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ABSTRACT

Rise of the Machines: The Effects of Labor-Saving Innovations on Jobs and Wages*

How do firms respond to technological advances that facilitate the automation of tasks? Which tasks will they automate, and what types of worker will be replaced as a result? We present a model that distinguishes between a task's engineering complexity and its training requirements. When two tasks are equally complex, firms will automate the task that requires more training and in which labor is hence more expensive. Under quite general conditions this leads to job polarization, a decline in middle wage jobs relative to both high and low wage jobs. Our theory explains recent and historical instances of job polarization as caused by labor-replacing technologies, such as computers, the electric motor, and the steam engine, respectively. The model makes novel predictions regarding occupational training requirements, which we find to be consistent with US data.

JEL Classification: E25, J23, J31, M53, O33

Keywords: automation, job polarization, technical change, wage inequality, training

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1 Introduction

How do firms respond to technological advances that facilitate the automation of a wide range of job tasks? Which tasks will they automate, and what types of worker will be replaced as a result? Recent empirical evidence has linked automation to job polarization: the decline in middle wage jobs relative to both high and low wage ones. For instance, advances in information and communication technology (ICT) have led to the replacement of workers predominantly in middle wage jobs.[1] Similarly, the adoption of steam power and later electricity in US manufacturing was associated with a replacement of middle wage workers such as artisans and the growth of low wage operator and high wage non-production jobs.[2] Steam power, the electric motor, and ICT have in common that they allowed firms to more cheaply automate (or mechanize) tasks.[3] However, so far there exists no economic model that predicts an inherent connection between automation and job polarization.

In this paper, we develop a model that analyzes how firms take advantage of a fall in the cost of automation. In doing so, we provide an explanation for why waves of automation tend to cause polarization of labor markets. We argue that recently observed patterns of task automation are consistent with the predictions of our model about automation choices made by firms. Moreover, the model has novel predictions regarding occupational training requirements, which we find to be consistent with US data.

Our model distinguishes between a task's engineering complexity and its training requirements. This distinction is motivated by the insight that what is complex from an engineering point of view is not necessarily difficult to humans, and vice versa (Moravec 1988). There are tasks that are easy to *any* worker but building a machine capable of performing them may be costly if not impossible. Occupations such as waiters, taxi drivers, or housekeepers are intensive in the use of vision, movement, and communication, which are complex functions from an engineering point of view. On the other hand, a task like bookkeeping requires knowledge of arithmetic which takes humans years to learn, but which is trivial from an engineering perspective.

We allow firms to optimally respond to a fall in the cost of automating tasks, rather than imposing that only a particular subset of tasks can be automated, as is typical in the existing literature (see the discussion below). When two tasks are equally complex, firms will automate the task that requires more training and in which labor is hence more expensive. Under reasonable assumptions about skill differences among workers, the equilibrium sorting of

[1] Recent job polarization was first documented for the US by Autor, Katz, and Kearney (2006), for the UK by Goos and Manning (2007), and for other European economies by Goos, Manning, and Salomons (2009). For evidence supporting the technological explanation of recent job polarization, see Autor, Levy, and Murnane (2003), Michaels, Natraj, and Reenen (2014), and Goos, Manning, and Salomons (2014).

[2] Katz and Margo (2013) show that from 1850 to 1880, US manufacturing witnessed a relative decline in middle wage jobs (artisans) compared to high wage jobs (non-production workers) and low wage jobs (operators), concurrent with the increased adoption of steam power. Gray (2013) finds that electrification in the US during the first half of the 20th century led to a fall in demand for dexterity-intensive tasks performed by middle wage workers, relative to manual and clerical tasks carried out by low and high wage workers, respectively. Previously, Goldin and Katz (1998) presented evidence suggesting that electrification in the US was an instance of skill-biased technical change. Their empirical work focussed on a high-vs.-low skill dichotomy which rules out "hollowing out" by construction.

[3] See for instance Atack, Bateman, and Weiss (1980) and Boff (1967).

workers to tasks is such that both low and high skill workers are shielded from increased automation. The replacement and subsequent reallocation of middle skill workers then leads to job polarization and faster wage growth at the extremes of the distribution than in the middle.

The dimension of worker skill that is critical in our model is the ability to learn.[4] If high skill workers learn faster, then workers at the bottom of the skill distribution will sort into tasks that require little or no training (fast food preparers, waiters). Middle skill workers, on the other hand, sort into tasks that require intermediate amounts of training (production workers, clerks). Low skill workers are shielded from automation not only because some of the tasks they perform are highly complex in engineering terms, but also because, holding complexity constant, firms will first automate the tasks performed by middle skill workers, since these workers are more expensive. High skill workers are protected from automation because they sort into tasks that are both highly complex and that require a large amount of training (lawyers, researchers).

Our model suggests an explanation for why innovations that facilitate automation—such as the steam engine, the electric motor, and ICT—tend to cause job polarization. It also helps to explain recent patterns of automation. There are job tasks that are currently not automated despite the fact that it would be feasible to do so. For example, fast food preparation takes place in a controlled environment and involves a limited number of simple steps. It is thus not surprising that technology exists that automates this process (Melendez 2013). However, fast food jobs do not currently appear to be at risk of being automated. Moreover, in manufacturing some tasks are automated that are arguably of higher complexity (Davidson 2012). Given that fast food preparation requires very little training, in particular compared to non-trivial assembly tasks in manufacturing, this pattern of automation choices is exactly what our model would predict.

In addition to explaining labor market polarization and patterns of task automation, our theory delivers several novel predictions regarding job training requirements. We measure training requirements in the US at the occupation level, using the Dictionary of Occupational Titles (DOT) combined with the 1971 April CPS, and the O*NET database combined with the 2008 ACS.[5] Consistent with the model we find a decrease between 1971 and 2008 in the employment shares of occupations requiring intermediate amounts of training.

The model also implies that firms' selective automation of tasks within occupations should lead to systematic shifts in training requirements. We find that training requirements fell on average in occupations of initially intermediate training intensity. Occupations that initially required no or little training did not experience a change in training requirements on average, and neither did the initially most training-intensive occupations. As an example for what this means in practice, consider the case of warehouses operated by an online delivery service. Storing items in such a way that they can be quickly found and retrieved is a non-trivial task that used to require non-negligible amounts of training. Warehouse organisation has now been automated: workers in these warehouses follow instructions given to them by software

[4]Our model also includes a second dimension of skill, which directly affects productivity.

[5]In these data, training requirements are measured as the time it takes the typical worker to become proficient in her job. This may include any occupation-specific knowledge acquired prior to entering the labor market—see Section 5.

through a hand-held device. In the remaining tasks—carrying, shelving, retrieving, packing and unpacking items—workers rely on what we call innate abilities such as walking and recognizing objects. Training requirements have been reduced to a minimum (O'Connor 2013).

We also find that employment and wage growth was lower in occupations that experienced larger decreases in training requirements, as should be the case if automation causes training requirements to fall.

Our paper is not the first to incorporate insights from computer science into the task framework of an economic model. In a seminal paper, Autor, Levy, and Murnane (2003, henceforth ALM) categorize tasks as routine and non-routine and document a shift of employment out of routine tasks. They call a task routine "if it can be accomplished by machines following explicit programmed rules." In contrast, non-routine tasks are "tasks for which rules are not sufficiently well understood to be specified in computer code and executed by machines." Recent technological progress demonstrates that many non-routine tasks can be automated after all, including driving a car, parts of legal research, and some types of medical diagnosis (Frey and Osborne 2013). We therefore advocate a task framework that accommodates the possibility that machine capabilities constantly expand. Complexity in our model is an objective, time-invariant measure of a task's intrinsic difficulty, so our model should be generalizable to different technological environments.[6] A second advantage of our approach over ALM's, we believe, is to allow firms a choice of which tasks to automate, rather than assuming that routine tasks are automated, and non-routine tasks are not. A task like fast food preparation might be considered routine in ALM's framework, so that the non-automation of this task poses a puzzle. Our model is consistent with this example.

Following ALM's observation that non-routine tasks vary by their skill intensity, Autor, Katz, and Kearney (2006) modify the ALM framework by dividing non-routine tasks into "abstract" (high skill) and "manual" (low skill) tasks. Our model is similarly multi-dimensional—for instance, there are complex tasks which require training, and complex tasks which do not. However, we do not impose that these tasks cannot be performed by machines. While they are assumed to be automatable in our model, we show that firms' *incentives* lead them to replace labor predominantly in other tasks, namely those initially carried out by middle skill workers.

Autor and Dorn (2013) document that many of the low wage occupations that have experienced increasing employment shares consist of low skill services. In our model, these occupations are represented by tasks in which workers rely on innate abilities and hence require no training, even though these tasks may be very complex. We have verified that low skill service occupations as classified by Autor and Dorn (2013) are indeed among the occupations with the lowest training requirements in our data. Thus, our model may be seen as an empirically plausible micro-foundation for the model of Autor and Dorn (2013).

We employ modelling tools developed in two papers that have analyzed the effects of "task-biased technical change" on the allocation of workers to tasks and on the wage distribution. Acemoglu and Autor (2011, AA) present a comparative statics exercise in which they assume

[6] We discuss approaches to measuring the complexity of tasks in Section 2.1.

that machines replace middle skill workers.[7] Similarly, Costinot and Vogel (2010, CV), among other things, investigate the effects of technical change that is biased in favor of workers at both extremes of the wage distribution.

Our paper differs from AA and CV in three important aspects. First, we allow for an endogenous response of firms to a fall in the cost of automating tasks. In our model, there is no assumption about which part of the skill distribution will be most affected by increased automation—instead, this is endogenous to the attributes of factors and their interaction with the characteristics of tasks. Second, our task framework has an explicit empirical interpretation, while AA and CV differentiate between tasks solely on the basis of which factors have a comparative advantage in them. This allows us to derive factors' productivity as functions of their attributes and the characteristics of tasks.[8] Third, while AA and CV feature one-dimensional task spaces, we emphasize the importance of two dimensions. In Section 4.1, we omit training requirements and are left with a single dimension of task differences, namely complexity, obtaining a model very similar to AA and CV. We show that this version of the model cannot be reconciled with the evidence of employment shifting to low wage occupations such as personal services that entail highly complex tasks.

We build on the literature on labor-replacing (or labor-saving) innovations, that is, innovations that allow for the replacement of labor at the task level, typically leading to a fall in the marginal product of labor relative to that of capital. Zeira (1998) presents a model in which economic development is characterized by the adoption of technologies that reduce labor requirements relative to capital requirements. Over time, an increasing number of tasks can be produced by new, more capital-intensive technologies. In an example which is closely related to our paper, new technologies only use capital, while old ones only use labor. We extend this type of setting by explicitly modeling the characteristics of tasks and thus the task-bias of technical change, as well as by allowing for heterogenous workers. Holmes and Mitchell (2008) present a model of firm organization where the problem of matching workers and machines to tasks is solved at the firm level. Their model admits a discrete set of worker types and machines replace low skill workers by assumption.

Another strand of papers analyzes the matching of workers with technologies of different vintages. Wage inequality results for instance when workers must acquire vintage-specific skills (Chari and Hopenhayn 1991) or machines are indivisible (Jovanovic 1998). Furthermore, skill or unskill bias of technical change can arise when new technologies require different learning investments than old ones, and when learning costs are a function of skill (Caselli 1999). We abstract from the issue of workers having to learn how to operate new technologies and focus instead on the problem of assigning workers and machines to tasks. However, as in Caselli (1999), differences in learning costs (here: costs of learning to perform tasks) across workers constitute the critical dimension of skills driving the results.

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[7] The model in AA builds on Dornbusch, Fischer, and Samuelson (1977).

[8] AA assume that tasks differ continuously and index them by $i \in [0,1]$. Given any two tasks, high skill workers are assumed to have a comparative advantage over middle skill and low skill ones (and middle skill workers over low skill ones) in the task that ranks higher on this index. Similarly, CV index tasks by $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. They assume a continuum of skills $s \in [\underline{s}, \bar{s}]$, but the assumption on comparative advantage are qualitatively the same as in Acemoglu and Autor (2011). In neither case does the task index have an explicit empirical interpretation.

The plan of the paper is as follows. Section 2 presents and solves the model. Section 3 discusses comparative statics, in particular how job assignment and the wage distribution change as a response to increased automation. We also present comparative statics for an increase in the supply of capital, and increase in skill supplies. Section 4 presents an alternative formulation of our model to make clear the connection with previous literature, and it contains two extensions to the model: endogenous capital accumulation and a fixed cost of technology adoption. Section 5 takes the novel implications of the model to the data. Section 6 concludes.

2 The Model

The model has one period that we interpret as a worker's lifetime.[9] There is a unique final good that is produced using a continuum of intermediate inputs, or *tasks*. These tasks are performed by workers of different skill levels and machines. Crucially, all factors of production are perfect substitutes at the task level. Although this may seem a strong assumption, the loss of generality is not substantial provided all tasks are essential in producing the final good, a condition that we shall maintain throughout.[10] Labor services as well as the economy's capital stock are supplied inelastically and all firms are perfectly competitive. Intermediate firms hire workers or capital to produce task output that is then sold to final good firms. Intermediate firms may need to train workers, and must transform generic capital into task-specific machines in order for these factors to be capable of performing tasks. Technologies for worker training and machine design are public knowledge.

2.1 Motivating the Task Framework

The task framework we employ in this paper seeks to incorporate insights from research into artificial intelligence, robotics, and cognitive science. Researchers in these fields have long been aware that some abilities that humans acquire quickly at an early age rely in fact on highly complex functions that are difficult if not impossible to reverse-engineer. In contrast, many abilities that humans must painstakingly acquire, such as mastery in arithmetic, are trivial from an engineering perspective. This observation has become known as Moravec's paradox: "[It] is comparatively easy to make computers exhibit adult-level performance in solving problems on intelligence tests or playing checkers, and difficult or impossible to give them the skills of a one-year-old when it comes to perception and mobility" (Moravec 1988, p.15). Moravec resolves the puzzle by considering the objective or intrinsic difficulty of a task, for instance the amount of information processing required, or the degrees of freedom and dexterity necessary to carry out a certain physical action. In terms of intrinsic difficulty, arithmetic is much easier than walking or face-to-face communication.[11] The reason that we are usually not aware of this fact, and why

[9]We discuss a dynamic (multi-period) version of the model in Section 4.2.

[10]In fact, when tasks are imperfect substitutes in producing the final good, factors of production will appear to be imperfect substitutes in the aggregate.

[11]A possible indicator of the intrinsic difficulty of a task, or its complexity, is the computer power a machine requires to perform the task. A common if imperfect measure of computer power is million instructions per second (MIPS, see Nordhaus (2007) for a discussion of MIPS as a measure of computing power). UNIVAC I, a computer

Moravec's observation at first seems puzzling, is that we rely on innate abilities[12] for functions like movement or perception, but have no such advantage when it comes to abstract tasks like arithmetic.[13]

Seeking to incorporate these insights into our task framework, we assume a division of the task space into *training-intensive* and *innate ability* tasks. Each subset of tasks is further differentiated by *complexity*. The amount of resources required to build a machine capable of performing a given task is always increasing in complexity. However, the amount of training a worker requires to be able to carry out a given task is increasing in complexity only in training-intensive tasks, which is motivated by the insight that humans are endowed with a set of capabilities which overcome engineering problems of varying degrees of complexity.[14]

A growing literature attempts to classify occupations by the difficulty of automating the tasks typically performed in them. Looking at this literature through the lens of our task framework, we think that the "routine" tasks described by ALM correspond to the lower range of our complexity measure. While ALM argued that non-routine tasks could not be automated, since the rules for executing them are not well understood, Frey and Osborne (2013) provide evidence that a growing set of non-routine tasks are at risk of automation—these are tasks of medium complexity in our framework. However, Frey and Osborne (2013) also identify tasks which are unlikely to be automated in the coming decades, since they pose engineering bottlenecks related to perception, "creative intelligence", and "social intelligence"—highly complex tasks in our framework.[15] Table 1 gives an overview of our task framework, relates it to the two task frameworks used in empirical work as just described, and provides examples.

Critically, for all levels of complexity we provide examples for tasks that require little or no training (innate ability); as well as for tasks that require training, whose duration is increasing in complexity (training-intensive). The extended ALM framework (Autor, Katz, and Kearney 2006) allows non-routine tasks to be either "abstract" or "manual" and assumes that

built in 1951 and able to perform arithmetic operations at a much faster rate than humans, performed at 0.002 MIPS. ASIMO, a robot introduced in 2000 that walks, recognizes faces and processes natural language, requires about 4,000 MIPS (for a comparison of various computers (including UNIVAC I) and processors by MIPS, see `http://en.wikipedia.org/wiki/Instructions_per_second`, retrieved on October 16, 2013; for technical details of ASIMO, see Sakagami, Watanabe, Aoyama, Matsunaga, Higaki, and Fujimura (2002)). By this measure, a set of tasks routinely performed by any three-year-old is two million times more complex than arithmetic.

[12] "Innateness" of a certain skill does not need to imply that humans are born with it; instead, the subsequent development of the skill could be genetically encoded. For a critical discussion of the concept of innateness, see Mameli and Bateson (2011).

[13] Moravec (1988, pp.15-16) provides an evolutionary explanation for this: "...survival in the fierce competition over such limited resources as space, food, or mates has often been awarded to the animal that could most quickly produce a correct action from inconclusive perceptions. Encoded in the large, highly evolved sensory and motor portions of the human brain is a billion years of experience about the nature of the world and how to survive in it. The deliberate process we call reasoning is, I believe, the thinnest veneer of human thought, effective only because it is supported by this much older and much more powerful, though usually unconscious, sensorimotor knowledge. We are all prodigious olympians in perceptual and motor areas, so good that we make the difficult look easy. Abstract thought, though, is a new trick, perhaps less than 100 thousand years old. We have not yet mastered it. It is not all that intrinsically difficult; it just seems so when we do it."

[14] Machines could be viewed as being endowed with some functions to the extent that materials have productive properties—take for instance copper with its electrical conductivity; but such endowments are usually highly specific and limited.

[15] Frey and Osborne (2013) calculate for three-digit occupations the probability that they will become automatable. They use the O*NET database to assess the extent to which occupational tasks feature bottlenecks to computerization.

Table 1: Two-Dimensional Task Framework: Examples and Relation to Literature

	COMPLEXITY		
	low	medium	high
ALM framework	*routine*	*non-routine*	
FO framework		*automatable*	*subject to bottlenecks*
INNATE ABILITY	crushing rocks fast food preparation	customer reception driving a car	child care event planning
TRAINING-INTENSIVE	bookkeeping weaving	pre-trial research trading stocks	arguing a legal case designing fashion

Note: The task dimensions used in this paper are shown in small capitals. Complexity is continuous in the model, but discretized here to facilitate examples and a comparison with the literature. The mappings between complexity and the task frameworks by Autor, Levy, and Murnane (2003, ALM) and Frey and Osborne (2013, FO) are illustrated in the two rows above the examples. See the text for a discussion of the distinction between "manual" and "cognitive" tasks made by Autor, Katz, and Kearney (2006).

these are performed by high and low skill workers, respectively, but not by machines. Our emphasis on training requirements also delivers a multi-dimensional task space. Moreover, it lets us derive factor assignment as an equilibrium outcome, rather than having to assume it.

2.2 Model Setup

Tasks are differentiated by *complexity*, denoted by $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, and by whether workers require training to complete the task, indicated by $\tau \in \{0, 1\}$. The expenditure required to convert one unit of capital into a machine capable of performing a task of complexity σ is given by $c_K\sigma$. Complexity σ is the task-specific component of design expenditure. It is a fundamental, time-invariant property of tasks. The design cost c_K is constant across tasks and falls over time as better technologies become available, leading to a flatter complexity-cost gradient.[16]

For workers, the complexity of a task does not necessarily affect the amount of training required. In particular, no training is required if completion of a task relies solely on functions that all workers are endowed with or have acquired at early age, regardless of the complexity of the task. This is true in the case of innate ability tasks ($\tau = 0$). Workers cannot rely on such endowments in the case of training-intensive tasks ($\tau = 1$). Training requirements do increase with complexity in training-intensive tasks: To become capable of performing the training-intensive task of complexity σ, a worker of type s requires an amount of training σ/s. Higher skilled workers face a flatter complexity-training gradient in training-intensive tasks. Note that the distinction of innate ability versus training-intensive is about the *extensive* margin of training, while the interaction of training-intensive with complexity concerns the *intensive* margin of training.[17]

For simplicity, we allow complexity to only affect design expenditure and training time required to complete a task. In general we could think of complexity as impacting the design expenditure and training time required to achieve a given level of productivity when completing a task. We present in the appendix an extended task model that features such an intensive margin of productivity, and in which design and training are chosen optimally. That model is based on an explicit characterization of the production process following Garicano (2000). It features a more sophisticated concept of complexity that is related to the predictability of a given production process. All our results apply to the more general model as well.

A machine produces A_K units of task output, regardless of the task's complexity. Hence, A_K can be viewed as task-neutral machine productivity. Since some of the hired capital is lost in machine design, a unit of capital produces an amount of output equal to $A_K(1 - c_K\sigma)$.[18]

All workers have a unit endowment of time, and produce A_N units of task output if they are able to spend all their time in production. A_N represents workers' task-neutral productivity, and can be seen as capturing general human capital as well as labor-augmenting technology.

[16]Strictly speaking, c_K is the design cost per unit of capital, per unit of the complexity measure.

[17]A more general model would allow τ to vary continuously. This would imply that the relationship between complexity and training requirements (holding skill constant) increases gradually as τ increases, rather than jumping from being flat to positive as we assume here. While the more general assumption seems intriguing, it would complicate the formal analysis dramatically.

[18]In reality, many of the innovations that lead to a fall in c_K may also cause a rise in A_K. However, as we show below, the comparative statics with respect to A_K are qualitatively the same as those with respect to c_K.

Workers of any type produce A_N units of task output in innate ability tasks ($\tau = 0$). Taking into account training time, a worker of type s produces $A_N(1 - \sigma/s)$ units of task output in training-intensive tasks ($\tau = 1$) of complexity σ.

The notation $s_K \equiv 1/c_K$, 'machine skill', will turn out to be more convenient. Let worker type range from $\underline{s} > 0$ to $\bar{s}$. We assume throughout that $s_K, \underline{s} \geq \overline{\sigma}$, so that machines and all worker types produce non-negative output in any task. Furthermore, we assume throughout that $s_K < \bar{s}$ for reasons discussed in Section 2.4.

Let $k_\tau(\sigma)$ denote the amount of capital used to produce task (σ, τ) and similarly let $n_\tau(s, \sigma)$ be the amount of type-s labor.[19] Given the task-specific productivity schedules for machines

$$\alpha^K(s_K, \sigma) = 1 - \sigma/s_K \tag{1}$$

and labor

$$\alpha_\tau^N(s, \sigma) = \begin{cases} 1 & \text{if } \tau = 0 \\ 1 - \sigma/s & \text{if } \tau = 1, \end{cases} \tag{2}$$

task output y can be written as

$$y_\tau(\sigma) = A_K \alpha^K(s_K, \sigma) k_\tau(\sigma) + A_N \int_{\underline{s}}^{\bar{s}} \alpha_\tau^N(s, \sigma) n_\tau(s, \sigma) ds. \tag{3}$$

Let Y denote the output of the final good. For tractability, we use a Cobb-Douglas production function,

$$\log Y = \frac{1}{\mu} \int_{\underline{\sigma}}^{\overline{\sigma}} \{\beta_0 \log y_0(\sigma) + \beta_1 \log y_1(\sigma)\} \, d\sigma. \tag{4}$$

Recall that the subscripts 0 and 1 indicate innate ability ($\tau = 0$) and training-intensive ($\tau = 1$) tasks, respectively. We impose $\sum_\tau \beta_\tau = 1$ and $\mu \equiv \overline{\sigma} - \underline{\sigma}$ to ensure constant returns to scale.

Let there be a mass K of machine capital and normalize the labor force to have unit mass. We assume a skill distribution that is continuous and without mass points. Let $V(s)$ denote the differentiable CDF, and $v(s)$ the PDF, with support $[\underline{s}, \bar{s}]$. Factor market clearing conditions are

$$v(s) = \sum_\tau \int_{\underline{\sigma}}^{\overline{\sigma}} n_\tau(s, \sigma) d\sigma \quad \textit{for all } s \in [\underline{s}, \bar{s}] \tag{5}$$

and

$$K = \sum_\tau \int_{\underline{\sigma}}^{\overline{\sigma}} k_\tau(\sigma) d\sigma. \tag{6}$$

Before characterizing the equilibrium of the model, we turn to a discussion of some of our assumptions.

[19] Throughout the paper we use a subscript to refer to the discrete dimension of the task space, τ, and write the continuous dimension σ as a function argument.

2.3 Discussion of the Model's Assumptions

Our assumption on task production functions and machine design imply that all costs related to automation are variable and that there are constant returns to scale. The assumption of variable costs captures the fact that the cost of employing capital is generally increasing in output, but to completely abstract from a fixed component is certainly restrictive. In particular, firms usually face large one-off expenses when installing new machinery. We therefore also wrote down a version of our model in which firms face a fixed cost of automation, which increases in complexity. Our results are robust to this extension as we discuss in Section 4.3.[20]

The concept of worker training we employ in our model refers to the acquisition of knowledge and capabilities required to perform a given task. This corresponds to the empirical measure of occupational training requirements, *specific vocational preparation* (SVP), that we use when taking the model to the data. SVP includes all occupation-specific training, whether it takes place outside or within the firm. Similarly, in our model, it is not relevant whether the training takes place within or outside the firm—this distinction is purely semantic. Training is task-specific but not specific to a given firm, since there are many firms producing each task. Firms do not literally pay for training (or design, for that matter), but training (and design) costs affect the firm through diminished productivity. Our model thus captures empirically relevant tradeoffs in a rather reduced-form way. The simplifications are arguably necessary for tractability. The question whether our results are robust to modelling training in a dynamic setting and under the frictions typically associated with training, is left for future research.[21]

While we believe that our task framework is an improvement over existing literature, there are several limitations. Technical change often leads to the introduction of new tasks and activities (flying airplanes, writing software). While our framework in principle allows for an endogenous task space, it does not suggest in what way technology might affect the set of tasks in the economy. Moreover, automation does not necessarily involve machines replicating exactly the steps that humans carry out in completing a given task. Instead, a task can be made less complex by moving it to a more controlled environment.[22] Our framework does not explicitly allow for this possibility, but our conclusions should still be broadly correct if the cost of moving a process to a more controlled environment is increasing in the processes' complexity. Furthermore, technological change tends to cause organizational change, but to keep the analysis tractable and to be able to focus on a single mechanism, we omit firm organization from the model. Finally, innovations such as ICT arguably make it easier to move tasks abroad. Our closed-economy setting abstracts from off-shoring and trade in tasks.

An important and distinguishing feature or our model is the assumption that machines could in principle perform any task. While this assumption is certainly not true at the moment,

[20] Another source of increasing returns could be the fact that the ideas and inventions leading to increased task automation are non-rival—given our static setting, we cannot address this issue, and this is left for future research.

[21] In a more realistic setting in which there is a distinction between general and firm-specific training there are additional complications in interpreting wage data, since observed wages may be net of training if training is firm-specific. Changes in the incidence of general relative to firm-specific training may then lead to changes in the observed wage distribution even in the absence of technical change or shifts in skill supplies.

[22] See ALM (p.1283) and Simon (1960, pp.33-35). A recent example is the new sorting machine employed by the New York Public Library (Taylor 2010).

it does not lead to counterfactual predictions, because comparative advantage ensures that some tasks will always be performed by humans. More importantly, recent technological progress suggests that machine capabilities might be expanding quite rapidly.[23] For our model to be useful as a guide to medium-term future developments in the economy, we think it best to make the most conservative assumption about what tasks are safe from automation.

2.4 Characterizing the Competitive Equilibrium

We normalize the price of the final good to one and denote the price of task (σ, τ) by $p_\tau(\sigma)$. Profits of final good firms are given by

$$\Pi = Y - \sum_\tau \int_{\underline{\sigma}}^{\overline{\sigma}} p_\tau(\sigma) y_\tau(\sigma) d\sigma,$$

and profits of intermediate producers of task (σ, τ) are

$$\Pi_\tau(\sigma) = p_\tau(\sigma) y_\tau(\sigma) - r k_\tau(\sigma) - \int_{\underline{s}}^{\overline{s}} w(s) n_\tau(s, \sigma) ds$$

where r is the rental rate of capital and $w(s)$ is the wage paid to a worker with skill s. Design and training costs are included in intermediate producers' profits in the sense that for each unit of capital or labor hired, a fraction may be lost in design or training, as captured by (1), (2), and (3).

As in Costinot and Vogel (2010), a competitive equilibrium is defined as an assignment of factors to tasks such that all firms maximize profits and markets clear. Profit-maximizing task demand by final good producers is

$$y_\tau(\sigma) = \frac{\beta_\tau}{\mu} \frac{Y}{p_\tau(\sigma)}. \tag{7}$$

Profit maximization by intermediates producers implies

$$\begin{array}{rcll} p_\tau(\sigma)\alpha^N(s,\sigma,\tau) & \leq & w(s)/A_N & \forall s \in [\underline{s}, \overline{s}], \\ p_\tau(\sigma)\alpha^K(s_K,\sigma) & \leq & r/A_K; & \\ p_\tau(\sigma)\alpha^N(s,\sigma,\tau) & = & w(s)/A_N & \textit{if } \; n_\tau(s,\sigma) > 0, \\ p_\tau(\sigma)\alpha^K(s_K,\sigma) & = & r/A_K & \textit{if } \; k_\tau(\sigma) > 0. \end{array} \tag{8}$$

Denote the economy's set of tasks by $T \equiv [\underline{\sigma}, \overline{\sigma}] \times \{0, 1\}$. Formally, a *competitive equilibrium* in this economy is a set of functions $y : T \to \mathbb{R}^+$ (task output); $k : T \to \mathbb{R}^+$ and $n : [\underline{s}, \overline{s}] \times T \to \mathbb{R}^+$ (factor assignment); $p : T \to \mathbb{R}^+$ (task prices); $w : [\underline{s}, \overline{s}] \to \mathbb{R}^+$ (wages); and a real number r

[23] See the discussion of Frey and Osborne (2013) above. Brynjolfsson and McAfee (2011, p.14) and Ford (2009) argue that machines can potentially substitute for humans in a much larger range of tasks than was thought possible not long ago, citing recent advances in pattern recognition (driverless cars), complex communication (machine translation), and combinations of the two (IBM's successful Jeopardy contestant Watson). Markoff (2012) provides an account of the increased flexibility, dexterity, and sophistication of production robots. Shein (2013) gives examples of robots being increasingly adopted in manual tasks, such as collecting items in a warehouse, or the pruning of grapes. An overview of recent developments in basic robotics research can be found in Nourbakhsh (2013).

(rental rate of capital) such that conditions (1) to (8) hold.

To be able to characterize the competitive equilibrium, we need to examine the properties of the productivity schedules α^K and α^N. In training-intensive tasks, workers face the same productivity schedule as machines, except for the skill parameter. Let $\breve{s} \in s_K \cup [\underline{s}, \bar{s}]$ and define

$$\alpha(\breve{s}, \sigma) \equiv 1 - \sigma/\breve{s} \quad \left(\equiv \alpha^K(\breve{s}, \sigma) \equiv \alpha_1^N(\breve{s}, \sigma)\right). \tag{9}$$

Note that $\alpha \in (0, 1)$. Furthermore, $\alpha_\sigma < 0$ and $\alpha_{\breve{s}} > 0$. Productivity declines in complexity since a larger design or training expense is incurred. Higher skilled factors are more productive since they incur a smaller design or training expense. To characterize comparative advantage, we rely on the following result.[24]

Lemma 1 *The productivity schedule $\alpha(\breve{s}, \sigma)$ is strictly log-supermodular.*

The log-supermodularity of the productivity schedule implies that in training-intensive tasks, factors with higher skill have a comparative advantage in more complex tasks, or

$$\breve{s}' > \breve{s}, \sigma' > \sigma \quad \Leftrightarrow \quad \frac{\alpha(\breve{s}', \sigma')}{\alpha(\breve{s}, \sigma')} > \frac{\alpha(\breve{s}', \sigma)}{\alpha(\breve{s}, \sigma)}.$$

For instance, high skill workers have a comparative advantage over low skill workers in more complex tasks; all workers with $s > s_K$ have a comparative advantage over machines in more complex tasks; and so on. The result is due to the fact that for higher skilled factors, training or design expenses increase less steeply with complexity.

Comparative advantage properties regarding training intensity are straightforward. Since α is increasing in $\breve{s}$, and because all workers have productivity one in all innate ability tasks, high skill workers have a comparative advantage over low skill workers in any training-intensive task. Furthermore, because machine productivity is the same in innate ability tasks as in training-intensive tasks if complexity is held constant, it follows that machines have a comparative advantage over all workers in any training-intensive task relative to the innate ability task with the same complexity. This seemingly trivial result has profound implications for the assignment of factors to tasks, and for the reallocation of factors in response to a fall in c_K (a rise in s_K). It is at the root of the job polarization result, as we will show in Section 3 below.

The equilibrium assignment of factors to tasks is determined by comparative advantage, which is a consequence of the zero-profit condition (8).[25] Because high skill workers have a comparative advantage in training-intensive tasks (holding complexity constant), in equilibrium the labor force is divided into a group of low skill workers performing innate ability tasks, and a

[24] All lemmas are proved in the appendix.

[25] To see how comparative advantage determines patterns of specialization, consider two firms, one producing training-intensive task σ, the other producing training-intensive task σ'. Suppose in equilibrium, firm σ is matched with workers of type s and firm σ' is matched with workers of type s'. Then (8) implies

$$\frac{\alpha(s', \sigma')}{\alpha(s, \sigma')} \geq \frac{\alpha(s', \sigma)}{\alpha(s, \sigma)},$$

which shows that type s (s') has a comparative advantage in task σ (σ').

group of high skill workers carrying out training-intensive tasks: there exists a marginal worker with skill s^*, the least skilled worker employed in training-intensive tasks. This is formally stated in part (a) of Lemma 2 below.

We focus on the empirically relevant case in which machines as well as workers perform both training-intensive and innate ability tasks. A *necessary* condition is that $s_K < \bar{s}$, since otherwise no workers would perform any training-intensive tasks. A simple *sufficient* condition is $\underline{\sigma} = 0$, which implies that if firms did not prefer to hire machines in the least-complex innate ability and training-intensive tasks, then they would not prefer to hire machines in any tasks, which would violate market clearing.[26] If $\underline{\sigma} > 0$, then capital must neither be very scarce nor extremely abundant. Machines will not perform any innate ability tasks if capital is very scarce, and workers will not carry out any training-intensive tasks if capital is extremely abundant. This is formally proven in the appendix. We assume throughout that parameters are such that the equilibrium corresponds to the empirically relevant case that we are interested in.

Machines are assigned to a subset of innate ability and training-intensive tasks that are relatively less complex, while low skill workers perform the remaining innate ability tasks: there is a threshold task σ_0^*, the marginal innate ability tasks, dividing the set of innate ability tasks into those performed by machines ($\sigma \leq \sigma_0^*$) and those carried out by low skill workers ($\sigma \geq \sigma_0^*$). Similarly, there is a marginal training-intensive task σ_1^* that divides the set of training-intensive tasks into those performed by machines ($\sigma \leq \sigma_1^*$) and those carried out by high skill workers ($\sigma \geq \sigma_1^*$). As in the case of the marginal worker, existence of these marginal tasks is of course a consequence of the comparative advantage properties discussed at the end of Section 2.2. These properties also imply $\sigma_0^* < \sigma_1^*$: the marginal training-intensive task is always more complex than the marginal innate ability task (recall that machines are relatively more productive in training-intensive tasks than workers, holding complexity constant); and $s^* > s_K$: it is always cheaper to train (though not to employ) the marginal worker than to design a machine in any task. These results are formally stated in part (b) of Lemma 2. An illustration of the equilibrium assignment is given in Figure 1.

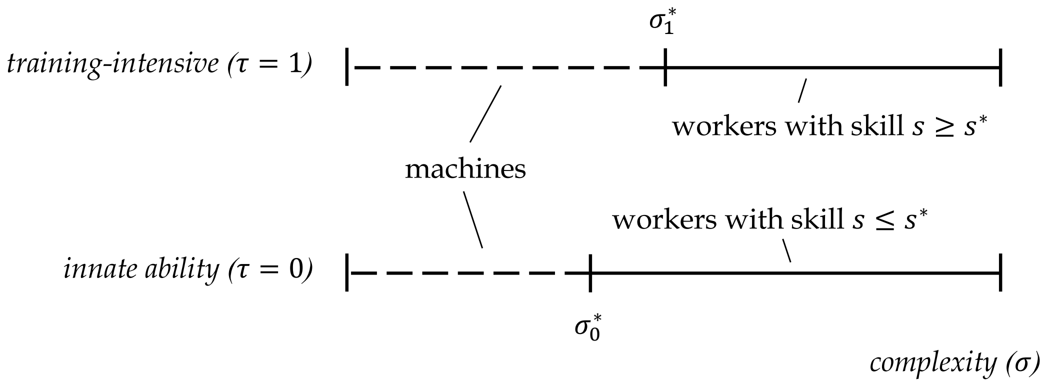
Lemma 2 (a) *In a competitive equilibrium, there exists an* $s^* \in (\underline{s}, \bar{s}]$ *such that*

- $n_0(s,\sigma) > 0$ *for some* σ *if and only if* $s \leq s^*$, *and*
- $n_1(s,\sigma) > 0$ *for some* σ *if and only if* $s \geq s^*$.

(b) *If* $k_0(\sigma) > 0$ *for some* σ, *then* $s^* > s_K$, *and there exist* $\sigma_0^*, \sigma_1^* \in [\underline{\sigma}, \bar{\sigma}]$ *with* $\sigma_0^* < \sigma_1^*$ *such that*

- $k_0(\sigma) > 0$ *if and only if* $\sigma \leq \sigma_0^*$;
- $k_1(\sigma) > 0$ *if and only if* $\sigma \leq \sigma_1^*$;
- $n_0(s,\sigma) > 0$ *if and only if* $s \leq s^*$ *and* $\sigma \geq \sigma_0^*$; *and*
- $n_1(s,\sigma) > 0$ *if and only if* $s \geq s^*$ *and* $\sigma \geq \sigma_1^*$.

Figure 1: Assignment of Labor and Capital to Tasks



The result as stated is general and does not rely on any restrictions on the model's parameters. Imposing the restrictions discussed above and in the appendix ensure that $k_0(\sigma) > 0$ for some σ.

An intuition for the equilibrium assignment may be gained by inspecting Figure 2, which plots the marginal cost of employing machines $r/[A_K\alpha(s_K,\sigma)]$ and that of the marginal worker $w(s^*)/[A_N\alpha_\tau^N(s^*,\sigma)]$ across the task space. Employing the marginal worker in innate ability tasks involves a constant level of marginal cost, no matter how complex the task. In contrast, the same worker's marginal cost is increasing in complexity in training-intensive tasks, as the worker spends more time training and less time producing the more complex the task. Machines are employed where their marginal cost is less than that of the marginal worker. The intersection of the two marginal cost curves is necessarily further to the right in training-intensive tasks—the level of complexity at which firms are indifferent between the two factors is larger in training intensive tasks, since in these tasks workers' marginal costs are increasing in complexity, too.

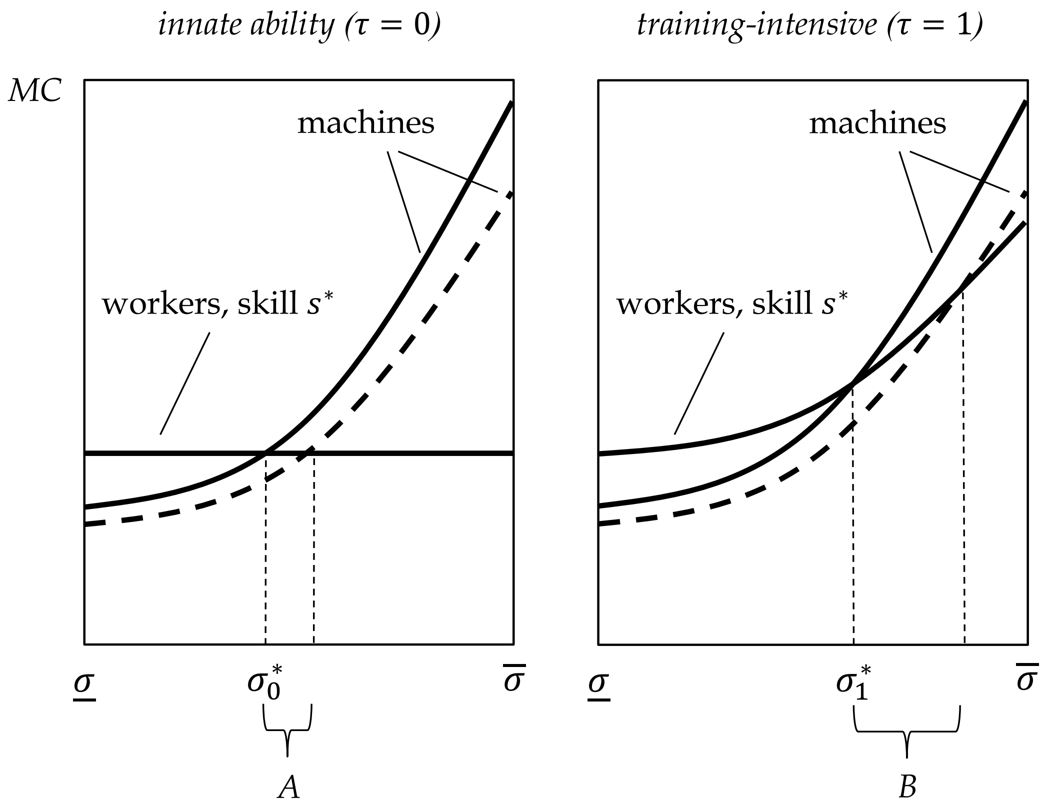
It remains to determine the assignment of low skill workers ($s \leq s^*$) to innate ability tasks ($\tau = 0, \sigma \geq \sigma_0^*$) and that of high skill workers ($s \geq s^*$) to training-intensive tasks ($\tau = 1, \sigma \geq \sigma_1^*$). The solution to the matching problem in innate ability tasks is indeterminate as all workers are equally productive in these tasks. However, knowledge of the assignment is not necessary to pin down task output and prices, as shown below. High skill workers are assigned to training-intensive tasks according to comparative advantage, with higher skilled workers carrying out more complex tasks. Formally, we have:

Lemma 3 *In a competitive equilibrium, if $s^* < \bar{s}$, there exists a continuous and strictly increasing matching function $M : [s^*, \bar{s}] \to [\sigma_1^*, \bar{\sigma}]$ such that $n_1(s,\sigma) > 0$ if and only if $M(s) = \sigma$. Furthermore, $M(s^*) = \sigma_1^*$ and $M(\bar{s}) = \bar{\sigma}$.*

This result is an application of Costinot and Vogel (2010), with the added complication that domain and range of the matching function are determined by the endogenous variables s^* and σ_1^*. The matching function is characterized by a system of differential equations. Using arguments along the lines of the proof of Lemma 2 in Costinot and Vogel (2010), it can be shown

[26] This case is drawn in Figure 2.

Figure 2: Marginal Cost, Factor Assignment, and Technical Change



The marginal cost of employing a given factor is plotted across the task space. Machines are employed up to the respective threshold tasks because their marginal cost is less than that of the marginal worker. The dashed lines represent the marginal cost of employing machines after a fall in the machine design cost, but holding the rental rate constant. (To simplify the figure, the marginal cost of employing the marginal worker in the least-complex training-intensive task is drawn to be equal to the marginal cost of workers in innate ability tasks, which amounts to setting $\underline{\sigma} = 0$. This is assumption is not made anywhere else in the paper.)

that the matching function satisfies

$$M'(s) = \frac{\mu}{\beta_1} \frac{w(s)v(s)}{Y}, \tag{10}$$

and that the wage schedule is given by

$$\frac{d \log w(s)}{ds} = \frac{\partial \log \alpha(s, M(s))}{\partial s}. \tag{11}$$

The last equation is due to the fact that in equilibrium, a firm producing training-intensive task σ chooses worker skill s to minimize marginal cost $w(s)/\alpha(s, \sigma)$. Once differentiability of the matching function has been established, (10) can easily be derived from the market clearing condition (5) given Lemma 2, and using (7) and (8).[27] Figure 3 illustrates how the matching function assigns workers to training-intensive tasks.

To pin down the endogenous variables σ_0^*, σ_1^*, and s^*, we use a set of no-arbitrage conditions. Firms producing the marginal tasks are indifferent between hiring labor or capital, and the marginal worker is indifferent between performing innate ability tasks or the marginal training-intensive tasks. Formally, the price and wage functions must be continuous, otherwise the zero-profit condition (8) could not hold. This is a well-known result in the literature on comparative-advantage-based assignment models. Hence, the no-arbitrage conditions for the marginal tasks are

$$\frac{r}{A_K \alpha(s_K, \sigma_0^*)} = \frac{w(s)}{A_N} \quad \textit{for all } s < s^* \tag{13}$$

and

$$\frac{r}{A_K \alpha(s_K, \sigma_1^*)} = \frac{w(s^*)}{A_N \alpha(s^*, \sigma_1^*)}, \tag{14}$$

and the no-arbitrage condition for the marginal worker is

$$w(s) = w(s^*) \quad \textit{for all } s \leq s^*. \tag{15}$$

The last result implies that there is a mass point at the lower end of the wage distribution. The mass point is a result of assuming task-neutral productivity A_N to be constant across workers. To avoid the mass point, we could instead assume that $A_N \equiv A_N(s)$ with $A_N'(s) \geq 0$. Equilibrium assignment and comparative statics results would be qualitatively the same, but the notation would be more tedious.

We can now complete the characterization of the competitive equilibrium by eliminating

[27] Lemma 2 and (5) imply

$$\int_{s^*}^{s} v(s')ds' = \int_{\sigma_1^*}^{\sigma} n_1(M^{-1}(\sigma'), \sigma')d\sigma'.$$

Changing variables on the RHS of the last expression and differentiating with respect to s yields

$$v(s) = n_1(s, M(s))M'(s),$$

and substituting (3) we obtain

$$M'(s) = \frac{\alpha(s, M(s))v(s)}{y(M(s))}. \tag{12}$$

After eliminating task output and price using (7) and (8), (10) follows.

factor prices from (13). Some algebra shows[28] that

$$r = \frac{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})}{\mu} \times \frac{Y}{K}. \tag{16}$$

This is of course the familiar result that with a Cobb-Douglas production function, factor prices equal the factor's share in output times total output per factor unit. In this case, the factor share is endogenously given by the (weighted) share of tasks to which the factor is assigned. A similar result obtains for the wage of workers employed in innate ability tasks,[29]

$$w(s^*) = \frac{\beta_0(\overline{\sigma} - \sigma_0^*)}{\mu} \times \frac{Y}{V(s^*)}. \tag{17}$$

With (16) and (17) in hand, we can eliminate factor prices from the marginal cost equalization condition (13). It is convenient to define $\theta \equiv A_K K / A_N$, the ratio of efficiency units of capital to those of labor (recall that the labor force is assumed to have unit mass). We obtain

$$\frac{\theta\alpha(s_K, \sigma_0^*)}{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})} = \frac{V(s^*)}{\beta_0(\overline{\sigma} - \sigma_0^*)}. \tag{18}$$

Also, combining conditions (13) to (15) yields

$$\alpha(s_K, \sigma_1^*) = \alpha(s_K, \sigma_0^*)\alpha(s^*, \sigma_1^*). \tag{19}$$

[28]To derive 16, note that by (7) and (8), we have

$$\frac{y_\tau(\sigma)}{y_\tau(\sigma')} = \frac{\alpha(s_K, \sigma)}{\alpha(s_K, \sigma')}, \qquad \frac{y_0(\widetilde{\sigma})}{y_1(\widetilde{\sigma}')} = \frac{\beta_0}{\beta_1}\frac{\alpha(s_K, \widetilde{\sigma})}{\alpha(s_K, \widetilde{\sigma}')}$$

for any tasks $(\sigma, \sigma', \widetilde{\sigma}, \widetilde{\sigma}')$ performed by machines. But (3), (3), and Lemma 2 imply

$$\frac{y_\tau(\sigma)}{y_\tau(\sigma')} = \frac{\alpha(s_K, \sigma)k_\tau(\sigma)}{\alpha(s_K, \sigma')k_\tau(\sigma')}, \qquad \frac{y_0(\widetilde{\sigma})}{y_1(\widetilde{\sigma}')} = \frac{\alpha(s_K, \widetilde{\sigma})k_0(\widetilde{\sigma})}{\alpha(s_K, \widetilde{\sigma}')k_0(\widetilde{\sigma}')}.$$

The previous two equations together give $k_\tau(\sigma) = k_\tau(\sigma')$ and $k_0(\widetilde{\sigma}) = \frac{\beta_0}{\beta_1}k_1(\widetilde{\sigma}')$. By (6) and Lemma 2,

$$k_\tau(\sigma) = \frac{\beta_\tau K}{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})}, \quad y_\tau(\sigma) = \frac{\beta_\tau A_K \alpha(s_K, \sigma) K}{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})} \quad \textit{for all } \sigma \in [\underline{\sigma}, \sigma_0^*].$$

Using these equations to solve for the task prices in (7), and plugging the obtained expression into (8), yields (16).

[29]Since in innate ability tasks, worker productivity does not vary across tasks nor types, all innate ability tasks with $\sigma \geq \sigma_0^*$ have the same price and all workers with $s < s^*$ earn a constant wage equal to $w(s^*)$ (as a result of the no-arbitrage condition for the marginal worker). As prices do not vary, neither does output. Under Lemma 2, integrating (5) yields

$$V(s^*) = \int_{\sigma_0^*}^{\overline{\sigma}} \int_{\underline{s}}^{s^*} n_0(s, \sigma) ds d\sigma,$$

but using (3) and the fact that task output is a constant y_0 results in

$$V(s^*) = (\overline{\sigma} - \sigma_0^*)y_0.$$

Using this equation to solve for the task prices in (7), and plugging the obtained expression into (8), yields (17).

Lastly, (10) and (17) imply

$$M'(s^*) = \frac{\beta_0(\overline{\sigma} - \sigma_0^*)}{\beta_1} \frac{v(s^*)}{V(s^*)}. \tag{20}$$

Equations (4), (10), (11), (18), (19), and (20) together with the boundary conditions $M(s^*) = \sigma_1^*$ and $M(\overline{s}) = \overline{\sigma}$, uniquely pin down the equilibrium objects σ_0^*, σ_1^*, s^*, w, and M. The comparative statics analysis makes extensive use of these expressions.

To conclude this section, we highlight two properties of the wage structure in our model. First, integrating (11) yields an expression for the wage differential between any two skill types that are both employed in training-intensive tasks,

$$\frac{w(s')}{w(s)} = \exp\left[\int_s^{s'} \frac{\partial}{\partial z} \log \alpha(z, M(z)) dz\right] \quad \textit{for all } s' \geq s \geq s^*. \tag{21}$$

This shows that wage inequality is fully characterized by the matching function (Sampson 2014). Second, adding (10) and (17) and integrating yields an expression for the average wage,

$$\mathrm{E}[w] = \frac{\beta_0(\overline{\sigma} - \sigma_0^*) + \beta_1(\overline{\sigma} - \sigma_1^*)}{\mu} \times Y. \tag{22}$$

Since the labor force is normalized to have measure one, this expression also gives the total wage bill. It follows that the labor share in the model is given by the (weighted) share of tasks performed by workers.

3 Comparative Statics

Having outlined the model and characterized its equilibrium in the previous section, we now move on to comparative statics exercises. Our main interest is in investigating the effects of a fall in the machine design cost, c_K. In addition we will analyze the effects of a greater supply of efficiency units of capital relative to labor, θ, and the effects of increased skill abundance. Our focus throughout continues to be the case in which all cutoff variables lie in the interior. Our comparative statics results carry over to the case in which machines do not perform any innate ability tasks.

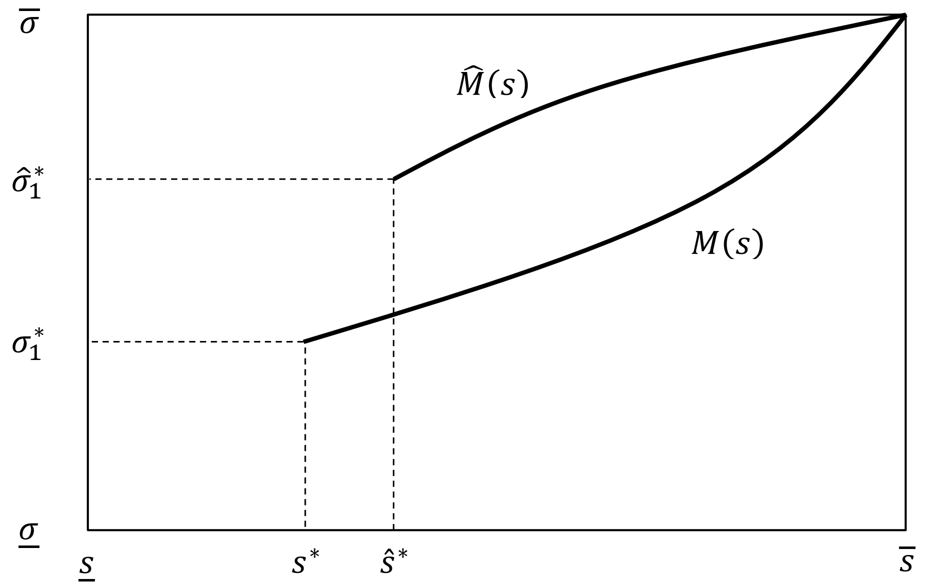
Consider first a fall in the machine design cost from c_K to $\widehat{c}_K$, so that $\widehat{s}_K > s_K$. Let M and $\widehat{M}$ be the corresponding matching functions, and similarly for σ_0^* and $\widehat{\sigma}_0^*$; σ_1^* and $\widehat{\sigma}_1^*$; and s^* and $\widehat{s}^*$. We now state the main result of the paper.[30]

Proposition 1 *Suppose the machine design cost falls, $\widehat{c}_K < c_K$ and so $\widehat{s}_K > s_K$. Then the marginal training-intensive task becomes more complex, $\widehat{\sigma}_1^* > \sigma_1^*$, the marginal worker becomes more skilled, $\widehat{s}^* > s^*$, and the matching function shifts up, $\widehat{M}(s) > M(s)$ for all $s \in [\widehat{s}^*, \overline{s})$.*

A fall in the machine design cost implies a rise in machine productivity and thus a fall in the marginal cost of employing machines in any task. The incentive to replace workers is larger in

[30] All propositions and corollaries are proved in the appendix.

Figure 3: Assignment of Workers to Training-Intensive Tasks and the Effects of Technical Change



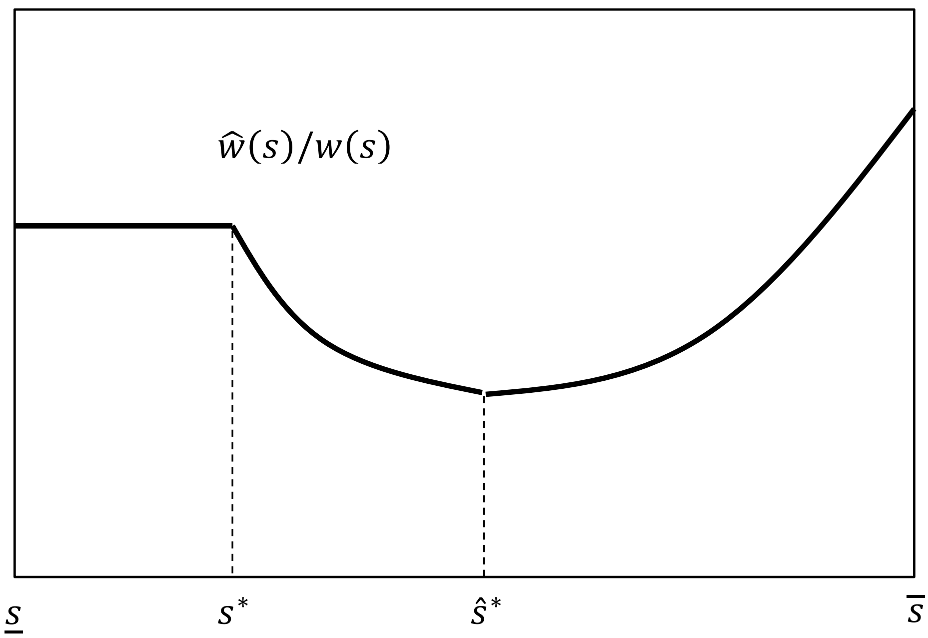
Complexity σ is plotted on the vertical axis, while skill level s is plotted on the horizontal axis. The upward shift of the matching function and the shift of its lower end to the north-east are brought about by a fall in the machine cost from c_K to $\hat{c}_K$ as stated in Proposition 1.

training-intensive tasks, where workers' marginal costs increase in complexity. This is illustrated by Figure 2 where the dashed lines represent the marginal cost of employing machines after the fall in the design cost, but holding constant factor prices and the assignment of labor to tasks. The increase in the level of complexity that is needed to make firms indifferent between machines and the marginal worker is larger in training-intensive tasks (distance A) than in innate ability tasks (distance B). This suggests that machine employment in training-intensive tasks increases by more than in innate ability tasks, even after general equilibrium effects are taken into account. Notice however that the effect of a fall in c_K on σ_0^* is ambiguous in general equilibrium. This is because the marginal cost of employing the marginal worker may fall.

The result that σ_0^* is always below σ_1^* together with the result that σ_0^* does not respond much to changes in the machine design cost, may justify the assumption made in Autor, Katz, and Kearney (2006) and Autor and Dorn (2013) that a set of tasks they call non-routine manual and that are performed by low skill workers, cannot be automated. While the corresponding tasks are automatable in principle in our model, we find that firms' *incentives* lead them to replace labor predominantly in tasks initially carried out by middle skill workers. Similarly, the assumption of Autor, Katz, and Kearney (2006) and Autor and Dorn (2013) that high skill abstract tasks cannot be performed by machines, receives some justification from our results, since highly complex, training-intensive tasks are safe from automation in our model. However, this is less and less true as c_K falls further.

As machines are newly adopted in a subset of training-intensive tasks, the workers initially

Figure 4: Changes in Wages as a Result of Technical Change



Changes in wages as a result of a fall in the machine design cost from c_K to $\hat{c}_K$. For each skill level s, the ratio of new to old wages is plotted. Workers with $s \in [\hat{s}^*, \bar{s}]$ remain in training-intensive tasks and experience a rise in the skill premium. Workers with $s \in [s^*, \hat{s}^*)$ switch to innate ability tasks and experience a fall in the skill premium. See Corollary 1 for details.

performing these tasks get replaced. Some of these workers upgrade to more complex training-intensive tasks—the matching function shifts up. Others downgrade to innate ability tasks. Thus, labor-replacing technical change causes job polarization. These effects are illustrated by Figure 3.

Proposition 1 has novel empirical implications regarding training requirements. Polarization by initial wages in our model is equivalent to polarization by training requirements, as employment shifts to innate ability tasks as well as to more-complex, training-intensive tasks. Furthermore, our model may speak to changes in training requirements at the level of an occupation. If occupations combine bundles of tasks, some of them innate ability, some of them training-intensive, then measured occupational training requirements could be indicative of the most complex training-intensive task within an occupation. A fall in the machine design cost will then cause training requirements to decrease in occupations with intermediate training requirements, as the most complex tasks in these occupations are automated. We will confront these implications with the data in Section 5.

How does a fall in c_K affect the wage distribution? The matching function is a sufficient statistic for inequality (Sampson 2014), so that the shift in the matching function contains all the required information for deriving changes in relative wages among workers who remain in training-intensive tasks.[31] Intuitively, as the upward shift implies skill downgrading by

[31]The results in this section rely on our assumptions about skill being one-dimensional (but including the case in

firms (but task upgrading for workers), the zero profit conditions imply that relatively low skill workers must have become relatively cheaper, or else their new employers would not be willing to absorb them. Hence the skill premium goes up for workers remaining in training-intensive tasks. Similar reasoning implies that workers who moved to innate ability tasks now earn relatively less than workers who were already performing these tasks. In sum, middle skill workers who are displaced by machines experience downward pressure on their wages, and they end up worse off in relative terms compared to high and low skill workers who are less affected by automation (or not at all). Wage inequality rises at the top, but falls at the bottom of the distribution, as illustrated by Figure 4. The formal result is as follows.

Corollary 1 *Suppose $\widehat{c}_K < c_K$. Wage inequality increases at the top of the distribution but decreases at the bottom. Formally,*

$$\frac{\widehat{w}(s')}{\widehat{w}(s)} > \frac{w(s')}{w(s)} \quad \text{for all } s' > s \geq \widehat{s}^*$$

and

$$\frac{\widehat{w}(s')}{\widehat{w}(s)} < \frac{w(s')}{w(s)} \quad \text{for all } s', s \textit{ such that } \widehat{s}^* > s' > s \geq s^*.$$

Relative wages are affected by technical change despite the fact that all factors are perfect substitutes at the task level. This is because tasks are q-complements in the production of the final good.[32] Intuitively, firms respond in two ways to the fall in the design cost. First, they upgrade existing machines. Second, they adopt machines in tasks previously performed by workers. The first effect on its own would lead to a rise in wages for all workers, because the increase in machines' task output raises the marginal product of all other tasks; moreover, relative wages would remain unchanged. The second effect, however, forces some workers to move to different tasks, putting downward pressure on their wages.[33] Since middle skill workers are most likely to be displaced by increased automation, their wages relative to low skill and high skill workers will decline.[34] Thus, whether technology substitutes for or complements a worker of given skill type (in terms of relative wage effects) depends on that worker's exposure to automation, which is endogenous in our model.[35]

which A_N is a monotonic function of s). With truly multidimensional skills, richer patterns of wage changes might occur.

[32] This means that the price of a task increases in the output of all other task. The mechanism described in this paragraph has been highlighted by Acemoglu and Autor (2011).

[33] The effect works mainly through changes in task prices. Physical productivity actually increases for middle skill workers who get reassigned to innate ability tasks.

[34] But middle skill workers' wages will not decline absolutely if the first effect dominates.

[35] To map the model's predictions for changes in wage inequality to the data, following Costinot and Vogel (2010) it is useful to distinguish between observable and unobservable skills. Our continuous skill index s is unlikely to be observed by the econometrician. Instead, assume that the labor force is partitioned according to some observable attribute e, which takes on a finite number of values and may index education or experience. Suppose further that high-s workers are disproportionately found in high-e groups. Formally, if $s' > s$ and $e' > e$, we require $v(s', e')v(s, e) \geq v(s, e')v(s', e)$. Costinot and Vogel (2010) show that an increase in wage inequality in the sense of Corollary 1 implies an increase in the premium paid to high-e workers as well as an increase in wage inequality among workers with the same e. In other words, the model predicts that if the machine design cost falls, both

Although the effect on the marginal innate ability task is uncertain, the overall weighted share of tasks performed by machines increases. By (22), this is equivalent to a decrease in the labor share.

Corollary 2 *Suppose* $\widehat{c}_K < c_K$*. The labor share decreases.*

The fact that the labor share decreases means that it is not possible to sign the effect of a fall in the design cost on wage levels. Equation (22) shows that the average wage is affected both by the decrease in workers' task shares and the increase in output, so that the overall change is ambiguous. Of course, wage levels may also differentially change by worker type. For instance, high and low skill workers may enjoy absolute wage gains, while middle skill workers may suffer absolute wage losses.

Our next comparative static exercise concerns the ratio of efficiency units of capital to those of labor, $\theta \equiv A_K K / A_N$.

Proposition 2 *Suppose that efficiency units of capital relative to those of labor become more abundant,* $\widehat{\theta} > \theta$*. Then the marginal training-intensive task becomes more complex,* $\widehat{\sigma}_1^* > \sigma_1^*$*, the marginal worker becomes more skilled,* $\widehat{s}^* > s^*$*, and the matching function shifts up,* $\widehat{M}(s) > M(s)$ *for all* $s \in [\widehat{s}^*, \bar{s})$*.*

The proposition suggests that the effects of an increase in θ are qualitatively identical to those of a decrease in c_K. It follows then that the implications for wage inequality and the labor share are analogous to Corollaries 1 and 2.

Corollary 3 *Suppose* $\widehat{\theta} > \theta$*. Wage inequality increases at the top of the distribution but decreases at the bottom. Formally,*

$$\frac{\widehat{w}(s')}{\widehat{w}(s)} > \frac{w(s')}{w(s)} \quad \text{for all } s' > s \geq \widehat{s}^*$$

and

$$\frac{\widehat{w}(s')}{\widehat{w}(s)} < \frac{w(s')}{w(s)} \quad \text{for all } s', s \text{ such that } \widehat{s}^* > s' > s \geq s^*.$$

Corollary 4 *Suppose* $\widehat{\theta} > \theta$*. The labor share decreases.*

Despite its similarity to Proposition 1, Proposition 2 is an important result that contains additional implications. Note that technical change may be such that c_K and θ both decline at the same time. In particular, human capital accumulation and labor-augmenting technical change lead to a higher A_N, which may more than offset increases in $A_K K$. Thus, Propositions 1 and 2 together show that an economy which experiences labor-replacing technical change (a fall in c_K) and labor-augmenting technical change (or increases in human capital) at the same time may not experience any secular changes in task assignment, wage inequality, and the labor share.

between and within (or residual) wage inequality will rise for the fraction of workers remaining in training-intensive tasks. On the other hand, within and between inequality falls for the set of workers below the new cutoff. This group includes stayers in as well as movers to innate ability tasks.

As our final comparative statics exercise we consider the effects of an increase in the relative supply of skills. As recently argued by (Goldin and Katz 2008), education may enter a race with technology such that increases in educational attainment may offset the rise in wage inequality caused by technical change. Here we focus on skill as the ability to acquire task-specific capabilities. Following Costinot and Vogel (2010), we say that $\widehat{V}$ is more skill abundant relative to V, or $\widehat{V} \succeq V$, if

$$\widehat{v}(s')v(s) \geq \widehat{v}(s)v(s') \quad \textit{for all } s' > s.$$

Such a shift in the skill distribution implies first-order stochastic dominance and hence an increase in the mean. The shift may be due, for instance, to an increase in average education levels, to the extent that general education leads to attainment of knowledge applicable to a wide range of job tasks. If so, then training costs for a given level of complexity will fall on average, exactly as occurs if $\mathrm{E}[s]$ decreases.

For simplicity, we restrict attention to distributions with common support, and we assume that $\widehat{v}(\bar{s}) > v(\bar{s})$. Characterizing comparative statics for changes in skill supplies is more challenging in our model than in the original Costinot-Vogel framework because domain and range of the matching function are endogenous. We are able to offer a partial result.

Proposition 3 *Suppose that skill becomes more abundant, $\widehat{V} \succeq V$ and $\widehat{v}(\bar{s}) > v(\bar{s})$. If this change in skill endowments induces an increase in the share of income accruing to labor, then the marginal training-intensive task becomes less complex, $\widehat{\sigma}_1^* < \sigma_1^*$; the marginal worker becomes more skilled, $\widehat{s}^* > s^*$; and the matching function shifts down, $\widehat{M}(s) < M(s)$ for all $s \in [\widehat{s}^*, \bar{s})$.*

Intuitively, such a change to the distribution of skills should raise the labor share, because the labor share equals the share of tasks performed by workers, and an increase in the average worker's productivity should induce more firms to hire labor. While the labor share always increased in numerical simulations that we performed, we are unable to prove the general result.[36]

The implications of Proposition 3 are as follows. Firms take advantage of the increased supply of skilled workers and engage in skill upgrading, which is equivalent to task downgrading for workers. This can be seen for training-intensive tasks by the downward shift of the matching function. For innate ability tasks, skill-upgrading is equivalent to the marginal worker becoming more skilled. Skill upgrading implies that the price of skill must have declined, so that the distribution of wages becomes more equal.

Corollary 5 *Suppose $\widehat{V} \succeq V$, and that the labor share increases as a result. Then wage inequality decreases globally: for all s, s' with $s' > s \geq s^*$,*

$$\frac{\widehat{w}(s)}{\widehat{w}(s')} > \frac{w(s)}{w(s')}.$$

[36]The labor share is given by $\int_{\underline{s}}^{\bar{s}} \frac{w(s)}{Y} dV(s)$. Because $\widehat{V}$ first-order stochastically dominates V and $w(s)/Y$ is an increasing function, we have $\int_{\underline{s}}^{\bar{s}} \frac{w(s)}{Y} d\widehat{V}(s) > \int_{\underline{s}}^{\bar{s}} \frac{w(s)}{Y} dV(s)$. Thus, for the labor share to *decrease*, there would need to be a sufficiently large decline in wage-output ratios for a subset of workers.

The result implies a fall in both within and between inequality. This is consistent with a fall in the college premium induced by an increase in the supply of college educated workers as occurred in the US in the 1970s (Acemoglu 2002). Thus, our model features a modified version of the "Race between Education and Technology", in the sense that education and technology have opposite effects on wage dispersion in the upper part of the distribution, but not in the lower part.

4 Alternative Assumptions and Extensions

In this section we discuss the robustness of our main comparative static result to an alternative formulation of the model that excludes innate ability tasks, and to two extensions of our model, one that features endogenous capital accumulation, and one in which the cost of machine design is fixed rather than variable.

4.1 The Importance of a Two-Dimensional Task Space: The Case $\beta_0 = 0$

We consider a version of our model that features a one-dimensional task space, in particular we assume that there are no innate ability tasks, $\beta_0 = 0$. This version of the model is very similar to the model of Costinot and Vogel (2010) with the addition of capital (and the simplification that the elasticity of substitution is unity). It is also very close to the Ricardian model of Acemoglu and Autor (2011), except that we assume a continuum of skills. An important feature of our model that distinguishes it from these two papers, even when $\beta_0 = 0$, is that our task index has an explicit empirical interpretation, namely the engineering complexity of a task.

The equilibrium assignment in such a model depends critically on the value of s_K.[37] If $s_K \leq \underline{s}$, then machines will perform a set of tasks at the lower end of the complexity range, up to a threshold σ^*. Workers sort into the remaining tasks, with higher skilled workers performing more complex tasks. Thus, there will be an increasing matching function $M(s)$ with properties $M(\underline{s}) = \sigma^*$ and $M(\bar{s}) = \bar{\sigma}$. In contrast, if $s_K \in (\underline{s}, \bar{s})$, then machines will perform an intermediate range of tasks between thresholds $\sigma^*_{\text{low}} < \sigma^*_{\text{up}}$. Workers with skill below (above) s_K sort into tasks below σ^*_{low} (above σ^*_{up}). Sorting is governed by two increasing matching functions M_{low} and M_{up} with properties $M_{\text{low}}(\underline{s}) = \underline{\sigma}$, $M_{\text{low}}(s_K) = \sigma^*_{\text{low}}$, $M_{\text{up}}(s_K) = \sigma^*_{\text{up}}$, and $M_{\text{up}}(\bar{s}) = \bar{\sigma}$.

This version of the model will not feature job polarization as a result of an increase in s_K if $s_K < \underline{s}$. In this case, the single task cutoff will increase, and the matching function will shift up. Hence, the model will feature task-upgrading from the workers' point of view and a global increase in wage inequality. Thus, for the model to feature job polarization it is required that $s_K \in (\underline{s}, \bar{s})$. This contrasts with our baseline model, which generates job polarization even if $s_K < \underline{s}$.[38]

More importantly, the one-dimensional model predicts counterfactual patterns of employment shifts. Job polarization in this version of the model would mean an increase in the

[37] Formal derivations and proofs of the results stated in this subsection are available upon request.

[38] In the baseline model, if s_K is initially very low, then it is likely that all innate ability tasks are performed by workers. We can then prove that an increase in s_K leads to job polarization. A proof is available upon request.

employment shares of the least-complex tasks. In reality, the low wage jobs whose employment shares have increased include many personal services that are highly complex, although they do not require much training. Hence it appears that for a model of job polarization to have a meaningful empirical interpretation, a two-dimensional task space is required.[39]

4.2 Making the Model Dynamic

Up to this point we have treated the economy's capital stock as exogenously given. To determine how endogenous capital accumulation would affect our comparative statics results, we assume that in the long run, the rental rate of capital is a constant pinned down by a time preference parameter[40] and that machines fully depreciate in every period. Furthermore, we assume that worker's knowledge depreciates fully in every period, or equivalently, there is an overlapping generations structure with each generation only working for one period. Suppose that the economy starts out in a steady state with the interest rate equal to its long-run value. Now recall that a fall in the machine design cost leads to a rise in the labor share. Furthermore, output must not decrease, since the economy's resource constraint is less tight. By (16), we have that the interest rate increases. Thus, in the long run, the capital stock must increase to bring the interest rate back down.

Proposition 2 shows that a rise in K and hence θ has qualitatively the same effects on the marginal tasks, the matching function, and wages, as a fall in the machine design cost c_K. This is because a higher supply of capital makes it cheaper to rent machines and thus encourages technology adoption. Thus, our predictions about the effects of a fall in c_K are not overturned with endogenous capital accumulation. In fact, the rise in the marginal training-intensive task, the upward shift of the matching function, the rise in the skill of the marginal worker, and the hollowing out of the wage distribution will be more pronounced in the long run as a result of the higher capital stock.

4.3 A Model with Fixed Costs

Our baseline model emphasizes that when a firm automates its production, total costs will generally be increasing in the firm's output and in the complexity of the processes required for production. While this in itself should be uncontroversial, our focus on variable costs with the implication of constant returns to scale is certainly restrictive. In particular, firms usually face large one-off expenses when adopting new technologies.[41] While such expenses would

[39] It may be tempting to prefer the version of our model with a one-dimensional task space because it matches the chronology of labor market changes observed in many countries, namely global task-upgrading and increases in wage inequality followed by job polarization and U-shaped wage growth. However, our baseline model can potentially generate rich patterns of changes in task assignment and wage inequality, as well, through the interaction of a fall in the machine design cost, increases in human capital, and changes in the skill distribution. It seems more likely that a combination of forces, rather than a single force, has been behind the labor market changes of the past decades.

[40] Alternatively, we could assume that the economy is open to world capital markets, where it is a price taker.

[41] For an example relating to recent advances in AI, consider the concept of 'machine learning', where a software requires a considerable amount of initial 'training' before becoming operational.

generally depend on the scale at which the firm plans to operate, it is useful to consider the extreme case of a fixed setup cost.

In the appendix we modify our baseline model such that firms wanting to automate production face a fixed cost (in units of the final good) which is increasing in the complexity of the task, but does not depend on the scale of production. We derive conditions ensuring an equilibrium assignment that is qualitatively the same as the one analyzed for the baseline model (see Figure 1). The marginal cost of using a machine must be sufficiently small, which can be achieved by making A_K very large, a realistic assumption; and the fixed cost must increase sufficiently in complexity. The model is much less tractable than the baseline model, and we are unable to derive general comparative statics results. Intuitively, when the fixed machine design cost falls, there is an incentive for firms to adopt machines in more-complex tasks. This incentive is stronger in training-intensive tasks: as complexity increases, the marginal cost of employing labor increases in training-intensive tasks but not in innate ability tasks. Thus, we would expect to see an increase in the share of workers performing innate-ability tasks. We have solved the model numerically and verified this intuition. We present results in the appendix.

5 Empirical Support for the Model's Predictions

The previous section suggests that any technological advance that facilitates automation of a wide range of tasks may lead to systematic shifts in task input, job polarization, a hollowing out of the wage distribution, and a fall in the labor share. In addition, the model also predicts which worker types will be replaced as more tasks are automated, and to which task a displaced worker gets reassigned. Our model is thus consistent with a list of stylized facts that have been documented in the literature of the past ten years.[42, 43, 44]

In this section we argue that our model also explains why historical waves of automation led to labor market polarization, something that existing models of job polarization fail to explain. Moreover, the model can be tested on data other than those that informed its construction. In particular, we argue that our model is consistent with employment shifts by occupational training requirements as well as recent changes in training requirements.

As for historical evidence, Katz and Margo (2013) find that from 1850 to 1880, US manufacturing witnessed a relative decline in middle skill jobs like artisans compared to high skill jobs (non-production workers) and low skill jobs (operatives), concurrent with the increased adoption of steam power. Gray (2013) shows that electrification in the US during the first half of the 20th century led to a fall in the demand for dexterity-intensive tasks performed by middle

[42] Wage polarization in the US has occurred in most of the past three decades, although not in the early 2000s (Autor 2014). Shierholz, Mishel, and Schmitt (2013) argue that the time series evidence does not support job polarization as a cause of wage polarization. However, Boehm (2014) and Cortes (forthcoming) find that the micro-level implications that job polarization has for wages are largely borne out by the data. Moreover, Autor and Dorn (2013) point out that the effects of job polarization on the wage structure are ambiguous when taking into account consumer responses to price changes brought about by task-biased technical change.

[43] See Karabarbounis and Neiman (2014) and Elsby, Hobijn, and Sahin (2013) on the decline of the labor share.

[44] Autor, Levy, and Murnane (2003) document a shift of employment out of routine tasks, which we consider to correspond to the lower range of our complexity measure.

skill workers, relative to manual and clerical tasks performed by low and high skill workers, respectively.[45]

The innovations that preceded these two historical instances of job polarization have in common that they facilitated a more wide-ranging automation (or mechanization) of tasks. The steam engine was instrumental in the increased mechanization of manufacturing because it provided a more reliable power source than water, and it allowed production to be located away from water, thus lowering transportation costs (Atack, Bateman, and Weiss 1980). Electricity facilitated automation because electric motors could be arranged more flexibly than steam engines (Boff 1967). But ICT facilitated automation, as well, in particular the automation of cognitive tasks as well as improved control of physical production processes. Thus, our model can account for three instances of job polarization by invoking the common feature of the technologies that arguably caused them, namely, making it easier to automate (or mechanize) tasks.

We now consider our model's novel implications regarding training requirements. In the model, training levels vary systematically with task characteristics. More-complex training-intensive tasks require more training. As a fall in the machine design cost triggers a reallocation of workers towards tasks of higher complexity on the one hand (the upward shift of the matching function) and towards innate ability tasks on the other (the rise in s^*), the model predicts a polarization of job training requirements. Furthermore, the model predicts that firms selectively automate tasks. Training-intensive tasks of intermediate complexity are more likely to be newly automated over time. We implement and test these predictions using data on occupational training requirements.

We view occupations as bundles of tasks, so that a given occupation may combine tasks from across the task space. Measures of occupational characteristics should be informative about which region of the task space features most prominently in a given occupation. Thus, occupations with low training requirements should be intensive in innate ability tasks; and occupations with very high training requirements should feature highly complex, training-intensive tasks.

To measure training requirements of occupations, we use the Fourth Edition Dictionary of Occupational Titles (DOT) in combination with the 1971 April Current Population Survey (CPS) (National Academy of Sciences 1981), and the US Department of Labor's O*NET database in combination with the 2008 American Community Survey (ACS). The information in the 2008 ACS refers to the previous year. Hence, our data cover the years 1971 and 2007. Since the 1971 April CPS lacks information on earnings, we also used the IPUMS 1970 census extract which contains earnings data pertaining to 1969.[46] We use David Dorn's three-digit occupation codes

[45] Bessen (2011) provides evidence on weavers employed at a 19th century Massachusetts firm that gradually increased the degree of mechanization during the period studied. Even though some of workers' skills were no longer needed as more tasks were automated, the tasks to which workers were reassigned required substantial on-the-job learning, much like the reassignment of workers to more-complex, training-intensive tasks in our model. Crucially, worker productivity in the remaining tasks increased, supporting the assumption of q-complementarity of tasks that underlies our model and previous task models in the literature. Note that we would not necessarily expect an aggregate phenomenon like job polarization to occur at the firm level.

[46] Because we have to merge separate data sets at the three-digit occupation level, we prefer using the census to the much smaller 1971 March CPS for obtaining earnings data.

throughout (Dorn 2009). Our analysis is based on a sample of all employed persons aged 17 to 65. To see whether our results are driven by changes in composition, we repeated the analysis using a sample of white males only. The results, available upon request, are qualitatively identical.

Both the DOT and O*NET contain the variable *Specific Vocational Preparation* (SVP), which indicates "the amount of time required to learn the techniques, acquire the information, and develop the facility needed for average performance in a specific job-worker situation. SVP includes training acquired in a school, work, military, institutional, or vocational environment, but excludes schooling without specific vocational content" (National Academy of Sciences 1981, p.21 in codebook). SVP is a bracketed variable and we use midpoints to convert it into training time measured in years. See the appendix for details.

The definition of SVP matches our concept of task-specific training more closely than years of education. This is because much of education, at least up to high school graduation, is general in nature and the skills acquired are portable across occupations. Also, the average level of education of workers in a given occupation may be affected by the supply of educated workers independently of actual training requirements—we provide evidence for this below. In professional occupations such as lawyers and physicians there is a clear mapping between years of schooling and training requirements, but in general this is not the case. In terms of our model, we think of general education as affecting the ability to acquire task-specific knowledge. Thus, years of schooling may proxy for s.

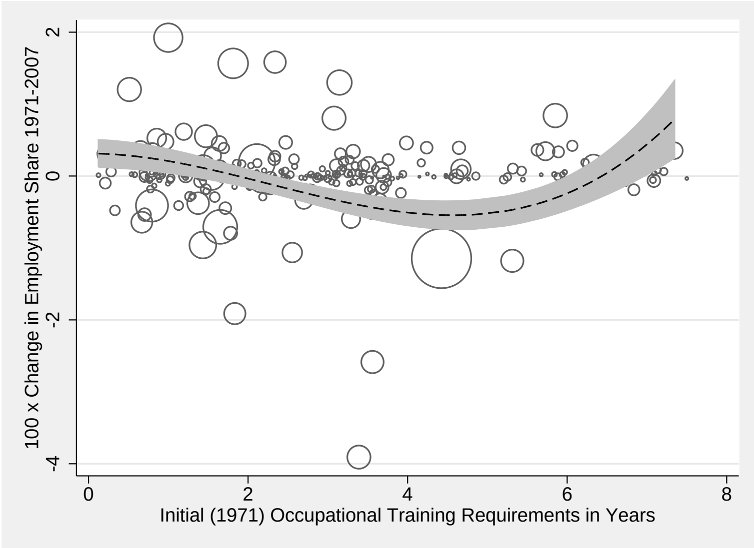
To test our prediction of polarization by initial training requirements, we plot in Figure 5 changes in occupational employment shares from 1971-2007 against initial occupational training requirements in years. We also fit a fractional polynomial. The U-shape of the polynomial is consistent with the model's prediction.

There is a noteworthy connection between our prediction and finding of polarization by training requirements and existing evidence. Autor and Dorn (2013) document that many of the low wage occupations that have experienced increasing employment shares consist of low skill services. In our model, these occupations are represented by tasks in which workers rely on innate abilities and hence require no training, even though these tasks may be very complex. We have verified that low skill service occupations as classified by Autor and Dorn (2013) are indeed among the occupations with the lowest training requirements in our data. Thus, our model may be seen as an empirically plausible micro-foundation for the model of Autor and Dorn (2013).

Given that occupations are arguably best viewed as bundles of tasks, it is not straightforward to derive predictions about changes in occupational training requirements if the precise mapping between tasks and occupations is not known. Let us assume that measured training requirements are indicative of the most complex training-intensive task within an occupation's bundle of tasks. Then we would expect training requirements to decrease in occupations with intermediate training requirements, as the most complex tasks in these occupations are automated.

Panel a) of Figure 6 shows that indeed, occupations with intermediate initial training requirements saw the largest declines in training requirements. These occupations include air traffic controllers, precision makers, insurance adjusters, and various engineering occupations

Figure 5: Changes in Occupational Employment 1971-2007 by Initial Training Requirements



We calculate training requirements using the variable *specific vocational preparation* (SVP) from the Dictionary of Occupational Titles and the O*NET database. Observations are weighted by average occupational employment shares. Fitted curves are fractional polynomials, drawn using Stata's *fpfitci* option.

(see Table A4), which appears consistent with our automation-based explanation.

Figure 6 also demonstrates once again that skills and tasks should be kept conceptually distinct, as argued for instance by Acemoglu and Autor (2011). Panel b) shows that average years of schooling increased in almost all occupations, and changes in years of schooling do not follow the same pattern as changes in training requirements. This is consistent with education and training requirements pertaining to different concepts, namely the skill of the worker and the characteristics of tasks, respectively. Moreover, skill upgrading across occupations is consistent with our model given the rise in the supply of skills over the past decades.

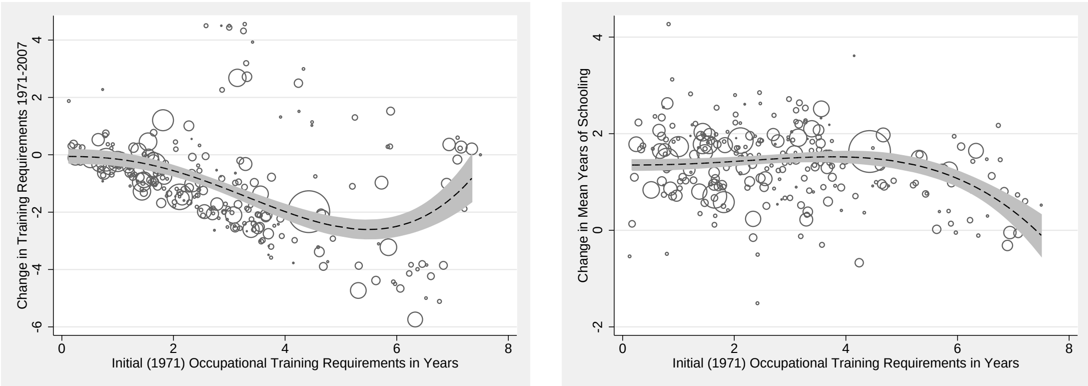
We also show the correlation between growth in occupational employment and changes in training requirements. The implications of our model for employment are perhaps less obvious. While larger decreases in training requirements should reflect more rapid automation and hence displacement of workers, it is not clear to what extent workers may be able to move towards other tasks within an occupation. Nevertheless, a regression of changes in log total hours on changes in log training requirements yields a coefficient of 0.33 (robust standard error 0.08). Raw data and fitted line are plotted in panel a) of Figure 7. Including changes in log years of education on the right hand side decreases the coefficient on training only slightly.[47]

Lastly, we consider how changes in training requirements correlate with changes in occupational mean wages. We obtain adjusted occupational mean log wages as the predicted values

[47] A positive and statistically significant relationship also exists between employment growth and changes in the level of training requirements; and between changes in occupational employment shares and changes in both the level and log of training requirements.

Figure 6: Changes in Training Requirements and Years of Schooling 1971-2007

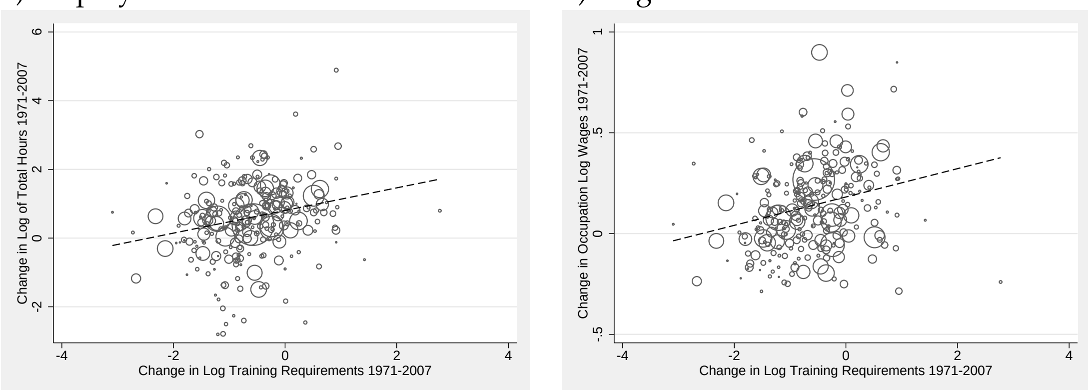
a) Training requirements b) Years of schooling



Observations are weighted by average occupational employment shares. Fitted curves are fractional polynomials, drawn using Stata's *fpfitci* option.

Figure 7: Employment Growth, Wage Growth, and Changes in Training Requirements

a) Employment b) Wages



Observations are weighted by average occupational employment shares. Occupational mean wages have been adjusted for potential experience, region of residence, gender, and race.

from a regression of log wages on occupation dummies, a quartic in potential experience, region dummies, and indicators for female and non-white, evaluated at sample means. A regression of changes in occupation log wages on changes in log training requirements yields a coefficient of 0.07 (standard error 0.026), see panel b) of Figure 7. Including changes in log years of education on the right hand side slightly increases the coefficient on training.

The finding is consistent with the model if we again interpret falls in training requirements as increased automation of tasks. For concreteness, consider an occupation whose task bundle initially includes training-intensive tasks with complexities between σ_1^* and $\sigma' > \widehat{\sigma}_1^*$. Let s' be the skill level of the worker initially performing task σ'. After the fall in machine design costs, all tasks in the interval $[\sigma_1^*, \widehat{\sigma}_1^*]$ are newly automated. Workers with skill levels between $\widehat{s}^*$ and some $s'' < s'$ will remain in the occupation. Figure 4 shows that these workers experience wage declines relative to most other workers.

6 Conclusion

In this paper we present a model addressing the following questions: How do firms respond to technological advances that facilitate the automation of a wide range of job tasks? Which tasks will they automate, and what types of worker will be replaced as a result? We build on insights from computer science to model the way in which tasks are differentiated, distinguishing between a tasks's engineering complexity and its training requirements. When two tasks are equally complex, firms will automate the task that requires more training and in which labor is hence more expensive. We show that this leads to job polarization, the decline in middle wage jobs relative to both high and low wage jobs.

Our model suggests a connection between automation (or mechanization) and job polarization. We do not need to invoke idiosyncratic features of ICT to explain recent job polarization. Moreover, we are able to explain historical episodes of polarization as caused by increased automation following the introduction of the steam engine and the electric motor. Finally, the focus on training requirements in our model leads to novel empirical implications that we find to be consistent with US data. Employment has shifted out of occupations featuring intermediate training requirements, and these were the same occupations that saw the largest decreases in training requirements.

Unlike previous theoretical work on job polarization, our model fully endogenizes firms' automation decisions. We thus provide a justification of the assumption made in the prior literature that "non-routine" manual and abstract tasks cannot be automated. While the corresponding tasks in our model are assumed to be automatable, we show that firms' *incentives* lead them to replace labor predominantly in other tasks, those performed by middle skill workers.

There are several promising avenues for further research. One could allow for changes in the economy's task mix or changes in firm organization resulting from technical change. Furthermore, it would be interesting to incorporate dynamic issues and market imperfections related to worker training. Another important question is how the effects of off-shoring and trade in tasks on task assignment and wage inequality (Grossman and Rossi-Hansberg 2008) may interact with those of labor-replacing technical change. Finally, it will be important to understand how labor market policies such as the minimum wage affect firms' incentives to automate tasks.

This paper informs predictions about the future of technical change and automation. Will job polarization continue, or will the next wave of automation threaten high or low skill workers instead? For instance, Autor (2014) conjectures that today's middle skill jobs will be more secure in coming decades, because they combine tasks from across the task spectrum that cannot be unbundled easily. In contrast, our model suggests that automation will lead to a continuing displacement of workers in the middle of the distribution, with the growth in low skill jobs more and more dominating that of high skill jobs. The term 'middle' will refer to increasingly skilled workers over time, as machines move into increasingly complex, training-intensive tasks.

References

ACEMOGLU, D. (2002): "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 40(1), 7–72.

ACEMOGLU, D., AND D. AUTOR (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 4, chap. 12, pp. 1043–1171. Elsevier.

ATACK, J., F. BATEMAN, AND T. WEISS (1980): "The Regional Diffusion and Adoption of the Steam Engine in American Manufacturing," *The Journal of Economic History*, 40(02), 281–308.

AUTOR, D. (2014): "Polanyi's Paradox and the Shape of Employment Growth," NBER Working Papers 20485, National Bureau of Economic Research, Inc.

AUTOR, D. H., AND D. DORN (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, 103(5), 1553–97.

AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2006): "The Polarization of the U.S. Labor Market," *American Economic Review*, 96(2), 189–194.

AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): "The Skill Content Of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, 118(4), 1279–1333.

BESSEN, J. (2011): "Was Mechanization De-Skilling? The Origins of Task-Biased Technical Change," Working Papers 1101, Research on Innovation.

BOEHM, M. J. (2014): "The Wage Effects of Job Polarization: Evidence from the Allocation of Talents," mimeo.

BOFF, R. B. D. (1967): "The Introduction of Electric Power in American Manufacturing," *The Economic History Review*, 20(3), pp. 509–518.

BRYNJOLFSSON, E., AND A. MCAFEE (2011): *Race Against The Machine: How the Digital Revolution is Accelerating Innovation, Driving Productivity, and Irreversibly Transforming Employment and the Economy*. Digital Frontier Press.

CASELLI, F. (1999): "Technological Revolutions," *American Economic Review*, 89(1), 78–102.

CHARI, V. V., AND H. HOPENHAYN (1991): "Vintage Human Capital, Growth, and the Diffusion of New Technology," *Journal of Political Economy*, 99(6), 1142–65.

CORTES, G. M. (forthcoming): "Where Have the Routine Workers Gone? A Study of Polarization Using Panel Data," *Journal of Labor Economics*.

COSTINOT, A., AND J. VOGEL (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, 118(4), 747–786.

DAVIDSON, A. (2012): "Making It in America," *The Atlantic*, Jan/Feb.

DORN, D. (2009): "Essays on Inequality, Spatial Interaction, and the Demand for Skills," Dissertation no. 3613, University of St. Gallen.

DORNBUSCH, R., S. FISCHER, AND P. A. SAMUELSON (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67(5), 823–39.

ELSBY, M., B. HOBIJN, AND A. SAHIN (2013): "The Decline of the U.S. Labor Share," *Brookings Papers on Economic Activity*, 47(2 (Fall)), 1–63.

FORD, M. (2009): *The Lights in the Tunnel: Automation, Accelerating Technology and the Economy of the Future*. CreateSpace Independent Publishing Platform.

FREY, C. B., AND M. A. OSBORNE (2013): "The future of employment: how susceptible are jobs to computerisation?," *Sept*, 17, 2013.

GARICANO, L. (2000): "Hierarchies and the Organization of Knowledge in Production," *Journal of Political Economy*, 108(5), 874–904.

GOLDIN, C., AND L. F. KATZ (1998): "The Origins Of Technology-Skill Complementarity," *The Quarterly Journal of Economics*, 113(3), 693–732.

——— (2008): *The Race between Education and Technology*. Harvard University Press.

GOOS, M., AND A. MANNING (2007): "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain," *The Review of Economics and Statistics*, 89(1), 118–133.

GOOS, M., A. MANNING, AND A. SALOMONS (2009): "Job Polarization in Europe," *American Economic Review*, 99(2), 58–63.

——— (2014): "Explaining Job Polarization: Routine-Biased Technological Change and Offshoring," *American Economic Review*, 104(8), 2509–26.

GRAY, R. (2013): "Taking technology to task: The skill content of technological change in early twentieth century United States," *Explorations in Economic History*, 50(3), 351–367.

GROSSMAN, G. M., AND E. ROSSI-HANSBERG (2008): "Trading Tasks: A Simple Theory of Offshoring," *American Economic Review*, 98(5), 1978–97.

HOLMES, T. J., AND M. F. MITCHELL (2008): "A theory of factor allocation and plant size," *RAND Journal of Economics*, 39(2), 329–351.

JOVANOVIC, B. (1998): "Vintage Capital and Inequality," *Review of Economic Dynamics*, 1(2), 497–530.

KARABARBOUNIS, L., AND B. NEIMAN (2014): "The Global Decline of the Labor Share," *The Quarterly Journal of Economics*, 129(1), 61–103.

KATZ, L. F., AND R. A. MARGO (2013): "Technical Change and the Relative Demand for Skilled Labor: The United States in Historical Perspective," in *Human Capital in History: The American Record*, NBER Chapters, pp. 15–57. National Bureau of Economic Research, Inc.

MAMELI, M., AND P. BATESON (2011): "An evaluation of the concept of innateness," *Phil. Trans. R. Soc. B*, 366, pp. 436–443.

MARKOFF, J. (2012): "Skilled Work, Without the Worker," *The New York Times*, Aug 18.

MELENDEZ, E. D. (2013): "Minimum Wage Jobs Won't Be Replaced With This Robot Anytime Soon," *The Huffington Post*, Aug 30.

MICHAELS, G., A. NATRAJ, AND J. V. REENEN (2014): "Has ICT Polarized Skill Demand? Evidence from Eleven Countries over Twenty-Five Years," *The Review of Economics and Statistics*, 96(1), 60–77.

MORAVEC, H. (1988): *Mind Children*. Harvard University Press.

NATIONAL ACADEMY OF SCIENCES (1981): "Dictionary of Occupational Titles (DOT): Part I - Current Population Survey, April 1971, Augmented With DOT Characteristics and Dictionary of Occupational Titles (DOT): Part II - Fourth Edition Dictionary of DOT Scores for 1970 Census Categories," .

NORDHAUS, W. D. (2007): "Two Centuries of Productivity Growth in Computing," *The Journal of Economic History*, 67(01), 128–159.

NOURBAKHSH, I. R. (2013): *Robot Futures*. MIT Press.

O'CONNOR, S. (2013): "Amazon unpacked," *The Financial Times*, Feb 8.

SAKAGAMI, Y., R. WATANABE, C. AOYAMA, S. MATSUNAGA, N. HIGAKI, AND K. FUJIMURA (2002): "The intelligent ASIMO: System overview and integration," Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems.

SAMPSON, T. (2014): "Selection into Trade and Wage Inequality," *American Economic Journal: Microeconomics*, 6(3), 157–202.

SHEIN, E. (2013): "Businesses adopting robots for new tasks," *Computerworld*, Aug 1.

SHIERHOLZ, H., L. MISHEL, AND J. SCHMITT (2013): "Dont Blame the Robots: Assessing the Job Polarization Explanation of Growing Wage Inequality," Discussion paper, Economic Policy Institute.

SIMON, H. (1960): "The corporation: Will it be managed by machines?," in *Management and corporations*, ed. by M. Anshen, and G. Bach, pp. 17–55. McGraw-Hill.

TAYLOR, K. (2010): "That Mighty Sorting Machine Is Certainly One for the Books," *The New York Times*, Apr 22.

ZEIRA, J. (1998): "Workers, Machines, And Economic Growth," *The Quarterly Journal of Economics*, 113(4), 1091–1117.

Appendices

Proofs of Propositions and Corollaries Stated in the Text

Proof of Proposition 1 We first show that in the absence of changes to the distribution of skills, a flattening (steepening) of the matching function at the upper end implies an upward (downward) shift of the matching function everywhere. Formally, if $\widehat{M}'(\bar{s}) < M'(\bar{s})$, then $\widehat{M}(s) < M(s)$ for all $s \in [\max\{s^*, \widehat{s}^*\}, \bar{s})$. For suppose that $\widehat{M}'(\bar{s}) < M'(\bar{s})$ and that there exists some $s' \in [\max\{s^*, \widehat{s}^*\}, \bar{s})$ such that $\widehat{M}(s') \leq M(s')$. Then there exists some $s'' \in [s', \bar{s})$ such that $\widehat{M}(s'') = M(s'')$, $\widehat{M}'(s'') \geq M'(s'')$, and $\widehat{M}(s) > M(s)$ for all $s \in (s'', \bar{s})$. We will show that this leads to a contradiction.

Integrating (11) yields an expression for the wage premium of the most skilled worker with respect to any other skill group employed in training-intensive tasks,

$$\frac{w(\bar{s})}{w(s)} = \omega(s; M), \quad s \geq s^*$$

where

$$\omega(s; M) \equiv \exp\left[\int_s^{\bar{s}} \frac{\partial}{\partial z} \log \alpha(z, M(z)) dz\right]. \tag{A1}$$

Because α is increasing in its first argument, ω is decreasing in s. Moreover, by the log-supermodularity of α, if $\widehat{M}(z) > M(z)$ for all $z \in (s, \bar{s})$ and any s that belongs to the domains of both $\widehat{M}$ and M, then $\omega(s; \widehat{M}) > \omega(s; M)$.

Plugging (A1) into (10), we obtain

$$\frac{M'(\bar{s})}{M'(s)} = \omega(s; M) \frac{v(\bar{s})}{v(s)}. \tag{A2}$$

Therefore,

$$\frac{\widehat{M}'(\bar{s})}{M'(\bar{s})} = \frac{\omega(s''; \widehat{M})}{\omega(s''; M)} \frac{\widehat{M}'(s'')}{M'(s'')}.$$

By the above arguments, the right side of the last equation is larger than one, so that we must have $\widehat{M}'(\bar{s}) > M'(\bar{s})$, a contradiction. A similar argument establishes that a steepening at the upper end leads to a downward shift everywhere.

Proof that $\widehat{\sigma}_1^* > \sigma_1^*$ First suppose $\widehat{\sigma}_1^* \leq \sigma_1^*$ and $\widehat{M}'(\bar{s}) \geq M'(\bar{s})$.

By (20) and (A2),

$$\frac{V(s^*)}{\bar{\sigma} - \sigma_0^*} \times \frac{M'(\bar{s})}{\omega(s^*; M)} = \frac{\beta_0 v(\bar{s})}{\beta_1}. \tag{A3}$$

This together with (18), implies

$$\frac{\theta \alpha(s_K, \sigma_0^*)}{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})} \times \frac{M'(\bar{s})}{\omega(s^*; M)} = \frac{v(\bar{s})}{\beta_1}. \tag{A4}$$

Suppose that $\widehat{s}^* \geq s^*$. Then (A3) implies that $\widehat{\sigma}_0^* < \sigma_0^*$, while (A4) implies $\widehat{\sigma}_0^* > \sigma_0^*$, a contradiction. So we must have $\widehat{s}^* < s^*$. If $\widehat{\sigma}_0^* \geq \sigma_0^*$, then from (19), $\widehat{s}^* > s^*$,[48] so it must be that $\widehat{\sigma}_0^* < \sigma_0^*$. Then by 19, $\alpha(\widehat{s}_K, \widehat{\sigma}_0^*) > \alpha(s_K, \sigma_0^*)$. This implies that the LHS of (18) increases, while the RHS decreases, a contradiction.

Next, suppose that $\widehat{\sigma}_1^* \leq \sigma_1^*$ and $\widehat{M}'(\bar{s}) < M'(\bar{s})$. We have shown that in this case the matching function shifts up, so we must have $\widehat{s}^* \leq s^*$. Then $\widehat{\sigma}_0^* < \sigma_0^*$ from (19). But we have just shown that it is impossible to have $\widehat{\sigma}_1^* \leq \sigma_1^*$, $\widehat{\sigma}_0^* < \sigma_0^*$, and $\widehat{s}^* \leq s^*$ at the same time. Thus we have established that $\widehat{\sigma}_1^* > \sigma_1^*$.

Proof that $\widehat{M}(s) > M(s)$ Suppose that $\widehat{M}'(\bar{s}) > M'(\bar{s})$, which we have shown implies $\widehat{M}(s) < M(s)$ and, by (A2), $\widehat{M}'(s) > M'(s)$ for all s belonging to the domains of both $\widehat{M}$ and M. As we have established that $\widehat{\sigma}_1^* > \sigma_1^*$, by the properties of the matching function we must have $\widehat{s}^* > s^*$. By (10), the wage share of a worker who is always assigned to training-intensive tasks has increased,

$$\frac{\widehat{w}(s)}{\widehat{Y}} = \frac{\beta_1}{\mu} \frac{\widehat{M}'(s)}{v(s)} > \frac{\beta_1}{\mu} \frac{M'(s)}{v(s)} = \frac{w(s)}{Y} \quad \forall s \in [\widehat{s}^*, \bar{s}].$$

But this means that the wage shares of all remaining workers have increased, as well,

$$\frac{\widehat{w}(s)}{\widehat{Y}} = \frac{\widehat{w}(\widehat{s}^*)}{\widehat{Y}} > \frac{w(\widehat{s}^*)}{Y} > \frac{w(s)}{Y} \quad \forall s \in [\underline{s}, \widehat{s}^*),$$

where the last inequality is due to (21). Therefore, the total labor share has increased,

$$\frac{\int_{\underline{s}}^{\bar{s}} \widehat{w}(s)v(s)ds}{\widehat{Y}} > \frac{\int_{\underline{s}}^{\bar{s}} w(s)v(s)ds}{Y}.$$

By (10) and (17), this implies $\beta_0\widehat{\sigma}_0^* + \beta_1\widehat{\sigma}_1^* < \beta_0\sigma_0^* + \beta_1\sigma_1^*$.

Now observe that if $\widehat{M}(s) < M(s)$ then $\omega(\widehat{s}^*; \widehat{M}) < \omega(s^*; M)$ since also $\widehat{s}^* > s^*$. By (A3), we must have $\widehat{\sigma}_0^* < \sigma_0^*$. But this means that (A4) can only hold if also the total labor share has decreased, $\beta_0\widehat{\sigma}_0^* + \beta_1\widehat{\sigma}_1^* > \beta_0\sigma_0^* + \beta_1\sigma_1^*$, a contradiction.

Proof that $\widehat{s}^* > s^*$ We begin by deriving an additional expression. Integrating (11) from s^* to s yields

$$w(s) = w(s^*) \exp\left[\int_{s^*}^{s} \frac{\partial}{\partial z} \log \alpha(z, M(z))dz\right].$$

Multiplying the last expression by $v(s)$, and integrating from s^* to $\bar{s}$, we obtain

$$\int_{s^*}^{\bar{s}} w(s)v(s)ds = w(s^*) \int_{s^*}^{\bar{s}} \exp\left[\int_{s^*}^{s} \frac{\partial}{\partial z} \log \alpha(z, M(z))dz\right] v(s)ds.$$

[48] To see this, rewrite (19) as

$$\frac{\alpha(s_K, \sigma_1^*)}{\alpha(s_K, \sigma_0^*)\alpha(s^*, \sigma_1^*)} = 1.$$

By the log-supermodularity of α, a rise in s_K leads the ratio $\alpha(s_K, \sigma_1^*)/\alpha(s_K, \sigma_0^*)$ to rise since $\sigma_1^* > \sigma_0^*$. Again due to log-supermodularity, the fall in σ_1^* raises the ratio $\alpha(s_K, \sigma_1^*)/\alpha(s^*, \sigma_1^*)$ since $s_K < s^*$. The rise in σ_0^* raises the LHS further. Therefore, s^* must increase.

Integrating (10) and plugging the result as well as (17) into the last expression, yields

$$\beta_1 \left(\overline{\sigma} - \sigma_1^*\right) = \frac{\beta_0 \left(\overline{\sigma} - \sigma_0^*\right)}{V(s^*)} \int_{s^*}^{\overline{s}} \exp\left[\int_{s^*}^{s} \frac{\partial}{\partial z} \log \alpha(z, M(z)) dz\right] v(s) ds,$$

which upon rearranging becomes

$$\frac{\beta_1}{\beta_0} \frac{\overline{\sigma} - \sigma_1^*}{\overline{\sigma} - \sigma_0^*} = \frac{1}{V(s^*)} \int_{s^*}^{\overline{s}} \exp\left[\int_{s^*}^{s} \frac{\partial}{\partial z} \log \alpha(z, M(z)) dz\right] v(s) ds. \tag{A5}$$

Suppose that $\widehat{s}^* \leq s^*$. Because of the upward shift of the matching function, the RHS of (A5) increases. For the LHS to increase, we must have $\widehat{\sigma}_0^* - \sigma_0^* > \widehat{\sigma}_1^* - \sigma_1^*$, or in calculus notation, $d\sigma_0^* > d\sigma_1^*$ and hence $d\sigma_0^* > 0$. Log-differentiating (19) yields

$$\begin{aligned} ds_K \tfrac{\partial}{\partial s} \log \alpha(s_K, \sigma_1^*) + d\sigma_1^* \tfrac{\partial}{\partial \sigma} \log \alpha(s_K, \sigma_1^*) \quad &= \quad ds_K \tfrac{\partial}{\partial s} \log \alpha(s_K, \sigma_0^*) + d\sigma_0^* \tfrac{\partial}{\partial \sigma} \log \alpha(s_K, \sigma_0^*) \\ &\quad + ds^* \tfrac{\partial}{\partial s} \log \alpha(s^*, \sigma_1^*) + d\sigma_1^* \tfrac{\partial}{\partial \sigma} \log \alpha(s^*, \sigma_1^*). \end{aligned}$$

Adding and subtracting $d\sigma_0^* \frac{\partial}{\partial \sigma} \log \alpha(s_K, \sigma_1^*)$ on the RHS and rearranging, we obtain

$$\begin{aligned} ds_K \tfrac{\partial}{\partial s} \left[\log \alpha(s_K, \sigma_1^*) - \log \alpha(s_K, \sigma_0^*)\right] \quad &= \quad \left(d\sigma_0^* - d\sigma_1^*\right) \tfrac{\partial}{\partial \sigma} \log \alpha(s_K, \sigma_1^*) \\ &\quad + d\sigma_0^* \tfrac{\partial}{\partial \sigma} \left[\log \alpha(s_K, \sigma_0^*) - \log \alpha(s_K, \sigma_1^*)\right] \\ &\quad + ds^* \tfrac{\partial}{\partial s} \log \alpha(s^*, \sigma_1^*) + d\sigma_1^* \tfrac{\partial}{\partial \sigma} \log \alpha(s^*, \sigma_1^*). \end{aligned}$$

The properties of α (signs of partials and log-supermodularity) and the fact that $\sigma_1^* > \sigma_0^*$ imply that the LHS is strictly positive while the RHS is strictly negative, a contradiction. ∎

Proof of Proposition 2 The proof follows similar steps as that of Proposition 1. To show that in the absence of changes to the distribution of skills, a flattening (steepening) of the matching function at the upper end implies an upward (downward) shift of the matching function everywhere, follow the exact same steps as in the proof of Proposition 1.

Proof that $\widehat{\sigma}_1^* > \sigma_1^*$ Follow almost identical steps as the corresponding part in the proof of Proposition 1.

Proof that $\widehat{M}(s) > M(s)$ The corresponding part in the proof of Proposition 1 applies.

Proof that $\widehat{s}^* > s^*$ The corresponding part in the proof of Proposition 1 applies, with the modification that in the log-differentiation of (19), the terms multiplied by ds_K drop out. ∎

Proof of Proposition 3 We proceed in three steps.

1. If the labor share increases, then the marginal training-intensive task becomes less complex. Formally, if $\beta_0\widehat{\sigma}_0^* + \beta_1\widehat{\sigma}_1^* < \beta_0\sigma_0^* + \beta_1\sigma_1^*$, then $\widehat{\sigma}_1^* < \sigma_1^*$. For suppose that $\beta_0\widehat{\sigma}_0^* + \beta_1\widehat{\sigma}_1^* < \beta_0\sigma_0^* + \beta_1\sigma_1^*$, but $\widehat{\sigma}_1^* \geq \sigma_1^*$. Then $\widehat{\sigma}_0^* < \sigma_0^*$. By (19), $\widehat{s}^* < s^*$. But by (18), $\widehat{s}^* > s^*$, a contradiction.

2. If the marginal training-intensive task becomes less complex, then the marginal worker becomes more skilled. Formally, if $\widehat{\sigma}_1^* < \sigma_1^*$, then $\widehat{s}^* > s^*$. For suppose that $\widehat{\sigma}_1^* < \sigma_1^*$ but $\widehat{s}^* \leq s^*$. Then (19) implies $\widehat{\sigma}_0^* < \sigma_0^*$. But since $\widehat{V}(\widehat{s}^*) < V(s^*)$, (18) implies $\widehat{\sigma}_0^* > \sigma_0^*$, a contradiction.

3. If at one point the new matching function is flatter and does not lie below the old matching function, then it lies above the old one everywhere to the left of this point. Formally, if $\widehat{M}'(s') \leq M'(s')$ and $\widehat{M}(s') \geq M(s')$ for some $s' \in (\max\{s^*, \widehat{s}^*\}, \bar{s}]$, then $\widehat{M}(s) \geq M(s)$ for all $s \in [\max\{s^*, \widehat{s}^*\}, s']$. For suppose that $\widehat{M}'(s') \leq M'(s')$ and $\widehat{M}(s') \geq M(s')$, and that there exists some $s'' \in [\max\{s^*, \widehat{s}^*\}, s')$ such that $\widehat{M}(s'') < M(s'')$. Then there exists some $s''' \in (s'', s')$ such that $\widehat{M}(s''') = M(s''')$, $\widehat{M}'(s''') > M'(s''')$, and $\widehat{M}(s) \geq M(s)$ for all $s \in [s''', s']$. By (10),

$$\frac{\widehat{M}'(s''')}{M'(s''')} = \frac{\widehat{w}(s''')/\widehat{w}(s')}{w(s''')/w(s')} \times \frac{\widehat{v}(s''')/\widehat{v}(s')}{v(s''')/v(s')} \times \frac{\widehat{M}'(s')}{M'(s')}.$$

Since $\widehat{V} \succeq V$, and because the upward shift of the matching function raises inequality and thus lowers the wage of type s''' relative to that of type s', the right side of the last equation is no greater than one, so that $\widehat{M}'(s''') \leq M'(s''')$, a contradiction.

Thus, we have shown that if the increase in skill abundance results in an increase in the labor share, then the lower endpoint of the matching function moves southeast (Steps 1 and 2). This means that the matching function must shift down everywhere, for if it shifted up at one point, it would shift up everywhere (Step 3), and it would be impossible for its lower endpoint to move southeast. ▮

Proof of Corollary 1 Integrating (11), the first part of the result is immediate given the shift in the matching function and the log-supermodularity of α. The second part follows since $\widehat{w}(s')/\widehat{w}(s) = 1$ but $w(s')/w(s) > 1$ for all such s', s. ▮

Proof of Corollary 2 Recall that the labor share is proportional to $\beta_0(\bar{\sigma} - \sigma_0^*) + \beta_1(\bar{\sigma} - \sigma_1^*)$. As $\widehat{\sigma}_1^* > \sigma_1^*$, the result is immediate if $\widehat{\sigma}_0^* \geq \sigma_0^*$. Then consider the case $\widehat{\sigma}_0^* < \sigma_0^*$. Rewrite (18) as

$$\theta\alpha(s_K, \sigma_0^*) = \frac{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\sigma_1^* - \underline{\sigma})}{\frac{\beta_0(\bar{\sigma} - \sigma_0^*)}{V(s^*)}}.$$

The LHS increases. If the denominator of the RHS increases, then so must the numerator, which is proportional to the capital share. Hence the labor share decreases. If the denominator of the RHS decreases, then the wage share of all workers falls, again implying a fall in the labor share. ▮

Proof of Corollary 3 Analogous to the proof of Corollary 1. ▮

Proof of Corollary 4 Analogous to the proof of Corollary 2. ▮

Proof of Corollary 5 Analogous to the proof of Corollary 1. ▮

Proofs of Remaining Results Stated in the Text

Sufficient Conditions for Existence of an Interior Equilibrium

We derive sufficient conditions ensuring that an interior equilibrium with $\sigma_0^*, \sigma_1^* \in (\underline{\sigma}, \overline{\sigma})$ and hence $s^* \in (\underline{s}, \overline{s})$ prevails. These conditions will consist of mild restrictions on the values that the economy's endowment of efficiency units of capital relative to labor, $A_K K / A_N \equiv \theta$ may take, given a particular choice of values $(\overline{s}, \underline{\sigma}, \overline{\sigma})$.

In any equilibrium in which $k_0(\sigma) = 0$ for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, we have by (8)

$$\begin{aligned} p_0(\underline{\sigma})\alpha(s_K, \underline{\sigma}) &\leq r/A_K \\ p_0(\underline{\sigma}) &= w(s^*)/A_N, \end{aligned}$$

which yields $\alpha(s_K, \underline{\sigma}) \leq [r/A_K]/[w(s^*)/A_N]$. Using (16) and (17) this inequality is shown to be equivalent to

$$\alpha(s_K, \underline{\sigma}) \leq \frac{\beta_1(\sigma_1^* - \underline{\sigma})}{\beta_0(\overline{\sigma} - \underline{\sigma})} \times \frac{V(s^*)}{\theta}.$$

The RHS of the last inequality is strictly less than $\beta_1/(\beta_0\theta)$, hence a sufficient condition to rule out any equilibrium in which $k_0(\sigma) = 0$ for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ is $\alpha(s_K, \underline{\sigma}) > \beta_1/(\beta_0\theta)$ or

$$\theta > \frac{\beta_1}{\beta_0} \frac{1}{\alpha(s_K, \underline{\sigma})}. \tag{A6}$$

And in any equilibrium in which $n_1(s, \sigma) = 0$ for all $s \in [\underline{s}, \overline{s}]$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ we have by (8)

$$\begin{aligned} p_1(\overline{\sigma})\alpha(c_K, \overline{\sigma}) &= r/A_K \\ p_1(\overline{\sigma})\alpha(\overline{s}, \overline{\sigma}) &\leq w(\overline{s})/A_N = w(s^*)/A_N, \end{aligned}$$

from which we obtain $\alpha(s_K, \overline{\sigma})/\alpha(\overline{s}, \overline{\sigma}) \geq [r/A_K]/[w(s^*)/A_N]$. Using (16) and (17) this inequality becomes

$$\frac{\alpha(s_K, \overline{\sigma})}{\alpha(\overline{s}, \overline{\sigma})} \geq \frac{\beta_0(\sigma_0^* - \underline{\sigma}) + \beta_1(\overline{\sigma} - \underline{\sigma})}{\beta_0(\overline{\sigma} - \sigma_0^*)} \times \frac{1}{\theta}.$$

The RHS of the last inequality is strictly greater than $\beta_1/(\beta_0\theta)$, hence a sufficient condition to rule out any equilibrium in which $n_1(s, \sigma) = 0$ for all $s \in [\underline{s}, \overline{s}]$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ is $\alpha(s_K, \overline{\sigma})/\alpha(\overline{s}, \overline{\sigma}) < \beta_1/(\beta_0\theta)$ or

$$\theta < \frac{\beta_1}{\beta_0} \frac{\alpha(\overline{s}, \overline{\sigma})}{\alpha(s_K, \overline{\sigma})}. \tag{A7}$$

Combining (A6) and (A7), we conclude that if

$$\theta \in S, \quad S \equiv \frac{\beta_1}{\beta_0} \left(\frac{1}{\alpha(s_K, \underline{\sigma})}, \frac{\alpha(\overline{s}, \overline{\sigma})}{\alpha(s_K, \overline{\sigma})} \right),$$

then the equilibrium is interior with $\sigma_0^*, \sigma_1^* \in (\underline{\sigma}, \overline{\sigma})$ and hence $s^* \in (\underline{s}, \overline{s})$. Existence of an interior equilibrium is ensured by choosing parameter values for $(\overline{s}, \underline{\sigma}, \overline{\sigma})$ such that S is a non-empty set. Our claim that the restrictions on θ are mild given a particular choice of $(\overline{s}, \underline{\sigma}, \overline{\sigma})$ is justified both for the baseline model and the more general production process in Appendix 6, because we can choose parameters such that $S \to \frac{\beta_1}{\beta_0}(1, \infty)$. In the case of the baseline model, we can set $\underline{\sigma} = 0$

and let $s_K \to \overline{\sigma}$. In the more general model, we can assume that $\underline{\sigma}$ is sufficiently small so that $F(Z;\underline{\sigma})$ is close to one even for very small Z; and that $\overline{\sigma}$ is sufficiently large so that $F(Z;\overline{\sigma})$ is close to zero even for very large Z, while at the same time $\overline{s}$ is large so that $\alpha(\overline{s},\overline{\sigma})$ stays finite.

Proofs of Lemmas Stated in the Text

Proof of Lemma 1 Follows from the definition of strict log-supermodularity and simple differentiation. ∎

Proof of Lemma 2 (a) For any vectors (s,σ) and (s',σ') such that $n_0(s,\sigma) > 0$ and $n_1(s',\sigma') > 0$ we have by the zero-profit condition (8) $p_0(\sigma) = w(s)/A_N$ and $p_0(\sigma) \leq w(s')/A_N$, or $w(s) \leq w(s')$, and

$$\begin{aligned} p_1(\sigma')\alpha(s',\sigma') &= w(s')/A_N, \\ p_1(\sigma')\alpha(s,\sigma') &\leq w(s)/A_N. \end{aligned}$$

Together these conditions imply $\alpha(s',\sigma')/\alpha(s,\sigma') \geq 1$. Since α is increasing in s we must have $s' \geq s$. Furthermore, it must be that $s^* > \underline{s}$, for suppose not. Then market clearing (3) implies that $k_0(\sigma) > 0$ for all σ (task output must be strictly positive due to the INADA properties of the Cobb-Douglas production function). By (8), for some (s,σ)

$$\begin{aligned} p_1(\sigma)\alpha(s,\sigma) &= w(s)/A_N, \\ p_1(\sigma)\alpha(s_K,\sigma) &\leq r/A_K, \end{aligned}$$

which yields

$$\frac{w(s)/A_N}{r/A_K} \leq \frac{\alpha(s,\sigma)}{\alpha(s_K,\sigma)}.$$

Furthermore, $p_0(\sigma)\alpha(s_K,\sigma) = r/A_K$ and $p_0(\sigma) \leq w(s)/A_N$. This yields

$$\frac{w(s)/A_N}{r/A_K} \geq \frac{1}{\alpha(s_K,\sigma)}.$$

Together with the previous result this implies $\alpha(s,\sigma) \geq 1$ which is impossible given (9).

(b) If $k_0(\sigma) > 0$, then by the zero-profit condition (8)

$$\frac{w(s^*)/A_N}{r/A_K} \geq \frac{1}{\alpha(s_K,\sigma)},$$

and there is some σ' such that $n_1(s^*,\sigma') > 0$ and hence by (8)

$$\frac{w(s^*)}{r/A_K} \leq \frac{\alpha(s^*,\sigma')}{\alpha(s_K,\sigma')}.$$

The previous two inequalities imply

$$\frac{\alpha(s^*,\sigma')}{\alpha(s_K,\sigma')} \geq \frac{1}{\alpha(s_K,\sigma)},$$

but since $\alpha(s_K,\sigma) < 1$, we have $\alpha(s^*,\sigma')/\alpha(s_K,\sigma') > 1$ which is only possible if $s^* > s_K$.

Next, observe that for any (σ,σ') and $s \leq s^*$ such that $k_0(\sigma) > 0$ and $n_0(s,\sigma') > 0$ we have by (8),

$$\begin{aligned} p_0(\sigma)\alpha(s_K,\sigma) &= r/A_K \\ p_0(\sigma) &\leq w(s)/A_N, \end{aligned}$$

and

$$\begin{aligned} p_0(\sigma')\alpha(s_K,\sigma') &\leq r/A_K \\ p_0(\sigma') &= w(s)/A_N, \end{aligned}$$

which yields $\alpha(s_K,\sigma) \geq \alpha(s_K,\sigma')$ and so $\sigma \leq \sigma'$. Thus we have established existence of σ_0^*.

Similarly, for any (σ,σ') and $s \geq s^*$ such that $k_1(\sigma) > 0$ and $n_1(s,\sigma') > 0$, we have by (8),

$$\begin{aligned} p_1(\sigma)\alpha(s_K,\sigma) &= r/A_K \\ p_1(\sigma)\alpha(s,\sigma) &\leq w(s)/A_N, \end{aligned}$$

and

$$\begin{aligned} p_1(\sigma')\alpha(s_K,\sigma') &\leq r/A_K \\ p_1(\sigma')\alpha(s,\sigma') &= w(s)/A_N, \end{aligned}$$

which yields

$$\frac{\alpha(s_K,\sigma)}{\alpha(s,\sigma)} \geq \frac{\alpha(s_K,\sigma')}{\alpha(s,\sigma')},$$

and so $\sigma \leq \sigma'$ by the log-supermodularity of α and since $s > s_K$. This establishes existence of σ_1^*.

Now, it must be that $\sigma_0^* < \sigma_1^*$, for suppose not. If $\sigma_0^* > \sigma_1^*$, then there exist (s,σ) such that $k_0(\sigma) > 0$, $k_1(\sigma) = 0$, $n_0(s,\sigma) = 0$, and $n_1(s,\sigma) > 0$. By (8),

$$\begin{aligned} p_0(\sigma)\alpha(s_K,\sigma) &= r/A_K \\ p_0(\sigma) &\leq w(s)/A_N, \end{aligned}$$

and

$$\begin{aligned} p_1(\sigma)\alpha(s_K,\sigma) &\leq r/A_K \\ p_1(\sigma)\alpha(s,\sigma) &= w(s)/A_N. \end{aligned}$$

This yields $\alpha(s,\sigma) \geq 1$ which contradicts (9). If $\sigma_0^* = \sigma_1^*$, then similar arguments lead to $\alpha(s,\sigma) = 1$, which also contradicts (9). ∎

Proof of Lemma 3 Given Lemma 2, the problem is to match workers of skill levels $s \in [s^*, \bar{s}]$ to tasks $\sigma \in [\sigma_1^*, \bar{\sigma}]$ in a setting identical to that in Costinot and Vogel (2010). Hence, the proof of Lemma 1 from their paper applies. ∎

An Extended Model of Task Production and Firms' Productivity Choices

Here we model the production process for tasks explicitly, following Garicano (2000). In order to produce, factors (workers, machines) must confront and solve problems. These problems are task-specific. There is a continuum of problems $Z \in [0, \infty)$ in each task, and problems are ordered by frequency. Thus, there exists a non-increasing probability density function for problems in each task.

Factors draw problems and produce if and only if they know the solution to the problem drawn. We assume that a mass A of problems is drawn, and A may vary across factors. Hence, the task-neutral productivity term introduced in Section 2.2 has a more precise interpretation in this context. Task output per factor unit is equal to A times the integral of the density function over the set of problems to which the factor knows the solution. To save on notation, we will assume here that all factors draw a unit mass of problems in all tasks, or $A = 1$. This does not affect any of the derivations and results below.

The distribution of problems in a task with complexity σ is given by the cumulative distribution function $F(Z; \sigma)$, which we assume to be continuously differentiable in both Z and the shift parameter σ. Let $\partial F / \partial \sigma < 0$, so that σ indexes first-order stochastic dominance. In terms of the examples mentioned in Section 2.2, waiting tables or managing an enterprise are are more complex (higher σ) than flipping burgers or weaving since the number of distinct problems typically encountered in the former set of tasks is higher than in the latter.

The probability density function corresponding to F is $f(Z; \sigma)$. Because F is continuously differentiable and Z indexes frequency, f is strictly decreasing in Z. We impose the following condition on the family of distributions $F(Z, \sigma)$.

Assumption 1 *$F(z, \sigma)$ is strictly log-supermodular.*

This assumption will give rise to the same comparative advantage properties as in the baseline model. One of the distributions satisfying Assumption 1 is the exponential distribution with mean σ.

Note that the distribution of problems depends only on σ and not on τ. As discussed above, training intensity is not an intrinsic property of a task, but arises from the fact that humans have evolved such that some tasks require less effort to master than others, even holding constant (objective) complexity. In this context, humans are assumed to be endowed with knowledge of the solutions to all problems in innate ability tasks.

We now characterize optimal training and design choices and derive equilibrium productivity of workers and machines. First observe that firms will equip factors with a set of knowledge $[0, z]$, since it can never be optimal not to know the solutions to the most frequent problems. As each worker is endowed with one efficiency unit of labor, after incurring learning costs $1 - z/s$ efficiency units are left for production, solving a fraction $F(z; \sigma)$ of problems drawn. Similarly, after the design cost, $1 - z/s_K$ units of capital are left, and the machine solves a fraction $F(z; \sigma)$ of problems drawn. Let the productivity level of an optimally trained worker of skill s in task $(\sigma, 1)$ be denoted by $\alpha^N(s, \sigma, 1)$, and similarly let $\alpha^K(s_K, \sigma)$ be the productivity level of an optimally designed machine. For simplicity, we omit the task-neutral productivity term A_K here, as it does not affect optimal machine design. Then we have

$$\alpha^N(s, \sigma, 1) \equiv \max_z F(z; \sigma) [1 - z/s],$$

$$\alpha^K(s_K, \sigma) \equiv \max_z F(z; \sigma) [1 - z/s_K],$$

A unique interior solution to the worker training and machine design problems always exists. Unlike in the baseline model, we do not require any restrictions on s_K and $\underline{s}$ in relation to $\overline{\sigma}$ to ensure that productivity is non-negative.[49] The optimal knowledge levels $z^N(s,\sigma)$ and $z^K(s_K,\sigma)$ are pinned down by the first-order conditions

$$\begin{aligned} f(z(s,\sigma);\sigma)\,[1-z(s,\sigma)/s] &= F(z(s,\sigma,\tau);\sigma)/s, \\ f(z(s_K,\sigma);\sigma)\,[1-z(s_K,\sigma)/s_K] &= F(z(s_K,\sigma);\sigma)/s_K. \end{aligned} \tag{A8}$$

Optimality requires that the benefit of learning the solution to an additional problem—the probability that the problem occurs times the number of efficiency units left for production, be equal to the cost of doing so—the number of efficiency units lost times the fraction of problems these efficiency units would have solved. Optimal worker and machine productivities are given by

$$\alpha^N(s,\sigma,\tau) = \begin{cases} F(z(s,\sigma,\tau);\sigma)\,[1-z(s,\sigma,\tau)/s] & \text{if } \tau = 1 \\ \\ 1 & \text{if } \tau = 0 \end{cases}$$

and

$$\alpha^K(s_K,\sigma) = F(z(s_K,\sigma);\sigma)\,[1-z(s_K,\sigma)/s_K]\,.$$

Let $\check{s}$ be an element in set $\check{S} = s_K \cup [\underline{s},\overline{s}]$. By the above results, we have that $\alpha^N(\check{s},\sigma,1) \equiv \alpha^K(\check{s},\sigma)$. Thus, workers and machines face the same productivity schedule in training-intensive tasks. We drop superscripts and define the function

$$\alpha(\check{s},\sigma) = F(z(\check{s},\sigma);\sigma)\left[1-\frac{1}{\check{s}}z(\check{s},\sigma)\right] \qquad \check{s} \in \check{S} = s_K \cup [\underline{s},\overline{s}], \tag{A9}$$

where $z(\check{s},\sigma)$ is implicitly given by (A8).

The qualitative properties of the productivity schedule $\alpha(\check{s},\sigma)$ are the same as in the baseline model. First notice that $\alpha \in (0,1)$ by (A9). Furthermore, from applying the envelope theorem to (A9) it follows that α is increasing in $\check{s}$ and decreasing in σ. Higher skilled factors are more productive since they face a lower learning/design cost, and productivity declines in complexity since a larger cost is incurred to achieve a given level of productivity.

To characterize comparative advantage, we again rely on log-supermodularity: Under Assumption 1, the productivity schedule $\alpha(\check{s},\sigma)$ is strictly log-supermodular. To show this, start by observing that $\alpha(\check{s},\sigma)$ is strictly log-supermodular if and only if

$$\frac{\partial^2}{\partial\check{s}\partial\sigma}\log\alpha(\check{s},\sigma) > 0.$$

Applying the envelope theorem to (A9) yields

$$\frac{\partial}{\partial\check{s}}\log\alpha(\check{s},\sigma) = \frac{z(\check{s},\sigma)}{(\check{s})^2-\check{s}z(\check{s},\sigma)}.$$

The RHS is an increasing function of $z(\check{s},\sigma)$, and so

$$\frac{\partial^2}{\partial\check{s}\partial\sigma}\log\alpha(\check{s},\sigma) > 0 \quad \Leftrightarrow \quad \frac{\partial}{\partial\sigma}z(\check{s},\sigma) > 0.$$

[49] A unique interior solution to the worker training problem exists if $\tau > 0$ because first, the problem is strictly concave as f is strictly decreasing; second, the derivative of the objective at $z = 0$ is strictly positive; finally, the value of the objective function becomes negative for a sufficiently large z. The same arguments also establish the result for the machine design problem.

Thus, α is log-supermodular if and only if optimal knowledge levels are increasing in σ. Differentiating the FOC (A8) yields

$$\frac{\partial}{\partial \sigma} z(\check{s}, \sigma) = \frac{F_\sigma \frac{1}{\check{s}} - f_\sigma \left[1 - \frac{1}{\check{s}} z\right]}{f_z \left[1 - \frac{1}{\check{s}} z\right] - 2f \frac{1}{\check{s}}}.$$

The denominator of the RHS is negative as $f_z < 0$, and so, using the FOC we find that

$$\frac{\partial}{\partial \sigma} z(\check{s}, \sigma) > 0 \qquad \Leftarrow \qquad F f_\sigma > F_\sigma f \quad \forall Z, \sigma > 0.$$

But this condition is equivalent to F being strictly log-supermodular.

Given that $\alpha(\check{s}, \sigma)$ has the same qualitative properties as the productivity schedule in the baseline model, assignment and comparative statics results are qualitatively the same, as well, since none of the results for the baseline model rely on the specific functional form.

A Model with Fixed Costs

Model setup

Worker training technologies are as in the baseline model. However, we now assume that an upfront expense of $\varphi(\sigma)$ is required to equip the firm's stock of machines with σ units of knowledge. This cost is independent of the size of the stock, as in the case of software. We make the critical assumption $\varphi' > 0$, and for simplicity we set $\varphi(\underline{\sigma}) = 0$ and $\varphi'' > 0$. In our numerical solutions we choose $\varphi(\sigma) = c_K(\sigma - \underline{\sigma})^2$ where c_K is the parameter capturing labor-replacing technical change. Machines capable of performing a task produce A_K units of task output—that is, machine productivity is independent of complexity. As in the baseline model, worker productivity is independent of complexity and worker skill in innate ability tasks, but in training-intensive tasks worker productivity is given by $\alpha(s, \sigma) \equiv 1 - \sigma/s$. We normalize task-neutral worker productivity to one to save on notation, $A_N = 1$.

We assume that each task is produced by a single monopolistic firm.[50] In contrast, final good firms are perfectly competitive just as in the baseline version of the model. The final good production function is now

$$Y = \left[\int_{\underline{\sigma}}^{\overline{\sigma}} \left\{ \beta_1 y_0(\sigma)^{\frac{\varepsilon-1}{\varepsilon}} + \beta_1 y_1(\sigma)^{\frac{\varepsilon-1}{\varepsilon}} \right\} d\sigma \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A10}$$

with $\varepsilon > 1$ and $\sum_\tau \beta_\tau = 1$ for CRS. Given profit maximization by final good firms, the CES production function yields the standard isoelastic input demand curve, inducing the well-known constant-markup pricing rule.

Standard arguments establish that equilibrium variable profits of the firm supplying task (σ, τ) are given by

$$\pi_\tau(\sigma, \chi) = \beta_\tau^\varepsilon \frac{(\varepsilon - 1)^{\varepsilon-1}}{\varepsilon^\varepsilon} \chi^{-(\varepsilon-1)} Y \tag{A11}$$

where χ is marginal cost which depends on the characteristics of tasks and factors employed. In particular, if employing labor of type s we have

$$\chi \equiv \chi(s, \sigma, \tau) = \begin{cases} w(s) & \text{if } \tau = 0 \\ w(s)/\alpha(s, \sigma) & \text{if } \tau = 1, \end{cases}$$

and if employing capital,

$$\chi = \frac{r}{A_K}.$$

Furthermore, equilibrium task output is

$$y_\tau(\sigma, \chi) = \beta_\tau^\varepsilon \left(\frac{\varepsilon - 1}{\varepsilon} \right)^\varepsilon \chi^{-\varepsilon} Y. \tag{A12}$$

Equilibrium assignment

We focus on equilibria in which $r/A_K < w(\bar{s})$, so that the marginal cost of using machines is less than that of employing labor of any type in any task.[51] In such an equilibrium, the assignment is

[50] Holmes and Mitchell (2008) present a more complex model where labor and machines are optimally assigned to tasks *within* monopolistic firms. We suspect that our results would hold in a version of that model as well.

[51] We do not derive formal conditions ensuring this property, but it will feature in our numerical solutions.

qualitatively the same as in the case we analyzed for the baseline model. Task producers employ the factor that delivers the highest total profits. Note that variable profits among firms that use machines are constant across tasks. In any equilibrium with the above characteristic there are a threshold tasks σ_0^*, σ_1^* such that it is optimal for firms to use machines in all innate ability tasks with $\sigma \leq \sigma_0^*$ and in all training-intensive tasks with $\sigma \leq \sigma_1^*$.[52] As in the baseline model, there is a cutoff s^* such that workers below the cutoff perform innate ability tasks, while workers above the cutoff carry out training intensive tasks, with higher skilled workers performing more complex tasks.

Solving the model

The threshold tasks are determined by no-arbitrage conditions. Profits from employing labor in these tasks must be equal to profits from using machines. This means that the difference in variable profits between machines and labor must equal the fixed cost of designing the machine,

$$\beta_0^\varepsilon \frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^\varepsilon} Y \left[\left(\frac{A_K}{r}\right)^{\varepsilon-1} - \left(\frac{1}{w(s^*)}\right)^{\varepsilon-1} \right] = \varphi(\sigma_0^*) \tag{A13}$$

and

$$\beta_1^\varepsilon \frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^\varepsilon} Y \left[\left(\frac{A_K}{r}\right)^{\varepsilon-1} - \left(\frac{\alpha(s^*,\sigma_1^*)}{w(s^*)}\right)^{\varepsilon-1} \right] = \varphi(\sigma_1^*). \tag{A14}$$

Worker assignment to training intensive tasks is assortative as in the baseline model, with the wage schedule given by

$$\frac{d \log w(s)}{ds} = \frac{\partial \log \alpha(s, M(s))}{\partial s} \tag{A15}$$

and the matching function satisfying

$$M'(s) = \frac{\alpha(s, M(s))^{1-\varepsilon} v(s) w(s)^\varepsilon}{\beta_1^\varepsilon \left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon Y} \tag{A16}$$

as well as the boundary conditions $M(s^*) = \sigma_1^*$ and $M(\bar{s}) = \bar{\sigma}$.

The model is closed by market clearing conditions and the production function. For capital markets to clear, we must have $\int_{\underline{\sigma}}^{\sigma_0^*} k_0(\sigma) d\sigma + \int_{\underline{\sigma}}^{\sigma_1^*} k_1(\sigma) d\sigma = K$. Because of our assumption of constant marginal costs of using machines regardless of a task's complexity, we have that task outputs are constant within innate ability tasks and within training-intensive tasks, and so are machine inputs. Thus, the capital market clearing condition becomes $(\sigma_0^* - \underline{\sigma})k_0 + (\sigma_0^* - \underline{\sigma})k_1 = K$ Using the task production function $y_\tau = A_K k_\tau$ and equilibrium task output (A12), we obtain $k_0/k_1 = (\beta_0/\beta_1)^\varepsilon$. Together with the market clearing condition and the task production function this implies

$$y_\tau(\sigma) = \frac{\beta_\tau^\varepsilon A_K K}{\sum_\tau \beta_\tau (\sigma_\tau^* - \underline{\sigma})} \quad \textit{for all } \sigma \in [\underline{\sigma}, \sigma_\tau^*]. \tag{A17}$$

[52]Notice that profits of firms employing type-s labor are flat in σ ($\tau = 0$) or a decreasing and convex function of σ approaching zero as σ gets large ($\tau = 1$). However, because fixed machine design costs are decreasing in σ, and because of the convexity of the cost function, profits of firms using machines become negative as σ gets large. Due to the properties of α, profits when employing labor are a convex function of σ, so that there exists a value of σ such that profits from employing typs-s labor are equal to profits from using machines.

Table A1: Parameter values for the model with fixed design costs

β_0	$=$	$1/3$
ε	$=$	2
σ	$\in$	$[0,1]$
s	$\in$	$[1.01,2]$
$v(s)$		see text
$A_K K$	$=$	1
$\phi(\sigma)$	$=$	$c_K\sigma^2$
c_K	$\in$	$[1,2]$

When firms employ labor in innate ability tasks, the task production function is $y_0(\sigma) = \int_{\underline{s}}^{s^*} n_0(s,\sigma)ds$. The market clearing condition is $\int_{\sigma_0^*}^{\overline{\sigma}}\int_{\underline{s}}^{s^*} n_0(s,\sigma)dsd\sigma = V(s^*)$, and so

$$y_0(\sigma) = \frac{V(s^*)}{\overline{\sigma}-\sigma_0^*} \quad \textit{for all } \sigma \in [\sigma_0^*, \overline{\sigma}]. \tag{A18}$$

Given (A16), (A17), and (A18), final good output must satisfy

$$\begin{aligned} Y^{\frac{\varepsilon-1}{\varepsilon}} \;&=\; \left[\beta_0^\varepsilon(\sigma_0^* - \underline{\sigma}) + \beta_1^\varepsilon(\sigma_1^* - \underline{\sigma})\right]^{\frac{1}{\varepsilon}} (A_K K)^{\frac{\varepsilon-1}{\varepsilon}} \\ &\quad + \beta_0[\overline{\sigma}-\sigma_0^*]^{\frac{1}{\varepsilon}} V(s^*)^{\frac{\varepsilon-1}{\varepsilon}} + \beta_1\int_{s^*}^{\overline{s}} M'(s)^{\frac{1}{\varepsilon}}\left[\alpha(s,M(s))v(s)\right]^{\frac{\varepsilon-1}{\varepsilon}} ds. \end{aligned} \tag{A19}$$

Plugging (A17) into (A12) yields an expression for the rental rate,

$$\frac{r}{A_K} = \frac{\varepsilon-1}{\varepsilon}\left[\beta_0^\varepsilon(\sigma_0^*-\underline{\sigma}) + \beta_1^\varepsilon(\sigma_1^*-\underline{\sigma})\right]^{\frac{1}{\varepsilon}}\left(\frac{Y}{A_K K}\right)^{\frac{1}{\varepsilon}}. \tag{A20}$$

Similarly, plugging (A18) into (A12) gives an expression for the wage paid to the marginal worker,

$$w(s^*) = \frac{\varepsilon-1}{\varepsilon}\beta_0(\overline{\sigma}-\sigma_0^*)^{\frac{1}{\varepsilon}}\left(\frac{Y}{V(s^*)}\right)^{\frac{1}{\varepsilon}}. \tag{A21}$$

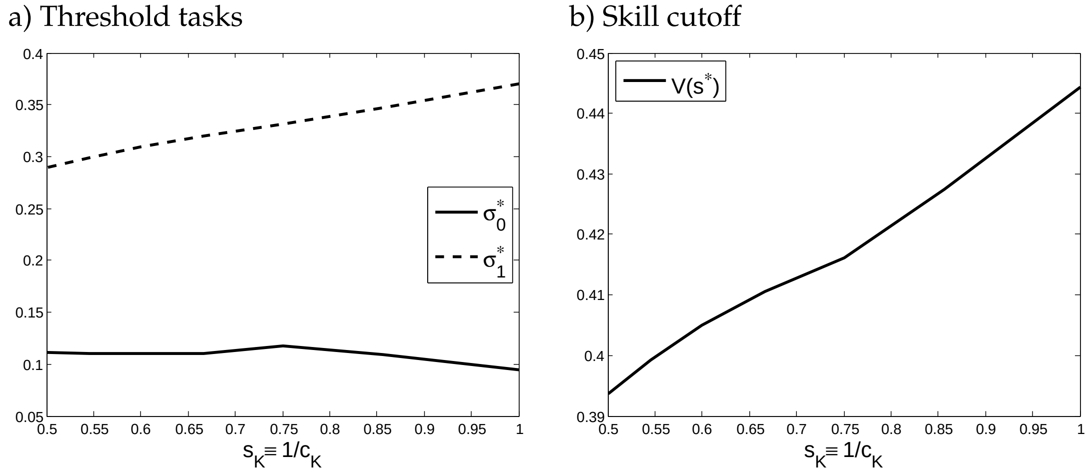
We solve the model by grid search. Given a guess of (s^*, σ_1^*), we solve for the matching function and the wage distribution, obtaining $w(s^*)/Y^{1/\varepsilon}$. Using this, we calculate σ_0^* from (A21) and Y from (A19), thus obtaining $w(s^*)$. We then calculate r/A_K from (A20). Finally, we check whether the no-arbitrage conditions (A13) and (A14) are satisfied.

Numerical solution

We solve the model for values of c_K ranging from one to two. Parameter values are given in Table A1. The skill distribution $v(s)$ is a truncated log-normal. We construct the distribution such that the corresponding (non-truncated) normal distribution has mean $\log \underline{s} + 1/3 * \log(\overline{s}/\underline{s})$ and standard deviation $1/6 * \log(\overline{s}/\underline{s})$. This implies that the original normal distribution is truncated at four (two) standard deviations above (below) the mean.

As in the baseline model, firms adopt machines more widely in training-intensive tasks as design becomes cheaper—the marginal training-intensive task becomes more complex. The

Figure A1: Threshold Tasks and the Skill Cutoff as Functions of the Design Cost



The figures show the changes in threshold tasks (a) and the skill cutoff (b) as machine design becomes cheaper, for the model with a fixed design cost. Panel b) plots the fraction of workers below the cutoff $V(s^*)$.

effect on the marginal innate ability task is ambiguous, however (see panel a) of Figure A1).

The skill cutoff increases as design gets cheaper, so that the employment share of innate ability tasks rises (see panel b) of Figure A1). But because σ_1^* also increases the matching function shifts up, implying a reallocation of workers to more complex training-intensive tasks. Market clearing implies a compression of the wage distribution in the lower part of the wage distribution but increasing dispersion in the upper part. Thus, the model with a fixed design cost features job and wage polarization just like the baseline model.

Data Sources and Measurement of Training Requirements

Data sources.—Our 1971 training measure comes from the Fourth Edition Dictionary of Occupational Titles (DOT), which is made available in combination with the 1971 April Current Population Survey (CPS) (National Academy of Sciences 1981). We obtain contemporaneous wage data from the IPUMS 1970 census extract (the processing of this data follows the procedure of Acemoglu and Autor (2011)). Our 2007 training measure comes from the Job Zones file in the O*NET database available at `http://www.onetcenter.org/database.html?p=2`. For contemporaneous micro data we use the IPUMS 2008 American Community Survey (ACS).

Measuring training requirements.—SVP (see definition in Section 5) is measured on a nine-point scale in the DOT. In the O*NET database, Job Zones are measured on a five-point scale which maps into the nine-point SVP scale. See Table A2 for the interpretation of the SVP scale and the mapping between SVP and Job Zones. In the DOT data, we convert SVP into Job Zones. We assign midpoints to consistently measure training requirements over time. We assign a conservative value to the highest category. See the last column in Table A2 for details.

Table A2: Measuring Training Requirements Based on SVP and Job Zones

	SVP	*Job Zone*	*Training time*
1	short demonstration	1	1.5 months
2	up to 30 days	1	1.5 months
3	30 days to 3 months	1	1.5 months
4	3 to 6 months	2	7.5 months
5	6 months to 1 year	2	7.5 months
6	1 to 2 years	3	1.5 years
7	2 to 4 years	4	3 years
8	4 to 10 years	5	7.5 years
9	over 10 years	5	7.5 years

The DOT variables, including SVP, in the 1971 April CPS extract vary at the level of 4,528 distinct occupations. For the occupation-level analysis, we collapse the CPS micro data to the three-digit occupation level using David Dorn's classification of occupations (Dorn 2009), weighting by the product of sampling weights and hours worked. The Job Zones variable in the O*NET database is available for 904 distinct occupations of the Standard Occupational Classification System (SOC). In the 2008 ACS data there are 443 distinct SOC occupations. We collapse the O*NET data to these 443 occupations and then merge it to the ACS data. For the occupation-level analysis, we collapse the ACS micro data to the three-digit occupation level in the same way as the CPS data.

Table A3 lists the twenty least and most training-intensive occupations (using David Dorn's classification) in 1971 and 2007. Table A4 lists the twenty occupations experiencing the largest declines and increases in training requirements.

Table A3: Least and Most Training-Intensive Occupations, 1971 & 2007

Occupation (occ1990dd grouping)	Training requirements in years (1971)
a) least training-intensive	
Public transportation attendants and inspectors	0.1
Packers and packagers by hand	0.2
Waiter/waitress	0.2
Mail carriers for postal service	0.3
Garage and service station related occupations	0.4
Bartenders	0.4
Messengers	0.4
Parking lot attendants	0.4
Cashiers	0.5
Child care workers	0.6
Misc material moving occupations	0.6
Taxi cab drivers and chauffeurs	0.7
Baggage porters	0.7
Housekeepers, maids, and lodging quarters cleaners	0.7
Typists	0.7
Mail and paper handlers	0.7
Proofreaders	0.7
Bus drivers	0.7
File clerks	0.7
Helpers, surveyors	0.8
b) most training-intensive	
Musician or composer	6.8
Mechanical engineers	6.8
Aerospace engineer	6.8
Electrical engineer	6.9
Biological scientists	6.9
Chemical engineers	7.0
Chemists	7.0
Managers in education and related fields	7.0
Petroleum, mining, and geological engineers	7.1
Architects	7.1
Subject instructors (HS/college)	7.1
Dentists	7.2
Veterinarians	7.2
Lawyers	7.2
Civil engineers	7.2
Clergy and religious workers	7.3
Psychologists	7.3
Physicians	7.3
Geologists	7.5
Physicists and astronomers	7.5

Occupation (occ1990dd grouping)	Training requirements in years (2007)
a) least training-intensive	
Waiter/waitress	0.1
Misc food prep workers	0.1
Ushers	0.1
Parking lot attendants	0.1
Kitchen workers	0.1
Furniture and wood finishers	0.1
Pressing machine operators (clothing)	0.1
Fishers, hunters, and kindred	0.1
Textile sewing machine operators	0.1
Graders and sorters of agricultural products	0.1
Garage and service station related occupations	0.1
Taxi cab drivers and chauffeurs	0.1
Animal caretakers, except farm	0.2
Butchers and meat cutters	0.3
Janitors	0.4
Sales demonstrators / promoters / models	0.4
Housekeepers, maids, and lodging quarters cleaners	0.4
Miners	0.4
Cashiers	0.4
Stock and inventory clerks	0.4
b) most training-intensive	
Other health and therapy	7.5
Psychologists	7.5
Physicians	7.5
Economists, market researchers, and survey researchers	7.5
Lawyers	7.5
Managers of medicine and health occupations	7.5
Physicians' assistants	7.5
Biological scientists	7.5
Medical scientists	7.5
Physical scientists, n.e.c.	7.5
Podiatrists	7.5
Veterinarians	7.5
Subject instructors (HS/college)	7.5
Dietitians and nutritionists	7.5
Urban and regional planners	7.5
Pharmacists	7.5
Librarians	7.5
Optometrists	7.5
Dentists	7.5
Physicists and astronomers	7.5

Table A4: Largest Decreases and Increases in Training Requirements, 1971-2007

Occupation (occ1990dd grouping)	Change in training requirements (years) 1971-2007	Training requirements in 1971 (years)
a) largest decreases in training requirements		
Carpenters	-5.7	6.4
Musician or composer	-5.1	6.8
Air traffic controllers	-5.0	6.5
Production supervisors or foremen	-4.7	5.4
Dental laboratory and medical appliance technicians	-4.7	5.9
Geologists	-4.5	7.5
Precision makers, repairers, and smiths	-4.4	5.9
Insurance adjusters, examiners, and investigators	-4.4	5.7
Civil engineers	-4.2	7.2
Recreation and fitness workers	-4.1	6.4
Chemical engineers	-4.0	7.0
Masons, tilers, and carpet installers	-3.9	4.7
Heating, air conditioning, and refigeration mechanics	-3.9	5.4
Electrical engineer	-3.9	6.9
Petroleum, mining, and geological engineers	-3.8	7.1
Aerospace engineer	-3.8	6.8
Mechanical engineers	-3.8	6.8
Explosives workers	-3.8	4.4
Patternmakers and model makers	-3.7	5.2
Molders, and casting machine operators	-3.6	4.2
b) largest increases in training requirements		
Primary school teachers	1.2	1.8
Operations and systems researchers and analysts	1.3	4.6
Agricultural and food scientists	1.3	4.7
Archivists and curators	1.5	4.5
Managers of medicine and health occupations	1.5	6.0
Public transportation attendants and inspectors	1.9	0.1
Therapists, n.e.c.	2.3	2.9
Proofreaders	2.3	0.7
Vocational and educational counselors	2.5	4.1
Registered nurses	2.7	3.1
Social workers	2.7	3.3
Social scientists, n.e.c.	3.0	4.2
Economists, market researchers, and survey researchers	3.2	4.3
Optometrists	3.9	3.6
Pharmacists	4.3	3.2
Librarians	4.4	3.1
Podiatrists	4.5	3.0
Physical scientists, n.e.c.	4.5	3.0
Other health and therapy	4.5	3.0
Dietitians and nutritionists	4.6	2.9