusions for the role of (initial) inequality for the process of development in the medium run, i.e., to examine the impact of an increase in ε on the growth rate $g_{t,0}(= Y_t/Y_0 - 1 = H_t/H_0 - 1)$ for $t \geq 2$, let

$$h_{t+3}^i = \tilde{h}_2(b_{t+1}^i) = \tilde{h}_2(\tilde{b}_1(b_t^i)) \equiv \tilde{h}_3(b_t^i), \tag{32}$$

where (26) has been used. Thus, recalling (20), the aggregate human capital stock in period 3 may be written as

$$H_3 = E\left[E\left[\lambda E\left[\tilde{h}_3\left(b_0^R - \varepsilon\right)\right] + (1-\lambda)E\left[\tilde{h}_3\left(b_0^P + \varepsilon\lambda/(1-\lambda)\right)\right]\right]\right] \equiv \hat{H}_3(\varepsilon). \tag{33}$$

19

As can be seen from comparing (33) with (30), the structure of the development process through wealth transmission and human capital investments remains similar from period 2 onwards. In fact, in line with Lemma 7, the following can be concluded.

Proposition 3. (Impact of higher equality in the medium run). *Under A1-A5, for all finite $t \geq 2$. (i) If $b''(I) \leq 0$, higher equality of initial family wealth has a positive impact on the subsequent growth rate $g_{t,0}$. (ii) If, however, $b''(I) > 0$, then the relationship between equality and $g_{t,0}$ may be negative.*

Proof. Note that (33) implies

$$\hat{H}_3'(\varepsilon) = \lambda E \left[E \left[E \left[\tilde{h}_3'(\breve{b}_0^P) + \tilde{h}_3'(\breve{b}_0^R) \right] \right] \right], \text{ where} \tag{34}$$

$$\tilde{h}_3'(b) = \tilde{h}_2'(\tilde{b}_1(b))\tilde{b}_1'(b), \tag{35}$$

according to the definition $\tilde{h}_3(b) = \tilde{h}_2(\tilde{b}_1(b))$ in (32). Since $\breve{b}_0^P < \breve{b}_0^R$, (34) implies that $\hat{H}_3'(\varepsilon) > (=, <)0$ if, for instance, for all realizations $a_0, a_1, a_2 \in \mathcal{A}$ at $t = 0, 1, 2$, and for all $b \in \mathbb{R}_+$, $\tilde{h}_3''(b) < (=, >)0$. Also note that (35) implies

$$\tilde{h}_3''(b) = \tilde{h}_2''(\tilde{b}_1(b))\tilde{b}_1'(b)^2 + \tilde{h}_2'(\tilde{b}_1(b))\tilde{b}_1''(b). \tag{36}$$

Recall that $\tilde{h}_2'(\cdot) \geq 0$, according to Lemma 5. First, suppose $b''(I) \leq 0$. Thus, $\tilde{h}_3''(b) \leq 0$, since $\tilde{b}_1''(\cdot) \leq 0$ and $\tilde{h}_2''(b) \leq 0$ in this case, according to part (i) of Lemma 4 and part (i) of Lemma 6, respectively. Using (36), this confirms part (i) of Proposition 3. In an analogous fashion, part (ii) of Proposition 3 follows from (36) together with part (ii) of Lemma 4 and part (ii) of Lemma 6. Thus, the impact of an increase in ε on H_3 is similar to its impact on H_2, where the latter has been established in Lemma 7. The impact of an increase in ε on H_4 and higher can be established in a completely analogous fashion, employing a very similar structure, which yields similar results. This concludes the proof. ∎

Proposition 3 shows that the medium run impact of higher equality on human capital accumulation and growth crucially depends on the properties of the saving

function. Whether part (i) or part (ii) of Proposition 3 is the relevant case is thus an empirical question, which is addressed in section 5.2.

4.2   Inequality and Aggregate Income in Stationary Equilibrium

What is the impact of initial inequality on the aggregate human capital stock and per capita income in the long run (i.e., as $t \to \infty$), denoted by H_∞ and $Y_\infty = H_\infty f(\bar{k})$, respectively? Answering this question requires an analysis of the long run behavior of wealth transfers within dynasties, which are governed by the Markov process $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$, defined by (13). Due to the uncertainty in the model, these transfers never reach steady state points as known from deterministic models. Therefore, the goal is to find stationary equilibria in the sense that, as $t \to \infty$, the *distribution* of b_t^i within dynasty i is time-invariant. The following discussion of such *stationary equilibria*, and its consequences for the relationship between inequality and per capita income in the long run deals with simple cases in a rather informal way. Both a more formal and more general treatment is provided in appendix B.

To focus the discussion on empirically plausible situations, suppose intergenerational transfers are zero for low levels of income.[27] Moreover, to prevent infinite wealth accumulation of rich dynasties, suppose that for high wealth levels b^i and for $a = \underline{a}, \bar{a}$, $\hat{b}(b^i, a)$ is strictly concave as function of b^i.[28] Simple cases which meet these two criteria are depicted in Fig. 1.[29] Appendix B shows that the main conclusions derived from Fig. 1 remain valid more generally.

<Figure 1>

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[27] For the US, for instance, it has been frequently confirmed that the mean savings rate of households in the lowest quintile of the income distribution is non-positive (e.g., Browning and Lusardi, 1996; Dynan et al., 2000).

[28] Note that according to (17) and (19), even if $b''(I) > 0$, for high I, this occurs under weak conditions since $h_{ee} < 0$.

[29] Note that for $b_{t+1}^i > 0$, the $\hat{b}-$curves for $\underline{a}$ and $\bar{a}$ do not intersect because $\hat{b}_a(b, a) > 0$ in this case. Also note that, under A2 and A5, $\hat{b}(b^i, a)$ is strictly increasing in b^i for $a = \underline{a}, \bar{a}$ if $b^i > \underline{b}_{\underline{a}}, \underline{b}_{\bar{a}}$, according to Lemma 3.

Panel (a) shows a situation in which, irrespective of initial wealth holdings, for any dynasty i, $b_t^i = 0$ as $t \to \infty$ with probability one. This is called a *trivial* stationary equilibrium. (In panel (a), this equilibrium is also globally stable.) Obviously, in this case, higher equality of initial wealth has no impact on per capita income in the long run, Y_∞.

In panel (b), if $b_0^i \leq c_{\bar{a}}$, again, wealth levels within dynasty i become zero with probability one in the long run (which now is a locally stable stationary equilibrium). If $b_0^i \geq c_{\underline{a}}$, then the distribution of b_t^i converges with probability one to a locally unique stable stationary equilibrium on the interval $[d_{\underline{a}}, d_{\bar{a}}]$. Thus, if $b_0^i \leq c_{\bar{a}}$ or if $b_0^i \geq c_{\underline{a}}$ for $i = R, P$, there is no impact of higher equality on Y_∞. If $b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$, then, as $t \to \infty$, both $b_t^i = 0$ or $b_t^i \in [d_{\underline{a}}, d_{\bar{a}}]$ is possible with positive probability.[30] Thus, according to the law of large numbers, if $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$, some fraction $q \in (0,1)$ of initially poor dynasties will end up with zero wealth in the long run. Also by the law of large numbers, note that this fraction q is increasing in the distance of b_0^P to $c_{\underline{a}}$ (or decreasing in $\left(b_0^P - c_{\bar{a}}\right)$, respectively). Thus, the lower $\left(c_{\underline{a}} - b_0^P\right)$ is, the larger the fraction of initially poor dynasties which transmit positive wealth levels in the long run. Hence, if $b_0^R > c_{\underline{a}}$ and $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$, then any redistribution to the poor which leaves wealth levels of initially rich individuals sufficiently high,[31] unambiguously leads to higher average human capital investments, H_∞, if $e'(b) > 0$ (as ensured under A1 and A3, according to Proposition 1), and thus, to a higher Y_∞. This case may be relevant in advanced countries, i.e., if there is a large group of sufficiently rich individuals. Whereas in this situation there is a negative link between inequality and Y_∞, there may be a positive link in other situations. To see this, suppose $b_0^i \in (0, c_{\bar{a}}]$ for $i = R, P$, such that the distribution of wealth levels of all dynasties converge with probability one to the trivial stationary equilibrium.[32] In this case, sufficient redistribution to the rich

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[30] For illuminating discussions of this stationary equilibrium indeterminacy in stochastic models, see, e.g., Laitner (1981) and Wang (1993).

[31] More precisely, this refers to any increase in ε small enough such that $\breve{b}_0^R = b_0^R - \varepsilon \geq c_{\underline{a}}$, implying that the distribution of wealth holdings of initially rich dynasties still converges with probability one to a locally unique stationary equilibrium on the interval $[d_{\underline{a}}, d_{\bar{a}}]$.

[32] This reflects a poverty trap of the kind often encountered in the literature on inequality and growth (which here is derived from a stochastic model).

may result in a situation in which wealth levels of at least some initially rich dynasties converge to the stationary equilibrium on the interval $[d_{\underline{a}}, d_{\bar{a}}]$, without affecting the long run wealth distribution of the initially poor (who end up with zero wealth anyway). Thus, if $e'(b) > 0$, Y_∞ is raised. Finally, if $b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$ for $i = R, P$, the impact of higher equality on Y_∞ is ambiguous. In sum, we may conclude the following from this discussion.

Proposition 4. (Impact of higher equality in the long run). *Suppose a stationary equilibrium of the Markov process $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$ exists. Then the relationship between equality and long run income, Y_∞, is generally ambiguous, irrespective of the shape of the saving function, $b(I)$.*

Thus, even if there is a systematic relationship between initial inequality and aggregate income in the short run and medium run (see Propositions 2 and 3 in section 4.1), one cannot draw general conclusions regarding the relationship between initial inequality and Y_∞ from the shape of the saving function. However, whereas higher inequality may help to overcome a poverty trap in poor economies, if anything, the analysis suggests that the relationship between initial inequality and long run income is negative for advanced countries.

5 Empirical Relevance

This section reviews empirical evidence regarding crucial features and results of the theory developed in this paper.

5.1 The Inequality-Growth Relation Revisited

The present paper has suggested a channel through which initial inequality may have a positive impact on the growth rate of aggregate income in the medium run. To address this result in the light of empirical evidence, first, it should be noted that the usual modelling approach in growth empirics relies on some hypothesis of conditional

convergence (e.g., Barro, 1991, 2000). That is, regression analysis regarding the determinants of economic growth, like inequality of income, usually control for the level of per capita income in some base year (properly instrumented) to account for the stage of development of an economy, like initial GDP, Y_0.[33] This focus on the transition path enables us to compare the hypotheses derived in section 4.1 (particularly regarding the medium run) with empirical evidence in this literature.

The earlier literature has suggested a negative effect of income inequality on growth (e.g., Alesina and Rodrik, 1994; Persson and Tabillini, 1994; Perotti, 1996), which however, is not based on panels. Using a new and comprehensive high-quality data set, Deininger and Squire (1998) and Banerjee and Duflo (2003) find practically none, whereas Li and Zou (1998) and Forbes (2000) report a positive and significant medium run effect.[34] The most interesting results for the present paper, however, emerge in a recent study of Barro (2000), who allows income inequality (measured by the Gini coefficient, using the same data set as Deininger and Squire, 1998) to interact with (the log of) per capita GDP in growth regressions. His evidence suggests a non-linear impact of inequality on GDP growth, with a negative relationship for less developed countries (with GDP per capita below around $2000) and a positive relationship for more advanced countries (with GDP per capita above $2000).[35] From these results, Barro (2000, p.18) concludes that "for rich countries, where credit constraints are less serious, the growth-promoting aspects of inequality may dominate". To the best of my knowledge, the only available theory which has been consistent with a potentially positive relationship between inequality and growth in advanced countries so far rests on the classical view: that is, higher inequality may enhance growth through *physical*

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[33] Usually, Y_0 has significant effects, suggesting that observed economies are not yet close to stationary equilibria.

[34] Banerjee and Duflo (2003) suggest that the change in inequality, in either direction, rather then its level is negatively associated with growth. Moreover, on basis of this finding, they argue that previous panel studies (particularly those relying on fixed effects) may have produced upward biased estimates of the effect of inequality on growth.

[35] For developing countries, a negative relationship between inequality and growth is consistent with the view that high inequality leads to social unrest (e.g., Alesina and Perotti, 1996), high fertility (e.g., Perotti, 1996), and low human capital investment (e.g., due to borrowing constraints; see Galor and Zeira, 1993), in turn, being associated with slow growth. Evidence by Perotti (1996) largely supports these channels.

capital accumulation. However, this mechanism crucially depends on the assumption of (at least partially) *closed* economies such that investment in physical capital is related to national savings. In contrast, the theory proposed in this paper rests on the role of intergenerational wealth transmission for *human* capital accumulation in a *small open* economy. Abstracting from borrowing constraints for human capital investments renders the model particularly relevant for advanced economies.

The remainder of this section provides empirical evidence for the forces suggested by the present analysis which are necessary to obtain a growth-promoting medium run effect of inequality.

5.2   Saving Behavior and Intergenerational Transfers

In the model, the amount of intergenerational transfers equals the amount of savings of adult individuals. In fact, empirically, savings seem to be strongly related to inter vivos gifts and bequests, thus lending support for a crucial assumption about preferences in the model. For instance, Kotlikoff and Summers (1981) emphasize the importance of intergenerational transfers for capital accumulation in the US, a hypothesis which has been frequently confirmed by later evidence. In particular, as argued by Menchik and David (1983) and Dynan et al. (2000, 2002), observed saving behavior in the US is empirically consistent with models hypothesizing a "joy of giving" motive for intergenerational transfers.[36] De Nardi (2003) calibrates an overlapping-generations model with voluntary and accidental bequests, arguing that voluntary bequests play a crucial role for explaining observed wealth concentration patterns not only in the US but also in Sweden.

According to the preceding analysis, if the marginal propensity to save for adults,

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[36]In contrast, the standard altruism (dynastic) model á la Barro (1974) seems to be inconsistent with the data. For instance, unlike predicted by the dynastic model, inheritances do not seem to compensate for earnings differences among siblings (e.g. Wilhelm, 1996). Moreover, as discussed in Carroll (2000), there does not seem to be an indication that the size of bequest is an increasing function of the ratio of parent's to child's lifetime income. Finally, whereas a one-dollar reduction in income of a recipient should raise inter vivos transfers from parents to child by one dollar according to the dynastic model, evidence by Altonji et al. (1997) suggests that transfers increase by just 13 cent on average, conditional on the event of a positive transfer having occurred.

$b'(\cdot)$, is increasing in income, then inequality may be positively related to growth in the medium run, despite diminishing marginal returns to human capital investments (Proposition 3). This is because under individual uncertainty of returns to human capital investments, (initial) wealth inequality affects human capital accumulation through intergenerational transfers.

Evidence on US saving and bequest behavior suggests that saving rates are strongly increasing in *lifetime* income.[37] For instance, Dynan et al. (2000) provide estimates (for the five-year period between 1984 and 1989) which imply that a \$10,000 increase in (permanent) income is associated with an increase in the saving rate in a range from over 1 to roughly 5 percent, depending on the database and instruments used.[38] Most importantly for the results of the theoretical model developed in the present paper, estimates from median regressions suggest that the *increase* in saving rates in response to an increase in income is approximately constant.[39] This implies that the marginal propensity to save is increasing as function of income, thus, being consistent with $b''(\cdot) > 0$. In a less recent study, Menchik and David (1983) directly focus on bequest behavior. Their evidence suggests that the marginal propensity to bequeath is increasing in lifetime earnings.

Remark 1. It is important to note the difference between $b''(\cdot) > 0$ on the one hand and an increasing *average* propensity of adults to save (which holds if $b(I)/I$ is increasing in I) on the other hand. For instance, following Galor and Zeira (1993), Moav (2002) and Galor and Moav (2004), among others, consider the saving function

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[37]There is overwhelming evidence for the hypothesis that saving rates rise strongly with *current* income (e.g., Browning and Lusardi, 1996). However, as pointed out by Friedman (1957), this finding may just reflect a response of savings behavior to changes in transitory income. That is, if income is temporarily high, savings increase and, analogously, if income is temporarily low, savings are reduced. Over the life-cycle, however, a positive relationship between current income and saving rates may still be consistent with a constant saving rate as function of lifetime (or permanent) income.

[38]Dynan et al. (2000) use three different databases to account for different measures of savings and use different instruments for permanent income like consumption, lagged and/or future earnings, and education. Using current income, a \$10,000 increase in income is associated with a 8 percent increase in the saving rate. This figure is considerably higher than for measures of permanent income, as expected.

[39]See Dynan et al. (2000, Fig. 1A-1D). This result is basically independent on the instrument used or when no instrumenting is done.

$b(I) = \alpha[I - \vartheta]$ if $I > \vartheta$ and $b(I) = 0$ otherwise, $\vartheta \geq 0$. Thus, if $I \geq \vartheta > 0$, the average propensity of adults to save, $b(I)/I = \alpha - \vartheta/I$, is increasing in I. Nevertheless, since $b''(I) \leq 0$ for $I > \vartheta$, this particular functional form rules out the potentially positive effect of inequality on medium run growth, suggested by part (ii) of Proposition 3. Appendix C further employs this saving function for an illustration of the analysis under a particular specification of preferences.

5.3 Educational Investment and Parental Income

Besides $b''(\cdot) > 0$, a second necessary condition for a positive relationship between inequality and medium run growth in the model is the positive relationship between family wealth and educational investments, $e'(\cdot) > 0$ (Proposition 1). This condition has been derived from the hypothesis that the variance of earnings increases with educational levels. In fact, there is overwhelming evidence for both this basic assumption and its implication.

First, evidence by Pereira and Martins (2002, 2003) for advanced European countries (in a period between 1980 and 1995) strongly suggests that the earnings dispersion is increasing in the level of schooling, confirming somewhat less recent evidence discussed in Levhari and Weiss (1974). Moreover, although credit constraints to finance higher education seem to be negligible in advanced countries, there is a strong positive relationship between parental social background and children's investment (or participation) in higher education. For instance, Manski (1992) finds that the percentage of children from low-income families in the US who graduate from high school is substantially lower than among high school graduates from other families. Similar patterns can be found in Germany. Egeln et al. (2003) report that 1996 even among those children who were eligible for university education (not more than roughly a third of all high school graduates in Germany), only 24 percent with a less favorable social background went to university, in contrast to 86 percent with a favorable social background.[40] This

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[40]In Germany, eligibility to attend university is exclusivley determined by high school performance. Egeln et al. (2003) also find that among *all* high school graduates only 8 percent with a less favorable social background went to university, in contrast to 72 percent of high school graduates with a favorable

comparison is striking, as it is rather implausible that heterogeneity in intellectual ability (which may partly be shaped by the social background) can account for this difference among those who have already acquired eligibility ("*Hochschulreife*").

In his review of US evidence based on econometric studies, Taubman (1989) concludes that estimates for the elasticity of years of schooling with respect to parental income are generally positive and range from 3 to 80 percent, after controlling for parents' education, father's occupation, and/or children's test scores on mental ability tests. Accounting for similar controls, also the correlation between children's adult earnings and their parents' income is highly positive (e.g. Behrman and Taubman, 1990). As concluded in the survey article by Solon (1999, p.1789): "Most of the evidence [...] indicates that intergenerational earnings elasticities are substantial and are larger than we used to think." The more recent literature on the relationship between a child's social environment and its educational attainment or earnings, respectively, is particularly concerned with the question to which degree such results are driven by genetic factors. For instance, Sacerdote (2002) finds that the effect of socioeconomic status on children's college attendance is just as large for adoptees as for children raised by biological parents. Plug and Vijverberg (2003) report higher effects of genetic factors (measured by parents' IQ) on the children's years of schooling and college attainment, although family income still has a large effect.

6   Concluding Remarks

After more than a decade of intensive research on the relationship between inequality and growth the debate is still ongoing. This paper has proposed a theory which is consistent with a positive medium run effect of inequality on growth in advanced countries, as found in Barro (2000). In contrast to the classical view, this alternative theory does not require any connection between national savings and physical capital investment in an economy, but rests on the role of intergenerational wealth transmission for

social background.

individual incentives to invest in risky and uninsurable human capital.

First, it has been shown that, under the fairly weak requirement of decreasing absolute risk aversion (defined over indirect utility), individual human capital investment is increasing in parental income. This prediction is largely supported by empirical evidence even in advanced countries (in which credit-market imperfections may play a minor role). Given this positive relationship between parental income and human capital investments, if the marginal return to education is diminishing, then the expected marginal return to education is higher for poorer individuals. Thus, in the *short run*, initial inequality of family wealth is typically negatively linked to the aggregate human capital stock, and thus, is an impediment for the short run process of development.

However, in the *medium run*, the effects of intergenerational wealth transmission may overturn this short run effect of inequality on growth if the marginal propensity to save is increasing in income. In fact, this property has frequently been confirmed. In contrast, if the marginal propensity to save is non-increasing, the model predicts that the relationship between inequality and medium run growth is typically negative.

For the *long run* (i.e., in stationary equilibrium), irrespective of the properties of the saving function, the relationship between inequality and per capita income (or the long run rate of growth, respectively, allowing for endogenous growth in a straightforward way) can go either way.

It is important to note, however, that even if the inequality-growth relationship turns out to be positive, the proposed theory does not suggest a rationale for inegalitarian policies. For instance, under uninsurable human capital risk, distortionary redistribution through the tax system may enhance risk-taking by providing insurance, as suggested by the literature on portfolio choice and taxation.[41] To examine the role of the shape of the saving function for implications of redistributive taxation in a similar context as analyzed in this paper is left for future research.

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[41] Moreover, since the model does not contain any "trickle-down" mechanism, a standard equity-growth trade-off arises such that optimal policy will crucially hinge on the social welfare function.

Appendix

A. Proof of Proposition 1

First, note that $e_t^i = e(b_t^i)$ is given by the first-order condition

$$
\begin{aligned}
0 &= E\left[v'\left(I(b_t^i, e_t^i, \tilde{a})\right)\left(\bar{w}h_e(e_t^i, \tilde{a}) - \bar{R}\right)\right], \text{ i.e.,} \tag{A.1}\\
\Xi(b_t^i, e_t^i) &\equiv E\left[v'\left(\bar{w}h(e_t^i, \tilde{a}) + \bar{R}\left(b_t^i - e_t^i\right)\right)\left(\bar{w}h_e(e_t^i, \tilde{a}) - \bar{R}\right)\right] = 0,
\end{aligned}
$$

according to (3) and (9). (An interior solution is ensured by assumption A1.) Due to $v''(I) < 0$ (recall Lemma 2) and $h_{ee} < 0$, we have $\Xi_e < 0$. Thus, according to the implicit function theorem, $e'(b_t^i) > 0$ if and only if $\Xi_b(b_t^i, e_t^i)|_{e_t^i = e(b_t^i)} > 0$. For notational simplicity, indices t and i are suppressed in the remainder of this proof. Moreover, define $\hat{I}(b, \tilde{a}) \equiv I(b, e(b), \tilde{a})$. Then, according to (A.1),

$$
\begin{aligned}
\Xi_b(b, e)|_{e=e(b)} &= E\left[v''\left(\hat{I}(b, \tilde{a})\right)\left(\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right)\right]\bar{R} \tag{A.2}\\
&= E\left[A\left(\hat{I}(b, \tilde{a})\right)v'\left(\hat{I}(b, \tilde{a})\right)\left(\bar{R} - \bar{w}h_e(e(b), \tilde{a})\right)\right]\bar{R},
\end{aligned}
$$

where $A(I) = -v''(I)/v'(I)$ has been used for the latter equation. Define a' as the realization of $\tilde{a}$ such that $\bar{w}h_e(e, a') - \bar{R} = 0$. We can write

$$
\begin{aligned}
&E\left[A\left(\hat{I}(b, \tilde{a})\right)v'\left(\hat{I}(b, \tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot, \tilde{a})\right)\right] \tag{A.3}\\
&= \int_{\underline{a}}^{a'} A\left(\hat{I}(b, \tilde{a})\right)v'\left(\hat{I}(b, \tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot, \tilde{a})\right)d\Phi(\tilde{a}) +\\
&\quad \int_{a'}^{\bar{a}} A\left(\hat{I}(b, \tilde{a})\right)v'\left(\hat{I}(b, \tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot, \tilde{a})\right)d\Phi(\tilde{a}).
\end{aligned}
$$

Recall from assumption A1 that $h_{ea} > 0$. Thus, by the definition of a', the first integral in (A.3) is positive, whereas the second one is negative. Moreover, note that since $h_a > 0$, $\hat{I}(b, a)$ is increasing in a, according to (3). Thus, $A\left(\hat{I}(b, a)\right)$ is strictly

decreasing in a under assumption A3. Hence, under A3,

$$A\left(\hat{I}(b,a')\right)\int_{\underline{a}}^{a'} v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R}-\bar{w}h_e(\cdot,\tilde{a})\right)d\Phi(\tilde{a}) \tag{A.4}$$

$$< \int_{\underline{a}}^{a'} A\left(\hat{I}(b,\tilde{a})\right)v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R}-\bar{w}h_e(\cdot,\tilde{a})\right)d\Phi(\tilde{a}),$$

$$A\left(\hat{I}(b,a')\right)\int_{a'}^{\bar{a}} v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R}-\bar{w}h_e(\cdot,\tilde{a})\right)d\Phi(\tilde{a}) \tag{A.5}$$

$$< \int_{a'}^{\bar{a}} A\left(\hat{I}(b,\tilde{a})\right)v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R}-\bar{w}h_e(\cdot,\tilde{a})\right)d\Phi(\tilde{a}).$$

Adding up (A.4) and (A.5) and using $A(I)v'(I)=-v''(I)$ yields

$$A\left(\hat{I}(b,a')\right)E\left[v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R}-\bar{w}h_e(\cdot,\tilde{a})\right)\right] < E\left[v''\left(\hat{I}(b,\tilde{a})\right)\left(\bar{w}h_e(\cdot,\tilde{a})-\bar{R}\right)\right]. \tag{A.6}$$

Under the optimal human capital investment, $e(b)$, the left-hand side of (A.6) is zero, according to (A.1). Thus, $E\left[v''(\hat{I}(b,\tilde{a}))\left(\bar{w}h_e(\cdot,\tilde{a})-\bar{R}\right)\right] > 0$, implying $\left.\Xi_b(b,e)\right|_{e=e(b)} > 0$, according to (A.2). Hence, $e'(b) > 0$. This concludes the proof. ∎

B. Stationary Equilibria

This appendix provides a more formal treatment of stationary equilibria and generalizes the cases discussed in section 4.2 (based on Fig. 1).

Let $P(b,\cdot)$ be the *transition function* of the Markov process $b_{t+1}^i = \hat{b}(b_t^i,\tilde{a})$, i.e., $P(b^i,\mathcal{Z})$ is the probability that b^i is in the set $\mathcal{Z}$ one period after it started in b^i. That is,

$$P(b^i,\mathcal{Z}) \equiv \Pr\{\tilde{a}:\hat{b}(b^i,\tilde{a})\in\mathcal{Z}\} \equiv \Pr\{\tilde{a}\in\mathcal{Z}_{b^i}\} = \int_{\mathcal{Z}_{b^i}}\Phi(\tilde{a}), \tag{A.7}$$

where $\mathcal{Z}_{b^i} \equiv \{\tilde{a}:\hat{b}(b^i,\tilde{a})\in\mathcal{Z}\}$ and $\mathcal{Z}$ is a Borel set in $\mathbb{R}_+$. Moreover, let $\mu_t^i(\mathcal{Z}) \equiv \Pr\{b_t^i\in\mathcal{Z}\}$ for all $\mathcal{Z}\subset\mathbb{R}_+$, $t=0,1,2,...$, be the *probability measure* associated with b_t^i. Thus, given initial wealth b_0^i of dynasty i, μ_0^i is given by $\mu_0^i([0,b^i)) = 0$ for all

$b^i \leq b^i_0$ and $\mu^i_0([b^i, \infty)) = 1$ otherwise. Starting from μ^i_0, the distribution of family wealth evolves according to $\mu^i_{t+1}(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu^i_t(db^i)$ for all $\mathcal{Z} \subset \mathbb{R}_+$. From this, we can define a stationary equilibrium as follows.[42]

Definition 1. (Stationary equilibrium). A *stationary equilibrium* for family wealth b^i of dynasty i is a probability measure μ^i such that $\mu^i(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu^i(db^i)$ for all $\mathcal{Z} \subset \mathbb{R}_+$. A *trivial stationary equilibrium* is a stationary equilibrium which is associated with a distribution of b^i such that all mass is concentrated on zero (i.e., $lim_{t\to\infty} \Pr\{b^i_t = 0\} = 1$).

Together with Definition 1, the next definition leads to an important existence result.

Definition 2. (Stable set). An interval $[x, y] \subset \mathbb{R}$ is called a *stable set* of the stochastic process $\hat{b}$ if (i) $\hat{b}(b^i, x) = x$, $\hat{b}(b^i, y) = y$, and (ii) $\hat{b}(b^i, x) < b^i$, $\hat{b}(b^i, y) > b^i$ for all $x < b^i < y$.

Lemma B.1. (Wang, 1993). *There is a unique stable stationary equilibrium on a stable set I. Moreover, the convergence to the stationary equilibrium is uniform on I.*

Proof. Brock and Mirman (1972), Wang (1993). ∎

Given these preliminaries, let $S_a \equiv \{b \in \mathbb{R}_{++} \big| \hat{b}(b, a) = 0\}$ be the set of strictly positive transfers received by a young individual, such that the optimal transfer as adult to her offspring is zero, $a \in \mathcal{A}$. Moreover, let $\underline{b}_a \equiv \max S_a$ be the largest element of such a set. Suppose that the following holds.

A6. $S_{\bar{a}}$ is non-empty, i.e., there exists $\underline{b}_{\bar{a}} > 0$.

Note that $\hat{b}_a(b, a) = b'\left(\hat{I}(b, a)\right)\hat{I}_a$, according to (12) and (13). Thus, using $\hat{I}_a > 0$ and Lemma 1, we have $\underline{b}_a > \underline{b}_{\bar{a}}$ for all $a \in [\underline{a}, \bar{a})$. That is, if an individual which has received a transfer b when young does not save as adult in the best state $\bar{a}$, neither she

[42]Definitions 1 and 2 closely follow Wang (1993).

does in any other state. Also note that, if $\underline{b}_a > 0$ exists for some a, then A2 and A5 imply that $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$, according to Lemma 3.

We are now ready to give a formal characterization of the result corresponding to panel (a) of Fig. 1.

Proposition B.1. *Under A2, A5 and A6. If $\hat{b}(b^i, \bar{a}) < b^i$ for all $b^i > 0$, the distribution of b^i globally converges to a trivial stationary equilibrium.*

Proof. First, note that A6, which says that $S_{\bar{a}}$ is non-empty, also implies that $S_{\underline{a}}$ is non-empty. Moreover, recall $\underline{b}_{\underline{a}} > \underline{b}_{\bar{a}} > 0$. Also recall that A2, A5 and A6 ensure $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$, according to Lemma 3. Thus, Proposition B.1 corresponds to the case in panel (a) of Fig. 1. From this figure, it is clear that there no stable set of the process $\hat{b}$ exists (recall Definition 2), and global convergence to a trivial stationary equilibrium (recall Definition 1) is obvious. ∎

To analyze more general situations than global convergence to a trivial stationary equilibrium, the next assumption prevents infinite wealth accumulation of rich dynasties.

A7. $lim_{b \to \infty} \hat{b}_b(b, a) = 0$ for all $a \in \mathcal{A}$.

Moreover, let $\Theta_a \equiv \{b \in \mathbb{R}_{++} \,\big|\, \hat{b}(b, a) = b\}$ be the set of strictly positive fixed points of $\hat{b}(b, a)$, $a \in \mathcal{A}$. In the remainder of this appendix, we focus on situations in which $\Theta_{\underline{a}}$ and $\Theta_{\bar{a}}$ have the following properties.

A8. (i) $\Theta_{\underline{a}}$ is non-empty.[43] (ii) $\Theta_{\underline{a}}$ and $\Theta_{\bar{a}}$ are finite. (iii) Let $c_{\bar{a}}, d_{\bar{a}}$ be two adjacent elements of $\Theta_{\bar{a}}$ such that $c_{\bar{a}} < d_{\bar{a}}$ and $\hat{b}_b(d_{\bar{a}}, \bar{a}) \leq 1$. Then there exists $c_{\underline{a}} \in \Theta_{\underline{a}}$ such that $c_{\underline{a}} \in (c_{\bar{a}}, d_{\bar{a}})$. (iv) Let $c_{\underline{a}}, d_{\underline{a}}$ be two adjacent elements of $\Theta_{\underline{a}}$ such that $c_{\underline{a}} < d_{\underline{a}}$ and $\hat{b}_b(d_{\underline{a}}, \underline{a}) \geq 1$. Then there exists $c_{\bar{a}} \in \Theta_{\bar{a}}$ such that $c_{\bar{a}} \in (c_{\underline{a}}, d_{\underline{a}})$.

<Figure 2>, <Figure 3>

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[43] Note that, under A6, part (i) of A8 implies that $\Theta_{\bar{a}}$ is non-empty as well.

It is easy to check that the case depicted in panel (b) of Fig. 1 is consistent with A6-A8.[44] Fig. 2 shows situations, which are consistent with A6 and A7, but inconsistent with some parts of A8, whereas panels (a)-(c) of Fig. 3 are, like panel (b) of Fig. 1, consistent with A6-A8.

Note that, by applying Definition 2, I_3 and I_6 in panels (a) and (b) of Fig. 3 are both stable sets, whereas in panel (c) I_3 is the unique stable set. Moreover, by replicating the arguments in Laitner (1981; section III), the following can be concluded from Fig. 3, starting with panels (a) and (b). In panel (a) of Fig. 3, if $b_0^i \in I_2 \cup I_3 \cup I_4$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1$. In panel (b), the same is true if $b_0^i \in I_2 \cup I_3$. In panel (a), if $b_0^i \in I_6 \cup I_7$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_6\} = 1$. In panel (b), the same is true if $b_0^i \in I_5 \cup I_6 \cup I_7$. In both panels (a) and (b), if $b_0^i \in I_0$, then $q \equiv \lim_{t\to\infty} \Pr\{b_t^i = 0\} = 1$, and, if $b_0^i \in I_1$, then $q \in (0,1)$ and $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1 - q$. Finally, if $b_0^i \in I_5$ in panel (a), then $b_t^i \in I_3$ or $b_t^i \in I_6$ with probability one. In panel (b), the same is true if $b_0^i \in I_4$. Now consider panel (c) of Fig. 3. If $b_0^i \in I_0 \cup I_1$, then $q = 1$. If $b_0^i \in I_2$, then $q \in (0,1)$ and $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1 - q$. Finally, if $b_0^i \in I_3 \cup I_4$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1$. Using this discussion, one can generalize the conclusions drawn from panel (b) of Fig. 1 discussed in the main text, in the following sense.

Proposition B.2. *Under A2, A5-A8. Let $c = \min \Theta_{\bar{a}}$ and $d = \min \Theta_{\underline{a}}$ (i.e., $c < d$).*

(i) For all $b_0^i \in [0, c]$, as $t \to \infty$, the distribution of b_t^i converges to a locally stable trivial stationary equilibrium.

(ii) For all $b_0^i \in (c, d)$, there is a positive probability $q \in (0, 1)$ that the distribution of b_t^i converges to a trivial stationary equilibrium, whereas with probability $1 - q$ it converges to a (unique and stable) stationary equilibrium on a stable set.

(iii) For all $b_0^i \in [d, \infty)$, the distribution of b^i converges to a (unique and stable) stationary equilibrium on a stable set.

Proof. Part (i) of Proposition B.2 can be deduced by similar arguments as in the proof of Proposition B.1. To prove parts (ii) and (iii), recall Definitions 1 and 2 and

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[44]Note that part (iv) of A8 is not relevant for panel (b) of Fig. 1, since $\hat{b}_b(d_{\underline{a}}, \underline{a}) < 1$.

verify from A2 and A5 (which ensure $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$) as well as A6-A8, that the result can directly be deduced by replicating the discussion of Fig. 3 above. ∎

C. A Simple Example

This appendix provides a simple illustration of the analysis by specifying preferences. Following Galor and Zeira (1993), Moav (2002) and Galor and Moav (2004), among others, the utility is given by

$$u(c_{t+1}^i, b_{t+1}^i) = (1 - \alpha) \ln c_{t+1}^i + \alpha \ln(\gamma + b_{t+1}^i), \quad 0 < \alpha < 1, \gamma \geq 0. \tag{A.8}$$

It is easy to check that assumption A2 and thus Lemma 1 and 2 holds. Using the first-order condition (6) for optimal savings as adult, (A.8) implies a saving function $b_{t+1}^i = b(I_{t+1}^i) = \alpha[I_{t+1}^i - \vartheta]$ if $I_{t+1}^i > \vartheta \equiv \gamma(1 - \alpha)/\alpha$ and $b_{t+1}^i = 0$ otherwise. (See also Remark 1 in section 5.2.) Moreover, indirect utility is given by $v(I_{t+1}^i) = 2\ln(I_{t+1}^i + \gamma) + \eta$, where $\eta \equiv \alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)$. Thus, the degree of absolute risk aversion is given by $A(I) = -v''(I)/v'(I) = 1/(I + \gamma)$. That is, $A(I)$ is strictly decreasing in I, in line with assumption A3. Hence, $e'(b) > 0$, according to Proposition 1, where $e(b)$ is given by

$$\int_{\mathcal{A}} \frac{\bar{w} h_e(e, \tilde{a}) - \bar{R}}{\bar{w} h(e, \tilde{a}) + \bar{R}(b - e)} d\Phi(\tilde{a}) = 0, \tag{A.9}$$

according to the first-order condition (A.1) for optimal educational investment. Using (A.9), tedious derivations reveal that the sign of $e''(b)$ is ambiguous, lending some justification to $|e''(b)| \approx 0$ (assumption A4). Regarding the short run, this means that $\hat{H}_1'(\varepsilon) > 0$ (Proposition 2) is likely to hold. For the remainder of this appendix, suppose $\gamma > 0$, i.e., $\vartheta > 0$. For the medium run and long run analysis, using (12), one then obtains

$$b_{t+1}^i = \hat{b}(b_t^i, a) = \tilde{b}_1(b_t^i) = \begin{cases} \alpha[\bar{w} h(e(b_t^i), a) + \bar{R}(b_t^i - e(b_t^i)) - \vartheta] & \text{if } b_t^i \geq \underline{b}_a, \\ 0 & \text{otherwise,} \end{cases} \tag{A.10}$$

where $\underline{b}_a$ is given by $\bar{w}h(e(\underline{b}_a),a) + \bar{R}\left(\underline{b}_a - e(\underline{b}_a)\right) = \vartheta$, $a \in \mathcal{A}$. Note that A5 implies that $\underline{b}_a$ is unique, and A6 implies that $\underline{b}_a > 0$. Thus, if $\alpha\left[\bar{w}h(e(b),a) + \bar{R}\left(b - e(b)\right) - \vartheta\right] < b$ for all $a \in \mathcal{A}$ and for all $b > 0$, Proposition A.1 applies, i.e., the distribution of b^i globally converges to a trivial stationary equilibrium. Moreover, it is easy to check that, under assumption A4, (A.10) implies

$$\hat{b}_{bb}(b_t^i, a) = \tilde{b}_1''(b) < 0 \text{ for all } b > \underline{b}_a, \, a \in \mathcal{A}. \tag{A.11}$$

Since $b''(I) = 0$ for $I > \vartheta$ under utility specification (A.8), $\tilde{b}_1''(b) < 0$ illustrates part (i) of Lemma 4. Thus, the impact of higher equality on medium run growth is positive, according to part (i) of Proposition 3. For the long run analysis, note that (A.11) is consistent with assumption A7. Moreover, if there exists a $b > 0$ such that $\alpha[\bar{w}h(e(b),\underline{a}) + \bar{R}\left(b - e(b)\right) - \vartheta] = b$, i.e., if part (i) of A8 holds, (A.11) implies that also parts (ii)-(iv) of A8 hold. This illustrates that, for the long run, the discussion of Fig. 1 in the main text applies (Proposition 4).

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**Figure 1:** Stationary equilibrium and convergence.

(a)

(b)
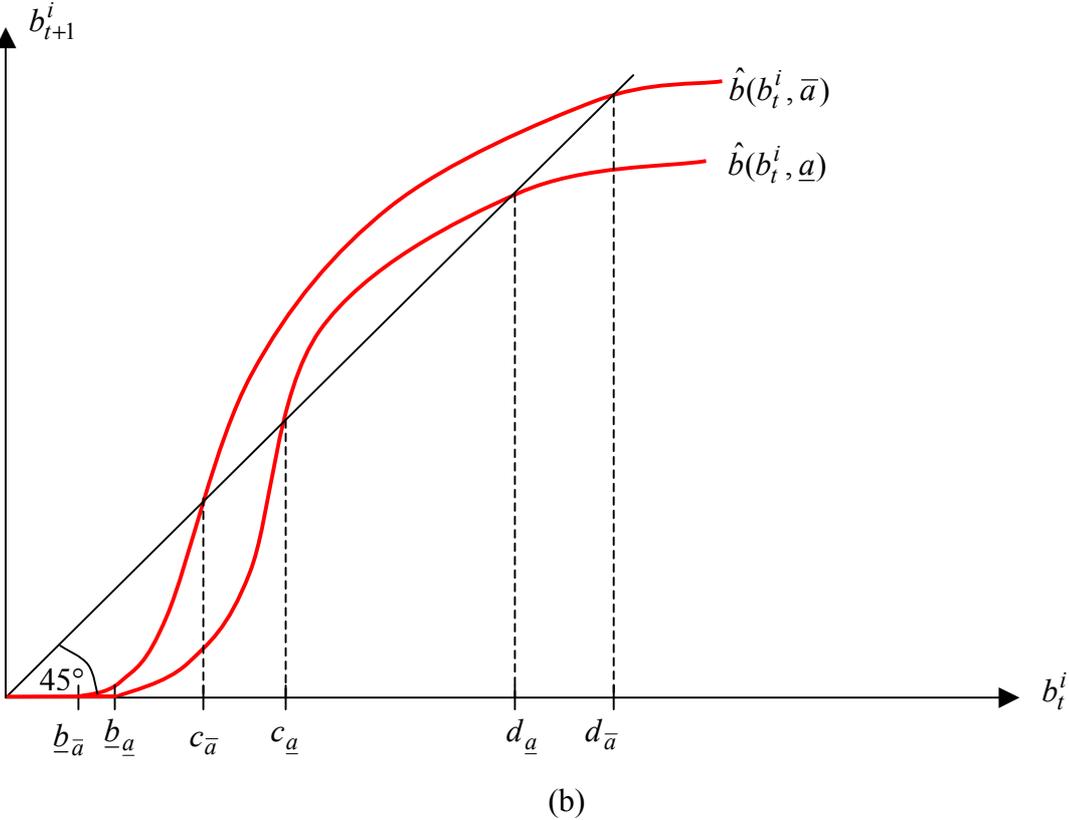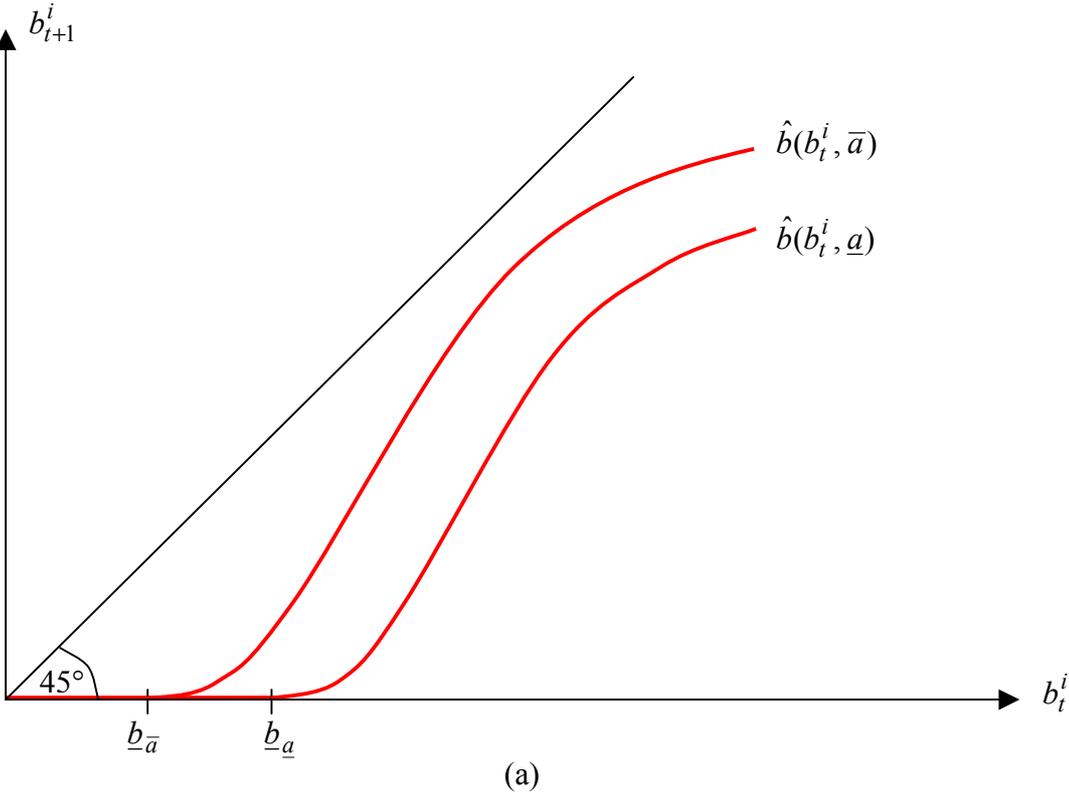
Figure 2: Inconsistency with assumption A8.
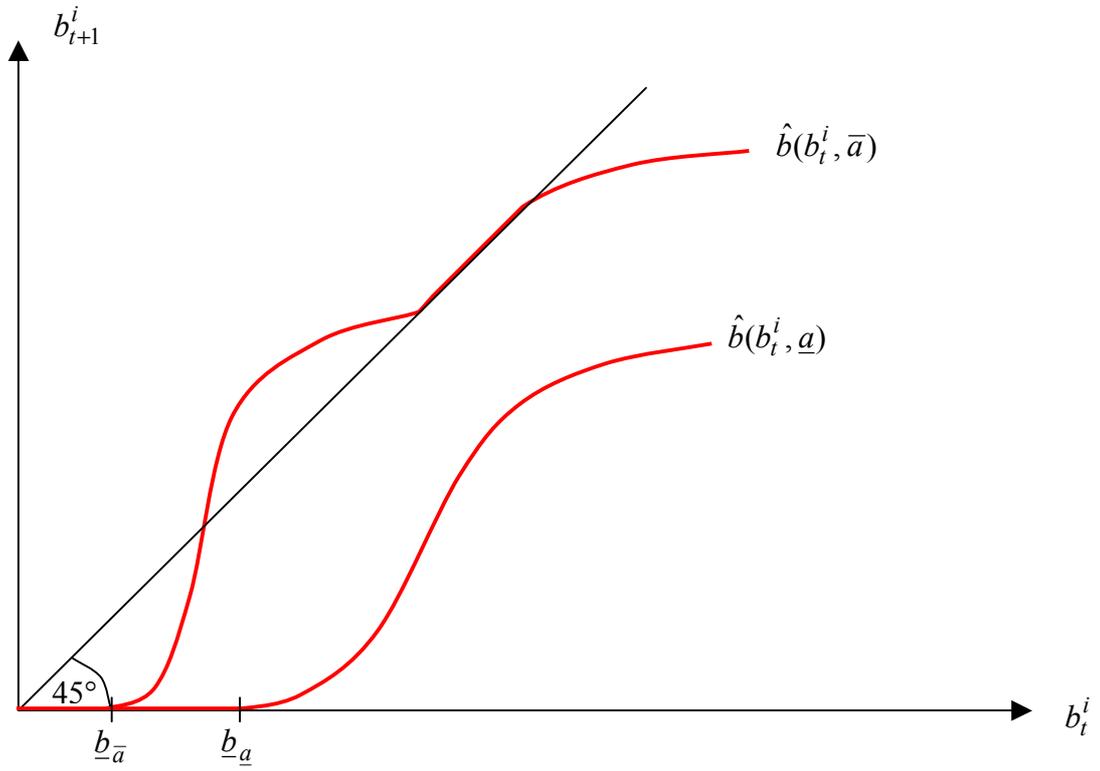
(a)
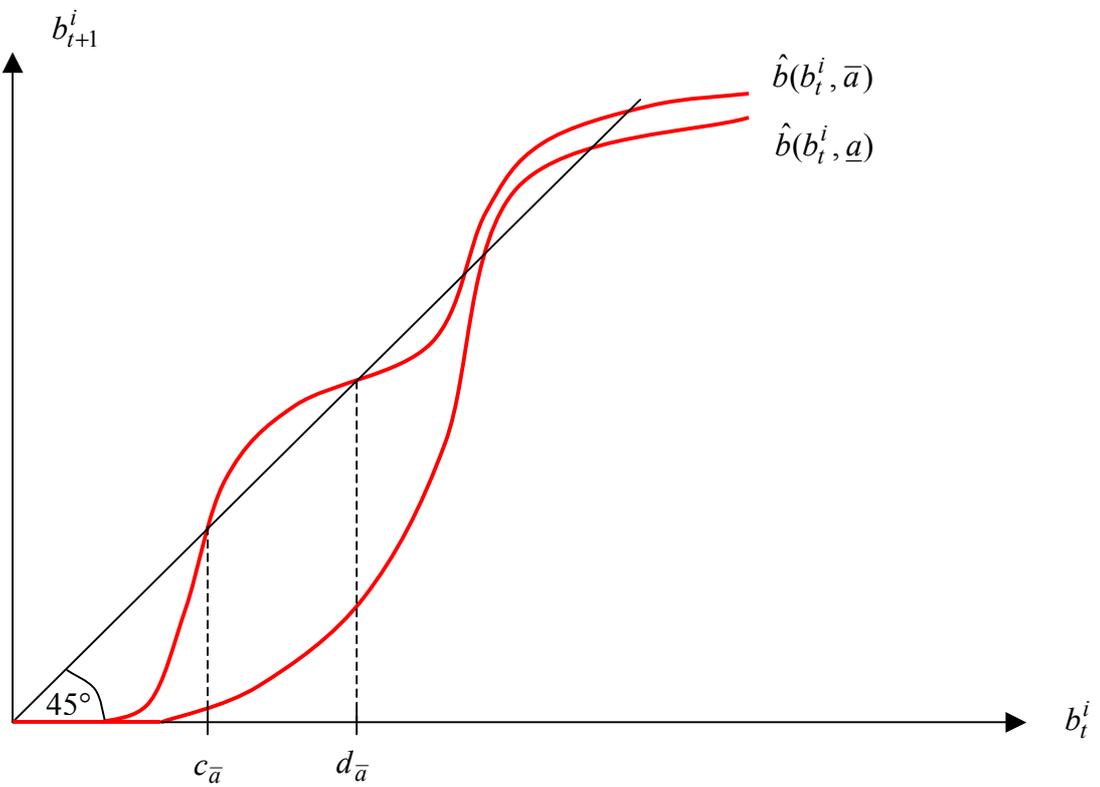
(b)

(Figure 2 continued)

(c)
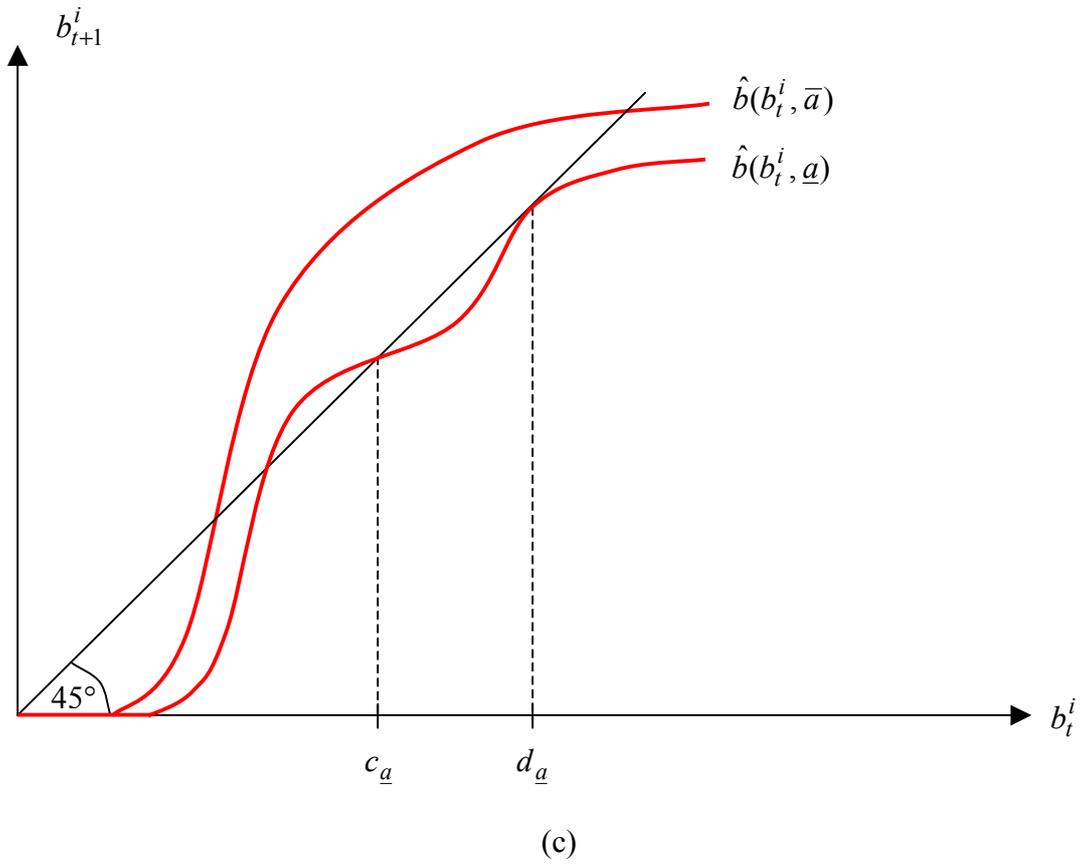
Figure 3: Consistency with assumptions A6-A8.

(a)

(b)
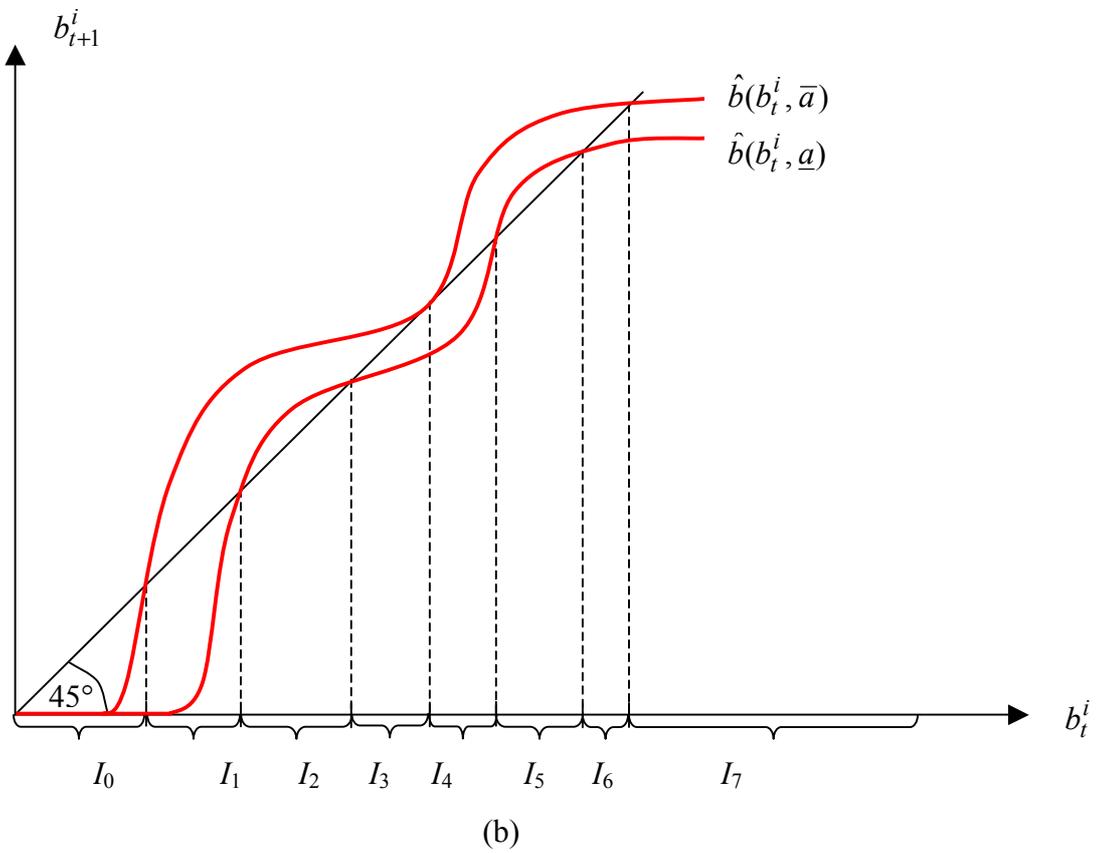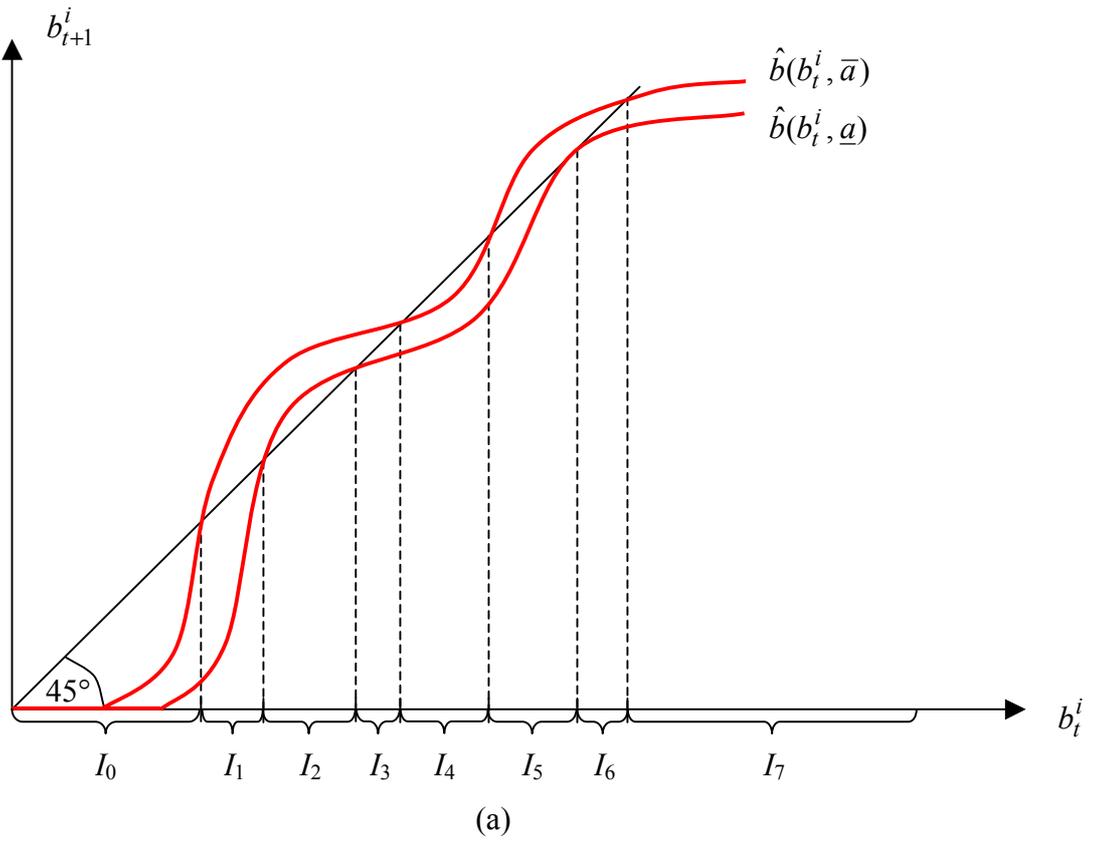
(Figure 3 continued)
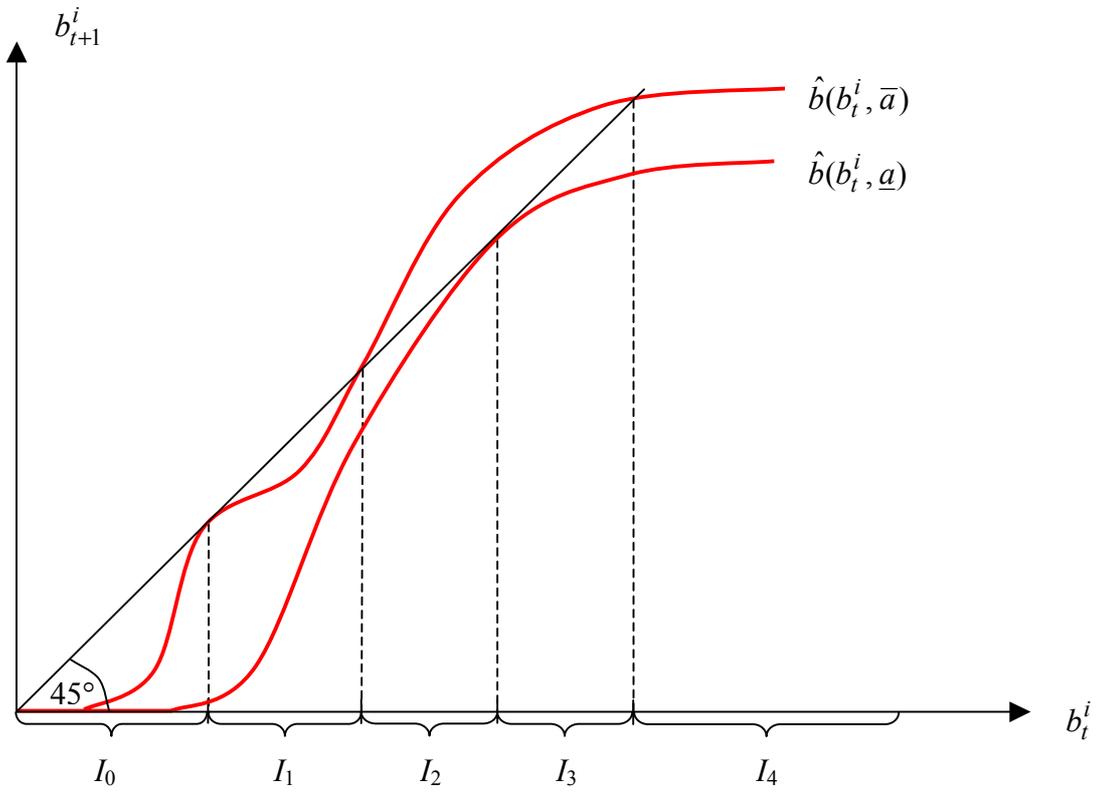
(c)

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