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A Centralized Approach to Modelling Collective Household Decisions: Some Preliminary Results

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ABSTRACT

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Empirical models of labour supply adopting the collective approach have commonly used the decentralized representation and a reduced form specification of the sharing rule. This procedure has two crucial drawbacks that in principle make it inappropriate for the very same type of applications that are thought to be mostly relevant for this family of models, i.e. tax reform simulations. The first problem concerns the decentralized representation. The possibility of decentralizing the maximization of household welfare rests on the convexity of the budget sets. However, both the actual tax systems and the tax reforms might imply significant non-convexities: this makes the decentralized representation in general inappropriate both for estimation and for simulation. The second problem concerns the specification of the sharing rule, which typically is not a structural one, but rather a reduced form, e.g. a combination of exogenous variables (wage rates, unearned incomes etc.). Such a specification might provide a reasonable approximation to the current intra-household allocation choices under the current tax rule, but in general it cannot be used for simulating the effects of a different tax rule. Analogously, the sharing rule in general will be different depending on whether both partners work or not. We propose – and illustrate with some preliminary results – a model that permits the estimation of a structural sharing rule.

JEL Classification: C35, D12, D13, H31, J22

Keywords: labour supply, tax reform evaluation, collective approach, decentralized representation, centralized representation, sharing rule, non-convex opportunity sets

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1. Introduction

Since the path-breaking contributions by Chiappori (1988, 1992), many authors have made attempts to estimate empirical versions of the collective labour supply model and use them for tax reform simulations. The most common approach is the so-called decentralized (sharing rule) representation of the collective household program: among others, see the applications and the survey by Vermeulen (2006) and Bloemen (2010). This representation exploits the fact that the collectively optimal intra-household allocation can be interpreted as the result of the partners' decentralized decisions complemented by intra-household transfers (a close analogy to the Second Welfare Theorem). The main motivation for adopting the decentralized representation is avoiding arbitrary assumptions upon the household's collective decision process. Typically, the private consumptions of the two partners are not observed. Therefore, private consumptions are defined as total consumption (observed) plus (or minus) a transfer. The so-called sharing rule is embodied into the transfer and reflects the intra-household distributive preferences and/or the partners' bargaining power. Under certain conditions, the sharing rule can be identified. Empirically, it is usually specified by a "flexible" reduced form including all exogenous variable that are thought to be relevant for the intra-household decision process. This procedure – that was proposed as a convenient one for estimation and simulation – has however two crucial drawbacks that in principle make it inappropriate for the very same type of applications that are thought to be mostly relevant for this family of models, i.e. tax reform simulations.

The first problem concerns the decentralized representation itself. The possibility of decentralizing the maximization of household welfare rests on the convexity of the budget sets. However, both the actual tax systems and the tax reforms might imply significant non-convexities: this makes the decentralized representation in general inappropriate both for estimation and for simulation. Surprisingly, most of the empirical applications barely mention the problem *en passant*, as if it were a theoretical *rarum*, while in fact the treatment of non-convexities has been a major focus of the empirical analysis of tax reforms. Vermeulen et al. (2006) address the issue in the context of a simulation exercise and adopt a calibration approach. Beninger (2008) makes an attempt to overcome the problem by adopting a centralized representation, but he treats the share of income among the partners as a parameter rather than as a decision. Bargain and Moreau (2013) adopt a Nash-bargaining approach but use a calibrated model.

The second problem concerns the specification of the sharing rule, which typically is not a structural one, but rather a reduced form, e.g. a combination of exogenous variables (wage rates, unearned incomes etc.). Such a specification might provide a reasonable approximation to the current intra-household allocation choices under the current tax rule, but in general it cannot be used for simulating the effects of a different tax rule (Donni 2003). Analogously, the sharing rule in general will be different depending on whether both partners work or not (Blundell et al. 2007).

In summary, if we want to develop empirical models that are useful for tax simulation and adopt the collective approach it seems that we cannot escape from (a) a centralized representation of household decisions and (b) a structural representation of the intra-household allocation program. Point (b) is necessary for the simulation of tax systems substantially different from the current one, while point (a) is required only under non-convex (current or simulated) budget sets.

In section 2 we use a very simple example to derive the structural form of the "sharing rule" and to show that in general it changes when the tax rule changes. In section 3 we examine different empirical strategies that could be used to specify and estimate a centralized and structural representation of the collective labour supply model. In section 4 we illustrate an empirical application.

2. Centralized and decentralized representation of the intra-household allocation

As a simple illustration, let us consider the following scenario with no taxes (or, equivalently, with proportional taxation) and no public goods. Leisure is assumed to be a private good. The representation is then conditional on these assumptions (Chiappori and Meghir 2014). Let l_i be leisure time of member $i=f,m$ and C_i be total expenditure or income of each member of the household couple.

$$\begin{aligned} U_f &= \left(l_f\right)^{b_f}\left(C_f\right)^{a_f} \\ U_m &= \left(l_m\right)^{b_m}\left(C_m\right)^{a_m} \end{aligned} \tag{1}$$

w_f = wife's wage rate
w_m = husband's wage rate
I_f = wife's exogenous income
I_m = husband's exogenous income
T = total available time for each partner
$W = \ln U_f + \pi \ln U_m$ = household's welfare function,
where π is the relative Pareto weight given to the husband's utility.

Equivalently, we might define $C_f = \mu C, C_m = (1-\mu)C$, where $\boldsymbol{\mu}$ is the consumption or resource share, and write the utility functions as

$$\begin{aligned} U_f &= \left(l_f\right)^{b_f}\left(\mu C\right)^{a_f} \\ U_m &= \left(l_m\right)^{b_m}\left((1-\mu)C\right)^{a_m} \end{aligned} \tag{2}$$

Note that μ is the result of household decisions: how to allocate consumption among the two partners. Instead π is a parameter of the household’s (social) preferences. Obviously, there is a relationship between π and μ that will be derived below.

The household solves the following centralized program

$$\begin{aligned} &\max W = \ln U_f + \pi \ln U_m \\ &s.t. \\ &C_f + C_m = w_f\left(T - l_f\right) + w_m\left(T - l_m\right) + I \end{aligned} \tag{3}$$

Assuming an interior solution, the FOCs are:

$$\begin{aligned} &a_f = \lambda C_f \\ &\pi a_m = \lambda C_m \\ &b_f = \lambda w_f l_f \\ &\pi b_m = \lambda w_m l_m \\ &C_f + C_m = w_f\left(T - l_f\right) + w_m\left(T - l_m\right) + I_f + I_m \end{aligned} \tag{4}$$

where λ is the Lagrange multiplier.

Therefore

$$a_f + \pi a_m + b_f + \pi b_m = \lambda\left(w_f l_f + w_m l_m + C_f + C_m\right) = \lambda\left[\left(w_f + w_m\right)T + I_f + I_m\right]$$

and

$$\begin{aligned}
l_f^* &= \frac{b_f}{a_f + \pi a_m + b_f + \pi b_m}\frac{\left(w_f + w_m\right)T + I_f + I_m}{w_f} \\
C_f^* &= \frac{a_f}{a_f + \pi a_m + b_f + \pi b_m}\left[\left(w_f + w_m\right)T + I_f + I_m\right] \\
l_m^* &= \frac{\pi b_m}{a_f + \pi a_m + b_f + \pi b_m}\frac{\left(w_f + w_m\right)T + I_f + I_m}{w_m} \\
C_m^* &= \frac{\pi a_m}{a_f + \pi a_m + b_f + \pi b_m}\left[\left(w_f + w_m\right)T + I_f + I_m\right].
\end{aligned} \tag{5}$$

Therefore, for example, the wife's share in consumption is:

$$\frac{C_f^*}{C^*} = \frac{a_f}{a_f + \pi a_m} \equiv \mu\,, \tag{6}$$

where one can see the relationship between μ and π.

The wife's leisure share is:

$$\frac{l_f^*}{l^*} = \frac{b_f w_m}{b_f w_m + \pi b_m w_f}\,. \tag{7}$$

Note that the above results can also be interpreted as the solution to a Nash-bargaining procedure where π represents the husband's bargaining power.

Turning to the decentralized representation, the partners maximize their own utility but their budget constraints include transfers that allow them to reach the same efficient solution attained by the centralized program.

$$\text{Max}\left\{U_f = \left(l_f\right)^{b_f}\left(C_f\right)^{a_f}\,\middle|\,C_f = w_f\left(T - l_f\right) + \phi\right\} \tag{8}$$

$$\text{Max}\left\{U_m = \left(l_m\right)^{b_m}\left(C_m\right)^{a_m}\,\middle|\,C_m = w_m\left(T - l_m\right) - \phi\right\},$$

where ϕ denotes the intra-household transfer.

Again, we only consider interior solutions:

$$
\begin{aligned}
l_f^+ &= \frac{b_f}{a_f + b_f} \frac{w_f T + I_f + \phi}{w_f}, \\
C_f^+ &= \frac{a_f}{a_f + b_f} \left[w_f T + I_f + \phi \right] \\
l_m^+ &= \frac{b_m}{a_m + b_m} \frac{w_m T + I_m - \phi}{w_m}, \\
C_m^+ &= \frac{a_m}{a_m + b_m} \left[w_m T + I_m - \phi \right]
\end{aligned} \tag{9}
$$

By imposing the decentralized solution to be equal to the centralized one, we can see what the appropriate transfer is. From

$$
C_f^+ = \frac{a_f}{a_f + b_f} \left[w_f T + I_f + \phi \right] = \frac{a_f}{a_f + \pi a_m + b_f + \pi b_m} \left[\left(w_f + w_m \right) T + I_f + I_m \right] = C_f^*
$$

we obtain:

$$
\phi^* = \frac{a_f + b_f}{a_f + \pi a_m + b_f + \pi b_m} \left[\left(w_f + w_m \right) T + I_f + I_m \right] - \left(w_f T + I_f \right). \tag{10}
$$

Note that if $\pi=0$ the sharing rule is given by the difference between full household labor and non-labor income and the income of the female. If the Pareto weight $\pi>0$, then the household budget is scaled up or down depending on the sign and size of the male specific coefficients a_m. and b_m.

The same result can be obtained by using proposition 2 in Browning et al. (2013). Namely, we substitute the solutions of the decentralized program into U_f and U_m and derive the indirect utilities V_f and V_m. Then, we maximize $W^+ \equiv \ln V_f + \pi \ln V_m$ with respect to ϕ.

In this particular example, μ depends only on preference parameters and is therefore unaffected by tax-transfer rules. This is an implication of the utility function, where we assumed independence between leisure and consumption. However, the sharing rule ϕ depends on full-incomes: therefore, it does depend on the tax-transfer rule at least to the extent that it affects full-incomes. Moreover, ϕ depends also on the participation status of the two partners. Let us suppose that the collective allocation requires the wife is not working, i.e.:

$$
\begin{aligned}
l_f^* &= T \\
C_f^* &= \frac{a_f}{a_f + \pi a_m + b_f + \pi b_m}\left[w_m T + I_f + I_m\right] \\
l_m^* &= \frac{\pi b_m}{a_f + \pi a_m + b_f + \pi b_m}\frac{w_m T + I_f + I_m}{w_m} \\
C_m^* &= \frac{\pi a_m}{a_f + \pi a_m + b_f + \pi b_m} w_m T + I_f + I_m .
\end{aligned}
\tag{11}
$$

Then, the transfer required to implement the optimal allocation as a decentralized solution turns out to be:

$$
\phi^* = \frac{a_f + b_f}{a_f + \pi a_m + b_f + \pi b_m}\left[w_m T + I_f + I_m\right] - \left(w_f T + I_f\right), \tag{12}
$$

which is different from what required when both partners are working. If a reform of the tax-transfer rule modifies the participation conditions, then also the sharing rule will change.

3. An empirical framework

In what follows we present an empirical framework for the analysis of collective household decisions adopting an explicit centralized representation.

We assume we have a sample of H households. For each household we observe:

h_g = hours of work of partner of gender *g = f, m*;

$l_g \equiv T - h_g$ = hours of leisure of partner of gender *g = f, m*;

C = household net disposable income;

w_g = wage rate (actual or potential) of partner of gender *g = f, m*;

I_g = exogenous income of partner of gender *g = f, m*;

h_g = hours of work of partner of gender *g = f, m*;

Z = vector of socio-demographic characteristics.

The opportunity set contains J "jobs" $\left(h_f(j), h_m(j), w_f(j), w_m(j)\right), j = 1,..., J$.

For each job we can compute the household net available income

$C(j) = f\left(h_f(j), h_m(j), w_f(j), w_m(j), I_f, I_m, Z\right)$

where f represents the tax-transfer rule (possibly dependent on characteristics *Z*) that transforms gross incomes into net available income.

Leisure and consumption are strictly private goods for the two partners. The household allocates $C(j)$ to individual consumptions $C_g(j), g = f, m.$ However, we only observe $C(j)$, not $C_g(j), g = f, m.$ As a matter of notational convenience we will write $C_f(j) = \mu C(j), C_m(j) = (1-\mu)C(j)$. It is convenient to represent the possible value of μ by a set of M discrete values $\in [0,1]$.

The alternatives among which the household can choose can now be represented as J$\times$M "packages" $\left(h_f(j), h_m(j), C(j), \mu(i)\right), j = 1,..., J, i = 1,..., M$.

When the household chooses package (j,i), the utility levels attained by the partners are respectively

$U_f\left(h_f(j), \mu(i)C(j), Z, \varepsilon_f(j,i); \theta_f\right)$

$U_m\left(h_m(j), (1-\mu(i))C(j), Z, \varepsilon_m(j,i); \theta_m\right)$

where $\varepsilon_f(j,i)$ and $\varepsilon_m(j,i)$ are random variables measuring unobserved characteristics of the package-household match and θ_f, θ_f are vectors of parameters to be estimated. The corresponding household welfare function is denoted by

$$W\left(U_f\left(h_f(j), \mu(i)C(j), Z, \varepsilon_f(j,i); \theta_f\right), U_m\left(h_m(j), (1-\mu(i))C(j), Z, \varepsilon_m(j,i); \theta_m\right), \varepsilon_W(j,i); \theta_W\right) \equiv \Omega_0(j,i;\theta)$$

where $\varepsilon_W(j,i)$ is a random variable measuring specific unobserved household welfare effects and θ_W is a vector of parameters specific to the household welfare criterion.

We write $P(j,i;\theta)$ to denote the probability that package (j,i) is chosen given the parameters θ. Since we assume μ is not observed, we must work with the marginal probability that job *j* is chosen, i.e. $P(j;\theta) = \sum_i P(j,i;\theta)$.

In what follows we propose various estimation strategies, all of them exploiting the fact that the choices *j* and *i* are generated by the same parameters.

3.1 A general simulated maximum likelihood procedure

The choice probabilities will depend on the joint distribution function of the three random variables $G\left(\varepsilon_f, \varepsilon_m, \varepsilon_W\right)$ and closed-form expressions in general will not be available for computing them. However they can be simulated as follows. For each household:

1) draw (from G) J $\times$ M realizations $\left(\varepsilon_f(1), \varepsilon_m(1), \varepsilon_W(1)\right), \left(\varepsilon_f(2), \varepsilon_m(2), \varepsilon_W(2)\right), \ldots, \left(\varepsilon_f(J\times M), \varepsilon_m(J\times M), \varepsilon_W(J\times M)\right)$;
2) compute $\Omega_0(j,i;\theta),\ j=1,\ldots,J, i=M$;
3) for any given (j,i) , compute $I\left[\Omega_0(j,i;\theta) \geq \Omega_0(t,k;\theta), t=1,\ldots,J, k=1,\ldots,M\right]$, where *I*[*s*] is the indicator function = 1 if *s* is true and = 0 if *s* is false;
4) repeat (1)-(3) R times and let $I_r(j,i;\theta)$ be the result of the r-th repetition at step 3.
5) compute the probability that package (j,i) is chosen as $P(j,i;\theta) = \sum_{r=1}^{R} \frac{1}{R} I_r(j,i;\theta)$;
6) compute the probability that job *j* is chosen as $P(j;\theta) = \sum_{i=1}^{M} P(j,i;\theta)$.

The steps (1) – (6) are embedded in an algorithm where the parameters θ are iteratively updated in order to maximize the sample log-likelihood function. It can be shown that the above procedure based on simulated choice probabilities leads to consistent estimates.

3.2 Special cases

The general procedure sketched in section 3.1 can be implemented in simpler or more special forms depending on specific assumptions on the distribution G or on the utility functions or on the welfare function.

Example 1

Let us suppose the household welfare function can be written as

$$W\left(U_f\left(h_f(j),\mu(i)C(j),Z,\varepsilon_f;\theta_f\right),U_m\left(h_m(j),(1-\mu(i))C(j),Z,\varepsilon_m;\theta_m\right);\theta_W\right)+\varepsilon_W$$
$$\equiv \Omega_1\left(j,i,\varepsilon_f,\varepsilon_m;\theta\right)+\varepsilon_W$$

and ε_W is i.i.d. Type I Extreme Value. Given the values of ε_f and ε_m the probability that package (j,i) is chosen is

$$P(j,i;\theta)=\frac{\exp\left\{\Omega_1\left(j,i,\varepsilon_f(j,i),\varepsilon_m(j,i);\theta\right)\right\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\left\{\Omega_1\left(t,k,\varepsilon_f(t,k),\varepsilon_m(t,k);\theta\right)\right\}}.$$

In order to get the unconditional probability we must integrate out the random variables ε_f and ε_m. To that purpose we can again adopt a simulation approach.

1) Let $G_1\left(\varepsilon_f,\varepsilon_m\right)$ be the joint distribution of ε_f and ε_m. Draw $J\times M$ realizations $\left(\varepsilon_f(1),\varepsilon_m(1)\right),\left(\varepsilon_f(2),\varepsilon_m(2)\right),\dots,\left(\varepsilon_f(J\times M),\varepsilon_m(J\times M)\right)$;
2) Repeat R times step (1) and let $\left(\varepsilon_{fr}(1),\varepsilon_{mr}(1)\right),\left(\varepsilon_{fr}(2),\varepsilon_{mr}(2)\right),\dots,\left(\varepsilon_{fr}(J\times M),\varepsilon_{mr}(J\times M)\right)$ be the r-th repetition;
3) Compute $P(j,i;\theta)=\sum_{r=1}^{R}\frac{1}{R}\frac{\exp\left\{\Omega_1\left(j,i,\varepsilon_{fr}(j,i),\varepsilon_{mr}(j,i);\theta\right)\right\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\left\{\Omega_1\left(t,k,\varepsilon_{fr}(t,k),\varepsilon_{mr}(t,k);\theta\right)\right\}}$
4) Compute the probability that job *j* is chosen as $P(j;\theta)=\sum_{k=1}^{M}P(j,k;\theta)$.

Example 2

Under the same assumptions of Example 1, we can drop the random variables ε_f and ε_m and write

$$W\left(U_f\left(h_f(j),\mu(i)C(j),Z;\theta_f\right),U_m\left(h_m(j),(1-\mu(i))C(j),Z;\theta_m\right);\theta_W\right)+\varepsilon_W\equiv\Omega_2\left(j,i;\theta\right)+\varepsilon_W$$

Then we have

$$P(j,i;\theta)=\frac{\exp\left\{\Omega_2\left(j,i;\theta\right)\right\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\left\{\Omega_2\left(t,k;\theta\right)\right\}}$$

$$P\left(j;\theta\right)=\sum_{i=1}^{M}P(j,i;\theta).$$

Example 3

We can drop the random variable ε_W and write

$$W\left(U_f\left(h_f(j),\mu(i)C(j),Z,\varepsilon_f;\theta_f\right),U_m\left(h_m(j),(1-\mu(i))C(j),Z,\varepsilon_m;\theta_m\right);\theta_W\right)\equiv\Omega_2\left(j,i,\varepsilon_f,\varepsilon_m;\theta\right).$$

1) draw J $\times$ M realizations $\left(\varepsilon_f(1),\varepsilon_m(1)\right),\left(\varepsilon_f(2),\varepsilon_m(2)\right),...,\left(\varepsilon_f(J\times M),\varepsilon_m(J\times M)\right)$;
2) compute $\Omega_2\left(j,i,\varepsilon_f,\varepsilon_m;\theta\right),\ j=1,...,J,i=M$;
3) for any given (j,i) compute

   $I\left[\Omega_2\left(j,i,\varepsilon_f,\varepsilon_m;\theta\right)\geq\Omega_2\left(t,k,\varepsilon_f,\varepsilon_m;\theta\right),t=1,...,J,k=1,...,M\right]$, where *I*[*s*] is the indicator function = 1 if *s* is true and = 0 if *s* is false;
4) repeat (1)-(3) R times and let $I_r(j,i;\theta)$ be the result of the r-th repetition at step 3.
5) compute the probability that package (j,i) is chosen as $P(j,i;\theta)=\sum_{r=1}^{R}\frac{1}{R}I_r(j,i;\theta)$;
6) compute the probability that job *j* is chosen as $P(j;\theta)=\sum_{i=1}^{M}P(j,i;\theta)$.

Example 4

Let us assume

$W = \ln U_f + \pi \ln U_m$

$U_f = u_f \left(h_f(j), \mu(i)C(j), Z; \theta_f \right) \varepsilon_f$

$U_m = u_m \left(h_m(j), (1-\mu(i))C(j), Z; \theta_m \right) \varepsilon_m$

where ε_f and ε_f are i.i.d. Type III Extreme Value. Therefore

$W = \dot{u}_f + \upsilon_f + \pi \dot{u}_m + \pi \upsilon_m$

where

$\dot{u} \equiv \ln(u)$.

Notice that υ_f, υ_m are i.i.d. Type I Extreme Value.

Given a vector $\upsilon_m = \{\upsilon_m(t,k), t = 1,...,J, k = 1,...,M\}$, the probability that package (j,i) is chosen is

$$P(j,i \setminus \upsilon_m) = \frac{\exp\{\dot{u}_f(j,i) + \pi\dot{u}_m(j,i) + \pi\upsilon_m(j,i)\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\{\dot{u}_f(t,k) + \pi\dot{u}_m(t,k) + \pi\upsilon_m(t,k)\}}$$

Letting $f\left(\upsilon_m\right)$ denote the p.d.f. of the vector υ_{ir}^m, we can write the joint probability as $P(j,i,\upsilon_m) = P(j,i \setminus \upsilon_m) f\left(\upsilon_m\right)$. In order to compute the unconditional probability $P\left(j,i\right)$ we must integrate out the random vector υ_m:

$$P(j,i) = \int_{\upsilon_m(1)} \int_{\upsilon_m(2)} \dots \int_{\upsilon_m(J\times M)} \frac{\exp\{\dot{u}_f(j,i) + \pi\dot{u}_m(j,i) + \pi\upsilon_m(j,i)\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\{\dot{u}_f(t,k) + \pi\dot{u}_m(t,k) + \pi\upsilon_m(t,k)\}} f\left(\upsilon_m\right) d\upsilon_m(1) d\upsilon_m(2) \dots d\upsilon_m(J\times M)$$

The above probability can be computed by simulation in the following way. From the Type I extreme value distribution we draw R random vectors $\upsilon_{m1}, \upsilon_{m2}, ..., \upsilon_{mR}$. Then we compute

$$P(j,i) \cong \frac{1}{R}\sum_{r=1}^{R} \frac{\exp\{\dot{u}_f(j,i) + \pi\dot{u}_m(j,i) + \pi\upsilon_{mr}(j,i)\}}{\sum_{k=1}^{M}\sum_{t=1}^{J}\exp\{\dot{u}_f(t,k) + \pi\dot{u}_m(t,k) + \pi\upsilon_{mr}(t,k)\}}.$$

3.3 Indirect Inference

An alternative to simulated maximum likelihood is the so-called Indirect Inference approach (Beninger et al. 2011).

i. Define the auxiliary model (AUX) as some model that is not the true one but we are able to write down the likelihood function for (e.g. the model that assume all agents face a convex budget). Call β the parameters of AUX.

ii. Define the true model (TRUE) as the structural model (e.g. the model also appropriate for non-convex budget sets). Call θ the parameters (to be estimated) of TRUE. Note: we must have $\dim(\beta) \geq \dim(\theta)$.

iii. Estimate AUX on the true data and obtain parameters β_0.

iv. Calibrate the true structural model with parameters $\hat{\theta}_0$ and generate a fake dataset (observed exogenous variables and simulated choices)

v. Estimate the AUX model on fake data and obtain $\hat{\beta}_0$

vi. Iterate steps iv – v to obtain $\left(\hat{\beta}_t, \hat{\theta}_t\right)$

vii. Stop the iterations when $\widehat{\beta}_t$ is close enough to β_0. The corresponding $\widehat{\theta}_t$ is the Indirect Inference estimate of θ. It can be shown to be consistent.

4. Empirical illustration

The empirical exercise illustrated here applies the procedure presented in Example 2 of section 3.2. Household *i* chooses job *j* (=1,…, *J*) and share μ_i in order to maximize:

$$W_i(j,\mu_i;\theta) = \ln V_i^F(j,\mu_i;\theta) + \ln V_i^M(j,\mu_i;\theta) + \varepsilon_i \tag{13}$$

where

$$\begin{aligned}
\ln V_i^F(j,\mu;\theta) &= \delta_F^{'} Z_{Fi}\left(\frac{\left(\mu C_i(j)\right)^{\alpha_F} - 1}{\alpha_F}\right) + \gamma_F^{'} Z_{Fi}\left(\frac{\left(L_F(j)\right)^{\beta_F} - 1}{\beta_F}\right)\\
\ln V_i^M(j,\mu;\theta) &= \delta_M^{'} Z_{Mi}\left(\frac{\left((1-\mu_i)C_i(j)\right)^{\alpha_M} - 1}{\alpha_M}\right) + \gamma_M^{'} Z_{Mi}\left(\frac{\left(L_M(j)\right)^{\beta_M} - 1}{\beta_M}\right)
\end{aligned} \tag{14}$$

$C_i(j)$ = household consumption

$L_g(j) = \dfrac{T - h_g(j)}{T}, g = F, M$

μ_i = the share of household income allocated to the wife. It is not a parameter, it is a choice variable, in principle different for different households.

Z_{si} = vector of characteristics, i.e.:

constant = 1,

ln(age) = natural logarithm of age,

ln(age)*ln(age),

#children 0-5 = number of children of age 0 – 5,

#children 6-10 = number of children of age 6 – 10,

#children 11-16 = number of children of age 11 – 16,

edu2 = 1[high school],

edu3 = 1[Bachelor],

edu4 = 1[Master/Ph.D.],

$\theta = \alpha_f, \beta_f, \delta_f, \gamma_f, \alpha_m, \beta_m, \delta_m, \gamma_m$ = parameters to be estimated.

ε_i = a Type I extreme value distributed random variable.

The estimation exercise uses a sample of 133250 couples drawn from the 2011 Population, Wage, Employment, Income and Wage registers of Statistics Norway.

The complete model is similar to Aaberge and Colombino (2013) and includes the simultaneous estimation of the earnings functions and of the opportunity density function. Here we only focus of the estimates of the preference parameters.

Note that in this example the preferences are separable in leisure and income. As a consequence, μ_i does not depend on the allocation of time (labour supply). Under the separability assumption, a non-structural specification of the sharing rule as a proportion of total household consumption would be rather harmless since the sharing decisions would not change across different tax-transfer regimes. We present all the same this case just as a first illustration of how the procedure works.

If we observed both the chosen job and the chosen share, we could estimate the parameters θ using the simulated probabilities

$$P_i(j,\mu_i;\theta) = \frac{\exp\{W_i(j,\mu_i;\theta)\}}{\sum_m \sum_k \exp\{W_i(k,m;\theta)\}} \tag{15}$$

However, we do not observe μ. Therefore we use

$$P_i(j;\theta) = \frac{\sum_m \exp\{W_i(j,m;\theta)\}}{\sum_m \sum_k \exp\{W_i(k,m;\theta)\}} \tag{16}$$

We adopt the following iterative procedure

a. Using (4) get an estimate of θ, say $\hat{\theta}$.
b. For each household we estimate the chosen value μ_i with

$$\hat{\mu}_i = \sum_\mu \mu \frac{\sum_k \exp\left\{W_i(k,\mu;\hat{\theta})\right\}}{\sum_m \sum_k \exp\left\{W_i(k,m;\hat{\theta})\right\}}$$

c. We re-estimate θ using

$$P_i(j,\hat{\mu}_i;\theta) = \frac{\exp\{W_i(j,\hat{\mu}_i;\theta)\}}{\sum_m \sum_k \exp\{W_i(k,m;\theta)\}} \tag{17}$$

We then iterate steps (a) - (c) until “convergence”.

Tables 1 and 2 show the initial and the final estimates of the preference parameters.

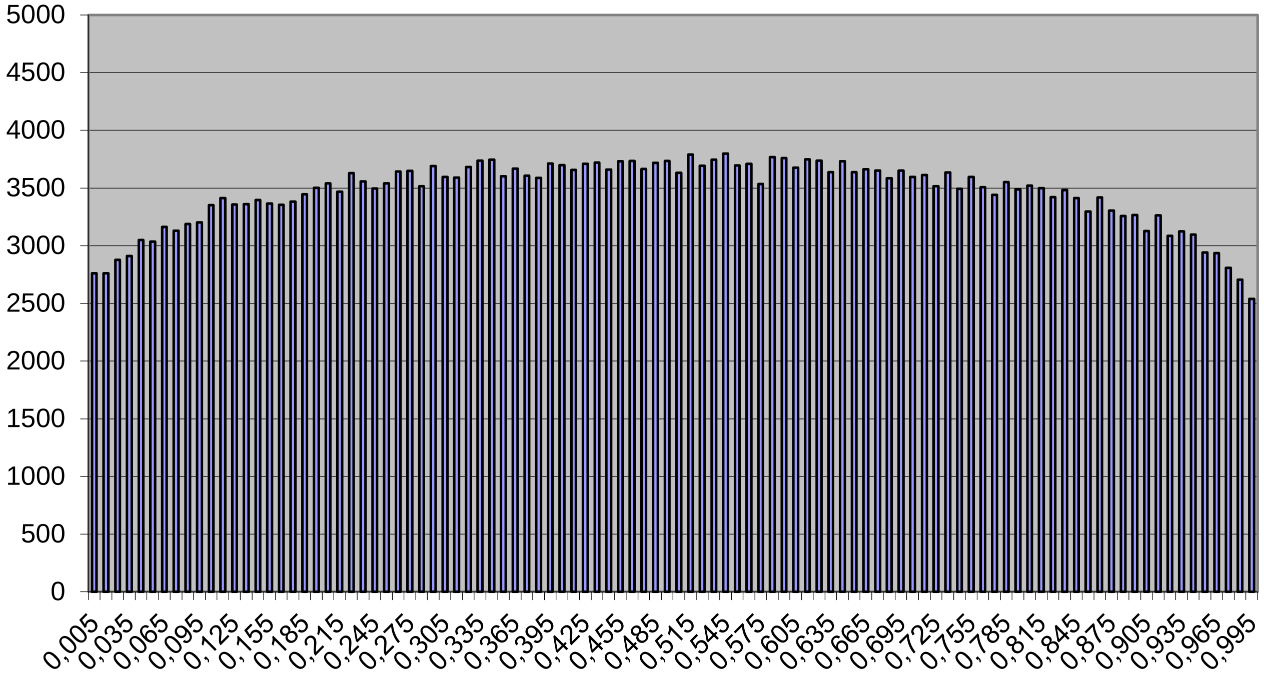
Given the above parameters one can compute the household specific values of μ. Graphs 1 and 2 show the frequency distribution of μ computed respectively with the initial and the final estimates of the preference parameters.

It can be shown that if the utility function is Cobb-Douglas (the limit case of our Box-Cox specification when α and β tend to 0), the exact distribution of μ is of the Beta type (see the Appendix). Although our estimated utility is not Cobb-Douglas (in fact the estimates – reported below – imply a much more pronounced concavity in C and L), the estimated distribution of μ looks close to a symmetric Beta distribution.

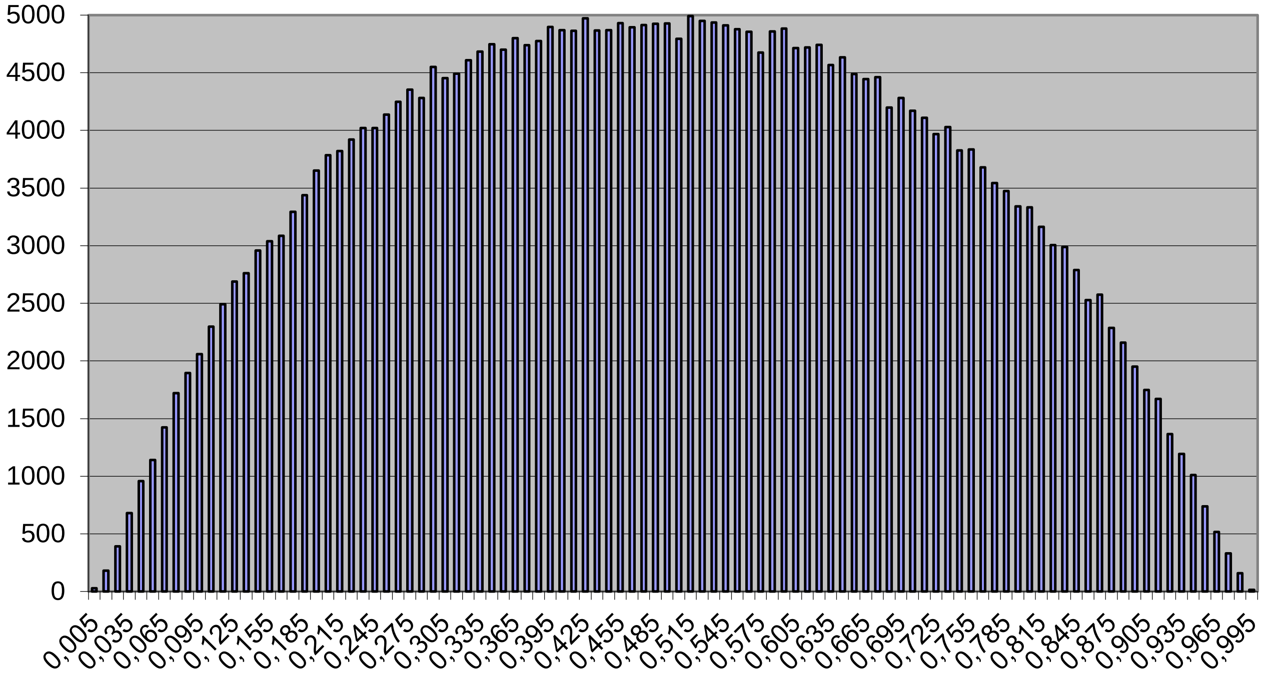
Table 1: Initial estimates				
	Female		Male	
Consumption parameters (α, δ)				
exponent (α)	0.458	36.6	0.444	35.5
constant	-12.611	-18.3	-10.673	-17.2
ln(age)	6.622	17.7	5.622	16.5
ln(age) * ln(age)	-0.857	-17.0	-0.727	-15.5
# children 0-5	0.022	4.6	0.024	5.0
# children 6-10	-0.003	-0.6	-0.001	-0.3
# children 11-16	0.019	4.7	0.014	3.4
edu2	0.056	8.9	0.062	10.4
edu3	0.126	16.1	0.126	18.2
edu4	0.166	16.5	0.174	13.8
Leisure Parameters (β, γ)				
exponent (β)	-9.278	-159.9	-11.928	-112.4
constant	17.402	18.2	0.605	1.9
ln(age)	-9.360	-17.9	-0.359	-2.1
ln(age) * ln(age)	1.314	18.3	0.064	2.7
# children 0-5	0.382	40.0	0.003	1.4
# children 6-10	0.157	21.6	-0.004	-1.9
# childrent 11-16	0.079	13.3	-0.006	-3.4
edu2	-0.299	-24.5	-0.026	-6.7
edu3	-0.490	-34.6	-0.030	-6.4
edu4	-0.618	-32.9	-0.049	-9.2

Table 2: Final estimates				
	Female		Male	
Consumption parameters (α, δ)				
exponent (α)	-0.368223	-46.81	-0.346795	-45.84
constant	-11.950024	-11.97	-10.056958	-10.64
ln(age)	6.579927	12.08	5.630644	10.75
ln(age) * ln(age)	-0.854352	-11.56	-0.730225	-10.12
# children 0-5	0.034956	4.91	0.027288	3.62
# children 6-10	0.008047	1.11	0.001656	0.22
# childrent 11-16	0.034471	5.04	0.037949	5.22
edu2	0.0995	9.96	0.100275	9.97
edu3	0.18887	14.25	0.203056	16.84
edu4	0.243339	13.11	0.304741	11.99
Leisure Parameters (β, γ)				
exponent (β)	-8.342091	-154.21	-10.350287	-114.93
constant	30.408136	23.56	3.235831	5.31
ln(age)	-16.171659	-22.97	-1.735691	-5.23
ln(age) * ln(age)	2.233168	23.23	0.257602	5.71
# children 0-5	0.520265	44.2	0.006801	1.87
# children 6-10	0.216043	23.2	-0.007389	-2.03
# childrent 11-16	0.105623	13.66	-0.015002	-4.33
edu2	-0.422304	-27.01	-0.064305	-8.91
edu3	-0.712491	-39	-0.084462	-9.96
edu4	-0.920317	-36.84	-0.131019	-13.24

Graph 1. Initial distribution of μ



Graph 2. Final distribution of μ



Appendix

Let's consider a special case of Example 2 where the systematic part of the utility function is Cobb-Douglas:

$W = a_f \ln(\mu C) + b_f \ln(L_f) + \pi a_m \ln((1-\mu)C) + \pi b_m \ln(L_m) + \varepsilon$

where ε is extreme value Type I.

Notice that this is a special case of the model of Section 4 (i.e. it is the limit case when the exponents of the Box-Cox terms tend to zero). For simplicity, we consider wage rates fixed across jobs, so we simply write:

$C = C(L_f, L_m)$.

We also assume a uniform opportunity density of hours between 0 and 1. Therefore the choice p.d.f. of $\left(\mu, L_f, L_m\right)$ is

$$P\left(\mu, L_f, L_m\right) = \frac{\exp\left\{a_f \ln(\mu C(L_f, L_m)) + b_f \ln(L_f) + \pi a_m \ln((1-\mu)C(L_f, L_m)) + \pi b_m \ln(L_m)\right\}}{\int_0^1\int_0^T\int_0^T \exp\left\{a_f \ln(xC(l_f, l_m)) + b_f \ln(l_f) + \pi a_m \ln((1-x)C(l_f, l_m)) + \pi b_m \ln(l_m) dx dl_f dl_m\right\}}$$

$$= \frac{\left(\mu^{a_f}(1-\mu)^{\pi a_m}\right) C(L_f, L_m))^{a_f+\pi a_m} L_f^{b_f} L_m^{\pi b_m}}{\left[\int_0^1 \left(x^{a_f}(1-x)^{\pi a_m}\right)dx\right] \int_0^T\int_0^T C(l_f, l_m))^{a_f+\pi a_m} l_f^{b_f} l_m^{\pi b_m} dl_f dl_m}.$$

$$= \frac{\left(\mu^{a_f}(1-\mu)^{\pi a_m}\right)}{\mathrm{B}(a_f+1, \pi a_m+1)} \times \frac{C(L_f, L_m))^{a_f+\pi a_m} L_f^{b_f} L_m^{\pi b_m}}{\int_0^T\int_0^T C(l_f, l_m))^{a_f+\pi a_m} l_f^{b_f} l_m^{\pi b_m} dl_f dl_m}$$

where B(.,.) is the Beta function.

Notice that

$$\frac{\left[\int_0^1 \left(x^{a_f}(1-x)^{\pi a_m}\right)dx\right] C(L_f, L_m))^{a_f+\pi a_m} L_f^{b_f} L_m^{\pi b_m}}{\left[\int_0^1 \left(x^{a_f}(1-x)^{\pi a_m}\right)dx\right] \int_0^T\int_0^T C(l_f, l_m))^{a_f+\pi a_m} l_f^{b_f} l_m^{\pi b_m} dl_f dl_m} = \frac{C(L_f, L_m))^{a_f+\pi a_m} L_f^{b_f} L_m^{\pi b_m}}{\int_0^T\int_0^T C(l_f, l_m))^{a_f+\pi a_m} l_f^{b_f} l_m^{\pi b_m} dl_f dl_m}$$

is the (marginal) p.d.f. $P(L_f, L_m)$. It does not involve μ and therefore it cannot be used to estimate all the parameters (this is not the case, in general, with our model).

The marginal p.d.f. of μ is

$$P(\mu) = \frac{\left(\mu^{a_f}(1-\mu)^{\pi a_m}\right)}{\mathrm{B}(a_f + 1, \pi a_m + 1)}$$

i.e. a Beta p.d.f.

Notice that $E(\mu) = \frac{a_f}{a_f + \pi a_m}$ as in the deterministic case.

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