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Sebastian Böhm
Volker Grossmann
Thomas M. Steger

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Forschungsinstitut
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Sebastian Böhm
University of Leipzig

Volker Grossmann
University of Fribourg, CESifo, IZA and CReAM

Thomas M. Steger
University of Leipzig and CESifo

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

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ABSTRACT

Does Public Education Expansion Lead to Trickle-Down Growth?*

The paper revisits the debate on trickle-down growth in view of the widely discussed evolution of the earnings and income distribution that followed a massive expansion of higher education. We propose a dynamic general equilibrium model to dynamically evaluate whether economic growth triggered by an increase in public education expenditure on behalf of those with high learning ability eventually trickles down to low-ability workers and serves them better than redistributive transfers. Our results suggest that, in the shorter run, low-skilled workers lose. They are better off from promoting equally sized redistributive transfers. In the longer run, however, low-skilled workers eventually benefit more from the education policy. Interestingly, although the expansion of education leads to sustained increases in the skill premium, income inequality follows an inverted U-shaped evolution.

JEL Classification: H20, J31, O30

Keywords: directed technological change, publicly financed education, redistributive transfers, transitional dynamics, trickle-down growth

Corresponding author:

Volker Grossmann
University of Fribourg
Department of Economics
Bd. de Pérolles 90
1700 Fribourg
Switzerland
E-mail: volker.grossmann@unifr.ch

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"Since 1979, our economy has more than doubled in size, but most of that growth has flowed to a fortunate few." (Barack Obama, December 4, 2013)

1 Introduction

Whether economic growth trickles down to the socially less fortunate has been a key debate for many decades in the US and elsewhere (e.g. Kuznets, 1955; Thornton, Agnello and Link, 1978; Hirsch, 1980; Aghion and Bolton, 1997; Piketty, 1997). In particular, social desirability and choices of growth-promoting policies may critically depend on their expected trickle-down effects. For instance, massive expansion of high school and college education throughout the 20th century has led to a surge in the relative supply of skilled labor (Goldin and Katz, 2008; Gordon, 2013). Goldin and Katz (2008) document the important role of the public sector for this development, particularly between 1950 and 1970.[1] Despite steady economic growth, however, median (full-time equivalent) earnings of males have almost stagnated from the 1970s onwards (e.g. Katz and Murphy, 1992; Acemoglu and Autor, 2012; DeNavas-Walt, Proctor and Smith, 2013). Moreover, earnings of less educated males fell considerably (Acemoglu and Autor, 2011, Tab. 1a). Thus, under the hypothesis that technological change has been endogenously skill-biased to the expansion of public education, the evidence suggests a pronounced equity-efficiency trade-off of this policy intervention.

In this paper, we propose a comprehensive dynamic general equilibrium framework with directed technical change, heterogenous agents and a key role of human capital for economic growth to evaluate the effects of public expenditure reforms on the evolution of living standards over time. In particular, we comparatively examine two public expenditure policies: public education finance on behalf of high-ability workers and income transfers towards low-ability workers who do not acquire more advanced education (e.g. because of limited ability). We investigate whether economic growth

[1] For instance, the fraction of college students in publicly controlled institutions gradually increased between 1900 and 1970. Between 1950 and 1970, it increased from 0.5 to almost 0.7 among students with four years of college attendance (Goldin and Katz, 2008; Fig. 7.7).

triggered by an increase in public education expenditure on behalf of those with high learning ability eventually trickles down to low-ability workers and serves them better than redistributive transfers. Relatedly, we examine whether expanding education of wealthy, high-ability households inevitably raises inequality of earnings and income over time.

Whether and when growth promoted by education expansion trickles down to low-skilled workers is a key question for at least three reasons. First, the evolution of the earnings distribution has recently provoked an intensive policy debate in the US and elsewhere (e.g. Stiglitz, 2012; Deaton, 2013; Mankiw, 2013; Piketty, 2014).[2] For instance, in his maybe most widely received speech of his US presidency (December 4, 2013), Barack Obama referred to it as "the defining challenge of our time", criticizing that "a trickle-down ideology became more prominent".[3] He also urged that "we need to set aside the belief that government cannot do anything about reducing inequality". In fact, the tax-transfer system in the US is rather unsuccessful to improve living standards of the working-poor, compared to other advanced countries (Gould and Wething, 2012). Second, upward social mobility has proved being severely limited by intergenerational transmission of learning ability and/or human capital, implying that a significant fraction of individuals may not acquire more than basic education for a long time to come.[4] It is thus important to know whether those individuals profit from publicly financed education expansion, particularly compared to the alternative policy of redistributive transfers which are directly targeted to less educated workers.[5] To focus our analysis on this issue we deliberately rule out social mobility in

[2] The earnings distribution has changed markedly also in Continental Europe, although later than in the US; see e.g. Dustmann, Ludsteck and Schönberg (2009) for evidence on Germany.

[3] See www.whitehouse.gov/the-press-office/2013/12/04/remarks-president-economic-mobility

[4] See e.g. Corak (2013). There is overwhelming evidence for the hypothesis that the education of parents affects the human capital level of children, even when controlling for family income. For instance, Plug and Vijverberg (2003) and Black, Devereux and Salvanes (2005) show that children of high-skilled parents have a higher probability of being high-skilled.

[5] There are, of course, many other policy options to improve economic situations of the poor which we do not consider because of our macroeconomic focus. For instance, there is a large literature on the effectiveness of programmes to promote rather basic education on behalf of low-income earners or the unemployed. Some of the evidence suggests that their success is very limited unless governments intervene at a very young age (see e.g. the survey by Cunha, Heckman, Lochner and Masterov, 2006). See, however, Osikominu (2013) for qualifying evidence on long term (versus short term) active labor

our model. Third, the literature on directed technological change, initiated by Von Weizsäcker (1966) and advanced by Acemoglu (1998, 2002), suggests to account for the possibility that an increase in the supply of human capital leads to skill-biased technological change, thus contributing to the differential evolution of living standards across individuals in the first place. Particularly, it is not evident whether and when workers with only basic education benefit from an increase in the economy's supply of human capital. It is therefore salient for addressing our research questions to capture the possibility that technological progress does not automatically benefit high-skilled and low-skilled labor in a similar fashion.[6]

To illustrate this point, we start out with a simple model without directed technical change where we allow for human capital externalities which benefit both types of workers alike. We then proceed to compare the speed of trickle-down of this model to that in a comprehensive framework with R&D-based directed technical change. Standard analyses of directed technological change models are inadequate to enter the trickle-down debate, because they exclusively focus on the long run and assume that skill supply is exogenous. For instance, as acknowledged by Autor and Acemoglu (2012), such analyses are unsuccessful to explain falling earnings at the bottom of the distribution of income. Rather, our goal is to dynamically evaluate the impact of an increase in public education expenditure that potentially affects both R&D and education decisions, is in line with the observed income dynamics in the last decades, and helps to predict and understand future dynamics.[7]

More specifically, our framework rests on the following features: (i) We focus on households which do not accumulate human capital, but may benefit from expansion of publicly financed education of others − either dynamically through trickle-down

market policy.

[6] In an interesting recent paper, Che and Zhang (2014) argue that the higher education expansion in China in the late 1990s had a causal positive effect on technological change particularly in human capital intensive industries, suggesting that technical change endogenously benefits primarily high-skilled workers.

[7] We employ the algorithm of Trimborn, Koch and Steger (2008) to analyze the transitional dynamics of the resulting non-linear, highly dimensional, saddle-point stable, differential-algebraic system. Despite the complexity of our model, the long run equilibrium can be derived and characterized analytically. This is important for calibrating the model and for understanding basic mechanisms.

growth or statically through complementarity of high-skilled and low-skilled labor; (ii) growth is endogenously driven by technological change which may complement different types of skills in a differential fashion; (iii) the government can extend redistributive transfers and promote economic growth by publicly financing education; (iv) there are distortionary taxes on (labor and capital) income and capital gains; (v) the accumulation of physical capital, human capital and R&D-based knowledge capital interact with public policy in determining the evolution of living standards over time.

Our key findings may be summarized as follows. First, when the government raises the fraction of tax revenue devoted to publicly finance education on behalf of high-ability individuals, net income and the wage rate of low-ability individuals first decrease compared to the baseline scenario without policy reform. Thus, consistent with empirical evidence, our analysis suggests that education expansion is followed by rising inequality and temporarily lower wages at the bottom of the earnings distribution. Later in the transition, the economic situation of the least educated improves and they eventually become better off than without education expansion. Second, an increase in the fraction of the tax revenue devoted to redistributive transfers rather than public education expenditure leads to short run gains but long run losses for this group. Thus, our analysis suggests a dynamic policy trade-off from the perspective of the socially less fortunate. This is not necessarily so in the simple model without directed technical change we analyze first (section 3): in this model, education expansion is always inferior to transfers from the perspective of low-ability workers in the case where there are no human capital externalities; if human capital externalities are sufficiently strong, the picture becomes qualitatively the one suggested by the comprehensive model. Examining the comprehensive model is more compelling though for the main argument and for a quantitative analysis because it allows for the possibility that education expansion triggers technological change which primarily benefits high-skilled workers. Third, our calibration to the US economy implies that it takes a long time until growth triggered by education expansion trickles-down to the poor and makes them better off than under redistributive transfers. Fourth, the speed of trickle-down is slower, the higher the (derived) elasticity of substitution between the two types of workers in the economy.

Fifth, promoting human capital accumulation implies that earnings inequality increases on impact and then further rises considerably over time. This also raises overall inequality of net income earlier in the transition. However, although remaining higher than under redistributive transfers, income inequality eventually decreases later in the transition because of (albeit limited) convergence of asset holdings between the two types of workers. In other words, education expansion leads to an inverted U-shaped "Kuznets curve" evolution of income inequality.

The paper is organized as follows. In section 2, we briefly discuss the related literature. Section 3 starts out with a simple model highlighting important features of our analysis. In section 4, we set up a comprehensive growth model designed for a quantitative analysis. Section 5 characterizes its equilibrium analytically. In section 6 we employ numerical analysis to dynamically evaluate the trickle down dynamics of policy reforms. Section 7 focusses on the evolution of the distribution of earnings and net income across different types of workers. The last section concludes.

2 Related Literature

We shall not attempt to review the vast literature on the interplay between economic growth and inequality. Rather, we selectively discuss the most related work. In their seminal paper, Galor and Zeira (1993) show that human capital investments are suboptimally low under credit constraints. According to their analysis, if the wedge between the borrowing and the lending rate is sufficiently large, not only is inequality harmful for growth but also may it increase over time (i.e., growth does not trickle down). Aghion and Bolton (1997), Piketty (1997) and Matsuyama (2000) examine the evolution of wealth distribution under imperfect credit market with fixed investment requirements for entrepreneurial projects. They identify conditions under which growth may trickle down and argue that (lump sum) wealth redistribution to the poor may speed up this process by mitigating credit constraints. In contrast to this literature, our focus is on the interplay between physical capital accumulation, human capital accumulation and technological change directed to different types of workers, while abstracting from credit constraints. In view of the minor role of credit constrains for education finance in the US (for a recent study, see e.g. Lochner and Monge-Naranjo, 2011), this appears to be a reasonable research strategy in our context. Moreover, we focus on publicly financed education and redistribution, financed by distortionary taxation.

Goldin and Katz (2008) argue that the evolution of skill premia can be explained by the pace at which the relative supply of skills keeps track with the relative demand for skills as driven by skill-biased technological change. However, as already pointed out by Acemoglu and Autor (2012), their analysis does not address the possible feedback effect of rising skill supply. Such effects result from education expansion via endogenously biased technological change, altering the relative demand for skills. Closest to our analysis, Acemoglu (1998, 2002) introduces the idea that the relative demand for different types of workers via technological change is endogenous to the supply of human capital. While he focusses on the long run effects of an exogenous increase in human capital, our interest lies in the transitional dynamics when both the formation of human capital and the extent and direction of technological change are endogenous to public policy reforms. Finally, Galor and Moav (2000) examine distributional effects of biased technological change in a dynamic model of endogenous skill supply. There are two main differences to our work. First, whereas Galor and Moav (2000) are interested in the evolution of wage inequality when the rate of (by assumption ability-biased) productivity growth starts below steady state, we evaluate public policy experiments. In particular, we consider the dynamic effects of a publicly financed expansion of education on behalf of high-ability individuals versus redistributive transfers on income dynamics. Second, in our model technological change is based on R&D decisions which potentially is skill-biased endogenously.

3 Simple Model

3.1 Set Up

Consider an infinite-horizon framework in continuous time. There are two types of labor, a unit mass of type$-h$ individuals with unit time endowment, capable of accumulating human capital by investing time for education, and a mass $\bar{l} > 0$ of type$-l$ individuals, inelastically supplying one unit of labor each period. For modern times, human capital accumulation of a representative type$-h$ individual may be interpreted as higher education attendance after high school graduation.[8] Ruling out social mobility captures intergenerational transmission of learning ability in a pointed form. The modeling choice is driven by our interest of trickle-down dynamics on behalf of those (type$-l$ individuals) with basic education only.

There is a homogenous consumption good with price normalized to unity. Final output is produced under perfect competition according to

$$Y = A\left[(H^Y)^{\frac{\psi-1}{\psi}} + (L^Y)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{1}$$

where H^Y and L^Y denote the amounts of $h-$type and $l-$type labor in manufacturing the numeraire good, $A > 0$ is total factor productivity, and $\psi > 0$ is the elasticity of substitution between the two types of labor. Let h denote the human capital level per type$-h$ individual. We allow for a human capital externality as a channel which may affect trickle-down growth; that is, A is a non-decreasing function of the human capital stock per $h-$type individual, h; we write

$$A = h^{\zeta},$$

$\zeta \geq 0$. In the special case $\zeta = 0$, there is no external effect of human capital accumulation on A and type$-l$ individuals are exclusively affected by an increase in h because of the complementarity of different types of labor in (1). The representative final good

[8] In the US, secondary graduation rates increased quickly through the 20th century and then stabilized (Goldin and Katz, 2008; Tab. 3.1, Fig. 6.1).

producer maximizes profits, taking both A and the wage rates as given.

Skill accumulation of type$-h$ individuals depends, first, on the time investment in education (Lucas, 1988). Second, it depends on the amount of publicly financed human capital ("teachers") per type$-h$ individual devoted to educational production. Moreover, it is characterized by intergenerational human capital transmission and depreciation over time. Let u and $1-u$ denote the fraction of time a type$-h$ individual supplies to the labor market and devotes to education, respectively. Let h^E denote the teaching input in educational production per type$-h$ individual. Their human capital stock evolves according to

$$\dot{h} = \xi(1-u)^{\beta}\left(h^E\right)^{\gamma} h^{\eta} - \delta_H h, \tag{2}$$

where $\delta_H > 0$ is the depreciation rate of human capital and the other parameters fulfill $\xi > 0$, $\beta \in (0,1)$, $\gamma > 0$, $\eta \geq 0$, $\gamma + \eta < 1$. $\beta < 1$ captures decreasing returns to time use in education. If $\eta > 0$, there is intergenerational human capital transmission. $\delta_H > 0$ and $\gamma + \eta < 1$ imply that, in the long run, the individual human capital level is stationary. Suppose that the teaching input is given by[9]

$$h^E = \vartheta h, \tag{3}$$

where $\vartheta > 0$ is the fraction of human capital devoted to education. In labor market equilibrium, $uh = H^Y + h^E$, i.e., $H^Y = (u-\vartheta)h$; moreover, $L^Y = \bar{l}$.

Our human capital accumulation process is similar to Lucas (1988), extended for publicly provided education. Substituting (3) into (2), we find $\dot{h} = \xi(1-u)^{\beta}\vartheta^{\gamma}h^{\gamma+\eta} - \delta_H h$. In Lucas (1988), $\beta = 1$ (constant rather than decreasing returns to time investment), $\gamma = 0$ (no publicly provided education), and $\eta = 1$ such that the stock of human capital per capita could grow with a positive rate even in the long run, which we rule out with our parameter restrictions.

Teaching input is publicly financed by income taxation. In each period, a fraction

[9] In any meaningful equilibrium, the fraction must be lower than the fraction of time devoted to labor market participation, $\vartheta < u$.

$s^E > 0$ of contemporaneous total tax revenue is used to publicly finance teachers in the education sector, endogenously determining policy parameter ϑ. Moreover, a constant fraction $s^T > 0$ of the tax revenue is devoted to finance transfers to individuals who own income below some income threshold, which may be thought of social welfare expenditure; $s^E + s^T \leq 1$. The possibility $s^E + s^T < 1$ allows for a third public spending category which may additively enter the utility function (like public expenditure for defense, the legal system, public order, and safety). Alternatively, the third category may be interpreted as government waste.

Let w_l denote the wage rate (and gross wage income) of type$-l$ individuals and w_h the wage rate per unit of human capital supplied by type$-h$ individuals; supplying a fraction u of their unit time endowment to the labor market, their gross wage income reads as $w_h uh$. We focus throughout on the case where type$-l$ individuals earn (endogenously) less than type$-h$ individuals at all times. Marginal tax rates on labor income are, if anything, higher for type$-h$ individuals. Formally, suppose that the marginal income tax rate is given by an increasing step-function $\tilde{\tau}(\cdot)$ fulfilling $\tilde{\tau}(w_h uh) \equiv \tau_h > \tau_l \equiv \tilde{\tau}(w_l)$. We focus on the case in which the step-function $\tilde{\tau}$ is such that τ_h and τ_l are time-invariant for the income ranges we consider.[10] Suppose that only type$-l$ individuals earn sufficiently little to be eligible for a transfer payment, denoted by T. Their income level then reads as $y_l := (1-\tau_l)w_l + T$, whereas after-tax income of type$-h$ individuals is given by $y_h := (1-\tau_h)w_h uh$.

Denote the level of consumption of a type$-j$ individual by c_j, $j \in \{h, l\}$. Let subscript t on a variable index time (suppressed if not leading to confusion). As there is no physical capital, individuals do not save, i.e. $c_{jt} = y_{jt}$ for all t, $j \in \{h, l\}$. Suppose that intertemporal utility of a type$-h$ individual is given by

$$U_h = \int_0^\infty \frac{(c_{ht})^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} \mathrm{dt} = \int_0^\infty \frac{((1-\tau_h) w_{ht} u_t h_t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} \mathrm{dt}. \tag{4}$$

The optimal sequence of time allocation, $\{u_t\}_{t=0}^\infty$, maximizes (4) subject to (2), taking

[10] Ensuring this outcome may require that the mapping from income brackets to marginal tax rates is adjusted when income levels grow, i.e. function $\tilde{\tau}(\cdot)$ is adjusted over time.

the path of h^E as given. The equilibrium analysis of the model proposed in this section is standard and relegated to an online-appendix.

3.2 Policy Evaluation

We now contrast the dynamic effects of an expansion of education (increase in s^E) and of higher transfers (increase in s^T) on income y_l of type$-l$ individuals. For given tax rates, an increase in s^E raises the fraction of human capital devoted to education, ϑ, whereas an increase in s^T raises transfer payment T. Throughout the paper, we maintain the assumption that individuals do not anticipate shocks in policy parameters.

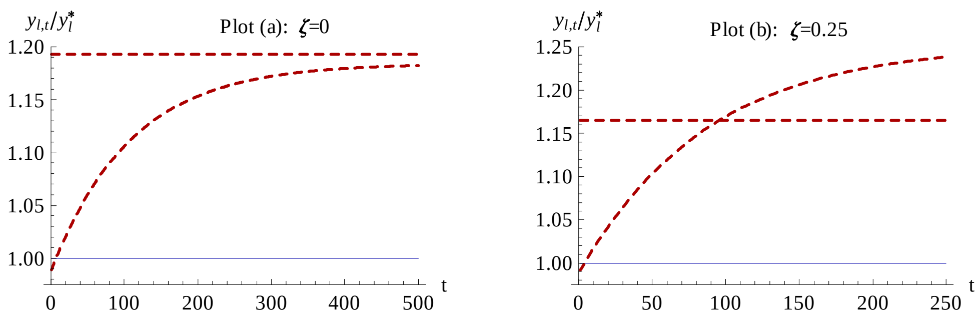

Figure 0: Time path of normalized income, y_l/y_l^*, in three scenarios: solid (blue) line: baseline scenario (s^T and s^E remain constant), horizontal dashed line: s^T increases by five percentage points), increasing dashed line: s^E increases by five percentage points. Set of parameters: $s^T = 0.07$, $s^E = 0.1$ (pre-shock levels), $\tau_h = 0.35$, $\tau_l = 0.17$, $\delta_H = 0.023$, $\xi = 0.84$, $\beta = 0.25$, $\eta = 0.35$, $\psi = 1.5$, $\rho = 0.02$, $\sigma = 1.91$, $\bar{l} = 0.15$. The calibration strategy is described in appendix.

Let y_l^* denote the net income (and consumption) of a type-l household in initial steady state (before the policy reform). Figure 0 illustrates the effects of increases in s^E ("education expansion") and in s^T ("redistribution extension") by five percentage points on normalized net income y_l/y_l^*. Panel (a) treats the case without human capital externality ($\zeta = 0$). The increasing (dashed) line shows that an increase in s^E leaves low-ability workers worse off early in the transition compared to the baseline scenario without policy reform. This reflects a reallocation of high-skilled labor away from final goods production (decrease in H^Y) towards the education sector, thereby depressing the marginal product of type$-l$ workers (decrease in $\partial Y/\partial L^Y$). Because of a complementarity between both types of labor in production function (1), later in the transition, y_l rises as human capital accumulates. However, for $\zeta = 0$, type$-l$ individuals turn out being worse off than under the alternative policy of raising s^T, which once and for all raises living standards of the recipients of transfer income, as indicated by the horizontal (dashed) line in panel (a). In Panel (b), we consider the same policy shocks for the case where there is a human capital externality ($\zeta = 0.25$). Thus, human capital accumulation triggered by expanding education now also increases total factor productivity. Now, although living standards again drop on impact in response to an increase in s^E, type$-l$ individuals become better off in the longer run, compared to the effect of increasing s^T. Comparing the results suggested by panel (a) and (b) of Figure 0 highlights the salient role of endogenous technological progress which we examine next in a more comprehensive way for the purpose of quantitative analysis.

4 Comprehensive Model

The model in the previous section is too simple for a quantitative policy evaluation. We next propose a comprehensive model with endogenous and directed technical change. It features may be viewed as a microfoundation of human capital externalities. Unlike in the simple model, however, education expansion does not automatically benefit low-skilled workers through increases in total factor productivity. Its effect runs through R&D investment which may be primarily directed to high-skilled intensive production. We also introduce savings and capital accumulation.

4.1 Firms

There is again a homogenous final good with price normalized to unity. Following Acemoglu (2002), final output is now produced under perfect competition according to

$$Y = \left[(X_H)^{\frac{\varepsilon-1}{\varepsilon}} + (X_L)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5}$$

$\varepsilon > 0$. X_L and X_H are composite intermediate inputs. They are also produced under perfect competition, combining capital goods ("machines") with human capital and low-skilled labor, respectively. Formally, we have

$$X_H = (H^X)^{1-\alpha} \int_0^{A_H} x_H(i)^\alpha di, \quad (6)$$

$$X_L = (L^X)^{1-\alpha} \int_0^{A_L} x_L(i)^\alpha di, \quad (7)$$

$0 < \alpha < 1$, where $x_H(i)$ and $x_L(i)$ are inputs of machines, indexed by i, which are complementary to the amount of human capital in this sector, H^X, and low-skilled labor, L^X, respectively. The mass ("number") of machines, A_H and A_L, expands through horizontal innovations, as introduced below. The initial number of both types of machines are given and positive; $A_{H,0} > 0$, $A_{L,0} > 0$.

In each machine sector there is one monopoly firm − the innovator or the buyer of a blueprint for a machine. They produce with a "one-to-one" constant-returns to scale technology by using one unit of final output to produce one machine unit. The total capital stock, K, in terms of the final good, thus reads as

$$K = \int_0^{A_H} x_H(i)di + \int_0^{A_L} x_L(i)di. \quad (8)$$

Machine investments are financed by bonds sold to households. In each machine sector there is a competitive fringe which can produce a perfect substitute for an existing machine (without violating patent rights) but is less productive: input coefficients are higher than that of the incumbents by a factor $\kappa \in (1, \frac{1}{\alpha}]$ in both sectors.[11] Parameter κ determines the price-setting power of firms and allows us to disentangle the price-mark up from output elasticities, which is important for a reasonable calibration of the model. Physical capital depreciates at rate $\delta_K \geq 0$.

There is free entry into two kinds of competitive R&D sectors. In one sector, a

[11] See Aghion and Howitt (2005), among others, for a similar way of capturing a competitive fringe.

representative R&D firm directs human capital to develop blueprints for new machines used to produce the human capital intensive composite input, X_H, the other sector to produce X_L. To each new idea a patent of infinite length is awarded. Following Jones (1995), ideas for new machines in the R&D sectors are generated according to

$$\dot{A}_H = \tilde{\nu}_H (A_H)^{\phi} H_H^A, \ \tilde{\nu}_H = \nu \cdot (H_H^A)^{-\theta}, \tag{9}$$

$$\dot{A}_L = \tilde{\nu}_L (A_L)^{\phi} H_L^A, \ \tilde{\nu}_L = \nu \cdot (H_L^A)^{-\theta}, \tag{10}$$

where H_H^A and H_L^A denote human capital input in the R&D sector directed to the human capital intensive and low-skilled intensive intermediate goods sector, respectively. $\nu > 0$ is a R&D productivity parameter. $\theta \in (0, 1)$ captures a negative R&D ("duplication") externality (Jones, 1995) which measures the gap between privately perceived constant R&D returns of human capital and socially decreasing returns. We assume that $\phi \in (0, 1)$. $\phi > 0$ captures a positive ("standing on shoulders") spillover effect.[12]

4.2 Households

There are again two types of individuals, indexed by $j \in \{l, h\}$, which differ with respect to their learning ability. The learning technology is identical to section 3: only type$-h$ individuals can accumulate human capital, according to (2). We now allow population sizes of both types, $N_h > 0$ and $N_l > 0$, to grow at the same and constant exponential rate, $n \geq 0$. We normalize the initial size of the type$-h$ population to unity, $N_{h,0} = 1$, and denote $N_{l,0} = \bar{l}$. Preferences of individuals of type $j \in \{l, h\}$ are represented by the standard utility function

$$U_j = \int_0^{\infty} \frac{(c_{jt})^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho - n)t} dt, \tag{11}$$

$\sigma > 0$, where c_{jt} is consumption of a type$-j$ individual at time t.

[12] Two remarks are in order. First, Acemoglu (1998, 2002) focusses on a "lab-equipment" version of the R&D process. Since empirically R&D costs are mainly salaries for R&D personnel, we prefer specifications (9) and (10). Second, $\phi < 1$ implies that growth is "semi-endogenous" (Jones, 1995), i.e. would cease in the long run if we population growth were absent.

Households can hold bonds − providing capital which serves as input for machine producers, and equity − thereby financing blueprints for machine producers. Financial markets are always in (no-arbitrage) equilibrium. Asset holdings in per member of dynasty j are denoted by a_j. Initial asset holdings are given by $a_{h,0} > 0$, $a_{l,0} > 0$. The interest rate for bonds is denoted by r. Dividends from equity holdings and bond holdings are taxed by the same constant rate τ_r. Maintaining the same labor income schedule as in section 3, assets accumulate according to

$$\dot{a}_h = y_h - c_h, \text{ with } y_h := [(1-\tau_r)r - n]a_h + (1-\tau_h)w_h uh, \tag{12}$$

$$\dot{a}_l = y_l - c_l, \text{ with } y_l := [(1-\tau_r)r - n]a_l + (1-\tau_l)w_l + T. \tag{13}$$

y_j again denotes net income of type $j \in \{l, h\}$. Capital gains are taxed with constant tax rate τ_g. Again, a fraction s^E of total tax revenue is devoted to publicly financing education on behalf of type$-h$ individuals and a fraction s^T finances transfers on behalf of type$-l$ individuals.

5 Equilibrium Analysis

This section derives important analytical results. In section 6, we will examine whether the calibrated model implies sufficiently strong trickle-down effects of an increase in education expenditure which eventually benefits the less fortunate better than extending redistribution. In addition to the evolution of net income of low-ability dynasties, we also consider that of their wage rate. In section 7, we study the dynamic effects of policy reforms on relative earnings and relative net income between the two types of individuals.

5.1 Preliminaries

The equilibrium definition is standard and relegated to the appendix. It turns out that, for the transversality conditions of household optimization problems to hold and intertemporal welfare levels U_h and U_l to be finite, we have to restrict the parameter space such that

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{n(1-\theta)}{1-\phi}. \tag{A1}$$

As will become apparent, g is the long run growth rate of individual consumption levels, individual income components, and knowledge measures A_H, A_L. Thus, in the long run, technological change turns out to be unbiased. In modern times and advanced economies, on average, the per capita income growth rate exceeds the population growth rate ($g > n$), implying

$$\phi > \theta. \tag{A2}$$

Profit maximization of non-R&D producers implies two intermediate results which relate to the previous literature, reminding us on the mechanics of directed technical change.

Lemma 1. *Define $\psi \equiv \alpha + \varepsilon(1-\alpha)$. The relative wage per unit of human capital between type$-h$ and type$-l$ individuals reads as*

$$\frac{w_h}{w_l} = \left(\frac{H^X}{L^X}\right)^{-\frac{1}{\psi}} \left(\frac{A_H}{A_L}\right)^{\frac{\psi-1}{\psi}}. \tag{14}$$

All proofs are relegated to the appendix. According to (14), ψ is the "derived" elasticity between high-skilled and low-skilled labor in production (Acemoglu, 2002). That is, for given productivity levels, an increase in relative amount of type$-h$ human capital devoted to manufacturing, H^X/L^X, by one percent reduces the relative wage rate, w_h/w_l, by $1/\psi$ percent. Notably, if $\varepsilon > 1$, then $\varepsilon > \psi > 1$; if $\varepsilon < 1$, then $\varepsilon < \psi < 1$.

Let P_H^X and P_L^X denote the price of the high-skilled intensive and low-skilled intensive composite intermediate good used in the final goods sector, respectively. An increase in the relative knowledge stock of the high-skilled intensive sector, A_H/A_L, has two counteracting effects on relative wage rate as given by (14). First, the relative productivity of type$-h$ human capital in the production of composite intermediates rises, w_h/w_l increases for a given relative price of intermediates, $P \equiv P_H^X/P_L^X$. Second, however, since relatively more of the high-skilled intensive composite good is produced when A_H/A_L rises, the relative price of composite goods, P, decreases for given labor inputs. Through this effect, the relative value of the marginal product of type$-h$ human capital declines. If and only if the elasticity of substitution between the composite intermediates is sufficiently high, $\varepsilon > \psi > 1$, the first effect dominates the second one (vice versa if $\varepsilon < \psi < 1$).

The next result provides insights on relative R&D incentives in the two R&D sectors. The respective profits of an intermediate good firms (symmetric within sectors) are denoted by π_H and π_L.

Lemma 2. *The relative instantaneous profit of machine producers reads as*

$$\frac{\pi_H}{\pi_L} = \left(\frac{A_H}{A_L}\right)^{-\frac{1}{\psi}} \left(\frac{H^X}{L^X}\right)^{\frac{\psi-1}{\psi}}. \tag{15}$$

There are counteracting effects of an increase in relative employment in composite input production, H^X/L^X, on relative R&D incentives. First, for a given relative price of the high-skilled intensive good, P, relative profits in the high-skilled intensive sector rise ("market size effect"). Second, however, P falls in response to an increase in relative output of the high-skilled intensive good ("price effect"). In the case where $\varepsilon > \psi > 1$, the first effect dominates the second one, and vice versa if $\varepsilon < \psi < 1$.

Moreover, as already discussed after Lemma 1, an increase in the relative knowledge stock of the high-skilled intensive sector, A_H/A_L, reduces the relative price P. Thus, relative profits π_H/π_L decline. The magnitude of the elasticity of π_H/π_L with respect to A_H/A_L is inversely related to the "derived" elasticity between high-skilled and low-skilled labor in production, ψ.

5.2 Balanced Growth Equilibrium

It turns out that restricting focus on the case in which the derived elasticity of substitution is bounded upwards,

$$\psi \leq \frac{2-\phi-\theta}{1-\theta}, \tag{A3}$$

is sufficient for existence and uniqueness of a balanced growth equilibrium. We focus on this case throughout. The balanced growth equilibrium is characterized by

Proposition 1. *Under (A1)-(A3), there exists a unique balanced growth equilibrium which can be characterized as follows:*

(i) c_h, c_l, a_h, a_l, A_H, A_L, w_h, w_l, T *grow with rate* g*;*

(ii) L^X, H^X, H_H^A, H_L^A, P_H^A, P_L^A *grow with rate* n*;*

(iii) X_H, X_L *grow with rate* $g+n$*;*

(iv) r, P_H^X, P_L^X, h, u *are stationary;*

(v) the fraction of time a type$-h$ *individual participates in the labor market is independent of policy parameters and reads as*

$$u = \frac{\rho - n + (\sigma-1)g + (1-\eta)\delta_H}{\rho - n + (\sigma-1)g + (1-\eta+\beta)\delta_H} \equiv u^*; \tag{16}$$

(vi) the human capital level per type$-h$ *individual is increasing in the fraction of human capital devoted to publicly financed teaching,* ϑ*, and independent of other policy parameters; it reads as*

$$h = \left[\frac{\xi(1-u^*)^\beta \vartheta^\gamma}{\delta_H}\right]^{\frac{1}{1-\gamma-\eta}} \equiv h^*. \tag{17}$$

According to (5), Proposition 1 implies that also per capita income grows at rate g in steady state. The result parallels the well-known property of semi-endogenous growth models that the economy's growth rate is policy-independent (e.g. Jones, 1995, 2005). Interestingly, taxation and public education policy have no effect on the time allocation of type$-h$ individuals (part (v) of Proposition 1). This is true even during the transition to the steady state (not shown). These result are implications of assuming time-invariant policy instruments and dynastic households.

An increase in the fraction of human capital demanded by the government for educating type$-h$ individuals, $\vartheta = h^E/h$ (triggered by an increase in tax revenue share s^E), raises the long run supply of human capital (part (v) of Proposition 1). However, if ϑ were too high, public education expansion could lower the supply of human capital to private firms per type$-h$ individual, $S \equiv (uh - h^E)$. The next result provides us with a condition ruling out this implausible outcome for the long run.

Corollary 1. *The long run supply of human capital per type-h individual, $S^* \equiv (u^* - \vartheta)h^*$, is increasing in ϑ if and only if*

$$\vartheta < \frac{\gamma u^*}{1-\eta}. \tag{A4}$$

Finally, in line with empirical estimates suggesting that the elasticity of substitution between high-skilled and low-skilled labor is larger than one (Johnson, 1997), we focus on the case where

$$\psi > 1. \tag{A5}$$

The subsequent propositions 2 and 3 show the effects of changes in tax revenue shares s^E and s^T on the steady state wage rate (and wage income) of type$-l$ individuals, w_l^*, and on the relative wage per unit of human capital between type$-h$ and type$-l$ individuals in steady state, w_h^*/w_l^*.

Proposition 2. *Under (A1)-(A5), the wage rate of type$-l$ individuals in the long run, w_l^*, is increasing in s^E (or ϑ), and independent of s^T.*

First, recall from (17) that an increase in ϑ raises the long run level of human capital per type$-h$ individual, h^*. Under (A4), in the long run, the amount of human capital devoted to production, H^X, thus rises, in turn raising the output level of the human capital intensive composite income, X_H. For given knowledge stocks, because of the complementarity of composite inputs in final goods production, this raises the price of the low-skilled labor intensive composite input, P_L^X. Moreover, as discussed after Lemma 2, for $\psi > 1$ (assumption (A5)), the "market size effect" of an increase in H^X/L^X on relative profits for high-skilled intensive production, π_H/π_L, dominates the "price effect". Thus, an increase in ϑ spurs innovation directed to type$-h$ human capital relatively more, i.e. A_H/A_L rises. As this also raises relative output X_H/X_L of the composite goods, P_L^X increases also through this effect. As a result, the value of the marginal product of low-skilled labor unambiguously increases in response to expanding education. Second, an increase in s^T, which finances the transfer to type$-l$ individuals, is neutral with respect to the allocation of human capital, therefore leaving w_l^* unaffected.

Proposition 3. *Under (A1)-(A5), the following holds for the relative wage per unit of human capital between type$-h$ and type$-l$ individuals in the long run, w_h^*/w_l^*.*

(i) If $\psi = \frac{2-\phi-\theta}{1-\theta}$ (i.e., (A3) holds with equality), w_h^*/w_l^* is independent of s^E (or ϑ); otherwise (if $\psi < \frac{2-\phi-\theta}{1-\theta}$), w_h^*/w_l^* is decreasing in ϑ,

(ii) w_h^*/w_l^* is independent of s^T.

Consider an (endogenous) increase in the type$-h$ human capital for the production of composite inputs, raising H^X/L^X in long run equilibrium. As in models with an exogenous educational composition of the workforce, an increase in H^X/L^X could be triggered by an increase in the supply of skilled labor. As suggested by the discussion of Proposition 2, an increase in H^X/L^X could be driven by a higher fraction of human capital demanded by the government for teaching, ϑ. Also recall that, for $\psi > 1$, an increase in H^X/L^X spurs innovation directed to type$-h$ human capital relatively more, thus raising A_H/A_L. According to Lemma 1, for $\psi > 1$, an increase in the "relative knowledge stock", A_H/A_L, would raise the relative wage rate per unit of type$-h$ human capital, w_h/w_l, for given H^X/L^X. However, under limited (derived) substitutability between type$-h$ and type$-l$ labor, ψ, as assumed in (A3), the effect is not large enough to overturn the negative impact of an increase of H^X/L^X (as triggered by a higher fraction of human capital demanded by the government for teaching, ϑ) on w_h/w_l for a given relative knowledge stock, A_H/A_L (see (14) in Lemma 1). If (A3) holds with equality, both effects exactly cancel.

6 Trickle Down Dynamics

Like in section 3, we examine the dynamic implications of two policy experiments on net income of type$-l$ individuals, i.e. an increase in the share of the total tax revenue devoted to publicly financed higher education, s^E, versus an equally sized increase in the share of the total tax revenue devoted to redistributive transfers, s^T. Moreover, we discuss our conclusions in several respects.

To this end, we apply the relaxation algorithm (Trimborn, Koch and Steger, 2008) which is designed to deal with highly-dimensional and non-linear differential-algebraic equation systems. A favorable feature of the relaxation algorithm is that it does not rely on linearization of the underlying dynamic system. As we focus on potentially large policy shocks and long term macroeconomic dynamics, the initial deviation from the final steady state may be quite large. The differential-algebraic system is summarized in the online-appendix.

6.1 Sketch of the Calibration Strategy

The details of the calibration strategy are laid out in the appendix. Importantly, we view a type$-l$ individual as representative for high school drop-outs and a type$-h$ individual as representing an "average" educated worker. The parameter values are based on observables for the US economy in the 2000s before the financial crisis 2007-2009 (including policy parameters), assuming that the US was in steady state initially (i.e. before the considered policy shocks). Some parameters $-$ the economy's growth rate (g), the population growth rate (n), the mark-up factor (κ), the elasticity of substitution between high-skilled and low-skilled labor (ψ) $-$ are observed directly. The other parameters are matched to endogenous observables like the full-time equivalent of relative wage income of the different types of workers ("skill premium" $\Omega \equiv w_h h / w_l$), the fraction of time which type$-h$ individuals supply to the labor market (u), the capital-output ratio (K/Y) and the interest rate (r). It turns out that the steady state values of individual asset holdings depends on its initial distribution. We therefore also calibrate the relative amount of asset holdings between the two types of households initially, $a_{h,0}/a_{l,0}$.

Parameter	Value	Parameter	Value	Parameter	Value
n	0.01	α	0.4	τ_g	0.1
g	0.02	ε	1.83	τ_l	0.17
ψ	1.5	δ_K	0.04	τ_h	0.35
θ	0.5	δ_H	0.03	τ_r	0.17
ϕ	0.75	κ	1.3	s_E	0.1
ρ	0.02	β	0.25	s_T	0.07
r	0.07	η	0.35	ϑ	0.04
σ	1.91	γ	0.25	$\bar{l}$	0.15
ν	0.2	ξ	0.15	$a_{h,0}/a_{l,0}$	5

Table 1: Baseline set of parameters.

6.2 Expanding Education versus Extending Redistribution

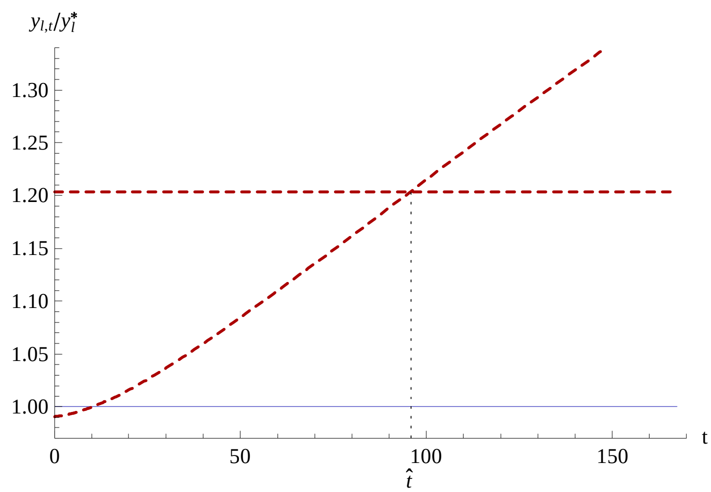

Figure 1: Time path of normalized income, y_l/y_l^*, in three scenarios: solid (blue) line: baseline scenario (s^T and s^E remain constant), horizontal dashed line: s^T increases by five percentage points), increasing dashed line: s^E increases by five percentage points. Parameter values as in Table 1.

In Figure 1, like for the simple model, we comparatively consider the effects of an increase in s^E and s^T by five percentage points on normalized net income of type$-l$

individuals, y_l/y_l^*, where superscript (*) again denotes initial steady state values. On impact, again, expanding education hurts the poor, whereas enhancing redistribution favors the poor, as compared to the baseline scenario. An increase in s^E diverts human capital (complementary to the type$-l$ workers) from manufacturing activity on impact (decrease in H^X) whereas increasing transfers leaves the human capital allocation unaffected. After about 11 years, net income of low-skilled workers in the scenario "education expansion" is equated with that in the baseline scenario. Eventually, unlike in panel (a) but like in panel (b) of Figure 0 displaying policy responses for the simple model, growth trickles down to the poor and makes them better off than under a "redistribution extension". (The mechanisms are discussed in detail in the next subsection.) Denote by $\hat{t}$ the time span (to be interpreted as the number of years) after a policy reform in $t = 0$ such that net income of low-skilled workers in the scenario "education expansion" is the same as in the scenario "redistribution extension" (and higher for $t > \hat{t}$). Figure 1 suggests that, in the US, $\hat{t} = 97$.

6.3 Decomposing the Effects of Expanding Education

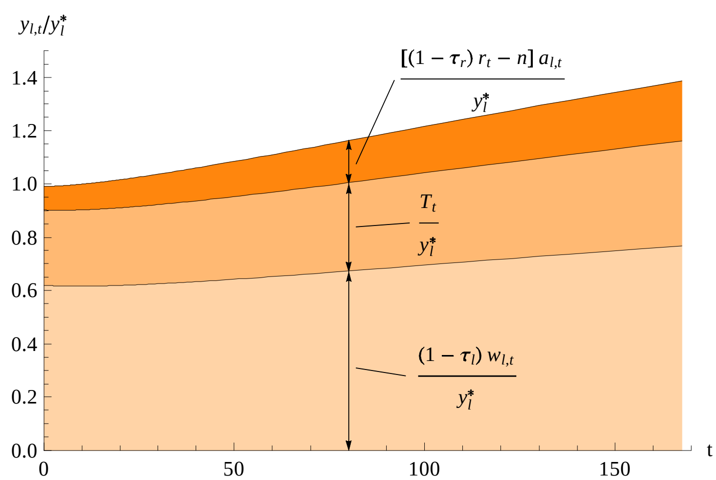

Figure 2: The time path of normalized income, y_l/y_l^*, and its additive components when s^E increases by five percentage points. Parameter values as in Table 1.

To gain more insights on why the poor are better off in the short run under extending redistributive transfers but are better off in the long run in case of expanding education, and to better understand the dynamic general equilibrium interactions, we consider a decomposition of normalized net income of a type$-l$ household in its additive components:

$$\frac{y_l}{y_l^*} = \frac{(1-\tau_l)w_l}{y_l^*} + \frac{[(1-\tau_r)r - n]\,a_l}{y_l^*} + \frac{T}{y_l^*}. \tag{18}$$

Assuming again that s^E is being increased from $s^E = 0.1$ to $s^E = 0.15$, Figure 2 displays the dynamic evolution of the three additive components as given by the right-hand side of (18), i.e. wage income net of taxes, $(1-\tau_l)w_l$, capital income net of taxes, $[(1-\tau_r)r - n]\,a_l$, and redistributive transfer, T, relative to total net income of a type$-l$ household in the long run, y_l^*. Apparently, the composition of income changes only slightly along the transition. Wage growth for type$-l$ workers and increased transfer income (which is driven by general economic growth) allows accumulation of assets.

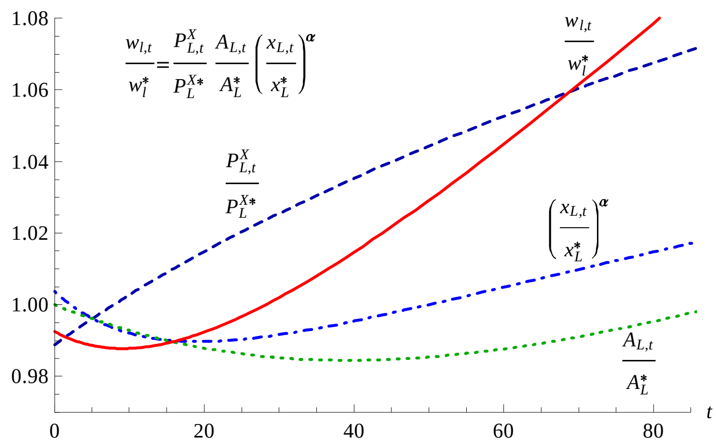

Figure 3: The time path of the normalized wage rate, w_l/w_l^*, and its multiplicative components when s^E increases by five percentage points. Parameter values as in Table 1.

It is apparent that the wage component dominates the other components. Thus, we decompose the wage component further. The solid (red) curve in Figure 3 displays the evolution of normalized wage income as given by[13]

$$\frac{w_l}{w_l^*} = \frac{P_L^X}{P_L^{X*}} \frac{A_L}{A_L^*} \left(\frac{x_L}{x_L^*}\right)^\alpha. \tag{19}$$

[13]The wage rate (value of the marginal product) of low-ability workers may, under within-group symmetry of machine producers, be expressed as $w_l = P_L^X (1-\alpha)(L^X)^{-\alpha} A_L (x_L)^\alpha$ (see (27) as derived in appendix).

It is driven by the three factors on the right-hand side of (19). Their evolution in response to an increase in s^E mirrors central human capital reallocation effects. First, a high-ability household devotes more time to education, i.e. $1-u$ increases. This implies a reduction in the total supply of skilled labor, $N_h uh$, on impact. Second, an increase in s^E allocates more skilled workers to the education sector (increase in teaching input h^E). Thus, skilled labor must be initially withdrawn from production of the human capital intensive good and from R&D. After the initial shock, total supply of skilled labor, $N_h uh$, increases because of human capital accumulation. The increased supply is then allocated to all uses of human capital during the transition.

As a consequence, normalized wage income drops downwards initially - the solid (red) curve starts below unity - and then starts to increase monotonically. The initial drop of w_l is driven by two opposing forces. First, the price of the low-skilled labor intensive composite (X_L-) input, P_L^X, drops on impact in response to a reallocation of human capital away from manufacturing. Second, the quantity of machines in this sector, with output x_L for all machine producers, goes up on impact despite the associated downward jump of price P_L^X. This initial response reflects the effect on the interest rate r (not shown), which declines on impact, in turn reducing marginal costs of machine producers. As human capital accumulates, there is a monotonic increase of P_L^X in the aftermath of the initial drop. As discussed after Proposition 2 for the long run, an increase in the amount of human capital used in production, H^X, leads to a higher level of the human capital intensive input, X_H. This pushes up the marginal product of the low-skilled intensive composite input, therefore raising P_L^X eventually. Moreover, in the first phase of the transition process after an increase in s^E, less R&D (directed to type$-l$ workers) is undertaken because of the reallocation of human capital towards educational production. A decrease in H_L^A depresses in turn the knowledge stock component A_L, as visualized by the downward sloping branch of the (green) dotted curve. However, eventually, as more human capital becomes available, more R&D is being undertaken than initially, which eventually also benefits low-skilled workers. Finally, the component $(x_L/x_L^*)^\alpha$ also follows a U-shaped evolution, partly driven by a similar evolution of A_L/A_L^*. The accumulation of machines in the low-skilled intensive sector is eventually fostered, as more machine types become available (A_L goes up) and as the low-skilled intensive good becomes more expensive (P_L^X rises).

In sum, the wage rate of low-skilled workers w_l and therefore net income y_l increase in the longer run in response to (i) rising prices of those goods that are produced low-skilled labor intensively (P_L^X), (ii) a more sophisticated state of technology due to more R&D targeted at the sector producing the low-skilled labor intensive input (raising A_L), and (iii) an accelerated accumulation of capital goods that are complementary to low-skilled labor (x_L). All of these mechanisms are fueled by the evolution of the supply and allocation of human capital over time.

6.4 Discussion

We now discuss the trickle-down dynamics by sensitivity analysis, by looking at consumption rather than income, and by examining alternative ways to change public expenditure for educational and redistributive purposes. To save space, the graphs supporting the following arguments and extended discussions are relegated to the online-appendix.

First, in addition to the extent and direction of endogenous technical change, we suspect the trickle-down growth mechanisms to critically depend on the (derived) elasticity of substitution between the two types of labor.

How does a change in ψ affect the time span $\hat{t}$ after which type$-l$ workers are better off if the government enhances public education (increasing s^E) compared to an increase in social transfers (increasing s^T)? For a derived elasticity of substitution, $\psi = 1.4$ (thus $\varepsilon = 1.67$ instead of $\varepsilon = 1.83$), type$-l$ individuals are better off from expanding education earlier than for the baseline calibration; we find that the threshold time span is $\hat{t} = 71$ (whereas $\hat{t} = 97$ for $\psi = 1.5$). The reason is simple: if both types of workers are better complements, type$-l$ individuals benefit more from human capital accumulation. In the case where $\psi = 1.6$ (i.e. $\varepsilon = 2$), $\hat{t}$ rises to 147 years. Further sensitivity analysis shows that the threshold time span $\hat{t}$ exists for reasonably high elasticities of substitution and is increasing in ψ throughout; $\hat{t}$ is convex for low values and concave for high values of ψ (see Figure A.1 in online-appendix).

Second, we may ask if the dynamic effects on the consumption level of type$-l$ individuals, c_l, eventually determining their welfare, is similar to the dynamic effects on net income, y_l. This is indeed the case (see Figure A.2). The initial drop in response to an increase in s^E is somewhat less pronounced, which reflects consumption smoothing. In the longer run, type$-l$ households eventually gain more from in increase in s^E than in s^T also in terms of consumption.

Third, so far we have focussed on an evaluation of changing s^E and s^T separately, necessarily being associated with a decrease in the government expenditure share of a third spending category. We may alternatively consider an increase (decrease) in the fraction of total tax revenue devoted to education while at the same time reducing (raising) the fraction devoted to transfers such that the sum of the two fractions, $s^E + s^T$, remains constant. We find that, again, a policy reform towards expanding education is harmful in the shorter run and improves situation of type$-l$ individuals in the longer run (see Figure A.3). Moreover, extending redistribution at the expense of education expenditure is beneficial in the shorter run but lowers y_l in the longer run even compared to the baseline scenario without policy reform.

Fourth, so far we have left the tax rates constant and focussed on a change in government expenditure shares, s^E and s^T. What happens if we increase tax rates to finance an increase in education expenditure or redistributive transfers? For instance, suppose we increase the tax rate on bond holdings, τ_r, to finance an increase in transfer T, while holding constant the government expenditure share on transfers, s^T. The fraction of human capital devoted to higher education, $\vartheta = h^E/h$ is unchanged as well. Such a policy reform benefits type$-l$ households on impact but soon becomes harmful even compared to the the baseline scenario without policy reform (see Figure A.4). The reason is the distortion of capital income taxation on savings and R&D investments by which higher transfers are financed. The fraction of human capital devoted to both kinds of R&D declines, eventually depressing net income y_l. Alternatively, we may finance an increase in ϑ by an increase in τ_r, while holding the education expenditure share s^E constant. As we start in long run equilibrium, transfers initially grow at rate g. We thus fix the growth-adjusted transfer $\tilde{T} \equiv Te^{-gt}$ at its initial level for this policy experiment. We find that type$-l$ households lose on impact because of the reallocation of human capital towards educational production (also displayed in Figure A.4). During the further transition, the distortion of R&D investments causes a further decline in y_l. Comparing both policies, again, expanding education is better in the longer run and worse in the shorter run than extending redistribution.

Fifth, examining a similar comparative policy evaluation to the previous one by raising the labor income tax rate of type$-h$ individuals, τ_h, instead of τ_r, suggests qualitatively similar dynamic effects than displayed in Figure 1 (see Figure A.5).

7 Inequality Dynamics

We finally discuss the dynamic implications of policy reforms on inequality. We consider the evolution of both the skill premium, $\Omega = w_h h/w_l$, and the relative net income between the two types of workers, y_h/y_l, again in response to rasing s^E and s^T by five percentage points.

7.1 Skill Premium

In Figure 4, the solid (blue) line displays the skill premium in the baseline scenario and the dashed (red) line shows its evolution in response to an increase in s^E. (The skill premium is unaffected by an increase in s^T.) Figure 4 demonstrates that expanding education raises earnings inequality in the short run as well as in the long run. The initial jump is driven by several reallocation effects discussed above which reduce employment in the human capital intensive production sector. The drop in H^X impacts directly on the relative wage rate, w_h/w_l, see (14). Thus, the relative wage rate jumps upwards. Along the transition, the increase in earnings inequality is mainly driven by an increase in the level of human capital per type$-h$ household, h, i.e. by an increase in human capital inequality across individuals.[14] In the long run, Ω is increased exclu-

[14] In the US average years of schooling increased steadily over the period 1880 to 1980 (Goldin and Katz, 2007, Figure 7). Rising human capital inequality as an explanation of an increasing skill

sively because of an increase in h, under the presumption of part (i) of Proposition 3, which applies for our preferred calibration.

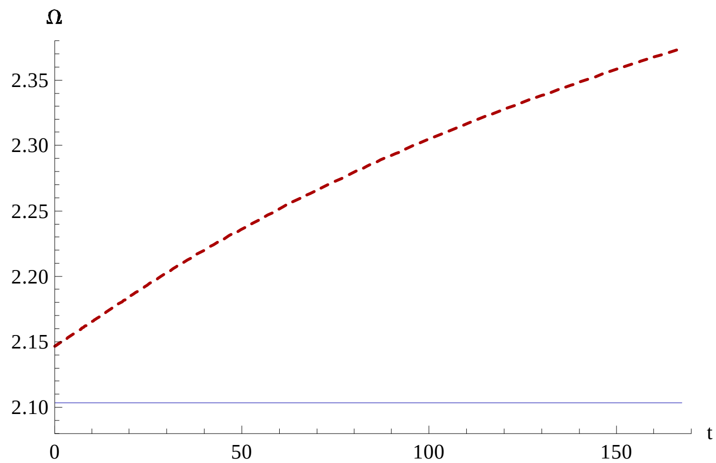

Figure 4: Time path of the skill premium $\Omega = w_h h / w_l$. Solid (blue) line: baseline scenario, and increase in s^T, increasing dashed line: s^E increases by five percentage points. Parameter values as in Table 1.

7.2 Income Inequality

Does rising earnings inequality in response to an expansion of education imply that also inequality of net income rises over time? At the first glance, given that also initial asset holdings are higher for type$-h$ individuals ($a_{h,0} > a_{l,0}$) and the long run interest rate is rather high for the calibration in Table 1 (particularly, $r > g$), we may suspect that this is the case. Interestingly, however, Figure 5 suggests that these conditions are not sufficient for relative net income between the two groups of households, y_h/y_l, to rise during the entire transition in response to an increase in s^E. Earlier in the transition y_h/y_l indeed rises, reflecting a rising skill premium over time. However, it turns out that type$-l$ individuals choose their consumption path such that they accumulate assets faster than type$-h$ individuals during the entire transition (not shown); that is, $\dot{a}_l/a_l > \dot{a}_h/a_h$. The implied (although less than full) convergence of asset holdings eventually drives down inequality of net income even below the initial level without

premium does not, in contrast to Acemoglu (2002), require the (derived) elasticity of skilled labor and unskilled labor, ψ, to be larger than 2.

policy reform! An increase in the fraction of tax revenue for redistributive transfers, s^T, drives down income inequality more substantially, however. Noteworthy and consistent with empirical evidence, in our calibrated model, the rich choose a higher savings rate (not shown), i.e. $\dot{a}_l/y_l < \dot{a}_h/y_h$.

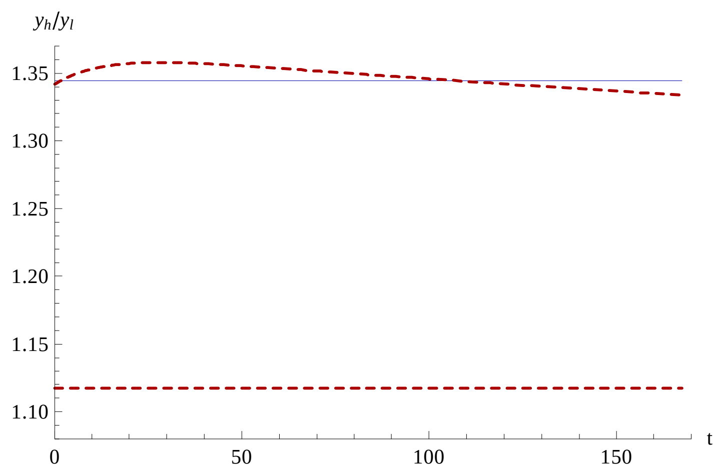

Figure 5: Time path of relative net income, y_h/y_l. Solid (blue) line: baseline scenario, increasing dashed line: s^T increases by five percentage points, increasing dashed line: s^E increases by five percentage points. Parameter values as in Table 1.

7.3 Discussion

Comparing the effects of the two considered policy options on income inequality, displayed in Figure 5, suggests an equity-efficiency trade-off. The dynamic, inverted U-shaped effect of expanding education, suggested by Figure 5, reminds us on the famous Kuznets curve and is intriguing in itself. It contributes to the recent debate, greatly popularized by Piketty (2014), on the past and future evolution of income inequality. Our analysis suggests that income inequality may eventually go down. The result is surprising to the extent that our setup is favorable for income inequality to rise over time in response to expanding education: it features divergence in earnings with higher earnings growth rates for the initially wealthy during the entire transition. Moreover, our analysis is consistent with the evidence by Piketty (2014) and Piketty and Zucman (2014) that $r > g$ prevails most of the time in history. Nevertheless, we have shown that these features do not allow us to draw strong conclusions on the future evolution of the income distribution, unlike suggested by Piketty (2014). It is interesting that the decline in inequality is not just a theoretical possibility but predicted by our preferred calibration to the US economy.

8 Conclusion

The goal of this paper was to understand whether and, if yes, when economic growth caused by an increase in public education expenditure on behalf of high-ability individuals trickles down to the least educated. We contrasted the dynamic effects of that kind of education expansion with those of an equally sized increase in redistributive transfers. In our dynamic general equilibrium model, public expenditures are financed by various distortionary income taxes, human capital accumulation is endogenous, and R&D-based technical change could be directed to complement either high-skilled or low-skilled labor. In the shorter run, the poor are better off from an increase in the fraction of government spending devoted to redistributive transfers and lose from expanding education. Consistent with empirical evidence for the US from the 1970s onwards, our analysis suggests that human capital accumulation is accompanied by falling or stagnating earnings of low-skilled individuals early in the transition phase and rising skill premia. In the longer run, however, our model predicts that low-ability workers eventually benefit more from promoting education of high-ability workers. The time span for this to happen critically depends on the elasticity of substitution between high-skilled and low-skilled workers. The higher this elasticity is, the faster the poor benefit from expanding education. The trickle-down effect is driven by an eventual increase in the level of human capital devoted towards R&D in the sector producing low-skilled intensive goods.

Moreover, our analysis suggests that expanding education on behalf of high-ability workers triggers an inverted U-shaped evolution of income inequality. The result is remarkable in view of the prediction of a considerable and sustained increase in the skill premium, the assumption that low-ability households start with lower initial asset holdings, and an interest rate which exceeds the long run growth rate of earnings

$(r > g)$. However, redistributive transfers are more successful in reducing inequality of net income compared to extending public education finance.

In sum, we identified two kinds of trade-offs regarding the alternative policies we consider which are potentially informative for the recent policy debate on income inequality. First, there is a dynamic trade-off with respect to absolute income of the poor. In the shorter run, low-ability households would always prefer higher transfers. If the goal of policy makers is, however, to improve absolute living standards of these households in the longer run, promoting education of high-ability workers is more promising. Second, there is a long run trade-off between the goal of raising living standards of the poor and reducing income inequality. Although both policy measures we considered lead to an eventual decline of net income dispersion, redistributive transfers are more successful in this respect.

These complex trade-offs call for a careful normative analysis which is left for future research. It also appears valuable to check robustness of our results in an alternative framework which highlights intergenerational conflicts resulting from the kind of policy trade-offs suggested by our analysis.

Appendix

Definition of Equilibrium (comprehensive model). Let P_H^X and P_L^X denote the price of the high-skilled intensive and low-skilled intensive composite intermediate good used in the final goods sector, respectively, and $p_H(i)$, $p_L(i)$ the prices of machine i in the respective composite input sector. Moreover, let P_H^A and P_L^A denote the present discounted value of the profit stream generated by an innovation in the low-skilled and high-skilled intensive sector, respectively. These are equal to equity prices. The exclusion of arbitrage possibilities in the financial market implies that the after-tax returns from equity (capital gains and dividends) in both sectors and bonds and must be equal; that is,

$$(1-\tau_g)\frac{\dot{P}_H^A}{P_H^A} + (1-\tau_r)\frac{\pi_H}{P_H^A} = (1-\tau_g)\frac{\dot{P}_L^A}{P_L^A} + (1-\tau_r)\frac{\pi_L}{P_L^A} = (1-\tau_r)r. \quad (20)$$

For given policy parameters $(\tau_g, \tau_r, \tau_h, \tau_l, s^E, s^T)$, an equilibrium consists of time paths for quantities $\{H_t^X, L_t^X, H_{Ht}^A,$

$H_{Lt}^A, h_t, u_t, h^E, X_{Ht}, X_{Lt}, \{x_{Ht}(i)\}_{i\in[0,A_{Ht}]}, \{x_{Lt}(i)\}_{i\in[0,A_{Lt}]}, A_{Ht}, A_{Lt}, c_{ht}, c_{lt}, a_{ht}, a_{lt}, T_t\}$

and prices $\{P_{Ht}^X, P_{Lt}^X, \{p_{Ht}(i)\}_{i\in[0,A_{Ht}]}, \{p_{Lt}(i)\}_{i\in[0,A_{Lt}]}, P_{Ht}^A, P_{Lt}^A, w_{ht}, w_{lt}, r_t\}$ such that

1. R&D firms and producers of the final good, the composite intermediate goods, and machines maximize profits;[15]

2. type$-h$ households maximize utility U_h s.t. (2) and (12); type$-l$ households maximize U_l s.t. (13);[16]

3. the no-arbitrage conditions (20) in the financial market hold;

4. the total value of assets (owned by households) fulfills

$$N_h a_h + N_l a_l = K + P_H^A A_H + P_L^A A_L, \tag{21}$$

where K is given by (8).

5. the labor markets for type–h and type–l workers clear:

$$\begin{aligned} H^X + H_H^A + H_L^A + N_h h^E &= N_h u h, && (22)\\ L^X &= N_l. && (23) \end{aligned}$$

6. The government budgets for transfers to type$-l$ individuals (T) and education (human capital devoted to education of type$-h$ individuals, as fraction ϑ of the total) are balanced each period.

Proof of Lemma 1. According to (5), inverse demand functions in the composite input sectors are given by

$$P_H^X = \frac{\partial Y}{\partial X_H} = \left(\frac{Y}{X_H}\right)^{\frac{1}{\varepsilon}}, \; P_L^X = \frac{\partial Y}{\partial X_L} = \left(\frac{Y}{X_L}\right)^{\frac{1}{\varepsilon}}. \tag{24}$$

[15] Condition 1 implies that the composite intermediate goods markets and the market for machines clear.

[16] Households also observe standard non-negativity constraints which lead to transversality conditions (see the proof of Proposition 1).

Thus, relative intermediate goods demand is given by

$$\frac{X_H}{X_L} = \left(\frac{P_H^X}{P_L^X}\right)^{-\varepsilon} \tag{25}$$

According to (7), the inverse demand for machine i in the human capital intensive sector is $p_H(i) = \alpha P_H^X (H^X/x_H(i))^{\alpha-1}$. Machine producers, being able to transform one unit of the final good to one unit of output, have marginal production costs equal to the sum of the interest rate and the capital depreciation rate, $r + \delta_K$. In absence of a competitive fringe, the incumbent's profit-maximizing price would be $(r + \delta_K)/\alpha$. A price equal to $\kappa(r + \delta_K)$ (the marginal cost of the competitive fringe) is the maximal price, however, a producer can set without losing the entire demand. Since $\kappa \leq 1/\alpha$, it is also the optimal price. Thus, with $p_H(i) = p_L(i) = \kappa(r + \delta_K)$ for all i,

$$x_H(i) = x_H = \left(\frac{\alpha P_H^X}{\kappa(r + \delta_K)}\right)^{\frac{1}{1-\alpha}} H^X \overset{(6)}{\Longrightarrow} X_H = A_H H^X \left(\frac{\alpha P_H^X}{\kappa(r + \delta_K)}\right)^{\frac{\alpha}{1-\alpha}}, \tag{26}$$

$$x_L(i) = x_L = \left(\frac{\alpha P_L^X}{\kappa(r + \delta_K)}\right)^{\frac{1}{1-\alpha}} L^X \overset{(7)}{\Longrightarrow} X_L = A_L L^X \left(\frac{\alpha P_L^X}{\kappa(r + \delta_K)}\right)^{\frac{\alpha}{1-\alpha}}, \tag{27}$$

Hence, relative supply of composite inputs is

$$\frac{X_H}{X_L} = \frac{A_H H^X}{A_L L^X} \left(\frac{P_H^X}{P_L^X}\right)^{\frac{\alpha}{1-\alpha}}. \tag{28}$$

Equating the right-hand sides of (25) and (28) and using $\psi = \alpha + \varepsilon(1 - \alpha)$ leads to an expression for the relative price of the composite inputs,

$$P \equiv \frac{P_H^X}{P_L^X} = \left(\frac{A_H H^X}{A_L L^X}\right)^{-\frac{1-\alpha}{\psi}}, \tag{29}$$

which is inversely related to the relative "efficiency units" of high-skilled to low-skilled labor in production activities, $\frac{A_H H^X}{A_L L^X}$.

According to (6) and (7), wage rates per unit of high-skilled and low-skilled labor are given by $w_h = P_H^X (1 - \alpha) X_H / H^X$ and $w_l = P_L^X (1 - \alpha) X_L / L^X$, respectively. Dividing both equations and using both (28) and (29) confirms (14). ■

Proof of Lemma 2: According to (26) and (27), the instantaneous profits of machine producers, $\pi_H = (\kappa - 1)(r + \delta_K)x_H$ and $\pi_L = (\kappa - 1)(r + \delta_K)x_L$, read as

$$\pi_H = (\kappa - 1)\left(\frac{\alpha}{\kappa}P_H^X\right)^{\frac{1}{1-\alpha}}(r + \delta_K)^{-\frac{\alpha}{1-\alpha}}H^X, \tag{30}$$

$$\pi_L = (\kappa - 1)\left(\frac{\alpha}{\kappa}P_L^X\right)^{\frac{1}{1-\alpha}}(r + \delta_K)^{-\frac{\alpha}{1-\alpha}}L^X. \tag{31}$$

Dividing both expressions, substituting (29) and noting from the definition of ψ that $\frac{\alpha}{1-\alpha} = \frac{\varepsilon-\psi}{\psi-1}$ confirms (15). ■

Proof of Proposition 1: First, we define $l^X \equiv L^X/N_h$, $h^X \equiv H^X/N_h$. We also define $h_k^A \equiv H_k^A/N_h$, $p_k^A \equiv P_k^A/N_h$, $k \in \{H, L\}$. With these definitions as well as expressions $N_l/N_h = \bar{l}$ and $h^E = \vartheta h$ from (3) we can rewrite labor market clearing conditions (22) and (23) as

$$\begin{aligned} h^X + h_H^A + h_L^A &= (u - \vartheta)h, && (32)\\ l^X &= \bar{l}. && (33) \end{aligned}$$

Moreover, let $\tilde{z}_t \equiv z_t e^{-gt}$ for $z \in \{T, c_h, c_l, a_h, a_l, w_h, w_l, A_h, A_l\}$. That is, if a variable z grows with rate g in the long run, then $\tilde{z}$ is stationary. Combining (8) and (21) and substituting both (26) and (27), we then have

$$\begin{aligned} \tilde{a}_h + \bar{l}\tilde{a}_l &= \tilde{A}_H\left(\frac{\alpha}{\kappa(r+\delta_K)}P_H^X\right)^{\frac{1}{1-\alpha}}h^X + \\ &\quad \tilde{A}_L\left(\frac{\alpha}{\kappa(r+\delta_K)}P_L^X\right)^{\frac{1}{1-\alpha}}\bar{l} + p_H^A\tilde{A}_H + p_L^A\tilde{A}_L. \end{aligned} \tag{34}$$

The representative R&D firm which directs R&D effort to the human capital intensive sector maximizes

$$P_H^A\dot{A}_H - w_h H_H^A = P_H^A\tilde{\nu}_H(A_H)^{\phi}H_H^A - w_h H_H^A, \tag{35}$$

taking A_H and $\tilde{\nu}_H$ as given. Analogously for the R&D sector targeted to machines which are complementary to low-skilled labor. Thus, using (9) and (10), we have

$$P_H^A \nu \left(A_H\right)^{\phi} \left(H_H^A\right)^{-\theta} = P_L^A \nu \left(A_L\right)^{\phi} \left(H_L^A\right)^{-\theta} = w_h. \tag{36}$$

From (36) we can write

$$p_H^A \nu \left(\tilde{A}_H\right)^{\phi-1} \left(h_H^A\right)^{-\theta} = \frac{\tilde{w}_h}{\tilde{A}_H}, \tag{37}$$

$$p_L^A \nu \left(\tilde{A}_L\right)^{\phi-1} \left(h_L^A\right)^{-\theta} = \frac{\tilde{w}_h}{\tilde{A}_L}. \tag{38}$$

We turn next to composite input prices. Combining (24) with (5) implies

$$P_H^X = \left[1 + \left(\frac{X_H}{X_L}\right)^{-\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}, \tag{39}$$

$$P_L^X = \left[1 + \left(\frac{X_H}{X_L}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}. \tag{40}$$

Substituting (29) into (25) we find

$$\frac{X_H}{X_L} = \left(\frac{A_H H^X}{A_L L^X}\right)^{\frac{\varepsilon(1-\alpha)}{\varepsilon(1-\alpha)+\alpha}}. \tag{41}$$

Substituting (41) into (39) and (40), and using $A_H/A_L = \tilde{A}_H/\tilde{A}_L$, $H^X/L^X = h^X/\bar{l}$ and $\psi = \varepsilon(1-\alpha)+\alpha$, we obtain

$$P_H^X = \left[1 + \left(\frac{\tilde{A}_H h^X}{\tilde{A}_L \bar{l}}\right)^{-\frac{\psi-1}{\psi}}\right]^{\frac{1}{\varepsilon-1}}, \tag{42}$$

$$P_L^X = \left[1 + \left(\frac{\tilde{A}_H h^X}{\tilde{A}_L \bar{l}}\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{1}{\varepsilon-1}}. \tag{43}$$

The current-value Hamiltonian which corresponds to the optimization problem of a type$-h$ household (see Definition 1) is given by

$$\begin{aligned}\mathcal{H}_h &= \frac{(c_h)^{1-\sigma}-1}{1-\sigma} + \mu\left\{\xi(1-u)^{\beta}(h^E)^{1-\beta}h^{\eta} - \delta h\right\} + \\ & \lambda\left\{\left[(1-\tau_r)r - n\right]a_h + (1-\tau_h)w_h u h - c_h\right\}, \end{aligned} \tag{44}$$

where μ and λ are multipliers (co-state variables) associated with constraints (2) and (12), respectively. Necessary optimality conditions are $\partial\mathcal{H}_h/\partial c_h = \partial\mathcal{H}_h/\partial u = 0$ (control variables), $\dot{\mu} = (\rho-n)\mu - \partial\mathcal{H}_h/\partial h$, $\dot{\lambda} = (\rho-n)\lambda - \partial\mathcal{H}_h/\partial a_h$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda = (c_h)^{-\sigma}, \tag{45}$$

$$\mu\xi\beta(1-u)^{\beta-1}(h^E)^{\gamma}h^{\eta} = \lambda\left(1-\tau_h\right)w_h h, \tag{46}$$

$$\frac{\dot{\mu}}{\mu} = \rho - n - \xi\eta\left(1-u\right)^{\beta}\left(h^E\right)^{\gamma}h^{\eta-1} + \delta_H - \frac{\lambda}{\mu}(1-\tau_h)w_h u, \tag{47}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau_r)r, \tag{48}$$

$$\lim_{t\to\infty}\mu_t e^{-(\rho-n)t}h_t = 0, \tag{49}$$

$$\lim_{t\to\infty}\lambda_t e^{-(\rho-n)t}a_{ht} = 0. \tag{50}$$

Differentiating (45) with respect to time and using (48) together with the definition of $\tilde{c}_h$, we obtain Euler equation

$$\frac{\dot{\tilde{c}}_h}{\tilde{c}_h} = \frac{(1-\tau_r)r-\rho}{\sigma} - g. \tag{51}$$

Next define $m_t \equiv \mu_t e^{(\sigma-1)gt}$. Combining (45) and (46) we can then write by using $h^E = \vartheta h$ and the definitions of $\tilde{c}_h$ and $\tilde{w}_h$:

$$m\xi\beta(1-u)^{\beta-1}\vartheta^{\gamma}h^{\eta+\gamma-1} = \left(\tilde{c}_h\right)^{-\sigma}(1-\tau_h)\tilde{w}_h, \tag{52}$$

whereas combining (46) and (47) and making use of (45) and (52),

$$\frac{\dot{m}}{m} = \delta_H + \rho - n + (\sigma - 1)g - \left[\eta\,(1-u) + u\beta\right]\xi(1-u)^{\beta-1}\vartheta^{\gamma}h^{\eta+\gamma-1}. \tag{53}$$

Moreover, (12) can be written as

$$\frac{\dot{\tilde{a}}_h}{\tilde{a}_h} = (1-\tau_r)r - n + (1-\tau_h)\frac{\tilde{w}_h uh}{\tilde{a}_h} - \frac{\tilde{c}_h}{\tilde{a}_h} - g. \tag{54}$$

For low-skilled types (who decide about their consumption profile only), we find analogously to (51) that

$$\frac{\dot{\tilde{c}}_l}{\tilde{c}_l} = \frac{(1-\tau_r)r - \rho}{\sigma} - g. \tag{55}$$

By using (13) we also obtain

$$\frac{\dot{\tilde{a}}_l}{\tilde{a}_l} = (1-\tau_r)r - n + (1-\tau_l)\frac{\tilde{w}_l}{\tilde{a}_l} - \frac{\tilde{c}_l}{\tilde{a}_l} + \frac{\tilde{T}}{\tilde{a}_l} - g. \tag{56}$$

Using $A_k = \tilde{A}_k e^{gt}$, $H_k^A = N_h h_k^A$, $k \in \{H, L\}$, as well as $N_{h,0} = 1$ and $g = \frac{(1-\theta)n}{1-\phi}$ we can also rewrite (9) and (10) as

$$\begin{aligned} \frac{\dot{\tilde{A}}_H}{\tilde{A}_H} &= \nu(\tilde{A}_H)^{\phi-1}\left(h_H^A\right)^{1-\theta} - g, && (57)\\ \frac{\dot{\tilde{A}}_L}{\tilde{A}_L} &= \nu(\tilde{A}_L)^{\phi-1}(h_L^A)^{1-\theta} - g. && (58) \end{aligned}$$

Recall that competitive wage rates read as $w_h = P_H^X(1-\alpha)X_H/H^X$ and $w_l = P_L^X(1-\alpha)X_L/L^X$. Combining these expressions with (26) and (27), respectively, we find for adjusted wage rates:

$$\begin{aligned} \tilde{w}_h &= (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}}\tilde{A}_H\left(P_H^X\right)^{\frac{1}{1-\alpha}}, && (59)\\ \tilde{w}_l &= (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}}\tilde{A}_L\left(P_L^X\right)^{\frac{1}{1-\alpha}}. && (60) \end{aligned}$$

Substituting (30) and (31) into (20) implies

$$\dot{p}_H^A + np_H^A = \frac{1-\tau_r}{1-\tau_g}\left(rp_H^A - \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}P_H^X\right)^{\frac{1}{1-\alpha}} h^X}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}}\right), \tag{61}$$

$$\dot{p}_L^A + np_L^A = \frac{1-\tau_r}{1-\tau_g}\left(rp_L^A - \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}P_L^X\right)^{\frac{1}{1-\alpha}} l^X}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}}\right). \tag{62}$$

Combining (8) with (26) and (27) we can write

$$K = A_H\left(\frac{\alpha P_H^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} H^X + A_L\left(\frac{\alpha P_L^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} L^X. \tag{63}$$

The total tax revenue ($\mathcal{TTR}$) is the sum of the revenue from taxation of labor income and returns to asset holding,

$$\mathcal{TTR} = \tau_h N_h w_h uh + \tau_l N_l w_l + \tau_r rK + \tau_g(\dot{P}_H^A A_H + \dot{P}_L^A A_L) + \tau_r(\pi_H A_H + \pi_L A_L). \tag{64}$$

Note that $\dot{P}_H^A/N_h = \dot{p}_H^A + np_H^A$ and $\dot{P}_L^A/N_h = \dot{p}_L^A + np_L^A$, as given by the right-hand side of (61) and (62), respectively. Thus, using $l^X = \bar{l}$ from (33), (30), (31) and (63) in (64) we obtain

$$\begin{aligned}\Xi \equiv \frac{\mathcal{TTR}}{N_h e^{gt}} &= \tau_h \tilde{w}_h uh + \tau_l \tilde{w}_l \bar{l} + \frac{(1-\tau_r)\tau_g}{1-\tau_g} r(p_H^A \tilde{A}_H + p_L^A \tilde{A}_L) + \\ &\frac{\tau_r - \tau_g}{1-\tau_g}\frac{(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}}\left(\left(P_H^X\right)^{\frac{1}{1-\alpha}} h^X \tilde{A}_H + \left(P_L^X\right)^{\frac{1}{1-\alpha}} \bar{l}\tilde{A}_L\right) \\ &\tau_r r\left(\tilde{A}_H\left(\frac{\alpha P_H^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} h^X + \tilde{A}_L\left(\frac{\alpha P_L^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} \bar{l}\right).\end{aligned} \tag{65}$$

The publicly financed expenditure is given by $w_h N_h h^E = s^E \mathcal{TTR}$. Thus, recalling $h^E = \vartheta h$, the fraction of tax revenue devoted to education, s^E, and the human capital share devoted to teaching, ϑ, are related according to

$$\vartheta \tilde{w}_h h = s^E \Xi. \tag{66}$$

Similarly, the aggregate transfer payments read as $N_l T = s^T \mathcal{T} \mathcal{T} \mathcal{R}$, implying that

$$\tilde{T} = \frac{s^T \Xi}{\bar{l}}, \tag{67}$$

The dynamical system, on which our numerical analysis is based, is given by (2), (32)-(34), (37), (38), (42), (43), (51)-(62), (66) and (67), with Ξ as given by (65).

To prove that a steady state with the properties stated in Proposition 1 exists, we need to show that h_k^A, p_k^A, P_k^X, $\tilde{A}_k$ ($k \in \{H, L\}$), l^X, h^X, u, h, $\tilde{T}$, $\tilde{c}_j$, $\tilde{a}_j$, $\tilde{w}_j$ ($j \in \{l, h\}$), r and m are stationary in the long run. To see this, we derive steady state values of the just derived dynamical system in what follow. First, set $\dot{h}/h = 0$ and $h^E = \vartheta h$ in (2) to find $\xi(1-u)^\beta \vartheta^\gamma h^{\eta+\gamma-1} = \delta_H$, which can also be rewritten as

$$h = \left[\frac{\xi(1-u)^\beta \vartheta^\gamma}{\delta_H}\right]^{\frac{1}{1-\gamma-\eta}}. \tag{68}$$

Using (68) in (53) and setting $\dot{m} = 0$ confirms (16), consistent with part (v). Evaluating (68) at $u = u^*$ then gives us (17), confirming part (vi). Note that h^* is indeed time-invariant (i.e., $\dot{h} = 0$ for $t \to \infty$), according to (16).

Next, set $\dot{\tilde{c}}_h = 0$ in (51) to find that

$$(1-\tau_r) r = \rho + \sigma g. \tag{69}$$

Thus, also $\dot{\tilde{c}}_l = 0$ holds, according to (55). Next, set $\dot{\tilde{A}}_H = \dot{\tilde{A}}_L = 0$ in (57) and (58) to obtain

$$\tilde{A}_H = \left(\frac{\nu \left(h_H^A\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}}, \tag{70}$$

$$\tilde{A}_L = \left(\frac{\nu \left(h_L^A\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}}, \tag{71}$$

respectively, or $\nu(\tilde{A}_k)^{\phi-1} = \left(h_k^A\right)^{\theta-1} g$, $k \in \{H, L\}$. Using the latter together with (59) in (37) and (38) yields

$$p_H^A = (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \left(P_H^X\right)^{\frac{1}{1-\alpha}} \frac{h_H^A}{g}, \tag{72}$$

$$p_L^A = (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \left(P_H^X\right)^{\frac{1}{1-\alpha}} \frac{h_L^A}{g}\frac{\tilde{A}_H}{\tilde{A}_L}, \tag{73}$$

respectively. Now substitute (69) and (72) into (61) and set $\dot{p}_H^A = 0$ to find

$$h_H^A = \Gamma(\tau_r, \tau_g) h^X, \tag{74}$$

where

$$\Gamma(\tau_r, \tau_g) \equiv \frac{1-\frac{1}{\kappa}}{\frac{1}{\alpha}-1} \frac{(1-\tau_r)g}{\rho+\sigma g-(1-\tau_g)n}. \tag{75}$$

Note that $\Gamma > 0$ under (A1). Similarly, substituting $l^X = \bar{l}$, (69) and (73) into (62) and setting $\dot{p}_L^A = 0$ we obtain

$$h_L^A = \frac{\Gamma(\tau_r, \tau_g)\bar{l}}{P^{\frac{1}{1-\alpha}} \frac{\tilde{A}_H}{\tilde{A}_L}}. \tag{76}$$

From (74) and (76) we get

$$\frac{h_H^A}{h_L^A} = \frac{\tilde{A}_H}{\tilde{A}_L} \frac{h^X}{\bar{l}} P^{\frac{1}{1-\alpha}}. \tag{77}$$

Moreover, (70) and (71) imply that

$$\frac{\tilde{A}_H}{\tilde{A}_L} = \left(\frac{h_H^A}{h_L^A}\right)^{\frac{1-\theta}{1-\phi}} = \left(\frac{h^X}{\bar{l}} P^{\frac{1}{1-\alpha}}\right)^{-\frac{1-\theta}{\phi-\theta}}, \tag{78}$$

where the latter equation follows after substituting (77).

Next, substitute $A_H/A_L = \tilde{A}_H/\tilde{A}_L$ as given by (78) into (29), and use $H^X/L^X = h^X/\bar{l}$ to obtain

$$P^{\frac{1}{1-\alpha}} = \left(\frac{h^X}{\bar{l}}\right)^{-\frac{1-\phi}{1-\theta-\psi(\phi-\theta)}}. \tag{79}$$

It is easy to show by recalling $\phi < 1$ that assumption (A3) implies $1-\theta > \psi(\phi-\theta)$.

Substituting (78) into (76) and using both (79) and $\frac{\alpha}{1-\alpha} = \frac{\varepsilon-\psi}{\psi-1}$ then leads to

$$h_L^A = \Gamma(\tau_r, \tau_g) \left(h^X\right)^{\varrho} \bar{l}^{1-\varrho}, \text{ where} \tag{80}$$

$$\varrho \equiv \frac{2-\phi-\psi+\theta(\psi-1)}{1-\theta-\psi(\phi-\theta)} \tag{81}$$

(thus $1-\varrho = \frac{(\psi-1)(1-\phi)}{1-\theta-\psi(\phi-\theta)}$). According to assumption (A3), $\varrho \geq 0$.

The supply of human capital to private firms per type-h individual (the right-hand side of ((32))) is, in the long run, given by

$$(u^* - \vartheta)h^* = (u^* - \vartheta) \left[\frac{\xi(1-u^*)^{\beta}\vartheta^{\gamma}}{\delta_H}\right]^{\frac{1}{1-\gamma-\eta}} \equiv S^*(\vartheta). \tag{82}$$

Denote the long run value of h^X by h^{X*}. Substituting (74) and (80) into labor market clearing condition (32) implies that h^{X*} is implicitly defined by

$$\left[1+\Gamma(\tau_r, \tau_g)\right] h^X = S^*(\vartheta) - \Gamma(\tau_r, \tau_g)(h^X)^{\varrho}\bar{l}^{1-\varrho}. \tag{83}$$

The left-hand side of (83) as a function of h^X is an increasing line through the origin. If $\varrho > 0$, the right-hand side of (83) is monotonically decreasing in h^X and goes to zero as $h^X \to \infty$. If $\varrho = 0$, it is a constant. Thus, whenever $\varrho \geq 0$, h^{X*} is unique.[17]

It is easy to check that (26), (27), (32), (42), (43), (51), (53), (54), (55), (56), (59), (60), (65)-(74), (80) and (83) are consistent with parts (i)-(iv) of Proposition 1.

Finally, it remains to be shown that the transversality conditions (49) and (50) hold under assumption (A1). Differentiating (46) with respect to time and using that $\dot{h} = \dot{u} = 0$ as well as $\dot{w}_h/w_h = g$ for $t \to \infty$ implies that, along a balanced growth path, $\dot{\mu}/\mu = \dot{\lambda}/\lambda + g$. From (45) and $\dot{c}_h/c_h = g$ for $t \to \infty$ we find $\dot{\lambda}/\lambda = -\sigma g$ and thus $\dot{\mu}/\mu = (1-\sigma)g$. As h becomes stationary, (49) holds iff $\lim_{t\to\infty} e^{[(1-\sigma)g+n-\rho]t} = 0$, i.e., iff (A1) holds. Similarly, using $\dot{\lambda}/\lambda = -\sigma g$ and the fact that a_h grows with rate g in the long run, we find that also (50) holds under (A1). The same holds analogously

[17] If $\varrho < 0$, meaning that assumption (A3) is violated, the right-hand side of (83) is strictly increasing and concave in h^X, goes to $-\infty$ for $h^X \to 0$ and to $S(\vartheta) > 0$ for $h^X \to \infty$. Thus, in this case, either two solutions or no solution for h^X as given by (83) exist.

for the transversality condition associated with a_l. This concludes the proof. ■

Proof of Corollary 1. Follows from (82) and part (v) of Proposition 1. ■

Proof of Proposition 2. First, it is useful to establish the following result.

Corollary A.1. *Under (A1)-(A5), h^{X*} is increasing in ϑ and independent of s^T.*

Proof. Apply the implicit function theorem to (83) and observe Corollary 1. ■

Next, using (74) and (80) we have

$$\frac{H_H^A}{H_L^A} = \frac{h_H^A}{h_L^A} = \left(\frac{h^X}{\bar{l}}\right)^{1-\varrho}. \tag{84}$$

Substituting (84) into $\tilde{A}_H/\tilde{A}_L = \left(h_H^A/h_L^A\right)^{\frac{1-\theta}{1-\phi}}$ (recall (78)) and using $1-\varrho = \frac{(\psi-1)(1-\phi)}{1-\theta-\psi(\phi-\theta)}$, according to (81), we obtain

$$\frac{A_H}{A_L} = \frac{\tilde{A}_H}{\tilde{A}_L} = \left(\frac{h^X}{\bar{l}}\right)^{\frac{(\psi-1)(1-\theta)}{1-\theta-\psi(\phi-\theta)}}. \tag{85}$$

Substituting (43) and (71) into (60) and using $1-\alpha = \frac{\psi-1}{\varepsilon-1}$, we find

$$\tilde{w}_l = (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\nu\left(h_L^A\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}} \left(1+\left(\frac{h^X}{\bar{l}}\right)^{\frac{1-\phi}{1-\theta-\psi(\phi-\theta)}}\right). \tag{86}$$

In view of Corollary A.1, the result follows from (86) by using the expression for h_L^A as given by (80) and recalling both $\varrho \geq 0$ and $\phi < 1$. ■

Proof of Proposition 3. Substituting $H^X/L^X = h^X/\bar{l}$ and (85) into (14) we obtain that

$$\frac{w_h}{w_l} = \left(\frac{h^X}{\bar{l}}\right)^{-\varrho}. \tag{87}$$

The result is confirmed by recalling that $\varepsilon > \psi > 1$ under assumption (A5), $\varrho \geq 0$ (where $\varrho = 0$ if (A3) holds with equality), $\phi < 1$, and Corollary A.1. ■

Calibration to the US Economy. We finally lay out how we derive the baseline calibration summarized in Table 1.

Consistent with average values for the period 1990-2006 (thereby averaging out business cycle phenomena in the period before the financial crisis started) from the Penn World Tables (PWT) 7.1 (Heston, Summers and Aten, 2012), we let the long-run average per capita income growth rate of the US economy, g, be equal to two percent. The average annual population growth rate was about one percent, $n = 0.01$. Thus, assuming that the US is in steady state, $g = \frac{n(1-\theta)}{1-\phi} = 0.02$ implies $\theta = 2\phi - 1$. Assumption (A2), $\phi > \theta$, is then equivalent to $\phi < 1$ and thus holds. Assuming an intermediate value $\theta = 0.5$, we arrive at $\phi = 0.75$. Our main conclusions are robust to variations in θ and ϕ which fulfill $\theta = 2\phi - 1$.

In his survey about skill biased technological change, Johnson (1997) argues that the elasticity of substitution between high-skilled and low-skilled labor is about 1.5. We thus take value $\psi = 1.5$ for our baseline calibration. Note that $\theta = 2\phi - 1$ and $\psi = 1.5$ jointly imply that (A3) holds with equality. Thus, according to part (v) of Proposition 1 and part (i) of Proposition 3, the long run effect of publicly financed education expansion (increase in ϑ) on the college premium, $\Omega^* \equiv w_h^* h^* / w_l^*$, is unambiguously positive.

Similar to Grossmann, Steger and Trimborn (2013), we use measures for the investment rate (sav) and the capital-output ratio to calibrate the depreciation rate of physical capital, δ_K. The investment rate is given by $sav = (\dot{K} + \delta_K K)/Y$. Using $\dot{K}/K = n + g$ and solving for δ_K yields $\delta_K = \frac{sav}{K/Y} - n - g$. Averaging over the period 1990-2006, sav is equal to about 21 percent, according to PWT 7.1. For the capital-output ratio, we take averages over the period 2002-2007 calculated from data of the US Bureau of Economic Analysis. The capital stock is proxied by the amount of total fixed assets (private and public structures, equipment and software). At current prices, this gives us $K/Y = 3$. Thus, the evidence suggests that $\delta_K = 0.04$, which is a standard value in the literature. For the mark-up factor on marginal costs of durable goods producers, κ, we take a typical value from the empirical literature, $\kappa = 1.3$ (Norrbin, 1993). For the output elasticity of capital goods, we choose $\alpha = 0.4$. Our conclusions are rather insensitive to alternative values in the typical range. With $\psi = 1.5$, we obtain an the elasticity of substitution between the inputs in final production of

$\varepsilon = \frac{\psi-\alpha}{1-\alpha} = 1.83$.

We calibrate variables related to type$-h$ and type$-l$ individuals by values for the "average" individual with at least high school diploma and high-school drop-outs, respectively. According to OECD (2013), the share of those among the 25-64 year old with less than upper secondary education in the year 2000 is 13 percent, suggesting $\bar{l} = N_{l,0}/N_{h,0} = 13/87 = 0.15$. Initial asset holdings, $a_{h,0}$, $a_{l,0}$, are calibrated as follows. Survey data for the year 2007 suggests that households headed by someone without a high school diploma (type$-l$ individuals) have, on average, a net worth of US$ 150,000 (in 2010 dollars). Moreover, the average asset holding of the other households (type$-h$ individuals) is approximately US$ 750,000. We thus assume for initial asset holdings that $a_{h,0}/a_{l,0} = 5$.[18]

For the tax rates, we take values from the 2000s prior to the financial crisis 2007-2009. As discussed in Grossmann, Steger and Trimborn (2010), the capital gains tax rate should be set to $\tau_g = 0.1$. Marginal labor income tax rates τ_l and τ_h are approximated by the US wage taxes in the 2000s for the ranges 20-40% and 200-400% of average gross earnings, respectively; this gives us $\tau_l = 0.17$ and $\tau_h = 0.35$. Moreover, we assume $\tau_r = 0.17$, which coincides with the US net personal capital income tax (equal to the net top statutory rate to be paid at the shareholder level, taking account of all types of reliefs and gross-up provisions at the shareholder level). Given the discount rate, ρ, and the long run interest rate, we can then derive a value for σ from the Keynes-Ramsey rule (which applies for individual consumption growth rates, see appendix), $g = \frac{(1-\tau_r)r-\rho}{\sigma}$. Using typical values $\rho = 0.02$ and $r = 0.07$ (Mehra and Prescott, 1985), we find $\sigma = 1.91$.

The fractions of tax revenue devoted to education, s^E, and redistributive transfers, s^T, are proxied by the recent shares of government spending to (higher) education and social welfare, respectively. According to OECD (2011), all US government bodies

[18] Those headed by a college graduate possess about US$ 1,15 million, whereas those households headed by high-school graduates and educated by some college possess about US$ 264,000 and US$ 384,000, respectively. See http://www.federalreserve.gov/econresdata/scf/scf_2010.htm. In the 2000s, about 40 percent of the 25-64 year olds in the US are tertiary-educated (OECD, 2013). Recall that $a_{h,0}$ and $a_{l,0}$ do not affect the the long run allocation and level of human capital.

combined spent 16.6 percent of its total expenditure on education in the year 2011. Unlike in our model, not all education spending is non-basic, of course. We assume, however, that the bulk of such welfare expenses benefits the "representative" low-ability individual;[19] according to www.usgovernmentspending.com, spending on social welfare in the 2000s is about 7-8 percent of total government spending. We set $s^E = 0.1$ and $s^T = 0.07$ in our baseline scenario and evaluate the effects of changes in these parameters. The implied fraction of human capital devoted to education of type$-h$ individuals is about four percent ($\vartheta = 0.04$). Our main conclusions turn out to be rather insensitive to the baseline set of policy parameters.

We confirmed that our results are rather insensitive to the configurations of parameters characterizing the educational production process, ξ, δ_H, β, γ, η, which match the two observables u^* and Ω^*. Like for the simple model, suppose $\beta = \gamma = 0.25$ for input elasticities and $\eta = 0.35$ for the degree to which human capital is transmitted over time. According to (17), parameter ξ, capturing the productivity of the educational production function, affects the long run human capital level per type$-h$ individual, h^*, independently of the long run fraction of time devoted to education, $1 - u^*$. It thus critically determines the long run skill premium, Ω^*. The human capital depreciation rate, δ_H, also affects the optimal (long run) time allocation of type$-h$ individuals. The representative type$-h$ individual attends school about 5-6 years out of 48 potential working years (between age 17 and 65), suggesting that $1 - u^* = 0.14$. Regarding the skill premium, we looked at the earnings distribution for those aged 25+ with at least high school diploma and without high school diploma (www.bls.gov/news.release/pdf/wkyeng.pdf). The relative median earnings between the two groups is 1.9 and the relative earnings at the 90th percentile about 2.1. We would like to measure relative average earnings to proxy Ω^* which are not available, however. As the earnings dispersion is less pronounced within the group of high school dropouts, we assume that the value for relative average earnings is higher than relative median earnings. We set $\xi = 0.15$ and $\delta_H = 0.03$ to simultaneously match $u^* = 0.86$ and $\Omega^* = 2.1$. Both values differ from those of the simple model; for Figure 0, ξ and δ_H

[19] According to our set up, high-ability individuals do not receive transfers.

matched u^* and Ω^* to that model.[20] We took over for the simple model the other parameters (common to both models) from the comprehensive model. The human capital depreciation rate is in the range of the estimated value in Heckman (1976), who finds that δ_H is between 0.7 and 4.7 percent.

Finally, R&D productivity parameter ν is confirmed to play a minor role for the important results of this paper and set to $\nu = 0.2$. Notably, the calibration implies a long run R&D intensity of 3.1 percent in the US, which is a reasonable value.[21]

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[20] In the simple model, both u^* and Ω^* turn out being independent of the degree of the human capital externality, ζ (see online-appendix). For Figure 0 we treated ζ as unknown and characterized the outcomes of policy shocks in terms of it.

[21] In the model, the R&D intensity equals total wage costs for researchers per unit of final output, $w_h(H_H^A + H_L^A)/Y$. Its steady state value is derived in the online-appendix.

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Online-Appendix

1. Equilibrium Analysis of the Simple Model (Section 3)

Education choice: The Hamiltonian of the intertemporal decision problem of a type$-h$ individual is given by

$$\mathcal{H} = \frac{\left[(1-\tau_h)w_h uh\right]^{1-\sigma}-1}{1-\sigma} + \mu\left[\xi(1-u)^{\beta}(h^E)^{1-\beta}h^{\eta} - \delta_H h\right], \tag{88}$$

where μ is the multiplier (co-state variable) associated with constraint (2). Necessary optimality conditions are $\partial\mathcal{H}_h/\partial u = 0$ (control variable) and $\dot{\mu} = \rho\mu - \partial\mathcal{H}/\partial h$ (state variable). Thus, using $h^E = \vartheta h$,

$$\mu = \frac{\left[(1-\tau_h)\, w_h\right]^{1-\sigma} h^{1-\sigma-\gamma-\eta} u^{-\sigma}}{\xi\beta(1-u)^{\beta-1}\vartheta^{\gamma}}, \tag{89}$$

$$\dot{\mu} = (\rho+\delta_H)\mu - \left[(1-\tau_h)\, w_h\right]^{1-\sigma} u^{1-\sigma} h^{-\sigma} - \xi\eta\mu\,(1-u)^{\beta}\,\vartheta^{\gamma} h^{\gamma+\eta-1}. \tag{90}$$

Substituting $\left[(1-\tau_h)\, w_h\right]^{1-\sigma} = \mu\xi\beta(1-u)^{\beta-1}\vartheta^{\gamma}h^{\sigma+\gamma+\eta-1}u^{\sigma}$ from (89) in (90), we have

$$\frac{\dot{\mu}}{\mu} = \rho + \delta_H - \xi\vartheta^{\gamma}h^{\gamma+\eta-1}(1-u)^{\beta-1}[\beta u + \eta(1-u)]. \tag{91}$$

Government budget conditions: Total tax revenue reads as $\tau_h w_h uh + \tau_l w_l \bar{l}$. Education expenditure $w_h h^E$ is a fraction s^E of it; thus,

$$w_h\vartheta h = s^E\left(\tau_h w_h uh + \tau_l w_l \bar{l}\right). \tag{92}$$

Total transfer payments, $T\bar{l}$, are a fraction s^T of total tax revenue; thus,

$$T = \frac{s^T}{\bar{l}}\left(\tau_h w_h u + \tau_l w_l \bar{l}\right). \tag{93}$$

Wage rates: Wage rates are given by the marginal productivity from the perspective of a final goods producer; according to (1),

$$
\begin{aligned}
w_h &= A\left[(H^Y)^{\frac{\psi-1}{\psi}} + (L^Y)^{\frac{\psi-1}{\psi}}\right]^{\frac{1}{\psi-1}} (H^Y)^{-\frac{1}{\psi}} \\
&= h^{\zeta}\left[1+\left(\frac{\bar{l}}{h(u-\vartheta)}\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{1}{\psi-1}}, \qquad (94)
\end{aligned}
$$

$$
\begin{aligned}
w_l &= A\left[(H^Y)^{\frac{\psi-1}{\psi}} + (L^Y)^{\frac{\psi-1}{\psi}}\right]^{\frac{1}{\psi-1}} (L^Y)^{-\frac{1}{\psi}} \\
&= h^{\zeta}\left[\left(\frac{h(u-\vartheta)}{\bar{l}}\right)^{\frac{\psi-1}{\psi}}+1\right]^{\frac{1}{\psi-1}}. \qquad (95)
\end{aligned}
$$

Steady state education choice: According to (2) and $h^E = \vartheta h$, in steady state with $\dot{h} = 0$, we find $\xi \vartheta^{\gamma} h^{\gamma+\eta-1} = \delta(1-u)^{-\beta}$. Using this in (91), setting $\dot{\mu} = 0$ and solving for u, we find that the long run fraction of time, u^*, allocated to production of final output is given by

$$
u^* = \frac{\rho + \delta_H(1-\eta)}{\rho + \delta_H(1+\beta-\eta)}. \qquad (96)
$$

u^* then determines the long run amount of human capital per type$-h$ individual,

$$
h^* = \left(\frac{\xi \vartheta^{\gamma}(1-u^*)^{\beta}}{\delta}\right)^{\frac{1}{1-\gamma-\eta}}. \qquad (97)
$$

2. The Comprehensive Model

Differential equations:

$$
\dot{h} = \xi(1-u)^{\beta} h^{\eta} - \delta h, \qquad (98)
$$

$$
\frac{\dot{\tilde{c}}_h}{\tilde{c}_h} = \frac{(1-\tau_r)r - \rho}{\sigma} - g, \qquad (99)
$$

$$
\frac{\dot{m}}{m} = \delta_H + \rho - n + (\sigma - 1)g - \left[\eta\,(1-u) + u\beta\right]\xi(1-u)^{\beta-1}\vartheta^{\gamma} h^{\eta+\gamma-1}, \qquad (100)
$$

$$\frac{\dot{\tilde{a}}_h}{\tilde{a}_h} = (1-\tau_r)r - n + (1-\tau_h)\frac{\tilde{w}_h uh}{\tilde{a}_h} - \frac{\tilde{c}_h}{\tilde{a}_h} - g, \tag{101}$$

$$\frac{\dot{\tilde{c}}_l}{\tilde{c}_l} = \frac{(1-\tau_r)r - \rho}{\sigma} - g, \tag{102}$$

$$\frac{\dot{\tilde{a}}_l}{\tilde{a}_l} = (1-\tau_r)r - n + (1-\tau_l)\frac{\tilde{w}_l}{\tilde{a}_l} - \frac{\tilde{c}_l}{\tilde{a}_l} + \frac{\tilde{T}}{\tilde{a}_l} - g, \tag{103}$$

$$\frac{\dot{\tilde{A}}_H}{\tilde{A}_H} = \nu(\tilde{A}_H)^{\phi-1}(h_H^A)^{1-\theta} - g, \tag{104}$$

$$\frac{\dot{\tilde{A}}_L}{\tilde{A}_L} = \nu(\tilde{A}_L)^{\phi-1}(h_L^A)^{1-\theta} - g, \tag{105}$$

$$\dot{p}_H^A = \frac{1-\tau_r}{1-\tau_g}\left(rp_H^A - \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}P_H^X\right)^{\frac{1}{1-\alpha}} h^X}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}}\right) - np_H^A, \tag{106}$$

$$\dot{p}_L^A = \frac{1-\tau_r}{1-\tau_g}\left(rp_L^A - \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}P_L^X\right)^{\frac{1}{1-\alpha}} l^X}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}}\right) - np_L^A. \tag{107}$$

Algebraic equations:

$$\begin{aligned}\tilde{a}_h + \bar{l}\tilde{a}_l = {} & \tilde{A}_H\left(\frac{\alpha P_H^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} h^X + \\ & \tilde{A}_L\left(\frac{\alpha P_L^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} \bar{l} + p_H^A\tilde{A}_H + p_L^A\tilde{A}_L, \end{aligned} \tag{108}$$

$$P_H^X = \left[1 + \left(\frac{\tilde{A}_H h^X}{\tilde{A}_L l^X}\right)^{-\frac{\psi-1}{\psi}}\right]^{\frac{1}{\varepsilon-1}}, \tag{109}$$

$$P_L^X = \left[1 + \left(\frac{\tilde{A}_H h^X}{\tilde{A}_L l^X}\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{1}{\varepsilon-1}}, \tag{110}$$

$$m\xi\beta(1-u)^{\beta-1}\vartheta^{\gamma}h^{\eta+\gamma-1} = (\tilde{c}_h)^{-\sigma}(1-\tau_h)\tilde{w}_h, \tag{111}$$

$$\tilde{w}_h = (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \tilde{A}_H \left(P_H^X\right)^{\frac{1}{1-\alpha}}, \tag{112}$$

$$\tilde{w}_l = (1-\alpha)\left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \tilde{A}_L \left(P_L^X\right)^{\frac{1}{1-\alpha}}, \tag{113}$$

$$p_H^A \nu \left(\tilde{A}_H\right)^{\phi-1} (h_H^A)^{-\theta} = \frac{\tilde{w}_h}{\tilde{A}_H}, \tag{114}$$

$$p_L^A \nu \left(\tilde{A}_L\right)^{\phi-1} (h_L^A)^{-\theta} = \frac{\tilde{w}_h}{\tilde{A}_L}, \tag{115}$$

$$h^X + h_H^A + h_L^A = (u-\vartheta)h, \tag{116}$$

$$l^X = \bar{l}. \tag{117}$$

$$\vartheta\tilde{w}_h h = s^E \Xi \tag{118}$$

$$\tilde{T} = \frac{s^T \Xi}{\bar{l}} \tag{119}$$

$$\begin{aligned}\Xi \equiv{}& \frac{TTR}{N_h e^{gt}} = \tau_h \tilde{w}_h u h + \tau_l \tilde{w}_l \bar{l} + \frac{(1-\tau_r)\tau_g}{1-\tau_g} r(p_H^A \tilde{A}_H + p_L^A \tilde{A}_L) + \\ & \frac{\tau_r - \tau_g}{1-\tau_g} \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}}{(r+\delta_K)^{\frac{\alpha}{1-\alpha}}} \left(\left(P_H^X\right)^{\frac{1}{1-\alpha}} h^X \tilde{A}_H + \left(P_L^X\right)^{\frac{1}{1-\alpha}} \bar{l}\tilde{A}_L\right) + \\ & \tau_r r \left(\tilde{A}_H \left(\frac{\alpha P_H^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} h^X + \tilde{A}_L \left(\frac{\alpha P_L^X}{\kappa(r+\delta_K)}\right)^{\frac{1}{1-\alpha}} \bar{l}\right). \end{aligned} \tag{120}$$

Steady state values:

$$h = \left[\frac{\xi(1-u)^{\beta}\vartheta^{\gamma}}{\delta_H}\right]^{\frac{1}{1-\gamma-\eta}}, \tag{121}$$

where

$$u = \frac{\rho - n + (\sigma-1)g + (1-\eta)\delta_H}{\rho - n + (\sigma-1)g + (1-\eta+\beta)\delta_H}. \tag{122}$$

The long run value of h^X is implicitly defined by

$$0 = (1+\Gamma)h^X + \Gamma(h^X)^{\varrho}\bar{l}^{1-\varrho} - S, \tag{123}$$

where

$$\Gamma = \frac{1-\frac{1}{\kappa}}{\frac{1}{\alpha}-1}\frac{(1-\tau_r)g}{\rho+\sigma g-(1-\tau_g)n}, \tag{124}$$

$$\varrho = \frac{2-\phi-\psi+\theta(\psi-1)}{1-\theta-\psi(\phi-\theta)}, \tag{125}$$

$$S = (u-\vartheta)\left[\frac{\xi(1-u)^{\beta}\vartheta^{\gamma}}{\delta_H}\right]^{\frac{1}{1-\gamma-\eta}}. \tag{126}$$

Using h^X we find long run values of h_H^A and h_L^A:

$$h_H^A = \Gamma h^X, \tag{127}$$

$$h_L^A = \Gamma\left(h^X\right)^{\varrho}\bar{l}^{1-\varrho}. \tag{128}$$

Setting $\dot{\tilde{A}}_H = \dot{\tilde{A}}_L = 0$ in (104) and (105) yields, by using the steady state values of h_H^A, h_L^A, the steady state values of $\tilde{A}_H$ and $\tilde{A}_L$:

$$\tilde{A}_H = \left(\frac{\nu\left(h_H^A\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}}, \tag{129}$$

$$\tilde{A}_L = \left(\frac{\nu\left(h_L^A\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}}, \tag{130}$$

Using the long run values of $\tilde{A}_H$, $\tilde{A}_L$, h^X as well as $l^X = \bar{l}$ in (109) and (110) gives us the long run values of P_H^X and P_L^X, respectively.

Setting $\dot{\tilde{c}}_h = 0$ (thus $\dot{\tilde{c}}_l = 0$) in (99), we obtain the steady state interest rate:

$$r = \frac{\rho + \sigma g}{1 - \tau_r}. \tag{131}$$

Using the long run values for $\tilde{A}_H$, $\tilde{A}_L$, P_H^X, P_L^X and r in (112) and (113) give us the long run values for $\tilde{w}_h$ and $\tilde{w}_l$, respectively.

Using the long run values for $\tilde{A}_H$, $\tilde{A}_L$, h_H^A, h_L^A and $\tilde{w}_h$ in (114) and (115) give us the long run values for p_H^A and p_L^A, respectively.

Using the long run values for h, u, $\tilde{w}_h$, $\tilde{w}_l$, $\tilde{A}_H$, $\tilde{A}_L$, P_H^X, P_L^X, p_H^A, p_L^A, h^X in (120) we obtain Ξ for policy parameters τ_h, τ_l, τ_g, τ_r. Thus, using $\tilde{w}_h h$ in (118) gives us the relationship between ϑ and s^E; (119) gives us the relationship between $\tilde{T}$ and s^T.

Finally, setting $\dot{\tilde{a}}_h = \dot{\tilde{a}}_l = 0$ in (101) and (103) implies

$$0 = (1 - \tau_r)r - n + (1 - \tau_h)\frac{\tilde{w}_h u h}{\tilde{a}_h} - \frac{\tilde{c}_h}{\tilde{a}_h} - g, \tag{132}$$

$$0 = (1 - \tau_r)r - n + (1 - \tau_l)\frac{\tilde{w}_l}{\tilde{a}_l} - \frac{\tilde{c}_l}{\tilde{a}_l} + \frac{\tilde{T}}{\tilde{a}_l} - g. \tag{133}$$

Using the long run values of $\tilde{w}_h$, $\tilde{w}_l$, u, h, r, ϑ and $\tilde{T}$, there are the four equations (108), (111), (132), (133) left for the five remaining unknown long run values of $\tilde{c}_h$, $\tilde{c}_l$, $\tilde{a}_h$, $\tilde{a}_l$, and m. The numerical implementation suggests that the long run values of these variables are unique under assumptions (A1)-(A5). Unlike the long run values of other variables, however, the respective values depend on initial conditions. The initial distribution of assets, characterized by initial conditions $a_{h,0}$, $a_{l,0}$, cannot be chosen independently of initial conditions $A_{H,0}$ and $A_{L,0}$. According to (108), in period 0, it must hold that

$$\begin{aligned} a_{h,0} + \bar{l} a_{l,0} \quad = \quad & A_{H,0} \left(\frac{\alpha P_{H,0}^X}{\kappa(r_0 + \delta_K)} \right)^{\frac{1}{1-\alpha}} h^X + \\ & A_{L,0} \left(\frac{\alpha P_{L,0}^X}{\kappa(r_0 + \delta_K)} \right)^{\frac{1}{1-\alpha}} \bar{l} + p_{H,0}^A A_{H,0} + p_{L,0}^A A_{L,0}. \end{aligned} \tag{134}$$

Let $\mathfrak{s} \equiv a_{h,0}/a_{l,0}$ denote initial assets of $h-$type individuals relative to those of $l-$type individuals. Thus, for given distributional parameter, $\mathfrak{s}$, initial assets read as

$$\begin{aligned} a_{l,0} &= \frac{1}{\mathfrak{s}+\bar{l}}\left(A_{H,0}\left(\frac{\alpha P^X_{H,0}}{\kappa(r_0+\delta_K)}\right)^{\frac{1}{1-\alpha}} h^X + \right. \\ &\left. A_{L,0}\left(\frac{\alpha P^X_{L,0}}{\kappa(r_0+\delta_K)}\right)^{\frac{1}{1-\alpha}} \bar{l} + p^A_{H,0}A_{H,0} + p^A_{L,0}A_{L,0}\right), \end{aligned} \tag{135}$$

according to (134), and $a_{h,0} = \mathfrak{s} a_{l,0}$.

R&D intensity: The model is calibrated such that the long run R&D intensity is two percent. The R&D intensity is given by

$$R\&D \equiv \frac{w_h(H^A_H + H^A_L)}{Y}. \tag{136}$$

Substituting (26) and (27) into (5) we obtain

$$Y = \left(\frac{\alpha}{\kappa(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}} \left[\left(A_L L^X \left(P^X_L\right)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(A_H H^X \left(P^X_H\right)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{137}$$

Using the expression (112) for $\tilde{w}_h$ and (137) in (136) we find

$$R\&D = \frac{(1-\alpha)\frac{\tilde{A}_H}{\tilde{A}_L}\left(P^X_H\right)^{\frac{1}{1-\alpha}}\left(h^A_H + h^A_L\right)}{\left[\left(l^X \left(P^X_L\right)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{\tilde{A}_H}{\tilde{A}_L} h^X \left(P^X_H\right)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}, \tag{138}$$

where P^X_H is given by (109), P^X_L is given by (110), $r = \frac{\rho+\sigma g}{1-\tau_r}$, h^X is given by (123) (observing (126) and (122)), h^A_H is given by (127), h^A_L is given by (128), $\tilde{A}_H/\tilde{A}_L$ is given by (85), $r = \frac{\rho+\sigma g}{1-\tau_r}$ and $l^X = \bar{l}$.

Additional trickle-down dynamic analyses (discussed in section 6.3)

- In section 6.1 we introduced the threshold time span $\hat{t}$ (see Figure 1), after which an increase in the share of tax revenue devoted to higher education, s^E, by five percentage points raises net income type$-l$ individuals, y_l, more than an equally sized increase in the share of tax revenue devoted to redistributive transfers, s^T. Figure A.1 shows $\hat{t}$ as a function of the (derived) elasticity of substitution between the two types of labor, ψ. As discussed as first point in section 6.3, a finite $\hat{t}$ always exists for the considered range of ψ; the function displayed in Figure A.1 is increasing, convex for low values of ψ, and concave for high values of ψ.

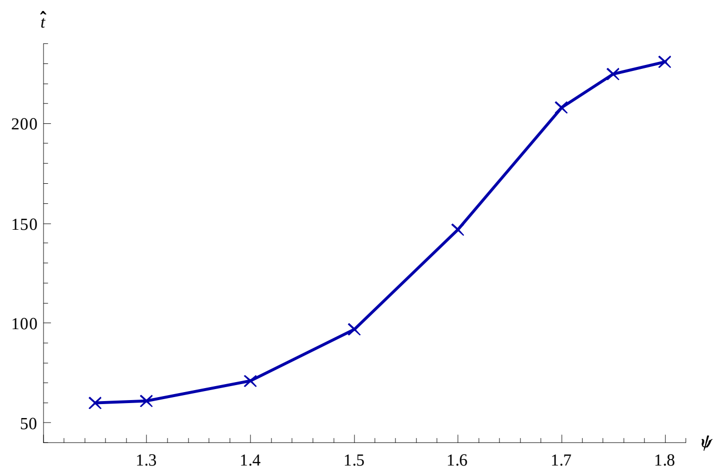

Figure A.1: Threshold time span $\hat{t}$ as a function of the derived elasticity of substitution between the two types of labor, ψ. Set of parameters as in Table 1.

- In Figure A.2, we show the evolution of consumption of a type$-l$ individual, c_l, again normalized to the intitial stady state level before the policy reform, c_l^*, in response to an increase in s^E and s^T separately by five percentage points. Qualitatively, the evolution of c_l/c_l^* is similar to the evolution of normalized net income, y_l/y_l^* shown in Figure 1. Quantitatively, we see a less pronounced initial impact of policy reforms compared to that on net income, reflecting consumption-smoothing behavior. After the "education reform" consumption levels of low-skilled workers are equated with those under the baseline scenario after 21 years. The intersection point with the scenario "redistribution reform" is 118 years.

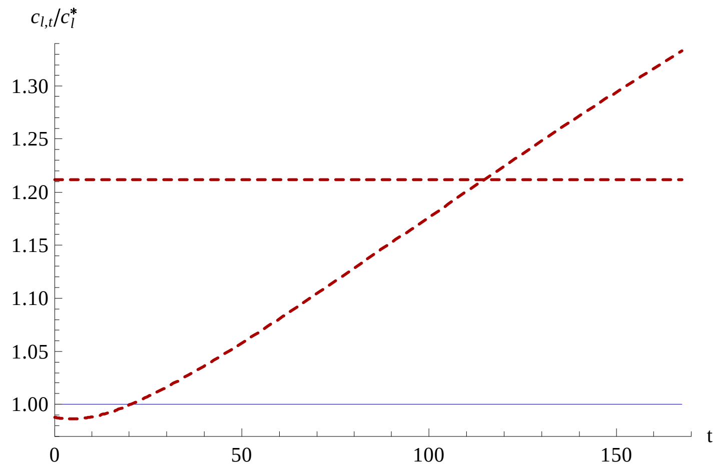

Figure A.2: Time path of normalized consumption, c_l/c_l^*, in three scenarios: solid (blue) line: baseline scenario (s^T and s^E remain constant), horizontal dashed line: s^T increases by five percentage points, increasing dashed line: s^E increases by five percentage points. Set of parameters as in Table 1.

- Alternatively to evaluating an increase in s^E and s^T separately, section 6.3 also discussed an increase (decrease) in the fraction of total tax revenue devoted to education, s^E, while at the same time reducing (raising) the fraction devoted to transfers, s^T, such that the sum of the two fractions, $s^E + s^T$, remains constant. The results from these policy reforms, changing both s^E and s^T simultaneously in opposite directions by one percentage points, are given in Figure A.3. We see that, on impact, total net income y_l decreases when expanding education and increases when extending redistribution. The transition paths after the initial response are in opposite directions. Again, a policy reform towards expanding education is harmful in the shorter run, but leads to trickle-down growth in the longer run. A policy reform towards extending redistribution which lowers the fraction of tax revenue devoted to education is beneficial in the shorter run, but harmful in the longer run. It takes about 90 years for an increase in s^E while reducing s^T to increase y_l compared to both the baseline scenarion without policy reform and the policy alternative to increase in s^E while reducing s^T.

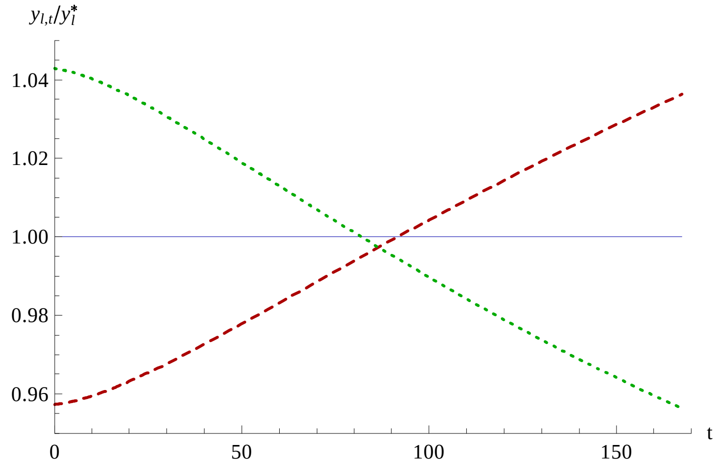

Figure A.3: Time path of normalized income, y_l/y_l^*, in three scenarios: solid (blue) line: baseline scenario (s^T and s^E remain constant), dotted (green) line: s^T increases by one percentage point and s^E decreases by one percentage point, dashed (red) line: s^E increases by one percentage point and s^T decreases by one percentage point. Set of parameters as in Table 1.

- We next examine the effect of an increase in the tax rate on bond holdings, τ_r, by one percentage point to $\tau_r = 0.18$ for two purposes: (i) to finance an increase in growth-adjusted transfer $\tilde{T}$, according to government budget constraint (67), while holding constant both the fraction of human capital devoted to higher education, ϑ and the government expenditure share on transfers s^T;[22] (ii) to finance an increase in ϑ, according to government budget constraint (66), fixing both the expenditure share for education, s^E, and the growth-adjusted transfer $\tilde{T}$ to its initial level.[23] In policy experiment (i), as displayed by the dotted line in Figure A.4, type$-l$ households gain on impact but soon lose even compared to the baseline scenario without policy reform. The underlying reason is a decline in the fraction of human capital devoted to both kinds of R&D (not shown), eventually depressing net income y_l. In this sense, increasing the tax rate on bond holdings distorts R&D investments. In policy experiment (ii), as displayed

[22] In experiment (i), the education expenditure share s^E may change along the transition; it is determined by (66).

[23] In experiment (ii), the expenditure share s^T may change along the transition; it is determined by (67).

by the dashed line in Figure A.4, type$-l$ households lose on impact. Again, like for Figure 1, the reason is human capital is reallocated towards educational production. Interestingly, and contrary to Figure 1, they lose even more in the longer run because of the distortive effect of raising τ_r on R&D investments. Nevertheless, after $\hat{t} = 57$ years they are better off in terms of net income from policy experiment (ii) as compared to (i).

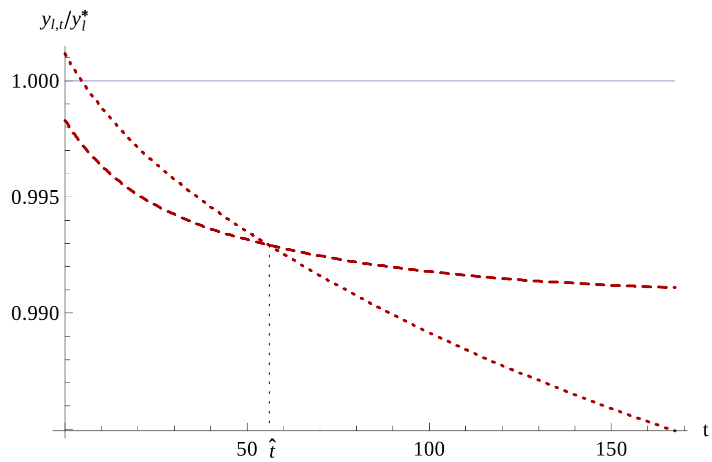

Figure A.4: Time path of normalized net income, y_l/y_l^*, in three scenarios: solid (blue) line: baseline scenario; dotted line: increase in τ_r by one percentage point is used to finance an increase in $\tilde{T}$, dashed line: increase in τ_r by one percentage point is used to finance an increase in ϑ. Set of parameters as in Table 1.

- Figure A.5 displays the same policy experiments than Figure A.4 except that now the labor income tax rate of type$-h$ individuals, τ_h, rather than τ_r is increased by one percentage point to $\tau_h = 0.36$. Qualitatively, the effect of extending transfers (experiment (i)) and expanding education (experiment (ii)) is similiar as in Figure 1; compare the horizontal and the increasing lines in Figure 1 and Figure A.5. The result is an implication of the fact that labor income taxation does not distort the allocation of human capital in our model. Compared to Figure 1, net income y_l becomes higher under education expansion (experiment (ii) rather than (i)) earlier than in Figure 1, with $\hat{t} = 55$ (rather than 97 years).

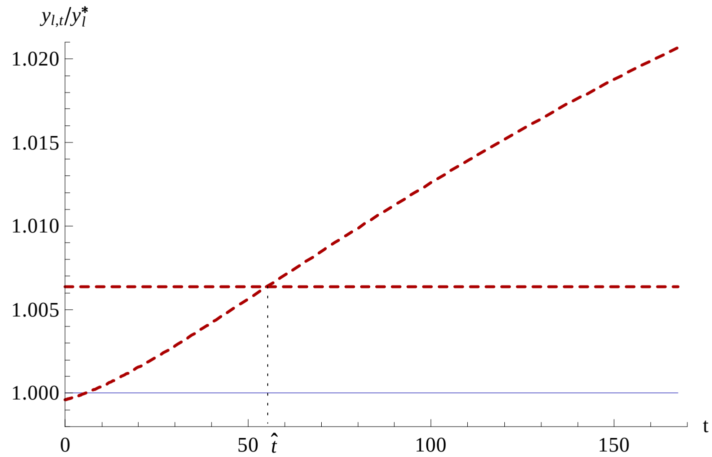

Figure A.5: Time path of normalized net income, y_l/y_l^*, in three scenarios: solid (blue) line: baseline scenario; horizontal dashed line: increase in τ_h by one percentage point is used to finance an increase in $\tilde{T}$, increasing dashed line: increase in τ_h by one percentage point is used to finance an increase in ϑ. Set of parameters as in Table 1.