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ABSTRACT

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This paper studies aggregate dynamics in a cobweb model where learning takes place through a selection mechanism, by which more successful firms are replicated at a higher rate. The structure of the model allows to characterize analytically the aggregate dynamics, and to compute the effect on welfare of alternative levels of selectivity. A central aspect is that greater selectivity, while bringing the distribution of firm types closer to the optimal one at a given date, tends to make the economy less stable at the aggregate level. As in Nelson and Winter (1982), firms differ in their labor/capital ratio. They do not choose it optimally, rather it is a characteristic of a firm. The distribution of firms evolves over time in a way that favors the most profitable firm types. Selection may be inadequate because firms are being selected on the basis of incorrect market signals. Selection itself may reinforce such mispricing, thus generating instability. I compare economies that differ in the volatility and persistence of their productivity shocks, as well as the elasticity of labor supply. The key findings are as follows. First, a trade-off arises since greater selection allows to better track shocks and limits mutational drift in firm types; on the other hand, selection may strengthen cobweb oscillatory dynamics. Second, there seems to be a value in maintaining a diverse "ecology of firms", in order to cope with future shocks. These observations explain the key results. Optimal selectivity is larger, the less "cobweb unstable" the economy, i.e. the more elastic the labor supply. Second, optimal selectivity is larger, the more persistent the aggregate productivity shocks. Finally, optimal selectivity is larger, the lower the variance of productivity innovations. The model can be extended to allow for firm entry and trend productivity growth, and a selection process with memory. Empirical evidence suggests that, in accordance to the model, countries with less regulated product markets exhibit lower aggregate inertia.

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1   Introduction

In most of macroeconomics, agents are considered as sufficiently intelligent to carry all required calculations and compute the rational expectations equilibrium (REE). An important literature, however, questions that assumption and tries to examine the extent to which the economy can "learn" such an equilibrium[1]. In many cases, for example, a reduced form law of motion for the variables of interest is postulated and the agents learn its parameters, typically by using least squares or Bayesian techniques.

This paper asks the following question: how does an economy behave, when learning takes place through a Darwinian selection mechanism by which less profitable firms are eliminated and more profitable ones replicate themselves? Does greater selection systematically bring the economy closer to the rational expectations equilibrium? When it does not, what are the dynamic properties of aggregate fluctuations, and how does welfare depend on the parameters that govern the selection process?

A naive "as if" argument would predict that the more selective the economy, the closer it is to the REE. Yet such an argument overlooks the fact that the market signals that are driving selection need not be the correct ones, because the environment in which selection takes place, as determined by the current value of the shocks and the current distribution of the individual firms' characteristics, is not the same that will apply to the firms that have been selected. Furthermore, selection of a given type of firm perturbs the market signals in such a way that mistakes in the selection process can be reinforced[2].

I study these issues in the context of a simple partial equilibrium economy

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[1]See for example Marcet and Sargent (1989), Evans and Honkahpoja (2001), Marcet and Nicolini (2003).

[2]This issue arises in Saint-Paul (2007), in a model where competing firms set their prices subject to mistakes.

with potential cobweb cycles. As in Nelson and Winter (1982), firms differ in their labor/capital ratio. They do not choose it optimally, rather it is a characteristic of a firm (that is, this behavior is hard wired in their DNA). Firms whose labor/capital ratio is further away from the profit-maximizing one are selected out. As a result the distribution of firms evolves over time in a way that favors the most profitable firm types. A single parameter, called selectivity, captures how stringly the most profitable types are favored.

Which firm type is most profitable depends on current wages and on the current realization of an aggregate productivity shocks. Therefore, selection may be inadequate because different wages and productivity levels will prevail in the future. Furthermore, excess selection may be destabilizing because it may induce a cobweb cycle: when wages are low the most labor-intensive firms are selected, which leads to too high wages and too high labor demand in the subsequent period, where the least labor-intensive firms will be selected, thus intensifying the cycle. Such cycles illustrate that firms are being selected on the basis of incorrect market signals and that selection itself also contributes to this mispricing.

I compare economies that differ in the volatility and persistence of their productivity shocks, as well as the elasticity of labor supply. For each of those economies I can characterize their aggregate dynamics as a function of their degree of selectivity. I can also compute aggregate welfare and the selectivity level which maximizes that welfare.

The key findings are as follows.

First, there is a trade-off due to the fact that, on the one hand, greater selection allows the economy to better track aggregate shocks (as long as they have some persistence) and limits the mutational drift in the cross-sectional variance of firm types; on the other hand, as just pointed out, selection may strengthen cobweb oscillatory dynamics, which leads to increased volatility and potentially unstable dynamics.

Second, there seems to be a value in maintaining a diverse "ecology of firms", because the firm types that will be more adequate in future (uncertain) environments have to be drawn from the pool of existing firms. If selection is too extreme in the current environment, the firms that are best adapted to a given future environmental change, yet performing poorly in present circumstances, will be very scarce, and it will take longer for the economy to produce a large number of such firms in the new environment.

These observations help us to understand the results. We find that

- Optimal selectivity is larger, the less "cobweb unstable" the economy, i.e. the more elastic the labor supply. This is because the more elastic the labor supply, the less distorted the wage signals may be and the more dampened the oscillations of the economy in response to an initial misalignment in wages. Therefore, the less destabilizing a given degree of selectivity will be. Indeed, if labor supply were infinitely elastic, that would pin down wages at their correct social value; they could not be distorted by wrong decisions on the demand side of the labor market.

- Optimal selectivity is larger, the more persistent the aggregate productivity shocks. This is because selection that takes place now affects the distribution of firms in the future. If shocks are more persistent firms that do better today are also more likely to do better in the future, hence selectivity is more valuable.

- Optimal selectivity is larger, the lower the variance of productivity innovations. This is the "biodiversity" effect. When productivity shocks are more volatile, the future is more uncertain and this makes it more valuable to keep a sufficient mass of firms of various types, because it is more likely that one of them will be the optimal one.

The model can also be extended to account for an endogenous capital

stock, economic growth, and a selection process "with memory", i.e. which rewards past performance in addition to just current performance. It is shown that the optimal selectivity level should go up (resp. down) with economic growth, if capital accumulation is unresponsive (resp. responsive) to growth. Also, my numerical simulations indicate that if selectivity is chosen optimally, faster growing economies will tend to have more volatile fluctuations. Finally, memory tends to raise the optimal level of selectivity, because it introduces another mechanism for raising inertia.

While selection is governed by a mechanical process, its parameters can be intuitively related to economic institutions. For example, we may think that greater selectivity is the outcome of more competitive markets or more stringent credit conditions. The results imply that which institutions work best at delivering a sound macroeconomic performance depends on the structure of the shocks and of the productivity growth process faced by the economy. I provide some suggestive evidence that the nexus between institutions, selectivity, and aggregate inertia may be at work in a cross-section of countries: I show that there exists a positive correlation between the ranking of a country in a number of indicators of product market regulation, on the one hand, and its inertia in the aggregate labor/capital ratio, on the other hand. This is consistent with the model provided one assumes that product market regulations reduce selectivity.

2   Related literature

Selection naturally intervenes in all models where some relevant dimension of economic activity is subject to an extensive margin (see for example Jovanovic (1982), Caballero and Hammour (1994), Mélitz (2003)). However, as long as agents optimize, such selection is just a by-product of the existence of nonconvexities and fixed costs. The assumption that firms are rational

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optimizing agents is not adequate to analyze the central role of selection as a mechanism for error correction in a capitalist economy[3]. By contrast in models of bounded rationality and adaptive learning, selection is an essential ingredient of the process by which the economy evolves.

This is not the first paper which studies those issues in the context of the cobweb model. Following the standard results of Ezekiel (1938) and Muth (1961), the literature has analyzed whether the cobweb cycle converges depending on learning processes (Carlson (1969), Bray and Savin (1986)). More recently, Arifovic (1994) has addressed the same issue using genetic algorithms, that is, applying Darwinian selection mechanisms to the learning strategies being used[4]. Her simulations indicate that, in the absence of shocks, the economy generally converges to the rational expectations equilibrium, even though the parameters may be such that it is cobweb-unstable. Franke (1998), building on this work, provides a number of interesting simulations that typically (but not systematically) imply that the economy does not deviate much from the Walrasian equilibrium.

Our simple framework allows us to parametrize selectivity by a single number, derive linear dynamics at the aggregate level, and compute the selectivity parameters that deliver the highest welfare. The price to be paid is that the strategies that the firms follow are fixed (as in Arifovic (1994)) so that the richer adaptive learning strategies of Franke (1998) are not consid-

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[3]As pointed out by Caballero and Hammour (1996), in general entry and exit will not be efficient (as compared to some optimal benchmark) if there are market failures; this is an instance of inadequate selection, but only as a consequence of the market failure.

[4]The Darwinian view according to which firms, rather than optimizing, are characterized by an array of fixed business strategies that one may interpret as their "DNA" is pervasive in the business literature. See for example Marks (2002).

To be sure, the strategies described in such a book are far more complex and qualitative than just deciding one's capital/labor ratio. Nevertheless they exemplify how capitalism is a trial-and-error process through which unprofitable behavioral rules are eventually abandoned, while profitable ones are replicated. This paper is a first step at analyzing the consequences of this trial-and-error process for aggregate dynamics.

ered[5]. Besides showing that greater selectivity reduces inertia and raises the likelihood of unstable aggregate oscillations, this paper's key contribution is to show how the optimal degree of selectivity depends on the economic environment, as defined by (i) the elasticities of supply and demand, (ii) the volatility and persistence of aggregate shocks, (iii) the variance of mutations, and (iv) the trend rate of productivity growth.

3   A simple selection process

There is a continuum of firms of total mass equal to one. Each firm i has one unit of capital. Its production function at date t is

$$y_{it} = A_t l_i^\alpha,$$

where A_t is an aggregate productivity shock. Firms do not optimize, instead their behavior is pinned down by their "type". Here this means that each firm pursues a fixed employment policy l_i. More generally this could stand for various aspects of the "DNA" of a firm, such as managerial practices, etc. Optimization then only takes place indirectly, through the way markets select firms.

At date t, total labor supply is such that

$$w_t = \omega L_t^\gamma,$$

where L_t is total employment and w_t is the wage.

Given w_t, the profit of a firm of type l at date t is given by

$$\pi_t(l) = A_t l^\alpha - w_t l.$$

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[5]Note however that Franke's results themselves, somewhat fascinatingly, show that a large proportion of firms survive following fixed output strategies, despite the other firms pursuing more flexible behavioural rules.

The most profitable firm type, at t, is the one such that

$$l = l_t^* = \left(\frac{\alpha A_t}{w_t} \right)^{\frac{1}{1-\alpha}}.$$

I assume that the distribution of firm type at t is given by $f_t(x)$, where $x = \ln l$. $f_t()$ evolves over time for two reasons. First, firm types are subject to random mutation. Second, selection by markets raises the frequency of the most profitable firms. This selection process is formalized as follows: the greater the distance between a firm type and the most profitable type, the more its frequency is reduced in the following period. More specifically, let $x_t^* = \ln l_t^*$ and let $\sigma_t^2 = Var_t(x)$ be the cross-sectional variance of the distribution of log firm size. At the end of period t, random mutation takes place, so that x is replaced by $x + \varepsilon$, where ε is a random noise with density $h(\varepsilon)$. That is, the distribution of firm types at the end of period t is given by

$$g_t(x) = \int_{-\infty}^{+\infty} f_t(y) h(x - y) dy.$$

After this mutation takes place, selection operates so as to favor the firm types that were closer to the most profitable one[6]. Hence the distribution at the beginning of $t + 1$ is given by

$$f_{t+1}(x) = \frac{g_t(x) \exp(-\delta \frac{(x-x_t^*)^2}{2})}{D_t}, \tag{1}$$

where $D_t = \int g_t(x) \exp(-\delta \frac{(x-x_t^*)^2}{2}) dx$. The parameter δ captures the intensity of selection. At $\delta = 0$ no selection takes place, and the distribution of firms keeps spreading under the influence of random mutations. At $\delta = +\infty$ only firms that have the optimal employment level survive, and next period all firms will have that type.

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[6]Strictly speaking, it would be more rigorous to assume that $f_t(x)$ is altered by a multiplicative factor which is increasing in $\pi_t(e^x)$, but replacing profits by the distance to the optimal employment level is a handy approximation.

The model is silent about how this selection process operates. There are three potential margins: imitation (by either new entrants or existing firms), exit, and growth of the most successful firms. The relative importance of these three margins is irrelevant here.

It is easy to see that if $f_t()$ is normal, i.e. $f_t(x) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp(-\frac{(x-\bar{x}_t)^2}{2\sigma_t^2})$, and if h is normal, that is $h(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_m} \exp(-\frac{\varepsilon^2}{2\sigma_m^2})$, then so is f_{t+1}. Furthermore the mean log employment level then evolves according to

$$\bar{x}_{t+1} = \frac{\bar{x}_t + \delta\left(\sigma_t^2 + \sigma_m^2\right) x_t^*}{1 + \delta\left(\sigma_t^2 + \sigma_m^2\right)}, \tag{2}$$

while the variance of the distribution evolves according to

$$\sigma_{t+1}^2 = \frac{\sigma_t^2 + \sigma_m^2}{1 + \delta\left(\sigma_t^2 + \sigma_m^2\right)}. \tag{3}$$

We are now in a position to solve for the equilibrium of this economy. It will be useful to use the following parameters:

$$\tau = \frac{\gamma}{1 - \alpha}$$

and

$$d = \delta\sigma_m^2.$$

We have that

$$x_t^* = \frac{1}{1 - \alpha}\left(\ln\alpha + a_t - \ln w_t\right), \tag{4}$$

where $a_t = \ln A_t$;

$$\ln L_t = \ln E_t l = E_t \ln l + \frac{\sigma_t^2}{2} = \bar{x}_t + \frac{\sigma_t^2}{2}. \tag{5}$$

This determines the wage at t :

$$\ln w_t = \gamma\left(\bar{x}_t + \frac{\sigma_t^2}{2}\right) + \ln\omega. \tag{6}$$

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Finally, substituting (6) into (4) and then into (2) we get

$$\bar{x}_{t+1} = b_t + \frac{\delta\left(\sigma_t^2 + \sigma_m^2\right)}{(1 + \delta\left(\sigma_t^2 + \sigma_m^2\right))(1 - \alpha)} a_t + \frac{1 - \alpha - \delta\gamma\left(\sigma_t^2 + \sigma_m^2\right)}{(1 + \delta\left(\sigma_t^2 + \sigma_m^2\right))(1 - \alpha)} \bar{x}_t, \quad (7)$$

where

$$b_t = \frac{\delta\left(\sigma_t^2 + \sigma_m^2\right)}{(1 + \delta\left(\sigma_t^2 + \sigma_m^2\right))(1 - \alpha)}\left(\ln\frac{\alpha}{\omega} - \gamma\frac{\sigma_t^2}{2}\right).$$

Equations (7) and (3) characterize the dynamics of the system. While they are non linear, the system is asymptotically univariate and linear. This is because (3) implies that σ_t^2 evolves deterministically and converges monotonically to

$$\sigma_\infty^2 = -\frac{\sigma_m^2}{2} + \frac{\sqrt{\sigma_m^4 + 4\sigma_m^2/\delta}}{2}. \quad (8)$$

This asymptotic cross-sectional dispersion of firms is larger, the greater the "mutation rate" σ_m^2 and the smaller the selectivity parameter δ. It becomes infinite as $\delta \to 0$ and nil as $\delta \to \infty$.

Also, the deterministic (log) employment component b_t converges to

$$b_\infty = \frac{\delta\sigma_\infty^2}{1 - \alpha}\left(\ln\frac{\alpha}{\omega} - \gamma\sigma_\infty^2/2\right) \quad (9)$$

Asymptotically, then, the evolution equation of $\bar{x}$ becomes

$$\bar{x}_{t+1} = b_\infty + \frac{\delta\sigma_\infty^2}{(1 - \alpha)} a_t + \theta\bar{x}_t, \quad (10)$$

where

$$\theta = \frac{\sigma_\infty^2}{\sigma_\infty^2 + \sigma_m^2} - \frac{\delta\gamma\sigma_\infty^2}{1 - \alpha} = \frac{\sqrt{1 + 4/d} - 1 - 2\tau}{1 + \sqrt{1 + 4/d}}. \quad (11)$$

This formula shows the first result of this paper:

PROPOSITION 1 – Assume the stochatic process for a_t is stationary. The dynamics of $\bar{x}_t$ are stable if and only if $\theta > -1$, or equivalently

$$\frac{1}{d} > \frac{\tau^2 - 1}{4}. \quad (12)$$

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Furthermore, if

$$\frac{1}{d} < \tau + \tau^2, \tag{13}$$

then the dynamics are oscillatory, i.e. $\theta < 0$.

PROOF – The AR1 term, $\frac{\sigma_\infty^2}{\sigma_\infty^2 + \sigma_m^2} - \frac{\delta\gamma\sigma_\infty^2}{1-\alpha}$, is always < 1. Dynamics are stable iff it is > -1. This is equivalent to $\delta < \frac{1-\alpha}{\gamma}\left(\frac{1}{\sigma_\infty^2+\sigma_m^2} + \frac{1}{\sigma_\infty^2}\right)$, which by (8) is equivalent to (12). Dynamics are oscillatory iff $\frac{\sigma_\infty^2}{\sigma_\infty^2+\sigma_m^2} - \frac{\delta\gamma\sigma_\infty^2}{1-\alpha} < 0$, i.e. $\delta < \frac{1-\alpha}{\gamma}\frac{1}{\sigma_\infty^2+\sigma_m^2}$, which is equivalent to (13). QED

The case where $\delta = +\infty$ and where there are no shocks delivers the standard cobweb cycle (Ezekiel, 1938): by selecting only the most profitable firm at t, markets set labor demand a $t+1$ at the level that corresponds to wages at t; if wages are high at t, employment is low at $t+1$, and wages are low at $t+1$. It is well known that this cycle converges if and only if $\gamma < 1 - \alpha$, i.e. $\tau < 1$. Indeed, if $\tau < 1$, (12) always holds. In the sequel I will label an eonomy such that the cobweb cycle converges "cobweb-stable". Furthermore, the greater τ, the greater the absolute value of the root of such a cycle. Hence τ is an index of cobweb instability.

As shown by Proposition 1, if the economy is cobweb-stable, then it is even more stable under finite selectivity. Otherwise, it will be stable provided selectivity remains below a given threshold. The economy is more stable, the lower its selectivity and the lower the mutation rate. Therefore, the selectivity threshold below which the economy is stable is lower, the greater the mutation rate. In generating instablility, mutation plays a somewhat similar role as selectivity. A greater mutation rate means that the pool of firms pursuing today's optimal policy rather than yesterday's will be larger, which increases the number of firms that will pursue this policy tomorrow, and therefore the likelihood of instability.

If we interpret, plausibly, economies with a larger δ or a larger mutation

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rate as being more of the "capitalist" kind, then Proposition 1 provides some foundations for the often heard claim that "capitalist economies are inherently instable". This instability comes from the fact that selection takes place on the basis of incorrect prices – that is, on the basis on wages at t, instead of the REE wages at $t+1$ – and by inducing too many firms to adopt the incorrect type, prices next period are made even more incorrect.

Suppose now a central planner wants to choose how capitalist the economy is, i.e. what the optimal value of δ should be. This central planner can be viewed as a metaphor for the outcome of "systems competition" à la Sinn (2003). That is, it is reasonable to assume that economies that do poorly will eventually adopt the institutions of economies that do well. By looking at the optimum, I am silent about the way this macro-selection process operates, as the model is equally silent about how the selectivity parameter δ relates to actual institutions.

Our central planner clearly faces a trade-off: the economy is more stable when δ is lower, on the other hand it takes longer for it to learn the correct price, and it is less reactive to shocks. Finally, a lower value of δ also raises the asymptotic dispersion of firm size because less selective pressure is exerted against mutations. The central planner would want all firms to choose the Walrasian employment level (which also maximizes total surplus) given by $x_t = \frac{1}{1-\alpha} \left(\ln \alpha + a_t - \ln w_t \right)$ and $\ln w_t = \gamma x_t$, that is

$$\tilde{x}_t = \frac{1}{1 - \alpha + \gamma}(\ln \alpha + a_t). \tag{14}$$

I assume that the central planner's welfare is given by

$$E \sum_{t=0}^{+\infty} \beta^t \ln(S_t + \Pi_t),$$

where S_t is the workers surplus and Π_t total profits. It is shown (Appendix 1) that maximizing this expression can be approximated by the following problem:

$$\min E \sum_{t=0}^{+\infty} \beta^t (\sigma_t^2 + (1+\tau)(\bar{x}_t - \tilde{x}_t)^2). \tag{15}$$

I assume that a_t follows an AR1 process

$$a_t = \rho a_{t-1} + \varepsilon_t, \tag{16}$$

with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Furthermore, the central planner acts "asymptotically" by minimizing the long-term welfare flow on the RHS of (15). That is, the government minimizes $\lim_{t\to\infty} \sigma_t^2 + (1+\tau)E(\bar{x}_t - \tilde{x}_t)^2$, which is equal to

$$\mathcal{L} = \sigma_\infty^2 + (1+\tau)((Eu)^2 + Var(u)), \tag{17}$$

where $u_t = \bar{x}_t - \tilde{x}_t$ is the "average log size gap" (ALSG), that is, the difference between the average log size of a firm in our economy and the (common to all firms) log size in the Walrasian economy. The preceding derivations allow us to characterize the law of motion for the ALSG:

$$u_{t+1} = \bar{u} - \frac{1}{1-\alpha+\gamma}\varepsilon_{t+1} + \frac{1-\rho}{1-\alpha+\gamma}a_t + \theta u_t, \tag{18}$$

where

$$\bar{u} = -\frac{\tau\delta}{2}\sigma_\infty^4. \tag{19}$$

From the RHS of (17), we see that the asymptotic losses to the social planner come from three sources:

1. The cross-sectional dispersion of firm size, given by

$$\sigma_\infty^2 = \frac{\sigma_m^2}{2}(-1 + \sqrt{1+4/d}). \tag{20}$$

This is larger, the larger the mutation rate and the smaller the selectivity level.

2. The average output gap, given by

$$Eu = \frac{\bar{u}}{1-\theta}. \tag{21}$$

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This formula tells us that the average mistake made by a firm setting employment in this economy compared to the Walrasian one is larger, the more cobweb-unstable (the larger τ) and the less selective (the lower δ) the economy. Why is that so? The average output gap is negative (meaning firms on average are too small compared to the Walrasian allocation) because log firm size dispersion per se tends to raise aggregate employment due to Jensen's inequality[7]. This tends to raise wages above the Walrasian level, which reduces average log firm employment. This effect is stronger, the greater the reaction of wages to employment, i.e. the greater the instability τ, and the greater the asymptotic cross sectional variance of log employment, i.e. the lower the selectivity parameter d.

3. The volatility of the ALSG, given by

$$Var(u) = \frac{\sigma_\varepsilon^2}{(1 - \alpha + \gamma)^2} \frac{2}{(1 + \rho)(1 + \theta)(1 - \rho\theta)}. \tag{22}$$

3.0.1 The effect of ρ and τ

Table 1 reports the optimal value of d as a function of ρ and τ for a typical set of simulations. The other parameters have been set to $\alpha = 0.5$, $\sigma_\varepsilon^2 = 0.02$ and $\sigma_m^2 = 0.05$. The optimal value of d is driven by the trade-off between long-term cross-sectional dispersion in firms' type and aggregate stability of employment dynamics. Two key patterns emerge.

First, optimal selectivity is larger, the more persistent the productivity shocks. This is because the more persistent those shocks, the more the most profitable firms at date t are likely to be the optimal firm type in the subsequent periods. Note that even if the shocks are not persistent some selectivity

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[7]By the same token, log firm size dispersion would tend to push aggregate employment above its Walrasian level. But we have proved that the social planner's objective can be expressed as a function of the cross-sectional deviation of log firm size and the absolute value of the output gap. Thus taking those into account we can ignore employment.

is optimal, since otherwise mutations would accumulate and the dispersion of firm types would become infinite, which is inefficient. Furthermore, as long as it is not strong enough to generate instability, some selectivity brings the firm types in line with the correct ones on average.

Second, the greater τ, i.e. the more cobweb-unstable the economy, the lower the selectivity level. The greater τ, the more wages react to a deviation in aggregate employment, and the greater the welfare loss in subsequent periods from picking the wrong firm type at a given date. Since selection never operates on the basis of the correct market signals, one is more willing to mitigate it when τ is larger. We can also note that when γ is large, selectivity is very inelastic to the degree of persistence in productivity shocks.

$\rho\backslash\gamma$	0	0.2	0.5	1	2	5
0.01	1.86	1.08	0.61	0.32	0.14	0.03
0.2	3.76	1.63	0.79	0.37	0.15	0.03
0.4	13.3	2.57	1.00	0.43	0.15	0.03
0.6	99	4.26	1.27	0.47	0.16	0.03
0.8	99	6.69	1.5	0.52	0.18	0.03
0.99	99	11.5	1.7	0.56	0.18	0.03

Table 1 – Optimal value of d as a function of ρ and γ, $\alpha = 0.5$, $\sigma_\varepsilon^2 = 0.02$, $\sigma_m^2 = 0.05$.

Table 2a reports the corresponding value of θ, the AR1 coefficient in aggregate employment dynamics. Note that this is not the autocorrelation in $\bar{x}_t$, but instead the part of it that is induced by selectivity (the other part comes from the autocorrelation in the shocks a). Under $\delta = +\infty$ it would be equal to $-\tau$. As Table 2a makes clear, this contribution is most of the time negative. Table 2b reports the autocorrelation of average log employment $\bar{x}_t$, which is equal to $\frac{\rho+\theta}{1+\rho\theta}$.[8] We see that in some cases it is optimal to pick d so as

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[8]This is due to the fact that if $\hat{x}_t = \bar{x}_t - E\bar{x}$, then $\hat{x}_{t+1} = \frac{d}{1-\alpha}\hat{a}_t + \theta\hat{x}_t$, so that $Var(\hat{x}_t) = \frac{d^2(1+\rho\theta)\sigma_\varepsilon^2}{(1-\alpha)^2(1-\rho)(1-\theta^2)}$ and $E\hat{x}_t\hat{x}_{t+1} = \theta Var(\hat{x}_t) + \frac{d^2\rho\sigma_\varepsilon^2}{(1-\alpha)^2(1-\rho)(1-\rho\theta)}$.

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to allow for negative autocorrelation. This is the case for ρ not too large and γ large. In other cases the optimal choice of d will increase the persistence of employment relative to its Walrasian counterpart (where it is simply equal to ρ). Note also that empirically observing a positive autocorrelation, i.e. $\frac{\rho+\theta}{1+\rho\theta} > 0$, does not preclude the endogenous dynamics of employment to be oscillatory, i.e. $\theta < 0$.

$\rho\backslash\gamma$	0	0.2	0.5	1	2	5
0.01	0.28	0.12	-0.07	-0.28	-0.54	-0.77
0.2	0.18	0.02	-0.15	-0.35	-0.59	-0.77
0.4	0.07	-0.07	-0.24	-0.42	-0.59	-0.77
0.6	0.01	-0.17	-0.32	-0.47	-0.65	-0.77
0.8	0.01	-0.24	-0.38	-0.51	-0.71	-0.77
0.99	0.01	-0.30	-0.41	-0.56	-0.71	-0.77

Table 2a – Persistence parameter θ at the optimal choice of d. Same parameters as Table 1.

$\rho\backslash\gamma$	0	0.2	0.5	1	2	5
0.01	0.29	0.13	-0.06	-0.27	-0.53	-0.77
0.2	0.37	0.22	0.05	0.16	-0.45	-0.68
0.4	0.45	0.33	0.18	0.03	-0.26	-0.54
0.6	0.61	0.47	0.35	0.18	-0.08	-0.32
0.8	0.8	0.70	0.61	0.48	0.22	0.07
0.99	0.99	0.98	0.98	0.97	0.94	0.92

Table 2b – Employment autocorrelation at the optimal choice of d. Same parameters as Table 1

Table 3 reports the corresponding asymptotic cross-sectional dispersion of firm size, σ_∞, while Table 4 reports the average ALSG. Given (20), the interpretation of Table 3 is straightforward. Table 4 confirms the negative effects of firm size dispersion which is discussed above. As selectivity reduces dispersion, it also reduces the size of the ALSG.

$\rho\backslash\gamma$	0	0.2	0.5	1	2	5
0.01	0.019	0.03	0.044	0.067	0.11	0.26
0.2	0.011	0.02	0.037	0.061	0.11	0.26
0.4	0.003	0.01	0.031	0.055	0.11	0.26
0.6	0.0005	0.009	0.026	0.052	0.10	0.26
0.8	0.0005	0.007	0.023	0.049	0.097	0.26
0.99	0.0005	0.004	0.021	0.046	0.097	0.26

Table 3 – Cross-sectional log employment variance at the optimal choice of d. Same parameters as Table 1

$\rho\backslash\gamma$	0	0.2	0.5	1	2	5
0.01	0	-0.4	-1.1	-2.2	-4.5	-11.8
0.2	0	-0.3	-0.9	-2.0	-4.3	-11.8
0.4	0	-0.2	-0.8	-1.8	-4.3	-11.8
0.6	0	-0.1	-0.6	-1.7	-4.0	-11.8
0.8	0	-0.1	-0.6	-1.6	-3.9	-11.8
0.99	0	-0.05	-0.5	-1.5	-3.9	-11.8

Table 4 – Average ALSG (%) at the optimal choice of d. Same parameters as Table 1

3.0.2 The effect of σ_m^2 and σ_ε^2.

How do the variances of mutations and aggregate shocks affect the optimal selectivity level? We can again answer this question by running numerical simulations. Also, in some special cases we can get an analytical solution.

For $\rho = \tau = 0$ we have that

$$Eu = 0$$

and

$$Var(u) = \frac{\sigma_\varepsilon^2}{(1-\alpha)^2}\frac{2}{1+\theta},$$

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while it is always true that
$$\sigma_\infty^2 = \frac{\theta\sigma_m^2}{1-\theta}.$$

Therefore, one replace the choice variable by θ and the asymptotic objective is
$$\min_\theta \frac{\theta\sigma_m^2}{2(1-\theta)} + \frac{2\sigma_\varepsilon^2}{(1-\alpha^2)(1+\theta)}.$$

The optimal value of θ is
$$\theta = \frac{2\sigma_\varepsilon - (1-\alpha)\sigma_m}{2\sigma_\varepsilon + (1-\alpha)\sigma_m}$$

and the corresponding value of d is[9]

$$
\begin{aligned}
d &= \left(\frac{\sigma_\varepsilon^2}{(1-\alpha)^2\sigma_m^2} - \frac{1}{4}\right)^{-1} \text{ if } 2\sigma_\varepsilon > (1-\alpha)\sigma_m \\
&= +\infty \text{ if } 2\sigma_\varepsilon \leq (1-\alpha)\sigma_m.
\end{aligned}
$$

This expression suggests that optimal selectivity is a decreasing function of the $\sigma_\varepsilon/\sigma_m$ ratio. The greater the variance of productivity shocks, the greater the "biodiversity" value of maintaining a large enough pool of firms at various employment level, in order to better react to future "changes in the environment", i.e. productivity shocks that call for a change in the optimal firm size. One can also show, based on the preceding formula, that $d\delta/d\sigma_m^2 > 0$. Thus selectivity clearly goes up with the variance of mutational shocks. Bigger mutations raise the long-term cross sectional dispersion of firm size, which induces an increase in selectivity so as to limit the associated welfare losses.

Are these results confirmed in more general parameter configurations? To answer that question we again have to use numerical simulations. Figure 1 shows the evolution of d as a function of σ_ε^2 for $\alpha = 0.5$ and twelve different

---

[9]As $\theta = \frac{\sqrt{1+4/d}-1}{1+\sqrt{1+4/d}}$, if the optimal θ is negative, it cannot be reached by picking d. I such cases, one cannot to better than setting $d = +\infty$.

set of parameter values for σ_m^2, ρ and γ. These values are reported in Table 6.

Simul. #	σ_m^2	ρ	γ
1	0.05	0.2	0.2
2	0.05	0.2	0.5
3	0.05	0.2	0.8
4	0.05	0.8	0.2
5	0.05	0.8	0.5
6	0.05	0.8	0.8
7	0.1	0.2	0.2
8	0.1	0.2	0.5
9	0.1	0.2	0.8
10	0.1	0.8	0.2
11	0.1	0.8	0.5
12	0.1	0.8	0.8

Table 6 – Parameter values for Figure 1

As Figure 1 reports an inverse measure of d $(1/(1+d))$, it confirms our intuition that selectivity falls with the variance of productivity shocks.

Figure 2 reports the evolution of δ as a function of σ_m^2. The corresponding parameter values are reported on Table 7. We see that selectivity goes up with σ_m^2 for low values of γ but falls with σ_m^2 for large values of γ.

Simul. #	σ_ε^2	ρ	γ
1	0.02	0.2	0.2
2	0.02	0.2	0.5
3	0.02	0.2	0.8
4	0.02	0.8	0.2
5	0.02	0.8	0.5
6	0.02	0.8	0.8
7	0.04	0.2	0.2
8	0.04	0.2	0.5
9	0.04	0.2	0.8
10	0.04	0.8	0.2
11	0.04	0.8	0.5
12	0.04	0.8	0.8

Table 7 – Parameter values for Figure 2

How can we explain this pattern? For a given δ, an increase in the size of mutations makes the cross-sectional distribution of firm size less persistent, which in turn reduces the persistence parameter θ. For $\theta < 0$ this makes aggregate dynamics more unstable, with the associated welfare losses captured by (22). This effect is stronger, the more negative θ, i.e. the greater τ. To offset it, one must pick a lower value of δ. This effect dominates for τ large enough, explaining the negative dependence of δ on σ_m^2. On the other hand, for τ low, the dynamics are unlikely to be oscillatory, and the contribution of σ_∞ dominates: selectivity goes up when mutations are larger, to offset the increase in the long run cross-sectional dispersion of firm size.

4   Extensions

4.1   Endogenous number of firms and the effect of growth

In the above model, the number of firms is fixed. In this section I make it more general by endogenizing the number of firms. This extension will then prove convenient to analyze the consequences of the introduction of an

exogenous growth trend in total factor productivity.

I assume that a fraction s of total output is saved (as in the Solow (1956) model), and that these savings are used to accumulate capital, which means more firms here. But instead of a linear relationship between savings and capital accumulation, I assume that it is non linear and subject to decreasing returns to scale, so that the following relationship holds:

$$K_{t+1} = \beta Y_t^{\psi}, \tag{23}$$

with $\psi \in [0,1]$. In general β should depend on s, but this dependency is immaterial, and we ignore it here. The above model is a special case for $s = 0, \beta = 1$ and $\psi = 0$. The ψ parameter captures how sensitive the number of firms is to the output level. The evolution of the distribution of firm size is unchanged from the previous analysis, meaning that the RHS of (1) drives the distribution of firm size among new entrants as well as survivors.

We clearly have that $Y_t = A_t K_t E_t l_i^{\alpha}$. Taking logs and denoting by $y_t = \ln Y_t$, this clearly implies that

$$y_t = a_t + k_t + \alpha \bar{x}_t + \alpha^2 \frac{\sigma_t^2}{2}. \tag{24}$$

Similarly, $L_t = K_t E_t l_i$ and therefore

$$\ln L_t = k_t + \bar{x}_t + \frac{\sigma_t^2}{2}.$$

This equation allows us to derive the equilibrium wage and profit-maximizing log firm size, that are respectively equal to

$$\ln w_t = \ln \omega + \gamma (k_t + \bar{x}_t + \frac{\sigma_t^2}{2})$$

and

$$x_t^* = \frac{1}{1-\alpha} (\ln \frac{\alpha}{\omega} + a_t - \gamma (k_t + \bar{x}_t + \frac{\sigma_t^2}{2})).$$

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Using the same derivations as in Section 3 we get the aymptotic law of motion for $\bar{x}_t$:

$$\bar{x}_{t+1} = (1 - \delta\sigma_\infty^2(1+\tau))\bar{x}_t - \tau\delta\sigma_\infty^2 k_t + \frac{\delta\sigma_\infty^2}{1-\alpha}a_t + b_\infty, \qquad (25)$$

where b_∞ is defined by (9).

Using (23) we also get an evolution equation for $k_t = \ln K_t$:

$$k_{t+1} = \psi a_t + \psi k_t + \psi\alpha\bar{x}_t + c_\infty, \qquad (26)$$

where
$$c_\infty = \ln\beta + \frac{\psi\alpha^2\sigma_\infty^2}{2}.$$

Proposition 2 characterizes the stability properties of the dynamical system (25-26), thus extending Proposition 1.

Proposition 2 – The dynamical system (25-26) is stable if and only if

$$\frac{1}{d} > \frac{\left(\frac{\tau+\gamma\psi}{1+\psi}\right)^2 - 1}{4}. \qquad (27)$$

Proof – See Appendix

We note that (27) is more likely to hold, the greater ψ : the more entry is sensitive to economic activity, the more stable the economy. Capital accumulation raises inertia, making it less likely that the economy oscillates.

The preceding extension can be applied to an economy where TFP a_t has a deterministic growth trend. More specifically, I assume that $a_t = a_{Ct} + gt$, where g is the trend growth rate and a_{Ct} the cyclical component. I assume that the law of motion for a_{Ct} is given by the same AR1 process as above, i.e. eq. (16).

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We want to know how the optimal selectivity depends on growth; we also want to know how this dependence affects the economy's cyclical properties. For this we first need to extend the welfare criterion derived in (17) to the case with an endogenous capital stock. This is done in the Appendix. Then, in order to compute social welfare, one needs to compute the stochastic steady state moments of the relevant vector $(k_t, \bar{x}_t, a_t)$. The relevant formulas are also derived in the Appendix. One can then use those formulas to compute the welfare-maximizing level of d. The next table shows how it depends on g and ψ. The other parameters were $\alpha = 0.5, \beta = 1, s = 0.5, \gamma = 0.5, \omega = 1, \rho = 0.01, \sigma_\varepsilon^2 = 0.02$ and $\sigma_m^2 = 0.05$.

$g \backslash \psi$	0	0.2	0.5	0.7	0.9	1	2/3
0	0.61	0.54	0.47	0.45	0.45	0.47	0.45
0.02	0.61	0.59	0.56	0.43	0.01	0.01	0.45
0.05	0.67	0.67	0.61	0.39	0.01	0.01	0.45
0.1	0.79	0.79	0.72	0.32	0.01	0.01	0.45
0.15	0.96	0.96	0.85	0.25	0.01	0.01	0.45
0.2	1.13	1.13	1.0	0.19	0.01	0.01	0.45

Table 8 – Optimal d as a function of g and ψ

To interpret Table 8, one should first note that the deterministic growth component of the average log firm size is equal to (see Appendix)

$$g_x = g \frac{1 - \psi(1 + \tau(1 - \alpha))}{(1 - \alpha)(1 + \tau - \psi(1 + \tau(1 - \alpha)))}.$$

This component is the same in our equilibrium and in the Walrasian one: our economy does not diverge from its Walrasian counterpart over time.

If $\psi < \frac{1}{1 + \tau(1 - \alpha)}$, g_x is positive and grows with g : the capital stock is not responsive enough to trend growth for the size distribution of firms to remain constant. Firm size grows over time. We have seen in Table 4 that firm size is typically smaller, on average, than the Walrasian benchmark. Since the distribution of firm size that produces at $t + 1$ is selected from the pool of

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firms at date t, an increase in g tends to widen the gap between the average firm size at t and its Walrasian counterpart. This raises the gain to the social planner of being more selective, because greater selectivity reduces the ALSG Eu in absolute value, as illustrated by (19) and (21).

For $\psi > \frac{1}{1+\tau(1-\alpha)}$, however, entry "overshoots" the level that would deliver a constant distribution of $\bar{x}$ through time. As a result, the average firm size shrinks over time, more so, the greater g. The lag in firm selection now tends to offset the fact that the ALSG is smaller in our equilibrium than in the Walrasian one. This offsetting factor is stronger, the greater g. Therefore, as g goes up, the ALSG shrinks in absolute value, which reduces the benefits of selectivity.

Finally, for $\psi = \frac{1}{1+\tau(1-\alpha)}$, both our economy and the Walrasian one are in a balanced growth path where the cross-sectional distribution of firm size (which is degenerate in the Walrasian case) has no deterministic trend. As a result the growth rate has no impact on the ALSG and consequently no impact on the optimal selectivity level. For our parameter values this corresponds to $\psi = 2/3$ and is reported in the last column of Table 8.

$g \backslash \psi$	0	0.2	0.5	0.7	0.9	1	2/3
0	0.006	0.010	0.023	0.039	0.072	0.103	0.035
0.02	0.006	0.011	0.023	0.039	0.076	0.159	0.035
0.05	0.006	0.011	0.024	0.039	0.076	0.159	0.035
0.1	0.007	0.012	0.025	0.038	0.076	0.159	0.035
0.15	0.008	0.013	0.026	0.037	0.076	0.159	0.035
0.2	0.009	0.014	0.026	0.036	0.076	0.159	0.035

Table 9 – Output volatility as a function of g and ψ

Table 9 reports the corresponding volatility of output. For $\psi < \frac{1}{1+\tau(1-\alpha)} = 2/3$, selectivity grows with g, and we observe that this is associated with an increase in volatility. For $\psi = 0$ this is due to the increase in the absolute value of the negative root, i.e. an amplification of the cobweb cycle, as

discussed above. For $\psi > 0$ the roots are complex and the results are more difficult to interpret. Overall, though, the results suggest that economies that grow faster should be more volatile, although this is not systematic (see column 5).

4.2   Selection with memory

I now study an extension of the model where selection takes place on the basis of current and past profitability, instead of current profitability alone. Intuitively, this means that investors have a "long memory" and that the firms that are selected for are those that have a greater profit over some period of time[10]. A natural way to formalize this is to assume that instead of (1) we have

$$f_{t+1}(x) = \frac{g_t(x) \exp(-\delta \frac{(x-\hat{x}_t)^2}{2})}{D_t},\tag{28}$$

,where the most selected type is a geometric weighted average of current and past most profitable types:

$$
\begin{aligned}
\hat{x}_t &= \lambda \hat{x}_{t-1} + (1-\lambda)x_t^* \\
&= \lambda \hat{x}_{t-1} + \frac{1-\lambda}{1-\alpha}\left(\ln \alpha + a_t - \ln w_t\right).
\end{aligned}
$$

λ is a parameter which captures the memory, or horizon, of the selection process. It is obvious that (3) is unchanged and that in (2) x_t^* has to be replaced by $\hat{x}_t$. Consequently, asymptotically we end up with a two dimensional system which evolves as

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[10]Keeping track of memory can be implemented in the economy through institutions such as money, or more generally, wealth. See Kocherlakota (1998).

$$\bar{x}_{t+1} = \left(\frac{-1 + \sqrt{1 + 4/d}}{1 + \sqrt{1 + 4/d}} \right) \bar{x}_t + \frac{d}{2}(-1 + \sqrt{1 + 4/d})\hat{x}_t$$

$$\hat{x}_{t+1} = \left(\lambda - \frac{(1-\lambda)\tau d}{2}(-1 + \sqrt{1 + 4/d}) \right) \hat{x}_t - \tau(1 - \lambda) \left(\frac{-1 + \sqrt{1 + 4/d}}{1 + \sqrt{1 + 4/d}} \right) \bar{x}_t$$

$$- \frac{\tau(1 - \lambda)\sigma_m^2}{4}(-1 + \sqrt{1 + 4/d}) + \frac{1 - \lambda}{1 - \alpha} (\ln \alpha + a_{t+1}) \, ;$$

We can already analyze how the stability of the system is affected by the selection process parameters (δ, λ). The following proposition can be proved.

Proposition 3 – A sufficient condition for the system to be stable is

$$\lambda > \frac{\tau - \sqrt{1 + 4/d}}{\tau + \sqrt{1 + 4/d}}. \tag{29}$$

PROOF – See Appendix 2.

We note that this condition is satisfied for any λ provided $\tau < \sqrt{1 + 4/d}$, which is equivalent to condition (12). Condition (12) is associated with the special case $\lambda = 0$. Intuitively, the longer the memory of the selection process, the less likely it is that the economy will oscillate and the more likely it is to be stable[11]. A longer memory thus enhances the range of selectivity values compatible with stability–the trade-off between selectivity and inertia is eased. As (29) makes clear, the maximum level of selectivity consistent with stability becomes infinite as λ converges to 1.

---

[11]From the Proof of Proposition 3, we can partition the (δ, λ) plane in various regions. Negative roots prevail for high values of δ and low values of λ. If the economy is cobweb unstable, there is a region where δ is too large and λ too low for these oscillations to be stable. But there is also a region where they are dampened. For λ larger and δ smaller, the roots are positive and the economy is always stable. Finally there is a region where the roots are complex, and in this case the economy is again stable, with dampened oscillations at a frequency lower than a 2-cycle.

It is then natural to expect that the optimal selectivity level will depend positively on the length of memory λ. The following Table reports simulations of the optimal d for different values of λ and γ, and it confirms this intuition.[12]

$\lambda\backslash\gamma$	0	0.2	0.5	1	2	5
0.0	1.86	1.08	0.61	0.32	0.14	0.03
0.2	5.25	2.33	1.13	0.56	0.25	0.06
0.4	99	13.3	3.17	1.28	0.52	0.14
0.6	99	99	99	6.69	1.56	0.35
0.8	99	99	99	99	99	2.125
0.99	99	99	99	99	99	99

Table 10 – Optimal selectivity d as a function of λ and γ, $\alpha = 0.5$, $\sigma_m^2 = 0.05$, $\sigma_\varepsilon^2 = 0.5$, $\rho = 0.01$

5 An empirical illustration

While the preceding discussion obviously rests on a very specific model, which ignores many features of an economy as well as many dimensions of a firm's characteristics upon which selection may take place, it is interesting to illustrate empirically one key proposition emerging from the above analysis, namely that there is a negative relationship between the intensity of selection and the degree of inertia at the macroeconomic level.

While how to measure selection in a real economy is open to discussion, I am more particularly interested in documenting the view that some institutions such as barriers to entry, lack of investor protection, etc, are likely to reduce the selection level. If the above analysis is correct, we should expect countries with more regulated markets to exhibit a higher level of macroeconomic inertia.

My strategy is to use the Penn World Table to estimate a simple version

---

[12]How to compute the welfare criterion is described in the Appendix.

of Equation (2) for a number of countries[13], and correlate the implied inertia in x (measured as the labor/capital ratio from the PWT) with some indices of product market regulation from the OECD product market regulation database. I remain relatively agnostic regarding the determinants of the preceding period's optimal value of x, and I proxy it by a set of three variables: TFP, the real wage, and the real exchange rate. The coefficient of interest is the degree of inertia, i.e. the coefficient on the lagged value of x. According to the model, it should be equal to

$$\theta' = \frac{1}{1 + \delta \left(\sigma_\infty^2 + \sigma_m^2 \right)} = \frac{\sqrt{1 + 4/d} - 1}{\sqrt{1 + 4/d} + 1}.$$

This is clearly decreasing from 1 to 0 as d goes up from zero to infinity.

I then rank the countries by this estimated persistence coefficient (hence more highly ranked countries are interpreted as being more selective) and correlate this with their rank in a number of OECD indices of product market regulation. The results are reported in Table 11.

Indicator	Rank correlation with θ'
State control	0.22 (0.1)
Barriers to entrepreneurship	0.24 (0.11)
Barriers to international trade	-0.15 (0.11)
Regulation of legal professions	0.14 (0.11)
Regulation of retail trade	0.35 (0.1)
Regulation of energy, transport and communication	0.28 (0.11)

Table 11 – Mean rank correlation between estimated inertia and OECD indicators of product market correlation. Standard deviations in parentheses. Both means and standard deviations estimated with bootstrap simulations to account for the sampling error in the estimation of θ'.

We see that the rank correlation, although not very large is generally positive, reaching 0.35 for retail trade. The only indicator that exhibits a

[13]See the Appendix for the actual specification and the estimated values for inertia.

negative correlation with inertia is that of barriers to international trade. However while this type of regulation forces domestic prices to deviate from international prices, there is no reason to believe that it would reduce the severity of selection among domestic firms, contrary to the other indicators. That this indicator, unlike the others, is not positively correlated with the degree of inertia, therefore comes as no surprise.

While these results hardly constitute a proof of the model (we can think of mechanisms other than selection through which regulation would affect inertia), they are consistent with its general message that there exists a link between selectivity at the micro level and inertia at the macro level.

6 Conclusion

In this paper I have developed a model that allows us to study the interactions between the intensity of selection at the microeconomic level and aggregate dynamics, as well as to discuss what level of selectivity is most desirable depending on the economic environment. This exercise has its limits because it assumes a mechanical rule for the evolution of the cross-sectional distribution of firm characteristics. This is the price to be paid for analytical transparency. An important direction for future research consists in providing foundations for the selection mechanism based on real world institutions such as the rules governing bankruptcies and corporate governance, as well as labor and product market regulations.

7 Appendix

7.1 Derivation of (15).

The workers' surplus is equal to

$$S_t = w_t L_t - \omega_t \frac{L_t^{1+\gamma}}{1+\gamma}.$$

Therefore

$$\Pi_t + S_t = Y_t - \omega_t \frac{L_t^{1+\gamma}}{1+\gamma}. \tag{30}$$

Furthermore

$$Y_t = A_t \int_{-\infty}^{+\infty} f_t(x) e^{\alpha x} dx.$$

Therefore

$$\begin{aligned} \ln Y_t &= a_t + \ln E l^\alpha \\ &= a_t + \alpha \bar{x}_t + \alpha^2 \frac{\sigma_t^2}{2}. \end{aligned}$$

Similarly

$$\ln L_t = \bar{x}_t + \sigma_t^2/2.$$

For convenience we rewrite (14):

$$\tilde{x}_t = \frac{1}{1-\alpha+\gamma}(\ln \alpha + a_t - z_t).$$

Using this and the preceding derivations, (30) can be rewritten as

$$\begin{aligned} \Pi_t + S_t &= A_t^{\frac{1+\gamma}{1-\alpha+\gamma}} \omega_t^{-\frac{\alpha}{1-\alpha+\gamma}} \alpha^{\frac{\alpha}{1-\alpha+\gamma}} \exp(\alpha(\bar{x}_t - \tilde{x}_t) + \alpha^2 \frac{\sigma_t^2}{2}) \\ &\quad - A_t^{\frac{1+\gamma}{1-\alpha+\gamma}} \omega_t^{-\frac{\alpha}{1-\alpha+\gamma}} \frac{\alpha^{\frac{1+\gamma}{1-\alpha+\gamma}}}{1+\gamma} \exp((1+\gamma)(\bar{x}_t - \tilde{x}_t + \frac{\sigma_t^2}{2})). \end{aligned}$$

This is equivalent to

$$
\begin{aligned}
\ln\left(\Pi_t + S_t\right) &= \frac{1+\gamma}{1-\alpha+\gamma}a_t - \frac{\alpha}{1-\alpha+\gamma} + \frac{\alpha}{1-\alpha+\gamma}\ln\alpha \\
&+ \phi(\bar{x}_t - \tilde{x}_t, \frac{\sigma_t^2}{2}),
\end{aligned}
$$

where

$$
\phi(u,v) = \ln\left[\exp(\alpha u + \alpha^2 v) - \frac{\alpha}{1+\gamma}\exp((1+\gamma)(u+v))\right].
$$

We note that for $v \geq 0$, $\phi(u,v) \leq \phi(0,0) = \ln(1 - \frac{\alpha}{1+\gamma})$, that $\phi'_u(0,0) = 0$, $\phi'_v(0,0) = -\frac{\alpha(1-\alpha)}{1-\frac{\alpha}{1+\gamma}} < 0$, and $\phi''_{u,u}(0,0) = -\frac{\alpha(1+\gamma)-\alpha^2}{1-\frac{\alpha}{1+\gamma}}$. Hence for $u, v << 1$

$$
\phi(u,v) \approx \ln(1 - \frac{\alpha}{1+\gamma}) - \frac{\alpha(1-\alpha)}{1-\frac{\alpha}{1+\gamma}}\left[v + (1+\tau)u^2/2\right].
$$

The other terms of the Taylor expansion are all negligible relative to either u or v. Maximization of $E\phi$ is therefore equivalent to minimizing $E(v + (1+\tau)u^2/2) = \frac{1}{2}(\sigma_\infty^2 + (1+\tau)Eu^2)$

7.2 Proof of Proposition 2

The relevant matrix to be studied is, from (25)-(26):

$$M = \begin{pmatrix} 1 - \delta\sigma_\infty^2(1+\tau) & -\tau\delta\sigma_\infty^2 \\ \psi\alpha & \psi \end{pmatrix}.$$

Let $x = \delta\sigma_\infty^2 \in [0,1]$. The eigenvalues μ of M are solution to

$$\mu^2 - [\psi + 1 - x(1+\tau)]\,\mu + \psi\,(1 - x(1+\gamma)) = 0. \tag{31}$$

The corresponding discriminant is

$$\Delta(x) = [\psi + 1 - x(1+\tau)]^2 - 4\psi\,(1 - x(1+\gamma)).$$

This quantity is positive iff

$$0 < x^2(1+\tau)^2 + 2x(\psi - (1+\tau) - \psi(\tau - 2\gamma)) + (1-\psi)^2$$

It is easy to see that $\psi - (1+\tau) - \psi(\tau - 2\gamma) = -b_2 < 0$ for all $\psi \in [0,1]$ and that $\Delta_2 = (\psi - (1+\tau) - \psi(\tau - 2\gamma))^2 - (1-\psi)^2(1+\tau)^2 \in (0, b_2^2)$. Therefore we have that $\Delta(x) < 0$ for $x \in (x_1, x_2)$, with $x_1 = \frac{b_2 - \sqrt{\Delta_2}}{(1+\tau)^2}$, $x_2 = \frac{b_2 + \sqrt{\Delta_2}}{(1+\tau)^2}$. Straightforward computations show that $0 < x_1 < x_2 < 1$. Furthermore, $x_2 < \frac{2(1+\tau - \psi + \psi(\tau - 2\gamma))}{(1+\tau)^2}$. Computations show that this quantity is smaller than

$$x^* = \frac{1 + \psi}{1 + \tau + \psi(1 + \gamma)}.$$

In the zone where $x \in (x_1, x_2)$, the two roots of (31) are complex conjugate since $\Delta(x) < 0$. Their module is equal to $\psi(1 - x(1+\gamma)) < 1$. Therefore the system is stable. In the zone where $x \notin (x_1, x_2)$, the roots of (31) are real and given by

$$\mu_1 = \frac{1 + \psi - x(1+\tau) - \sqrt{\Delta(x)}}{2},$$

$$\mu_2 = \frac{1 + \psi - x(1+\tau) + \sqrt{\Delta(x)}}{2}.$$

We can check that $\mu_2 < 1$. As for μ_1, it must be greater than -1 for the system to be stable. This is equivalent to

$$3 + \psi - x(1+\tau) > \sqrt{\Delta(x)}.$$

This inequality is violated if $x \geq \frac{3+\psi}{1+\tau}$. Suppose that $x < \frac{3+\psi}{1+\tau}$. Then the preceding inequality is equivalent to

$$(3 + \psi - x(1+\tau))^2 > \Delta(x).$$

Rearranging, this latter condition is equivalent to

$$x < x^*. \tag{32}$$

Since, on the one hand, $x^* < \frac{3+\psi}{1+\tau}$, and, on the other hand, $x^* > x_2$ and the system is stable for $x \in (x_1, x_2)$, it follows that the system is stable iff (32) holds. Then, we note that $x = \frac{-d+\sqrt{d^2+4d}}{2}$; substituting into (??) and rearranging, we get that the system is stable iff

$$\frac{1}{d} > \frac{\left(\frac{\tau+\gamma\psi}{1+\psi}\right)^2 - 1}{4}.$$

As $\tau > \gamma$, this is more likely to hold, the greater ψ.

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7.3 Computing social welfare in the model with endogenous capital

Relative to Appendix 1, consumption now differs from output. We have to subtract savings from it. Therefore, the flow of total welfare is given by

$$\ln\left(\Pi_t + S_t - sY_t\right) = \ln((1-s)Y_t - \omega\frac{L_t^{1+\gamma}}{1+\gamma}).$$

We follow the approach of Appendix 1 and express welfare as a function of the moments of the deviation between the endogenous variables $\bar{x}_t, k_t$ and their Walrasian counterparts, denoted by $\tilde{x}_t$ and $\tilde{k}_t$. In the Walrasian equilibrium with wages equal to $\tilde{w}_t$, we clearly have that $\tilde{x}_t = \frac{1}{1-\alpha}\left(\ln\alpha + a_t - \ln\tilde{w}_t\right)$ and $\ln\tilde{w}_t = \ln\omega + \gamma(k_t + x_t)$. Therefore,

$$\tilde{x}_t = \frac{1}{1-\alpha+\gamma}\left(\ln\frac{\alpha}{\omega} + a_t - \gamma\tilde{k}_t\right). \tag{33}$$

The law of motion for capital in the Walrasian equilibrium is

$$\tilde{k}_{t+1} = \ln\beta + \psi\tilde{k}_t + \psi a_t + \psi\alpha\tilde{x}_t. \tag{34}$$

For convenience we reproduce our dynamical system:

$$\bar{x}_{t+1} = (1 - \delta\sigma_\infty^2(1+\tau))\bar{x}_t - \tau\delta\sigma_\infty^2 k_t + \frac{\delta\sigma_\infty^2}{1-\alpha}a_t + b_\infty, \tag{35}$$

$$k_{t+1} = \psi a_t + \psi k_t + \psi\alpha\bar{x}_t + c_\infty. \tag{36}$$

It is easy to compute the trend growth rates for $\bar{x}$ and k from those equations. One gets

$$g_x = g\frac{1 - \psi(1+\tau(1-\alpha))}{(1-\alpha)(1+\tau - \psi(1+\tau(1-\alpha)))}$$

and

$$g_k = g \frac{\psi(1 + \tau(1 - \alpha))}{(1 - \alpha)(1 + \tau - \psi(1 + \tau(1 - \alpha)))} > 0.$$

One can also check from (33) and (34) that the trend growth rates of $\tilde{x}_t$ and $\tilde{k}_{t+1}$ are the same.

Let $\hat{x} = \bar{x} - \tilde{x}$ and $\hat{k} = k - \tilde{k}$. Then we have that

$$\begin{aligned} Y_t &= \exp(k_t + a_t + \alpha\bar{x}_t + \alpha^2\sigma_\infty^2/2) \\ &= \exp(\tilde{k}_t + a_t + \alpha\tilde{x}_t)\exp(\hat{k}_t + \alpha\hat{x}_t + \alpha^2\sigma_\infty^2/2). \end{aligned}$$

Similarly,

$$L_t^{1+\gamma} = \exp((1 + \gamma)(\tilde{k}_t + \tilde{x}_t))\exp((1 + \gamma)(\hat{k}_t + \hat{x}_t + \sigma_\infty^2/2)).$$

Now using (33) we get that

$$\tilde{k}_t + a_t + \alpha\tilde{x}_t = \frac{\alpha}{1 - \alpha + \gamma}\ln\frac{\alpha}{\omega} + M_t$$

and

$$(1 + \gamma)(\tilde{k}_t + \tilde{x}_t) = \frac{1 + \gamma}{1 - \alpha + \gamma}\ln\frac{\alpha}{\omega} + M_t,$$

where

$$M_t = \frac{1 + \gamma}{1 - \alpha + \gamma}a_t + \frac{(1 + \gamma)(1 - \alpha)}{1 - \alpha + \gamma}\tilde{k}_t.$$

Clearly, then,

$$(1-s)Y_t - \omega\frac{L_t^{1+\gamma}}{1 + \gamma} = \left(\frac{\alpha}{\omega}\right)^{\frac{\alpha}{1-\alpha+\gamma}}\exp M_t.\left[\begin{array}{c}(1 - s)\exp(k_t + a_t + \alpha\bar{x}_t + \alpha^2\sigma_\infty^2/2) \\ -\frac{\alpha}{1+\gamma}\exp((1 + \gamma)(\hat{k}_t + \hat{x}_t + \sigma_\infty^2/2))\end{array}\right].$$

Only the expression in brackets depends on δ. Therefore, maximizing $\ln(\Pi_t + S_t - sY_t)$ is equivalent to maximizing that term. Denoting this term

by $\exp(\phi)$, computing its second-order Taylor expansion and keeping only terms of order 0,1, and 2 in $(\hat{k}, \hat{x}, \sigma_\infty)$ we get

$$\exp\phi \approx \left(1 - s - \frac{\alpha}{1+\gamma}\right)\left(1 - a_1\hat{k} + a_2\hat{x} + a_3\sigma_\infty^2 + a_4\hat{k}^2 + a_5\hat{x}^2 + a_6\hat{x}\hat{k}\right),$$

$$(37)$$

where

$$a_1 = \frac{1 - s - \alpha}{1 - s - \alpha/(1+\gamma)}$$

$$a_2 = \frac{-\alpha s}{1 - s - \alpha/(1+\gamma)}$$

$$a_3 = \frac{\alpha^2(1-s) - \alpha}{2(1 - s - \alpha/(1+\gamma))}$$

$$a_4 = \frac{1 - s - \alpha(1+\gamma)}{2(1 - s - \alpha/(1+\gamma))}$$

$$a_5 = \frac{\alpha^2(1-s) - \alpha(1+\gamma)}{2(1 - s - \alpha/(1+\gamma))}$$

$$a_6 = \frac{-\alpha(s+\gamma)}{1 - s - \alpha/(1+\gamma)}.$$

Applying again a second-order Taylor expansion for $\ln(1+x)$ to (37) and taking expectations we get, neglecting again terms of order greater than 2:

$$\begin{aligned} E\phi &\approx \ln\left(1 - s - \frac{\alpha}{1+\gamma}\right) + a_1 E\hat{k} + a_2 E\hat{x} + a_3\sigma_\infty^2 \qquad (38)\\ &+ \left(a_5 - \frac{a_2^2}{2}\right)E\hat{x}^2 + \left(a_4 - \frac{a_1^2}{2}\right)E\hat{k}^2 + (a_6 - a_1 a_2)E\hat{k}\hat{x}. \end{aligned}$$

This is the quantity being maximized in the numerical simulations.

To compute the moments that appear in the RHS of (38) we use the following 4-dimensional system

$$\hat{k}_{t+1} = \psi\hat{k}_t + \psi\alpha\hat{x}_t + \psi\alpha^2\frac{\sigma_\infty^2}{2} \qquad (39)$$

$$\hat{x}_{t+1} = \left(-\frac{\tau \delta \sigma_\infty^4}{2} + \frac{\psi\gamma - 1}{1 - \alpha + \gamma} g \right) + \left(\frac{\psi\gamma}{1 - \alpha + \gamma} \Delta a_{Ct} - \frac{1}{1 - \alpha + \gamma} \Delta a_{Ct+1} \right)$$
$$-\tau\delta\sigma_\infty^2 \hat{k}_t + (1 - \delta\sigma_\infty^2(1+\tau))\hat{x}_t + \frac{\psi\gamma}{1 - \alpha + \gamma} \Delta\tilde{k}_t + \frac{\psi\gamma\alpha}{1 - \alpha + \gamma} \Delta\tilde{x}_t \qquad (40)$$

$$\Delta\tilde{k}_{t+1} = \psi\Delta\tilde{k}_t + \psi\alpha\Delta\tilde{x}_t + \psi(g + \Delta a_{Ct}) \qquad (41)$$

$$\Delta\tilde{x}_{t+1} = \frac{1 - \psi\gamma}{1 - \alpha + \gamma} g + \left(\frac{1}{1 - \alpha + \gamma} \Delta a_{Ct+1} - \frac{\psi\gamma}{1 - \alpha + \gamma} \Delta a_{Ct} \right) \qquad (42)$$
$$- \frac{\psi\gamma}{1 - \alpha + \gamma} \Delta\tilde{k}_t - \frac{\psi\gamma\alpha}{1 - \alpha + \gamma} \Delta\tilde{k}_t$$

To get (39), subtract (34) from (36). To get (41), subtract (34) lagged once from itself. To get (42), subtract (33) at t from itself at $t + 1$, then replace $\Delta\tilde{k}_{t+1}$ in the resulting expression with the RHS of (41). To get (40), subtract (33) at $t+1$ from (35), then replace $\Delta\tilde{x}_{t+1}$ in the resulting expression with the RHS of (42).

This system can be rewritten in matrix form as

$$V_{t+1} = AV_t + B + \eta_{t+1},$$

where

$$V = (\hat{x}, \hat{k}, \Delta\tilde{x}, \Delta\hat{k})',$$

$$\eta = \begin{pmatrix} \frac{\psi\gamma}{1-\alpha+\gamma}\Delta a_{Ct} - \frac{1}{1-\alpha+\gamma}\Delta a_{Ct+1} \\ 0 \\ \frac{1}{1-\alpha+\gamma}\Delta a_{Ct+1} - \frac{\psi\gamma}{1-\alpha+\gamma}\Delta a_{Ct} \\ \psi\Delta a_{Ct} \end{pmatrix}',$$

and the coefficients of matrices A and B are obtained straightforwardly from the above expressions.

It can be checked that, denoting by L the lag operator,

$$\eta = (1 - \rho L)^{-1}\beta(L)\varepsilon,$$

where $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2$; $\beta_0 = (-\frac{1}{1-\alpha+\gamma}, 0, \frac{1}{1-\alpha+\gamma}, 0)'$; $\beta_1 = (\frac{1+\psi\gamma}{1-\alpha+\gamma}, 0, -\frac{1+\psi\gamma}{1-\alpha+\gamma}, \psi)'$; $\beta_2 = (-\frac{\psi\gamma}{1-\alpha+\gamma}, 0, \frac{\psi\gamma}{1-\alpha+\gamma}, -\psi)'$.

Therefore,

$$
\begin{aligned}
V_t - EV &= (I - A)^{-1}(1 - \rho L)^{-1}\beta(L)\varepsilon_t \\
&= \sum_{j=0}^{+\infty} A^j \beta_0 \varepsilon_{t-j} + \sum_{j=1}^{+\infty} A^{j-1}(\beta_1 + \rho\beta_0)\varepsilon_{t-j} + \sum_{j=0}^{+\infty} A^j \left(\sum_{i=2}^{+\infty} \rho^i \beta(\rho^{-1})\varepsilon_{t-i-j} \right) \\
&= Q(L)\varepsilon_t.
\end{aligned}
$$

The last term can be computed as $\sum_{k=2}^{+\infty}(\rho I - A)^{-1}(\rho^{k-1}I - A^{k-1})\rho^2\beta(\rho^{-1})$. Altogether, this expansion allows us to get all the coefficients Q_i of Q and then to compute

$$Var(V) = \left(\sum_{j=0}^{+\infty} Q_i Q_i' \right) \sigma_\varepsilon^2.$$

Furthermore,

$$EV = (I - A)^{-1}B.$$

This allows us to get all the moments in (37). Next, to compute to compute the moments of the equilibrium, we rewrite (35)-(36) as well as the law of motion for a as

$$v_{t+1} = Mv_t + C + N\varepsilon_t + Zt,$$

where $v = (\bar{x}, k, a)'$. The variance-covariance matrix of the cyclical component in v, Ω, is solution to

$$\Omega = M\Omega M' + NN'\sigma_\varepsilon^2.$$

By rewriting (24) as $y_t = Dv_t + E$, we then compute detrended output volatility as $\sigma_y^2 = D\Omega D'$.

7.4 Proof of Proposition 3

The characteristic equation for the eigenvalues of matrix A is

$$x^2 - x(\lambda Q + \theta) + \frac{\lambda}{1+\delta} = 0,$$

where

$$Q = 1 + \frac{\delta \gamma}{(1+\delta)(1-\alpha)}.$$

Note that if the roots are complex their common module is $\left(\frac{\lambda}{1+\delta}\right)^{1/2} < 1$. Therefore the dynamics are stable. Let us characterize this regime first. Roots are complex iff

$$(\lambda Q + \theta)^2 < \frac{4\lambda}{1+\delta}. \tag{43}$$

This will be the case provided λ lies between the roots of

$$h(\lambda) = \lambda^2 Q^2 + 2\lambda \left(Q\theta - \frac{2}{1+\delta} \right) + \theta^2 = 0.$$

We note that if $Q\theta - \frac{2}{1+\delta} < 0$ and if these roots are real, they are both positive, and one of them is lower than one since their product is $\theta^2 < 1$. Furthermore, $h(1) = Q^2 + \left(2Q\theta - \frac{4}{1+\delta}\right) + \theta^2 = (Q+\theta)^2 - \frac{4}{1+\delta} = (1 - \frac{1}{1+\delta})^2 > 0$. Therefore both roots, if they exist are between 0 and 1.

Next, we note that $Q\theta = \left(1 + \frac{\delta\gamma}{(1+\delta)(1-\alpha)}\right) \frac{1}{1+\delta} \left(1 - \frac{\delta\gamma}{1-\alpha}\right)$, which, if it is positive, is lower than $\left(1 + \frac{\delta\gamma}{(1-\alpha)}\right) \frac{1}{1+\delta} \left(1 - \frac{\delta\gamma}{1-\alpha}\right) < \frac{1}{1+\delta} < \frac{2}{1+\delta}$. Thus we always have that $Q\theta - \frac{2}{1+\delta} < 0$. Next, the roots are real if and only if

$$\left(Q\theta - \frac{2}{1+\delta} \right)^2 > \theta^2 Q^2,$$

or equivalently

$$\frac{2}{1+\delta} - Q\theta > |\theta| Q.$$

This clearly always holds for $\theta \leq 0$. Furthermore, for $\theta > 0$, it is equivalent to $\theta Q < \frac{1}{1+\delta}$, which we just proved above. This proves that for any δ,

there exists an interior interval of values of λ included in $[0, 1]$, $[\lambda_c^-(\delta), \lambda_c^+(\delta)]$, over which (43) holds and therefore the eigenvalues of A are complex. Furthermore, we can check that for $\delta = 0$, $Q = 1$ and $\theta = 1/(1 + \delta)$, and we have a double root equal to 1. On the other hand for $\delta \to \infty$, we have $Q \to 1 + \frac{\gamma}{1-\alpha}$ and $\theta \to -\frac{\gamma}{1-\alpha}$, and we get the double root $-\theta/Q = \frac{\gamma}{1-\alpha+\gamma}$. Hence the banana-shaped lens on Figure 1.

Let us now turn to the regime where the eigenvalues are real, i.e. where (43) is violated. We have to distiguinsh between two cases.

Case 1:

$$(\lambda Q + \theta) > 0.$$

This inequality holds iff

$$\lambda > \frac{\delta\gamma - (1 - \alpha)}{(1 + \delta)(1 - \alpha) + \delta\gamma} = \lambda_{pos}(\delta).$$

This defines a threshold for λ which is increasing in γ and converges to $\frac{\gamma}{1-\alpha+\gamma}$ as $\delta \to \infty$. Then both eigenvalues are positive, and the largest one is

$$x_1 = \frac{\lambda Q + \theta + \sqrt{\Delta}}{2},$$

where $\Delta = (\lambda Q + \theta)^2 - \frac{4\lambda}{1+\delta}$. For the system to be stable we need that $x_1 < 1$, i.e.

$$\sqrt{\Delta} < 2 - \lambda Q - \theta.$$

A necessary condition is that $2 - \lambda Q - \theta \geq 0$. This is equivalent to $\lambda \leq \frac{(1+2\delta)(1-\alpha)+\delta\gamma}{(1+\delta)(1-\alpha)+\delta\gamma}$, which always holds since the RHS is ≥ 1. Given that, the preceding inequality holds iff

$$\Delta = (\lambda Q + \theta)^2 - \frac{4\lambda}{1 + \delta} < (2 - \lambda Q - \theta)^2,$$

or equivalently

$$-\frac{\lambda}{1 + \delta} < 1 - (\lambda Q + \theta),$$

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but

$$1 - (\lambda Q + \theta) = 1 - \lambda + (1 - \lambda) \frac{\delta\gamma}{(1 + \delta)(1 - \alpha)} - \frac{1}{1 + \delta} > -\frac{\lambda}{1 + \delta}.$$

This proves that dynamics are stable in the zone where the eigenvalues are positive.

Next, note that at the frontier of this zone, we have $\lambda Q + \theta = 0$, implying that (43) holds. Therefore,

$$\lambda_c^-(\delta) < \lambda_{pos}(\delta) < \lambda_c^+(\delta) \text{ for } \lambda_{pos}(\delta) > 0.$$

This means that for $\delta > (1 - \alpha)/\gamma$, the eigen values are complex, and stable, for $\lambda_c^-(\delta) < \lambda < \lambda_c^+(\delta)$, and positive, and stable for $\lambda > \lambda_c^+(\delta)$, while they are negative for $\lambda < \lambda_c^-(\delta)$. On the other hand, for $\delta < (1 - \alpha)/\gamma$, the eigenvalues are complex and stable for $\lambda_c^-(\delta) < \lambda < \lambda_c^+(\delta)$, and positive and stable for $\lambda < \lambda_c^-(\delta)$ and $\lambda > \lambda_c^+(\delta)$.

Case 2:

$$(\lambda Q + \theta) < 0,$$

i.e.

$$\lambda < \frac{\delta\gamma - (1 - \alpha)}{(1 + \delta)(1 - \alpha) + \delta\gamma}.$$

Then both roots are negative, and the largest one in absolute value is

$$x_2 = \frac{\lambda Q + \theta - \sqrt{\Delta}}{2}.$$

We have $x_2 > -1$ iff

$$\sqrt{\Delta} < 2 + \lambda Q + \theta.$$

A necessary condition is $2 + \lambda Q + \theta \geq 0$. This necessary condition is satisfied iff

$$\lambda \geq \frac{\delta\gamma - (1-\alpha)(3+2\delta)}{(1+\delta)(1-\alpha) + \delta\gamma}.$$

If this condition is violated, the dynamics are necessarily unstable. If it holds, then they are stable if

$$\Delta < (2 + \lambda Q + \theta)^2.$$

This is equivalent to

$$-\frac{\lambda}{1+\delta} < 1 + \lambda Q + \theta,$$

or equivalently

$$\lambda > \frac{\delta\gamma - (2+\delta)(1-\alpha)}{\delta\gamma + (2+\delta)(1-\alpha)} = \lambda_{stab}(\delta). \tag{44}$$

We note that

$$\frac{\delta\gamma - (1-\alpha)(3+2\delta)}{(1+\delta)(1-\alpha) + \delta\gamma} < \lambda_{stab}(\delta) < \lambda_{pos}(\delta).$$

Furthemore, at $\lambda = \lambda_{stab}(\delta) > 0$, we have that $-\frac{\lambda}{1+\delta} = 1 + \lambda Q + \theta$, implying that $(\lambda Q + \theta)^2 = \left(1 + \frac{\lambda}{1+\delta}\right)^2 > \frac{4\lambda}{1+\delta}$. Therefore (43) holds, implying (since $\lambda_{stab}(\delta) < \lambda_{pos}(\delta) < \lambda_c^+(\delta)$), that we must have

$$\lambda_{stab}(\delta) < \lambda_c^-(\delta).$$

Therefore, if $\delta > \frac{3(1-\alpha)}{\gamma - 2(1-\alpha)}$, the eigenvalues are negative and unstable for $\lambda < \lambda_{stab}(\delta)$, negative and stable for $\lambda_{stab}(\delta) < \lambda < \lambda_c^-(\delta)$, complex and stable for $\lambda_c^-(\delta) < \lambda < \lambda_c^+(\delta)$, and positive and stable for $\lambda > \lambda_c^+(\delta)$. On the other hand, if $\frac{1-\alpha}{\gamma} < \delta < \frac{3(1-\alpha)}{\gamma - 2(1-\alpha)}$, the eigenvalues are negative and stable for $\lambda < \lambda_c^-(\delta)$, complex and stable for $\lambda_c^-(\delta) < \lambda < \lambda_c^+(\delta)$, and positive and stable for $\lambda > \lambda_c^+(\delta)$.

This completes the proof of Proposition 3.

7.5 Computing welfare in the case with memory

We rewrite the dynamical system as

$$
\begin{aligned}
\bar{x}_{t+1} &= c\bar{x}_t + (1-c)\hat{x}_t \\
\hat{x}_{t+1} &= \left(\lambda - (1-\lambda)\tau(1-c)\right)\hat{x}_t - \tau(1-\lambda)c\bar{x}_t \\
&\quad -\frac{\tau(1-\lambda)\sigma_m^2}{4}\left(-1+\sqrt{1+4/d}\right) + \frac{1-\lambda}{1-\alpha}\left(\ln\alpha + a_{t+1}\right);
\end{aligned}
$$

where $c = \left(\frac{-1+\sqrt{1+4/d}}{1+\sqrt{1+4/d}}\right)$.

Let $u_t = \bar{x}_t - \tilde{x}_t$ and $v_t = \hat{x}_t - \tilde{x}_t$. Let $y_t = (u_t\ v_t)'$. Then

$$
y_{t+1} = Ay_t + w_{t+1},
$$

where

$$
A = \begin{pmatrix} c & 1-c \\ -\tau(1-\lambda)c & \lambda - (1-\lambda)\tau(1-c) \end{pmatrix}
$$

and

$$
w_{t+1} = \begin{pmatrix} -\frac{1}{1-\alpha+\gamma}(a_{t+1}-a_t) \\ -\frac{\tau(1-\lambda)\sigma_m^2}{4}\left(-1+\sqrt{1+4/d}\right) + \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma}(a_{t+1}-a_t) \end{pmatrix}.
$$

We have that

$$
Ew = \begin{pmatrix} -\frac{g}{1-\alpha+\gamma} \\ -\frac{\tau(1-\lambda)\sigma_m^2}{4}\left(-1+\sqrt{1+4/d}\right) + \frac{g(\tau-\lambda(1+\tau))}{1-\alpha+\gamma} \end{pmatrix}
$$

and

$$
Ey = (I-A)^{-1}Ew.
$$

Let $\hat{w} = a - Ew$ and $\hat{y} = y - Ey$. Then $\hat{y}_{t+1} = A\hat{y}_t + \hat{w}_t$. Furthermore,

$$
\hat{w}_{t+1} = \begin{pmatrix} -\frac{1}{1-\alpha+\gamma} \\ \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{pmatrix} \left((\rho-1)a_{Ct} + \varepsilon_{t+1}\right).
$$

This allows us to compute the following quantities

$$E\hat{w}a_C = \begin{pmatrix} -\frac{1}{1-\alpha+\gamma} \\ \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{pmatrix} \left((\rho-1)\rho\sigma_a^2 + \sigma_\varepsilon^2 \right)$$

$$= \begin{pmatrix} -\frac{1}{1-\alpha+\gamma} \\ \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{pmatrix} \frac{\sigma_\varepsilon^2}{1+\rho}.$$

$$E\hat{y}a_C = (I - \rho A)^{-1} E\hat{w}a_C.$$

$$E\hat{y}_t\hat{w}'_{t+1} = (1-\rho)\, E\hat{y}a_C \left(\begin{array}{cc} \frac{1}{1-\alpha+\gamma} & -\frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{array} \right) = M.$$

$$E\hat{w}\hat{w}' = \left((1-\rho)^2\, \sigma_a^2 + \sigma_\varepsilon^2 \right) \begin{pmatrix} -\frac{1}{1-\alpha+\gamma} \\ \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{pmatrix} \left(\begin{array}{cc} -\frac{1}{1-\alpha+\gamma} & \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{array} \right)$$

$$= \frac{2\sigma_\varepsilon^2}{1+\rho} \begin{pmatrix} -\frac{1}{1-\alpha+\gamma} \\ \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{pmatrix} \left(\begin{array}{cc} -\frac{1}{1-\alpha+\gamma} & \frac{\tau-\lambda(1+\tau)}{1-\alpha+\gamma} \end{array} \right) = N.$$

We then get that the variance-covariance matrix of $\hat{y}, V$, is the solution to the linear equation

$$V = AVA' + N + AM + M'A',$$

which can be solved by vectorization.

The asymptotic social welfare is then given by (17), i.e.

$$-E(\sigma_\infty^2 + (1+\tau)(\bar{x}_t - \tilde{x}_t)^2) = -\sigma_\infty^2 - (1+\tau)\left[(Ey)_1 + V_{11} \right].$$

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7.6 Regression results for estimating the persistence parameter

Country	θ'	S.E.	N	R^2
Australia	0.2	0.11	60	0.19
Austria	0.5	0.12	60	0.63
Belgium	0.36	0.12	60	0.4
Canada	0.32	0.12	60	0.25
Denmark	0.38	0.12	60	0.61
Finland	0.57	0.11	60	0.64
France	0.61	0.11	60	0.71
Germany	0.20	0.17	40	0.46
Greece	0.78	0.09	59	0.66
Iceland	0.34	0.14	54	0.26
Ireland	0.54	0.11	60	0.49
Italy	0.72	0.08	60	0.77
Japan	0.74	0.08	60	0.76
Korea	0.3	0.14	47	0.22

Country	θ'	S.E.	N	R^2
Netherlands	0.42	0.12	60	0.62
New Zealand	0.35	0.13	60	0.18
Norway	0.46	0.12	60	0.63
Portugal	0.07	0.14	60	0.18
Spain	0.68	0.09	60	0.71
Sweden	0.37	0.12	60	0.55
Switzerland	0.66	0.1	60	0.69
United Kingdom	0.48	0.11	60	0.41
United States	0.00	0.18	60	0.11

Table A1 – Regression results for estimating the persistence parameter in the aggregate labor/capital ratio. The specification that was estimated for each country was $\Delta x = \theta' \Delta x(-1) + a_0 \Delta w(-1) + a_1 \Delta a(-1) + a_2 \Delta e(-1) + C$, with Δ denoting the first difference operator, $x = \ln L/K$, $w = \ln W/L$, $a = \ln A, e = \ln RER$. The data used was the Penn World Table, with a = TFP L = Total yearly hours worked = number of persons engaged *

average annual hours worked, K = Total capital stock (national accounts), w = real wage (=share of labor compensation (national accounts) * real GDP (national accounts) / total hours worked), RER = real exchange rate = price level of GDP, PPP, output side, US 2005 $= 1$. Estimation was conducted by country, and countries outside the OECD or with fewer than 40 observations were dropped.

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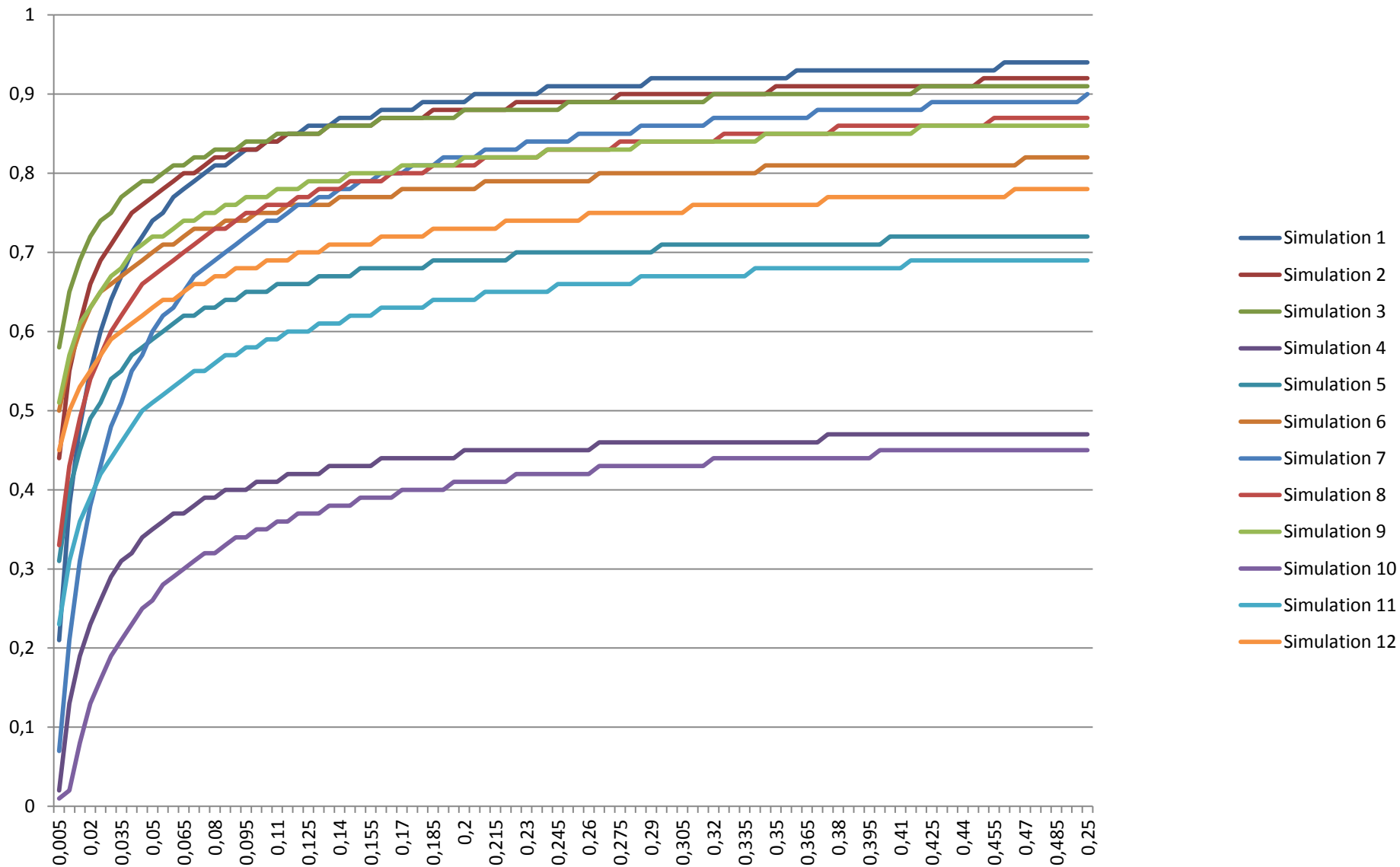
Figure 1 -- Effect of the variance of aggregate shocks on the optimal selectivity level ($1/(1+d)$) on the vertical axis
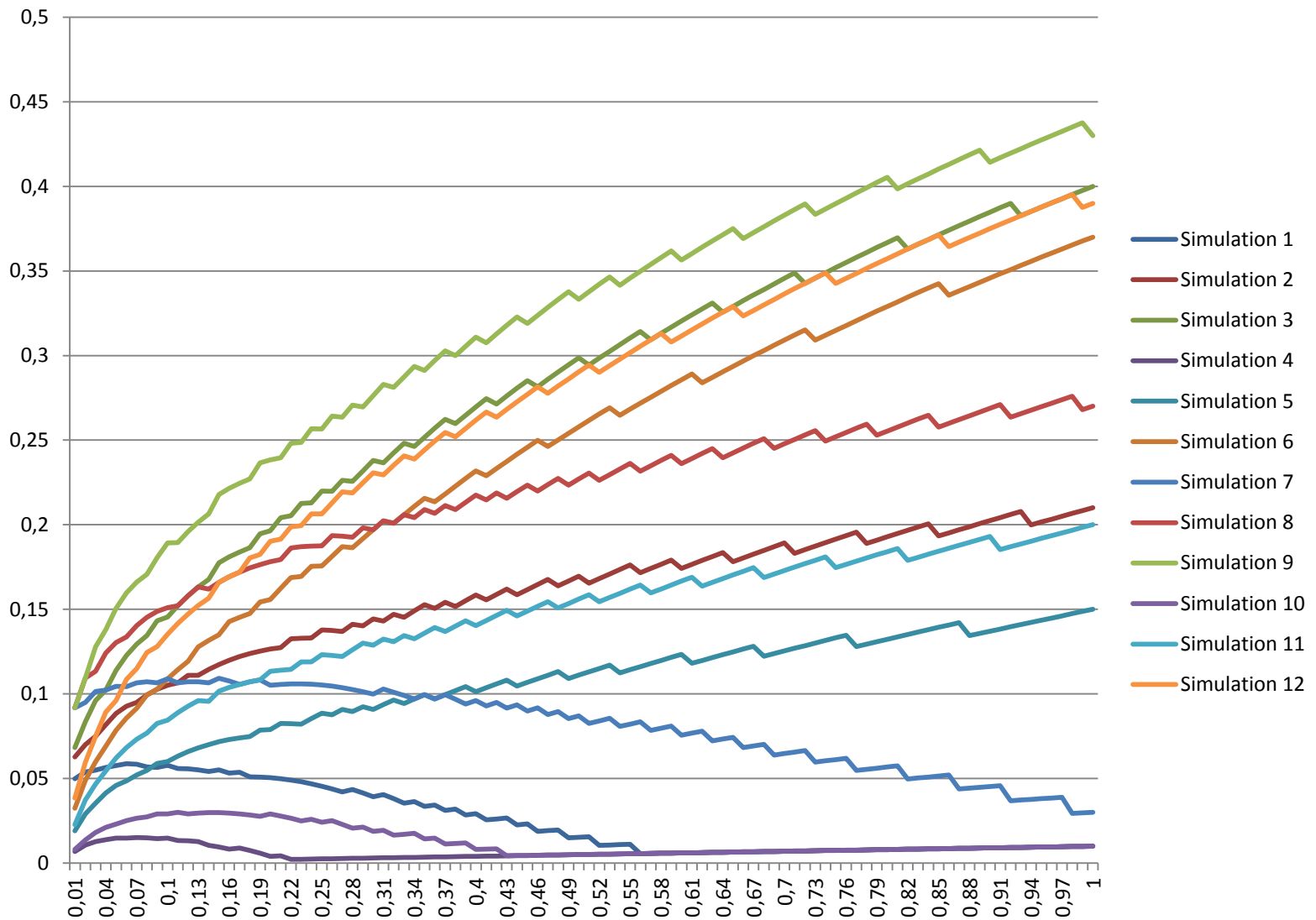
Figure 2 -- Effect of the mutational variance on optimal selectivity ($(1/(1+\delta)$ on the vertical axis)