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ABSTRACT

Unemployment and Endogenous Reallocation over the Business Cycle*

We build an analytically and computationally tractable stochastic equilibrium model of unemployment in heterogeneous labor markets. Facing search frictions within markets and reallocation frictions between markets, workers endogenously separate from employment and endogenously reallocate between markets, in response to changing aggregate and local conditions. Empirically, using the 1986-2008 SIPP panels, we document the occupational mobility patterns of the unemployed, finding notably that occupational change of unemployed workers is procyclical. The heterogeneous-market model yields highly volatile countercyclical unemployment, and is simultaneously consistent with procyclical reallocation, countercyclical separations and a negatively-sloped Beveridge curve. Moreover, the model exhibits unemployment duration dependence, which (when calibrated to long-term averages) responds realistically to the business cycle, creating substantial longer-term unemployment in downturns. Finally, the model is also consistent with different employment and reallocation outcomes as workers gain experience in the labor market, on average and over the business cycle.

JEL Classification: E24, E30, J62, J63, J64

Keywords: unemployment, business cycle, search, endogenous separations, reallocation, occupational mobility

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1 Introduction

The Great Recession has revived an important debate about the extent and nature of the misallocation of unemployed workers across labor markets. Central to this debate is the notion that some labor markets offer better employment prospects than others and that some unemployed workers would benefit if they were to reallocate.[1] In this paper we investigate to what extent unemployed workers are able and willing to change markets in response to changes in aggregate or local conditions and how this affects the cyclical behavior of unemployment. We do this in a fully-fledged equilibrium business cycle model in which frictional labor markets are heterogeneous and workers' reallocation choices across these markets are endogenous.

Our paper makes five contributions. First, we show that our heterogenous market model remains tractable and easy to compute. Decisions are easily characterized and comparative statics can be done analytically, laying bare the underlying forces in the model. Second, we operationalize workers' 'changes of markets' by changes of occupations and document the reallocation behavior of the unemployed in the data, in particular over the business cycle. We use a subset of the data to ground our theory quantitatively and another subset to test it. Third, using the documented reallocation behavior, we can decompose aggregate unemployment into three components. Reallocation unemployment, caused by a time-consuming process of determining in which other occupation to look for jobs. Search unemployment, caused by frictions that make it time-costly to successfully find a vacancy in a given occupation. Rest unemployment, when unemployed workers remain attached to an occupation that currently provides few employment opportunities, but is sufficiently likely to recover in the future; this can make it optimal for workers to wait for conditions to improve and jobs to arrive.[2] As in Alvarez and Shimer (2011), but inferred from different dimensions of the data, we find that rest unemployment is quantitatively very important. We subsequently show it is also an important force shaping cyclical patterns of unemployment.

Fourth, given the calibrated sources of unemployment and the presence of occupation-specific human capital, the model delivers differences in *unemployment outcomes* between young and prime-aged workers very much in line with the data. Most related, Kambourov and Manovskii (2009a) link occupational human capital, heterogeneous outcomes inside an occupation and occupational mobility to inequality in *wage outcomes*, and Alvarez and Shimer (2009) link sectoral human capital to rest unemployment of, in particular, experienced workers. Taking all types of unemployment into account at different points of the life cycle, we find that the model can reproduce the decrease in occupational mobility in unemployment of prime-aged workers relative to young workers, and can explain *quantitatively* their longer durations in unemployment and their decreased inflow to unemployment. Fifth,

[1] See, for example, Elsby, et al. (2010) and Barnichon and Figura (2011), who measure the extent to which matching efficiency has decrease during the Great Recession; and Şahin, et al. (2012) and Herz and Van Rens (2011) who measure the extent of mismatch.

[2] Jovanovic (1987), Hamilton (1988), Gouge and King (1997) and Alvarez and Shimer (2011) provide earlier studies of rest unemployment. Rest unemployment in our model is conceptually close to unemployment arising in stock-flow matching models. One can think of our rest unemployed workers as those who are actively look for job openings, for example by visiting job centres (as in the UK), by inquiring at firms and through acquaintances, but who find out that there are no jobs available for them. Instead of starting to look for jobs in additional occupations or locations, these workers prefer to wait and repeat the same process for the same market in the next period. When a market receives shocks that make hiring profitable again, a flow of newly created vacancies will start matching with the existing stock of unemployed. See Coles and Smith (1998) and Ebrahimy and Shimer (2010) for examples of stock-flow matching models.

to our knowledge, we provide the first fully-fletched quantitative business-cycle investigation of rest, search and reallocation unemployment in an equilibrium model with many (i.e. more than two) markets, where mobility between markets is endogenous. We find that the heterogeneity in markets is very helpful. The model delivers the sought-after amplification of unemployment fluctuations, and moreover exhibits countercyclical separations and procyclical reallocations. Crucially, it does so without counterfactual implications for the Beveridge curve, usually associated with endogenous separations, or lowering the correlation between aggregate job finding and labor market tightness. Moreover, the cyclical behavior of the distribution of unemployment durations in the model is also in line with the data, with a quantitatively similar increase in the proportion of long-term unemployment spells in recessions. The cyclical volatilities of the separations, job finding and reallocation rates for young and prime-aged workers are also in line with data.

Our theory combines the ideas originally set out by Lucas and Prescott (1974), in the setup of Alvarez and Veracierto (2000), and Mortensen and Pissarides (1994). It considers an economy with different occupations. Within every occupation a set of 'islands' represents the various labor markets attached to the occupation. On each island unemployed workers can decide to search and apply for existing job opportunities, become rest unemployed and wait for jobs to arrive, or reallocate to a different occupation. Employed workers can decide to separate from their employers and become unemployed. Search frictions on each island are modelled through a matching function that governs the meeting process of workers and firms. Reallocation frictions across occupations are modelled through a time and resource consuming process, where new islands in other occupations are randomly drawn from the set of all islands. Our framework contrasts with most of the literature as it considers search unemployment separately from reallocation and rest unemployment, and because it analyses the business cycle implications of rest and reallocation unemployment, in addition to search unemployment.[3]

Our paper belongs to the emerging literature that investigates the role of heterogeneity (e.g. across workers and/or firms) in aggregate unemployment fluctuations.[4] Though our economy is subject to both aggregate productivity shocks and shocks to the productivity of the worker in his occupation (referred to as worker-occupation specific or idiosyncratic productivity), the model stays tractable and is easily computed, because the equilibrium has a block recursive structure. This means that equilibrium mobility decisions only depend on the aggregate and idiosyncratic productivity states.[5]

[3] In island models, the term search unemployment is often used to denote the unemployment experienced when moving to a new island. In our model, to distinguish search *within* an island or occupation from search *across* occupations, we label the latter *reallocation unemployment*, and the former *search unemployment*.

[4] Closely related papers are Alvarez and Shimer (2011), Lkhagvasuren (2012), Şahin et al. (2012), and Herz and Van Rens (2012), who all consider heterogeneity and unemployment across sectors, occupations or locations (or combinations thereof). The first two consider steady state settings; Lkhagvasuren (2012) computes the effect of a one-time unexpected permanent shock to aggregate productivity for geographical mobility and unemployment dispersion. The latter two papers map detailed data on vacancies and unemployment, using non-arbitrage relations (from the perspective of the market or the planner) to measure mismatch. More broadly related papers are Birchenall (2010), who constructs an island model with demand uncertainty and reallocation; and Garin, et al. (2011), who study net sectoral mobility in a two-island model. More generally, this paper contributes to the literature that analysis the quantitative impact of heterogeneity on aggregate unemployment fluctuations. Recent example are Menzio and Shi (2010), Chassamboulli (2011), Robin (2011), Bils, et al. (2012), Coles and Mortensen (2012), Elsby and Michaels (2012), Hawkins (2011), Kaas and Kircher (2012), Lise and Robin (2012) and Moscarini and Postel-Vinay (2012).

[5] Shi (2009) and Menzio and Shi (2011) were the first to formally apply this concept to solve a steady state model, resp. a business cycle model, with on-the-job search and an infinite dimensional state space. Hoffmann and Shi (2012) study the transition path of a long-run shift from the manufacturing to the service sector, employing a block recursive structure.

Key behind this last result is that, on each island, the production and matching technologies are constant returns to scale. This is also a key building block in the standard Pissarides model, implying that job finding rates are a function only of productivity, not of unemployment or employment. In our model, this property is used to its full extent to keep the heterogeneity tractable. On every island, vacancy posting, search, reallocation and separations decisions are functions only of the relevant productivities, not of the number of employed and unemployed on the island, or much more importantly, on all other islands.[6] In the absence of these constant returns to scale technologies, the distribution of workers over islands typically will matter, rendering the characterisation and computation of the equilibrium rather involved. As a result studies with endogenous mobility between heterogeneous frictional markets have mostly been confined to steady state analysis.[7] Alternatively, as in Shimer (2007) and Mortensen (2009), one can abstract from endogeneity of mobility between markets, and focus on how the business cycle shapes match formation when it changes the numbers of unemployed and vacancies which are unequally spread over islands such that only the short side succeeds in its entirety to match. Here, because of the aforementioned structure, we are able to deal with both aggregate fluctuations and endogenous mobility.

In the analytical and quantitative sections we establish that separation and reallocation decisions can be summarized by reservation properties. The reallocation cutoff is completely analogous to the reservation wage in a McCall-type model, where here workers consider the expected life-time values of unemployment across islands. Additionally, search frictions on islands imply that there is a non-trivial separation decision, which typically results in a separation cutoff similar to the one in Mortensen and Pissarides (1994). These two cutoffs are not necessarily equal and their relative position is a crucial determinant of outcomes in our economy, shaping the relative importance of search, rest and reallocation unemployment. Search unemployment occurs in islands with productivities higher than both cutoffs; reallocation unemployment, occurs when unemployed workers are in islands below the reallocation cutoff, and rest unemployment, when workers are in islands below the separation cutoff but above the reallocation cutoff (if there are any such islands).

We are able to study analytically the behavior of these cutoffs to changes in aggregate productivity. We show that their implied cyclical behavior is affected by the relative importance of search, rest and reallocation unemployment. Search unemployment creates, perhaps surprisingly, more *procyclical* reallocation behavior, while more reallocation unemployment could lead to a less procyclical reallocation rate. We also show that rest unemployment affects the countercyclicality of separations: when separated workers expect to be rehired eventually in the same occupation, a potentially procyclical value of reallocation does not weigh heavily in the decision to separate. Given the different implications of the three types of unemployment, it is of first-order importance to quantify the empirical relevance of search, rest and reallocation unemployment. The detailed patterns underlying reallocation in unemployment, which we briefly discuss next, helps us to do so.

We document new evidence on the patterns of worker reallocation *through unemployment* across occupations in the US, using the Survey of Income Program Participation (SIPP) for the period 1986-

[6] The constant returns to scale assumption brings our model mathematically closer to the one-person one-island model of Jovanovic (1987), though in our model, importantly, the value of a non-resting worker is shaped by the search frictions on the island.

[7] An interesting exception is Veracierto (2008), who takes an approximation around the steady state that reduces the heterogeneous island problem into a linear-quadratic problem.

2011. Existing literature has mainly focused on overall occupational mobility or only on the occupational mobility of the employed.[8] We find that: (i) The extent of occupational mobility by unemployed workers is high and decreases with age. (ii) Occupational mobility increases with unemployment duration, but rather moderately, such that even at high unemployment durations a large proportion of unemployed workers find a job in their own occupation. (iii) Occupational mobility is associated with higher re-employment wages. (iv) After changing occupations workers often change occupations again in a subsequent unemployment spell. However, if the worker did not changed occupation, he/she is more likely to stay in the same occupation after a subsequent unemployment spell. (v) The occupational mobility of the unemployed is procyclical and positively correlated with the rate at which unemployed workers find jobs in a different occupation.

In the calibrated model, we use the evidence on reallocation, net of cyclical patterns, to inform us about the costs and benefits of reallocation and, more generally, the environment in which workers make their reallocation decisions. In turn this helps us determine which type of unemployment is quantitatively important. We find that a large part of unemployment is rest unemployment, in line with Alvarez and Shimer (2011) who use industry wage dynamics data to determine the relative importance of rest unemployment. Further, the calibrated model has significant search unemployment, but not much reallocation unemployment. The latter occurs as the time spent searching across occupations is small, usually less than a month.[9]

Rest unemployment is prominent in the calibration because it can reproduce a number of unemployment and reallocation patterns in the data in a mutually consistent way, while search and reallocation unemployment by themselves cannot. First, rest unemployment is fully able to reconcile the coexistence of large reallocation flows and the substantial empirical proportion of long-duration unemployed workers who ultimately remain in their occupation. A rest unemployed worker remains attached to an occupation because a volatile enough worker-occupation specific productivity process gives him enough upside there. Ultimately, the shock process will push many of these workers to reallocate, after further adverse shocks, while at the same time, a considerable amount of long-term unemployed workers will still be rehired in the same occupation, after favorable shocks: for those, waiting was worthwhile ex post. Without rest unemployment, it appears much harder to resolve the tension between sizeable reallocation flows and long-term occupational staying.[10]

Second, within a SIPP panel 40% of the unemployed who found a job in their old occupation are observed to change occupations when they become unemployed a second time. In the absence of rest unemployment, the forces of selection would be much stronger: unemployed occupational stayers would be in the better markets, and hence likely to remain in their occupation even when becoming unemployed a second time. With rest unemployment, many of the unemployed workers who found a job in their previous occupation would be instead in markets which have recovered just enough for firms to start hiring workers again. When these workers separate again, they likely

[8] See Moscarini and Thomssom (2007), Moscarini and Vella (2008) and Kambourov and Manovskii (2008) for recent examples.

[9] This does not mean that reallocation frictions are small (the fixed cost of reallocation is substantial in the calibrated model), or that reallocation is unimportant, as almost all the discussion in this paper suggests the opposite.

[10] Without rest unemployment, overall reallocation requires a volatile enough process, while occupational staying even after long unemployment durations, i.e. in the most unattractive markets, would require a very persistent process. Otherwise reallocations would be triggered too frequently after long unemployment durations, leaving too little occupational stayers at high durations.

become rest unemployed again, and will be a few adverse shocks away from reallocation. Third, rest unemployment can explain how unemployment incidence is concentrated in a subset of workers. With rest unemployment, much of the unemployment is found in markets close to the separation cutoff, and rehired workers need only a few adverse shocks to become unemployed again.

We calibrate our model using averages based on the entire time series (spanning 1986-2011), apart from the driving aggregate productivity process. The model generates *pro*cyclical reallocations in unemployment, and *counter*cyclical unemployment. Importantly, the model also produces sufficient amplification of productivity shocks. The cyclical volatilities of unemployment, job finding and separation rates are very close to their empirical counterparts. These features are important, and perhaps surprising, since it is well known that the canonical search and matching model, as described in Pissarides (2001), has difficulty in matching these aspects of the data (see Shimer, 2005, and Costain and Reiter, 2008), and most of its extensions are able to reproduce some but not all of features we discuss.

The model also does well along the dimensions that the canonical representative-market model is able to capture. It produces a high correlation between job finding and tightness and the strongly downward-sloping Beveridge curve.[11] It is also noteworthy that endogenous separations do not lead to a breakdown of the Beveridge curve, while this usually causes trouble in the standard representative-market model. The main reason is that our model exhibits 'market selection'. Those workers separating are largely confined to labor markets in which conditions are poor, where there is little incentive to hire workers, and thus the inflow into unemployment triggers little or no additional vacancy posting. In other markets vacancies are still posted, and workers still find jobs, implying that in the aggregate we do not observe the entire labor market shutting down.

Most of the cyclical behavior can be understood directly from the countercyclical endogenous separation and procyclical reallocation responses to aggregate productivity shocks. Rest unemployment occurs at idiosyncratic productivities below the separation and above the reallocation cutoffs, and is the most important driver of cyclical unemployment. As aggregate productivity drops, the separation cutoff in the calibration rises, and more employed workers separate into rest unemployment. Simultaneously, the reallocation cutoff in the calibration drops. This means an increase in the range of idiosyncratic productivities at which rest unemployment occurs and, as a result, rest unemployed workers take longer to become either search or reallocation unemployed. This causes the overall job finding rate to drop. In addition, in islands marginally above the separation cutoff, employment surpluses are small and sensitive to the aggregate productivity, which further decreases the aggregate job finding rate. The countercyclical separation cutoff and procyclical reallocation cutoff, and the associated rest unemployment, thus produce cyclical fluctuations in the unemployment, separation, reallocation and job finding rates that are in line with the data. Moreover, the increased distance between the two cutoffs also increases longer-duration unemployment in equal magnitudes as in the data during downturns. These features show that the model captures well the untargeted disaggregated patterns underlying observed unemployment.

The empirical evidence on occupational mobility drawn from the SIPP also suggests that workers' occupational attachment vary distinctly with occupational experience and age. To analyse whether

[11] This suggests that the results in Shimer (2007) and Mortensen (2009) are robust to endogenous reallocation. Behind this is the importance of rest unemployment: unemployed workers remain, at least initially, attached to their previous occupation, and do not separate to move to a different market immediately.

our model, and the idiosyncratic shock process that drives much of unemployment and reallocation, can also reproduce *quantitatively* these different outcomes for workers of different ages (and simultaneously perform well over the business cycle), we let workers gain occupational specific human capital, which they loose by changing occupations, along the lines of Rogerson (2005), Kambourov and Manovskii (2009a) and Alvarez and Shimer (2009).[12] In particular, Alvarez and Shimer (2009) relate sector-specific human capital to rest unemployment of experienced workers in a steady state environment, focusing on net mobility and abstracting from unemployment of inexperienced workers and other causes of unemployment.

We consider occupational gross mobility patterns, allow search, rest and reallocation unemployment at all human capital levels, and quantitatively link the idiosyncratic shock process to the reallocation, separation and job finding rates, unemployment duration dependence, and reallocation rates at different unemployment durations for young and prime aged workers. Even with different human capital levels our model remains tractable, due to its 'block recursive' structure. Our model with different human capital levels can simultaneously reproduce, on one hand, the empirical age-differences in transition and unemployment rates in response to large returns to occupational experience, and on the other hand, the empirical sensitivity of unemployment and transition rates in response to the smaller business cycle shocks. Both life-cycle and business-cycle patterns are shaped by the distribution of workers over different labor markets (each associated with a value of idiosyncratic productivity and human capital capital in a worker's occupation), and thus *driven by the same idiosyncratic shock process*. Unemployment fluctuations over the business cycle are driven by the mass of workers close to the countercyclical separation cutoff, and the procyclical reallocation cutoff below it.

It is worthwhile mentioning that our heterogeneous market model does not require the usual proximity of the average value of unemployment to the average value of production to create amplification. Our estimated unemployment benefits flows are close to Hall and Milgrom (2008), rather than Hagedorn and Manovskii (2009). For the same reason, our setting with heterogeneous markets also appears to provide a solution to the trade-off, present in representative-market search models, between the degree of wage growth dispersion (which is allowed to trigger only the empirical extent of separations), and the size of cyclical fluctuations (Bils et al. 2011). Finally, we emphasize that this paper studies the reallocation behavior of the unemployed and how it shapes their outcomes. It is not a theory of aggregate reallocation flows between sectors or occupations – these should also include the flows of employed workers.

The rest of the paper is organised as follows. In the next section we present our motivating evidence. In Sections 3-5 we develop the model and discuss the implications of theory. Section 6 contains our quantitative analysis and Section 7 concludes. Proofs are relegated to the Appendix or to the Supplementary Appendix.

2 Patterns of Worker Reallocation Through Unemployment

We now present evidence on the occupational reallocation behavior *of unemployed workers*. We consider the extent of occupational mobility, its relation with unemployment spell durations, the subse-

[12] Rogerson (2005) focusses on the role of the life-cycle in observed net mobility between sectors. See also Wong (2012). Albrecht et al. (1998) build an early model that considers the interaction between sector specific human capital with rest and reallocation unemployment.

quent outcomes for reallocating workers, and its cyclical patterns. These patterns will help ground our model as much as possible in the actual reallocation behavior of the unemployed as observed on the micro-level. We also think these patterns are interesting and informative in themselves. There is no reason to believe that the much more extensively studied occupational mobility patterns for employed workers (or all workers together), are the same for the unemployed. This caution particularly applies to the cyclical pattern of reallocation of the unemployed.

For these purposes, we use the Survey of Income Program Participation (SIPP) for the period 1986-2011. The SIPP, administrated by US Census Bureau, provides demographic data on a reasonably large number of individuals of all ages at a moment in time. It follows them typically for 2.5 to 4 years, depending on the panel, while keeping track of the individuals' labor market status, including workers' occupations and matches with firms. From this sample, we consider all workers between 16 and 65 years of age who are not in self-employment, government employment or in the armed forces. In the Supplementary Appendix we provide further details of the data used, and its construction; here we present the main results.

To study occupational reallocation we consider those unemployed workers that after experiencing an employment to unemployment transition reported an *uninterrupted* spell of unemployment that ended in employment. We consider a worker to be unemployed when he/she is unattached to a job and looking for work.[13] We compare their occupation at re-employment with the previously held occupations, focusing on 'major' occupational groups.[14] Our focus is also on gross occupational mobility. Given that in the SIPP gross mobility flows are typically 9 times larger than net mobility flows, and that we are interested in understanding how reallocation frictions affect decisions and outcomes for individual workers in unemployment, rather than the effect of occupational shifts on aggregate output, we find the former better suited for the problem at hand.[15]

The Extent of Occupational Mobility To measure the extent of occupational mobility in unemployment, we calculate the proportion of the inflow into unemployment (or equivalently, outflow from unemployment) that will eventually re-enter employment in a different occupation, and denote it by Cm. For brevity, we will simply refer to the group of unemployed workers who find a job in a different occupation as 'occupational movers' and their counterparts as 'occupational stayers'.

Occupational mobility through unemployment is high, on average 50% of workers will find a job that is classified in a different major occupational group. In table 1 we collect the extent of occupational mobility, also for various demographic groups. We see mobility is broadly similar across gender and eduction (using the entire sample period and pooling all the panels). Occupational mobil-

[13] Under this assumption we exclude those workers that are still with a job but on layoff and hence reduce the effects of firms recalling laid-off workers on our analysis. We miss, however, those workers on layoff who were not recalled and changed employers and include those workers that experienced a permanent separation and where recall by their previous employers. Moscarini and Fujita (2012), who provide an analysis of recall unemployment using the SIPP for a similar period, find that the latter types of workers represent a small proportion of those workers without a current job (on layoff or permanently separated). See the Supplementary Appendix for the employment status classification used in the SIPP.

[14] We also have performed our analysis by comparing the occupation at re-employment with the entire pool of occupations reported in all observed employment spells prior to the unemployment spell and for finer levels of aggregation, finding similar results.

[15] To measure the relative size of gross mobility flows we use $M_o = \frac{min\{I_o, O_o\}}{|I_o - O_o|}$, where I_o denotes the inflow of workers into an occupation o and O_o the outflow from that occupation. For each of the three sets of panels sharing the same occupational classification (see the Supplementary Appendix for details), we rank each occupation by the size of their M_o and compute the median value. The number reported above is the average of these median values.

Table 1: Proportion of completed unemployment spells ending with an occupation change

Major Occupational Groups

	all	male	female	high school	college
young ($20 < \text{age} \leq 30$y)	0.525	0.541	0.506	0.537	0.526
prime ($35 < \text{age} \leq 55$)	0.445	0.446	0.443	0.469	0.453
all working ages	0.500	0.511	0.486	0.519	0.493

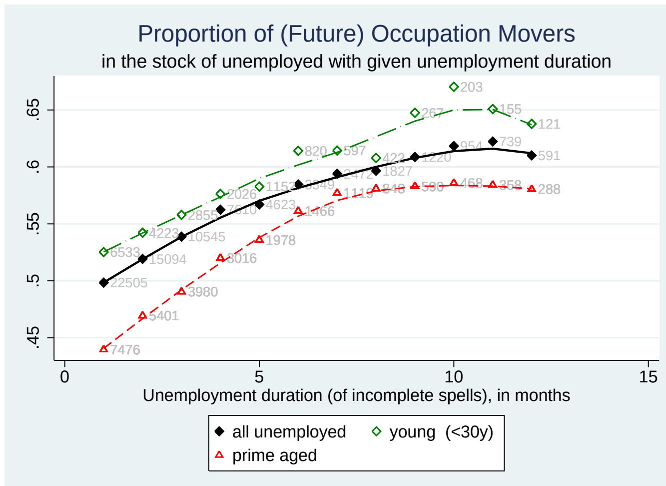

Figure 1: Extent of occupational mobility by unemployment duration - Major occupational groups

ity declines with age: prime-aged workers (35-55yo) change occupations about 15% less than young workers (20-30yo) at the end of their unemployment spell.

From the definition of the occupational categories, these occupational movements appear largely "horizontal". For example, a worker could change from performing a "sales" related job to performing (after re-employment) a "transportation and material moving" related job. These occupational movements also appear somewhat random as their transition matrices show no significant clustering outside the diagonal (see the Supplementary Appendix).

Now consider the *monthly outflow rates* for the subset of all unemployed who are occupational movers, Um_t, resp. Us_t for occupational stayers. Formally, the outflow rate of occupational movers, Pm_t, equals UEm_{t+1}/Um_t, where UEm_{t+1} denotes the number of unemployed workers at month t that found a job in a different occupation the following month, $t+1$. Ps_t is the analogous outflow rate for stayers. We find that on average occupational stayers tend to leave unemployment faster than occupational movers, with $Ps = 0.300$ versus $Pm = 0.255$. Interestingly, this pattern occurs across occupational categories, and age. For young workers, for example, the monthly outflow rate for movers $Pm = 0.275$, while for stayers $Ps = 0.322$. For prime age workers we obtain that $Pm = 0.215$ and $Ps = 0.269$.[16]

[16]Combining the above measures we obtain $Jfm_t = Pm_t Cm_t$ and $Jfs_t = Ps_t(1 - Cm_t)$, which describe the proportion of unemployed workers at time t that found a job in a different (same) occupation between t and $t+1$. Overall the above results show that both occupational movers and stayers account for a significant proportion of the total monthly outflow from unemployment; 50.1% of the aggregate job finding rate, $Jfm + Jfs$, occurs with a major occupational change. Bart Hobijn has kindly provided us with his own calculations of the latter measure using the CPS for the period 1986-2011. He finds that on average 48.2% of all those unemployed workers who became employed at any given month changed their major occupations. This calculation, however, combines those unemployed workers that, before finding a job, did not transit between unemployment and non-participation (or vice-versa) and those that did, were our analysis is based only on the former set of workers. By combining these two types of workers we obtain that 41.7% those unemployed workers who became employed at any given month changed major occupation. This comparison is evidence

Table 2: Monthly median re-employment wage changes (%) - Major occupational groups.

Age Group	Aggregate	Occupational Change	Occupational Stay
All	-6.060	-1.378	-7.773
Young	2.223	10.106	0.128
Prime	-9.102	-7.648	-10.061
Old	-14.357	-15.998	-10.456

While a comparison of the average outflow rates shows that occupational movers spend longer in unemployment, it does not tell us precisely how occupational mobility evolves with unemployment duration. Figure 1 shows that as on-going unemployment spells become longer, more of the remaining unemployed will end up in a new occupation (though this relation becomes non-monotone close to 12 months duration). This increase, however, is very moderate. As a result, the proportion of workers who will be occupational stayers remains high, even at high unemployment durations (close to 40% at 12 months). The slow increase with duration of the proportion of occupational movers in the stock of remaining unemployed is not driven by a composition effect over age: it also appears when we look at the subset of young and prime-aged workers. Their profiles appear to be vertical translations, shifted up for young workers, relative to the profile for all, and shifted down for prime-aged workers.

This pattern reflects the reallocation choices of workers and hence can be informative about the underlying environment in which unemployed workers trade off the cost and benefits of reallocation. An immediate and unambiguous conclusion based on these data alone, however, is not straightforward. Does the long time spent in unemployment for occupational movers reflect that reallocation itself takes time? Or is it that after a job separation labor market conditions need to worsen further before a worker looks for jobs in other occupations, so those who find jobs quickly have been first searching in their old occupation? These two explanations imply different sources behind unemployment and its cyclical behavior. With the model in hand, we hope to uncover these sources.

Post-reallocation Labor Market Outcomes We now analyse the outcomes after a reallocation to explore further the potential gains of reallocation and the potential forces in the way of it.

First, consider the gains of reallocation. Perhaps the most direct measure is to compare a worker's wages after unemployment with the same worker's wages before unemployment, for the set of workers who reallocate, and the set of those who stay in their occupation.[17] Table 2 considers the average of the monthly median re-employment wage changes for workers that experienced an unemployment spell leading to either an occupational or a non-occupational change.[18] The monthly median re-employment wage growth is overall negative. Re-employment wage growth is higher when the workers ended their unemployment spell with an occupational change than when they stayed in the same occupation. This feature is prevalent at an aggregate level and for young and prime age workers. Further these pat-

that the extent of occupational reallocation through unemployed we find in the SIPP is also present in the CPS.

[17] To emphasize, we are considering the gains of reallocation for a specific subset of workers, those who go through unemployment. Potentially different patterns for job-to-job or firm-to-firm moves of employed workers with an occupational change or stay concern a very different subset of workers in a very different choice environment. One should therefore be careful drawing parallels. For recent work looking at wage changes with occupational changes of employed workers, see Groes et al. (2012).

[18] The wage change is computed using the average wage earned in the job immediately prior and the average wage earned in the job that followed immediately after the unemployment spell. These results are based on the 1996 - 2008 panels as they provide more reliable estimates of re-employment wage changes due to their larger sample size. As from 1996 the sample size of each panel increased from, approximately, 15,000 to 40,000 individuals.

Table 3: Outflow rates of the repeat unemployed

Age Group	stay after stay	stay after move	move after move	move after stay
All	0.294	0.309	0.281	0.263
Young	0.307	0.348	0.290	0.305
Prime	0.284	0.305	0.274	0.194

terns survive when we consider the average median re-employment wage changes based on the entire panel rather than in each month.[19] Since workers who became unemployed *and* changed occupations appear to gain (lose less) in wages relative to workers who became unemployed and stayed in their occupation, reallocation appears to benefit workers afterwards.

To get a sense to what degree workers display attachment to their occupations, we analyse what happens if a recently unemployed worker becomes unemployed again.[20] We refer to the statistics concerning these two subsequent unemployment spells as 'repeat mobility' statistics, and the workers in underlying these statistics as 'repeat unemployed'. When analysing repeat-unemployed workers who did *not* change occupation in their first spell, we find that they are more likely to remain in their occupation at the end of their second unemployment spell. A little under 40% (36%-40%) of these workers move occupations in the second unemployment spells. This percentage is lower for prime-aged workers (35%), and higher for young workers (44%), and it compares to the proportion of 50% for occupational mobility over all unemployment spells. Likewise, we can calculate a corresponding statistic for repeat-unemployed workers who *changed* occupations in their initial spell. We find that these workers in a larger proportion move to yet another occupation in their second spell of unemployment. Unlike the other statistics, the precise percentage of this proportion is sensitive to the definition of occupations held before the first occupational move in the data, ranging from 46%-56%.[21]

Now, consider the *outflow rates* from unemployment in the second spell, as function of occupational mobility in the previous and the current unemployment spell, as displayed in table 3. Note that the average outflow rates of occupational movers (resp. stayers) in their second spell are close to the average outflows rates of movers (resp. stayers) in all spells. The average outflow rate of movers among the repeat unemployed is slightly higher than the average overall outflow rate of movers, reflecting that more young workers are among the repeat movers. In this respect, repeat-unemployed workers appear to be similar to the typical unemployed worker. A couple of patterns are particularly interesting: (i) the highest outflow rate occurs among repeat unemployed who only moved occupations in their previous spell (in particular for the young); and (ii), the lowest outflow rate occurs for those

[19]When considering all the panels the results also show that the overall average monthly median and panel median are higher for workers that change occupation and for the young group. However, for the prime age workers re-employment wage growth is slightly higher for those that did not change occupation.

[20]Even though this worker could be from a selected subset of workers, we can gain insights from these statistics; the model subsequently will allow us to get more traction on the selection issue.

[21]The issue here is the treatment of occupations held preceding the first of the two unemployment spells in question. If returns to those occupations are counted as occupational stays, then this statistic is 46%, if we consider these transitions as 'ambiguous' and leave them out of the sample, then we get 56%. A reason for this sensitivity is that we are purposefully very harsh in selecting repeated spells of unemployment and the number of spells is somewhat small (though the fact that we have the entire panel from 1986-2011 at hand, helps). Since we need to assign the second spell to occupational move/stay, the spell needs to be complete; then, since short spells are more likely to be stays, we require that at the beginning of the second spell, the worker remains in sample for at least another year. This issue does not change much the proportion of stayers in an unemployment spell following an spell with an occupational stay, and does not affect overall mobility statistics.

workers who stayed in their occupation in the previous spell but change occupations in the current spell. This number appears driven mostly by prime-aged workers. These differences appear consistent with a degree of attachment to an old occupation acquired over time, which is not as strong in the new occupation after reallocation; the first pattern may also reflect gains in job finding rates after reallocation. Without getting traction on the underlying fundamentals (and any selection issues arising from these), this is speculative – but, with the model in hand, we can shed more light on these matters.

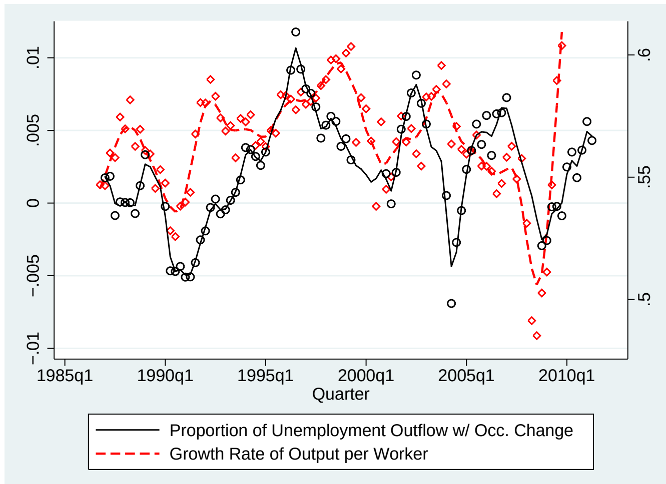

Figure 2: Moving average of the growth rate of output per worker and the log series of Cm- Major occupational groups.

Business Cycle Patterns We now turn to analyse the behavior of the proportion of occupational movers in the outflow from unemployment, and outflow rates Pm_t and Ps_t over the business cycle. Figure 2 shows a (centered) 5-quarters moving average of the proportion of workers starting in a new occupation of all unemployed workers gaining employment in quarter t, together with the growth rate of output per worker at this time. It shows that the occupational mobility of the unemployed is procyclical, it tends to be higher when the economy is growing faster. The correlation of the proportion with the growth rate of output per worker is 0.36. Table 4 shows that the volatility and cyclicality of Cm, Pm and Ps.

Table 4: Composition and Outflow Rates of Occupational Movers/Stayers over the Business Cycle

	Jf	Cm	Pm	Ps	u	output/worker	output
Standard Deviation	0.097	0.028	0.101	0.099	0.129	0.009	0.016
Autocorrelation	0.928	0.850	0.947	0.943	0.966	0.695	0.871
Corr. w/ output/worker	0.453	0.256	0.464	0.433	-0.524	1.000	0.834
Correlation w/ u	-0.773	-0.255	-0.854	-0.646	1.000	-0.524	-0.816

It appears that the claim that unemployed workers find it better to reallocate in bad times, perhaps building on an intuition that in downturns the opportunity cost of reallocation is lower, is not supported by observed behavior in the data. This raises two questions we take up in the quantitative section of the paper. How we can rationalize the pattern we actually observe among occupational movers/stayers, while keeping consistent with other important labor market facts? What further implications does the procyclicality of reallocation have for unemployment fluctuations and its many facets?

Perspective The patterns documented above show that (i) occupational mobility among the unemployed is high, decreases with age and increases with unemployment duration. (ii) There is evidence of gains to reallocation as well as occupational attachment. (iii) Occupational mobility is procyclical. These provide a new set of facts on the occupational mobility outcomes of the unemployed (exclusively), in particular of the cyclical behavior of their occupational mobility.

In the context of (our) existing knowledge, these mobility patterns are in line with and complement those documented in Murphy and Topel (1987) for *sectoral* mobility in the US using the CPS for the period 1970-1985. They show that the incidence of unemployment is significantly higher for those workers who change sectors and that inter-sectoral mobility is procyclical for these workers. Our results are also in line with recent unpublished work by Bart Hobijn, which is perhaps closest related to us. He documents the procyclicality of the proportion of occupational movers out of unemployment in hires using the CPS for the period 1986-2011.

The analysis of Kambourov and Manovskii (2008), using the PSID, Xiong (2008), using the SIPP, and Longhi and Taylor (2011), using the Labour Force Survey for the UK, also find that the extent of occupational mobility through unemployment is high (though their focus is not on behavior over the business cycle). Fujita and Moscarini (2012), using the SIPP, have found that those worker that experienced unemployment after being permanently separated from their previous jobs are much more likely to make an (3-digit) occupational change than those that were on layoff and recalled within 3 months. They also find that the likelihood of experiencing occupational change in this context increases with unemployment duration.[22]

In the next sections we construct and study, analytically and quantitatively, a business cycle model of the labor market that aims to contribute to our understanding of unemployment with particular focus on the role of reallocation in unemployment and its behavior over the business cycle. While occupational human capital is a key input in these matters, for ease of exposition we first present the model without it. We discuss the implications of this feature later in the paper. In the Appendix we show that our results generalize to this case.

3 Model

Time is discrete, and goes on forever; it is denoted by t. There is a finite number of occupations indexed by $o = 1, ..., O$. Within each occupation there is a continuum of infinitely lived risk-neutral workers. At any time t workers in an occupation o differ in their occupation-specific productivities, z_{ot}. These productivities are specific to the current match between the worker and occupation and evolve over time.[23] A worker can be either employed or unemployed in an occupation. An unemployed

[22]Faberman and Kudlyak (2012) with data from an on-line job-search website, find that workers apply more to vacancies outside their usual occupational field as their spell duration increases.

[23]Other than a match specific component, we think that the productivity of a worker in an occupation also depends on several other factors, not explicitly modelled, such as the sector and geographical location of this worker. For example, the productivity of a chemical engineer working in the photo industry in Rochester, US, can differ from the productivity of another, equally qualified, chemical engineer working in the oil industry in Texas, US. Moreover, these productivities can evolve differently over time. Our occupation specific productivity tries to capture these features in a tractable way.

worker receives b each period. The wages of employed workers are determined below. There is also a continuum of risk-neutral firms that live forever that are attached to each occupation.[24] A firm has one position and needs a worker to produce a good with a production function $y(p_t, z_{ot})$, where p_t is the aggregate productivity shock which impacts all occupations in the economy. We assume that the production function is continuous differentiable and strictly increasing in all arguments. Both types of productivities are drawn from bounded intervals and follow stationary Markov-processes. Further, the set of productivities z and the stochastic process governing their evolution are identical across workers and occupations. Assume that all agents discount the future using the same discount factor β.

A firm in occupation o can find a worker with productivity z_{ot} by posting a vacancy. Posting a vacancy costs k per period. Only unemployed workers can decide to search for vacant jobs. Once a match is formed, firms pay workers according to the posted contract, until the match is broken up. The latter can happen with an exogenous (and constant) probability δ, but can also occur if the worker and the firm decide to do so. Once the match is broken, the worker becomes unemployed in his current occupation and the firm has to decide to reopen the vacancy. A worker that separates from his current employer (voluntarily or not) stays unemployed in his occupation until the end of the period.

Instead of applying for jobs in their current occupations, unemployed workers can also decide to start a reallocation process towards another occupation by paying a cost c and sampling a new z in a different occupation. The new productivity z is an i.i.d. draw with cdf $F(z)$, while any new occupation is drawn independently of z and has a $1/(O-1)$ probability of being sampled. To simplify the analysis we assume that once a worker samples a new productivity, he does not recall his previous one. In essence, occupational reallocation allows the worker to re-start the z process at a new value of z that is independent of his previous one. Our notation of z_o is meant to capture both the value of z and the identity of the occupation, (z, o), and anticipates that the relevant component is z.

Over time workers can sample from the set of occupations with replacement, such that it is possible to re-visit previously performed occupations each time with a different z.[25] Further, workers who have sampled a new z cannot immediately apply for jobs in the new occupation, and must sit out the rest of the period unemployed. Once the period is over, workers can again decide whether to apply for jobs in the new occupation or sample another z in a different occupation. A worker, however, will not be considered to have undergone an occupational reallocation until he has finalised the reallocation process by finding a job in a new occupation. This distinction is made in order to measure occupational reallocation in the quantitative section of the paper in the same way as we did in Section 2.

Given the above considerations, Figure 3 summarises the timing of the events within a period in a given occupation. A period is subdivided into four stages: separation, reallocation, search and matching, and production. Let $\mathcal{E}_t^x$ denote the joint distribution of z-productivities for unemployed and employed workers over all occupations at the beginning of stage x in period t. The state space for a worker with productivity z_{ot} at the beginning of stage x is then described by the vector $\Omega_t^x = \{es_t, o_t, p_t, z_{ot}, \mathcal{E}_t^x\}$, where es_t captures the worker's employment status and o_t his occupational at-

[24] Equivalently one can think of a continuum of firms attached to a particular z within an occupation.

[25] One justification is, again that the same occupation has many locations (or sectors) with productivities independently drawn from $F(z)$. The worker samples one cross-product between location and occupation at a time, and hence can return to an occupation and still get a new draw. In his current occupation, we assume that the z-process, in reduced form, captures the best the worker can do without switching occupations.

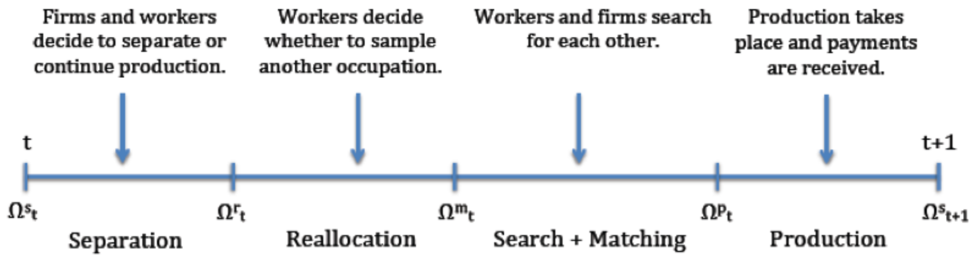

Figure 3: Timing of events within a period

tachment. To keep the notation as simple as possible, we will leave Ω^x_t only with its time subscript, though it should be understood that it captures all states relevant for the worker. Tractability will arise because we can show that the equilibrium decision rules have a relevant state space described solely by $\{p_t, z_t\}$ and the employment status, for completeness we present the setup of the model using the general state space described by Ω.

Posting and Matching In each occupation firms post contracts to which they are committed. Unemployed workers and advertising firms then match with frictions as in Moen (1997). In particular, for each productivity z_o in an occupation o there is a continuum of sub-markets, one for each expected lifetime value $\tilde{W}$ that could potentially be offered by a vacant firm.[26] After firms have posted a contract in the sub-market of their choice, unemployed workers with productivity z_o can choose which appropriate sub-market to visit. Once in their preferred sub-market j, workers and firms meet according to a constant returns to scale matching function $m(u_j, v_j)$, where u_j is the measure of workers searching in sub-market j, and v_j the measure of firms which have posted a contract in this sub-market.

From the above matching function one can easily derived the workers' job finding rate $\lambda(\theta_j) = m(1, v_j/u_j)$ and the vacancy filling rate $q(\theta_j) = m(u_j/v_j, 1)$, where labor market tightness is given by $\theta_j = v_j/u_j$. The matching function and the job finding and vacancy filling rates are assumed to have the following properties: (i) they are twice-differentiable functions, (ii) non-negative on the relevant domain, (iii) $m(0,0) = 0$, (iv) $q(\theta)$ is strictly decreasing, and (v) $\lambda(\theta)$ is strictly increasing and concave.[27]

Worker's problem Consider the value function of an unemployed worker having productivity z_{ot} in occupation o at the beginning of the production stage, $W^U(\Omega^p_t) = b + \beta\mathbb{E}[W^R(\Omega^r_{t+1})]$, where Ω^p_t summarises the worker's state vector at the production stage. The value of unemployment consists of the flow benefit of unemployment b this period, plus the discounted expected value of being unemployed at the beginning of next period's reallocation stage,

$$W^R(\Omega^r_{t+1}) = \max_{\rho(\Omega^r_{t+1})} \{\rho(\Omega^r_{t+1})R(\Omega^r_{t+1}) + (1-\rho(\Omega^r_{t+1}))\mathbb{E}[S(\Omega^m_{t+1}) + W^U(\Omega^m_{t+1})]\}, \tag{1}$$

[26] This modelling choice is without a loss of generality. In principle, we could allow workers with different z_o productivities to visit the same sub-market and firms to offer menus which specify the $\tilde{W}$ for the contingency that a firm matches with a worker with productivity z_o. One can show that these menus will be chosen such that in equilibrium only one type of worker will visit each sub-market.

[27] We impose two restrictions on beliefs off-the-equilibrium path. Workers believe that, if they go to a sub-market that is inactive on the equilibrium path, firms will show up in such measure to have zero profit in expectation. Firms believe that, if they post in an inactive sub-market, a measure of workers will show up, to make *the measure of deviating firms* indifferent between entering or not. We assume, for convenience, that the zero-profit condition also holds for deviations of a single agent: loosely, the number of vacancies or unemployed, and therefore the tightness will be believed to adjust to make the zero-profit equation hold.

where $\rho(\Omega^r_{t+1})$ takes the value of one when the worker decides to reallocate and samples a new z in a different occupation and take the value of zero otherwise. Value $R(.)$ denotes the expected benefit of sampling a new productivity $z'_{o'}$ in a different occupation. Given that workers who sample a new z have to sit out the rest of the period unemployed, this benefit is given by $R(\Omega^r_{t+1}) = -c + \mathbb{E}_{\tilde{\Omega}^p_{t+1}}[W^U(\tilde{\Omega}^p_{t+1})]$, where $\tilde{\Omega}^p_t$ refers to the states associated with the different values of z and o that a worker can sample.

The worker's expected value of staying and searching in his old occupation is given by $\mathbb{E}[S(\Omega^m_{t+1}) + W^U(\Omega^m_{t+1})]$. In this case, $W^U(\Omega^m_{t+1}) = \mathbb{E}[W^U(\Omega^p_{t+1})]$ describes the expected value of not finding a job, while $S(\Omega^m_{t+1})$ summarizes the expected value *added* of finding a new job. The reallocation decision is captured by the choice between $R(\Omega^r_{t+1})$ and the expected payoff of search in the current occupation.

To derive $S(.)$ recall that $\lambda(\theta(\Omega^m_t, W_f))$ denotes the probability with which a worker with productivity z_o meets a firm f in the sub-market associated with the promised value W_f and tightness $\theta(\Omega^m_t)$. Further, let $\alpha(W_f)$ denote the probability of visiting such a sub-market. From the set $\mathcal{W}$ of promised values which are offered in equilibrium by firms for a given z_o, workers only visit with positive probability those sub-markets for which the associated W_f satisfies

$$W_f \in \arg\max_{\mathcal{W}} \lambda(\theta(\Omega^m_{t+1}, W_f))(W_f - W^U(\Omega^m_{t+1})) \equiv S(\Omega^m_{t+1}). \tag{2}$$

When the set $\mathcal{W}$ is empty, the expected value added of finding a job is zero and the worker is indifferent between visiting any sub-market.

Now consider the value function at the beginning of the production stage of an employed worker with productivity z_{ot} in a contract that currently has a value $\tilde{W}_f(\Omega^p_t)$. Similar arguments as before imply that

$$\tilde{W}_f(\Omega^p_t) = w_{ft} + \beta\mathbb{E}\Big[\max_{d(\Omega^s_{t+1})}\{(1 - d(\Omega^s_{t+1}))\tilde{W}_f(\Omega^s_{t+1}) + d(\Omega^s_{t+1})W^U(\Omega^s_{t+1})\}\Big], \tag{3}$$

where $d(\Omega^s_{t+1})$ take the value of δ when $\tilde{W}_f(\Omega^s_{t+1}) \geq W^U(\Omega^s_{t+1})$ and the value of one otherwise. In equation (3), the wage payment w_{ft} at firm f is contingent on state Ω^p_t, while the second term describes the worker's option to quit into unemployment in the separation stage the next period. Note that $W^U(\Omega^s_{t+1}) = \mathbb{E}[W^U(\Omega^p_{t+1})]$ as a worker who separates must stay unemployed for the rest of the period and $\tilde{W}_f(\Omega^s_{t+1}) = \mathbb{E}[\tilde{W}_f(\Omega^p_{t+1})]$ as the match will be preserved after the separation stage.

Firm's problem Consider a firm f in occupation o, currently employing a worker with productivity z_{ot} who has been promised a value $\tilde{W}_f(\Omega^p_t) \geq W^U(\Omega^p_t)$. Noting that the state space for this firm is the same as for the worker and given by Ω^x_t, the expected lifetime discounted profit of the firm can be described recursively as

$$\begin{aligned} J(\Omega^p_t; \tilde{W}_f(\Omega^p_t)) \quad = \quad \max\Big\{ & y(p_t, z_{ot}) - w_{ft} + \beta\mathbb{E}\Big[\max_{\sigma(\Omega^s_{t+1})}\Big\{(1 - \sigma(\Omega^s_{t+1}))J(\Omega^s_{t+1}; \tilde{W}_f(\Omega^s_{t+1})) \\ & + \sigma(\Omega^s_{t+1})\tilde{V}(\Omega^s_{t+1})\Big\}\Big]\Big\}, \end{aligned} \tag{4}$$

where $\sigma(\Omega^s_{t+1})$ takes the value of δ when $J(\Omega^s_{t+1}; \tilde{W}_f(\Omega^s_{t+1})) \geq \tilde{V}(\Omega^s_{t+1})$ and the value of one otherwise, $\tilde{V}(\Omega^s_{t+1}) = \max\{\overline{V}(\Omega^s_{t+1}), 0\}$ and $\overline{V}(\Omega^s_{t+1})$ denotes the maximum value of an unfilled vacancy in occupation o at the beginning of next period $t+1$. Hence (4) takes into account that the firm could decide to target its vacancy to workers of with a different productivity in the same occupation or withdraw the vacancy from the economy and obtain zero profits.

The first maximisation in (4) is over the wage payment w_{ft} and the promised lifetime utility to the worker $\tilde{W}_f(\Omega^p_{t+1})$. The second maximisation refers to the firm's layoff decision. The solution to (4) then gives the wage payments during the match (for each realisation of Ω^p_t for all t). In turn these wages determine the expected lifetime profits at any moment during the relation, and importantly also at the start of the relationship, where the promised value to the worker is $\tilde{W}_f$.

Equation (4) is subject to the restriction that the wage paid today and tomorrow's promised values have to add up to today's promised value $\tilde{W}_f(\Omega^p_t)$, according to equation (3). Moreover, the workers' option to quit into unemployment, and the firm's option to lay off the worker imply the following participation constraints

$$\Big(J(\Omega^s_{t+1};\tilde{W}_f(\Omega^s_{t+1})) - \tilde{V}(\Omega^s_{t+1})\Big) \geq 0 \quad \text{and} \quad \Big(\tilde{W}_f(\Omega^s_{t+1}) - W^U(\Omega^s_{t+1})\Big) \geq 0. \tag{5}$$

Now consider a firm posting a vacancy in occupation o. Given cost k and knowing Ω^m_t, a firm must choose which unemployed workers to target. In particular, for each z_o a firm has to decide which $\tilde{W}_f$ to post given the associated job filling probability, $q(\theta(\Omega^m_t, \tilde{W}_f))$. This probability summarises the pricing behaviour of other firms and the visiting strategies of workers. Along the same lines as above, the expected value of a vacancy targeting workers of productivity z_o solves the Bellman equation

$$V(\Omega^m_t) = -k + \max_{\tilde{W}_f}\Big\{q(\theta(\Omega^m_t, \tilde{W}_f))J(\Omega^m_t, \tilde{W}_f) + (1 - q(\theta(\Omega^m_t, \tilde{W}_f)))\tilde{V}(\Omega^p_t)\Big\}. \tag{6}$$

We assume that there is free entry of firms posting vacancies within any occupation. This implies that $V(\Omega^x_t) = 0$ for all those Ω^x_t and $\tilde{W}_f$ that yield a $\theta(\Omega^x_t, \tilde{W}_f) > 0$, and $V(\Omega^x_t) \leq 0$ for all those Ω^x_t and $\tilde{W}_f$ that yield a $\theta(\Omega^x_t, \tilde{W}_f) \leq 0$ at any stage x in period t. In the former case, the free entry condition then simplifies (6) to $k = \max_{\tilde{W}_f} q(\theta(\Omega^m_t, \tilde{W}_f))J(\Omega^m_t, \tilde{W}_f)$.

Worker flows Until now, we have taken as given the state vectors $\Omega^s_t, \Omega^r_t, \Omega^m_t, \Omega^p_t$ and their evolution to discuss agents' optimal decisions. As mentioned earlier p_t, z_{ot} follow exogenous processes that are common across all workers and occupations. However, the evolution of the number of unemployed and employed workers is a result of optimal vacancy posting, visiting strategies, separation and reallocation decisions. In the Supplementary Appendix we provide a derivation of how these measures evolve.

4 Equilibrium

We look for an equilibrium in which the value functions and decisions of workers and firms in any occupation only depend on the productivities p_t and z_t and workers' employment status. To solve for equilibria of this type it is convenient to label the labor market attached to a particular value of z_o in an occupation o an 'island'. Hence, in any period t an island, within a given occupation and associated with the value z_o, is composed of unemployed workers with idiosyncratic productivity equal to z_{ot}, vacant firms which have decided to attract unemployed workers of productivity z_{ot}, and existing job matches with current productivity z_{ot}. When workers move to a new occupation, they sample one from the measure of islands in that occupation, and become an unemployed worker on that island after reallocation. When a worker's z-productivity is shocked, it is as if he has moved islands within his occupation.[28]

[28]The island-structure we set up also can be applied to study *net* mobility between occupations, sectors, etc.; in this case, the mapping between islands and e.g. an occupation could be one-to-one. All results that we derive subsequently would hold for such a

Under these considerations the following describe the candidate equilibrium value functions of agents in any occupation

$$W^U(p,z_o) = b + \beta \mathbb{E}_{p',z'_o}\Bigg[\max_{\rho(p',z'_o)}\Big\{\rho(p',z'_o)\Big[-c + \sum_{o'\neq o}\int W^U(p',\tilde{z_{o'}})\frac{dF(\tilde{z})}{O-1}\Big] + \tag{7}$$

$$(1-\rho(p',z'_o))\Big[\max_{W^{E\prime}}\Big\{\lambda(\theta(p',z'_o,W^{E\prime}))W^{E\prime} + (1-\lambda(\theta(p',z'_o,W^{E\prime})))W^U(p',z'_o)\Big\}\Big]\Big\}\Bigg]$$

$$W^E(p,z_o) = w(p,z_o) + \beta \mathbb{E}_{p',z'_o}\max_{d(p',z'_o)}\Big\{(1-d(p',z'_o))W^E(p',z'_o) + d(p',z'_o)W^U(p',z'_o)\Big\} \tag{8}$$

$$J(p,z_o,\tilde{W}^E) = \max_{\{w,\tilde{W}^{E\prime}(p',z'_o)\}}\Big\{y(p,z_o) - w + \beta\mathbb{E}_{p',z'_o}\max_{\sigma(p',z'_o)}\{(1-\sigma(p',z'_o))J(p',z'_o,\tilde{W}^{E\prime}(p',z'_o))\}\Big\} \tag{9}$$

$$V(p,z_o,\tilde{W}) = -k + q(\theta(p,z_o,\tilde{W}))J(p,z_o,\tilde{W}) \leq 0, \tag{10}$$

where we have left implicit the time subscripts denoting the following period with a prime, $\tilde{z}$ in (7) refers to new draws of z in different occupations, $\tilde{W}^E$, w and $\tilde{W}^{E\prime}$ must satisfy (8) and the maximisation in (9) is subject to the participation constraints in (5).

In this type of equilibria we do not need to keep track of the measures of unemployed and employed workers within occupations or their flows between occupations to derive agents' decision rules. In turn, this implies that equilibrium outcomes can now be derived in two steps. In the first step, decision rules are solved independently of the heterogeneity distribution that exists across islands using the above four value functions. Once those decision rules are determined, we fully describe the dynamics of these distributions using the workers' flow equations. Shi (2009) and Menzio and Shi (2011, 2012) call this type of equilibrium 'block recursive'.[29]

Definition A Block Recursive Equilibrium (BRE) is a set of value functions $W^U(p,z_o)$, $W^E(p,z_o)$, $J(p,z_o,W^E)$, policy functions $d(p,z_o)$, $\rho(p,z_o)$, $\alpha(p,z_o)$ (resp. separation, reallocation and visiting strategies), firms' policy functions $\tilde{W}_f(p,z_o)$, $\sigma(p,z_o,W^E)$, $w(p,z_o,W^E)$, $\tilde{W}^{E\prime}(p,z_o,W^E)$ (resp. contract posted, layoff decision, wages paid, and continuation values promised), tightness function $\theta(\tilde{W},p,z_o)$, laws of motion of z_o, p for all islands and occupations, and a law of motion on the distribution of unemployed and employed workers over islands and occupations $\tilde{u}(.) : \mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$ and $\tilde{e}(.) : \mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$, such that *(1)* the value functions and decision rules follow from the firm's and worker's problems outlined above; *(2)* tightnesses $\theta(p,z_o,\tilde{W})$ are consistent with free entry, with zero expected profits determining $\theta(p,z_o,\tilde{W})$ for all $\tilde{W}$ at which positive ex-post profits exist, also off the equilibrium path; $\theta = 0$ otherwise; *(3)* distributions evolve according to workers' and firms' decisions. A more detailed definition and the flow equations underlying the evolution of the distributions are in the Supplementary Appendix.[30]

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setting as well.

[29]This recursive property is common in many search models when markets are segmented per type and there is constant returns to scale in the key 'production' functions.

[30]We have not fully specified wages here; this is because wages are not uniquely pinned in the equilibrium, whereas allocations are. Firms and workers, faced with the same discount rates, can substitute across time wage payments, without affecting separations and application decisions, as long as the ex ante value of the match remains the same. If one additionally assumes that over time, the values in existing matches are kept identical to values in starting matches with the same state variables, the equilibrium wages paid are exactly in line with the Nash Bargaining solution under the Hosios' condition.

Characterization We start the characterisation of equilibria by showing that if there are positive gains to form a productive match in an island, firms offer a unique $\tilde{W}_f$ with associate tightness $\tilde{\theta}(p, z_o)$ in the matching stage. Consider an island characterised by state vector (p, z_o). For any promised value W^E, the joint value of the match is defined as $W^E + J(p, z_o, W^E) \equiv \tilde{M}(p, z_o, W^E)$. Lemma 1 shows that under risk neutrality the value of a job match is constant in W^E and J decreases one-to-one with W^E.

Lemma 1. *The joint value $\tilde{M}(p, z_o, W^E)$ is constant in $W^E \geq W^U(p, z_o)$ and hence we can uniquely define $M(p, z_o) \stackrel{def}{=} \tilde{M}(p, z_o, W^E)$, $\forall\ M(p, z_o) \geq W^E \geq W^U(p, z_o)$ on this domain. Further, $J_W(p, z_o, W^E) = -1$, $\forall\ M(p, z_o) > W^E > W^U(p, z_o)$, and endogenous match breakup decisions are efficient from the perspective of the firm-worker match.*

The proof of Lemma 1 crucially relies on the firms' ability to offer workers inter-temporal wage transfers such that the value of the job match is not affected by the (initial) promised value. Note that Lemma 1 implies that no firm will post vacancies in islands for which $M(p, z_o) - W^U(p, z_o) \leq 0$. Lemma 2 now shows that in any island z_o, for which $M(p, z_o) - W^U(p, z_o) > 0$, firms offer a unique $\tilde{W}_f$ in the matching stage and there is a unique θ associated with it.

Lemma 2. *Assume free entry of firms, and $J_W(p, z_o, W^E) = -1$ for each p, z_o, and a matching function that exhibits constant returns to scale, with a vacancy filling function $q(\theta)$ that is nonnegative and strictly decreasing, while the job finding function $\lambda(\theta)$ is nonnegative, strictly increasing and concave. If the elasticity of the vacancy filling rate is weakly negative in θ, there exists a unique $\theta^*(p, z_o)$ and $W^*(p, z_o)$ that solve* (2)*, subject to* (6)*.*

The requirement that the elasticity of the job filling rate with respect to θ is non-positive is automatically satisfied when $q(\theta)$ is log concave, as is the case with the Cobb-Douglas and urn ball matching functions. Both matching functions imply that the job finding and vacancy filling rates have the properties described in Lemma 2 and hence guarantee a unique pair $\tilde{W}_f, \theta$. To simplify the analysis that follows, we assume a Cobb-Douglas matching function as it implies a constant $\varepsilon_{q,\theta}(\theta)$. Using η to denote the (constant) elasticity of the job finding rate with respect to θ, we find the well-known division of the surplus according to the Hosios' (1991) rule

$$\eta(W^E - W^U(p, z_o)) - (1 - \eta)J(p, z_o, W^E) = 0. \tag{11}$$

Since in every island there is at most one $\tilde{W}_f$ offered in the matching stage, the visiting strategy of an unemployed worker is to visit the sub-market associated with $\tilde{W}_f$ with probability one when $S(p, z_o) > 0$ and to randomly visit any sub-market (in the island) when $S(p, z_o) = 0$ (or not visit any submarket at all).

Given Lemmas 1 and 2, we can move forward to prove existence and characterize the reallocation and separation decisions rules, in addition to the application policy rule.

Existence To show existence of equilibrium, once again consider an island characterised by state vector (p, z_o). In addition, noting that the shock process for z_o does not depend on the island identity, we conjecture at this point that all values and decisions at $z_o = z_{o'}$ are in fact identical at differing o, o', and verify whether this is indeed the case. It is useful to consider the operator T that maps the value function $\tilde{M}(p, z, n)$ for $n = 0, 1$ into the same function space, such that $\tilde{M}(p, z, 0) = M(p, z)$,

$\tilde{M}(p,z,1) = W^U(p,z)$ and

$$T(\tilde{M}(p,z,0)) = y(p,z) + \beta \mathbb{E}_{p',z'}\Big[\max_{d^T}\{(1-d^T)M(p',z') + d^T W^U(p',z')\}\Big],$$

$$T(\tilde{M}(p,z,1)) = b + \beta \mathbb{E}_{p',z'}\Big[\max_{\rho^T}\{(\rho^T\Big(\int W^U(p',\tilde{z})dF(\tilde{z}) - c\Big) + (1-\rho^T)(S^T(p',z') + W^U(p',z'))\}\Big],$$

where by virtue of the free entry condition

$$S^T(p',z') \stackrel{def}{=} \max_{\theta(p',z')}\Big\{\lambda(\theta(p',z'))\Big(M(p',z') - W^U(p',z')\Big) - \theta(p',z')k\Big\}.$$

A fixed point $\tilde{M}(p,z,n)$, $n = 0,1$ describes the problem faced by unemployed workers and firm-worker matches with productivity z in any occupation o. Further, since the identity of the occupation does not affect the value function $\tilde{M}(p,z,n)$ for $n = 0,1$, a fixed point of $\tilde{M}(p,z,n)$, $n = 0,1$ also describes the problem faced by agents in the decentralised economy. In the proof of Proposition 1 we show that all equilibrium functions and the evolution of the economy can be derived completely from the fixed point of the mapping T. For that purpose, we make the following assumption.

Assumption 1. $F(z'|p,z) < F(z'|p,\tilde{z})$, *for all* z', p *if* $z > \tilde{z}$.

Thus, a higher occupational-specific productivity today leads, on average, to a higher productivity tomorrow and hence the ranking of productivities z within an occupation is, in this sense, persistent. The next result derives the essential properties of T.

Lemma 3. *T is (i) a well-defined operator mapping functions from the closed space of bounded continuous functions $\tilde{M}$ into itself, (ii) a contraction and (iii) maps functions $M(p,z)$ and $W^U(p,z)$ that are increasing in z into itself.*

Given this Lemma, and Banach's Fixed Point Theorem, a unique fixed point $(M(p,z), W^U(p,z))$ of the mapping T exists. We can reverse our steps, and derive all equilibrium value functions and decision rules from this fixed point. We have the following: $\tilde{W}(p,z) = M(p,z) - J(p,z,\tilde{W})$ and $J(p,z,\tilde{W}) = (1-\eta)(M(p,z) - W^U(p,z)) = k/q(\theta(p,z,\tilde{W})), \tilde{W}(p,z)$, then $J(p,z,\tilde{W})$ and $\theta(p,z,\tilde{W})$ can be constructed from $M(p,z)$ and $W^U(p,z)$. Completing these steps, assuming a tie breaking rule in favor of the status quo, we then have existence and uniqueness of a block recursive equilibrium which are 'inherited' from the existence and uniqueness of the fixed point of the mapping T. Subsequently, with help of Proposition 2, we can show that the equilibrium is also unique in the more general class of equilibria that respect the standard 'subgame-perfection-like' off-equilibrium belief restriction made in competitive and directed search models.

Proposition 1. *A Block Recursive Equilibrium exists and it is the unique equilibrium (up to the timing of transfers within the match that are irrelevant for both ex ante values and the optimal policies of separation, application, reallocation and posting).*

Since the remaining verification of equilibrium is mechanical, reversing the steps in the construction of mapping T, it is in the Supplementary Appendix. There we also show the more general uniqueness proof.

Reservation Cutoffs for Separations $z^s(p)$ and Reallocation $z^r(p)$ Lemma 3 tells us that $W^U(p,z)$ is increasing in z, and hence the optimal reallocation policy has a reservation property, as a function of p: reallocate if and only if $z < z^r(p)$, where $z^r(p)$ satisfies $W^U(p, z^r(p)) = \int W^U(p,\tilde{z})dF(\tilde{z}) - c$.

We can derive a similar property for separations, under a sufficient condition that is easily satisfied – when being in a job today before the matching stage makes it more likely to be in the job as well tomorrow, compared to being unemployed today but applying for this job.

Lemma 4. *If $\delta + \lambda(\theta(p, z)) < 1$, for all p, z in equilibrium, then difference $M(p, z) - W^U(p, z)$ associated with the fixed point of T is increasing in z.*

Hence (for non-trivial cases), there exists a unique cutoff $z^s(p)$ that depends only on the aggregate productivity p, such that $d(p, z_0) = \sigma(p, z_0) = 1$ for all $z_o < z^s(p)$ in any occupation, and $d(p, z_o) = \sigma(p, z_o) = 0$ for all $z_o \geq z^s(p)$. Additionally, Lemma 4 and equations (10) and (11) together imply that labor market tightness $\theta(p, z)$ and the job finding rate $\lambda(\theta(p, z))$ are also increasing functions of z if $\delta + \lambda(\theta(p, z)) < 1$.

Thus two functions $z^s(p)$, $z^r(p)$ capture all separation and reallocation behavior in our economy. In a way, our model with Pissarides-type search on each island, and McCall-type random search across islands is not more difficult than its two parts: instead randomly drawing from a wage distribution, a worker now draws randomly from the distribution of unemployment values, which in turn is calculated with the same ease as in the Pissarides model, where the unemployment rate is not a state variable in the value of unemployment. Underlying this tractability, which is present in the representative-market Pissarides model but here is used on each and every island, i.e. at every (z, o), is constancy in the returns to scale: in the production function, in the matching function, and in the vacancy production function.

Planner's Problem and Efficiency The social planner, currently in the production stage, in this economy solves the problem of maximising total discounted output by choosing separations, reallocations, applications and vacancy creation decisions for each pair (z, o) in any period t. Namely,

$$\max_{\{d(.),\rho(.),v(.),\alpha(.)\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \sum_{o=1}^{O} \int_{\underline{z}}^{\overline{z}} \left[u_t(z_o)b + e_t(z_o)y(p_t, z_o) - (c\rho(.)u_t(z_o) + kv(.))\right] dz_o,\right]$$

where choices depend on (p_t, z_t, Ω_t), where Ω is the entire worker distribution at the time of decision making. The planner's problem is subject to initial conditions (p_0, Ω_0), and laws of motion

$$\begin{aligned} u_{t+1}(z_o)dz_o &= \int_{\underline{z}}^{\overline{z}} \left[(1 - \lambda(\theta(.)))(1 - \rho(.))u_t(\tilde{z}_o) + d(.)e(\tilde{z}_o)\right] dF(z_o|\tilde{z}_o)d\tilde{z}_o \\ &\quad + \sum_{\forall \tilde{o} \neq o} \Big[\int_{\underline{z}}^{\overline{z}} \rho(.)u_t(\tilde{z}_{\tilde{o}})d\tilde{z}_{\tilde{o}}\Big] \frac{dF(z_o)}{O-1} \\ e_{t+1}(z_o)dz_o &= \int_{\underline{z}}^{\overline{z}} \left[\lambda(\theta(p_t, \tilde{z}_o))(1 - \rho(p_t, \tilde{z}_o))u_t(\tilde{z}_o) + (1 - d(p_t, \tilde{z}_o))e_t(\tilde{z}_o)\right] dF(z_o|\tilde{z}_o)d\tilde{z}_o \end{aligned}$$

where $u_t(z_o)$ and $e_t(z_o)$ denote the number of unemployed and employed workers in island z_o in occupation o at period t, $v(p_t, z_o, \Omega_t)$ denotes the number of vacancies posted for unemployed workers in this island at time t and $\theta(p_t, z_o, \Omega_t) = v(p_t, z_o, \Omega_t)/(1 - \rho(p_t, z_o, \Omega_t))u_t(z_o)$ denotes labor market tightness. As with tightnesses, the planner's choice variables $\rho(p_t, z_o, \Omega_t), d(p_t, z_o, \Omega_t)$ are continuous choice variables in $[0, 1]$: the planner can decide on the proportion of workers at a certain z_o-productivity to separate, or reallocate. A full derivation of the workers' flow equations can be found in the Supplementary Appendix.

Proposition 2. *The equilibrium identified in Proposition 1 is constrained efficient.*

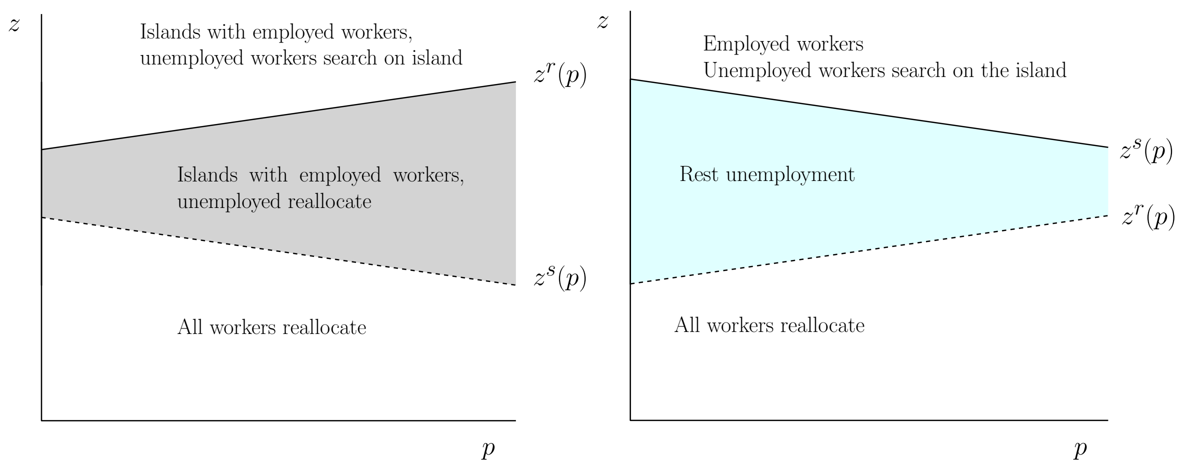
This immediately implies that the planner's decisions (when considering tightnesses instead of number of vacancies) only vary with (p, z), not with the distribution of workers over employment status, or islands. The crucial insight behind Proposition 2 is that the social planner's value functions are linear in the number of unemployed and employed on each island. The remaining dependence on p and z is equivalent to the one derived from the fixed point of T. Given the value functions of unemployed workers and firm-worker matches, the outcome at the matching stage is efficient and the Hosios' (1991) condition is thus satisfied. Proposition 2 also implies that workers' reallocation decisions are efficient. This is intuitive as the value of an unemployed worker cannot move to another occupation equals the shadow value of this worker in the problem of a similarly restricted social planner. One step backwards, suppose one and only one reallocation is allowed: then reallocation decisions are made by comparing the expected value of unemployment at other islands in different occupations with the value of unemployment on the current island and occupation, which are identical for planner and worker. Iterating on this backwards, the coincidence of planner's values and workers' value of unemployment is preserved, ad infinitum into the fixed point.

Occupational Human Capital The empirical evidence presented in Section 2 points to age effects in the reallocation patterns of workers across occupations. Further, we also know that unemployment rate differs with age, and so do the underlying inflows and outflows into unemployment. We would like to investigate the role of occupation-specific human capital in heterogeneous occupational markets (and endogenous reallocation between these), along all these dimensions. To capture these features, we allow the production function $y(p, x, z)$ to depend positively on occupation-specific human capital x, which the worker acquires with productive time spent in an occupation, and loses when moving to another occupation. In the Appendix, we prove existence, uniqueness, and efficiency for this generalized setting. Again, the block-recursive structure will be very helpful here, as decisions do not depend on the distribution of workers (now also over human capital levels).

5 Implications of Agents' Decisions

Aggregate outcomes are determined by the interplay between the decisions agents make given their economic environment, and how this environment evolves, in part as a result of decisions made. Keeping track of the evolving distributions over state variables requires us to go beyond just analytics, and is detailed in the next section. However, we can already learn important lessons by focussing entirely on decisions and how these vary with the state variables and parameters. This can be done analytically, in simplified settings. The block recursive property is very helpful here: decisions depend on the aggregate productivity state p, on the individual-level employment status, z-productivity and human capital level of the worker himself, but not the individual state variables of other workers.

We first consider the separation and reallocation cutoff, for now ignoring (again) different levels of human capital. Figure 4 depict the two situations that can arise. Figure 4a shows that the cutoff for reallocation can lie above the cutoff for separation. This implies that unemployed workers reallocate at z-productivities at which existing matches still produce a surplus. Employed workers who fall below the separation cutoff become unemployed will start looking for jobs in other occupations. As a result, no new matches are created below the reallocation cutoff, but existing matches above z^s



(a) $z^r > z^s$ (b) $z^s > z^r$

Figure 4: Relative positions of the reservation productivities

remain productive. On the other hand, Figure 4b shows that the cutoff for separations can as well lie above the cutoff for reallocation. This implies that separating workers, at least initially, do not look for jobs in other occupations. Following Alvarez and Shimer (2011) we call this 'rest unemployment', reflecting that for z-productivities (islands) below separation cutoff z^s, the probability of finding a job is very low – in the model, starkly, zero. Only after the z-productivity has declined further, to below reallocation cutoff z^r, will workers look for jobs in other occupations; otherwise, they hang on for a possible improvement in the z-productivity in their old occupation at which they can find a job again. Above both cutoffs unemployed workers remain in their occupation and flow back to employment over time.

An important issue in this paper is the cyclicality of reallocation and separation. The countercyclicality of separation rate in the aggregate naturally depends on how aggregate productivity affect the separation decision. Are workers and firms less picky about there z-productivity *for employment* in good times? A (sufficiently) negative slope of $z^s(p)$, as depicted in Figure 4, will imply countercyclical separations. Likewise, procyclicality of reallocations occurs when $z^r(p)$ is (sufficiently) positively sloped in p, also as depicted in Figure 4. We now investigate the slopes of the productivity cutoffs analytically by employing the same device as used in analytical investigations of the Pissarides model: a situation in which aggregate productivity is believed to be fixed at p forever, but a one-time only, unexpected, and permanent change in p occurs.[31] Additionally, we assume (for now) that z-productivities are permanently fixed.

The Cyclicality of Reallocation in Unemployment The reservation cutoff occurs where the value of reallocation, $R(p)$, equals the value of staying $W^U(p, z^r(p))$. In this simplified setting, the interesting case is where a worker decides between looking for a job in his current occupation or in other occupation (i.e. there is no rest unemployment: z productivities are permanent). We can write the value

[31] See also Shimer (2005), Mortensen and Nagypal (2007), and Hagedorn and Manovskii (2008). Since the equilibrium value and policy functions only depend on p and z, analysing the change in the expected value of unemployment and joint value of the match after a one-time productivity shock is equivalent to compare those values at the steady states associated with each productivity level. This follows as the value and policy functions jump immediately to their steady state level, while the distribution of unemployed and employed over islands takes time to adjust.

of unemployment at z-productivities that do not trigger reallocation, and the value of reallocation

$$W^U(p,z) = \frac{b + \beta\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z))}{1-\beta}. \; \forall z \geq z^r(p) \tag{12}$$

$$R(p) = \int_{\underline{z}}^{\bar{z}} \max\{W^U(p,z), W^U(p,z^r(p))\} dF(z) - c. \tag{13}$$

Using the free entry condition, together with optimal vacancy posting (or Nash Bargaining with the Hosios condition), this results in the following equality (with details of the derivation in the Supplementary Appendix)

$$\frac{(1-\eta)k}{\eta}\left(\hat{\beta}\int_{\underline{z}}^{\bar{z}} \max\{\theta(p,z), \theta(p,z^r(p))\} dF(z)\right) - c(1-\beta) = \frac{(1-\eta)k}{\eta}\theta(p,z^r(p)). \tag{14}$$

In this form, equation (14) captures a slightly more general situation: for $\hat{\beta} = \beta$, it captures the one period delay in unemployment before the worker can start applying after reallocation. If this delay is absent (and the worker can do his sampling within the same period), $\hat{\beta} = 1$, if the delay is τ periods instead, $\hat{\beta} = \beta^\tau$. With the Pissarides wage equation in hand,

$$w(p,z) = (1-\eta)y(p,z) + \eta b + \beta(1-\eta)\theta(p,z)k, \tag{15}$$

together with the free entry condition, we can derive [32]

$$\frac{d\theta(p,z)}{dj} = \frac{\theta(p,z)}{w(p,z)-b}\frac{dy_j(p,z)}{dj}, \;\; \text{for } j = p, z. \tag{16}$$

From this, we can derive the slope of $z^r(p)$:

$$\frac{dz^r}{dp} = \frac{\hat{\beta}F(z^r)\frac{y_p(p,z^r)}{y_z(p,z^r)} + \hat{\beta}\int_{z^r}^{\bar{z}} \frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\frac{y_p(p,z)}{y_z(p,z^r)} dF(z) - \frac{y_p(p,z^r)}{y_z(p,z^r)}}{1-\hat{\beta}F(z^r)} \tag{17}$$

It is perhaps also insightful to compare labor markets with search frictions with their perfectly competitive counterpart (under the same constant returns to scale production function), where we keep the reallocation frictions – including the time cost of reallocation– constant, but $\lambda(.) = 1$ always. Since $y_p(p,z)$ is weakly increasing in z, and $\theta(p,z)/(w(p,z)-b)$ is increasing in z, the integral is larger than $(1 - F(z^r)\frac{y_p(p,z^r)}{y_z(p,z^r)})$, and hence we find the following:

Proposition 3. *Given an unexpected, permanent increase in p,*

1. *For $\hat{\beta}$ close enough to 1, or reallocation cost c high enough, then $\frac{dz^r(p)}{dp} > 0$, i.e. reallocation is procyclical, when $\frac{dy(p,z)}{dpdz} \geq 0$;*
2. *Search frictions make reallocations more procyclical than in the frictionless competitive benchmark with the same cutoff $z^r(p)$;*
3. *For $\hat{\beta}$ small enough, reallocation becomes countercyclical.*

The proof of this proposition, intermediate steps and the equations in the text are in the Appendix. This means that procyclicality with respect to aggregate productivity comes in part from the complementary on the production function, which would be present in frictionless competitive settings as well, but is *amplified* by the presence of search unemployment, rather than dampened, as perhaps intuition would lead to expect. Unemployment caused entirely by the time cost of reallocating, on the other hand, does have a dampening or offsetting effect on the cyclicality of reallocation. To see the argument for the

[32] A formal derivation of this equation can be found in the Supplementary Appendix.

amplification of search unemployment, note that in the equilibrium the trade-off between wages and job finding is optimally resolved. The envelope condition then tells us that on the margin, we can pay out any productivity gains in wages, $y_z(p,z)$

$$(1-\beta)\frac{dW^U(p,z)}{dz} = \frac{\theta(p,z)}{w(p,z)-b}\frac{(1-\eta)k}{\eta}y_z(p,z) = \frac{\beta\lambda(\theta)}{1-\beta(1-\delta)+\beta\lambda(\theta)}y_z(p,z), \tag{18}$$

while in the competitive benchmark, employment would be found instantly after sitting out one period of unemployment, $\lambda(.) = 1$, and hence the corresponding derivative reduces to

$$(1-\beta)\frac{dW_c^U(p,z)}{dz} = y_z(p,z), \tag{19}$$

Now, we can see how search creates an additional complementarity between p and z: the gains *in employment* are proportional to the competitive case, but these are premultiplied by a term capturing the probability of *becoming employed*, which is *higher at higher* z.

All in all, the cyclicality of reallocation of unemployed workers really depends on the underlying causes of unemployment, and their relative importance: the implications for the cyclicality of the longer unemployment spells of occupational movers differ among cases where this is caused mostly by search unemployment, a time cost of reallocation, or, more generally also, the time in rest unemployment before the worker starts looking in other occupations. This means that the quantitative investigation must be judicious about the quantitative importance of search, rest and reallocation unemployment.

Cyclicality of Job Separations The reallocation decision interacts with the separation decision. If $z^r > z^s$, workers separate endogenously to reallocate; on the other hand, if $z^s > z^r$, workers separate into rest unemployment. In the case of permanent z, and a one-time unexpected permanent shock to p, these two cases are very different. First, consider 'rest unemployment', in this case, workers compare productivity in the match with *'productivity in unemployment'*, where all resting workers get b forever. At separation indifference $y(p, z^s(p)) = b$, and hence

$$\frac{dz^s(p)}{dp} = -\frac{y_p(p,z^s(p))}{y_z(p,z^s(p))} < 0; \tag{20}$$

separations are countercyclical.

In the second case of $z^r > z^s$, workers compare productivity in the match with productivity *after unemployment*, after reallocation (taking into account a limited time of 'productivity in unemployment'.)

Lemma 5. *With permanent occupational specific productivities* z*, and* $z^s(p) < z^r(p)$ *for* p*, it holds that*

$$-\frac{y_p(p,z^s(p))}{y_p(p,z^r(p))} + \frac{\beta\lambda(\theta(p,z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z^r(p)))}\left(1+\frac{y_z(p,z^r(p))}{y_p(p,z^r(p))}\frac{dz^r(p)}{dp}\right) = \frac{y_z(p,z^s(p))}{y_p(p,z^r(p))}\frac{dz^s(p)}{dp}. \tag{21}$$

The first term on the LHS captures the same effect in the case of rest unemployment. Indeed if $\lambda(.) = 0$, the two implied slopes of the separation cutoff would be identical. The entire second term on the LHS captures the change in the gains of reallocation, $dR(p)/dp$, which become more prominent, the less time is spent in unemployment when keeping constant $dz^r(p)/dp$. The more responsive the gains of reallocation are to the business cycle, the more separations are pushed towards a less countercyclical, perhaps even procyclical territory.

Incorporating endogenous reallocation thus affects the cyclicality of separations differently depending on which kind of unemployment is prominent. Thus, relative to theories that create amplification of unemployment fluctuations with endogenous separations, but without reallocation (or with exogenous reallocation, as Shimer, 2007), allowing for endogenous reallocations can dampen the cyclicality of separations and therefore unemployment fluctuations; however, crucially, this force decreases with the importance of rest unemployment. Once again, the underlying cause of unemployment shapes cyclical behavior, here of separations.

The Occurrence of Rest Unemployment Since the relative importance of search, rest and reallocation unemployment affects the behavior of unemployment over the business cycle (and over the life cycle), it is important to understand how the model and its parameters affect the different kinds of unemployment. Reallocation unemployment is straightforwardly linked to reallocation technology, i.e. the time and resource costs of sampling. Search unemployment is directly linked to the matching function and vacancy costs. For rest unemployment, the link with the model environment is perhaps a little less direct. Let us highlight some of the forces and tradeoffs that affect the relative position of the reservation productivities $z^r(p)$ and $z^s(p)$, to understand the conditions under which rest unemployment is more likely to occur. For simplicity, we do this for a constant level of aggregate productivity p, while instead incorporating time-variation in the occupation specific productivities, z. Allowing for the latter is crucial to understand the occurrence of rest unemployment because a worker decides to stay unemployed in his occupation, even though there are no jobs currently available for him, when there is a high enough probability that his productivity will become sufficiently high in the future. Since the arguments below hold for any p, they tell us how the entire reservation functions $z^r(p)$ and $z^s(p)$ move relative to each other.

An analytically tractable way to allow for time-varying productivities z, is to introduce a shock that triggers a new z, randomly redrawn from $F(z)$.[33] Using this setup we study how the expected lifetime values of remaining in an occupation, or sampling a new island in a different occupation, are affected by permanent and unexpected changes in the sampling cost c, unemployment benefit flow b, and the degree of persistence of the z shocks. Given that we hold p constant, we abuse notation slightly and re-label $z^s(p)$ and $z^r(p)$ such that z^s describes the reservation productivity below which workers separate and z^r the reservation productivity below which workers start the reallocation process, and drop the explicit reference to p elsewhere.

In this environment, the value of sampling is given by $R = \mathbb{E}[W^U(z)] - c$ in (13). Noting that the expected value of unemployment does not change if the z-shock arrives at the beginning of period $t+1$ or at the beginning of the production stage in period t, the expected value of an unemployed worker with productivity z (measured at the production stage) is given by

$$W^U(z) = \gamma\Big(b + \beta \max\Big\{R, W^U + \max\{\lambda(\theta)(1-\eta)(M(z) - W^U(z)), 0\}\Big\}\Big) + (1-\gamma)\mathbb{E}[W^U(z)], \quad (22)$$

where $(1-\gamma)$ is the probability of drawing a new productivity z. Equation (22) shows that there are two ways in which an unemployed worker below z^s can return to production. *Passively*, he can wait

[33] This is a process similar to the one in Mortensen and Pissarides (1994), but now it shocks workers' occupational specific productivities instead of firms. In the calibration below we will use an autoregressive process for these shocks. In our calibration, the same properties as derived below apply.

until his current z-productivity increases exogenously. Or, *actively*, by paying c and sampling a new productivity in a different occupation. In the former case, $\max\{.\} = W^U(z^s)$, while in the latter case $\max\{.\} = R$.[34] Letting $W^U(z^s) \equiv W^s$, the difference $W^s - R$ then captures the net gain of passively waiting unemployed for one period over actively sampling a new z immediately. If $W^s \geq R$, then $z^s \geq z^r$ and there is rest unemployment; while if $R > W^s$, then $z^r > z^s$, and endogenously separated workers reallocate immediately. From the expression for R and (22) it follows that

$$(1-\beta\gamma A(z^r))(W^s - R) = (1-\beta\gamma)c - \beta\gamma \int_{z^r} \lambda(\theta(z))(1-\eta)(M(z) - W^U(z))dF(z), \tag{23}$$

where $A(z^r) = F(z^r)$ if $z^r > z^s \geq \underline{z}$, and $A(z^r) = 1$ if $z^r \leq z^s$. The key trade-off in rest unemployment is between waiting, and (immediate) reallocating. The net gain of waiting instead of reallocating is that it saves the reallocation cost c, in the first term on the RHS of (23); on the other side, if the current z-productivity persist, the worker has missed his opportunity to have sampled another occupation and start applying for jobs if the productivity in the other occupation is high enough, in the second term on the RHS. Changing the sampling cost, the unemployment benefit flows, or the persistence of the occupational specific productivity will affect the relative gain of waiting. In the following lemma we derive the direction of the change, where we need to take fully into account the feedback effect of these changes on the surpluses $M(z) - W^U(z)$ as well.

Lemma 6. *The expected values of sampling, waiting, job surplus and unemployment, and the reallocation and separation reservation productivities, respond to changes in parameters as follows*

$$\frac{d(W^s - R)}{dc} > 0, \frac{d(W^s - R)}{db} > 0, \frac{d(W^s - R)}{d\gamma} < 0.$$

Raising the cost of reallocation changes raises the relative gains of waiting.[35] A rise in b lowers the effective cost of waiting for the productivity to improve exogenously (through $(1-\gamma)$), and thus drives up $W^s - R$, and decreases the surplus of employment at every z-productivity. It is worth noting that an increase in b *unambiguously*, independent of e.g. assumptions on the production function, leads to an increase in $W^s - R$. Finally, a rise in persistence γ decreases the upside of waiting, i.e. experiencing a z-shock without paying c, while it increases the value of being at good z-productivities, thus decreasing the relative value of waiting. In the Appendix, we further prove that an increase in $W^s - R$ leads to an increase in $z^s - z^r$ when z^r and z^s are interior: for a sufficiently large c, b and persistence γ, rest unemployment occurs.

Rest Unemployment and Occupational Human Capital Occupational human capital makes a worker more productive in his current occupation. As a result, workers are willing to accept employment at worse productivities because then they can capitalize immediately on a good shock. They are

[34] The cost of sampling is important here. If there was only the time cost of sampling alternative occupations an unemployed worker on an inactive island has no opportunity cost of sampling new islands.

[35] The direct effect is very intuitive. To illustrate the complications arising from the feedback effect: an increase in the sampling cost also leads to a larger surplus, $M(z) - W^U(z)$, making employed workers less likely to separate and pushes to reduce rest unemployment. This occurs because a bad productivity shock affects employed and unemployed workers differently. If an employed worker receives a low productivity, he has to wait one period before deciding whether to sample a new z; while if an unemployed worker receives the same productivity shock, he can sample a new island immediately. This implies that the value of waiting for an exogenous increase in z has increased and the utility loss experienced by an employed worker is smaller than that of an unemployed worker, raising the value of employment relative to unemployment. In the proof of Lemma 6 we show that the first effect dominates, leading to a increase in $W^s - R$ after an increase in c.

also willing to hang on longer in their occupation unemployed, because they know that at a given *productive* z, they now can find jobs more quickly and receive higher wages. In the next lemma, we show that the second force is stronger: an increase in occupational human capital, in some sense, increases the attachment to the current occupation more than to employment.

Lemma 7. *Consider a setting where* p *is fixed,* z *redrawn with probability* $(1-\gamma)$*, and production is given by* $y = xz$*. Consider an unexpected, one-time, permanent increase in occupation-specific human capital* x *from* $x = 1$*. Then*

$$\frac{d(W^s(x) - R)}{dx} > 0 \tag{24}$$

Moreover, we also show that this means that the cutoffs the difference $z^r - z^s$ becomes smaller when human capital increases and z^r and z^s are interior. More occupational human capital thus can lead to rest unemployment.[36] This increase in rest unemployment (or, if $z^r > z^s$, a diminished difference between cutoffs) can potentially explain differences in unemployment and reallocation in unemployment over the life cycle as well. We will discuss this more, as well as the importance of the different kinds of unemployment, and the resulting quantitative implications for the business and life-cycle, in the section below.

6 Quantitative Analysis

To study aggregate outcomes of unemployment and reallocation, we compute and calibrate the version of the model that incorporates occupational human capital. Below we will discuss our calibration strategy, the resulting parameters and fit, the implications for rest unemployment, and the implications for unemployment of human capital accumulation and business cycle shocks.

The block recursive structure implies our model has essentially three superimposed layers, where each layer is affected only by the layer(s) below it. The exogenous shock processes constitute the lowest layer. They are the deep causes of differences among workers in the cross section and over time. These exogenous processes are (i) the aggregate productivity process; (ii) the worker-occupation specific productivity process that creates a stationary cross-sectional distribution of 'islands'; (iii) the birth and death of workers, in conjunction with the stochastic acquisition of occupation-specific human capital for employed workers. We parameterize these processes below.

Workers and firms optimization decisions in response to these exogenous processes form the middle layer of the model. Key are the separation and reallocation decisions, derived without reference to the distribution of workers over islands and -also in the more general case- characterized by simple z-reservation productivity rules that depend on aggregate productivity and human capital. In the previous section, for simplified settings, we were able to characterize analytically the behavior of these functions. Here, the shock processes are generalized to capture further aspects of the data (discussed further below).

The evolution of the distribution of workers over islands, occupations, human capital levels and

[36] It furthermore hints that in case of more general autoregressive shock processes, *more* rest unemployment can be created when x increases. Being rest unemployed means receiving b until either the worker experiences a $z < z^r$, in which case the continuation value is R, unchanged; or experiencing $z > z^s$, in this case, more human capital has increased the continuation value has increased along the lines of Lemma 7. This hints that the value of unemployment in rest unemployment, keeping the chance of returning above z^s constant, is raised.

employment status forms the final layer. It is determined by the above shock processes *and* the workers' decisions in response. Aggregate statistics are affected by the levels of worker-occupation specific productivity and human capital of unemployed workers, which, because of endogenous separation and reallocation, are the levels of a selected subset of workers in all markets.

Calibration Strategy To capture the heterogeneity among labor markets, in 'space' and in time, we model the worker-occupation specific productivity process as an AR(1) process with autoregressive parameter ρ_z and dispersion parameter σ_z. The reallocation between different markets is obstructed by the fixed cost c, identical for all workers. Occupation-specific human capital acquisition creates increasing attachment to occupations, captured by a three-level human capital process, in which the next level is stochastically acquired after five years on average. The productivity of the initial level of human capital is normalized to one, the productivity in the second stage and third stage are parameterized by x_2 and x_3.

Since selection among markets and human capital acquisition moves the average productivity away from one, we add a normalization parameter z_{norm} that moves the z-distribution downwards and will be set such that measured total productivity in the model averages one. This facilitates the comparison with standard calibrations, in which average productivity typically equals one. The remaining parameters are inherited from the standard calibrated Pissarides model as in Shimer (2005) and Hagedorn and Manovskii (2008). Namely the vacancy cost k, the elasticity of the matching function η, the flow benefits in unemployment b, the exogenous separation rate δ, and the persistence and dispersion parameters of the aggregate productivity process, ρ_p and σ_p, which we assume also follows an AR(1) process.

Before delving into the details of the estimation, let us note the choices that are made from the outset. The model period is one week. We set the discount rate β and the exit rate from the labor force d to match a yearly real interest rate of 4% and an average working life of 40 years. Young, unemployed workers replace those that leave the labor force. The production function is multiplicative and given by $y = pxz$, chosen to keep close to a 'Mincerian' formulation in which a time effect, worker-occupation specific effect, and a function of experience are additive in logs.

The above choices mean we have a set $\Theta = \{\delta, k, b, \rho_p, \sigma_p, \eta, z_{norm}, \rho_z, \sigma_z, c, x_2, x_3\}$, with 12 parameters, to estimate. We estimate these parameters by minimizing the distance between a set of simulated moments from the model and their counterparts in the data.

Targeted Moments We target 20 moments based on the long-run behavior of the economy. These are described in Table 5. The parameters the model shares with the standard Pissarides model, δ, k, b, ρ_p, σ_p and η, are informed by moments that are also typically used in the conventional 'representative-market' calibrations of this model. In our setting, however, the mapping from these data moments to the parameters is affected by the heterogeneity and endogenous nature of reallocations and separations.

The monthly aggregate job finding rate, for example, typically informs us about the search frictions and vacancy cost k. Here, however, it is affected negatively by the mass of rest unemployed workers. The average unemployment rate, given the job finding rate, informs us about the separation rate, but now there are two sources of separation, exogenous separation with parameter δ, and endogenous separations. Since the endogeneity of separations is an interesting dimension of our model, we will discuss this in more detail below. The empirical elasticity of the aggregate job finding rate, which we take to be 0.5, in the range of Petrongolo and Pissarides (2001), relates to η, the elasticity of the matching function in each island, but the aggregation might drive a wedge between the two. Furthermore, the total aggregate productivity per worker in the model is partially endogenous, because workers reallocation to more productive markets is endogenous, as is workers separation in less productive markets.

A feature of our calibration is that we use several age-related moments to help in the estimation of b, were the link between the two arises due to the accumulation of occupational human capital. For example, if the percentage change in surplus for a given change in human capital is large, workers become much more attractive to employ, become more attachment to their occupations and face better labor market prospects (wages, job finding and separation rates, etc), keeping everything else constant. Hence, for given productivity changes, the relative change in surplus depends one the average size of the surplus, which is a direct function of b. The observed extent to which reallocation decreases with age and the extent to which prime age workers have a lower unemployment rate, gives information about the size of the surplus, and thus about b. We find that it will be a considerable success if the estimated value of b driven by the large productivity gains from experience accumulation over the life cycle, is also consistent with the cyclical volatility of unemployment found in models where the unemployment fluctuations are driven by aggregate productivity fluctuations of a much smaller magnitude.

The returns to experience in the model, x_2 and x_3, are closely linked to the measured returns to occupation-specific experience. However, young workers tend to select good islands (with high z) to work on, but their worker-occupation specific productivities can have a (potentially slow) tendency to revert to the mean. Thus measured returns are a result of two opposing forces: human capital acquisition and worker-occupation specific productivity mean reversion. Since occupation selection occurs both in model and in data, we use the OLS measured returns to occupational experience for both. Further, as the panel structure in the SIPP is relatively short, it is difficult to estimate returns to occupational experience accurately in this data. For this reason we use the OLS estimates for 1-digit occupations reported in Kambourov and Manovskii (2009b), which are a 15% return to five years of occupational experience, and a 23% return to ten years of occupational experience.

The worker-occupation specific productivity parameters ρ_z, σ_z and the reallocation cost c are further informed by the remaining set of 10 moments. The extent of duration dependence (captured in the 3 unemployment survival moments) informs us about the persistence of the z-process: the higher is the persistence is this process, the higher is the degree of duration dependence.[37] A lower reallocation cost naturally leads to more reallocation, and hence the average level of reallocation helps determine c. The difference in reallocation rates between prime-aged and young workers, informs us about how much higher in the z-distribution are the cutoffs productivities for young workers. The higher the dispersion of the z-process, the lower, *relative to the z-productivity differences*, are the differences in productivities derived form occupational human capital. This implies that a higher σ_z brings closer together the separation and reallocation flow patterns for young and prime-aged workers.

The remainder set of moments are meant to capture the empirical dynamics of the reallocation

[37] Because of seam bias in the SIPP, we prefer to pick the cumulative survival rates at intervals of 4 months, the length of a data wave.

Table 5: Targeted Moments. Data and Model Comparison

Moments	Data	Model		Data	Model
ave. job finding rate	0.269	0.258	**Unemployment Survival**		
matching function: $\widehat{\eta}$	0.500	0.512	≤ 4 months	0.373	0.395
Aggregate Productivity			≤ 8 months	0.130	0.133
y (normalised)	1	0.999	≤ 12 months	0.055	0.047
ρ_y	0.691	0.689	**Cm by Unemployment Duration**		
σ_y	0.009	0.009	C_m 1 month	0.519	0.405
Life-cycle Moments			C_m 3 months	0.556	0.536
$urate_y$	0.067	0.074	C_m 6 months	0.582	0.584
$urate_p$	0.040	0.043	C_m 9 months	0.614	0.625
$l(cm_y/cm_p)$	0.134	0.146	C_m 12 months	0.610	0.640
Returns to Occupational Tenure			**Mobility in Subsequent Unemp. Spells**		
5 years	0.154	0.140	c_s after occ. stay	0.621	0.610
10 years	0.232	0.242	**Employed with Us-spell within 3 years**		
			proportion of empl.	0.1441	0.167

process and its interaction with separations and unemployment as closely as possible. First, we would like to capture how the empirical reallocation process changes with unemployment duration as analysed in section 2. Concretely, we look at the slope of the $c_m(d)$ profile; when combined with the cumulative survival function, it also tells us about the size of the outflow of the subset of occupational movers, resp. stayers, from unemployment as a function of duration. It is of particular interest that long-term unemployment arises among both (those who are ex-post assigned to) occupational stayers and occupational movers.

Second, we also would like to capture the subsequent outcomes after unemployed workers have found a job in the same occupation reported in section 2, with the aim of incorporating the persistence of occupational attachment among the selected group of occupational stayers. This can further inform us about the persistence of the z-process, and the role of occupational human capital in creating addition attachment to occupations.[38] Third, we add the empirical proportion of employed workers who will experience an unemployment spell during the subsequent three years. Given a fixed number of spells, this gives a sense about how much an unemployment spell 'begets' subsequent unemployment spells. This moment will give additional information on the importance of endogenous separations and rest unemployment, which we discuss in more detail below.

Fit of the Model We take logarithms of the moments discussed above and minimize the distance between logged data moments and their simulated model counterparts (using the identity matrix as weighting matrix). In Table 5, we report the fit between the model and the targeted moments. The model appears to fit the data remarkably well, given the extent of over-identification, with 20 moments informing 12 parameters. In fact, for the moments capturing unemployment duration dependence, reallocation patterns with spell duration, repeat mobility and the concentration of unemployment incidence, the simulated moments are close to the data, even though all this, including the underlying

[38]It is important to emphasize that those workers who experience a second spell of unemployment within a SIPP panel are not necessarily a random selection from workers who experience *an* unemployment spell, but neither will they be in the model: separations are more likely in worse labor markets. We use the model, and the implied selection as a measurement tool: in generating the statistics, we take care to calculate these in the model precisely as we calculate these in the data: in simulated SIPP-like panels of 4 years. We also calculated these statistics in 3.5-year panels instead; this doesn't appear to make much difference.

endogeneity of reallocations and separations, is driven mostly by just three parameters: ρ_z, σ_z and c. The moments used commonly in the standard calibrations of the Pissarides model are matched closely. Unemployment among the young is a bit larger than in the data, while for prime-aged workers it is closely matched. Together these two moments also imply an overall unemployment rate close to the data. The OLS returns to occupational experience are aligned with the data as well, as is the differential occupational mobility pattern between the prime-aged and the young workers.

The model captures the duration dependence of unemployment very well, creating close to the empirical amount of long-term unemployment, and at the same time it produces a large amount of occupational mobility. All this is achieved with a large proportion of occupational stayers among the unemployed at all durations. A too-quick conjecture that substantial occupational mobility is principally inconsistent with long-term unemployment, in particular for occupational stayers, is wrong. On the contrary, substantial mobility does not imply a reallocation cost so low that all bound-to-be long-term unemployed workers find it too profitable to move to better markets, leaving only as stayers those with the fastest job finding rates. Moreover, as spell duration goes up beyond two months, the proportion of eventual occupational movers in the stock goes up in the model, at a similarly slow rate as in the data (while the magnitude of this proportion is also in line with the data).

The model also captures well the evolution of occupational mobility with the progression of unemployment duration, except at the very beginning: the proportion of occupational movers at after one month of unemployment is too low in the model. This feature is due to several aspects of the model: (i) Occupational sampling with a uniform time cost per sample for all workers suppresses very early outflows into employment for occupational movers; (ii) Exogenous and immediate separations of workers with high z-productivities, lead to a set of occupational stayers quickly dipping in and out of unemployment, who with a notice period before the actual separation would conceivably be moving job-to-job; (iii) Since the endogenous separation productivity cutoff is also a vacancy posting cutoff, this induces sometimes too quick rehiring upon a small positive z-shock. These forces overemphasize occupational staying and de-emphasize occupational moving at the lowest unemployment spell lengths, but, importantly, the aforementioned effects diminish quickly with unemployment spell duration. Hence, we are able to match closely the proportions of occupational movers for durations above two months. As spells longer than a month contribute most to the stock of unemployment and its fluctuations, we chose our targets to emphasize these spells.[39]

The model is also able to match the proportion of employed workers that will become unemployed within a three-year window very closely. This implies that the model can produce a concentration of unemployment incidence in line with the data, which is driven by a z-shock process that is also consistent with the other targeted reallocation and unemployment patterns. Furthermore, the 'repeat mobility' of those unemployed that recently were occupational stayer in their recent previous spell is also closely matched. Those workers are staying with their occupation in 61% of the cases in

[39] Some additional realism, by adding measurement errors or heterogeneity in the reallocation cost, could improve the fit at these very low unemployment durations. We have opted to keep the model as clean and parsimonious as possible. In our calibration, one key result is that the outflow rate for occupational stayers and movers become closer at longer durations, and thus, for example, measurement error in occupational assignment will bring the outflow rates closer at low durations, while not changing much the outflows at higher durations. Low reallocation costs will trigger quicker reallocation for some, instead of an interval of z-productivities in which worker are 'resting' that is identical for each and every worker.

the model, versus 62% in the data. Overall, we think it remarkable that (i) the unemployment flow and occupation flow behavior during unemployment spells, (ii) the occupational mobility in subsequent unemployment spells and, (iii) the incidence of unemployment spells across workers, all can be approximated closely by incorporating a small number of further parameters to the standard model which add cross-sectional and time-series heterogeneity in individual workers' occupation-specific productivity.

We now turn to discuss in more detail some important implications of the estimation.

Duration Dependence The negative duration dependence of unemployment in the data is closely linked to the persistence of the z-process in the model, as persistence of a low z-productivity maps into persistence of a low job finding probability. However, the mapping between duration dependence and the z-process is affected by endogenous reallocation and endogenous separations. Endogenous separation affects duration dependence because it determines the distribution of z-productivities with which the unemployed start their spell. Reallocation affects duration dependence because it consumes the unemployed's time, and because it resets the z of the reallocating unemployed from a lower to a higher z-productivity.

An important consideration is that the conditions for duration dependence can be generated ex ante, i.e. before the beginning of the unemployment spell, or ex post during the unemployment spell. To which extent these two channels are relevant in our calibration, depends on the (endogenous) relative position of z^r and z^s, and on the degree to which separations are endogenous. To see this, abstract from differences in the acquired human capital for a moment. In principle, heterogeneity in the initial z-productivity at the 'birth' of the unemployment spell could drive all of the observed pattern of duration dependence. This, though, requires that separations are occurring exogenously across a range of z-productivities. On the other hand, if all separations are endogenous then all duration dependence must be created ex post, by workers receiving different shocks. In this case, with some persistence in the z-process, the distribution of z-productivities of the unemployed will fan out with unemployment duration from a distribution concentrated around z^s. Those who are shocked back above z^s can be hired into employment with positive probability; those who are shocked to lower z-productivities, will come closer to z^r – making, on one hand, reallocation more likely, and, on the other hand, requiring more positive shocks to recover to above z^s, when not reallocating. Since at higher unemployment durations, workers are on average further away from z^s, the outflow from unemployment will be lower; more so for occupational stayers.

Attachment in Employment or in Unemployment (Rest Unemployment)? To understand the forces behind duration dependence, but also for life cycle and business cycle implications, it is important to establish whether z^s is above or below z^r. If $z^r > z^s$, *employed* workers stay attached to occupations at z-productivities that would make unemployed workers leave. Workers, in some sense, are relatively more attached to employment than to their occupation. If, $z^s > z^r$ on the other hand, *un*employed workers are more attached to their occupation than to their employment matches. In the theory section, we have argued that this relative position depends on the z-process, and given this process, in a most straightforward way, on the size of the reallocation cost c: a larger c pushes z^r down, also relatively to z^s.

The targeted moments argue in favor of $z^s > z^r$, i.e. the prevalence of rest unemployment, in at least three ways. First, consider the extent of reallocation in combination with the cumulative survival in unemployment and its composition. As argued above, this implies that there is a significant amount of occupational stayers at high unemployment durations. If $z^r > z^s$, the long-term unemployed among the occupational stayers must be in islands persistently close to z^r: as soon as workers are shocked below z^r, they reallocate. This leads to a drop in the outflow into employment in the same occupation, and translates into a drop, with duration, in the (ex-post) constructed stock of eventual occupational stayers. A slow decrease in this stock then implies a shock process that pushes very few of the unemployed workers with a z just above z^r below it, even at long durations. However, this implies that long-term unemployment among stayers is in direct tension with the overall extent of reallocation. The latter requires that enough mass of workers becomes unemployed below z^r, which requires that enough workers move over time from a z from the entire range above z^r to a z below z^r. When $z^s > z^r$, this tension diminishes, as in this case long-term unemployed stayers can be found in the entire interval between z^s and z^r, while in this interval these workers all face the same zero outflow probability to employment.

Second, consider the repeat mobility patterns (while abstracting from human capital accumulation for now). Those workers who have two unemployment spells within a SIPP(-like) window and are occupational stayers in the first spell, must come from the workers in the interval $[z^r, \bar{z}]$. If $z^r > z^s$, a larger difference between z^r and z^s implies that a higher number of occupational stayers comes from the best z-productivities. This makes it more likely, everything else equal, that these workers will remain in their occupation in a second unemployment spell, in particular when it occurs within a relatively short period. If $z^s > z^r$, and all separations are endogenous (occurring at z^s), the subsequent reallocation behavior of occupational stayers looks like the unconditional reallocation behavior of all unemployed. The degree of similarity of the repeat mobility behavior of occupational stayers to the overall mobility of all unemployed workers in the data is linked closely to the position of z^r relative to z^s in the model, and the extent of endogenous separations.[40]

Third, consider the distribution of unemployment spells across workers. In the data, current unemployment appears to beget future unemployment for the same worker. If $z^r > z^s$, those employed workers below z^r are more at risk of an endogenous separation, and after reallocating these workers will end up above z^r, where they will face a strictly lower probability of endogenous separation. In this case, where employed workers remain attached to an occupation and unemployed workers are more free to move and hence are more picky about their occupation, current unemployment makes future unemployment *less* likely. If on the other hand, $z^s > z^r$ and the extent of endogenous separations is significant (necessary in this case to get the large extent of occupational mobility), occupational stayers will be hired back at often marginal productivities, and are consequently more likely to separate again in the future. Likewise, the post-reallocation unemployment risk does not diminish to the same extent as for the case $z^r > z^s$: occupational movers will be distributed over the entire range of z-productivities above z^s, instead of subset of best z-productivities. Thus, the extent of rest unemployment (with $z^s > z^r$) is closely linked to the degree of concentration of unemployment spells across workers.

The same moments that point in favor of 'attached unemployment' also imply that time spent in

[40] Acquired human capital also creates a degree of repeat staying, taken into account in the calibration.

the actual reallocation process should be relatively small. The model is flexible to accommodate a large time spent in sampling different occupations, by, for example, locating both reallocation and separation cutoffs very high in the stationary z-distribution, while scaling down the downward pull of mean reversion.[41] In the estimation, we observe a relative small amount of reallocation unemployment. The moderate increase in the $c_m(d)$ with duration is created by rest unemployment, which emphasizes the option value of recovery and is naturally consistent with a stock of occupational stayers –who are rewarded for waiting– at long durations. It appears that replacing rest unemployment with reallocation unemployment would easily create a too-steep profile of $c_m(d)$. The explicit time spent reallocating can hamper the early outflow to employment of occupational movers, while reallocations that began at high unemployment duration would weigh in the measured stock of occupational movers from the beginning, contributing zero outflow probability for extended time. To match the overall decline in the cumulative survival at lower durations, occupational stayers need to flow out faster, steepening the $c_m(d)$ profile.[42] Further, to prevent late-duration starts of reallocation, the shock process would again need to be very persistent, creating the inconsistencies with repeat mobility discussed above, as well as the overall level of reallocation.[43]

Endogenous Separations Endogenously separated workers face different outcomes than exogenously separated workers. In the calibration, with $z^s > z^r$, endogenously separated workers become rest unemployed rather than search unemployed; their fate as occupational stayers or movers will only be determined later, depending on the further realizations of the z-process. Thus the extent of rest unemployment is directly linked to the importance of endogenous separations, as is the concentration of unemployment spells across workers. With rest unemployment, endogenously separated workers are the ones most likely to reallocate, since of all separations, these separations occur closest to z^r. Then, the extent of endogenous separations must be large, because of the extent of reallocation in the data, the significant concentration of unemployment spells, and the other reasons laid out above. In the calibration, as a result, much of unemployment takes place (endogenously) between these cutoffs $z^s(p, x)$ and $z^r(p, x)$, and the way these cutoffs vary with aggregate conditions and human capital accumulated play a important role in shaping the different unemployment outcomes over the life cycle and business cycle.

Parameters Table 6 reports the resulting parameter values implied by the calibration. The value of b represents 74% of total average output, y, in line with Hall and Milgrom (2008), though estimated using different information. Reallocation, from the value of c and the sampling process, costs in expectation about 10 months of output. Our calibration also implies that the z-productivity process is, in

[41] In this case, mean-reversion will not draw productivities below these cutoffs any faster than it would do when these cutoffs would be situated lower in the distribution. This argument applies as long as the cutoffs would be placed in the upper half of the distribution in either case: where there is downward pull, instead of upward pull of mean reversion. In our calibration, cutoffs are approximately in the upper half of the z-distribution.

[42] Additionally, to make the outflow rates of occupational stayers low enough at long durations, search or rest unemployment should be extensive, which by the same token also adds additional time to the outflow rates of occupational movers who have moved to a better, but barely acceptable productivity in a new occupation – making it harder to match the survival hazard without giving up on the duration profile $c_m(d)$.

[43] Since these reasons would be applicable, a fortiori, also if we added further time costs of reallocation by increasing the time interval between sampling of occupations, we have not included an additional parameter in the estimation that would capture the additional length of time of sampling above one period.

Table 6: Calibrated Parameters

δ	k	b	c	η	ρ_p	σ_p	ρ_z	σ_z	$\underline{z}_{corr}$	x_2	x_3
0.0002	3.582	0.738	14.54	0.048	0.981	0.003	0.9992	0.0078	0.513	1.104	1.487

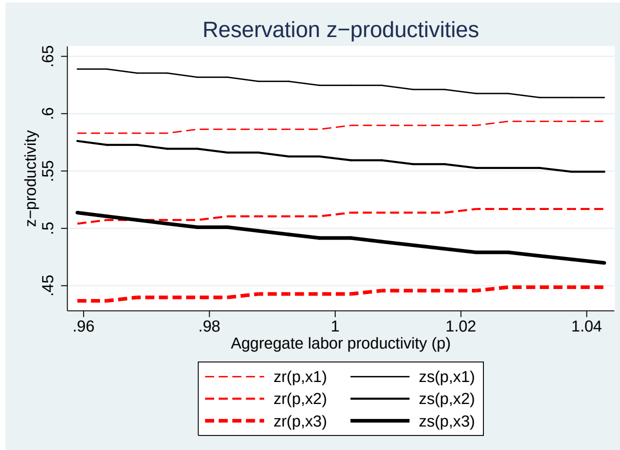


Figure 5: Reservation functions by occupational human capital

relative terms (i.e. in the log), more persistent than the aggregate shock process driving the business cycle.[44] However, the larger variance of the shock process creates significantly more dispersion across z-productivities and more extensive movements measured in absolute terms. The component of exogenous separations, δ, is small. Finally, observe that the parameter linked to the actual returns to tenure at 10 years is significantly higher at ten years than the OLS returns: the best z-productivities mean revert over time, and at the same time, experienced workers find it still profitable to work at worse z-productivities, dampening the *composite* xz-productivity on which measurement of the returns to occupational tenure is based.[45]

6.1 Quantitative Implications

In this section we analyse the business cycle and the life cycle implications of our calibrated model, and also look at 'repeat mobility' patterns produced by the model.

Business Cycle Patterns To start, we consider the decisions rules that result from our calibration. Figure 5 depicts the three sets of $z^s(p)$ and $z^r(p)$ functions, each corresponding to an occupational human capital level. The job separation productivity cutoff is downward sloping and the reallocation productivity cutoff is upward sloping, making reallocations procyclical and separations countercyclical at all occupational human capital levels. Further, since $z^s > z^r$ in all the three cases, rest unemployment occurs at each level of occupational human capital. This implies very limited feedback effect from reallocations to separations, rendering the latter always countercyclical.

Cyclical fluctuations in unemployment are driven by both fluctuations in workers' inflows and outflows, whose cyclical response we can be already gauged in Figure 5. In a downturn, larger inflows arise because the separation cutoff increases when p decreases, leading those matches closely above the margin to break up. Decreased outflows in a recession occur due to three reasons. First, surpluses

[44]The high persistence of z is in line with the high persistence typically estimated for individual earnings processes, and also, for example, with the autoregressive component of the process in Bils et al. (2012).

[45]The increasing range of viable z-productivities with experience implies that the model is also consistent with an increasing variance of earnings across workers with experience.

Table 7: Logged and HP-filtered Business Cycle Statistics. Data and Model

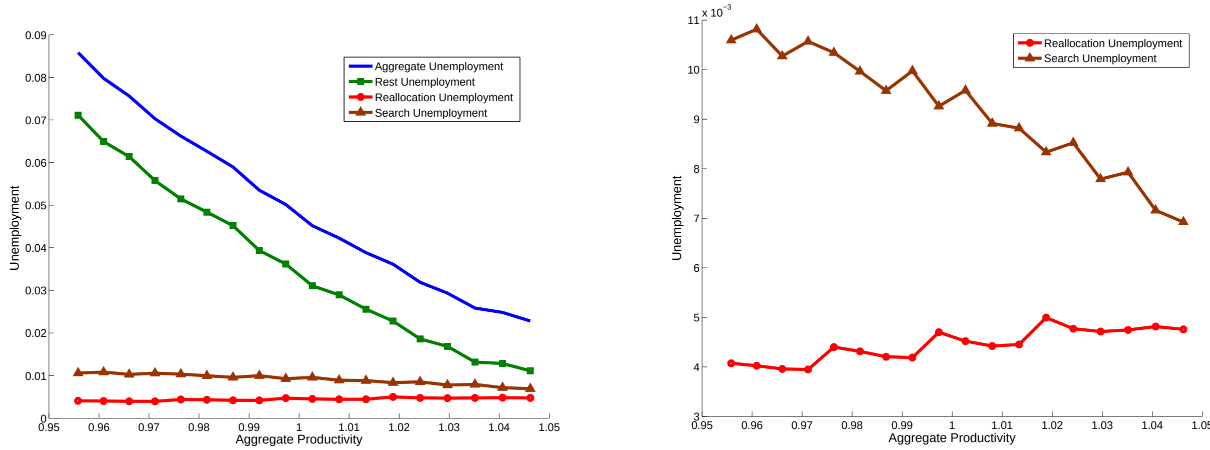
	Data: 1986-2011							**Model**						
	u	v	θ	s	f	y	Cm	u	v	θ	s	f	y	Cm
σ	0.13	0.11	0.23	0.13	0.10	0.01	0.03	0.15	0.08	0.21	0.10	0.13	0.01	0.06
ρ_{t-1}	0.97	0.93	0.94	0.83	0.93	0.69	0.85	0.74	0.25	0.64	0.06	0.55	0.69	0.81
	Correlation Matrix													
u	1.00	-0.84	-0.96	0.67	-0.78	-0.52	-0.26	1.00	-0.61	-0.96	0.32	-0.80	-0.98	-0.77
v		1.00	0.95	-0.69	0.46	0.66	0.30		1.00	0.82	-0.38	0.67	0.68	0.6
θ			1.00	-0.72	0.47	0.63	0.30			1.00	-0.37	0.83	0.97	0.71
s				1.00	-0.67	-0.68	-0.32				1.00	-0.79	-0.39	-0.27
f					1.00	0.45	0.40					1.00	0.84	0.59
y						1.00	0.26						1.00	0.72

at a given z-productivity that remains above z^s becomes smaller, leading to less vacancy posting, and lower labor market tightnesses. Second, this also means that reallocating workers who arrive in a market above z^r will take longer to find a job in a different occupation. Third, existing rest unemployed workers will tend to persist in this state longer as they will need more improvements to their z-productivities to rise above z^s and experience positive job finding rates in the same occupation, while at the same time they face a lower z^r making them less likely to end their rest unemployment by reallocation.

Table 7 shows the resulting business cycle patterns. The model successfully reproduces many of the key labor market statistics, amplifying productivity shocks sufficiently for all variables, in particular unemployment, and creating procyclical reallocations, while separations and unemployment are countercyclical. We now analyse in detail these and the underlying patterns.

First note that we do not achieve these successes by worsening the performance of the model along dimensions that are very successfully matched in the standard search and matching model. Shimer (2007) and Mortensen (2009) show that aggregation across heterogeneous markets can still produce aggregate behavior that is fundamental in the representative-market Pissarides model. With endogenous vacancy creation separately in each market, and endogenous separation and reallocation, we show that this conclusion still holds in our calibration with significant rest unemployment. In particular, the model does produce a high (and positive) correlation between aggregate labor market tightness and the aggregate job finding rate. Likewise, even though endogenous separations typically creates a trade off with achieving a strong negative correlation between unemployment and vacancies (the Beveridge curve), this does not appear the case in our model. In our calibration, this correlation is -0.61, relative to -0.84 in the data.[46] The underlying reason is the unprofitability of creating additional vacancies for newly unemployed workers in precisely those markets for which existing matches are terminated. This is in contrast to most other models with endogenous separations, which are typically build with a 'representative market', where – since at any point in time it must be profitable to hire some workers – it must also still be profitable to hire these newly-unemployed workers. There, espe-

[46]The discreteness of the p-productivity and z-productivity grids creates some jumpiness in vacancies. For example, when aggregate productivity moves one grid point up, it increases vacancies discretely, reducing their autocorrelation and the absolute value of the v-u correlation. We find that increasing the grid size improves these statistics, and hence with more computational power and time, we expect that these correlations can be even closer. One needs a very fine grid for smooth cutoffs, as these cutoffs are located in a small subset of the z-productivity space, making even a detailed grid, in a sense, relatively coarse. It is not immediately beneficial to make the grid finer around the cutoffs only, z-productivities away from the cutoffs are still relevant for search unemployment, option values and returns to tenure.



(a) Unemployment decomposition (b) Search and Reallocation unemployment

Figure 6: Unemployment decomposition and aggregate productivity

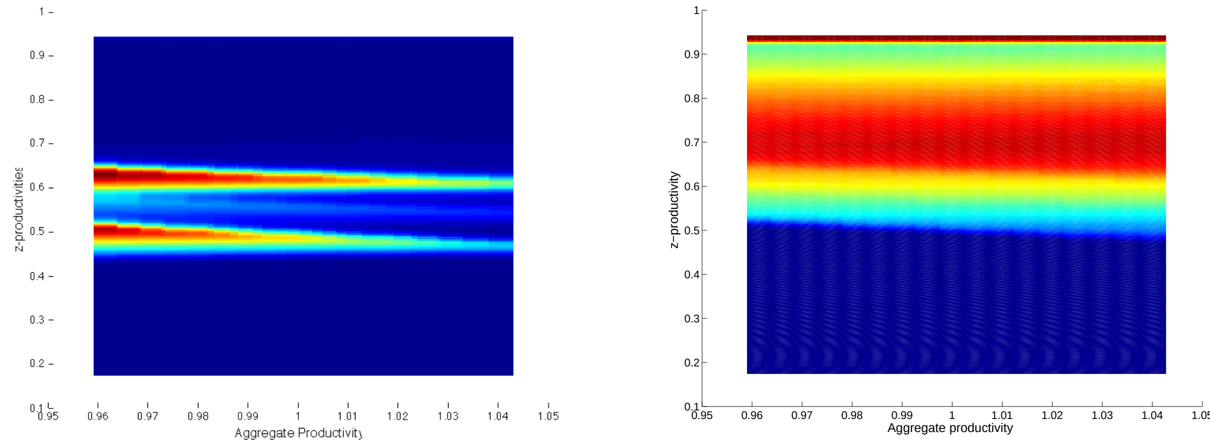
cially when the tightness is independent of the unemployment rate (when the usual constant returns to scale assumptions are made), additional vacancy posting is triggered when unemployment inflow is higher, which presses the Beveridge curve in the direction of becoming positively-sloped.[47] Heterogeneity of markets, and separation occurring in the worst markets, can naturally 'prevent' such a counterfactual response.

The model is successful in creating the somewhat elusive amplification of productivity shocks for unemployment fluctuations. The standard deviation of logged and HP-filtered unemployment is 0.15 versus 0.13 in the data. It does so while the flow benefits of unemployment are only 74% of average productivity (close to with the preferred value of Hall and Milgrom, 2008), and meanwhile consistent with substantial returns to human capital, which would conceivably create trouble in small-surplus explanations of this volatility. Moreover, the reallocation rate of the unemployed is procyclical, with a larger standard deviation and more persistence than output and less than half as volatile as unemployment fluctuations in model and data, though the model over-predicts reallocation volatility to some extent.

To gain insight into the underlying patterns, we now decompose aggregate unemployment over the business cycle. The resulting decomposition is depicted in Figure 6a. It shows that rest unemployment is predominant over the entire business cycle. On average, rest unemployment constitutes a little below 68.6% of aggregate unemployment, while search and reallocation unemployment account for 20.6% and 10.8%, respectively. Alvarez and Shimer (2011) have recently evaluated the importance of rest unemployment in US data. Using a very different estimation procedure and relying mostly on wage data and sectoral mobility, they also obtain that rest unemployment explains around three quarters of aggregate unemployment. They attribute the remainder to reallocation unemployment. Our analysis shows that once we incorporate search unemployment, it takes up more than 60% of the remaining non-rest unemployment, diminishing the importance of reallocation unemployment, as we have defined it (the time spent in unemployment because of a time-costly reallocation technology).

Assessing how the contributions of rest, search and reallocation unemployment change with ag-

[47] Adding on-the-job search can be helpful in this respect. See Fujita and Ramey (2012).



(a) Distribution of unemployed workers

(b) Distribution of employed workers

Figure 7: Distribution of workers over island and aggregate productivities

gregate productivity, we see that changes in the amount of rest unemployment are the largest drivers of the unemployment fluctuations over the business cycle. When the unemployment rate is high, close to 9%, rest unemployment in the calibrated model contributes a full 83% of it; when the unemployment rate is below 3%, its contribution drops to 49% of total unemployment. Search unemployment is countercyclical as well, though it does not change as much in absolute terms, while reallocation unemployment is small but procyclical.

Why is the overall contribution of search and reallocation unemployment to aggregate unemployment so much lower than that of rest unemployment? Figure 7 helps us understand the underlying reasons. It depicts a heat map of the density of unemployed and employed workers over values of z and p. Figure 7a shows that indeed most of the unemployed are situated in productivities between $z^s(p, x_1)$ and $z^r(p, x_1)$, and between $z^s(p, x_3)$ and $z^r(p, x_3)$, and also (less visible) just above separation productivity cutoffs $z^s(p, x_1)$ and $z^s(p, x_3)$. The same happens for $z^s(p, x_2)$ and $z^r(p, x_2)$, but because there is a smaller amount of workers in this transitory state, this is less visible in the figure.

When p decreases, the mass of workers caught between the new wider 'band' between z^s and z^r increases substantially. Those employed workers that are just above the separation cutoff z^s now fall below this cutoff and separate into rest unemployment. This increases the overall inflow into unemployment and decreases the outflow. Additionally the mass of employed at risk of separation is higher in bad times, as shown in Figure 7b. Together these forces imply that as p decreases, the mass of unemployed workers just below the cutoff z^s, increases substantially. The aggregate job finding rate also decreases after a negative productivity shock as existing unemployed workers with z-productivities further below z^s now face a higher expected unemployment duration. This occurs for two reasons: (i) these workers need a sequences of positive shocks, which occurs with a lower probability, to get an actual positive job finding rate above the now-higher z^s, and, (ii) because the reallocation cutoff has decreased, they also would need a sequences of negative shocks before their z-productivity decreases sufficiently such that it is now worthwhile reallocating, after which they would find jobs with a decreased probability as well. As a result, more workers will on average spend more time unemployed between the cutoffs.

Table 8: Semi-elasticities of the share of unemployed workers by duration

Unemp. Duration	Mean		Std. Deviation		Semi-elas. wrt outpw		Semi-elas. wrt unemp	
	Model	Data	Model	Data	Model	Data	Model	Data
unemp < 3m	0.53	0.47	0.080	0.071	4.17	3.78	-0.28	-0.24
unemp < 5m	0.74	0.72	0.081	0.085	3.45	1.46	-0.24	-0.23
unemp $5-8$m	0.17	0.19	0.041	0.033	-2.14	-1.22	0.14	0.13
unemp $9-12$m	0.06	0.05	0.027	0.028	-0.93	-0.52	0.07	0.09

Search unemployment does not appear to respond as much with aggregate productivity in the decomposition due to a composition effect. As productivity improves, islands where many unemployed were previously searching for jobs are often those closely above the separation cutoff. An increase in aggregate productivity improves the job finding rates at these islands substantially, lowering search unemployment. However, the same increase in aggregate productivity implies that those rest unemployed workers on islands which were just below the separation cutoff before, but after the productivity increase above it, now become search unemployed, dampening the overall decrease in search unemployment.

The cyclical response of rest unemployment, and largely of unemployment in general, is determined by the steepness of the reservation functions $z^s(p,x)$ and $z^r(p,x)$, by the z-shock process, which drives both separations and reallocations, and by the distribution of employed and unemployed workers over z-productivities. Table 7 shows that the calibrated model creates substantial volatility in the aggregate separation rate. As can be expected from the decision rule $z^s(p,x)$, the separation rate has the right cyclical properties: it is negatively correlated with productivity and reallocation, and positively with unemployment.[48] Note that our heterogeneous-market model does not appear to exhibit the unfortunate trade-off between wage (growth) dispersion and, on the other hand, separation and unemployment fluctuations, laid bare in the standard represented-market Pissarides model in Bils et al. (2011). In our model, we are close to the empirical fluctuations in the unemployment and separation rates and, at the same time, wages can vary from below 0.65 to 0.95 for inexperienced workers and from 0.7 to 1.4 for experienced workers.

Now consider the aggregate job finding rate. In this case the model generates a slightly higher volatility than in the data. Again, as aggregate productivity improves and the band between z^s and z^r narrows, there will be a faster outflow from unemployment. This leads to a strong negative correlation between the job finding rate and unemployment (around -0.8 in both model and data), a positive correlation with output as in the data, and with the proportion of occupational movers in the outflow from unemployment (0.6 in the model versus 0.4 in the data). Moreover, the job finding rate is strongly negatively correlated with the separation rate (at -0.8 in the model vs. -0.7 in the data). Overall, it appears that the model not only produces sufficient amplification of productivity shocks for unemployment fluctuations, but also by-and-large allocates correctly the relative importance of outflows and inflows in determining unemployment fluctuations.

Since our framework presents a theory of unemployment fluctuations delivered to a large extent

[48]The coarseness of the grid means that the autocorrelation of separations is low, and correlations in the model not as high as in the data. When considering measures that are less sensitive to noise, e.g. the semi-elasticity of the separation rate with respect to contemporaneous productivity, we also find a strong relationship: the model gives rise to three-quarters of the empirical relation (-0.06, against -0.08 in the data).

by the widening and contracting interval between z^s and z^r as times get worse, resp. better; we use the model's implications on the duration distribution of unemployment spells to further test our theory. Above, we have discussed that the interval between cutoffs creates duration dependence in unemployment, which we calibrate to the data. As the interval of rest unemployment between z^s and z^r widens with adverse productivity shocks, we naturally expect more long-term unemployment, but is the increase as strong as in the data? Table 8 evaluates the ability of the model to reproduce the shifts in the incomplete unemployment duration distribution with the business cycle. First, note that the model matches the incomplete duration distribution well.[49] Untargeted, the shares of the different durations in the unemployment duration distribution exhibit nearly the same extent of fluctuations in the model as in the data. HP-filtering the shares, and calculating the semi-elasticity of the shares with respect to the aggregate (filtered) unemployment rate, we see that the model reproduces nearly perfectly the movements of the duration shares with the overall unemployment rate in the data. Thus, as the unemployment rate increases by 10% (i.e. about half a percentage point), it increases the *share* of incomplete unemployment spells between five and eight months by 1.4 *percentage points* (versus 1.3 percentage points in the data), while it increases the share of unemployment spells by 0.7 percentage points (versus 0.9 in the data). Thus, the widening of the band creates an increase in long-term unemployment with the overall unemployment rate that is very close to the data.[50]

Overall, given the slopes of the $z^s(p, x)$ and $z^r(p, x)$, and the resulting 'band' of rest unemployment between these cutoffs, which narrows in good times and widens in bad times, the model can reproduce to a large extent, the business cycle volatilities of the unemployment, job finding, reallocation and separation rates; and, it can reproduce the shifts in unemployment duration distribution over the business cycle.

Life Cycle Patterns As workers age, they are likely to spent long enough time in their occupations to acquire some specific human capital. The latter affects their separation, job finding and reallocations rates. We now evaluate the ability of the calibrated model to reproduce the separation, job finding and reallocation patterns observed across age groups, taking averages over the business cycle. We discuss the business cycle patterns across age groups below.

At this stage, it is important to reiterate that all differences between the two age groups we consider, young (20-30yo) and prime-aged (35-55yo) workers, are created by adding just two additional occupation-specific productivity levels to the model. Moreover, besides the observed returns to occupational experience, we have used only two specific life-cycle moments in our estimation: the difference in occupational mobility at two months unemployment duration, and the difference in unemploy-

[49] This is almost implied by targeting the cumulative survival rate. The main statistics are the proportions of unemployment spells 0-4 months, 5-8 months, 9-12 months, since this division of durations also takes care of seam bias. We observe that the average shares are well-matched. Additionally, we have added the measure of the share of very short durations, 0-2 months. This measure is not robust to seam bias, nor to any non-reporting of very short unemployment spells in the SIPP. As a result, we see that our model generates a measure of unemployment spells with very short duration that does not quite match the on in the SIPP (overall 0.53 in the model vs. 0.45 in the SIPP). To the extent that the inclusion of spells which will be completed within a month, simply shifts up the average share of short spells ($<$ 3months), using the semi-elasticity instead of the elasticity is also helpful. This being said, we still find that the model makes predictions for the cyclical behavior of very short spells that are in line with the data.

[50] Given the tight match of the responsiveness with respect to the unemployment rate, it is then not surprising that the model overpredicts the semi-elasticity of the shares with respect to output. In the data, at times when productivity has recovered but unemployment is still high, the share of long-term unemployment is also still high. This leads to a measured responsiveness of duration shares lower in the data than in the model, where, almost by construction, productivity and unemployment move much closer together.

Table 9: Life-cycle Separation and Job Finding Rates

	f data	f model	s data	s model
young (20-30yo)	0.288	0.302	0.011	0.018
prime (35-55yo)	0.234	0.239	0.006	0.011
all	0.269	0.259	0.009	0.013

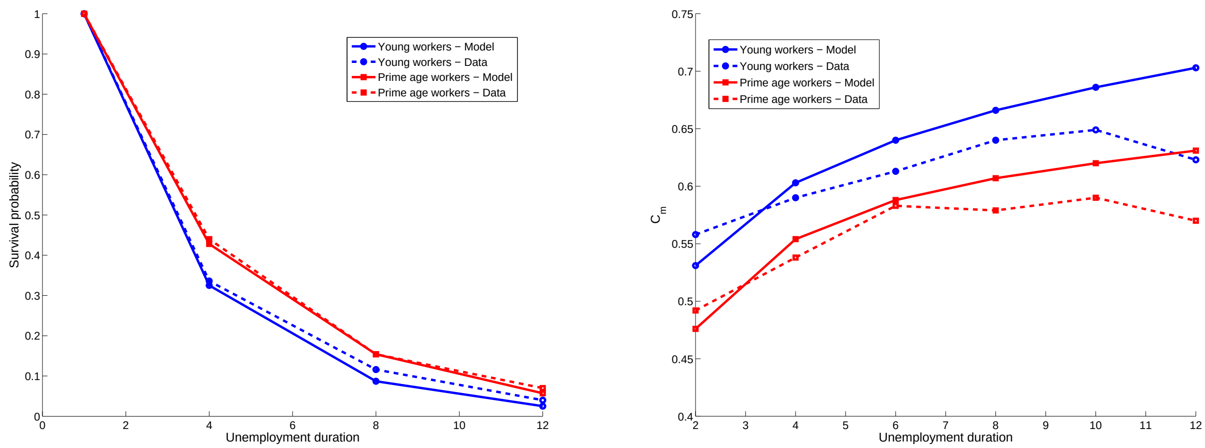
ment rates across these age groups. All the other differences between young and prime-aged workers are un-targeted outcomes of our model.

Table 9 shows the separation and job finding rates from the model and their counterparts in the data. Both the separation rates and job finding rates are lower for prime-aged workers than for young workers, by more or less the same amount in the data and in the model. While we target differences in unemployment rates across age groups, this does not determine separation and job finding rates, as one degree of freedom remains for given unemployment rates, which the model fills in very successfully.[51] In levels, the model closely reproduces the job finding rates with ages, but overestimates the separation rates. This is a straightforward consequence of our abstraction: in this model, as in many other models, the only way to enter unemployment during the working life is from employment. In reality, workers enter unemployment from non-participation as well (see, for example, Elsby et al. 2012). Since workers who come from non-participation contribute to the unemployment rate, which we target, but are not necessarily are recorded in the separation rate into unemployment, we naturally expect the model to produce a higher separation rate.

To understand why the model is able to generate these outcomes, consider again Figure 5. We see that the separation cutoff $z^s(p,x)$ is lower for higher occupational human capital x, and the distance between $z^r(p,x)$ and $z^s(p,x)$ is larger as we increase x. These differences, captured by the shifts in the productivity cutoffs, can explain both the lower inflow and outflow from unemployment across age groups. The inflow into unemployment is lower for experienced workers for two reasons: perhaps most importantly, the mass of experienced employed workers close to their z^s-cutoff is lower because these employed workers are distributed across more z-productivities, and hence spread more thinly, as implied by Figure 7b. Second, more technically, at low z-productivities the pull of mean-reversion becomes weaker, leading to comparatively less and slower transitions to z-productivities in which there is a danger of endogenous separations, and then to z-productivities that will trigger a separation. Once unemployed, the greater distance between $z^s(p,x)$ and $z^r(p,x)$, in combination with the slower pull of mean-reversion, lead the unemployed experienced worker to spend more time on average in unemployment, immediately implying a lower job finding rate for more experienced workers.

Given the estimated importance of rest unemployment, the lower job finding rate, the separation and also the reallocation rate of the prime-aged all have to do with the distance between the cutoffs z^s and z^r, and where these cutoffs lie in the z-spectrum. When discussing our estimation procedure above, we have highlighted how the z-shock process creates duration dependence and affects the

[51] It is perhaps also interesting to note that in a representative-market setting the explanation for the decline in job finding rate over the life cycle is not straightforward, and part of on-going research. The finite horizon of a working life can contribute to the decline, (Cheron et al. 2012, Menzio et al. 2012), however, this effect is relevant mostly in the last ten years of the working life. Esteban-Pretel and Fujimoto (2011) and Gorry (2012) have proposed that as workers age they can better distinguish good from bad jobs, leading them to reject bad jobs more often as they become prime-aged, reducing the separation rate and job finding rate. Here, in a heterogeneous market setting with endogenous reallocation, occupational returns to tenure can explain both.



(a) Survival functions by age groups (b) Occ. mobility and unemp. duration by age groups

Figure 8: Unemployment duration and occupational mobility by age groups

proportion of future occupational movers in the unemployment pool. It is an interesting test of the model to see whether the differences in the distance and location of the productivity cutoffs between less and more experienced workers imply we are able to reproduce the empirical differences with age both in the survival function with unemployment duration, and in the proportion of future reallocators, evolving with spell duration.

Figure 8a shows that the cumulative survival in unemployment for both young and prime-aged workers is very similar in the model and in the data. For both age groups in the data, finding a job becomes less likely when unemployment duration grows longer, but the job finding probability is higher for younger workers at any given duration. The model reproduces this feature. Duration dependence occurs as the distribution of unemployed workers shifts downwards away from z^s with increased unemployment duration. Intuitively, at any spell duration inexperienced workers are closer to an exit from the rest unemployment 'band' (either through the bottom, into reallocation, or through the top, into employment in the same occupation), and hence job finding rates are higher at any unemployment duration. For young workers, relative to prime-aged workers, this implies the model generates a higher outflow rate for occupational stayers (and movers) of about 2%-4% at almost all durations. This is very much in line with the data, and hence the implied cumulative survival functions for the two age groups closely track the empirical ones.

A smaller 'band' between z^s and z^r does not only affect the job finding rates (with duration), it also makes reallocation more likely. We have targeted the difference in the proportion of occupational movers in the stock of the two-month unemployed between young and prime-aged workers. We now analyse the ability of the model to reproduce the subsequent evolution of occupation mobility as unemployment duration increases, for young and prime-aged workers. Figure 8b show that the model produces a persistent, slightly growing difference in the occupational mobility with duration between the two age groups, which stays close to the corresponding difference in the data. In levels, the reallocation patterns for these age groups follows closely the data (maximally deviating 3-4 percentage points) between 2 and 10 months; thereafter, the empirical pattern is non-monotone, especially for the young. This captures that a larger proportion of those workers who will be unemployed for over a year tend to eventually find a job in their own occupation, while the calibration predicts a monotone relationship that is flattening out.[52] Nevertheless, eventual occupational staying remains elevated in the model even among those young unemployed for close to a year. At 12 months duration the remaining stock of young unemployed has more than 30% stayers, and close to 40% for the corresponding prime-aged workers, both in the model and in the data.

In summary, the model produces age differences in the unemployment, separation and job finding rates close to do the data, while at the same time the returns to occupational tenure are substantial: those who become unemployed at later age are a subset of workers who have been 'unlucky' in the realization of the z-productivity. Importantly, our model suggest that unemployment of the young is not driven by a different mechanism than prime-aged unemployment. Rather, the strength of the forces within the mechanism are quantitatively different. Rest unemployment also shapes unemployment outcomes for the young, however, the difference between their separation and reallocation cutoff is on average smaller, which simultaneously can rationalize higher outflow rates – while keeping duration dependence as in the data– and higher reallocation rates, while keeping a large proportion of occupational stayers at high unemployment duration.

Repeat Mobility Patterns In Section 2, we analysed how subsequent outcomes in unemployment depend on reallocation outcomes in a previous unemployment spell. In the calibration, we have used one of these statistics, the reallocation outcome of unemployed workers who were previously occupational stayers, to inform us about the shock process. We now analyse the ability of the model to reproduce the remaining (un-targeted) statistics described in Section 2.

First, consider the measure of subsequent occupational staying of those who in the previous unemployment spell were occupational movers. As argued in Section 2, this measure (unlike the other statistics) is sensitive to its definition, ranging from 44%-54%. The corresponding statistic in the model is 57%, close to the higher incarnation of this measure, which acknowledges returns to occupations which were held before the previous occupation as also occupational stays. Selection is important here, because of the extent of endogenous separations: those occupational movers who become unemployed again are a group, of those who found themselves -again- at marginal z-productivities and therefore do not necessarily stay in their new occupation.

Second, consider the life cycle behavior of the measure of subsequent occupational staying of those who were occupational stayers before. For young workers, we observe 58% of unemployed workers staying in their occupation at the end of the second spell, versus 56% in the data. For prime-aged workers we observe 62% versus 65% in the data. Thus, in both model and in the data, the increase in occupational staying with age is relatively small, though somewhat stronger in the data. The reason that in our model occupational moving remains elevated, even at prime age, is again consistent with the endogeneity of separations. Those who become unemployed twice in a relatively short period are likely to have a marginal z-productivity, which often leads to occupational mobility, also for experienced workers.[53]

[52] Theoretically, the model can produce a non-monotone relationship between reallocation and duration. This occurs when at the highest human capital levels workers become very, very attached to their occupation, and these workers form a significant part of the long-term unemployed. However, with three levels of occupational human capital in the calibration, and significant mobility even for the prime-aged, this behavior is not exhibited.

[53] In the model, there is no life cycle profile behavior for the repeat unemployed who were occupational movers in their first spell,

Third, consider the unemployment outflow rates of workers who have a second spell of unemployment within a SIPP panel (or pseudo-SIPP panel, in the model). In the data, two statistics were highlighted. (i) Those workers who have changed occupations in the first spell and are subsequently staying in their now-current occupation have the highest outflow rates. The model reproduces precisely this behavior: the outflow rate of occupational staying after occupational moving before is 31.5%, and the difference between this outflow rate and the outflow rate of those who stayed in their occupation twice in two sequential unemployment spells is between 2.5-4% in the data versus 2.5-5% in the model, for both young and prime-aged workers. In the model, this occurs mainly because after losing any occupational human capital at the end of the first spell, these workers now face a smaller 'band' of rest unemployment. This implies that ex-post occupational stayers who were occupational movers in the previous spell spend less time between the two productivity cutoffs on average, and thus, equivalently, have a higher outflow rate. (ii) Of those workers who change occupations the second time around, those who were occupational stayers in their first spell have a lower outflow flow in the second spell. The model also reproduces this, with a 4% difference in the model versus a 2% difference in the data, and for prime-aged workers a larger 5%, and 8% difference, resp. in model and data. This occurs because those who are occupational stayers the first time around are more likely to face the large 'band' of rest unemployment in the second spell, while those that moved the first time around, face the smaller 'band' associated with low levels of occupational human capital.

In summary, we interpret the repeat mobility patterns produced in the calibrated model and their closeness to the data as further pieces of evidence supporting that we have captured well the environment in which workers make their reallocation decisions. As a result of this environment rest unemployment and endogenous separations underlie an important part of the behavior of unemployment in our model.

The Business Cycle and the Life Cycle We now consider the business cycle implications of the model for young and prime-aged workers. These are completely untargeted moments, not even the volatility averaged over the ages is targeted, nor the level of transition rates, averaged over time. In the data, the volatilities of the job finding and separation rates of both age categories are rather close to each other, as can be seen in table 10.[54] Thus, a good theory about the impact of the business cycle on the unemployment of young and prime-aged workers should incorporate substantial age-differences in transition rates averaged over the business cycle, but simultaneously predict business cycle responses of these that are reasonably similar to each other. In general, these observations appear to suggest that the fluctuations in the transition rates is shaped by a process shared across age groups, as in our model.[55] Focussing on the two separate age categories, the model is able to reproduce the similarity in their business cycle volatilities reasonably well, in table 10, though the young experience a bit less volatility than in the data. This indicates that the smaller (resp. larger) 'band' of rest unemployment for inexperienced (resp. experienced) workers, which explain the difference in average separation and job finding rates across age groups, also captures a larger part of the business cycle behavior per

because both young and prime-aged were reset to the initial human capital level, and thus look the same from that moment on.

[54]One must not overlook that these are relative volatilities taken with respect to a base that differs between young and prime-aged workers: both *average* separation rates and job finding rates are higher for young workers.

[55]Given the different unemployment rates, and different average transition rates, fluctuations in e.g. hours over the business cycle will be different for different age groups. See e.g. Gomme et al. (2005).

Table 10: Volatilities of Separation, Job finding and Reallocation Rates, per age group

σ:	s model	s data	f model	f data	c_m model	c_m data
Young	0.093	0.133	0.080	0.107	0.051	0.040
Prime	0.113	0.127	0.090	0.100	0.066	0.045

age group in the data. Specifically, when hit by a recession, the increase in the width of the 'bands' between separation and reallocation cutoffs for all experience groups works to create more and longer-duration unemployment for young and prime-aged workers alike. This also produces a more volatile reallocation rate for prime-aged workers in the model, which is also the case in the data, though not as distinct as in the model.

Finally, when aggregate productivity p is low, the distance between z^s and z^r for inexperienced workers is substantial, leading to long-term unemployment for young workers that is driven mostly by an increase in rest unemployment. Indeed, when decomposing the three types of unemployment among young and prime age workers, we find that for both age groups rest unemployment is the most prominent type of unemployment (accounting for a little over 70% for both groups), both increasing in tandem in the decomposition when times are bad, and decreasing when times are good.[56]

7 Conclusions

We have presented a tractable equilibrium framework with heterogenous labor markets to study the evolution of unemployment over the business cycle. Our analysis is motivated by new evidence on the occupational reallocation patterns of unemployed workers. This evidence allowed us to estimate an idiosyncratic shock process that is very important in shaping the types of unemployment workers experience. We focused on workers' decisions to search, reallocate and separate in response to this process; these are described by simple cutoff rules, and vary with aggregate productivity and human capital. The three kinds of unemployment, search, rest and reallocation, are determined by the (relative) position of these cutoffs. Both the life cycle and business cycle patterns are shaped, in an intuitive way, by the response of these cutoffs to aggregate productivity and human capital accumulated.

Our quantitative evaluation shows that our model successfully captures important business cycle and life cycle patterns among the unemployed in the US. In particular, the procylicality of reallocations, the high cyclical volatility of unemployment, the Beveridge curve and countercyclicality of separations. It highlights the importance of occupational human capital in explaining life cycle differences in workers' search, reallocation and separation outcomes. It also reveals that rest unemployment is not only a major source of unemployment, but can both intuitively and quantitatively be linked to many of the observed patterns over the business cycle and across age groups.

[56] In the Supplementary Appendix we provide a graphical decomposition of unemployment by age groups.

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Appendix

In this section, we have collected the proofs of the lemmas, propositions and results in the paper. We show all results (except for the reservation property in separation) in section 4 for the general case, including human capital. The value functions, for this case are given by

$$W^U(p,x,z) = b + \beta \mathbb{E}_{p',z'}\Bigg[\max_{\rho(p',x,z')}\Bigg\{\rho(p',x,z')\left[-c+\int_{\underline{z}}^{\overline{z}} W^U(p',x_1,\tilde{z})dF(\tilde{z})\right]+ \tag{25}$$

$$(1-\rho(p',x,z'))\left[\max_{W^{E\prime}}\Big\{\lambda(\theta(p',x,z',W^{E\prime}))W^{E\prime}+(1-\lambda(\theta(p,x,z,W^{E\prime})))W^U(p',x,z')\Big\}\right]\Bigg\}\Bigg]$$

$$W^E(p,x,z) = w(p,x,z) \tag{26}$$

$$+\beta \mathbb{E}_{p',x',z'}\Big\{\left[\max_{d(p',x',z')}(1-d(p',x',z'))W^E(p',x',z')+d(p',x',z')W^U(p',x',z')\right]$$

$$J(p,x,z,\tilde{W}^E) = \max_{\{w,\tilde{W}^{E\prime}\}}\Big\{y(p,x,z)-w+\beta\mathbb{E}_{p',x',z'}\left[\max_{\sigma(p',x',z')}\{(1-\sigma(p',x',z'))J(p',x',z',\tilde{W}^{E\prime})\}\right]\Big\} \tag{27}$$

$$V(p,x,z,\tilde{W}) = -k + q(\theta(p,x,z,,\tilde{W}))J(p,x,z,\tilde{W}) = 0, \tag{28}$$

where we have left implicit the time subscripts denoting the following period with a prime, $\tilde{z}$ refers to new draws of z in different occupations, $\tilde{W}^E$, w and $W^{E\prime}$ must satisfy (26) and the first maximization in (27) is subject to the participation constraint (5).

Proof of Lemma 1 Fix any occupation o and consider a firm that promised $W \geq W^U(p,x,z_o)$ to the worker with productivity z_o, delivers this value in such a way that his profit $J(p,x,z_o,W)$ is maximized, i.e. solving (4). Now consider an alternative offer $\hat{W} \neq W$, which is also acceptable to the unemployed worker, and likewise maximizes the profit given $\hat{W}$ for the firm, $J(p,x,z_o,\hat{W})$. Then an alternative policy that delivers W by using the optimal policy for $\hat{W}$, but transfers additionally $W-\hat{W}$ to the worker in the first period must be weakly less optimal, which using the risk neutrality of the worker, results in

$$J(p,x,z_o,W) \geq J(p,x,z_o,\hat{W}) - (W-\hat{W})$$

Likewise, an analogue reasoning implies $J(p,x,z_o,\hat{W}) \geq J(p,x,z_o,W)-(\hat{W}-W)$, which together with the previous equation implies

$$J(p,x,z_o,W) \geq J(p,x,z_o,\hat{W})-(W-\hat{W}) \geq J(p,x,z_o,W)-(\hat{W}-W)--(W-\hat{W}),$$

and hence it must be that $J(p,x,z_o,W) = J(p,x,z_o,\hat{W})-(W-\hat{W})$, for *all* $M(p,x,z_o) \geq W,\hat{W} \geq W^U$. Differentiability of J with slope -1 follows immediately. Moreover, $M(p,x,z_o,W) = W + J(p,x,z_o,\hat{W})+\hat{W}-W = M(p,x,z_o,\hat{W}) \equiv M(p,x,z_o)$. Finally, if $W'(p',x,z_o') < W^U(p',x,z_o')$ is offered tomorrow while $M(p',x,z_o') > W^U(p',x,z_o')$, it is a profitable deviation to offer $W^U(p',x,z_o')$, since $M(p',x,z_o')-W^U(p',z_o') = J(p',z_o',W^U(p',x,z_o')) > 0$ is feasible. This completes the proof of Lemma 1.

Proof of Lemma 2 Fix any occupation o and consider an island (x,z_o) for those who of human capital x, with occupation-worker-match specific productivity z_o, such that $M(p,x,z_o)-W^U(p,x,z_o) > 0$. Since we confine ourselves to this island, with known continuation values $J(p,x,z_o,W)$ and

$W^U(p,x,z_o)$ in the production stage, we drop the dependence on p,x,z_o for ease of notation. Free entry implies $k = q(\theta)J(W) \Rightarrow \frac{dW}{d\theta} < 0$. Notice that it follows that the maximand of workers in (2), subject to (6) is continuous in W, and provided $M > W^U$, has a zero at $W = M$ and at $W = W^U$, and a strictly positive value for intermediate W: hence the problem has an interior maximum on $[W^U, M]$. What remains to be shown is that the first order conditions are sufficient for the maximum, and the set of maximizers is singular.

Solving the worker's problem of posting an optimal value subject to tightness implied by the free entry condition yields the following first order conditions (with multiplier μ):

$$\begin{aligned}\lambda'(\theta)[W - W^U] - \mu q'(\theta)J(W) = 0 \\ \lambda(\theta) - \mu q(\theta)J'(W) = 0 \\ k - q(\theta)J(W) = 0\end{aligned}$$

Using the constant returns to scale property of the matching function, one has $q(\theta) = \lambda(\theta)/\theta$. This implies, combining the three equations above, to solve out μ and $J(W)$,

$$0 = \lambda'(\theta)[W(\theta) - W^U] + \frac{\theta q'(\theta)}{q(\theta)}k \equiv G(\theta),$$

where we have written W as a function of θ, as implied by the free entry condition. Then, one can derive $G'(\theta)$ as

$$G'(\theta) = \lambda''(\theta)[W(\theta) - W^U] + \lambda'(\theta)W'(\theta) + \frac{d\varepsilon_{q,\theta}(\theta)}{d\theta},$$

where $\varepsilon_{q,\theta}(\theta)$ denotes the elasticity of the vacancy filling rate with respect to θ and

$$\frac{d\varepsilon_{q,\theta}(\theta)}{d\theta} = \frac{q'(\theta)k}{q(\theta)} + \frac{\theta[q''(\theta)q(\theta) - q'(\theta)^2]k}{q(\theta)^2}.$$

Since the first two terms in the RHS are strictly negative, G' is strictly negative when $\varepsilon_{q,\theta}(\theta) \leq 0$. The latter then guarantees there is a unique $\tilde{W}_f$ and corresponding θ that maximizes the worker's problem. This completes the proof of Lemma 2.

Proof of Lemma 3 First we show that the operator T maps continuous functions into continuous functions. Note that $\theta \in [0,1]$, for all p,x,z and $W^U(p,x,z)$, $M(p,x,z)$ and $\lambda(\theta)$ are continuous functions. The Theorem of the Maximum then implies that $S(p,z)$ is also a continuous function. That T maps continuous functions into continuous functions then follows as the $\max\{M(p',x',z'), W^U(p',x',z')\}$ is also a continuous function. Moreover, since the domain of p,x,z is bounded, the resulting continuous functions are also bounded.

To show that T defines a contraction, consider two functions $\tilde{M}, \tilde{M}'$, such that $\|\tilde{M} - \tilde{M}'\|_{\sup} < \varepsilon$. Then it follows that $\|W^U(p,x,z) - W^{U\prime}(p,x,z)\|_{\sup} < \varepsilon$ and $\|M(p,x,z) - M'(p,x,z)\|_{\sup} < \varepsilon$, where W^U, M are part of $\tilde{M}$ as defined in the text. Since $\|\max\{a,b\} - \max\{a',b'\}\| < \max\{\|a - a'\|, \|b - b'\|\}$, as long as the terms over which to maximize do not change by more than ε in absolute value, the maximized value does not change by more ε. The only maximization for which it is nontrivial to establish this is $\max\{\int W^U(p,x_1,z)dF(z) - c, S(p,x,z) + W^U(p,x,z)\}$. The first part can be established readily: $\|\int \Big(W^U(p,x_1,z) - W^{U\prime}(p,x,z)\Big)dF(z)\| < \varepsilon$. We now show that this property holds for $\|S(p,x,z) + W^U(p,x,z) - S'(p,x,z) - W^{U\prime}(p,x,z)\|$.

Consider first the case that $M - W > M' - W'$. Then, we must have $\varepsilon > W' - W \geq M' - M > -\varepsilon$.

Construct $M'' = W' + (M - W) > M'$ and $W'' = M' - (M - W) < W'$. Call $S(M - W)$ the maximized surplus $\max_\theta\{\lambda(\theta)(M - W) - \theta k\}$ and θ the maximizer; likewise $S(M' - W')$ and θ'. Then

$$
\begin{aligned}
-\varepsilon < S(M' - W'') + W'' - S(M - W) - W &\leq S(M' - W') + W' - S(M - W) - W \\
&\leq S(M'' - W') + W' - S(M - W) - W < \varepsilon
\end{aligned}
$$

where $S(M' - W'') = S(M - W) = S(M'' - W')$ by construction. Note that the outer inequalities follow because $M - M' > -\varepsilon, W' - W < \varepsilon$.

Likewise, consider the case where $M' - W' > M - W \geq 0$. Then

$$
\begin{aligned}
\varepsilon > S(M' - W'') + W'' - S(M - W) - W &> S(M' - W') + W' - S(M - W) - W \\
&> S(M'' - W') + W' - S(M - W) - W > -\varepsilon
\end{aligned}
$$

Hence $\|S(p, z) + W^U(p, x, z) - S'(p, x, z) - W^{U\prime}(p, z)\| < \varepsilon$. It then follows that $\|T(\tilde{M}(p, x, z, 1)) - T(\tilde{M}'(p, x, z, 1)\| < \beta\varepsilon$ for all p, z, and $\|\tilde{M} - \tilde{M}'\| < \varepsilon$. Hence, the operator is a contraction.

It is now trivial to show that if M and W^U are increasing in z, T maps them into increasing functions. This follows since the $\max\{M(p', x', z'), W^U(p', x', z')\}$ is also an increasing function. Assumption 1 is needed so higher z today implies (on average) higher z tomorrow. Since the value of reallocation is constant in z, the reservation policy for reallocation follows immediately. This completes the proof of Lemma 3.

Proof of Lemma 4 T maps the subspace of functions $\tilde{M}$ into itself with $M(p, z)$ increasing weakly faster in z than $W^U(p, z)$. To show this take $(M(p, z), W^u(p, z)$ such that $M(p, z) - W^U(p, z)$ is weakly increasing in z and z^s denote the reservation productivity such that for and $z < z^s$ a firm-worker match decide to terminate the match, and investigate whether the mapping T preserves the increasing difference. Using $\max_\theta\{\lambda(\theta)(M - W^U) - \theta k\} = \lambda(\theta^*)(M - W^U) - \lambda'(\theta^*)(M - W^U)\theta^* = \lambda(\theta^*)(1 - \eta)(M - W^U)$, we construct the following difference

$$
\begin{aligned}
&T\tilde{M}(p, z, 0) - T\tilde{M}(p, z, 1) = \\
&\quad y(p, z) - b + \beta\mathbb{E}_{p', z'}\Big[(1 - \delta)\max\{M(p', z') - W^U(p', z'), 0\} - \\
&\quad \max\Big\{\int W^U(p', \tilde{z})dF(\tilde{z}) - c - W^U(p', z'), \lambda(\theta^*)(1 - \eta)\big(M(p', z') - W^U(p', z')\big)\Big\}\Big].
\end{aligned}
\tag{29}
$$

The first part of the proof shows the conditions under which $T\tilde{M}(p, z, 0) - T\tilde{M}(p, z, 1)$ is weakly increasing in z. Because the elements of the our relevant domain are restricted to have $W^u(p, z)$ increasing in z, and $M(p, z) - W^u(p, z)$ increasing in z, we can start to study the value of the term under the expectation sign, by cutting a number of different cases to consider depending on where z' is relative to the implied reservation cutoffs.

– *Case 1.* Consider the range of tomorrow's $z' \in [\underline{z}(p'), z^r(p',))$, where $z^r(p') < z^s(p'$. In this case, the term under the expectation sign in the above equation reduces to $-\int W^U(p', x_1, \tilde{z})dF(\tilde{z}) + c + W^U(p', z')$, which is increasing in z'.

– *Case 2.* Now suppose tomorrow's $z' \in [z^r(p'), z^s(p'))$. In this case, the term under the expectation sign becomes zero (as $M(p', z') - W^U(p', z') = 0$), and is therefore constant in z'.

– *Case 3.* Next suppose that $z' \in [z^s(p'), z^r(p'))$. In this case, the entire term under the expectation sign reduces to

$$(1-\delta)(M(p',z')-W^U(p',z'))-\int W^U(p',\tilde{z})dF(\tilde{z})+c+W^U(p',z'),$$

and, once again, is weakly increasing in z', because by supposition $M(p',z')-W^U(p',z')$ is weakly increasing in z', and so is $W^u(p',z')$ by lemma 3.

– *Case 4.* Finally consider the range of $z' \geq \max\{z^r(p'),z^s(p')\}$, such that in this range employed workers do not quit nor reallocate. In this case the term under the expectation sign equals

$$(1-\delta)[M(p',z')-W^U(p',z')]-\lambda(\theta^*(p',z'))(1-\eta)[M(p',z')-W^U(p',z')]. \tag{30}$$

It is easy to show using the free entry condition that $d/d(M-W)(\lambda(\theta^*(p',z'))(1-\eta)[M(p',z')-W^U(p',z')])=\lambda(\theta^*(p',z'))$, and hence that the derivative of (30) with respect to z' is positive whenever $1-\delta-\lambda(\theta^*)\geq 0$.

Given assumption 1, the independence of z of x,p, and the just-established increasingness in z' of the term under the expectation sign, given any p', it follows that the integral in (29) is increasing in today's z. Together with $y(p,z)$ increasing in z, it must be that $T\tilde{M}(p,z,1)-T\tilde{M}(p,z,0)$ is also increasing in z.

To establish that the fixed point also has increasing differences in z between the first and second coordinate, we have to show that space of this functions is closed in the space of bounded and continuous functions. In particular, consider the set of functions $F \stackrel{def}{=} \{f\in\mathcal{C}|f:X\times Y\to\mathbb{R}^2, |f(x,y,1)-f(x,y,2)| \text{ increasing in } y\}$, where $f(.,.,1),f(.,.,2)$ denote the first and second coordinate, respectively, and $\mathcal{C}$ the metric space of bounded and continuous functions endowed with the sup-norm.

The next step in the proof is to show that fixed point of $T\tilde{M}(p,z,0)-T\tilde{M}(p,z,1)$ is also weakly increasing in z. To show we first establish the following result.

Lemma A.1: *F is a closed set in $\mathcal{C}$*

Proof. Consider an $f'\notin F$ that is the limit of a sequence $\{f_n\}, f_n\in F, \forall n\in\mathbb{N}$. Then there exists an $y_1<y$ such that $f'(x,y_1,1)-f'(x,y_1,2)>f'(x,y,1)-f'(x,y,2)$, while $f_n(x,y_1,1)-f_n(x,y_1,2)\leq f_n(x,y,1)-f_n(x,y,2)$, for every n. Define a sequence $\{s_n\}$ with $s_n=f_n(x,y_1,1)-f_n(x,y_1,2)-(f_n(x,y,1)-f_n(x,y,2))$. Then $s_n\geq 0,\ \forall n\in\mathbb{N}$. A standard result in real analysis guarantees that for any limit s of this sequence, $s_n\to s$, it holds that $s\geq 0$. Hence $f'(x,y_1,1)-f'(x,y_1,2)\leq f'(x,y,1)-f'(x,y,2)$, contradicting the premise. □

Thus, the fixed point exhibits this property as well and the optimal quit policy is a reservation-z policy given $1-\delta-\lambda(\theta^*)>0$. Since $y(p,z)$ is strictly increasing in z, the fixed point difference $M-W^u$ must also be strictly increasing in z. Furthermore, since $\lambda(\theta)$ is concave and positively valued, $\lambda'(\theta)(M-W^U)=k$ implies that job finding rate is also (weakly) increasing in z. This completes the proof of Lemma 4.

Proof of Proposition 2 Consider the mapping T^{SP}, where the values are measured at the production stage of the period, and the following period is denoted by a prime, and the entire aggregate state of aggregate productivity p, and the distribution of workers over employment status, human capital level, and individual occupation-specific productivities z is summarized in Ω^j, where j is s,r,m,p for the corresponding four stages of a period, and $P(x'|x)$ denotes the Markov transition probability for the human capital process.

$$
\begin{aligned}
T^{SP}W^{SP}(\Omega^p) = & \max_{\{d(p',x',z',\Omega^{s\prime}),\rho(p',x',z',\Omega^{r\prime}),v(p',x',z',\Omega^{m\prime})\}} \sum_X \sum_{o=1}^{O} \int_{\underline{z}}^{\overline{z}} (u(x,z_o)b + e(x,z_o)y(p,x,z_o))dz_o \\
& + \beta \mathbb{E}_{p',x',z'} \Bigg[- \Bigg(c \sum_X \sum_{o=1}^{O} \int_{\underline{z}}^{\overline{z}} \rho(p',x',z_o',\Omega^{r\prime})u(x',z_o')dz_o' + k \sum_X \sum_{o=1}^{O} \int_{\underline{z}}^{\overline{z}} v(p',x',z_o',\Omega^{m\prime})dz_o' \Bigg) \\
& + W^{SP}(\Omega^{p\prime}) \Bigg]
\end{aligned}
$$

subject to laws of motion as detailed in the Supplementary Appendix, here we highlight the laws of motion from production stage to production stage, to give an impression.

$$
\begin{aligned}
u'(x',z_o)dz_o = & \sum_X \Bigg(\int_{\underline{z}}^{\overline{z}} [(1-\lambda(\theta(p',x',z_o,\Omega^{s\prime})))(1-\rho(p,x,\tilde{z}_o,\Omega^{r\prime}))u(x,\tilde{z}_o) \\
& + d(p',x',z_o,\Omega^{s\prime})e(x,\tilde{z}_o)]\, dF(z_o|\tilde{z}_o)d\tilde{z}_o \Bigg) P(x'|x) \\
& + (\mathbf{1}_{x'=x_1}) \sum_{\tilde{x}\in X} \sum_{x\in X} \sum_{\tilde{o}\neq o} \Big[\int_{\underline{z}}^{\overline{z}} \rho(p',\tilde{x},\hat{z}_o)dF(\hat{z}_o|\tilde{z}_o)u(x,\tilde{z}_o)d\tilde{z}_o \Big] P(\tilde{x}|x) \frac{dF(z_o)}{O-1}
\end{aligned}
$$

$$
\begin{aligned}
e'(x,z_o)dz_o = \int_{\underline{z}}^{\overline{z}} \Bigg[& \lambda(\theta(p',x,z_o,\Omega^{m\prime}))(1-\rho(p',x,z_o,\Omega^{r\prime}))u(x,\tilde{z}_o) \\
& + \sum_{\tilde{x}\in X} ((1-d(p,x,z_o,\Omega^{s\prime}))\mathbb{P}(x|\tilde{x})e(\tilde{x},\tilde{z}_o)) \Bigg] dF(z_o|\tilde{z}_o)d\tilde{z}_o
\end{aligned}
$$

For each island (x, z_o) the social planner must decide whether to (i) reallocate workers, $\rho(.)$, (ii) break up job matches, $d(.)$, and (iii) set the number of vacancies for the unemployed, $v(.)$, given aggregate productivity p and the values of z, and potentially the distribution of workers over individual states. With $v(.) = \theta(.)(1-\rho(.))u(x, z_o)$, we can substitute the vacancy creation decision by a decision on labour market tightness.

The next step is to show that as W^{SP} is linear in $u(x, z)$ and $e(x, z)$, where $u(x, z) = \sum_{o\in O} u(x, z_0)$ and $e(x, z)$ is analogously defined, T^{SP} maps these linear functions into a function that is likewise linear in these variables. Linearity of W^{SP} implies that we can define $W^u(p, x, z)$ and $M(p, x, z)$ such that W^{SP} can be written as

$$
W^{SP}(p,\Omega) = \sum_X \int_{\underline{z}}^{\overline{z}} \left(W^U(p,x,z)u(x,z) + M(p,x,z)e(x,z) \right) dz.
$$

Moreover, substituting in the flow equations, under linearity (combined with the random sampling technology, also captured in the flow equations) the expected value of sampling a new z in a different occupation for u workers is $\int_{\underline{z}}^{\overline{z}} W^U(p,\tilde{z})ud\tilde{z} - uc$ and hence we can write

$$
\begin{aligned}
&T^{SP}W^{SP}(p,\Omega) = \\
&\quad \max_{d(.),\rho(.),\theta(.)} \sum_X \int_{\underline{z}}^{\overline{z}} \Bigg(u(x,z)b + \beta \mathbb{E}_{p',x',z'} \Bigg[\left(\int_{\underline{z}}^{\overline{z}} W^U(p',x_1,\tilde{z})d\tilde{z} - c \right) \rho(p',x',z')u(x,z') \\
&\qquad + (1-\rho(p',x',z')) \Big[\lambda\left(\theta(p',x',z')\right) M(p',x',z') - \theta(p',x',z')k \\
&\qquad\quad + (1-\lambda\left(\theta(p',x',z')\right)) W^U(p',x',z') \Big] \Bigg| p,x,z \Bigg] u(x,z) \\
&\qquad + e(x,z)y(p,x,z) + \beta \mathbb{E}_{p',x',z'} \Bigg[\Big[(1-d(p',x',z'))M(p',x',z') + \\
&\qquad\quad + d(p',x',z')W^U(p',x',z') \Big] \Bigg| p,x,z \Bigg] e(x,z) \quad \Bigg) dz,
\end{aligned}
$$

where implicitly we have already used the notion that when W^{SP} is linear in $e(x,z)$ and $u(x,z)$, then decisions $\rho(.), d(.), \theta(.)$ are only functions of (p,x,z). Further, we can completely isolate the terms with $u(x,z)$ and $e(x,z)$ and take the maximization over the remaining terms such that

$$
T^{SP}W^{SP}(p,\Omega) = \sum_X \int_{\underline{z}}^{\overline{z}} \left[W^U_{max}(p,x,z)u(x,z) + M_{max}(p,x,z)e(x,z) \right] dz
$$

where

$$
\begin{aligned}
&W^U_{max}(p,x,z) = \\
&\quad \max_{\rho(p',x',z'),v(p',x',z')} \Bigg\{ b + \beta \mathbb{E}_{p',x',z'} \Bigg[\left(\int_{\underline{z}}^{\overline{z}} W^U(p',x_1,\tilde{z})d\tilde{z} - c \right) \rho(p',x',z') \\
&\quad + (1-\rho(p,x',z')) \left[\lambda(\theta(p',x',z') \left[M(p',x',z') - W^U(p',x',z') \right] - \theta(p',x',z')k + W^U(p',x',z') \right] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&M_{max}(p,x,z) = \\
&\quad \max_{d(p',x',z')} \Bigg\{ y(p,x,z) + \beta \mathbb{E}_{p',x',z'} \Bigg[\Big(d(p',x',z')W^U(p',x',z') + (1-d(p',x',z'))M(p',x',z') \Big) \Bigg] \Bigg\}
\end{aligned}
$$

The maximized value depends only on p and z_o, and hence T^{SP} maps a value function that is linear in $u(z_o)$ and $e(z_o)$ into a value function with the same properties. Moreover, using the definitions of W^U_{max} and M_{max} it follows that from the fixed point of the mapping T^{SP} we can derive a W^{U*}_{max} and M^*_{max} that constitutes a fixed point to T, and vice versa. Hence, the allocations of the fixed point of T are allocations of the fixed point of T^{SP}, and hence the equilibrium allocation is the efficient allocation. This completes the proof of Proposition 2.

Proof of Proposition 3 The reservation island productivity for the competitive and search case, satisfies, respectively,

$$b+\beta\int_{\underline{z}}^{\bar{z}}\frac{\max\{y(p,z),y(p,z_c^r)\}}{1-\beta}dF(z)-\frac{y(p,z_c^r)}{1-\beta}-c_c=0 \tag{31}$$

$$\frac{(1-\eta)k}{\eta}\left(\beta\int_{\underline{z}}^{\bar{z}}\frac{\max\{\theta(p,z),\theta(p,z^r)\}}{1-\beta}dF(z)-\frac{\theta(p,z^r)}{1-\beta}\right)-c_s=0 \tag{32}$$

Using (16), the response of the reservation island productivity, for the competitive, and the frictional case, is then given by

$$\frac{dz_c^r}{dp}=\frac{\beta F(z_c^r)\frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}+\beta\int_{z_c^r}^{\bar{z}}\frac{y_p(p,z)}{y_z(p,z_c^r)}dF(z)-\frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}}{1-\beta F(z_c^r)} \tag{33}$$

$$\frac{dz^r}{dp}=\frac{\beta F(z^r)\frac{y_p(p,z^r)}{y_z(p,z^r)}+\beta\int_{z^r}^{\bar{z}}\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\frac{y_p(p,z)}{y_z(p,z^r)}dF(z)-\frac{y_p(p,z^r)}{y_z(p,z^r)}}{1-\beta F(z^r)} \tag{34}$$

Choosing c_c, c_s appropriately such that $z_c^r = z^r$, the above expressions imply that $\frac{dz^r}{dp} > \frac{dz_c^r}{dp}$ if $\frac{\theta(p,z)}{w(p,z)-b} > \frac{\theta(p,z^r)}{w(p,z^r)-b},\ \forall\, z > z^r$. Hence we now need to show that $\frac{\theta(p,z)}{w(p,z)-b}$ is increasing in z.

$$\frac{d\left(\frac{\theta(p,z)}{w(p,z)-b}\right)}{dz}=\frac{\theta y_z(p,z)}{(w-b)^2}-\theta\left(\frac{(1-\eta)+(1-\eta)\beta\frac{\theta}{w-b}k}{(w-b)^2}\right)y_z(p,z),$$

which has the same sign as $\eta-(1-\eta)\beta k\frac{\theta}{w-b}$ and the same sign as

$$\begin{aligned}&\eta(1-\eta)(y(p,z)-b)+\eta(1-\eta)\beta\theta k-(1-\eta)\beta\theta k\\&\qquad=(1-\eta)(\eta(y(p,z)-b)-(1-\eta)\beta\theta k).\end{aligned}$$

But $\eta(y(p,z)-b)-(1-\eta)\beta\theta k = y(p,z)-w > 0$ and thus we have established Part *1* of the Proposition.

For Part *2*, note that modularity implies that $y_p(p,z)=y_p(p,\tilde{z}),\ \forall z>\tilde{z}$; while supermodularity implies $y_p(p,z)\geq y_p(p,\tilde{z}),\ \forall z>\tilde{z}$. Hence modularity implies

$$\frac{dz_c^r}{dp}=\frac{1}{1-\beta F(z_c^r)}\frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}\left(\beta F(z_c^r)+\beta\int_{z_c^r}^{\bar{z}}\frac{y_p(p,z)}{y_p(p,z_c^r)}dF(z)-1\right)<0,\ \forall\,\beta<1.$$

In the case with frictions,

$$\frac{dz^r}{dp}=\frac{1}{1-\beta F(z^r)}\frac{y_p(p,z^r)}{y_z(p,z^r)}\left(\beta F(z^r)+\beta\int_{z^r}^{\bar{z}}\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\frac{y_p(p,z)}{y_p(p,z^r)}dF(z)-1\right).$$

If we can show that the integral becomes large enough, for c large enough, to dominate the other terms, we have established the claim. First note that $\frac{y_p(p,z)}{y_p(p,z^r)}$ is weakly larger than 1, for $z>z^r$ by the (super)modularity of the production function. Next consider the term $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}$. Note that

$$\lim_{z\downarrow y^{-1}(b;p)}\frac{\theta(p,z)}{w(p,z)-b}=\frac{\lambda(\theta(p,z))}{1-\beta+\beta\lambda(\theta(p,z))}=0,$$

because $\theta(p,z)\downarrow 0$, as $y(p,z^r)\downarrow b$. Hence, fixing a z such that $y(p,z)>b$, $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\rightarrow\infty$, as $y(p,z^r)\downarrow b$. Since this holds for any z over which is integrated, the integral term becomes unboundedly large, making dz^r/dp strictly positive if reservation z^r is low enough. Since the integral rises continuously but slower in z^r than the also continuous term $\frac{\theta(p,z^r)}{1-\beta}$, it can be readily be established that z^r depends continuously on c, and strictly negatively so as long as $y(p, z^r) > b$ and $F(z)$ has full support. Moreover, for some $\bar{c}$ large enough, $y(p, \underline{z}^r) = b$. Hence, as $c \uparrow \underline{z}^r$, $\frac{dz^r}{dp} > 0$. This completes the proof of Proposition 3.

Proof of Lemma 5 Note that $R(p) = \frac{b+\beta\theta(p,z^r(p))k(1-\eta)/\eta}{1-\beta}$. The derivative of this function with respect to p equals

$$\frac{\beta k(1-\eta)}{(1-\beta)\eta} \frac{\theta}{w(p, z^r(p)) - b} \left(y_p(p, z^r(p)) + y_z(p, z^r(p)) \frac{dz^r(p)}{dp} \right). \tag{35}$$

Since $w(p, z^r(p)) - b = (W^E(p, z^r(p)) - W^U(p, z^r(p)))(1 - \beta(1-\delta) + \beta\lambda(\theta(p, z^r(p))))$ and $\frac{\theta\beta k(1-\eta)}{(1-\beta)\eta} = \beta\lambda(\theta(p, z^r(p)))(W^E(p, z^r(p)) - W^U(p, z^r(p))$, we find that (35) reduces to

$$\frac{\beta\lambda(\theta(p, z^r(p)))}{1 - \beta(1-\delta) + \beta\lambda(\theta(p, z^r(p))} \left(y_p(p, z^r(p)) + y_z(p, z^r(p)) \frac{dz^r(p)}{dp} \right). \tag{36}$$

From the cutoff condition for separation, we find $(1-\beta)R(p) = y(p, z^s(p))$. Taking the derivative with respect to p implies the left side equals (36) and the right side equals $y_p(p, z^s(p)) + y_z(p, z^s(p)) \frac{dz^s(p)}{dp}$. Rearranging yields (21). This completes the proof of Lemma 5.

Proof of Lemma 6 First, we state the detailed version of lemma 6, as lemma 8.

Lemma 8. *The expected values of sampling, waiting, job surplus and unemployment, and the reallocation and separation reservation productivities, respond to changes in parameters as follows*

1. *(i)* $\frac{d(W^s - R)}{dc} > 0$, *(ii)* $\frac{d(M(z) - W^U(z))}{dc} > 0$ *for all active islands; and (iii)* $z^r - z^s$ *is decreasing in* c, *strictly if* $z^r > z^s$.
2. *(i)* $\frac{d(W^s - R)}{db} > 0$, *(ii)* $\frac{d(M(z) - W^U(z))}{db} < 0$ *for all active islands; and (iii)* $z^r - z^s$ *is decreasing in* b *(while both* z^r *and* z^s *are increasing in* b*).*
3. *(i)* $\frac{d(W^s - R)}{d\gamma} < 0$, *(ii) there exists a cutoff* $z^\gamma > \max\{z^r, z^s\}$ *such that for all* $z > z^\gamma$, $\frac{d(M(z) - W^U(z))}{d\gamma} > 0$ *and* $\frac{dW^U(z)}{d\gamma} > 0$*; while* $\frac{d(M(z) - W^U(z))}{d\gamma} < 0$ *and* $\frac{dW^U(z)}{d\gamma} < 0$ *for* $z^\gamma > z > \max\{z^r, z^s\}$*. In expectation,* $\frac{d(\mathbb{E}_z[M(z) - W^U(z)])}{d\gamma} > 0$ *and* $\frac{d\mathbb{E}_z[W^U(z)]}{d\gamma} > 0$*. And (iii), if* $z^r > z^s$ *and* $R - W^s$ *is not too large, or if* $z^s > z^r$*, then* $z^r - z^s$ *is increasing in* γ*.*

First, consider the link between $W^s - R$ and $z^s - z^r$: the difference $W^s - R$ directly affects the distance between z^s and z^r. An increase in the former often leads directly to an increase in the latter. To see this formally, denote the parameter of interest generically by ω; for example, c, b or γ. The reservation productivities for separation and reallocation then implicitly satisfy

$$M(\omega, z^s(\omega)) - W^s(\omega) = 0 \tag{37}$$

$$\lambda(\theta(\omega, z^r(\omega)))(1-\eta)(M(\omega, z^r(\omega)) - W^s) + (W^s(\omega) - R(\omega)) = 0, \tag{38}$$

where (38) only applies when $R(\omega) > W^s(\omega)$ since with the assumed process for z we have that $W^s(\omega) > R(\omega)$ implies $z^r(\omega) = \underline{z}$.

To obtain the derivatives of $z^s(\omega)$ and $z^r(\omega)$ wrt ω, we can take the derivative of (37)-(38), where we make explicit the dependence on ω if and only if the derivative is taken with respect to it.

$$\begin{aligned}\frac{dz^s(\omega)}{d\omega} &= -\frac{d}{d\omega}\bigg[z^s - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega,z)-W(\omega,z), W^s(\omega)-R(\omega)\}] \\ &\quad + \beta\gamma(W^s - R)\bigg] - \beta\gamma\frac{d(W^s(\omega)-R(\omega))}{d\omega} \qquad (39)\\ \frac{dz^r(\omega)}{d\omega} &= -\frac{d}{d\omega}\bigg[z^r - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega,z)-W(\omega,z), W^s(\omega)-R(\omega)\}] \\ &\quad + \beta\gamma(1-\lambda(\theta)(1-\eta))(M(z^r)-W^s)\bigg] - \frac{1-\beta\gamma(1-\lambda(\theta))}{\lambda(\theta)}\frac{d(W^s(\omega)-R(\omega))}{d\omega}. \qquad (40)\end{aligned}$$

These equations imply that the sign of the derivative of $z^r - z^s$ with respect to b or c is the opposite of the sign of the corresponding derivatives of $W^s - R$. This is because $\beta\gamma < \frac{1-\beta\gamma(1-\lambda(\theta))}{\lambda(\theta)}$ and, when taking derivatives, the differential terms within the squared brackets are identical.

We divide the proof into three sections. To simplify notation we consider the transformation $y = y(z)$, where $y(.)$ is the common production function, and let F denote the cdf of y. Accordingly, let $y^r = y(z^r)$ and $y^s = y(z^s)$.

Comparative statics wrt c Consider the difference $W^s - R$ and values of c such that $R \geq W^s$. In this case we have that

$$W^s = (1-\gamma)(R+c) + \gamma(b+\beta R),$$
$$W^s - R = -\gamma(1-\beta)R + (1-\gamma)c + \gamma b.$$

Suppose towards a contradiction that $d(W^s - R)/dc < 0$. The above equations imply that $\frac{dR}{dc} > \frac{(1-\gamma)}{\gamma(1-\beta)} > 0$. We will proceed by showing that under $d(W^s - R)/dc < 0$ both the expected surplus (after a z-shock) and the surplus for active islands (those with productivities that entail positive surplus) decrease, which implies that the value of unemployment decreases, which in turn implies $\frac{dR}{dc} < 0$, which is our contradiction.

Consider an active island with $W^U(y) > R$, the surplus on this island is given by

$$\begin{aligned}M(y) - W^U(y) &= \gamma(y - b + \beta(1-\lambda(\theta(y))(1-\eta))(M(y)-W^U(y))) \\ &\quad + (1-\gamma)(\mathbb{E}_y[M(y)-W^U(y)] + (y - \mathbb{E}[y])), \qquad (41)\end{aligned}$$

where $\mathbb{E}_y[M(y)-W^U(y)]$ describes the expected surplus after a z-shock (after the search stage). Note that $\frac{d}{d(M(y)-W^U(y))}(\lambda(\theta(y))(1-\eta)(M(y)-W^U(y))) = \lambda(\theta(y))$, since (dropping the y argument for brevity) $(1-\eta)(M-W^U) = \frac{(1-\eta)}{\eta}J = \frac{1-\eta}{\eta}\frac{k}{q(\theta)}$, and hence $\lambda(\theta)(1-\eta)(M-W^U) = \frac{1-\eta}{\eta}k\theta$. Moreover, $\frac{d\theta}{d(M-W^U)} = \frac{\eta}{1-\eta}\frac{\lambda(\theta)}{k}$. Putting the last two expressions together, we find that the above derivative equals $\lambda(\theta)$. From (41), it follows that

$$0 < \frac{d(M-W^U)}{d(\mathbb{E}_y[M(y)-W^U(y)])} = \frac{1-\gamma}{1-\gamma\beta(1-\lambda(\theta))} < 1. \qquad (42)$$

Expected match surplus measured after the search stage is

$$\mathbb{E}_y[M(y) - W^U(y)] = \int_{y^r} y - b + \beta(1-\lambda(\theta(y))(1-\eta))(M(y) - W^U(y))dF(y) + \int_{y^s}^{y^r} y - b + \beta(M(y) - R)dF(y) + \int^{y^s} y - b + \beta(W^s - R)dF(y), \tag{43}$$

note that the $(1-\gamma)$ shock integrates out. The third term of the expression above is decreasing in c, by our contradiction supposition. The second term, $\int_{y^s}^{y^r}[M(y) - W^U(y)]dF(y)$, can be rewritten as

$$M - W^s = \gamma(y - b + \beta(M - W^s + W^s - R)) + (1-\gamma)(\mathbb{E}_y[M(y) - W^U(y)] + y - \mathbb{E}[y]),$$

and rearranging yields

$$M - W^s = \frac{\gamma}{1-\gamma\beta}(y - b + \beta(W^s - R)) + \frac{1-\gamma}{1-\gamma\beta}\mathbb{E}_y[M(y) - W^U(y)],$$

where $\frac{\gamma}{1-\gamma\beta}(y - b + \beta(W^s - R))$ is decreasing. For the first term, note that $M(y) - W^U(y)$ responds to c through $\mathbb{E}_y[M(y) - W^U(y)]$, from (42). Combining all the elements (42), (43) and the last two equations, we find that

$$\begin{aligned}
\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} &= \int_{y^r} \frac{(1-\gamma)\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} \\
&\quad + (F(y^r) - F(y^s))\left(\frac{\gamma\beta}{1-\gamma\beta}\frac{d(W^s - R)}{dc} + \frac{1-\gamma}{1-\gamma\beta}\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc}\right) \\
&\quad + F(y^s)\beta\frac{d(W^s - R)}{dc}
\end{aligned}$$

$$\iff \frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} = C \cdot \frac{d(W^s - R)}{dc} < 0, \tag{44}$$

where C is a positive constant. From this it follows that $\frac{d[M(y) - W^U(y)]}{dc} < 0$, by (42).

Next consider $\frac{dW^U}{dc}$, and $\frac{d\mathbb{E}[W^U]}{dc}$. For $y \leq y^r$, $W^U(y) = W^s = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta\mathbb{E}[W^U] - \beta c)$. For $y > y^r$, $W^U(y) = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(\lambda(\theta(y))(1-\eta)(M(y) - W^U(y)) + \beta W^U(y)))$. It follows that $\mathbb{E}[W^U] = F(y^r)(b + \beta\mathbb{E}[W^U] - \beta c) + \int_{y^r}(b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y)) + \beta W^U(y))dF(y)$. Combining the latter equation with

$$W^U = \frac{1-\gamma}{1-\beta\gamma}\mathbb{E}[W^U] + \frac{\gamma}{1-\beta\gamma}(b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y))),$$

we have that

$$\begin{aligned}
&\left(1 - \beta F(y^r) - \beta\frac{1-\gamma}{1-\beta\gamma}(1 - F(y^r))\right)\mathbb{E}[W^U] \\
&\quad = F(y^r)(b - \beta c) + \int_{y^r}\frac{b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y))}{1-\beta\gamma}dF(y)
\end{aligned}$$

Taking the derivative with respect to c, we find that both the first and second terms on the RHS are negative, the latter because we have established that $\frac{d(M(y) - W^U(y))}{dc} < 0$. It then follows that $\frac{d\mathbb{E}[W^U]}{dc} < 0$, which implies that $\frac{dR}{dc} = \frac{d\mathbb{E}[W^U]}{dc} - 1 < 0$, which contradicts our premise.

Now consider values of c such that $R < W^s$. Here there is rest unemployment. In this case, $W^s = \gamma(b + \beta W^s) + (1-\gamma)\mathbb{E}[W^U]$ and $\frac{dW^s}{dc} = 0$, since workers in islands with productivities $y \leq y^s$

will never reallocate. Doing so implies paying a cost $c > 0$ and randomly drawing a new island from the productivity distribution, while by not sampling a worker obtains (with probability $1-\gamma$) a free draw from the productivity distribution. Hence, $d(R - W^s)/dc = dR/dc$. Noting that workers in islands $y > y^s$ prefer employment in their current occupation, the above arguments imply W^U is independent of the value of sampling for any y. It then follows that $\frac{dR}{dc} = \frac{d\mathbb{E}[W^U]}{dc} - 1 = -1 < 0$, which contradicts our premise.

Comparative Statics with respect to b. Here we proceed in the same way as in the previous case. Once again consider the difference $W^s - R$ such that $R \geq W^s$. Writing W^s and W^U, for islands above the separation reservation productivity, as

$$W^s = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(R - W^s)) + \gamma\beta W^s \tag{45}$$

$$W^U(y) = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(\lambda(\theta)(1-\eta)(M(y) - W^U(y)))) + \gamma\beta W^U(y), \tag{46}$$

we find that $W^s - \mathbb{E}[W^U] = \int_{y^r}(W^s - W^U(y))dF(y)$, which in turn implies

$$W^s - R = \frac{1}{1-\gamma\beta F(y^r)}\left(-\beta\gamma\int_{y^r}\lambda(\theta)(1-\eta)(M(y) - W^U(y))dF(y) + (1-\gamma\beta)c\right). \tag{47}$$

That is, the difference between waiting one period to sample and sampling a new island now is the forgone possibility of searching for a job in the new island next period, but on the other hand, the sampling cost only has to be incurred next period with probability γ, and discounted at rate β.

Next consider the relationship between $M(y) - W^U(y)$ and $\mathbb{E}[M(y) - W^U(y)]$. From (41) and (42) , we find that

$$\frac{d(M(y) - W^U(y))}{db} = \frac{1-\gamma}{1-\gamma\beta(1-\lambda(\theta))}\frac{d\mathbb{E}[M(y) - W^U(y)]}{db} - \frac{\gamma}{1-\gamma\beta(1-\lambda(\theta))}. \tag{48}$$

Note that $\frac{d(M(y)-W^U(y))}{db}$ must have the same sign for all y, which is positive if and only if

$$\frac{d\mathbb{E}[M(y) - W^U(y)]}{db} > \frac{\gamma}{1-\gamma}.$$

Towards a contradiction, suppose $\frac{d(W^s-R)}{db} < 0$. Then, we have $\frac{d(W^s-R)}{db} = \frac{d(W^s-\mathbb{E}[W^U])}{db}$, which equals $\frac{d}{db}\left(-\int_{y^r}\max\{W^U(y) - W^s, 0\}dF(y)\right)$. By the envelope condition, the effect $\frac{dy^r}{db}$ disappears. By the previous argument and (45) subtracted by (46), it follows that $\frac{d(M(y)-W^U(y))}{db} > 0$ and by (48), $\frac{d\mathbb{E}[M(y)-W^U(y)]}{db} > 0$.

Along the lines of (44), we find

$$\begin{aligned}
\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{db} = -1 &+ \int_{y^r}\frac{\beta(1-\lambda(\theta(y))) - \gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{db} \\
&- \int_{y^r}\frac{\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{db} \\
&+ (F(y^r) - F(y^s))\left(\frac{\gamma\beta^2}{1-\gamma\beta}\frac{d(W^s - R)}{db} + \frac{\beta(1-\gamma)}{1-\gamma\beta}\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{db}\right) \\
&- (F(y^r) - F(y^s))\frac{\gamma\beta}{1-\gamma\beta} + F(y^s)\beta\frac{d(W^s - R)}{db}
\end{aligned} \tag{49}$$

$$\implies \frac{d\mathbb{E}_y[M(y) - W^U(y)]}{db} = C_2 \cdot \frac{d(W^s - R)}{db} - C_3 < 0,$$

with C_2, C_3 are positive-valued terms. This is the desired contradiction.

Next consider the case that $W^s > R$. Then, equation (47) becomes instead

$$W^s - \mathbb{E}[W^U] = -\frac{\beta\gamma}{1-\beta\gamma}\int_{y^s}\lambda(\theta)(1-\eta)(M(y)-W^U(y))dF(y) \tag{50}$$

Similarly, if we start from the premise that $\frac{d(W^s-R)}{db} < 0$, this will imply again by (48) that $\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} > 0$. Note that in this case, in equation (43) reduces to

$$\mathbb{E}_y[M(y)-W^U(y)] = \int_{y^s} y - b + \beta\lambda(\theta(y))(1-\eta)(M(y)-W^U(y))dF(y), \tag{51}$$

and (49) reduces to

$$\begin{aligned}\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} &= -1 + \int_{y^s}\frac{\beta(1-\lambda(\theta(y))) - \gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} \\ &\quad - \int_{y^s}\frac{\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db},\end{aligned} \tag{52}$$

which again implies that $\frac{d\mathbb{E}[M-W^U]}{db} < 0$, a contradiction.

Comparative statics with respect to γ As in the previous cases we start with the case where $R > W^s$. Towards a contradiction, assume that $\frac{d(W^s-R)}{d\gamma} > 0$. From equation (47), we find that

$$\begin{aligned}\frac{d(W^s-R)}{d\gamma} =& \frac{\beta F(y^r)}{1-\beta\gamma}(W^s-R) + \frac{1}{1-\beta\gamma F(y^r)}\left(-\int_{y^r}\lambda(\theta)(1-\eta)(M(y)-W^U(y)dF(y) - \beta c\right) \\ & - \int_{y^r}\beta\gamma\lambda(\theta)\frac{d(M(y)-W^U(y))}{d\gamma}dF(y)\end{aligned} \tag{53}$$

From our premise it follows that

$$\begin{aligned}-\int_{y^r}\beta\gamma\lambda(\theta)\frac{d(M(y)-W^U(y))}{d\gamma}dF(y) \geq& \frac{\beta F(y^r)}{1-\beta\gamma}(R-W^s) \\ & + \frac{1}{1-\beta\gamma F(y^r)}\left(\int_{y^r}\lambda(\theta)(1-\eta)(M(y)-W^U(y))dF(y) + \beta c\right) > 0\end{aligned} \tag{54}$$

Now, let us look at the implications for $\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}$. We can rewrite (43), bringing tomorrow's continuation values to the LHS as

$$\begin{aligned}(1-\beta)\mathbb{E}_y[M(y)-W^U(y)] =& \int_{y^r} y - b - \beta\lambda(\theta(y))(1-\eta)(M(y)-W^U(y))dF(y) \\ & + \int_{y^s}^{y^r} y - b + \beta(W^s-R))dF(y) \\ & + \int^{y^s} y - b + \beta(W^s-R) - \beta(M(y)-W^s)dF(y)\end{aligned} \tag{55}$$

Taking derivatives with respect to γ, we find

$$
\begin{aligned}
(1-\beta)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{d\gamma} =& -\beta\int_{y^r}\lambda(\theta(y))\frac{d(M(y)-W^U(y))}{d\gamma}dF(y) \\
&+\int_{y^s}^{y^r}\beta\frac{d(W^s-R)}{d\gamma})dF(y) \\
&+\int^{y^s}\beta\frac{d(W^s-R)}{d\gamma}-\beta\frac{d(M(y)-W^s)}{d\gamma}dF(y) \\
&>0
\end{aligned}
\tag{56}
$$

For $y<y^s$ it holds that

$$
\begin{aligned}
\frac{d(M(y)-W^s)}{d\gamma} =&(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}+\gamma\beta\frac{d(W^s-R)}{d\gamma} \\
&+(y-b+\beta(W^s-R)-\mathbb{E}[M(y)-W^U(y)]).
\end{aligned}
\tag{57}
$$

The first two terms on the RHS are positive, the last term on the RHS negative. In the RHS of (56) all terms are positive, except for $F(y^s)(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}$ and $F(y^s)\gamma\beta\frac{d(W^s-R)}{d\gamma}$ associated with $\frac{d(M(y)-W^s)}{d\gamma}$. However, one can see that $-F(y^s)\gamma\beta\frac{d(W^s-R)}{d\gamma}$ is more than offset by $\beta\frac{d(W^s-R)}{d\gamma}$ on the same line, while we can bring $F(y^s)(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}$ to the LHS, to find that $\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}$ is premultiplied by $(1-F(y^s)\beta\gamma)>0$. Hence, it follows that $\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}>0$.

From $M(y)-W^U(y)=(1-\gamma)\mathbb{E}[M(y)-W^U(y)]+\gamma(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^U(y))$, it follows that for $y>y^r$

$$
\begin{aligned}
\beta\gamma\lambda(\theta)\frac{dM(y)-W^U(y)}{d\gamma} =&\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\bigg(\Big(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^U(y)) \\
&-\mathbb{E}[M(y)-W^U(y)]\Big)+(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}
\end{aligned}
\tag{58}
$$

Integrating this term over all $y>y^r$, we have

$$
\begin{aligned}
\beta\gamma\int_{y^r}\lambda(\theta(y))\frac{d(M(y)-W^U(y))}{d\gamma}dF(y) \geq& \frac{\beta\gamma\lambda(\theta(y^r))}{1-\beta\gamma+\beta\gamma\lambda(\theta(y^r))}\bigg(\int_{y^r}(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}dF(y) \\
&+\frac{1}{\gamma}\int_{y^r}M(y)-W^U(y)-\mathbb{E}[M(y)-W^U(y)]dF(y)\bigg)>0,
\end{aligned}
\tag{59}
$$

where the last inequality follows from the fact that $M(y)-W^U(y)-\mathbb{E}[M(y)-W^U(y)]$ is increasing in y, and $\frac{\beta\lambda(\theta(y))}{1-\beta\gamma+\beta\gamma\lambda(\theta(y))}$ similarly is increasing in y. Then $\int_{y^r}M(y)-W^U(y)-\mathbb{E}[M(y)-W^U(y)]dF(y)$ is larger than zero. The LHS of (59) is positive, but this contradicts our premise in (54).

For the case that $W^s>R$, we can derive directly that

$$
(1-\beta)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{d\gamma}=-\beta\int_{y^r}\lambda(\theta(y))\frac{d(M(y)-W^U(y))}{d\gamma}dF(y); \tag{60}
$$

with this in hand, we can derive from (58) that

$$-(1-\beta)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{d\gamma}=\int_{y^s}\left(\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\bigg(\Big(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^U(y))-\mathbb{E}[M(y)-W^U(y)]\Big)+(1-\gamma)\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}\bigg)\right)dF(y). \tag{61}$$

Isolating $\frac{d\mathbb{E}[M(y)-W^U(y)]}{d\gamma}$ on the LHS, we find

$$\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{d\gamma}\left((1-\beta)+\frac{F(y^s)\beta\gamma\lambda(\theta)(1-\gamma)}{1-\beta\gamma+\beta\gamma\lambda(\theta)}\right)=-\int_{y^s}\left(\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\Big(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^U(y))-\mathbb{E}[M(y)-W^U(y)]\Big)\right)dF(y)<0. \tag{62}$$

From (62), it follows that $\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{d\gamma}<0$, and therefore that, from (23) - for the case in which $W^s>R$- , the above equation, and equation (60), W^s-R is decreasing in γ:

$$\frac{d(W^s-R)}{d\gamma}=\frac{d(W^s-\mathbb{E}[W^U])}{d\gamma}=-\frac{1}{1-\beta}\int_{y^s}\gamma\lambda\frac{d(M(y)-W^U(y))}{d\gamma}<0. \tag{63}$$

Proof of lemma 7 In the same setting as in Lemma 6, we introduce human capital x premultiplying productivity y. We once again let y be distributed with cdf F. Normalize (without a loss of generality, for the results we are deriving here) $x=1$. If we have an incremental improvement in x that is occupational specific, the value of sampling will stay constant, at $R=\mathbb{E}_y[W^U(1,y)]-c$, where now we denote $W^U(x,y)$ by the productivity component that is enjoyed by every worker on the island. However, the value of W^s increases, since $W^s=(1-\gamma)\mathbb{E}[W^U(x,y)]+\gamma(b+\beta\max\{R,W^s\}$ implies that W^s is increasing in $\mathbb{E}[W^U(x,y)]$.

Suppose that $R>W^s$. The value of unemployment for the cases in which $y\geq y^r(x)$ and $y^r>y\geq y^s$ is given by

$$W^U(x,y)=(1-\gamma)\mathbb{E}_y[W^U(x,y)]+\gamma(b+\beta\lambda(\theta)(1-\eta)(M(x,y)-W^U(x,y))+\beta W^U(x,y))$$
$$W^s(x)=(1-\gamma)\mathbb{E}_y[W^U(x,y)]+\gamma(b+\beta R).$$

When comparing the expected value of separating with the expected value of sampling (and hence, a reset to $x=1$), we see that the difference $W^s(x)-\mathbb{E}_y[W^U(1,y)]$ is given by

$$(W^s(x)-\mathbb{E}_y[W^U(1,y)])=(1-\gamma)(\mathbb{E}_y[W^U(x,y)]+C,$$

where C denotes those terms that do not depends on x. As a result, $\frac{d(W^s(x)-\mathbb{E}_y[W^U(1,y)])}{dx}=(1-\gamma)\frac{d\mathbb{E}_y[W^U(x,y)]}{dx}$. Rewriting $\mathbb{E}[W^U(x,y)]$, using $W^U(x,y)=(1-\gamma)\mathbb{E}[W^U(x,y)]+\gamma(b+\beta\big(\lambda(\theta)(1-\eta)(M(x,y)-W^U(x,y))\big)+\beta W^U(x,y)$ we find

$$\mathbb{E}[W^U(x,y)]=(1-F(y^r(x))(b+\beta R)+\int_{y^r}\Big[b+\beta\lambda(\theta)(M(x,y)-W^U(x,y))+\beta W^U(x,y)\Big]dF(y),$$

from which it follows that

$$\mathbb{E}[W^U(x,y)]\Big(1-\frac{\beta(1-\gamma)}{1-\beta\gamma}(1-F(y^r(x)))\Big)= \tag{64}$$
$$F(y^r(x)(b+\beta R)+\frac{\beta\gamma}{1-\beta\gamma}b(1-F(y^r(x))+\frac{\beta}{1-\beta\gamma}\int_{y^r(x)}\Big[\lambda(\theta(x,y))(1-\eta)(M(x,y)-W^U(x,y))\Big]dF(y).$$

In turn, (using the envelope condition, which implies that the term premultiplying $dy^r(x)/dx$ again equals zero), this means

$$\frac{d\mathbb{E}_y[W^U(x,y)]}{dx}= \tag{65}$$
$$\frac{\beta}{(1-\beta\gamma)-\beta(1-\gamma)(1-F(y^r(x)))}\frac{d}{dx}\Big(\int_{y^r}\Big[\lambda(\theta(x,y))(1-\eta)(M(x,y)-W^U(x,y))\Big]dF(y)\Big).$$

Let us now look at the behavior of the expected surplus, from

$$\mathbb{E}_y[M(x,y)-W^U(x,y)]=\int_{\underline{y}}^{\bar{y}}(yx-b)dF(y)+\int_{y^r(x)}\beta\lambda(\theta(x,y))(1-\eta)(M(x,y)-W^U(x,y))dF(y)$$
$$+\int_{y^s(x)}^{y^r(x)}\beta(M(x,y)-W^s)dF(y)+\beta\int^{y^r(x)}(W^s(x)-R)dF(y). \tag{66}$$

The surplus for islands with $y\geq y^r(x)$ and $y^r>y\geq y^s$, respectively, behaves as

$$\frac{d(M(x,y)-W^U(x,y))}{dx}=(1-\gamma)\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+(1-\gamma)(y-\mathbb{E}[y])+\gamma y$$
$$+\beta\gamma(1-\lambda(\theta(x,y)))\frac{d(M(x,y)-W^U(x,y))}{dx} \tag{67}$$
$$\frac{d(M(x,y)-W^s(x))}{dx}=(1-\gamma)\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+(1-\gamma)(y-\mathbb{E}[y])+\gamma y$$
$$+\beta\gamma\frac{d(M(x,y)-W^s(x))}{dx}+\beta\gamma\frac{d(W^s(x)-R)}{dx} \tag{68}$$

The derivative of $\int_{y^r}\beta\lambda(\theta(x,y))(1-\eta)(M(x,y)-W^U(x,y))dF(y)$ wrt to x then equals

$$\int_{y^r}\left(\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda(\theta(x,y)))}\left(\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\lambda(\theta(x,y))\gamma}{1-\beta\gamma(1-\lambda(\theta(x,y)))}y\right)\,dF(y), \tag{69}$$

where we note that $\int_{y^r}\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda(\theta(x,y)))}dF(y)\leq\frac{\beta\lambda(\theta(x,\bar{y}))(1-\gamma)}{1-\beta\gamma(1-\lambda(x,\bar{y}))}(1-F(y^r))<1-F(y^r)$.

We now consider the behavior of the second line in (66). The derivative of $\beta(W^s(x)-R)$ is given by

$$\frac{\beta(1-\gamma)}{1-\beta+\beta(1-\gamma)F(y^r(x))}\times \tag{70}$$
$$\int_{y^r}\left(\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda(\theta(x,y)))}\quad\left(\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\lambda(\theta(x,y))\gamma}{1-\beta\gamma(1-\lambda(\theta))}y\right)dF(y).$$

The derivative of $\beta(M(x,y)-W^U(x,y))+\beta(W^s(x)-R)$ with respect to x is then

$$\frac{\beta(1-\gamma)}{1-\beta\gamma}\left(\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\gamma}{1-\beta\gamma}y+\left(\frac{1}{1-\beta\gamma}\frac{\beta(1-\gamma)}{1-\beta+\beta(1-\gamma)F(y^r)}\cdot\right.$$
$$\left.\int_{y^r}\left(\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda(\theta(x,y)))}\left(\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\lambda(\theta(x,y))\gamma}{1-\beta\gamma(1-\lambda(\theta(x,y)))}y\right)\,dF(y)\right) \tag{71}$$

We want to make sure that all terms premultiplying $\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]$ on the RHS do not add up to a number larger than 1. First, note that the terms premultiplying the derivative of the expected surplus in (71) are larger than in (70). Hence if we can show that by replacing the premultiplication term in (70) with the corresponding term in (71), we obtain that the entire term premultiplying the derivative of the expected surplus on the RHS is less than one, we have established this step of the proof. The contribution of these premultiplication terms in the second term on the RHS of (66) is smaller than $\beta(1-\gamma)(1-F(y^r(x)))$. Hence, if

$$\left(\beta\frac{d(M(x,y)-W^U(x,y))}{dx}+\beta\frac{d(M(x,y)-W^U(x,y))}{dx}\right)F(y^r(x))<1-\beta(1-\gamma)(1-F(y^r(x))) \tag{72}$$

we have established the desired property. Starting from collecting the terms premultiplying the derivative of the expected surplus, $\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]$, and substituting these into the LHS of (72), we can develop

$$F(y^r)\left(\frac{\beta(1-\gamma)(1-\beta+\beta(1-\gamma)F(y^r(x)))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}+\frac{\beta(1-\gamma)(\beta(1-\gamma)(1-F(y^r(x))))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}\right)$$
$$=\frac{\beta(1-\gamma)(1-\beta+\beta(1-\gamma))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}F(y^r(x)) \tag{73}$$

The RHS of (72) can be rewritten as $1-\beta+\beta\gamma)+\beta(1-\gamma)F(y^r(x))$. We will show that $\beta\gamma+\beta(1-\gamma)F(y^r(x))$ are larger than (73), from which the desired result follows (as the remaining term, $1-\beta$, is larger than zero, and therefore means that the desired inequality is additionally slack).

$$\beta\gamma>\frac{\beta\gamma(\beta(1-\gamma))\Big(1-\beta+\beta(1-\gamma)\Big)F(y^r(x))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))} \tag{74}$$

$$\beta(1-\gamma)F(y^r(x))>\frac{\beta(1-\gamma)(1-\gamma\beta)\big(1-\beta+\beta(1-\gamma)\big)F(y^r(x))}{(1-\gamma\beta)(1-\beta+\beta F(y^r(x)))} \tag{75}$$

Adding up (74) and (75), we find that the RHS equals precisely the term in (73).

Bringing all terms involving $\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}+y-\mathbb{E}[y]$, it is now straightward to see that the remaining terms on the RHS premultiplying y, are positive. (Integrating terms $y-\mathbb{E}[y]$, will also yield a positive term.) Hence, $\frac{d\mathbb{E}_y[M(x,y)-W^U(x,y)]}{dx}>0$. It follows from (67) that $\frac{d(M(x,y)-W^U(x,y))}{dx}>0$, and therefore, by (65), $\frac{d\mathbb{E}_y[W^U(x,y)]}{dx}>0$, and subsequently, $\frac{d(W^s(x)-R)}{dx}>0$.

Consider next the case that $W^s>R$. In this case again $\frac{d(R-W^s(x))}{dx}=(1-\gamma)\frac{d\mathbb{E}_y[W^U(x,y)]}{dx}$. In this case

$$(1-\beta)\frac{d\mathbb{E}_y[W^U(x,y)]}{dx}=\int_{y^s(x)}\beta\lambda(\theta(x,y))\frac{d(M(x,y)-W^U(x,y))}{dx}dF(y) \tag{76}$$

The surplus $M(x,y) - W^U(x,y)$ responds to changes in x is given by

$$\frac{d(M(x,y) - W^U(x,y))}{dx} = (1-\gamma)\frac{d\mathbb{E}_y[M(x,y) - W^U(x,y)]}{dx} + (1-\gamma)(\mathbb{E}_y[y] - y) + \gamma(y + \beta(1-\lambda(x,y))\left(\frac{d(M(x,y) - W^U(x,y))}{dx}\right), \quad (77)$$

while the expected surplus evolves according to

$$\frac{d\mathbb{E}_y[M(x,y) - W^U(x,y)]}{dx} = \int_{y^s(x)} y + \beta(1-\lambda(x,y))\frac{d(M(x,y) - W^U(x,y))}{dx}, \quad (78)$$

Substituting (77) into (78), it follows that $\frac{d\mathbb{E}_y[M(x,y) - W^U(x,y)]}{dx} > 0$, from which in turn it follows that (76) is also positive.

Finally, let us look at the implications for the cutoff in terms of island productivities $y^s(x), y^r(x)$. Consider first the case that $y^r(x) > y^s(x)$. The reservation quality for separation and reallocation satisfy implicitly, respectively

$$M(x, y^s(x)) - W^s = 0 \quad (79)$$

$$\lambda(\theta(x, y^r(x)))(1-\eta)(M(x, y^r(x)) - W^U(x, y^r(x))) + (W^s(x) - R) = 0. \quad (80)$$

We can see this defines $y^r(x), y^s(x)$ as implicit functions of $M(x,y) - W^s(x)$ and $W^s - R$. The first term is given by

$$M(x, y^s(x)) - W^s = xy^s(x) - b + \beta(1-\gamma)\mathbb{E}_y[\max\{M(x,y) - W^U(x,y), W^s - R\}] + \beta\gamma(W^s(x) - R) \quad (81)$$

$$M(x, y^r(x)) - W^s = xy^r(x) - b + \beta(1-\gamma)\mathbb{E}_y[\max\{M(x,y) - W^U(x,y), W^s - R\}] + \beta\gamma(1 - \lambda(\theta(x, y^r(x))(1-\eta)(M(x, y^r(x)) - W^s(x)). \quad (82)$$

Taking derivatives with respect to x (taking into account the implicit relationship $y^s(x), y^r(x)$), we find

$$y^s(x) + \beta(1-\gamma)\frac{d}{dx}\Big(\mathbb{E}_y[\max\{M(x,y) - W^U(x,y), W^s - R\}]\Big) + \beta\gamma\frac{d(W^s - R)}{dx} + x\frac{dy^s(x)}{dx} = 0 \quad (83)$$

$$\frac{\lambda(\theta)}{1 - \beta\gamma(1-\lambda(\theta))}\left(y^r(x) + \beta(1-\gamma)\frac{d}{dx}\Big(\mathbb{E}_y[\max\{M(x,y) - W^U(x,y), W^s - R\}]\Big) + x\frac{dy^r(x)}{dx}\right) + \frac{d(W^s - R)}{dx} = 0 \quad (84)$$

Since $\frac{d(W^s - R)}{dx} > 0$, this implies that

$$y^s(x) + x\frac{dy^s(x)}{dx} \geq y^r(x) + x\frac{dy^r(x)}{dx} + \frac{1-\beta\gamma}{\lambda(\theta)}\frac{d(W^s - R)}{dx}, \quad (85)$$

which implies that, evaluated at $x = 1$,

$$\frac{dy^s(x)}{dx} - \frac{dy^r(x)}{dx} > y^r(x) - y^s(x).$$

This means that for $y^r > y^s$, more occupational human capital brings closer the two cutoffs. For $y^r < y^s$, it holds in this simplified setting that y^r jumps to the corner, $y^r = \underline{y}$, while y^s is lowered for an increase in x.

Supplementary Appendix

A Data Construction

The SIPP is a longitudinal data set based on a representative sample of the US civilian non-institutionalized population. It is divided into multi-year panels. Each panel comprise a new sample of individuals and is subdivided into four rotation groups. Individuals in a given rotation group are interviewed every four months such that information for each rotation group is collected each month. At each interview individuals are asked, among other things, about their employment status as well as their occupations and industrial sectors during employment in the last four months.[57]

There are several advantages of using the SIPP to other data sets like the Current Population Survey (CPS) or the Panel Study of Income Dynamics (PSID), which also have been used to measure labor market flows and/or occupational and sectoral mobility. The SIPP's longitudinal dimension, high frequency interview schedule and explicit aim to collect information on worker turnover allows us to construct reliable measures of occupational mobility and labor market flows. Further, its panel dimension allows us, compared to the CPS, to follow workers over time and construct uninterrupted spells of unemployment that started with an employment to unemployment transitions and ended in a transition to employment. It's panel dimension also allows us to analyse these workers' occupational mobility patterns conditional on unemployment duration and their post occupational (in) mobility outcomes as outlined in Section 2.[58]

We consider the period 1986 - 2011. To cover this period we use the 1986-1988, 1990-1993, 1996, 2001, 2004 and 2008 panels. Although the SIPP started in 1984, our period of study reflects two considerations. The first one is methodological. Since 1986 the US Census Bureau has been using dependent interviewing in the SIPP's survey design, which helps to reduce measurement error problems. The second reason is that such a period allows us to study the behaviour of unemployment, labor market flows between unemployment and employment and occupational mobility during two recessions, the Great Moderation period and the Great Recession.

For the panels 1986-1988 and 1990-1993 we have used the Full Panel files as the basic data sets, but appended the monthly weights obtained from the individual waves. We have used the Full Panel files as the individual waves do not have clear indicators of the job identifier. Since the US Census Bureau does not provide the Full Panel file for the 1989 data set, which was discontinued and only three waves are available, we opted for not using this data set. This is at a minor cost as the 1988 panel covers up to September 1989 and the 1990 panel collects data as from October 1989. For the panels 1996, 2001, 2004, 2008 there are no Full Panel files, but one can easily construct the full panel by appending the individual wave information using the individual identifier "lgtkey". In this case, the job identifier information is clearly specified.

Two important differences between the post and pre-1996 panels are worth noting. The pre-1996 panels have an overlapping structure and a smaller sample size. Starting with the 1996 panel the sample size of each panel doubled in size and the overlapping structure was dropped. To overcome these

[57] See http://www.census.gov/sipp/ for a detailed description of the data set.

[58] See Mazumder (2007), Fujita, Nekarda and Ramey (2007) and Nagypal (2008) for recent studies that document labor market flows and Xiong (2008) and Moscarini and Fujita (2012) for studies that document occupational mobility using the SIPP. To our knowledge there is no study that uses the SIPP to jointly study labor market flow, occupational mobility and its cyclical patterns.

differences and make the sample sizes somehow comparable, we constructed our pre-1996 indicators by obtaining the average value of the indicators obtained from each of the overlapping panels. On the other hand, the SIPP's sample design implies that in *all* panels the first and last three months have less than 4 rotation groups and hence a smaller sample size. For this reason we only consider months that have information for all 4 rotation groups. The data also shows the presence of seams effects between waves. To reduce the seam bias we average the value of the indicator over the four months that involve the seam. Our indicators are based on the employment status variable at the second week of each month, "wesr2" for the 1986-1988 and 1990-1993 panels and "rwkesr2" for the 1996-2008 panels. The choice of the second week is to approximate the CPS reference week when possible.[59]

For the 1986-2008 panels, a worker is considered employed if he/she was *attached* to a job. Namely if the individual was (1) with job/business - working, (2) with job/business - not on layoff, absent without pay and (3) with job/business - on layoff, absent without pay. A worker is considered unemployed if he/she was *not attached* to a job and looking for work. Namely if the individual was with (4) no job/business - looking for work or on layoff. A worker is then considered out of the labor force (non-participant) if he/she was with (5) no job/business - not looking for work and not on layoff.

The SIPP collects information on a maximum of two jobs an individual might hold simultaneously. For each of these jobs we have information on, among other things, hours worked, total earnings, 3-digit occupation and 3-digit industry codes. If the individual did hold two jobs simultaneously, we consider the main job as the one in which the worker spent more hours. We break a possible tie in hours by using total earnings. The job with the highest total earnings will then be considered the main job. In most cases individuals report to work in one job at any given moment. In the vast majority of cases in which individuals report two jobs, the hours worked are sufficient to identify the main job. Once the main job is identified, the worker is assigned the corresponding two, three or four digit occupation.[60]

The SIPP uses the Standard Occupational Code (SOC). The 1986-1993 panels use the 1980 SOC classification, while the 1996 and 2001 panels use the 1990 SOC classifications. These two classifications differ only slightly between them. The 2004 and 2008 panels use the 2000 SOC classification, which differs more substantially from the previous classifications. Since we find continuity in both the levels and cyclical patterns, we consider the full 1986-2010 period as our benchmark. At each step, we calculate a separate set of statistics spanning the 1986-2001 panels for robustness purposes, but have not find substantial differences, unless explicitly noted. We aggregate the information on "broad" occupations (3-digit occupations) provided by the SIPP into "minor" and "major" occupational categories. Table A.1 shows the categories that constitute major occupations for the 1980, 1990 and 2000 SOC classification.[61]

Using the derived labor market status indicators and main job indicators we measure occupational mobility in two ways: (i) by comparing the reported occupation at re-employment with the one per-

[59] See Fujita, Nekarda and Ramey (2007) for a similar approach. We have also performed our analysis by constructing the labor market status of a worker based on the employment status monthly recode variable for all panels and our results do not change.

[60] For the 1990-1993 panels we correct the job identifier variable following the procedure suggested by Stinson (2003).

[61] In any of these classifications we have not included the Armed Forces. The 1980 and 1990 classifications can be found in http://www.census.gov/hhes/www/ioindex/pdfio/techpaper2000.pdf. The 2000 classification can be found in http://www.bls.gov/soc/socguide.htm. Additional information about these classifications can be found at http://www.census.gov/hhes/www/ioindex/faqs.html.

formed immediately before the unemployment spell and (ii) by comparing the reported occupation at re-employment with all those occupations the individual had performed in past jobs. This distinction only had a minor effect our results. The results presented in the paper are based on the first method for two reasons. For (time-averaged) statistics calculated over the full SIPP dataset (1986-2011) we want to focus on the set of unemployment spells that we can assign the occupational mobility unambiguously. For time series (cyclical patterns), we do not take into account occupations not immediately preceding the unemployment spell, as the length of the available data history varies between the start and end of a panel, and could create spurious patterns. Since the occupational data is collected only when the worker is employed, this procedure is valid only for job changes (with an intervening unemployment spell) after the first observed employment spell. For these cases, we assume that after an employment spell, the unemployed worker retains the occupation of the last job and stays with it until he/she re-enters employment, were the worker might perform a new occupation. Under this procedure we have allowed the unemployed worker to keep his/her occupation when he/she undergoes an intervening spell of non-participation that leads back to unemployment. If this spell of non-participation leads directly to employment, however, we do not count this change as it does not involve an unemployment to employment transition. We also have allowed the worker to retain his/her occupation if the employment spell is followed by a spell of non-participation that leads into unemployment. In summary, the worker retains his/her occupation for transitions of the type: E-U-E, E-U-NP-U-E, E-NP-U-E or combinations of these; and does not retain his/her occupation for transitions of the type: E-NP-E, E-U-NP-E or combinations of these. For unemployment spells that precede the first employment spell we have not imputed an occupation and left it as missing to avoid over representing non-occupational movers in our sample. Tables A.2, A.3 and A.4 report the transition matrices of occupational mobility for major occupational groups, where we have use the numbering in Table A.1 to identify major occupations.

We construct monthly time series for the unemployment rate, employment to unemployment transition rate (job separation rate), unemployment to employment transition rate (job finding rate), and the components of the decomposition of the job finding rate described in the main text. Since there are months for which the SIPP does not provide data and we do not take into account months with less than 4 rotation groups, we have breaks in our time series. To cover the missing observations we interpolate the series using the TRAMO (Time Series Regression with ARIMA Noise, Missing Observations and Outliers) procedure developed by Gomez and Maravall (1999).[62] The periods with breaks are between 1989Q3-1989Q4, 1995Q4-1996Q1, 1999Q4-2000Q4, 2003Q4-2004Q1 and 2007Q4-2008Q2. To construct Cm, however, we left out the last 8 months of each panel. Not doing so would have biased downward this measure towards the end of the panel, as occupational stayers have a higher outflow rate than occupational movers.

Given the interpolated series, we seasonally adjust them using the Census Bureau X12 program. The cyclical components of these series are obtained by detrending the log of each of these series based on quarterly averages and using the HP filter with smoothing parameter 1600. The same filter is applied to the simulated series in the quantitative section of the paper. Our working series are not adjusted for time aggregation error. The main reason for this choice is that when using the now 'stan-

[62] See also Fujita, Nekarda and Ramey (2007) for a similar procedure using the SIPP.

dard' method to correct for time aggregation bias proposed by Shimer (2012) and extended by Elsby et al. (2009) and Fujita and Ramey (2009), one can only get closed form solution for the adjusted job finding and separation rates when only considering changes between two states (for example, employment and unemployment). Correcting for time aggregation when taking into account for occupational changes then becomes extremely cumbersome. Using Fujita and Ramey's (2009) extension, however, we find that time aggregation has little effect on the cyclical behaviour of the aggregate job finding and separation rates in the SIPP.[63] Table A.5 presents a correlation matrix of the cyclical components of our main reallocation measures for Major occupational categories and compares them with the cyclical components of unemployment, output per worker and output.[64]

B Omitted Theoretical Material

B.1 Derivation of Workers Flows

Fix an occupation o and productivity z_o. Assume that firms offer the same $\tilde{W}_f^*(\Omega_t^m)$ during the matching stage to all unemployed workers with productivity z_o. As shown later, this will be indeed the case in equilibrium. Let $u_t^s(z_o)$ and $e_t^s(z_o)$ denote the number of unemployed and employed workers with productivity z_o at the beginning of period t.

Given $u_t^s(z_o)$ and $e_t^s(z_o)$, the number of unemployed workers at the beginning of the reallocation stage is given by

$$u_t^r(z_o) = d(\Omega_t^s)e_t^s(z_o) + u_t^s(z_o),$$

The first term takes into account that a measure $\delta e_t^s(z_o)$ of employed workers gets displaced when $\tilde{W}_f^*(\Omega_t^s) \geq W^U(\Omega_t^s)$, while all employed workers quit to unemployment if $\tilde{W}_f^*(\Omega_t^s) < W^U(\Omega_t^s)$. The number of unemployed $u_t^r(z_o)$ is obtained by summing this flow to the number of unemployed at the beginning of the period. The number of employed at the beginning of the reallocation stage is simply $e_t^r(z_o) = e_t^s(z_o)(1 - d(\Omega_t^s))$.

Now consider the number of unemployed and employed workers at the beginning of the matching stage. It is important to remember that only those unemployed workers at the beginning of the period, $u_t^s(z_o)$, are allowed to sample a new z in a different occupation. Therefore, the outflow of workers with productivity z_o is given by $\rho(\Omega_t^r)u_t^s(z_o)$. Hence the number of unemployed workers at the beginning of the matching stage is given by

$$u_t^m(z_o) = d(\Omega_t^s)e_t^s(z_o) + (1 - \rho(\Omega_t^r))u_t^s(z_o),$$

where only $(1 - \rho(\Omega_t^r))u_t^s(z_o)$ unemployed workers can apply for jobs during the matching stage and the newly unemployed workers, $d(\Omega_t^s)e_t^s(z_o)$, must stay unemployed for the rest of the period. The number of employed workers at the beginning of the matching period is the same as the number of employed workers at the beginning of the reallocation stage; that is, $e_t^m(z_o) = e_t^s(z_o)(1 - d(\Omega_t^s))$.

The number of unemployed workers at the beginning of the production stage is given by

$$u_t^p(z_o) = (1 - \lambda(\theta(\Omega_t^m)))(1 - \rho(\Omega_t^r))u_t^s(z_o) + d(\Omega_t^s)e_t^s(z_o),$$

[63] Fujita et al. (2007) arrived to a similar conclusion when analysing aggregate job finding and separations rates using the SIPP for the period 1983-2003.

[64] Output refers to the seasonally adjusted series of non-farm business output provided by the BLS. Output per worker ($outpw$) is constructed using this output measure and the seasonally adjusted employment series from the CPS obtained from the BLS website, http://www.bls.gov.

where the first term refers to those unemployed workers that did not get a job during the matching stage. The second term refers to those employed workers that got displaced or decided to quit their jobs. The number of employed workers at the beginning of the production stage is given by

$$e_t^p(z_o) = \lambda(\theta(\Omega_t^m))(1-\rho(\Omega_t^r))u_t^s(z_o) + e_t^s(z_o)(1-d(\Omega_t^s)).$$

At the beginning of next period the number of unemployed workers with productivity z_o is given by

$$u_{t+1}^s(z_o)dz_o = \int_{\underline{z}}^{\overline{z}} u_t^p(\tilde{z}_o)dF(z_o|\tilde{z}_o)d\tilde{z}_o + \sum_{\forall\tilde{o}\neq o}\Big[\int_{\underline{z}}^{\overline{z}} \rho(\tilde{\Omega}_t^r)u_t(\tilde{z}_{\tilde{o}})d\tilde{z}_{\tilde{o}}\Big]\frac{dF(z_o)}{O-1},$$

where $\tilde{\Omega}_t^r$ refers to the state space of workers with different productivities $\tilde{z}$ and occupations $\tilde{o}$ at the start of the reallocation stage. The first term refers to all those unemployed workers with productivities $\tilde{z}_o \neq z_o$ in occupation o in period t that obtained productivity z_o at the start of next period due to the exogenous process govern the evolution of z. The second term refers to all those unemployed workers that sampled the pair (z, o) from other occupations.

Note that to derive the expression for $u_{t+1}^s(z_o)$ we have assumed that when a worker samples a new productivity in a different occupation during period t, the value of the new productivity is the one with which he starts the following period. This assumption is made purely for convenience as it will make it easier to describe the planners' problem. It is made without a loss of generality since there are no decisions taken during the production stage and workers draw new productivities z in an i.i.d fashion from the stationary distribution F_z.

Finally, the number of employed workers at the beginning of next period is given by

$$e_{t+1}^s(z_o)dz_o = \int_{\underline{z}}^{\overline{z}} e_t^p(\tilde{z}_o)dF(z_o|\tilde{z}_o)d\tilde{z}_o,$$

where the first term in $e_t^p(\tilde{z}_o)$ refers to those newly employed workers with productivities $\tilde{z}_o \neq z_o$ in occupation o in period t that obtain productivity z_o at the start of next period. The second term refers to those workers with productivities $\tilde{z}_o \neq z_o$ in occupation o in period t that started the period employed and obtained productivity z_o at the start of next period.

B.2 Full Equilibrium Definition

Definition A Block Recursive Equilibrium (BRE) is a set of value functions $W^U(p, x, z_o)$, $W^E(p, x, z_o)$, $J(p, x, z_o, W^E)$, workers' policy functions $d(p, x, z_o)$, $\rho(p, x, z_o)$, $\alpha(p, x, z_o)$ (resp. separation, reallocation and visiting strategies), firms' policy functions $\tilde{W}_f(p, x, z_o)$, $\sigma(p, x, z_o, W^E)$, $w(p, x, z_o, W^E)$, $\tilde{W}^{E\prime}(p, x, z_o, W^E)$ (resp. contract posted, layoff decision, wages paid, and continuation values promised), tightness function $\theta(\tilde{W}, p, x, z_o)$, matching probabilities $\lambda(\theta), q(\theta)$, laws of motion of z_{ot}, p_t for all islands and occupations, a law of motion for human capital acquisition x, and a law of motion on the distribution of unemployed and employed workers over islands and occupations $\tilde{u}(.) : \mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$ and $\tilde{e}(.) : \mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$, such that:

(i) $\theta(p, x, z_o, \tilde{W})$ results from the free entry condition $V(p, x, z_o, \tilde{W}) = 0$ if $\theta(p, x, z_o, \tilde{W}) > 0$ and $V(p, x, z_o, \tilde{W}) \leq 0$ if $\theta(p, x, z_o, \tilde{W}) = 0$, defined in (10), and given value function $J(p, x, z_o, \tilde{W})$.

(ii) Matching probabilities $\lambda(.)$ and $q(.)$ are only functions of labor market tightness $\theta(.)$, according to the definitions in section 3.

(iii) Given firms' policy functions, laws of motion of z_{ot}, p_t and x, and implied matching probabilities from $\lambda(.)$, the value functions W^E and W^U satisfy (8) and (7), while $d(.), \rho(.), \alpha(.)$ are the associated policy functions.

(iv) Given workers optimal separation, reallocation and application strategies, implied by $W^E(.)$ and $W^U(.)$, and the laws of motions on p_t, z_{ot} and human capital x, firms' maximisation problem is solved by $J(.)$, with associated policy functions $\{\sigma(.), w(.), \tilde{W}^{E\prime}(.)\}$.

(v) $\tilde{W}^{E\prime}(p, x, z_o) = W^E(p, x, z_o)$.

(vi) $\tilde{u}$ and $\tilde{e}$ map initial distributions of unemployed and employed workers (respectively) over islands and occupations into next period's distribution of unemployed and employed workers over islands and occupations, according to the above policy functions and exogenous separations, and then according to the flow equations.

B.3 Proof of Proposition 1

The proof is basically an exercise to construct candidate equilibrium functions from the fixed point value and policy functions of T, and then verify these satisfy all equilibrium conditions. Since the identity of an occupation does not affect the value functions or the policy functions we drop the subscript o. From the fixed point functions $M(p, x, z)$ and $W^U(p, x, z)$ with policy functions $\gamma_\theta^T(p, x, z)$ and $\gamma_W^T(p, x, z)$ define the function $J(p, x, z, W) = \max\{M(p, x, z) - W, 0\}$, and the functions $\theta(p, x, z, W)$ and $V(p, x, z, W)$ from $0 = V(p, x, z, W) = -k + q(\theta(p, x, z, W))J(p, x, z, W)$. Also define $W^E(p, x, z) = M(p, x, z) - k/q(\gamma_\theta^T(p, x, z)) = \gamma_W^T(p, x, z)$ if $M(p, x, z) > W^U(p, x, z)$, and $W^E(p, x, z) = M(p, x, z)$ if $M(p, x, z) \leq W^U(p, x, z)$, using $W^U(p, x, z)$ from the fixed point. Finally, define $\delta(p, x, z) = \delta^T(p, x, z)$, $\sigma(p, x, z) = \delta^T(p, x, z)$, $\rho(p, x, z) = \rho^T(p, x, z)$, $\tilde{W}^{E\prime}(p', x, z') = \gamma_W^T(p', x, z')$, $\tilde{W}^f = \gamma_W^T(p, x, z)$ and $w(p, x, z)$ derived from (8) given all other functions.

Now (8) is satisfied by construction. Given the construction of $J(p, x, z, W)$, $\theta(p, x, z, W)$ indeed satisfies the free entry condition. $J(p, x, z, W)$ is satisfied if we ignore the maximization problem. However, $w(p, x, z, W^E)$ and $\tilde{W}^{E\prime}(p', x, z'|p, z, W^E)$ satisfying (8) all yield the same $J(p, x, z, W^E)$ as long as $M(p, x, z) \geq W^E > W^U(p, x, z)$ and

$$M(p', x, z') \geq \tilde{W}^{E\prime}(p', x, z'|p, z, W^E) \geq W^U(p, x, z),$$

which is indeed the case. Hence, $J(p, x, z, W)$ also satisfies (9), provided the separation decisions coincide, which is the case as the matches are broken up if and only if it is efficient to do so according to $M(p, x, z)$ and $W^U(p, x, z)$.

Given the constructed $W^U(p, x, z)$, the constructed $\rho(p, x, z)$ also solves the maximization decision in the decentralized setting. Finally, we have to verify $W^U(p, x, z)$. It is easy to see that this occurs if $S(p, x, z) = S^T(p, x, z)$. Consider the unemployed worker's application maximization problem that gives $S(p, x, z)$,

$$\max_{\{\tilde{\theta}(p,x,z), \tilde{W}(p,x,z)\}} \lambda(\tilde{\theta}(p, x, z))(\tilde{W}(p, x, z) - \tilde{W}^U(p, x, z)),$$

subject to

$$J(p, x, z, \tilde{W}(p, x, z))q(\tilde{\theta}(p, x, z)) - k = 0.$$

From Lemma 1, we know that $\tilde{W}(p, x, z) = M(p, x, z) - J(p, x, z, W(p, x, z))$. Substitute in the latter equation to eliminate J, and we see that the maximization problem for $S^T(p, x, z)$ is equivalent to the problem for the worker in the competitive equilibrium. Finally, $\tilde{W}^f(p, x, z)$ is consistent with profit maximization and thus here with the free entry condition, since any $W \in [W^U(p, x, z), M(p, x, z)]$ by construction of $\theta(p, x, z, W)$ is made consistent with free entry. Hence, the constructed value functions and decision rules satisfy all conditions of the equilibrium, and the implied evolution of the distribution of employed and unemployed workers will also be the same.

Uniqueness follows from the same procedure in the opposite direction, by contradiction. Suppose the block recursive equilibrium is not unique. Then a second set of functions exists that satisfy the equilibrium conditions. Construct $\hat{M}$ and $\hat{W}^U$ from these. Since in any equilibrium the breakup decisions have to be efficient and the reallocation and job application decisions are captured in T, $\hat{M}$ and $\hat{W}^U$ must be a fixed point of T, contradicting the uniqueness of the fixed point established by Banach's Fixed Point Theorem: hence, there is a unique BRE.

BRE gives unique equilibrium allocation We can show that in any equilibrium decisions and values are only functions of (p, x, z). One can show that in our BRE fixed point, the policy correspondence has a single element for every state vector: there is a unique policy function. Suppose there is an equilibrium in which values and decisions do not depend only on (p, x, z), then they depend on a third factor – like the entire distribution of workers of employment status and productivities, or its entire history of observables, H_t. Consider the associated value functions in the alternative equilibrium, which we denote with state vector (p, x, z, H_t) to make this additional factor explicit. Consider two cases: first, suppose in the alternative equilibrium all value functions are the same as in the BRE, i.e. $V(p, x, z, H_t) = V(p, x, z)$, but decisions differ at the same (p, x, z). This violates the property that, in our setting, all maximizers in the BRE fixed point are unique. Hence, we are left with the second case: a value function must differ depending on other variables at the same (p, x, z), and at least one value function will therefore differ from the BRE value function $V(p, x, z)$. Then also, in particular, it can easily be shown that the values of unemployment must differ. Denote the non-BRE value function that differs from the BRE at some (p, x, z) by $W^U(p, x, z, H_t)$, while we refer to the BRE equilibrium value function as $W^U(p, x, z)$. In Proposition 2, we show that the BRE is constrained efficient; since that proof nowhere relies on the uniqueness of the BRE in the broader set of all equilibria (the uniqueness of the equilibrium within the class of BRE is establish just above), we can use the proved results of Proposition 2 here.

We can use the fact that $W^U(p, x, z)$ is the best the worker can do in any case, including the market equilibrium (from Proposition 2). This implies that $W^U(p, x, z) - W^U(p, x, z, H_t) > 2\varepsilon > 0$, for some $\varepsilon > 0$. Now, write the implied sequential problem, with the maximizing policies dictated by the two equilibria; take a time T such that $\beta^{T'}\mathbb{E}[W^U(p(H_{T'}), x(H_{T'}), z(H_{T'}), H_{T'}) - W^U(p(H_{T'}), x(H_{T'}), z(h_{T'}))] < \varepsilon$, over all histories $H_{T'}$; and all subsequent $T' \geq T$. Since payments are bounded, and $\beta < 1$, this T must exist. Consider the alternative policy $\breve{W}^U(p, x, z, H_t)$ where from $0, \dots, T$, the worker applies to a market with a tightness $(\theta(p, x, z), W(p, x, z))$ and reallocation policy function dictated by the block recursive policy functions, and also otherwise follows the separation policies dictated by the BRE until becoming unemployed. Thus, the probability of a match, and the total discounted payments during the match are the same as in the block-recursive equilibrium. By our off-equilibrium restriction workers believe that they can visit any submarket that is consistent with zero expected profits, and here this is the case, since firms are expected to do the same as in the block recursive equilibrium. Note that the constant returns to scale in matching, production, and vacancy posting is crucial here. Construct the value associated with this deviating behavior. This value must lie within ε of $W^U(p,x,z)$, since the only difference with the block recursive policies occur after time T. Within the (supposed) non-BRE equilibrium, the workers choose differently from the constructed deviating profile and must do weakly better. Hence, the value $W(p,x,z,H_t) > W^U(p,x,z) - \epsilon$, which leads us to our desire contradiction.

B.4 Derivation of the 'Pissarides wage equation'

Given that an employed worker value in steady state is

$$W^E(p,z) = w(p,z) + \beta(1-\delta)W^E(p,z) + \beta\delta W^U(p,z),$$

then

$$W^E(p,z) - W^U(p,z) = w(p,z) - b - \beta\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z)) + \beta(1-\delta)(W^E(p,z) - W^U(p,z)),$$

or

$$W^E(p,z) - W^U(p,z) = \frac{w(p,z)-b}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z))}.$$

From the combination of the free entry condition and the Hosios condition, we have

$$\eta\frac{w(p,z)-b}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z))} = (1-\eta)k/q(\theta(p,z)). \tag{86}$$

Moreover, from the value of the firm, we have

$$\frac{k}{q(\theta(p,z))} = \frac{y(p,z)-w(p,z)}{1-\beta(1-\delta)}$$

Solving the latter equation for $w(z)$ gives

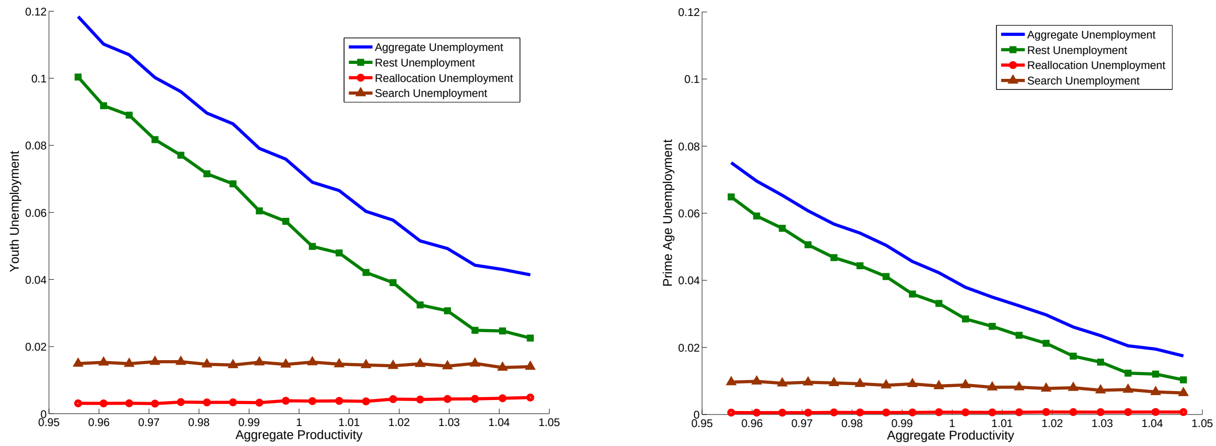
$$w(p,z) = y(p,z) - \frac{k}{q(\theta(p,z))}(1-\beta(1-\delta)).$$

Substituting this in (86), we find

$$\eta(y(p,z)-b) - \frac{k}{q(\theta(p,z))}(1-\beta(1-\delta)) - \beta\theta(p,z)(1-\eta)k = 0.$$

If we replace the middle term with $y(p,z) - w(p,z)$, we get the desired wage equation.

C Omitted Material from the Quantitative Section



(a) Unemployment decomposition - Young Workers (b) Unemployment decomposition - Prime Age Workers

Figure 9: Unemployment decomposition and aggregate productivity by age groups

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D Occupational Mobility Matrices

Table A1: Major Occupational Categories

	1980	1990	2000
1	Executive, Admin., Managerial	Executive, Admin., Managerial	Management
2	Professional Speciality	Professional Speciality	Business and Financial Operations
3	Technicians and Related Support	Technicians and Related Support	Computer and Mathematical
4	Sales	Sales	Architecture and Engineering
5	Admin. Support, incl Clerical	Admin. Support, incl Clerical	Life, Physical, and Social Science
6	Private Household	Private Household	Community and Social Services
7	Protective Services	Protective Services	Legal
8	Service, except Protective - Household	Service, except Protective - Household	Education, Training, and Library
9	Farm, Forestry and Fishing	Farm, Forestry and Fishing	Arts, Design, Ent., Sports, Media
10	Precision Production, Craft and Repair	Precision Production, Craft and Repair	Healthcare Practitioners and Tech.
11	Machine Oper., Assemblers, Insp	Machine Oper., Assemblers, Insp	Healthcare Support
12	Transportation and Material Moving	Transportation and Material Moving	Protective Service
13	Handlers, Equip. Cleaners, Helpers, Lab.	Handlers, Equip. Cleaners, Helpers, Lab.	Food Preparation, Serving Rel.
14			Building, Grounds Cleaning, Maint.
15			Personal Care and Service
16			Sales and Related
17			Office, Admin. Support
18			Farming, Fishing, and Forestry
19			Construction and Extraction
20			Installation, Maintenance, and Repair
21			Production
22			Transportation and Material Moving

Table A.2: Transition Matrix 1980 SOC, 1986-1993

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	**0.365**	0.062	0.014	0.143	0.196	0.001	0.006	0.073	0.038	0.031	0.020	0.040	0.011
2	0.064	**0.529**	0.053	0.074	0.094	0.000	0.005	0.057	0.065	0.026	0.012	0.018	0.006
3	0.036	0.130	**0.430**	0.037	0.101	0.000	0.000	0.095	0.083	0.046	0.008	0.026	0.008
4	0.046	0.025	0.005	**0.450**	0.125	0.006	0.007	0.143	0.041	0.059	0.016	0.063	0.015
5	0.055	0.021	0.016	0.121	**0.537**	0.002	0.003	0.107	0.030	0.046	0.013	0.047	0.004
6	0.000	0.014	0.000	0.122	0.081	**0.360**	0.000	0.323	0.048	0.000	0.015	0.037	0.000
7	0.011	0.008	0.000	0.091	0.030	0.000	**0.273**	0.156	0.091	0.065	0.156	0.091	0.030
8	0.017	0.020	0.005	0.118	0.068	0.005	0.012	**0.499**	0.057	0.059	0.024	0.098	0.017
9	0.014	0.011	0.009	0.050	0.024	0.000	0.015	0.057	**0.563**	0.086	0.038	0.110	0.024
10	0.009	0.014	0.005	0.060	0.050	0.003	0.005	0.099	0.111	**0.489**	0.041	0.089	0.025
11	0.004	0.006	0.000	0.040	0.020	0.005	0.021	0.056	0.113	0.058	**0.548**	0.104	0.026
12	0.010	0.012	0.009	0.068	0.069	0.001	0.012	0.095	0.122	0.100	0.048	**0.413**	0.041
13	0.000	0.009	0.000	0.031	0.026	0.002	0.003	0.075	0.053	0.075	0.040	0.105	**0.581**

Table A.3: Transition Matrix 1990 SOC, 1996-2001

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	**0.355**	0.091	0.017	0.126	0.180	0.000	0.009	0.086	0.012	0.040	0.026	0.012	0.044
2	0.076	**0.520**	0.057	0.070	0.090	0.000	0.003	0.087	0.006	0.028	0.024	0.004	0.034
3	0.020	0.125	**0.370**	0.052	0.137	0.000	0.011	0.102	0.018	0.083	0.047	0.016	0.019
4	0.055	0.032	0.012	**0.401**	0.157	0.004	0.008	0.154	0.019	0.037	0.037	0.017	0.067
5	0.074	0.032	0.024	0.151	**0.473**	0.005	0.010	0.115	0.010	0.027	0.036	0.009	0.034
6	0.032	0.028	0.000	0.182	0.120	**0.191**	0.000	0.271	0.023	0.013	0.042	0.000	0.097
7	0.016	0.026	0.000	0.155	0.051	0.000	**0.330**	0.068	0.033	0.066	0.077	0.013	0.166
8	0.024	0.020	0.007	0.143	0.066	0.009	0.008	**0.514**	0.018	0.045	0.045	0.023	0.080
9	0.026	0.014	0.009	0.040	0.066	0.000	0.013	0.088	**0.395**	0.141	0.120	0.035	0.053
10	0.022	0.024	0.009	0.075	0.045	0.001	0.006	0.072	0.018	**0.440**	0.104	0.052	0.131
11	0.020	0.005	0.023	0.061	0.064	0.010	0.010	0.123	0.043	0.119	**0.353**	0.046	0.124
12	0.018	0.008	0.004	0.035	0.048	0.000	0.005	0.077	0.034	0.115	0.089	**0.456**	0.109
13	0.017	0.006	0.003	0.098	0.050	0.001	0.002	0.123	0.064	0.125	0.108	0.069	**0.334**

Table A.4: Transition Matrix 2000 SOC, 2004-2008

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	**0.309**	0.081	0.030	0.014	0.004	0.008	0.008	0.005	0.013	0.015	0.023	0.007	0.048	0.005	0.024	0.153	0.126	0.012	0.021	0.009	0.060	0.026
2	0.074	**0.258**	0.031	0.021	0.000	0.000	0.000	0.029	0.003	0.016	0.045	0.007	0.024	0.017	0.000	0.085	0.298	0.008	0.000	0.013	0.013	0.057
3	0.016	0.055	**0.378**	0.026	0.000	0.000	0.000	0.019	0.011	0.000	0.000	0.007	0.052	0.000	0.000	0.093	0.110	0.002	0.004	0.062	0.119	0.045
4	0.061	0.053	0.015	**0.457**	0.009	0.000	0.016	0.000	0.016	0.014	0.014	0.000	0.000	0.009	0.000	0.039	0.041	0.016	0.115	0.066	0.016	0.044
5	0.070	0.012	0.041	0.281	**0.253**	0.028	0.000	0.000	0.000	0.070	0.042	0.000	0.060	0.000	0.000	0.034	0.070	0.000	0.000	0.000	0.040	0.000
6	0.081	0.000	0.000	0.000	0.026	**0.298**	0.000	0.034	0.091	0.000	0.030	0.000	0.055	0.000	0.026	0.071	0.181	0.000	0.000	0.000	0.019	0.088
7	0.028	0.160	0.000	0.000	0.000	0.052	**0.544**	0.000	0.000	0.000	0.000	0.000	0.036	0.000	0.000	0.040	0.140	0.000	0.000	0.000	0.000	0.000
8	0.037	0.094	0.009	0.010	0.012	0.009	0.000	**0.384**	0.000	0.019	0.019	0.000	0.043	0.000	0.043	0.126	0.156	0.000	0.005	0.000	0.000	0.034
9	0.074	0.011	0.000	0.012	0.000	0.021	0.000	0.068	**0.453**	0.014	0.010	0.030	0.024	0.000	0.005	0.111	0.101	0.000	0.018	0.000	0.009	0.038
10	0.029	0.013	0.000	0.000	0.006	0.036	0.000	0.025	0.000	**0.556**	0.099	0.000	0.037	0.007	0.000	0.029	0.141	0.000	0.000	0.000	0.020	0.000
11	0.009	0.000	0.000	0.000	0.000	0.004	0.000	0.016	0.003	0.069	**0.444**	0.012	0.043	0.066	0.098	0.056	0.096	0.000	0.007	0.013	0.035	0.030
12	0.000	0.020	0.000	0.000	0.000	0.016	0.000	0.000	0.000	0.000	0.020	**0.370**	0.086	0.018	0.038	0.128	0.090	0.000	0.070	0.000	0.023	0.123
13	0.011	0.004	0.002	0.001	0.001	0.000	0.000	0.005	0.004	0.005	0.011	0.003	**0.466**	0.028	0.018	0.153	0.097	0.004	0.037	0.015	0.037	0.098
14	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.006	0.006	0.003	0.014	0.049	**0.366**	0.014	0.121	0.065	0.045	0.096	0.016	0.097	0.095
15	0.023	0.012	0.000	0.000	0.000	0.007	0.000	0.019	0.011	0.011	0.058	0.020	0.088	0.044	**0.321**	0.119	0.169	0.000	0.007	0.008	0.022	0.060
16	0.033	0.017	0.016	0.004	0.005	0.002	0.001	0.008	0.010	0.004	0.016	0.010	0.127	0.023	0.023	**0.412**	0.164	0.001	0.021	0.014	0.048	0.040
17	0.027	0.042	0.012	0.013	0.001	0.005	0.007	0.010	0.018	0.010	0.014	0.005	0.067	0.033	0.013	0.137	**0.441**	0.008	0.019	0.017	0.044	0.058
18	0.000	0.000	0.007	0.000	0.005	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.002	0.051	0.013	0.023	0.015	**0.637**	0.046	0.018	0.065	0.115
19	0.006	0.001	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.010	0.034	0.035	0.002	0.019	0.023	0.011	**0.648**	0.049	0.061	0.096
20	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.006	0.000	0.000	0.009	0.002	0.028	0.079	0.000	0.061	0.041	0.015	0.130	**0.422**	0.094	0.098
21	0.017	0.006	0.012	0.011	0.010	0.001	0.000	0.006	0.005	0.000	0.012	0.007	0.049	0.046	0.012	0.060	0.066	0.015	0.090	0.019	**0.409**	0.147
22	0.003	0.006	0.000	0.000	0.000	0.002	0.000	0.003	0.002	0.004	0.008	0.022	0.060	0.050	0.005	0.072	0.083	0.025	0.086	0.025	0.097	**0.449**