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December 2010

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

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Discussion Paper No. 5416
December 2010

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December 2010

ABSTRACT

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A large part of the literature on frictional matching in the labor market assumes bilateral meetings between workers and firms. This ignores the frictions that arise when workers and firms meet in a multilateral way and cannot coordinate their application and hiring decisions. I analyze the magnitude of these frictions. For this purpose, I present an equilibrium search model of the labor market with an endogenous number of contacts between workers and firms. Workers contact firms by applying to vacancies, whereas firms contact applicants by interviewing them. Sending more applications and interviewing more applicants are both costly activities but increase the probability to match. In equilibrium, contract dispersion arises endogenously and workers spread their applications over the different types of contracts. Estimation of the model on the Employment Opportunities Pilot Projects data set provides values for the fundamental parameters of the model, including the cost of an application, the cost of an interview, and the value of non-market time. These estimates are used to determine the loss in social surplus compared to a Walrasian world. Frictions on the worker and the firm side each cause approximately half of the 4.7% loss. There is a potential role for activating labor market policies, because I show that for the estimated parameter values welfare is improved if unemployed workers increase their search intensity.

JEL Classification: J64, J31, E24, D83

Keywords: labor, search, recruitment, frictions, efficiency

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* This paper was originally titled *Simultaneous Search and Recruitment*. I am very grateful to Philipp Kircher for feedback at various stages of this project. I thank Jim Albrecht, Martin Burda, Gabriele Camera, Jason Faberman, Giuseppe Ferrero, Manolis Galenianos, Pieter Gautier, Veronica Guerrieri, Ali Hortacsu, Andrey Launov, Guido Menzio, Dale Mortensen, Jean-Marc Robin, Richard Rogerson, Shouyong Shi, Robert Shimer, and many others for valuable comments. This paper has benefited from presentations at the University of Iowa, the European Meeting of the Econometric Society, the Tinbergen Institute, the Search and Matching Workshop at the Philadelphia FED, the Fifth IZA Conference on Labor Market Policy Evaluation at Georgetown University, Chicago Booth, the University of Chicago, Purdue University, the Stockholm School of Economics, the University of Illinois at Urbana-Champaign, the University of Toronto, Australian National University, the University of Melbourne, the Midwest Economic Theory Meeting, the Workshop on Labour Market Search at the University of Konstanz, the Annual Meeting of the Society of Economic Dynamics, the NBER Summer Institute, Queen's University and the University of Western Ontario.

1 Introduction

1.1 Motivation and Summary

In most labor markets, the interaction between firms and workers is of a decentralized nature. In such a world, matching is often subject to frictions in the sense that it takes time and/or effort to find the right trading partner. Various authors have proposed equilibrium models which account for such frictions, with Mortensen & Pissarides (1994) and Burdett & Mortensen (1998) being prominent examples. These models have been used to analyze a wide spectrum of topics, such as the effects of various government policies.[1]

An important assumption in most of this literature - either explicit or implicit - is that meetings between workers and firms are bilateral. In reality however, this is rarely the case. Firms typically receive multiple applications after posting a vacancy and need time to select the worker that they wish to hire. This screening may cause a significant delay between the moment at which a worker applies to the job and the point at which the firm makes the hiring decision.[2] Such delays provide the workers with an incentive to send multiple applications at once in order to reduce the risk of remaining unemployed for a prolonged period of time.[3] Hence, multilateral meetings - rather than bilateral meetings - are a natural feature of decentralized labor markets.

The distinction between both types of meetings is important since they have different implications for the nature of the competition between agents. With bilateral meetings, a firm and a worker only compare each other to the option value of continued search when deciding whether to match or not. When meetings are multilateral instead, an applicant that would have gotten the job if he had been the only one meeting the firm, may now be rejected just because another applicant is considered slightly more qualified. Likewise, a firm wanting to hire a certain worker may now fail to succeed just because the worker prefers a position at a different firm. Hence, multilateral meetings may introduce additional frictions into the matching process.

These frictions get magnified when sending applications and screening workers are costly activities. A firm turning down the application of a worker imposes a negative externality since the worker had spent valuable resources on applying. Similarly, a worker rejecting a job offer imposes a negative externality on the firm by causing the resources spent on screening to be wasted. These externalities can potentially have important implications for understanding labor market outcomes or for the desirability of government programs that aim to increase the search intensity of unemployed workers. It is therefore crucial to understand the magnitude of these externalities and of the costs that generate them.

[1] See e.g. Lise et al. (2005), Cahuc et al. (2008), and Albrecht et al. (2009).

[2] For example, van Ours & Ridder (1992) study vacancy durations in the Dutch labor market and find that after most applications have arrived, it still takes a firm on average about 3 months to fill the vacancy. Larger values are found for positions requiring university degrees.

[3] See for example Morgan & Manning (1985).

In this paper, I analyze these issues, both theoretically and empirically by endogenizing the matching function. I present an equilibrium model with multilateral meetings between workers and firms in which firms post contracts to attract applicants. Unemployed workers observe these contracts and decide where to apply. An important novelty compared to existing models is that both workers and firms can choose how many agents on the other side of the market they want to contact. Sending many applications reduces the worker's probability of remaining unemployed, but is costly. Therefore, a worker will typically decide to apply to multiple but not all firms. A similar structure holds for the firms. A firm contacts applicants by inviting them for an interview. Interviewing applicants is a costly but necessary activity before job offers can be made, hence it reduces the probability for a firm to remain unmatched. As a result, firms typically choose to interview multiple but not all applicants.

A lack of coordination between individual workers and firms will cause the limited number of contacts on both sides of the market to result in the two types of frictions outlined above. Throughout this paper, I will refer to the first friction (two workers may apply to the same firm; only one can get the job and the other one has wasted a costly application) as the *search friction*. I call the corresponding friction on the firm side (two firms may interview the same candidate; only one can hire him and the other firm has wasted a costly interview) the *recruitment friction*.[4]

Together, these frictions are responsible for the externalities in the labor market. The two fundamental parameters of the model, i.e. the cost of an application and the cost of an interview, determine the magnitude of the frictions and the externalities. For example, if the cost of an application is really low, workers will decide to apply a lot and the search friction will be limited. However, firms now face a lot of competition for each worker, so the recruitment friction will be severe. On the other hand, if the cost of an interview is sufficiently close to zero, firms would be able to contact all applicants and the recruitment friction would be absent.

After specifying the model, I characterize the equilibrium. I show that different types of contracts are offered by the firm. Some firms offer low wages but contact many applicants, while other firms do the opposite. The number of contract types is equal to the maximum number of applications that workers send in any given period and firms are indifferent between all types. Workers face a trade-off between the wage and the job offer probability. Applications to low wage firms are more likely to turn into job offers than applications to high wage firms. I show that workers maximize the payoff from their application portfolio by spreading their applications over the different types of contracts.

The model is estimated on the Employment Opportunities Pilot Projects data set, which contains detailed information on the recruitment processes of a sample of firms. The estimation provides values for the search cost and the recruitment cost, as well as the workers' value of non-market

[4]Albrecht et al. (2004, 2006) discuss both frictions in a theoretical setting, but call them the 'urn-ball friction' and the 'multiple-application friction' respectively. See also Gautier & Moraga-Gonzalez (2005).

time. An additional application is found to cost a worker 0.73 hours of his time. Firms incur a cost equal to 1.97 hours of production for each interview. The value of non-market time is estimated at 0.950. I discuss that this high estimate is necessary to explain why firms do not spent a larger amount of resources on recruitment.

By simulating the equilibrium for different values of the two cost parameters, the market equilibrium can be compared with worlds in which one of the frictions or both of the frictions are absent. Social surplus is 4.7% lower in the market equilibrium than in a Walrasian world. Search and recruitment frictions each contribute approximately the same amount to this output loss. Each is also responsible for roughly half of the frictional unemployment arising in equilibrium.

Finally, I consider a social planner's problem to analyze the effect of UI eligibility rules that specify a minimum search intensity. The planner chooses a certain minimum level, after which workers decide whether they comply or not. I show that if unemployed workers increase their search intensity in response to these rules, firms indeed need to spend more resources on recruitment. Nevertheless, their matching probability decreases due to the increased competition that they face for any given candidate. However, for modest increases in the search intensity, social surplus rises.

1.2 Related Literature

This paper adds to the literature in multiple ways. First of all, the paper extends the literature on micro-founded matching technologies by analyzing a model with an arbitrary number of contacts on both the supply and the demand side. Let A denote the number of applications per worker and let I be the number of interviews per firm.[5] Various authors have analyzed different combinations of A and I. For example, the seminal stable matching outcome of Gale & Shapley (1962) can be interpreted as a model with $A \to \infty$ and $I \to \infty$.[6] This case, which has been applied extensively in the analysis of many different markets, would allow all workers in my model to apply to all firms and all firms to contact all workers.[7]

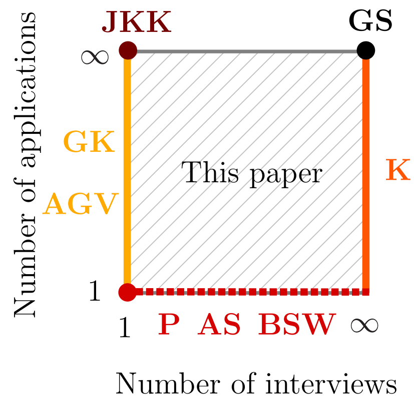
The other extreme is a situation with $A = 1$ and $I = 1$, i.e. all workers send one application and all firms can contact one worker. This is the matching technology explored by e.g. Peters (1991, 2000), Acemoglu & Shimer (2000) and Burdett et al. (2001). Julien et al. (2000) study a model in which workers apply to all firms, and firms can bid for the services of one worker, i.e. $A \to \infty$ and $I = 1$. Albrecht et al. (2004, 2006) and Galenianos & Kircher (2009) also impose $I = 1$, but study the case in which workers send multiple applications simultaneously, i.e. $A > 1$.[8] Gautier &

[5] In the model described in this paper, both variables are endogenous and have a stochastic nature (see section 2). I abstract from that here in order to keep the comparison with the existing literature as clear as possible.

[6] The paper by Gale & Shapley (1962) includes a lot more heterogeneity on both sides of the market than most other models. I do not consider that aspect in this discussion.

[7] Parallel to the search literature, a large literature on matching has been developed with extensions, refinements and applications of the original model by Gale & Shapley (1962). See Roth (2008) for a recent overview.

[8] In a technical appendix to their paper, Albrecht et al. (2006) briefly discuss what happens if firms can contact a second applicant ($I = 2$).



Overview of related matching technologies. The vertical axis shows the number of applications A that workers send out. The horizontal axis displays the number of interviews I that a firm can conduct. GS = Gale & Shapley (1962), P = Peters (1991, 2000), JKK = Julien et al. (2000), AS = Acemoglu & Shimer (2000), BSW = Burdett et al. (2001), AGV = Albrecht et al. (2004, 2006), GK = Galenianos & Kircher (2009), K = Kircher (2009). In Peters (1991, 2000), Acemoglu & Shimer (2000) and Burdett et al. (2001), I is assumed to be 1, but the first interview always leads to a match, so the equilibrium is the same for $I > 1$ as indicated by the dashed line.

Figure 1: Related literature

Moraga-Gonzalez (2005), Kaas (2010) and Gautier et al. (2009) study a similar matching process in a random search setting. Finally, Kircher (2009) studies a model with $A > 1$ applications in which firms can contact all their applicants ($I \to \infty$).

By allowing for arbitrary values of A and I, the model presented here includes the matching technologies in all the cited papers as special cases. Moreover, it goes beyond these papers by exploring combinations of A and I that have not been studied before. This is summarized in figure 1. A related but equally important novelty is that the aggregate matching function in this paper is endogenous. While most of the literature simply imposes a particular functional form as a technological constraint, the matching function in this paper arises as the result of optimal decision making on both sides of the market given the costs that agents face to meet a trading partner. In that sense, the paper is related to Lagos (2000).

The empirical analysis in section 4 will indicate which values of A and I are consistent with the data and which values of the cost parameters correspond to them. As argued, these values can be used to assess the efficiency of the decentralized equilibrium and the magnitude of the search and recruitment friction. I do so in section 4.6. In this sense, the paper is related to contributions by e.g. Ridder & van den Berg (2003), Flinn (2006), and Gautier et al. (2009), who all empirically assess the matching frictions and/or efficiency in the labor market. Knowledge of A and I is also relevant for further research along the lines of Eeckhout & Kircher (2009) who show that a seller's optimal sales mechanism depends on whether the meetings with buyers are bilateral (i.e. $A = I = 1$)

or multilateral ($A > 1, I > 1$).

Finally, this paper adds to the literature on recruitment decisions by firms. Barron et al. (1985), Barron et al. (1987) and Burdett & Cunningham (1998) study the determinants of employer search measures using the same data set as this paper. van Ours & Ridder (1992, 1993) and Abbring & van Ours (1994) analyze Dutch recruitment data and find evidence for simultaneous rather than sequential search by firms. All these papers have in common that they consider partial equilibrium models. Villena-Roldan (2008) develops an equilibrium model in which workers are heterogeneous and firms choose how many applicants to screen. However, all workers apply only once, which makes the model unsuitable for studying recruitment frictions.

The structure of the paper is as follows. Section 2 introduces the model in a static setting. The dynamic model is presented in section 3. Section 4 reports the results of the empirical analysis and section 5 concludes. Proofs are relegated to the appendix.

2 Static Model

This section introduces a simple model of two-sided simultaneous search. In order to develop intuition for the key mechanisms, a number of strong assumptions are made. In particular, 1) the model is static, 2) the number of applications per worker is exogenous, and 3) workers are homogeneous in their productivity. These assumptions will be relaxed in the dynamic model in the next section. That model will embed the static model presented here in the sense that the agents solve it in each period.

2.1 Setting

Consider a labor market with a mass u of unemployed workers and a positive measure v of firms, determined by free entry. Both types of agents are risk-neutral. All workers supply one indivisible unit of labor, while each firm has a position that can be filled by exactly one worker. Workers aim to match with firms with vacancies by sending applications. Both firms and workers are homogeneous in terms of their productivity while matched.

The period consists of several phases. At the beginning of the period, firms decide whether they want to enter the market by creating a vacancy. The vacancy cost is $k_V > 0$. In order to attract applications from workers, entering firms post a contract c. Each firm commits to its contract for the remainder of the period. A contract consists of two elements, both of which are choice variables to the firm.

The first component of the contract is the fraction $w \in \mathcal{W} \equiv [0,1]$ of output that the firm promises to pay to the worker that it hires. Specifying a worker's compensation as a piece rate rather than a fixed amount will later prove notationally convenient, but is without loss of generality in the setup that I consider. Therefore, I will typically refer to w as the worker's wage. The second component of the contract is a recruitment technology $r \in \mathcal{R} \equiv [1, \infty)$, which can be interpreted as a measure for the amount of time that the firm will spend on interviewing workers, or alternatively as a measure for the number of recruiters that the firm hires. The frictions in the labor market are less severe if the firm spends a lot of time on interviewing, as I will explain below. A higher value of r therefore increases the matching probability of both the firm and the workers applying to the firm. However, choosing a higher value for r is also more costly for the firm. I assume that the cost of recruitment technology r equals $k_R(r-1) \geq 0$.[9] Summarizing, a contract c is pair (r, w). For future reference, let $\mathcal{C} = \mathcal{R} \times \mathcal{W}$ denote the set of possible contracts and $v(c)$ the mass of firms offering a contract c, with $v = \int_{\mathcal{C}} v(c)\, dc$.

The next phase is the search phase, during which unemployed workers can search for a job by sending applications. Workers observe all posted contracts and base their search decision on this information. Since there is a delay between sending the application letters and learning their result, workers may have an incentive to send multiple applications simultaneously (see Morgan & Manning, 1985). Sending multiple applications is more costly than applying only once, since each application takes time and effort, but has two positive effects. First, it reduces the risk of not getting any job offer and remaining unemployed. Second, if there is wage dispersion in equilibrium, it increases the probability to get a high wage offer. For the moment, I assume that all workers send A applications in each period, where A is a finite integer larger than 1. Workers decide to which contracts they wish to apply.

After the applications are sent, the recruitment phase starts. During the recruitment phase, each of the firms will interview a number of applicants. The number of interviews R that a firm can conduct (its 'interview capacity') is a random variable which follows a distribution around the firm's recruitment technology r. The intuition for this setup is as follows. In reality, a firm can roughly determine the number of applicants it is able to interview by hiring more or fewer recruiters or by allocating more or less time to recruitment. However, the exact capacity often also depends on factors that cannot be anticipated by the firm. For example, some recruiters may unexpectedly not be available at the moment of hiring, or the screening of a certain candidate takes longer or shorter than foreseen. Having a stochastic element in the recruitment capacity captures this. For computational reasons, it is convenient to have a distribution that generates closed-form expressions for variables like the matching probability. As I will show in section 2.3, a geometric distribution does precisely this. Therefore, I assume that the actual interview capacity follows a geometric distribution with parameter $\frac{r-1}{r} \in [0, 1]$. Hence, the probability that the firm can interview $R \in \mathbb{N}_1$ applicants is given by $\left(\frac{r-1}{r}\right)^{R-1} \frac{1}{r}$, which implies that the expected capacity is exactly equal to the firm's investment,

[9] The first unit of capacity is free, since it assumed to be included in the entry cost. This assumption is not restrictive, but rules out a situation in which a large k_R dissuades firms from interviewing any candidates which would lead to the collapse of the market. As will become clear, the model now converges to the model in which all firms interview one applicant, as described in Galenianos & Kircher (2009), for $k_R \to \infty$.

i.e. $E[R] = r$.[10]

The firm's interview capacity R is realized at the beginning of the recruitment phase and therefore not observed by the workers. Note that the actual number of interviews a firm conducts may be constrained by the number of applicants. If the firm receives a applications, the number of interviews taking place will be $\min\{a, R\}$. If the number of applicants exceeds the interview capacity, candidates are chosen randomly since they all seem identical to the firm. Remaining applicants are rejected. During the interview the firm learns the outcome of a stochastic process that is iid over all firm-worker pairs. With probability $\phi \in (0,1]$, the firm and the worker form a good match and are indeed able to produce output. I call these candidates 'qualified'. With probability $1-\phi$ however, the match is considered a bad match which would be completely unproductive. The firm rejects these unqualified applicants.

In the next phase, matches are formed between workers and firms. In order to understand the mechanism governing this process, consider workers and firms as nodes in a bipartite network. The applications that are sent create links between the workers and the firms. The rejection of some applicants by the firms during the recruitment phase destroys some of these links. The matching is now assumed to be stable on the remaining network in the Gale & Shapley (1962) sense.[11] Hence, matches form such that no firm remains unmatched while one of its qualified candidates is hired by another firm at a lower wage or remains unemployed. If this were not true, both the firm and the worker could do better by deviating and forming a match together. Stability can be motivated by a process in which firms offer their job sequentially to the candidates, and workers are free to reconsider their options.[12] Ties are broken randomly.

In the last phase, production and consumption take place. Each worker-firm pair creates Y units of output, of which the worker consumes a fraction w. The firm obtains the remaining fraction $1-w$ of output. To let the notation be as similar as possible to the dynamic model that I introduce in the next section, I denote the worker's payoff given a wage w by $V_E(w) = \omega_0 + wY$, for arbitrary $\omega_0 \geq 0$, and the firm's payoff by $V_F(w) = (1-w)Y$. Unemployed workers receive a payoff from unemployed benefits and/or household production. I denote this outside option by $V_U \geq 0$. To prevent a collapse of the market, I impose that the surplus from a match is positive, i.e. $V_U < \omega_0 + Y$.

[10] Compared to a situation in which firms can interview a deterministic number of applicants $\widetilde{R}$, the approach followed here has a second advantage. Estimating the continuous parameter r is computationally much easier than estimating the discrete parameter $\widetilde{R}$.

[11] In the Gale & Shapley (1962) model, the network is complete. This paper allows for an arbitrary level of sparseness.

[12] In more detail: each firm randomly selects one of its qualified applicants, if any, and offers him the job. The applicant in question might get multiple offers. He compares them, tentatively accepts the best one and rejects all others. If a firm gets rejected, it can make new job offers if it still has other qualified applicants. After this, workers again compare all offers, potentially including the one tentatively accepted in the previous round, accept the best one for the moment and reject all other. The process continues until no firm can make an offer anymore. All offers that are tentatively accepted at the end of this process become matches between the worker and firm in question. This is known as the Deferred Acceptance Process, first described by Gale & Shapley (1962). For finite economies it converges in finite time. Since the labor market described here contains a continuum of agents, I impose stability by assumption following Kircher (2009).

In the next section, I derive the workers' and firms' optimal strategies. As standard in the literature, I impose symmetry, i.e. identical agents have identical strategies. Further, I require the strategies to be anonymous, in the sense that they cannot be conditioned on the identity of a specific worker or firm.

2.2 Expected Payoffs

Consider a worker who has observed all posted contracts and has to decide where to apply. This decision is not trivial, since the worker does not only care about the wage that he can earn at a particular firm, but also about the probability that the firm will hire him. The worker realizes that relatively many workers may apply to firms which post high wages or large investments in recruitment, which implies that the matching probability at those firms will be lower. This provides the worker an incentive to consider firms with lower wages or recruitment investments as well. Although the worker can distinguish firms that post different contracts, the anonymity assumption implies that the worker applies to all firms that post the same contract with equal probability. The application strategy of the worker can therefore be written as

$$\mathbf{c} = (c_1, \ldots, c_A) = ((r_1, w_1), \ldots, (r_A, w_A)),$$

where c_i indicates to what contract the worker sends his i-th application. It will convenient to assume - without loss of generality - that $w_1 \leq \ldots \leq w_A$ and that the worker accepts the contract with the higher index in case of a tie. I denote the distribution of workers' strategies by G. In other words $G(\widetilde{\mathbf{c}})$ denotes the probability that an unemployed worker sends his first application to a contract (r, w) below $\widetilde{c}_1 = (\widetilde{r}_1, \widetilde{w}_1)$, his second application to a contract below $\widetilde{c}_2$, et cetera. The support of G is denoted by $\mathcal{G}$.

Let $\psi(c)$ denote the endogenous probability that an application to a firm posting $c = (r, w)$ results in a job offer. The result of each application is independent of the result of the workers' other applications. Therefore, the worker's job finding probability $\Psi_0(\mathbf{c})$, which equals the probability that the application strategy $\mathbf{c}$ results in at least one job offer, is given by

$$\Psi_0(\mathbf{c}) = 1 - \prod_{i=1}^{A} (1 - \psi(c_i)). \tag{1}$$

A worker accepts the job offer that gives him the highest wage. Hence, he ends up in a position paying w_i if he gets a job offer from that firm and if none of the applications to higher wages resulted in an offer. This happens with probability $\prod_{j=i+1}^{A} (1 - \psi(c_j)) \psi(c_i)$. The worker will then receive the value of employment at the wage w_i, i.e. $V_E(w_i)$, but has to give up his outside option, i.e. the value of unemployment V_U. Summing over all possible values $i \in \{1, \ldots A\}$ gives the expected payoff of the strategy $\mathbf{c}$.

$$V_S(\mathbf{c}) = V_U + \sum_{i=1}^{A} \prod_{j=i+1}^{A} (1-\psi(c_j))\,\psi(c_i)\,(V_E(w_i) - V_U)\,. \tag{2}$$

Workers choose the application strategy $\mathbf{c}$ that maximizes their expected payoff, i.e.

$$V_S^* = \max_{\mathbf{c}} V_S(\mathbf{c}) \tag{3}$$

Firms with vacancies post a contract c in order to maximize their expected payoff. Denote the distribution of contracts by $F(c)$, its density by $f(c)$ and its support by $\mathcal{F}$. Let $\eta(c)$ denote the hiring probability for a firm posting c, which indirectly depends on the number of firms $v(c)$ posting this specific contract. Then, firms solve

$$\max_{c} V_V(c)\,,$$

where

$$V_V(c) = -k_V - k_R(r-1) + \eta(c)\,V_F(w)\,. \tag{4}$$

The free entry condition implies that firms will continue to enter and post a contract c until the expected payoff of this strategy becomes zero. Hence, in equilibrium we must have

$$V_V(c) \leq 0, \tag{5}$$

and $v(c) \geq 0$, with complementary slackness, for all $c \in \mathcal{C}$.

2.3 Queue Lengths

After specifying the expected payoffs, I now turn to the matching probabilities for the workers and the firms. For this, consider a group of firms all posting same contract c and the group of workers each sending one or more applications to these firms. The anonymity assumption implies that the workers apply to each of the firms with equal probability. Hence, the number of applicants that a firm faces is a random variable. If the number of workers and firms were finite, it would follow a binomial distribution (see Albrecht et al., 2004). With a continuum of agents on both sides of the market, as I consider here, the number of applicants per firm converges to the Poisson distribution. This distribution can by characterized by one parameter, the queue length $\lambda(c)$, which equals the ratio of the total number of applications sent and the total number of firms at a given contract c.

Before I describe the matching probabilities in the model presented here, it is instructive to briefly consider a few special cases, which already have been studied in the literature. For example, consider a world in which all workers send one application and all firms interview one applicant.

Then it can be shown that the queue length $\lambda(c)$ determines the matching probabilities for the workers and the firms. After all, firms can be certain that their job offer will be accepted and hire thus as long as they have at least one qualified applicant, i.e. with probability $\phi\left(1-e^{-\lambda(c)}\right)$. Since there are $\lambda(c)$ as many workers as firms, an individual worker finds a job with probability $\phi\left(1-e^{-\lambda(c)}\right)/\lambda(c)$ (compare to e.g. Acemoglu & Shimer, 2000 and Burdett et al., 2001).

If workers start to send multiple applications, the recruitment technology of the firm becomes more important. If firms can still only interview one applicant, recruitment frictions arise because of congestion. A firm may offer the job to an applicant who rejects the offer because he got a better offer somewhere else. Hence, firms do not only care about the probability to have at least one applicant, but also about the (endogenous) probability that their job offer will get accepted. I denote this probability by $1-\Psi(c)$. Hence, firms now match with probability $\phi(1-\Psi(c))\left(1-e^{-\lambda(c)}\right)$, while the workers' job offer probability still equals $\phi\left(1-e^{-\lambda(c)}\right)/\lambda(c)$. This is the case analyzed by Galenianos & Kircher (2009), which corresponds with $k_R \to \infty$ (i.e. $r=1$) in the model presented in this paper.

On the other hand, if $k_R = 0$ (i.e. $r \to \infty$), firms can freely interview all their applicants. In this case, recruitment frictions do not play a role. The firm may still offer the job to a worker who rejects it because of a better offer somewhere else, but this worker does not cause congestion since the firm can now go back to any other applicants it had. Hence, the firm will match as long as it has at least one qualified applicant without better offers. Kircher (2009) shows that the expected number of such applicants per firm follows a Poisson distribution with mean

$$\mu(c) = \phi(1-\Psi(c))\lambda(c). \tag{6}$$

Clearly, $\mu(c) \leq \lambda(c)$. Both firms and workers only care about the queue length $\mu(c)$ now. The firm's hiring probability equals $1-e^{-\mu(c)}$ and the workers' job offer probability $\phi\left(1-e^{-\mu(c)}\right)/\mu(c)$.

For intermediate values of k_R, firms will typically interview neither one nor all applicants. The actual number of interviews that a firm conducts depends on its number of applicants and its interview capacity, both of which are endogenous random variables. However, the following lemma shows that a simple expression can be derived for the expected number of interviews.

Lemma 1. *A firm posting a contract $c=(r,w)$ and with queue length $\lambda(c)$ will in expectation interview $r\left(1-e^{-\lambda(c)/r}\right)$ applicants.*

Note that the expected number of interviews equals $1-e^{-\lambda(c)}$ for $r=1$ and converges to $\lambda(c)$ for $r \to \infty$, exactly in line with the discussion above. If the firm interviews multiple candidates, it may be able to make a second offer if the first one gets rejected, provided it still has other qualified applicants. However, if the firm runs out of qualified applicants, it will remain unmatched for the remainder of the period, even if it still has applicants which it has not interviewed yet. Hence, congestion caused by recruitment frictions still arises, but to a lesser extent than for $k_R \to \infty$.

As I will show below, the firms' hiring probability and the workers' job offer probability are determined by a third queue length in this case. I denote this queue length by $\kappa(c)$. In order to distinguish between the three different queue lengths, I call $\lambda(c)$ the *gross queue length*, $\mu(c)$ the *net queue length,* and $\kappa(c)$ the *effective queue length*.[13] The effective queue length $\kappa(c)$ is defined as a weighted average of the gross and the net queue length, with $\frac{1}{r}$ and $\frac{r-1}{r}$ being the respective weights, i.e.

$$\kappa(c) = \frac{1}{r}\lambda(c) + \frac{r-1}{r}\mu(c). \tag{7}$$

Note that this definition implies that $\kappa(c) = \lambda(c)$ if $r = 1$ and $\kappa(c) \to \mu(c)$ if $r \to \infty$.

In order to derive expressions for the workers' job offer probability $\psi(c)$ and the firms' hiring probability $\eta(c)$, I consider a firm posting a contract c and I assume for the moment that the gross queue length $\lambda(c)$ and the net queue length $\mu(c)$ are exogenously given. This also pins down the queue length $\lambda(c) - \mu(c)$ of applicants that will never match with the firm. The actual number of applicants of each type follows a Poisson distribution. Given realizations for the number of qualified applicants without better offers and for the number of other applicants, it is straightforward to calculate the matching probabilities. Taking expectations then yields the following result.

Proposition 1. *A firm posting $c = (r, w)$ with gross queue length $\lambda(c)$, net queue length $\mu(c)$, and effective queue length $\kappa(c)$ hires with probability*

$$\eta(c) = \frac{\mu(c)}{\kappa(c)}\left(1 - e^{-\kappa(c)}\right). \tag{8}$$

A worker that applies to the firm gets a job offer with probability

$$\psi(c) = \frac{\phi}{\kappa(c)}\left(1 - e^{-\kappa(c)}\right). \tag{9}$$

By convention, $\eta(c) = 0$ and $\psi(c) = \phi$ if $\kappa(c) = 0$.

The lemma shows that the job offer probability $\psi(c)$ indeed solely depends on $\kappa(c)$. The firm's hiring probability $\eta(c)$ depends on $\kappa(c)$ and, through $\mu(c)$, on the probability $1 - \Psi(c)$ that its job offer will get accepted. Hence, the matching probabilities have the same structure as in the special cases discussed above, i.e. they only depend on r and w through the queue lengths. This similarity in the structure is an important result. It implies that in the derivation of the equilibrium, we can build on the insights gained by Galenianos & Kircher (2009) and Kircher (2009), which means that the model is tractable despite its additional richness. Note further that equations (8) and (9) indeed collapse to the special cases for $r = 1$ (i.e. $k_R \to \infty$) and $r \to \infty$ (i.e. $k_R = 0$).

Note that the scope of proposition 1 and lemma 1 is not limited to directed search models, since they describes the matching technology. The applications that workers have sent during the

[13] Kircher (2009) calls $\mu(c)$ the *effective queue length*, since it is the relevant notion in the world that he describes.

application phase have created a network. The structure that is imposed implies that this network can be fully characterized by the queue lengths. The lemma and the proposition take these variables as given and therefore hold for other application processes, e.g. random search, as well. The application process however matters for the equilibrium expressions for the queue lengths. For example, under random search the gross queue length would be independent of the wage offered by a firm. This is not the case in the directed search setting of this paper. Firms that post higher wages or better recruitment technologies attract more applicants. Formally, the gross queue length is defined by the following integral equation

$$v\int_0^w \int_1^r \lambda\left(\widetilde{r},\widetilde{w}\right) f\left(\widetilde{r},\widetilde{w}\right) d\widetilde{r} d\widetilde{w} = u \sum_{i=1}^{A} G_i\left(c\right) \; \forall c \in \mathcal{C}, \tag{10}$$

where $G_i(c)$ denotes the marginal distribution of G with respect to c_i. The right hand side denotes the total mass of applications that are sent by the workers to contracts no higher than c. The left hand side represents the mass of applications received by firms posting a contract no higher than c. Both masses need to be the same for each possible c.

Equation (6) shows that once the gross queue length $\lambda(c)$ is known, the acceptance probability is the only endogenous variable still required to calculate the net queue length $\mu(c)$. Consider a worker who gets a job offer with his i-th application. Let $\hat{G}_i(\mathbf{c}_{-i};c)$ denote the conditional distribution of the remaining applications, given that the i-th application was sent to contract c. The worker will accept the offer w if and only if all applications sent to higher wages do not result in a job offer. This is the case with probability $\prod_{j=i+1}^{A}(1-\psi(c_j))$. Finally, let $\hat{P}(i;w)$ denote the conditional probability that the worker who got the offer w applied there with his i-th application. Then the acceptance probability equals

$$1-\Psi(c) = \sum_{i=1}^{A} \hat{P}(i;w) \int \prod_{j=i+1}^{A} \left(1-\psi(w_j)\right) d\hat{G}_i\left(\mathbf{c}_{-i};c\right). \tag{11}$$

2.4 Equilibrium Definition and Summary

Before defining the equilibrium, consider the out-of-equilibrium beliefs. It is necessary to specify the firms' beliefs about the workers' response to a deviation in the contract posting. In particular, a deviating firm has to anticipate how many workers will apply, since that will affect the hiring probability. As standard in literature, I use the market utility condition. The market utility is defined as the highest level of utility from search that an unemployed worker can achieve by following the optimal application strategy. This is V_S^*, as defined in equation (3). The market utility condition postulates that workers will apply to the deviating firm in a way that provides them with exactly the same utility as they could have obtained by applying to non-deviating firms. The intuition is that if the deviant would provide the workers with a higher level of utility, all workers would have an incentive to apply to this firm, which would drive up the queue length and reduces the expected payoff. The reverse holds if the deviant offered a lower level of utility. Hence, $\kappa(c)$ has to be such that the market utility condition holds.

I now define an equilibrium as follows:

Definition 1. A static equilibrium is a tuple $\{v, F, G\}$ such that there exists $\kappa(\cdot)$ satisfying

1. Profit Maximization: $V_V(c) = V_V^* \equiv \max_{c'} V_V(c')$ for all $c \in \mathcal{F}$;
2. Free Entry: $V_V^* = 0$ if $v > 0$ and $V_V^* \leq 0$ if $v = 0$;
3. Utility Maximization: $V_S(\mathbf{c}) = V_S^* \equiv \max_{\mathbf{c}'} V_S(\mathbf{c}')$ for all $\mathbf{c} \in \mathcal{G}$.
4. Consistency: $\kappa(\cdot)$ is consistent with equations (6) to (11) and fulfills the market utility condition.

The first and third condition respectively guarantee optimal behavior by firms and workers. Firms with vacancies choose the contract c that maximizes their discounted future payoff. Workers choose the application portfolio $\mathbf{c}$ that maximizes their value of search. The second condition represents free entry. Firms continue to post a certain contract until its value becomes zero. If no firm posts a specific contract, it must be the case that the associated value is non-positive. The last condition ensures that the effective queue length satisfies its requirements.

In the next subsections, I analyze the existence of an equilibrium. I start by analyzing the worker's optimal strategy, after which I turn to the firm's decision problem. I derive the following result.

Summary 1. *An equilibrium exists and has the following properties: A different types of contracts are posted; firms that post a higher wage invest less in recruitment; each worker applies exactly once to each type of contract.*

2.5 Workers' Application Behavior

An unemployed worker observes all contracts posted by the firms and has to decide to which firms he wishes to apply. When considering a firm, the worker is concerned about two things: the wage w that he can earn in the firm and the probability $\psi(c)$ that the firm will hire him. Hence, while observing the contracts, the wage is of direct interest to him, whereas he only cares about the recruitment technology through the job offer probability. This probability is a function of the effective queue length $\kappa(c)$, as shown in equation (9). Hence, everything else being equal, the worker prefers firms with short effective queue lengths. Whether the short queue lengths are the result of only few other workers applying to the same firm, i.e. a low $\lambda(c)$, or of a good recruitment technology, i.e. a high r, is irrelevant to the worker.[14]

Since workers only care about the wage and the job offer probabilities, an equilibrium relation exists between these two variables. This relationship has to be negative, i.e. applications to firms that offer higher wages are less likely to result in a job offer. After all, a worker would not apply to a certain firm if he can get a higher wage with a larger probability somewhere else. So, after observing the contracts, the worker knows in equilibrium what job offer probability is associated with each one of them. He then chooses the application portfolio that maximizes his expected lifetime utility. Hence, his A applications must solve

$$\max_{(c_1,\ldots,c_A)\in\mathcal{F}^A} V_U + \sum_{i=1}^{A}\prod_{j=i+1}^{A}\left(1-\psi(c_j)\right)\psi(c_i)\left(V_E(w_i)-V_U\right). \tag{12}$$

Note that this optimization problem has a recursive structure. The application to a firm offering w_i is only relevant if all applications to higher wage firms, i.e. the applications to wages $w_{i+1},\ldots,w_A$, fail to result in a job offer. This simplifies the derivation of the workers' optimal strategy considerably. Define V_a as the maximum value for the worker provided by the first a applications and let $V_0 = V_U$. Then the worker's optimal choice for the i-th application ($i \in \{1,\ldots,A\}$) solves

$$\max_{c\in\mathcal{F}} \psi(c)\,V_E(w) + (1-\psi(c))\,V_{i-1}. \tag{13}$$

This recursive structure provides several important insights. First, the worker's decision where to apply does not depend on the exact number of applications that he sends. This simplifies the implementation of heterogeneity in the number of applications. I discuss this in more detail in section 3. Second, note that this recursive equation describes the equilibrium relationship between the job offer probability $\psi(c)$ and the wage w. Since $\psi(c)$ only depends on $\kappa(c)$, it also defines a relationship between $\kappa(c)$ and w. This implies that in equilibrium $\kappa(c)$ and $\psi(c)$ only depend on r through w. The intuition for this result is as follows: if two firms offer the same wage but a different recruitment technology, then the one with the better technology is more attractive and will receive more applications. Hence, the gross queue length $\lambda(c)$ will go up until workers are again indifferent between both firms. This will be the case if the effective queue length $\kappa(c)$ is the same.

Now, let $\bar{w}_i$ denote the highest wage that yields utility V_i if the outside option equals V_{i-1}, i.e.

$$\bar{w}_i = \sup\{w \in \mathcal{W} \,|\, \psi(c)\,V_E(w) + (1-\psi(c))\,V_{i-1} = V_i\} \;\forall i \in \{1,\ldots,A\}$$

Additionally, define $\bar{w}_0$ as the lowest wage that would receive applications. The following lemma

[14] Since r does not enter the worker's decision problem directly, the derivation of the worker's optimal search strategy is analogous to the analysis in Galenianos & Kircher (2009) and Kircher (2009). I therefore keep the exposition brief and refer to their papers for additional details. See also Chade & Smith (2006).

states that the values $\bar{w}_i$ bound the interval to which application i can be sent in equilibrium.

Lemma 2. *The optimal application strategy for any worker is to send application $i \in \{1, \ldots, A\}$ to a wage in the interval $\mathcal{W}_i \equiv [\bar{w}_{i-1}, \bar{w}_i]$.*

This lemma confirms that workers do not necessarily want to send all their applications to the firm offering the highest wage. Specifically, workers send their first application to a low wage, although higher wages may be offered as well. The reason is that workers also care about the probability to be hired and a high wage implies a low job offer probability. The lemma also reveals a second characteristic of the equilibrium. Note that the first application provides the worker with a value V_1. The second application is sent to a wage w in the interval $\mathcal{W}_2$. By the definition of $\bar{w}_1$, this application by itself provides a value that is at most V_1 (if $w = \bar{w}_1$) and typically strictly less (if $w < \bar{w}_1$). The reason why workers nevertheless want to send their second application to this interval rather than to the interval $\mathcal{W}_1$ is the fact that the outside option is different. The outside option to the first application is the value of unemployment V_U, whereas failure of the second application still yields $V_1 > V_U$ to the worker. Because of this better safety net, the worker is willing to take more risk with his second application, i.e. apply to a firm that offers a higher wage but a lower job offer probability.

Since lemma 2 restricts the workers' application behavior, it has implications for the job offer probability $\psi(c)$ and the effective queue length $\kappa(c)$. For example, no worker will apply to wages below $\bar{w}_0$. Therefore, the queue length at firms offering such wages equals 0 and the corresponding job offer probability equals ϕ.[15] Firms posting wages in $[\bar{w}_0, \bar{w}_1]$ receive the first application of each worker. This application must yield a utility V_1 to the worker, hence the job offer probability is such that the condition $V_1 = \psi(c) V_E(w) + (1 - \psi(c)) V_0$ holds. In a similar manner conditions for the remaining intervals can be derived. This is summarized in the following lemma.

Lemma 3. *In any equilibrium, the job offer probability $\psi(c)$ satisfies the following conditions*

$$\begin{aligned} \psi(c) &= \phi \quad \forall w \in [0, \bar{w}_0], r \in \mathcal{R} && (14) \\ \psi(c) V_E(w) + (1 - \psi(c)) V_{i-1} &= V_i \quad \forall w \in [\bar{w}_{i-1}, \bar{w}_i], r \in \mathcal{R}, i \in \{1, \ldots, A\}, && (15) \end{aligned}$$

for some tuple $(V_1, \ldots, V_A)$, $V_0 = V_U$, $\bar{w}_0 = V_E^{-1}(V_1)$, and $\bar{w}_i = V_E^{-1}\left(\frac{V_i^2 - V_{i-1}V_{i+1}}{2V_i - V_{i-1} - V_{i+1}}\right)$. By equation (9)*, the conditions* (14) *and* (15) *also determine the effective queue length $\kappa(c)$.*

Given a vector of values $(V_0, \ldots, V_A)$, this lemma describes the workers' application behavior at all possible contracts, including the ones not offered in equilibrium. This will prove to be convenient in the derivation of the firms' optimal strategies in the next subsection.

[15] A deviant applying to a firm posting a wage below $\bar{w}_0$ faces no competition from other workers and will get an offer if he is qualified for the job.

2.6 Firms' Contract Posting

I now analyze the strategy of a firm. Consider a tuple $(V_0,\ldots,V_A)$ and the associated cut-off values $(\bar{w}_0,\ldots,\bar{w}_{A-1})$, which by lemma 3 describe the application behavior of workers. Firms have to decide what contract $c=(r,w)$ to post, taking into account this application behavior as well as the strategies followed by the other firms. The free entry condition implies that firms will continue to choose c until the expected payoff of this choice becomes zero. Hence, in equilibrium we must have

$$-k_V - k_R\,(r-1) + \eta\,(c)\,V_F\,(w) \leq 0,$$

and $v\,(c)\geq 0$, with complementary slackness, for all $c\in\mathcal{C}$.

Lemma 3 implies that we can distinguish between A different types of firms. Specifically, I define a firm to be of type $i\in\{1,\ldots,A\}$ if it posts a wage in the interval $\mathcal{W}_i=[\bar{w}_{i-1},\bar{w}_i]$ and receives the i-th application of a worker.[16] In choosing its optimal strategy, a firm of type i must take into account that a worker may reject its wage offer because he got a better offer at a different firm. Clearly, the probability $\Psi\,(c)$ that the worker rejects the offer is lower for higher type firms and it is zero for the firms offering the highest wages. Moreover, by lemma 3, we know that this probability is constant within the interval $\mathcal{W}_i$, since workers send one application to a given interval only. Individual firms take $\Psi\,(c)$ as given, since it only depends on the behavior of workers and higher type firms. Denote the rejection probability in interval $\mathcal{W}_i$ by Ψ_i. Substituting this variable and the equations (6) and (7) into (8) shows that for all wages in the interval $\mathcal{W}_i$ the hiring probability $\eta\,(c)$ can be written as

$$\eta\,(c)=\frac{r\phi\,(1-\Psi_i)}{1+(r-1)\,\phi\,(1-\Psi_i)}\left(1-e^{-\kappa(c)}\right). \tag{16}$$

Note that the first factor on the right hand side is independent of the exact value of $w\in\mathcal{W}_i$, but only depends on the recruitment technology r. On the other hand, the second factor only depends on the contract through the effective queue length $\kappa\,(c)$.

By substituting (16) into the firm's objective function, we can write the optimization problem of a firm of type i as

$$\begin{aligned}
&\max_{w\in\mathcal{W}_i,r\in\mathcal{R}} \frac{r\phi\,(1-\Psi_i)\left(1-e^{-\kappa(c)}\right)V_F\,(w)}{1+(r-1)\,\phi\,(1-\Psi_i)} - k_V - k_R\,(r-1)\\
\text{s.t.}\quad &\psi\,(c)\,V_E\,(w)+(1-\psi\,(c))\,V_{i-1}=V_i,
\end{aligned}$$

where the constraint specifies the level of utility that the firm has to provide to the workers in order to attract applicants. This optimization problem can be solved by exploiting the fact that there exists

[16] Posting a wage in the interval $[0,\bar{w}_0]$ does not lead to any applications and can therefore not be part of an equilibrium strategy.

a one-to-one relationship between the wage and the effective queue length. Each wage w provides the firm with a certain queue length κ. Rather than by choosing a wage, the firm can therefore also solve this optimization problem by choosing an effective queue length. First, using the worker's payoff function and equation (9), I rewrite the constraint as

$$w(\kappa) \equiv \frac{1}{Y}\left(V_{i-1} + \frac{\kappa}{\phi}\frac{V_i - V_{i-1}}{1-e^{-\kappa}} - \omega_0\right),$$

which specifies the wage that the firm needs to post in order to attract a certain queue κ. Evaluating the firm's payoff function in this expression yields

$$V_F(w(\kappa)) = V_E(1) - V_{i-1} - \frac{\kappa}{\phi}\frac{V_i - V_{i-1}}{1-e^{-\kappa}}$$

Substituting this into the objective function eliminates the wage. Instead, we get an expression that depends on the effective queue length κ and the recruitment technology r only. To be precise, the firm solves

$$\max_{\kappa\in\mathcal{K}_i, r\in\mathcal{R}} \frac{r\phi(1-\Psi_i)\left(\left(1-e^{-\kappa(c)}\right)(V_E(1)-V_{i-1}) - \frac{\kappa}{\phi}(V_i - V_{i-1})\right)}{1+(r-1)\phi(1-\Psi_i)} - k_V - k_R(r-1) \tag{17}$$

where $\mathcal{K}_i \equiv [\bar{\kappa}_{i-1}, \bar{\kappa}_i]$ and $\bar{\kappa}_i$ is the effective queue length that corresponds to $\bar{w}_i$.

We can now show that in equilibrium firms of different types will never offer the same wage. As a result, wage dispersion is a fundamental characteristic of any equilibrium.

Lemma 4. *In any equilibrium, for all $i, j \in \{1,\ldots,A\}$, firms of type i and $j \neq i$ do not post the same contract. In particular, they do not post the same wage.*

The fact that in equilibrium κ does not directly depend on r makes it possible to maximize the objective function over both variables independently. One can show that the objective function is strictly concave in both $\kappa \in \mathcal{K}_i$ and $r \in \mathcal{R}$. This implies that all firms of type i choose the same optimal queue length κ_i^* and recruitment technology r_i^*, which are determined by either the solution to the first order conditions or the boundary values. I summarize this in the following lemma.

Lemma 5. *Consider a firm of type $i \in \{1,\ldots,A\}$. In any equilibrium, the firm's optimal effective queue length κ_i^* equals*

$$\kappa_i^* = \max\left\{\log\frac{\phi(V_E(1)-V_{i-1})}{V_i - V_{i-1}}, \bar{\kappa}_{i-1}\right\} \tag{18}$$

and the firm's optimal recruitment technology r_a^* is given by

$$r_i^* = \max\left\{\sqrt{\frac{1-\phi(1-\Psi_i)}{\phi(1-\Psi_i)}\frac{k_V - k_R}{k_R}}, 1\right\} \tag{19}$$

This lemma provides several important insights. First of all, note that in equilibrium exactly A different contracts will be offered. Hence, $\mathcal{F} = \{c_1^*, \ldots, c_A^*\}$, where w_i^* is the wage that corresponds to κ_i^*. Workers will send one application to each of the A types of contracts. As the lemma shows, firms that post low wages and are therefore likely to be rejected by applicants (i.e. Ψ_i is large), will buy more interview capacity than firms that offer higher wages. Hence, firms face a trade-off between paying a high wage and investing a lot in recruitment. Further, equation (19) reveals that firms also invest more in recruitment if only few candidates are qualified (i.e. ϕ is small), or if the cost per interview k_R is small compared to the fixed cost of opening a vacancy k_V, which is in line with intuition.

Note that $\bar{\kappa}_0 = 0$, which implies that the optimal queue length κ_1^* for the lowest wage firms is given by the interior solution $\log \frac{\phi(V_E(1)-V_0)}{V_1-V_0}$. In general, it is not straightforward to derive whether the interior or the boundary solution prevails for κ_i^*, $i \in \{2, \ldots, A\}$. Only in special cases such a result can be obtained. Galenianos & Kircher (2009) show that in a model with one interview per firm ($k_R \to \infty$), the optimal queue length equals the boundary value $\bar{\kappa}_{i-1}$ for $i \in \{2, \ldots, A\}$. In a similar model but with an unlimited number of interviews ($k_R = 0$), the optimal queue length for a type i firm is defined by the solution to the first order conditions, as shown by Kircher (2009). Such a general result cannot be derived for the model presented here. Numerical simulations however indicate that for a large range of parameter values for k_R, the boundary solutions arise. Only for k_R very close to zero, the interior solution may occur.

Each optimal effective queue length κ_i^* is associated with a gross queue length λ_i^* and a net queue length μ_i^*. By combining equations (6) and (19), we can express the optimal recruitment technology as a function of these queue lengths:

$$r_i^* = \max\left\{\sqrt{\frac{\lambda_i^* - \mu_i^*}{\mu_i^*}\frac{k_V - k_R}{k_R}}, 1\right\}.$$

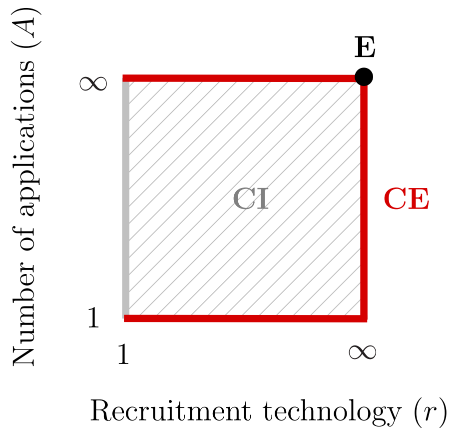
2.7 Equilibrium Outcome and Efficiency

The previous subsections have derived some important characteristics of the equilibrium. In particular, the optimal recruitment technology for a firm posting the i-th contract has been determined. After substituting this expression into the firm's problem, the model is isomorphic to Galenianos & Kircher (2005, 2009). Hence, an equilibrium can be shown to exist.

Proposition 2. *A static equilibrium exists. It satisfies the properties derived in lemma 2 to 5.*

As already shown by Galenianos & Kircher (2005, 2009), it is not straightforward to prove uniqueness of the equilibrium. Numerical simulations however indicate that the equilibrium is in fact unique for a wide range of parameter values.

For the efficiency properties of the equilibrium we can also build on the insights from the existing literature. I summarize them here and in figure 2.



Efficiency properties of the equilibrium. The vertical axis shows the number of applications A that workers send out. The horizontal axis displays the recruitment technology r of the firms. E = efficient (Walrasian) equilibrium, CE = constrained efficient equilibrium, CI = constrained inefficient equilibrium.

Figure 2: Efficiency

Summary 2. *The equilibrium exhibits efficient (Walrasian) exchange for $A \to \infty$ and $k_R = 0$ (i.e. $r \to \infty$). If $A = 1$, $A \to \infty$ or $k_R = 0$, the equilibrium is constrained efficient. For other combinations of A and k_R the equilibrium is constrained inefficient.*

It is well-known that standard directed search models with one application per worker are constrained efficient in the sense that a social planner subject to the same frictions as the decentralized market cannot increase welfare (see e.g. Moen, 1997). This changes when workers send multiple applications. Albrecht et al. (2006) and Galenianos & Kircher (2009) show that if firms can contact only one applicant, the decentralized market equilibrium is not constrained efficient. The inefficiencies however disappear if $A \to \infty$, as follows from Julien et al. (2000) and Kaas (2010).[17] Kircher (2009) discusses that for general A constrained efficiency is only obtained if firms can contact all applicants, i.e. in terms of the current model $r \to \infty$ or, equivalently, $k_R = 0$.

The intuition for these results is as follows. Recall that while applying workers only care about the effective queue length $\kappa(c)$ at a given firm. Hence, if a firm changes the wage it posts, workers adjust their application behavior until the new effective queue length at the deviant is such that the market utility condition holds. However, firms do not only care about the arrival rate of applicants, but also about whether one of the contacted applicants accepts the job offer. The probability of retaining a contacted worker does not depend on the number of applicants that the firm has received, so in general the firm is not able to control both dimensions with the one pricing instrument that it

[17] Both papers consider a world in which firms can contact one worker only. It is however straightforward to show that their efficiency results extend to environments in which multiple job offers are possible.

has, i.e. the wage that it posts. Only in special cases such efficient pricing is possible. If $A = 1$, job offers will always be accepted and firms only need to focus on the arrival rate. Reversely, if $A \to \infty$, the wage set by the firm plays no role in the arrival rate of applicants. Finally, if $r \to \infty$, the probability to have a job offer accepted directly depends on the queue length, since the firm can offer the job to all workers until one accepts. In terms of the model in this paper, $\kappa(c)$ and $\mu(c)$ coincide in that case, making efficient pricing possible.[18]

To conclude, note that if both the recruitment technology r and the number of applications tend to infinity, i.e. $r \to \infty$ and $A \to \infty$, the Walrasian outcome is obtained, as shown by Kircher (2009). This suggests a natural procedure to investigate the magnitude of the frictions in the model. Given parameter values, total social surplus in the market equilibrium can be calculated. This can be compared to the surplus generated in the Walrasian outcome. The difference is then a measure for the loss in surplus due to the frictions. I follow this approach in section 4.6 after discussing and estimating the dynamic version of the model.

3 Dynamic Model

In this section, I extend the simple static model of the previous subsection to a dynamic framework that can be confronted with the data. I endogenize the workers' values of employment and unemployment, the unemployment rate u, and the workers' search intensity. I further introduce worker heterogeneity to avoid that too much of the wage dispersion in the data would be attributed to the frictions.

3.1 Setting

The key elements of the model are the same as in the previous section. However, a few elements differ. I discuss each one of them in this subsection.

First, time is discrete and continues forever. Both firms and workers maximize the sum of expected future payoffs, discounted at rate $\beta_0 \in (0,1)$. Firms are infinitely-lived, but an exogenous fraction τ of the workers retires at the end of each period. Retired workers have a zero payoff forever after and are replaced by new unemployed workers in order to keep the size of the labor force constant.[19] Besides retirement, job destruction is a second source of separations. At the end of each period, an exogenous fraction $\delta \in (0,1)$ of the matches gets dissolved. Employed workers hit by the shock flow back to unemployment. In the next period they can - like all other unemployed workers - search for a new job. Hence, a worker who is employed in the current production phase

[18] See section 3.3 in Galenianos & Kircher (2009) and section VI in Kircher (2009) for a detailed discussion of this issue.

[19] Retirement is important to keep the cross-sectional distribution of worker types stationary. See below for more details.

may be employed at a different firm in the production phase of the next period after experiencing a very short unemployment spell. Discreteness of the data may cause this to look like a job-to-job transition, even though the model does not allow for on-the-job search in the classic sense.[20] The labor market is assumed to be in steady state, implying that the unemployment rate at the beginning of the production phase must satisfy

$$u\Psi_0 = (1-u)(\delta+\tau-\delta\tau)(1-\Psi_0),$$

where Ψ_0 denotes a worker's matching probability.

Another new element is that the number of applications per worker is no longer exogenously given. Instead, workers choose how many firms they wish to contact. Since optimization over a discrete variable is cumbersome, I choose a specification that is similar to the one described in for the firms' recruitment technology.[21] Unemployed workers choose a search intensity $\alpha \in \mathbb{R}_+$ which is a measure for the time they allocate to searching a job. The search intensity determines the expected number of applications that a worker can send. However, as with the interview capacity, there is a stochastic element as well, e.g. because a particular application takes more or less time than initially expected. Hence, the actual number of applications follows a certain distribution that depends on α. I denote the fraction of workers with search intensity α that can send $a \in \mathbb{N}_0$ applications by $p(a|\alpha)$ and the cumulative distribution function by $P(a|\alpha) = \sum_{i=0}^{a} p(i|\alpha)$. In the empirical part I will assume a particular functional form for $p(a|\alpha)$, but in this section I keep the analysis as general as possible by only making three assumptions: i) first order stochastic dominance in α to make α interpretable as a search intensity; ii) $P(1|\alpha) < 1$ for all $\alpha > 0$ to generate simultaneous search; iii) the support of the distribution is bounded to avoid an infinite number of contracts. In particular, workers send at most A applications per period, where A is a potentially large but finite integer. I denote the equilibrium distribution of search intensities by $H(\alpha)$ and its support by $\mathcal{H}$.

A last new feature is worker heterogeneity. While firms continue to be homogeneous, I allow workers to differ in their human capital, which will determine the amount of output that is created in a match. Workers enter the labor market with human capital equal to y_0, which is a draw from a distribution $F_y(y)$ with support equal to (a subset of) $(0,\infty)$. During his life, a worker experiences deterministic growth of his human capital according to $y_{t+1} = (1+\theta)\,y_t$.[22] I assume that $\beta_0(1-\tau)(1+\theta) < 1$.

Without further assumptions, the presence of heterogeneity among workers would render the model intractable. The reason is that when firms face applicants with different productivity levels, the firm's decision problem becomes much more complex. Always interviewing the most skilled

[20] See Menzio & Shi (2009) for on-the-job search in a directed search model with one application per period. The combination of simultaneous search and on-the-job search is computationally challenging.

[21] See also Kaas (2010) for a similar setup.

[22] It is important to allow for human capital growth since a Mincer equation reveals a strong influence of a worker's age on his wage. See section 4.2 for details.

applicant is generally not the optimal strategy, since that implies that the competition for these workers will be much higher than for slightly less productive workers. Instead, firms will need to diversify and interview multiple types of applicants with positive probability, while rejecting applicants with much lower productivity. This implies that the firm's strategy is a function of its total pool of applicants, which can be composed in infinitely many ways.

To maintain tractability, I therefore make a few important assumptions. First, a worker's human capital y is public information and a different submarket exists for each type, such that workers with different skills do not compete with each other.[23] I impose this structure exogenously, but it can be generated endogenously by assuming that firms that create a vacancy need to choose and post a production technology that allows them to match with one specific type of workers only. I further assume that there is a continuum of workers of each type, such that the expressions for the matching probabilities given in proposition 1 hold in each submarket.

Finally, I assume that all relevant periodical costs and payoffs are scaled linearly by the worker's type. This guarantees that all Bellman equations are linear in y and therefore that the exact same contracts $((r_1,w_1),\dots,(r_A,w_A))$ are posted in each submarket. Hence, the structure is as follows. In each submarket, the output of a match equals the amount of human capital y possessed by the worker. The firm pays a fraction w of the output to the worker and keeps the remainder. Hence, the periodical payoff of the worker is wy, while the firm obtains $(1-w)y$. Creating a job for a highly productive worker is more costly than creating a job for a low-skilled worker, such that a firm opening a vacancy in submarket y incurs an entry cost $k_V y$. Likewise, the recruitment cost equals $k_R(r-1)y$. Unemployed workers receive a payoff from household production and/or unemployment benefits equal to hy, where $h \in (0,1)$, and incur a search cost $k_A \alpha y$.

3.2 Payoffs

In the static model, the workers' and firms' payoff functions from being matched were taken as exogenous. In this section I endogenize them by deriving Bellman equations. The assumption that all costs and payoffs, as well as the change in human capital, are proportional to y implies that the Bellman equations are proportional to y as well. To simplify notation, I therefore specify the Bellman equations in efficiency units, i.e. per unit of y.

Consider first a worker's value $V_E(w)$ of being employed at a wage w at the beginning of the production stage. The worker will have a periodical payoff equal to his wage. At the end of this period, he will lose his job without retiring with probability $\delta(1-\tau)$. In that case he will be searching for a new job at the beginning of the next period and receive his value of search $(1+\theta)V_S$, which I derive below. If the match does not get dissolved, the worker obtains $(1+\theta)V_E(w)$. Hence,

[23] See van den Berg & Ridder (1998) for a similar approach.

the the worker's value function satisfies the following Bellman equation

$$V_E(w) = w + \beta((1-\delta)V_E(w) + \delta V_S),$$

where $\beta = \beta_0(1-\tau)(1+\theta)$. Note that $V_E(w)$ indeed satisfies the linear structure assumed in the previous section, with $Y = \frac{1}{1-\beta(1-\delta)}$ and $\omega_0 = \frac{\beta\delta V_S}{1-\beta(1-\delta)}$.

Next, consider the search decision of a worker looking for a job. In the previous section, the optimal application portfolio was derived for an exogenous number of applications A. Now, workers jointly choose their search intensity α and the optimal application portfolio $\mathbf{c}$, which results in an equilibrium distribution of workers' strategies $J(\alpha, c)$ with support $\mathcal{J}$. The recursive structure (13) of the application problem however implies that an individual worker's choice of which jobs to apply to, i.e. his choice of $\mathbf{c}$, is independent of his choice of search intensity α. Hence, we can write $J(\alpha, c) = G(\mathbf{c})H(\alpha)$ with $\mathcal{J} = \mathcal{G} \times \mathcal{H}$. Consequently, the workers' value of search can can now be calculated by conditioning on the number of applications a first and subsequently taking the expectation over a.[24] Hence, the updated version of equation (2) in efficiency units equals

$$V_S(\alpha, \mathbf{c}) = V_U + \sum_{a=1}^{A} p(a|\alpha) \sum_{i=1}^{a} \prod_{j=i+1}^{a} (1-\psi(c_j))\psi(c_i)(V_E(w_i) - V_U). \tag{2'}$$

Workers maximize this expression over α and c, such that the value of search equals

$$V_S = \max_{\alpha, \mathbf{c}} V_S(\alpha, \mathbf{c}) - k_A \alpha.$$

Using this, we can now derive the Bellman equation for the workers' value of unemployment. During the production phase an unemployed worker receives a payoff h. In the next period, he obtains the value of search $(1+\theta)V_S$ if he does not retire. Hence,

$$V_U = h + \beta V_S.$$

Next, I turn to the firm's Bellman equations. A firm employing a worker at a wage w obtains a periodical payoff $1-w$. The match between the worker and the firm continues in the next period

[24] Likewise, the gross queue length $\lambda(c)$ is now defined by

$$v\int_0^w \int_1^r \lambda(\tilde{r}, \tilde{w}) f(\tilde{r}, \tilde{w}) d\tilde{r} d\tilde{w} = u \int_{\mathcal{H}} \sum_{i=1}^{A} (1 - P(i-1|\alpha)) G_i(c) dH(\alpha) \ \forall c \in \mathcal{C}, \tag{10'}$$

and the acceptance probability equals

$$1 - \Psi(c) = \sum_{i=1}^{A} \sum_{a=i}^{A} \mathcal{P}(a, i; w) \int \prod_{j=i+1}^{a} (1 - \psi(w_j)) d\hat{G}_i(\mathbf{c}_{-i}; c), \tag{11'}$$

where $\mathcal{P}(a, i; w)$ denotes the conditional probability that the worker who got the offer w sent there his i-th out of a applications.

if no job destruction shock or retirement takes place. Otherwise, the firm obtains the value of a vacancy, which in equilibrium equals zero due to free entry. Therefore,

$$V_F(w) = 1 - w + \beta(1-\delta)V_F(w),$$

which is consistent with the specification $V_F(w) = (1-w)Y$ of the previous section. Finally, a firm with a vacancy posting a contract c, gets a payoff $V_V(c)$ as specified in equation (4).

3.3 Equilibrium

The updated equilibrium definition is as follows:

Definition 2. A dynamic equilibrium is a tuple $\{u, v, F, G, H\}$ such that there exists $\kappa(\cdot)$ satisfying

1. Profit Maximization: $V_V(c) = V_V^* \equiv \max_{c'} V_V(c')$ for all $c \in \mathcal{F}$;
2. Free Entry: $V_V^* = 0$ if $v > 0$ and $V_V^* \leq 0$ if $v = 0$;
3. Utility Maximization: $V_S(\alpha, \mathbf{c}) = V_S^* \equiv \max_{\alpha', \mathbf{c}'} V_S(\alpha', \mathbf{c}')$ for all $\mathbf{c} \in \mathcal{G}$ and $\alpha \in \mathcal{H}$.
4. Consistency: $\kappa(\cdot)$ is consistent with equations (6) to (9), (10′) and (11′), and fulfills the market utility condition.
5. Steady State: $u\Psi_0 = (1-u)(\delta + \tau - \delta\tau)(1-\Psi_0)$ with

$$\Psi_0 = \int_{\mathcal{G}} \int_{\mathcal{H}} \left(1 - \sum_{a=0}^{A} p(a|\alpha) \prod_{i=1}^{a} (1 - \psi(c_i))\right) dH(\alpha)\, dG(c).$$

The first two conditions concern the firm's behavior and are the same as for the static equilibrium. The utility maximization condition now includes the choice of search intensity. The fourth condition guarantees consistency of $\kappa(\cdot)$ with the updated definitions of $\lambda(c)$ and $\Psi(c)$. The last condition equates the number of workers entering and leaving unemployment.

It is straightforward to see that the new model elements introduced in this section do not fundamentally change the structure of the equilibrium. Even though the current framework is dynamic and contains worker heterogeneity, workers and firms in each submarket solve the static model of the previous section period by period. The equilibrium properties derived in the static version held for any $V_E(\cdot)$, V_U, and u, so they continue to hold for the endogenized versions of these objects. The following corollary summarizes this.

Corollary 1. *In any dynamic equilibrium, lemma 2 to 5 hold.*

Hence, the heterogeneity in the number of applications per worker change the exact equilibrium contracts that are offered and the number of agents trading at each of the contracts, but do not change the fact that i) still A different contracts are offered; ii) workers applying A times send one application to each type of contract; and iii) unemployed who send $a < A$ applications, apply to the a types of contracts offering the lowest wages. Consequently, a worker choosing his search intensity solves $\max_{\alpha} V_S(\alpha) - k_A \alpha$, where

$$V_S(\alpha) = \sum_{i=0}^{A} p(i|\alpha) V_i. \tag{20}$$

The solution to this optimization problem is unique if $V_S(\alpha)$ is strictly concave in α. Note that $V_S(\alpha)$ can be rewritten as

$$V_S(\alpha) = V_0 + \sum_{i=1}^{A} (1 - P(i-1|\alpha))(V_i - V_{i-1}).$$

Hence, strict concavity requires

$$\frac{d^2 V_S(\alpha)}{d\alpha^2} = -\sum_{i=1}^{A} \frac{d^2 P(i-1|\alpha)}{d\alpha^2}(V_i - V_{i-1}) < 0.$$

In that case, all workers choose the same level of search intensity, as determined by the first order condition of the optimization problem. Because of the boundedness of $V_S(\alpha)$ concavity holds for sufficiently large α. However, in general it is not obvious that the second derivative is strictly negative for all α. In particular, it may be positive for low values of α, as pointed out by Kaas (2010) in a similar setting. In that case, the equilibrium may require that the workers mix between $\alpha = 0$ and the α implied by the first order condition. Since the sign of the second derivative depends on the endogenous objects V_i, it is not obvious which distribution functions give strict concavity. In the empirical part, I will therefore choose a particular distribution, assume that the condition is satisfied in the equilibrium calculation, and verify this assumption ex post.

Endogenous search intensity may be a source of multiplicity of the equilibrium. For example, if the cost of search k_A is low, workers will typically apply a lot. This forces firms to offer high wages, which may provide workers with an incentive to indeed send many applications. The reverse holds if the cost of an application is high. Such multiplicity is a potential problem in the estimation. I solve this by estimating the search intensity α rather than the search cost k_A, as I will explain in section 4.4.

To complete the description of the equilibrium, consider the unemployment rate u. Using the intermediate value theorem, it is straightforward to check that given the workers' and firms' optimal strategies, there is a unique value of u that satisfies the fifth condition of the equilibrium definition above. With this result, one can now proof the existence of a dynamic equilibrium.

Proposition 3. *A dynamic equilibrium exists. It satisfies the properties derived in lemma 2 to 5.*

4 Estimation

4.1 Data

The model discussed above has implications for several key variables at the firm level. For given parameter values, it determines - in expectation - 1) the number of applicants that a firm has, 2) the number of interviews that the firm conducts, 3) the number of job offers that the firm makes, 4) the wage that the firm pays, 5) the amount of time that the firm spends on screening and 6) the firm's vacancy duration. Unfortunately, the availability of microdata containing information on each of these variables is very limited. To my knowledge, the only exception is the Employment Opportunities Pilot Projects (EOPP) data set, which I therefore use in this paper. It contains the information from a two-wave longitudinal survey that was designed to evaluate the impact of US labor market programs developed by the Office of the Assistant Secretary for Policy, Evaluation and Research, and funded by the Department of State's Employment and Training Administration.[25]

The Employment Opportunities Pilot Project was introduced in the summer of 1979. It consisted of an intensive job search program combined with a work and training program, organized at 10 pilots sites throughout the country. Each pilot site consisted of a small number of neighboring counties. The program was aimed at unemployed workers with a low family income, and tried to place eligible workers in private-market jobs at one of the pilot sites during a job search assistance program. If these attempts failed, the worker was offered a federally-assisted work or training position. The program was in full operation by the summer of 1980, but was phased out during 1981 by the new Administration.

In order to evaluate the program, a survey was sent to firms at the ten pilot sites and twenty control sites which where selected on the basis of their similarity to the pilot sites. Table 1 lists all sites included in the final data set.[26] The first wave of the survey took place between March and June 1980. The second wave was conducted between February and July 1982 and aimed to re-interview all respondents to the first survey. The response rate was about 70%.

The data set is not representative for the US labor market as a whole. Workers in the sample are relatively young and due to the nature of the labor market program, low incomes are overrepresented.[27] Moreover, the pilot sites are disproportionally concentrated in Gulf Coast cities and underrepresent cities in the Northeast of the US. Further, the data set does not include workers in

[25] Several other authors have used the EOPP data set, e.g. Barron et al. (1985), Barron et al. (1987), and Burdett & Cunningham (1998). See these papers for additional information about the survey. The papers are different from mine in the sense that they do not use a two-sided equilibrium search model to analyze the data.

[26] A few sites were excluded from the final data set for various exogenous reasons.

[27] Note that this is not a concern if we want to study the effect of frictions on the desirability of active labor market programs. These programs are often targeted at unemployed workers, among which young, low wage workers are typically overrepresented as well. Nevertheless, it would certainly be interesting to estimate the model on a more representative sample of workers or on data from specific, well-defined submarkets, like e.g. the job market for economists, once the data is available. For future research in this area, surveys like the one conducted Hall & Krueger (2008) could be an important source of information.

government and non-profit organizations. The probability for a firm in one of the sites to be included in the survey depended on its size and location and varied between 0.006 for the smallest establishments to close to 1 for establishments with more than 200 employees (see Barron et al., 1985, for more details). The data set contains sample weights to account for the heterogeneity in the sampling probability.

The survey sent to the firms included various questions on the recruitment process for the last hired worker. The second survey was a lot more comprehensive than the first one. In particular, the first survey did not include a question on the number of applicants per vacancy. For this reason, I only utilize the 1982 data in this paper. This data set contains information on all six variables listed above for the firm's most recent hire, conditional on the hiring taking place between January 1980 and September 1981.[28] Further, the data set contains a number of worker characteristics, like gender, age, education level, sector, location, and occupation.[29] I use these variables to control for productivity differences, as I explain in more detail in the next subsection.

Hence, each observation in the data is a tuple $(z_A, z_I, z_O, z_W, z_T, z_V, x)$, respectively denoting 1) the number of applicants z_A at a given firm, 2) the number of interviews z_I the firm conducted, 3) the number of job offers z_O it made, 4) the wage z_W it paid to the worker it hired, 5) the amount of time z_T that the firm spent screening applicants, 6) the time z_V that elapsed between posting and filling the vacancy, and 7) the characteristics x of the worker. I restrict the sample by selecting workers who are 18 years or older and by omitting observations with missing or unreliable values.[30] Information on the number of applicants, interviews and job offers is missing much more often than information on the wages and the worker characteristics.[31] I therefore create two samples: a large sample with 2156 observations which I will use to control for worker heterogeneity in the next subsection, and a smaller subset with 640 observations which I will use for the estimation of the frictional model.

Table 1 displays some descriptive statistics for both samples.[32] The table shows that there is considerable variation in the number of applicants and the number of interviews per firm. The variation in the number of job offers is somewhat smaller: most firms only need to make one job offer to hire a worker. Firms spend on average about 6 hours on screening workers and fill vacancies

[28] The data set contains information on both the last subsidized and the last non-subsidized hire. I restrict the sample to the latter group.

[29] Although sometimes seen as job characteristics, I choose to interpret the last three variables as worker characteristics. Workers have a choice regarding the sector, the region and the occupation in which they work, hence their choice contains information about their productivity. The sector is coded according to the SIC 1972 (4 digits) and occupations are coded according to the DOT 1977. I use the first digit of both.

[30] I omit outliers (top 0.5% of the distribution) in the number of applications, interviews and job offers. This does not affect the results.

[31] The data set includes two wage variables. The first one reports the wage of the individual at the survey date, but is by definition only available if the workers is still employed at the firm. The second wage variable, which reports the wage that the firm would offer to a new hire in the same position at the moment of the survey. Conditional on observing both, the correlation between the two variables is more than 90%. I choose the second variable since it is observed more often and because it excludes the influence of wage increases since the moment of hiring.

[32] Section 4.5 presents some histograms of the data, while discussing the goodness of fit of the model.

relatively quickly. The average vacancy duration is 17 days and about 90% of the firms find a worker within a month.

4.2 Worker Heterogeneity

The data does not directly report the workers' level of human capital. I therefore assume that each worker's human capital is a function of his observed characteristics x and a random component $\varepsilon_W \sim N\left(-\frac{\sigma_W^2}{2}, \sigma_W^2\right)$ in the following way

$$\log y = \gamma_0 + x'\gamma + \varepsilon_W,$$

where γ_0 is a constant and γ is a vector of coefficients common to all workers.[33] In the estimation, x will include the worker's gender, his age, his education level, as well as indicators for the sector, the location and the type of occupation.

As described in the previous section, y is assumed to be public knowledge to all agents in the model. Hence, the realization of ε_W is unknown to the econometrician, but known by all workers and firms. Given this structure, a worker with human capital y who matches with a firm posting (r_j, w_j) gets a wage equal to $z_W = w_j y$. Hence, it must hold that

$$\begin{aligned} \log z_W &= \log w_j + \log y \\ &= \log w_j + \gamma_0 + x'\gamma + \varepsilon_W. \end{aligned} \tag{21}$$

Equation (21) is a standard Mincer equation, augmented with the term $\log w_j$, which captures the worker's success in the matching process. This term depends on the parameters of the frictional model, but only serves as the intercept in the equation and is orthogonal to x. The coefficients γ can therefore be estimated directly by regressing $\log z_W$ on x and a constant. Subsequently, the residuals $z_R = \log z_W - x'\widehat{\gamma}$ can be calculated and can be used to estimate γ_0 and the variance σ_W^2 of the random productivity component along with the other parameters of the frictional model, as I will do in the next subsection.[34]

I present the results of the productivity regression in table 2. The estimates for γ are very much in line with what is commonly found in this type of Mincer regressions. The wage is increasing in the education level and is higher for men than for women. Age has a statistically significant but quantitatively limited effect on the wage. The estimated coefficient corresponds to an annual growth

[33] Setting the mean of ε_W equal to $-\frac{\sigma_W^2}{2}$ guarantees that the measurement error does not distort the mean of the human capital, i.e. $E[y] = \exp(\gamma_0 + x'\gamma)$. Estimating the model with $E[\varepsilon_w] = 0$ does not yield different results.

[34] The equilibrium in the frictional model actually depends on γ through the coefficient for age. However, in the estimation of the frictional model I will exogenously fix β after which the dependence disappears. See the next subsection for details. The sequential approach also yields larger standard errors for $\widehat{\gamma}$ than joint estimation, but since the productivity coefficients γ are not the main parameters of interest the computational advantage of a sequential approach outweighs the loss in precision. The estimation of the frictional model is unaffected.

Variable	Mean	Std.dev.	Variable	Mean
Recruitment variables			*Education*	
Number of applicants	14.06	22.66	Grade school	0.019
Number of interviews	4.97	6.80	Some high school	0.136
Number of offers	1.20	0.60	High school	0.543
Wage	4.11	1.60	Some college	0.169
Hours of screening	6.16	7.48	College	0.056
Vacancy duration	17.48	27.03	Other education	0.076
Age			*Location*	
Age	26.74	9.31	Columbus, OH	0.079
			Corpus Christi, TX	0.052
Gender			Baton Rouge, LA	0.038
Male	0.519		Mobile, AL	0.044
Female	0.481		Pike, KY	0.005
			Weld, CO	0.014
Sector			Marathon, WI	0.019
Extraction	0.014		Balance of state, WA	0.019
Construction	0.109		Balance of state, MO	0.011
Manufacturing	0.076		Toledo, OH	0.055
Transport	0.056		Cincinnati, OH	0.076
Wholesale	0.121		San Antonio, TX	0.117
Retail	0.277		Beaumont/Port Arthur, TX	0.027
Financial	0.084		Birmingham, AL	0.100
Service	0.263		Buchanan/Dickenson, VA	0.003
			Alamosa, CO	0.024
Occupation			Outagamie, WI	0.027
Prof. / Techn. / Managerial	0.059		Skagit/Whatcom, WA	0.024
Clerical / Sales	0.460		St. Francoise, MO	0.015
Service	0.128		New Orleans, LA	0.052
Agricultural	0.005		Lake Charles/Lafayette, LA	0.038
Processing	0.018		Pensacola, FL	0.024
Machine trades	0.081		Harlan, KY	0.004
Benchwork	0.028		Logan/El Paso, CO	0.034
Structural work	0.112		Winnebago, WI	0.022
Miscellaneous	0.111		Skamania, WA	0.014
			Grundy, MO	0.007
			Dayton, OH	0.059

Descriptive statistics for the variables used in the estimation. The values for the number of applicants, interviews and offers are based on 640 observations. For all other variables the sample size is 2156 observations. Wages are measured in 1982 dollars. 1 dollar in 1982 has the same buying power as $2.22 in 2009 (Bureau of Labor Statistics, 2009). Age and experience are measured in years. The gender, education, sector, occupation, and location variables are binary.

Table 1: Descriptive statistics

in human capital of about 0.66%. The relatively high coefficients for the extraction and construction industry are present in the raw data as well and is most likely the result of the specific sample of workers being selected. Some modest regional variation in human capital exists.

4.3 Parametrization

Despite its richness along several dimensions, the EOPP data does not contain sufficient information for non-parametric identification of all elements of the model. I will therefore choose some parameter values and functional forms exogenously. I describe these assumptions in this subsection.

Application Distribution. With respect to the number of applications, I assume a truncated Poisson distribution with the search intensity α as its parameter. Hence, the fraction of workers $p(a)$ that sends a applications satisfies:

$$p(a|\alpha) = \begin{cases} e^{-\alpha}\frac{\alpha^a}{a!} & \text{for } a \in \{0,\ldots,A-1\} \\ 1-\sum_{i=0}^{A-1} e^{-\alpha}\frac{\alpha^i}{i!} & \text{for } a = A, \end{cases}$$

where A is a finite integer and $\alpha > 0$. Note that if $\alpha \to 0$, all workers send zero applications. On the other hand, if $\alpha \to \infty$, all workers apply A times. The distribution converges to a regular Poisson distribution for $A \to \infty$. The theoretical analysis in the previous sections shows that calculation of the equilibrium requires solving for A different wage levels. In order to avoid dimensionality problems, I will set A equal to 15 in the estimation.

A Poisson distribution is a natural choice, since it is the discrete time equivalent of the Poisson process typically used in continuous time models. Further, it has the attractive feature that it has discrete support while depending on a continuous parameter. Estimation of such a continuous parameter is significantly less cumbersome than estimating a discrete number of applications. As mentioned before, endogenous search intensity may be a source of multiplicity of the equilibrium. In order to avoid problems with the estimation, I therefore do not estimate the search cost k_A directly, but I treat the search intensity α as a parameter and estimate that instead. The search cost can then be calculated ex post by equating it to the marginal benefit of an extra unit of search intensity.

Period Length. In theory, the data contains some information on the length of a period. To see this, suppose that the number of applications that workers send is deterministic. Then, compare a period of one week in which workers send one application to a period of two weeks in which workers send two applications. In both cases, the gross queue length λ is the same given a fixed number of vacancies, but other equilibrium outcomes are very different. In the first case, no congestion occurs, so firms have less incentive to interview many candidates. Moreover, no wage dispersion will be present in the data after controlling for observed and unobserved productivity differences. This is all not true in the second case. However, identification along these dimensions cannot be established

	Estimate	Std.err.		Estimate	Std.err.
Demographics			*Location*		
Male	0.165	0.023	Corpus Christi, TX	-0.004	0.038
Age	0.007	0.001	Baton Rouge, LA	0.017	0.042
			Mobile, AL	-0.036	0.047
Education			Pike, KY	0.073	0.071
Some high school	-0.092	0.105	Weld, CO	-0.049	0.053
High school	0.022	0.104	Marathon, WI	0.168	0.079
Some college	0.077	0.107	Balance of state, WA	0.124	0.049
College	0.199	0.114	Balance of state, MO	-0.036	0.040
Other education	0.037	0.109	Toledo, OH	0.083	0.075
			Cincinnati, OH	0.050	0.048
Sector			San Antonio, TX	-0.052	0.050
Extraction	0.348	0.053	Beaumont/Port Arthur, TX	0.049	0.071
Construction	0.186	0.050	Birmingham, AL	0.028	0.043
Manufacturing	0.103	0.043	Buchanan/Dickenson, VA	0.119	0.076
Transport	0.038	0.047	Alamosa, CO	0.015	0.068
Wholesale	-0.003	0.034	Outagamie, WI	0.013	0.047
Retail	-0.100	0.026	Skagit/Whatcom, WA	0.172	0.055
Financial	0.080	0.042	St. Francoise, MO	-0.025	0.073
			New Orleans, LA	0.129	0.051
Occupation			Lake Charles/Lafayette, LA	0.110	0.073
Clerical/sales	-0.089	0.051	Pensacola, FL	-0.172	0.058
Service	-0.239	0.059	Harlan, KY	-0.073	0.067
Agricultural	-0.239	0.137	Logan/El Paso, CO	-0.128	0.055
Processing	-0.058	0.099	Winnebago, WI	-0.025	0.056
Machine trades	-0.125	0.068	Skamania, WA	0.160	0.070
Benchwork	-0.119	0.071	Grundy, MO	-0.046	0.051
Structural work	-0.047	0.067	Dayton, OH	0.016	0.041
Miscellaneous	-0.123	0.059			
			Statistics		
Constant			Number of observations	2156	
Constant	1.143	0.130	R^2	0.396	

Results of the productivity regression. Age is measured in years. The gender, education, sector and labor market variables are dummies. The reference category is female, grade school, service sector, professional/technical/managerial occupations, and Columbus, OH.

Table 2: Regression results

easily and would be very indirect at best. Therefore, I leave this issue for further research and follow the standard practice in the literature by fixing the length exogenously to one month.

Discounting and Retirement. I exogenously fix the retirement probability at 2.5% per year, i.e. $\tau = 0.0021$, which corresponds to an average career of 40 years. Further, I set β equal to $0.93^{1/12}$. Given the assumed retirement probability and the human capital growth rate estimated in section 4.2, this value is consistent with an annual discount rate β_0 of 0.948.

Job Destruction. In order to determine the job destruction probability, I use an approach similar to Shimer (2005, 2007), taking into account the new elements of my model. Recall that unemployment evolves according to

$$u_{t+1} = u_t\left(1 - \Psi_{0,t+1}\right) + u_{t+1}^s,$$

where $u_{t+1}^s \equiv (1-u_t)\left(\delta_t + \tau - \delta_t \tau\right)\left(1 - \Psi_{0,t+1}\right)$ denotes the number of short-term unemployed workers at time $t+1$, i.e. the number of workers who were not unemployed yet at time t. Using data from the Current Population Survey, Shimer (2005, 2007) constructs time series for both u_t and u_t^s. These can be used to calculate the workers' matching probability $\Psi_{0,t+1}$ according to

$$\Psi_{0,t+1} = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}. \tag{22}$$

An estimate for the job destruction rate can then be obtained via

$$\delta_t = \frac{1}{1-\tau}\left(\frac{u_{t+1}^s}{1-u_t} \times \frac{u_t}{u_{t+1} - u_{t+1}^s} - \tau\right).$$

After averaging over the relevant time interval and taking into account the age structure of the sample, I find $\delta = 0.063$.[35]

Vacancy Creation. The vacancy creation cost k_V affects several equilibrium outcomes. First, a higher value will imply that a lower number of vacancies will be opened in the market. Unfortunately however, no precise information on the number of vacancies in the US around 1980 is available. Davis et al. (2009) create a times series by detrending the Conference Board's Help-Wanted Index using an HP filter, and rescale the deviations such that they match the mean of the more direct measure of the vacancy rate in the Job Openings and Labor Turnover Survey (JOLTS) in

[35] It is important to stress that δ cannot directly be compared to the estimates for the 'employment-exit probability' by Shimer (2007). First, δ only implies a transition from employment to the pool of workers who will search for a job in the next period. Unemployment only follows in case that search fails, i.e. with probability $1-\Psi_0$. Second, Shimer (2007) needs to control for time aggregation because he uses a continuous-time model. Here such a correction is not necessary since the period length in my model corresponds to the frequency of the data (1 month). Finally, I control for the age structure by calculating δ for seven different age cohorts and averaging over them using the corresponding sample fractions. Ideally, one would control for other sample characteristics (education, location, occupation) in a similar way, but such data is hard to obtain.

the overlapping period (2001 - 2007). This gives vacancy rates of close to 3% for the relevant time period. However, my definition of a vacancy is most likely wider than the one in the Help-Wanted Index or JOLTS, since in my model any match requires a preceding vacancy. I therefore determine k_V through its effect on the worker's matching probability Ψ_0. With fewer vacancies available, it will be harder for workers to find a job. An estimate for the workers' matching probability can be obtained with equation (22), which yields 0.413. In the estimation of the model, I will constrain k_V such that the matching probability implied by the model equals this value.

Note that the values of τ, δ and Ψ_0 jointly imply a steady state unemployment rate of $u = 0.084$. This is close to the value observed in the data, i.e. 0.082, meaning that the steady assumption is reasonable despite the recessive state of the US economy in 1980 and 1981. Note further that the values imply that in each period a fraction $u + (1-u)(\delta + \tau - \delta\tau) = 0.144$ of the labor force is searching for a new job. A fraction $(\delta + \tau - \delta\tau)\Psi_0 = 0.027$ of the employed workers move from one job to another without intermittent unemployment spell. This is in consistent with estimates of monthly job-to-job transition rates by e.g. Fallick & Fleischman (2004), Nagypal (2008) and Moscarini & Thomsson (2007) which range from 2.2% to 3.2%. The number of matches in each period is equal to $(u + (1-u)(\delta + \tau - \delta\tau))\Psi_0 = 0.059$, indicating that JOLTS indeed does not fully capture the equivalent of a vacancy in my model.

4.4 Estimation Strategy

After making the above assumptions, several parameters remain, including the search cost k_A and the interview cost k_R. In this subsection, I will discuss how I use maximum likelihood to obtain estimates for these parameters. The analysis in section 2 shows that in equilibrium we can distinguish between A different firms types in each submarket, each associated with a specific wage w_j and recruitment technology r_j, $j \in \{1, \ldots, A\}$. The data does not tell us the type of a certain firm. The likelihood of an observation therefore needs to be calculated by initially conditioning on each possible type and subsequently taking the expectation with respect to the type. Note further that a firm is only included in the data set if it manages to match with a worker. This creates a selection issue: firms with many applicants or firms with a high acceptance probability are more likely to be observed. Hence, the likelihood also needs to be conditioned on the hiring probability η_j of a firm. I now discuss each element of an observation separately.

Applicants. A firm of type j faces a number of applicants that is the realization of a Poisson distribution. The mean is equal to the firm's gross queue length λ_j, which is determined in equilibrium by its choice of (r_j, w_j). The firm therefore has exactly z_A applicants with probability $L_{A,j}(z_A) \equiv e^{-\lambda_j} \frac{\lambda_j^{z_A}}{z_A!}$. For the labor market as a whole, the number of applications received by the firms has to equal the number of applications that are sent by the workers. The latter figure depends on the number of searchers, which I fixed, and the distribution of applications $P(a|\alpha)$. Hence, the data on z_A identifies

α.

Interviews. The number of interviews that a firm conducts depends on its number of applicants and its interview capacity. If we observe that the number of interviews z_I is strictly smaller than the number of applicants z_A, it follows that the number of interviews was restricted by the capacity. The capacity follows a geometric distribution with mean r_j, hence the probability that the firm has a capacity of z_I equals $\left(\frac{r_j-1}{r_j}\right)^{z_I-1}\left(\frac{1}{r_j}\right)$. In case we observe that the firm interviews all applicants (i.e. $z_I = z_A$), the capacity must have been at least z_A. The probability of this event is given by $\left(\frac{r_j-1}{r_j}\right)^{z_I-1}$. Hence, the likelihood contribution equals $L_{I,j}(z_I|z_A) \equiv \left(\frac{r_j-1}{r_j}\right)^{z_I-1}\left(\frac{1}{r_j}\right)^{\mathbb{I}\{z_I<z_A\}}$, where $\mathbb{I}\{z_I < z_A\}$ is an indicator function which equals 1 if the condition $z_I < z_A$ is satisfied and 0 otherwise. A high value for z_I is more likely when r_j is high. Hence, z_I contains information on r_j. Note that r_j depends on k_R, k_V, ϕ and Ψ_j. Since k_V is determined by the number of vacancies, and Ψ_j is determined by the structure of the equilibrium, z_I identifies k_R for a given value of ϕ. Specifically, a higher value of k_R leads to fewer interviews.

Job Offers. The number of job offers that a firm needs to make in order to attract a worker follows a geometric distribution as well. The probability of success (i.e. the worker accepts the offer) is $1-\Psi_j$. However, acceptance needs to occur before the firm runs out of qualified applicants. The conditional probability that the firm hires with the z_O-th offer and had $z_O \leq q \leq z_I$ qualified candidates is given by $L_{O,j}(z_O|z_I) = \Psi_j^{z_O-1}(1-\Psi_j)\sum_{q=z_O}^{z_I}\binom{z_I}{q}\phi^q(1-\phi)^{z_I-q}$.

Wage. The variance σ_W^2 of the unobserved productivity component follows from the wage residuals. Note that if a residual z_R is observed for a worker who is employed at a type j firm, the worker's unobserved productivity component must equal $\widehat{\varepsilon}_{W,j} = z_R - \gamma_0 - \log w_j$, where γ_0 is a constant to be estimated. The likelihood of this is given by the normal density $f_\varepsilon(\widehat{\varepsilon}_{W,j})$. Given the other parameters, the equilibrium wage rates $(w_1^*, \ldots, w_A^*)$ are fixed. We do not observe which of these wage rates the worker earns, but after conditioning on each possible value and averaging, the identification of σ_W is standard.

Screening Time. The cost of recruitment as implied by the model equals $k_R(r_j-1)$ for a firm of type j. In order to convert this to time, I assume that a firm-worker pair produces output during 2000 hours a year, which corresponds to 50 weeks of 5 days of 8 hours. Note that if firms behave according to the model, only $A = 15$ different values would be observed for the variable z_T. I attribute the excess variation in the data to measurement error. Hence, I assume that when a firm chooses a recruitment technology r, its reported screening time z_T satisfies

$$\log z_T = \log k_R(r-1) + \varepsilon_T,$$

where $\varepsilon_T \sim N\left(-\frac{\sigma_T^2}{2}, \sigma_T^2\right)$. Hence, the likelihood contribution of an observation z_T equals $f_\varepsilon\left(\widehat{\varepsilon}_{T,j}\right)$ where $\widehat{\varepsilon}_{T,j} = \log z_T - \log k_R\left(r-1\right)$ and the variation in $\widehat{\varepsilon}_{T,j}$ helps to identify σ_T.

Note that not only the variation, but also the level of the observed screening times is informative of the parameters of the model. How much time firms are willing to spend on recruitment depends on the surplus from a match. This surplus is smaller for higher values of the household production h, since firms are forced to pay higher wages in that case. The observations z_T therefore contain information on h.

Vacancy Durations. The last remaining parameter is the probability ϕ that an interviewed candidate is found to be qualified. The data from the current period does not identify this probability, since the number of qualified applicants is unobserved and since each observation is conditional on matching. Fortunately, an estimate for ϕ can be obtained by using the information on vacancy durations. Assuming that firms keep their vacancy open until it is filled and post the same contract throughout, the model implies that vacancy durations follow a geometric distribution with parameter equal to the periodical hiring probability η_j, which is increasing in ϕ. In order to make the data consistent with the model, I group the observed vacancy durations into bins of 30 days, i.e. the assumed period length. Hence, $\widehat{z}_V = i$ iff $z_V \in (30(i-1), 30i]$. The probability to observe a vacancy duration of $\widehat{z}_V$ periods conditional on matching in the current period, then equals $L_{V,j}\left(\widehat{z}_V\right) = \left(1-\eta_j\right)^{\widehat{z}_V - 1}$.

Summarizing, the likelihood of an observation is given by

$$L = \frac{\sum_{j=1}^{A} v_j L_{A,j}\left(z_A\right) L_{I,j}\left(z_I|z_A\right) L_{O,j}\left(z_O|z_I\right) L_{V,j}\left(\widehat{z}_V\right) f_\varepsilon\left(\widehat{\varepsilon}_{W,j}\right) f_\varepsilon\left(\widehat{\varepsilon}_{T,j}\right)}{\sum_{j=1}^{A} v_j \eta_j},$$

where v_j denotes the measure of firms of type j. I maximize the log of this likelihood, summed over all observations. The standard errors are calculated with the delta method from the inverse of the Hessian.

4.5 Estimation Results

The results of the estimation are presented in table 3 and figure 3. Panel 1 of figure 3 displays the contracts (r, w) that are posted in equilibrium. There is contract dispersion and, in line with the theory, I find that that firms that post higher wages invest less in recruitment.

The second panel of figure 3 shows the density of the posted wages. It is worth noting that this density is downward sloping, in contrast to what is obtained in estimations of the Burdett & Mortensen (1998) model. The distribution of the wages earned by employed workers is not shown, but it is virtually identical to the distribution of the posted wages, since all firms match with probability close to 1, as can been seen in the last panel. Note further that the estimated amount of frictional wage dispersion is small. Multiple causes underlie this result. First, limited wage dispersion is a standard finding in well calibrated search models of the labor market, as shown by Hornstein et al. (2010).[36] Second, I limit the amount of dispersion by allowing for unobserved heterogeneity in human capital in a very flexible way. The model attributes a large part of the remaining variation in the wages after controlling for worker characteristics to the unobserved productivity component ε rather than to frictions: I find $\sigma_W = 0.233$. Third, I obtain a high estimate for the value of home production, which restricts the amount of wage dispersion that can arise. To be precise, I get $h = 0.950$. This is roughly in line with the value that Hagedorn & Manovskii (2008) find in their calibration, but higher than the value used by Shimer (2005) (0.40) or the value found by Hall & Milgrom (2008) (0.71).

The mechanism behind this result is the following. I find that screening is not very costly. An extra unit of interview capacity requires an additional investment of $k_R = 0.012$ (i.e. 1.2% of monthly production), which corresponds to 1.97 hours of output or, equivalently, 2% of the vacancy creation cost $k_V = 0.529$. Nevertheless, firms interview only a few candidates. The average firm chooses $r = 4.091$, which implies that the total cost of recruitment amounts to only 6.09 hours of production. For this to be optimal behavior, a firm's payoff from a match cannot be too large, implying that the outside option of the workers must be quite high. Recall also that I use data from a sample of relatively low-skilled workers during a recession in the US labor market. Hence, a small gap between labor productivity and productivity at home is not unreasonable.

Despite the small amount of wage dispersion, workers are not indifferent between the different contracts. In particular, the job offer probability varies considerably across firms, with applications to the lowest wage firms being almost four times as likely to result in an offer as applications to the highest wage firms. Workers send on average 4.852 applications. In order to determine the cost of an application, we first need to verify that the gains from search as defined in equation (20) are strictly concave in α. Figure 4 shows that this is indeed the case: the marginal benefits $V_S'(\alpha)$ are downward sloping. This implies that all workers choose their search intensity by equating marginal benefit to marginal cost. Hence, the marginal cost k_A of an extra unit of search intensity equals 0.004. This is equivalent to 0.73 hours of household production. The total time spent on search by an unemployed worker amounts to 3.56 hours per month, which is a bit lower than other estimates in the literature.[37] Recall that a worker matches with probability 0.413, as was imposed exogenously to aid the identification of the model.

The average firm hires with probability 0.911, which is well in line with the fraction of firms

[36] The exact magnitude of frictional wage dispersion is still an open question in the literature, which I do not aim to answer here. Different assumptions about productivity and wages can of course be made. For example, adding on-the-job search would probably help in generating more wage dispersion, but is left for future research because of the associated computational complexities. However, it is important to stress that changing the model along such dimensions has a limited effect on the estimates for its key parameters since wage data is not the main source of their identification.

[37] For example, Krueger & Mueller (2008) find a value of 16 hours per month using data from the American Time Use Surveys. Note that their estimate concerns a different time period and a different sample of workers.

	Estimate	Std.err.
Model parameters		
α	4.852	0.145
k_R	0.012	0.001
h	0.950	0.005
ϕ	0.887	0.021
γ_0	1.205	0.010
σ_W	0.233	0.007
σ_T	1.123	0.032
Statistics		
Observations	640	
Avg log-likelihood	-9.531	

Estimation results obtained with maximum likelihood.

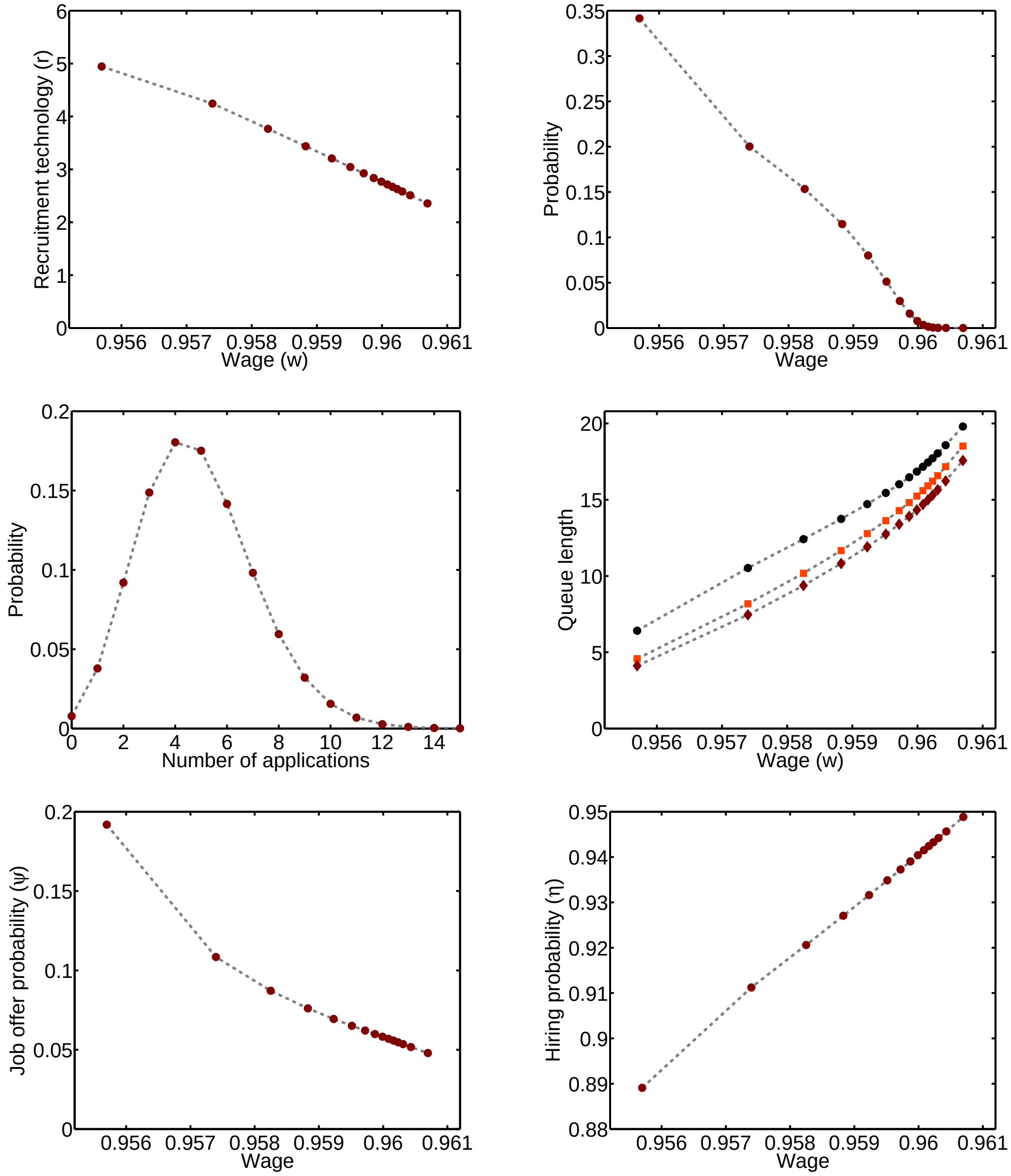
Table 3: Estimation results

in the data that fill their vacancy within a month. Correspondingly, I find that the probability ϕ that a worker is qualified equals 0.887. This high value is consistent with the fact that firms match with large probability, even though they interview relatively few applicants. Finally, the standard deviation of the measurement error in the screening time is estimated at 1.123.

In order to consider the fit of the model, I use the estimated parameter values to simulate data for each of the dependent variables. Figure 5 displays the densities of the simulated data together with those of the actual data. It shows that in general the model matches the data quite well. The fit of the number of job offers and the vacancy durations is particularly good. For the applicants, interviews and screening time, the model has some difficulties fitting the large amount of dispersion present in the data. It predicts too few very low and very high values. However, the fit is generally good around the median in each case. For the unobserved wage component $\log(w) + \varepsilon$, the model slightly underestimates the height of the mode. Note that the graph confirms that once the unobserved productivity component ε is taken into account the amount of wage dispersion implied by the model is not smaller than in the data.

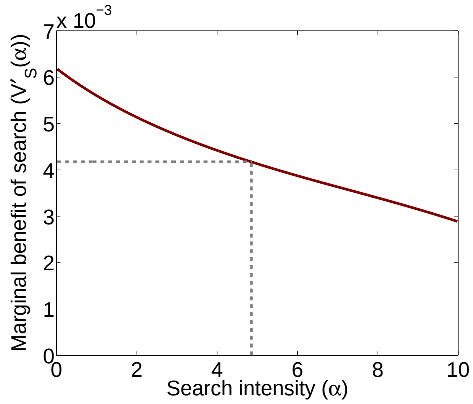
4.6 Magnitude of Frictions

The estimation results indicate that both sending applications and conducting job interviews are costly activities. This means that both search and recruitment frictions arise, causing the externalities described above. I analyze the magnitude of these frictions by considering the effect of an exogenous change in the search cost k_A and/or recruitment cost k_R on social surplus. Because of the efficiency unit assumption, a change in one of the parameters will change the equilibrium in any



Equilibrium outcomes. The panels display (numbered from left to right, top to bottom): 1) Equilibrium contracts (r,w) posted in equilibrium; 2) Fraction of firms posting a certain wage; 3) Density of the number of applications sent by workers; 4) Queue lengths λ (black circles), μ (maroon diamonds) and κ (orange squares) as a function of the wage; 5) Workers' job offer probability as a function of the wage; 6) Firms' hiring probability as a function of the wage. The dashed lines connecting the dots serve illustrative purposes only.

Figure 3: Equilibrium outcomes



The maroon solid line represents the marginal benefits of search for an individual worker, given equilibrium behavior by the firms and the other workers. The worker equates marginal benefit to marginal cost (horizontal light-gray line) in order to determine his search intensity (vertical light-gray line).

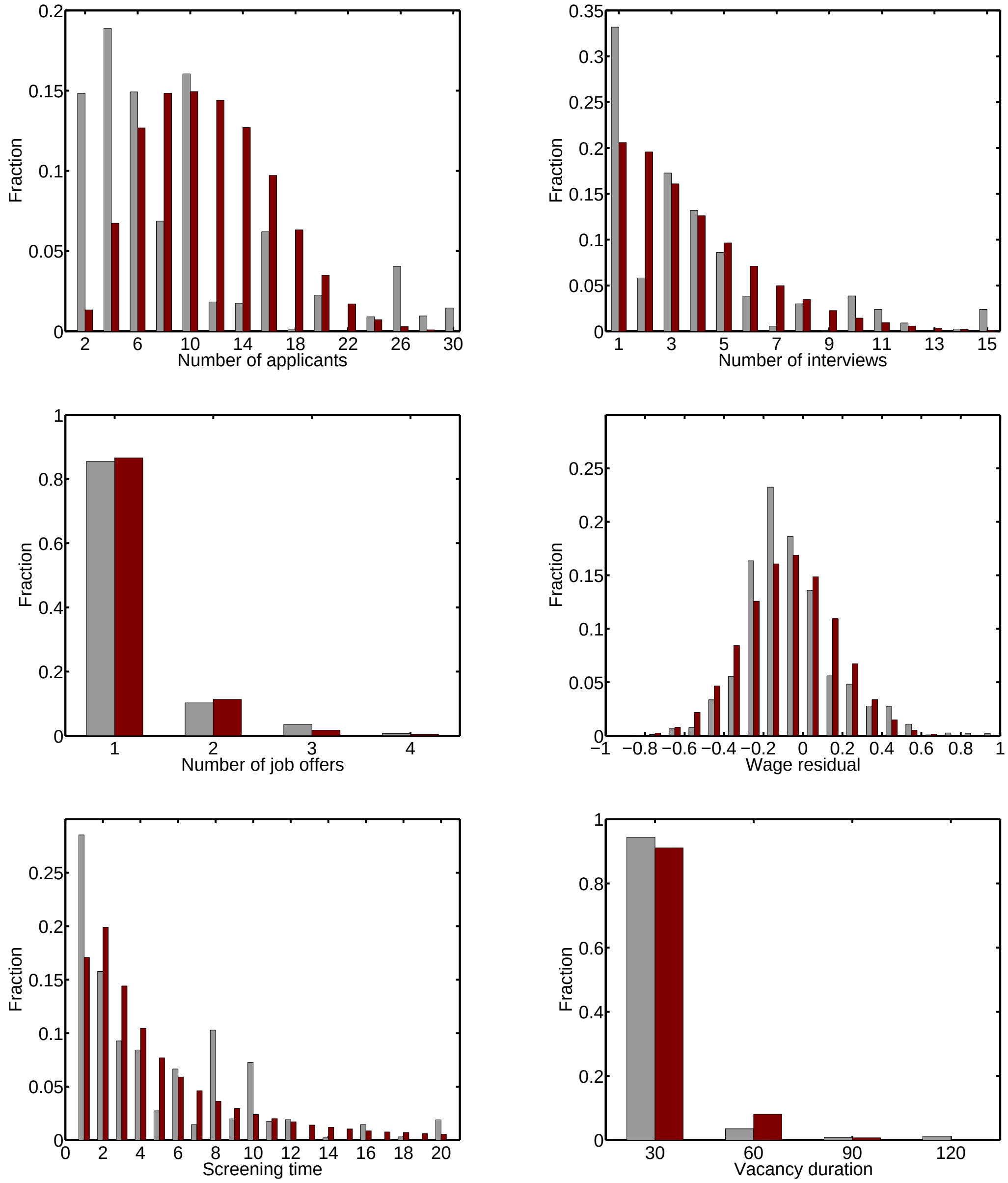
Figure 4: Costs and benefits of search

submarket in the same way. As a result, it is not necessary to aggregate across the submarkets. We can focus on one particular submarket instead.

The Bellman equation for social surplus depends on a single state variable, the unemployment rate. I start from the unemployment rate arising in the decentralized equilibrium, which I label u_0, and consider how it evolves over time as the agents in the model adjust their strategies in response to the change in the cost parameters.

In each period t, the main component of output is the production in the labor market by matched firm-worker pairs. There is a mass $1-u_t$ of such pairs, each producing 1 unit. Unemployed worker on the other hand produce in the household. So far, it was irrelevant what components are included in h. When considering welfare however, it is not the worker's private value of non-market time but the social value that is the relevant measure. In particular, if h includes unemployment benefits, then those do not add to total output, because they are simply a transfer. However, the fraction of h that consists of unemployment benefits cannot be identified with the model and data used in this paper. Therefore, I assume that unemployment benefits equal $b = 0.4$, roughly in line with the highest replacement rates in the US. Household production and the worker's value of leisure then add up to $h-b$.

In the beginning of the next period, a mass $u_t + (1-u_t)(\delta + \tau - \delta\tau)$ of workers can search for a new job. They incur the search cost αk_A. A mass $v_{t+1,j}$ of vacancies enters the market, posts a contract (r_j, w_j), and pays the associated cost $(r_j - 1)k_R + k_V$. Together, this leads to a new unemployment rate u_{t+1} at the beginning of the next production phase. Hence, the Bellman equation



Fit of the model for the distribution of the number of applicants per firm, the number of interviews per firm, the number of job offers per firm, the posted wages, the screening time and the vacancy duration. Light-gray bars represent the data and maroon bars the model. All graphs are conditional on the firm hiring a worker. The wage residual panel shows the distribution of $\log(w) + \varepsilon$, i.e. the unexplained component in the wages after controlling for the effect of the observable characteristics x. The bin width is 2 for the applications, 1 for the interviews, offers and screening time, 0.1 for the wage residuals, and 1 period (30 days) for the vacancy durations. The graphs do not display the entire domain of the distributions; outliers are omitted.

Figure 5: Fit of the model

equals

$$S(u_t) = (1-u_t) + u_t(h-b) + \beta_0(1+\theta) \times \left[-\left(u_t + (1-u_t)(\delta+\tau-\delta\tau)\right)\alpha k_A - \sum_{j=1}^{A} v_{t+1,j}\left((r_j-1)k_R + k_V\right) + S(u_{t+1}) \right], \qquad (23)$$

where

$$u_{t+1} = \left(u_t + (1-u_t)(\delta+\tau-\delta\tau)\right)(1-\Psi_0)$$

and α, v_j, r_j and Ψ_0 follow from the workers' and firms' decision problems and the equilibrium structure.

I solve (23) for three different combinations of k_A and k_R. The results of this exercise are summarized in table 4. In the first scenario ('estimated equilibrium'), I keep both cost parameters at their estimated values. Since the market was in steady state, $u_t = u_0$ for all $t \geq 1$ in that case. Social surplus equals 234.74.

In the second scenario ('free recruitment'), I set the recruitment cost equal to zero, while the application cost stays at its estimated value. This leads to an equilibrium in which firms send a finite number of applications, but firms contact all applicants. Hence, more matches are formed for any given level of search intensity, which means that the job offer and hiring probability go up. In general, the effect of such an increase in the matching probability on the level of search intensity that workers choose is not obvious.[38] Here I however find that workers start to apply slightly less. Nevertheless, the periodical matching probability for the workers increases by almost 20 percentage points. The unemployment rate decreases over time by about 50% (4.2 percentage points). Firms match with larger probability and slightly fewer vacancies are opened Social surplus is now 240.66, approximately 2.5% higher than in the estimated equilibrium.

In the last scenario ('Walrasian outcome'), I also eliminate the application cost, such that both $k_R = 0$ and $k_A = 0$. In this Walrasian world, the maximum level of social surplus is obtained. Frictions are completely eliminated and the matching process is fully efficient. Both firms with vacancies and unemployed workers match with probability 1. As a result, unemployment disappears completely in the limit. The number of opened vacancies is at the same level as in the estimated equilibrium.[39] This scenario yields a social surplus of 245.69, which is 2.1% higher than in the scenario with free recruitment and 4.7% higher than in the estimated equilibrium.[40]

[38] See Shimer (2004) for a discussion of this issue.

[39] Recall that u and v are measured at different moments in a model period: v is the number of vacancies posted in the entry phase, whereas u equals the number of workers still without a job at the beginning of the production phase. In a Walrasian world, the number of unmatched vacancies at the beginning of the production phase obviously equals zero as well.

[40] In both scenarios, the increase in surplus consists of two effects: 1) the direct effect of the elimination of the cost (while holding the equilibrium constant) and 2) the indirect effect via a change in the equilibrium outcomes. The indirect effect greatly dominates both times, as can be shown by calculating the surplus while only taking the direct effect into

	k_A	k_R	α	r	u	v	Ψ_0	η	S
Estimated equilibrium	0.004	0.012	4.852	4.091	0.084	0.065	0.413	0.911	234.74
Free recruitment	0.004	0	4.44	∞	0.042	0.063	0.596	0.987	240.66
Walrasian outcome	0	0	∞	∞	0	0.065	1	1	245.69

Equilibrium outcomes for various combinations of the search cost k_A and the recruitment cost k_R. The reported values for the recruitment technology r and the hiring probability η are averages across vacancies. The reported values for u and v are the limit values.

Table 4: Magnitude of the frictions

These results demonstrate the importance of the frictions, not only regarding unemployment but also with respect to social surplus. A reduction in the cost of search and/or recruitment can generate considerable welfare gains. The results further indicate that recruitment frictions are roughly equally important as search frictions in understanding frictional unemployment or the welfare loss compared to a Walrasian world. This suggests that studies which only include search frictions but ignore recruitment frictions potentially draw wrong inference about e.g. the desirability of certain labor market policies.

4.7 Policy

As a final exercise, I analyze the effects of unemployment insurance (UI) eligibility criteria. O'Leary (2006) describes how many states in the US have moved away from specifying explicit criteria on the search intensity of unemployed workers in the law, because firms "do not want repetitive and burdensome employment applications that are filed merely to meet the UI work search requirements."[41] In order to assess whether this issue arises in the estimated equilibrium, I consider a scenario in which a government sets the minimum search intensity $\underline{\alpha}$ that is required for workers to be eligible for UI benefits. Everything else is determined by the decentralized market.[42] In particular, workers choose whether they want to comply with the rules, or whether they rather prefer to forgo the benefits. Starting from the estimated equilibrium, I determine how the labor market evolves after the implementation of the policy and which effect this has on social surplus as defined in (23). I do this for minimum search intensities between 0 and 10. The results are displayed in

account. Starting from the estimated equilibrium, elimination of the recruitment cost would yield a surplus of 235.34. Likewise, the direct effect of the elimination of the search cost increases social surplus from 240.66 to 241.16.

[41] Similar considerations can be found in different contexts. High-school students in the UK are restricted in the number of college applications that they can send. Until 2008, students could send six applications. This was reduced to five, after colleges expressed their preference for such a change in a survey, because "allowing up to six applications was burdensome, particularly as many students' fifth and sixth applications were often poorly considered" (Department for Education and Skills, 2006).

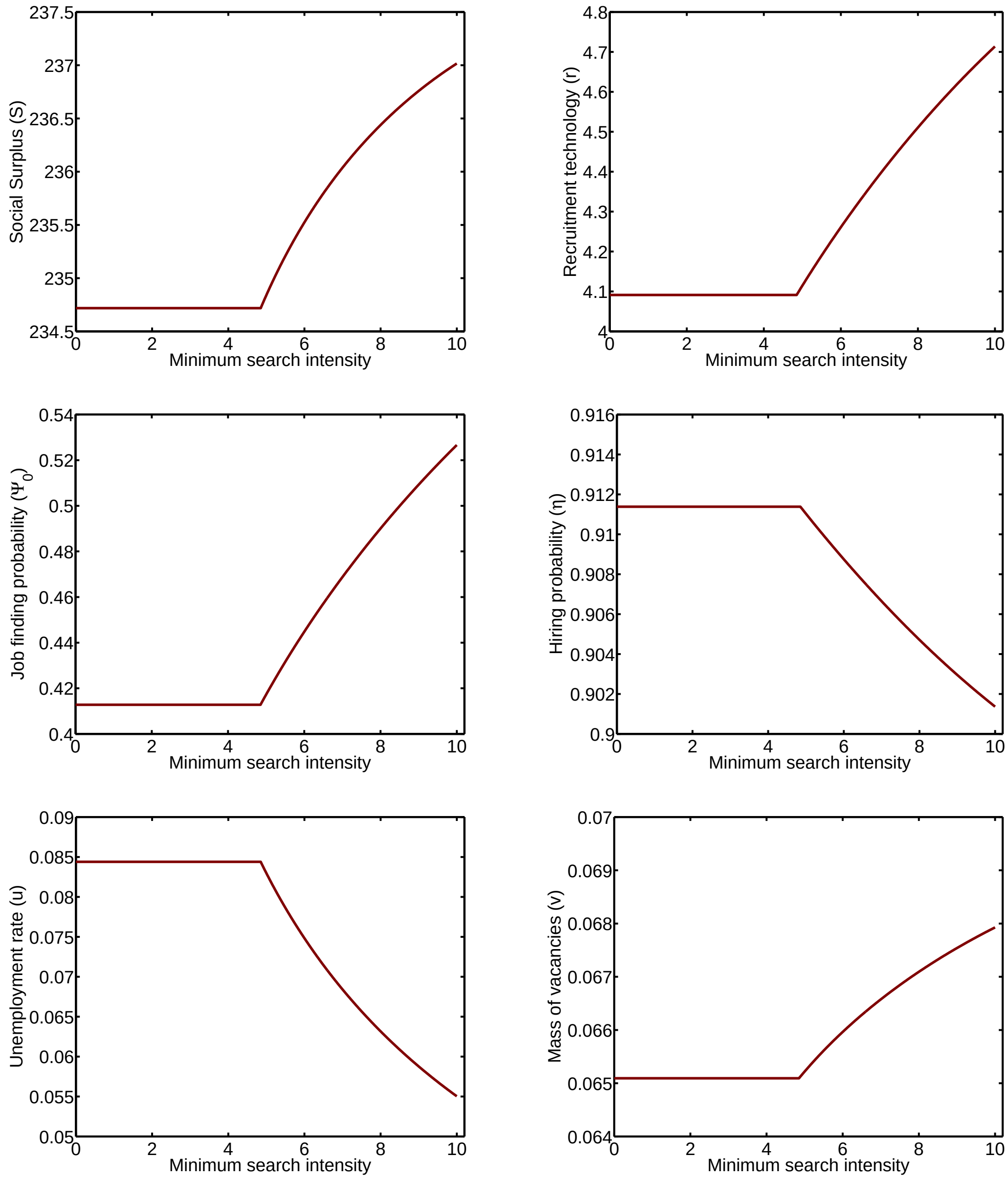
[42] Note that in real life, the rules are often stated in terms of the number of applications instead of the search intensity. I abstract from that here.

figure 6.

Two results are immediate. First, if the minimum search intensity is not binding because it is set below the estimated market level of 4.852, the equilibrium does not change. Therefore, I focus on higher values of $\underline{\alpha}$ in the discussion below. Second, for any reasonable value of $\underline{\alpha}$, workers always decide to comply, since the gain from getting unemployment benefits ($b = 0.4$) is much larger than the cost of a few more applications ($k_A = 0.004$). Other results are less straightforward because of the large number of externalities included in the model. The top right panel of figure 6 shows that firms respond to the increase in search intensity by investing more in recruitment. For example, setting $\underline{\alpha}$ equal to 10 causes firms to choose a 15% higher value of r. This is consistent with the fact that any job offer is now more likely to be rejected. The increase in r is however not sufficient to completely offset the increase in competition, as shown in the fourth panel of the figure: the increase in the workers' search intensity *reduces* the matching probability of a firm.

Hence, the firms are now worse off than before along two dimensions, which seems to be in line with the quote from O'Leary (2006). However, we cannot ignore the effects of the policy on the equilibrium wages. It turns out that that effect goes in the opposite direction. Wages decrease as a result of the more intensive search by the workers (not shown in the figure). In fact, this effect dominates, which translates into additional entry of vacancies (bottom right panel). Workers benefit from the increase in the number of vacancies since it increases their job finding probability Ψ_0 (third panel). As a result, unemployment falls. This change is considerable: when $\underline{\alpha}$ is set at 10, the unemployment rate goes down by almost three percentage points. An unemployed worker is nevertheless worse off than before, in the sense that the value of unemployment V_U decreases. The lower wages and the higher search costs dominate the positive effect of the better matching possibilities. However, the key question in analyzing the policy is what happens to social surplus, which includes all gains and losses from the policy. As the top left panel shows, this welfare measure goes up by about 1%. Although the program aggravates the negative externalities and leads to higher recruitment costs for the firms, the positive externalities that result from the increase in the workers' search intensity dominate.

Note that output does not reach a maximum for $\underline{\alpha}$ in the interval (4.852, 10). In principle, one can extend the analysis to find the optimal level for the minimum level of search. However, a lot of caution is required in such an exercise. First, the potential existence of multiple equilibria raises the question how the market will respond to the policy. For a small change, convergence to an equilibrium 'nearby' seem a reasonable assumption. For larger changes, such an assumption is much more questionable. Second, for large values of $\underline{\alpha}$, the assumption that $A = 15$ becomes restrictive. This limits the negative impact of the recruitment friction, which leads to an overestimate of the surplus for such high values of $\underline{\alpha}$. Third, the optimal level of $\underline{\alpha}$ strongly depends on the level of unemployment benefits b, which is a variable that was not estimated. For example, if $b < 0.06$ while maintaining $h = 0.950$, the conclusion would be the opposite. In that case, an increase in search



Equilibrium outcomes for the poliy change. The displayed values for r and η are averages across vacancies, while u and v are limit values, reached only in the new steady state.

Figure 6: Effects of policy

intensity decreases social surplus, even though it still leads to a higher number of matches and a reduction of unemployment. The intuition for this result is clear: if workers are almost equally productive at home as in a job, the social gains from search are very small. The main conclusion from the exercise here should therefore be as follows: given the estimated parameter values, a small increase in the required search intensity is likely to increase steady state output.

5 Conclusion

This paper analyzes the magnitude of frictions in a labor market in which a lack of coordination results in multilateral meetings between workers and firms. For this purpose, a directed search model is presented in which both workers and firms decide how many agents on the other side of the market they want to contact. Firms post a wage and recruitment technology that determines how many applicants they will interview. After observing these contracts, workers decide how many times and to which firms to apply. Since workers typically send multiple applications, any firm faces the risk that its job offer gets rejected. Likewise, a worker's application may be unsuccesful because the firm decides to hire a different candidate. In such an environment, it is crucial to understand the costs associated with applying and interviewing, since they determine the magnitude of the frictions and the negative externalities caused by them.

I show that contract dispersion is a fundamental feature of the equilibrium. Some firms offer low wages but make large investments in recruitment, while other firms do the opposite. The number of contract types is equal to the maximum number of applications that workers may send in any given period and firms are indifferent between all types. Workers face a trade-off between the wage and the probability to get a job offer. Applications to low wage firms are more likely to turn into job offers than applications to high wage firms. It is shown that workers maximize the payoff from their application portfolio by spreading their applications over the different types of contracts.

Estimation of the model then provides values for the cost of search and recruitment. An additional application is estimated to cost the worker 0.73 hours of his time, while firms incur a cost equal to 1.97 hours of production for each interview. By simulating the equilibrium for different values of the two cost parameters, the market equilibrium is compared to the frictionless, Walrasian outcome. Social surplus is 4.7% lower in the market equilibrium than in a Walrasian world. Search and recruitment frictions are each responsible for approximately half of this loss in surplus. Finally, I argue that there is a potential role for active labor market programs, since programs that marginally increase the level of search intensity of unemployed workers increase social surplus, even though they negatively affect the firms' hiring probability.

An interesting avenue for future work would be to allow for more productivity differences among workers and firms. This would make the model suitable to study settings in which heterogeneity plays a larger role than in the sample that I use here. In such an environment, firms' choices regarding the recruitment technology will most likely also have important implications for the amount of sorting in the labor market. For such work however, the availability of high-quality data on the decisions of both firms and workers will be essential.

A Proofs

A.1 Proof of Lemma 1

Proof. Consider a firm with queue length λ and recruitment technology r. The number of applicants i follows a Poisson distribution with mean λ, while the interview capacity j follows geometric distribution with parameter $\rho \equiv \frac{r-1}{r}$. The actual number of interviews is equal to $\min\{i,j\}$. Hence, the expected number of interviews equals

$$\begin{aligned}
\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\min\{i,j\}\,e^{-\lambda}\frac{\lambda^i}{j!}\left(\frac{r-1}{r}\right)^{j-1}\frac{1}{r} &= \sum_{i=0}^{\infty}e^{-\lambda}\frac{\lambda^i}{i!}\sum_{j=1}^{i-1}j\rho^{j-1}\frac{1}{r}+\sum_{i=0}^{\infty}ie^{-\lambda}\frac{\lambda^i}{i!}\sum_{j=i}^{\infty}\rho^{j-1}\frac{1}{r} \\
&= r\sum_{i=0}^{\infty}e^{-\lambda}\frac{\lambda^i}{i!}\left(1-\rho^{i-1}\left(\rho+\frac{i}{r}\right)\right)+\sum_{i=0}^{\infty}ie^{-\lambda}\frac{\lambda^i}{i!}\rho^{i-1} \\
&= r\left(1-e^{-\lambda/r}-\frac{\lambda}{r}e^{-\lambda/r}\right)+\lambda e^{-\lambda/r} \\
&= r\left(1-e^{-\lambda/r}\right).
\end{aligned}$$

□

A.2 Proof of Lemma 1

Proof. Consider the job offer probability ψ first.[43] Let $\rho = \frac{r-1}{r}$. When a worker applies to a firm, he will in general face a number of competitors, i.e. other applicants at the same firm. Let m denote the number of competitors who are qualified and will accept the job if offered to them. On the other hand, let n denote the number of competitors who are either unqualified or will reject the job in favor of a better offer. In order to simplify terminology, I refer to the first type of competitors as 'direct competitors'. The second type is called 'indirect competitors', since they only reduce the worker's matching probability by causing congestion.

A number of applicants will be selected for an interview. Each applicant is equally likely to be selected and to be qualified. Therefore, I assume without loss of generality that if the firm has multiple qualified candidates, it makes offers among them in the same order as in which it conducted the interviews. Hence, one can interpret the recruitment as a process with multiple rounds, which ends when the firm selects a qualified applicant who accepts the job, or when the maximum possible number of interviews has been held.

For the worker it is not relevant in which round he gets the job offer. Getting a job offer in round i and getting one in round j are mutually exclusive events, since the firm will offer the job to each worker at most once. Hence, the probability that the worker gets a job offer equals the probability

[43] In this proof, I omit the dependence of the job offer probability, the matching probability and the queue lengths on the contract c in order to keep notation as simple as possible.

that he gets an offer in round i, summed over all possible i. In order for the worker to get a job offer in round i, several things must happen: (i) the firm must be able to interview at least i applicants, (ii) only indirect competitors must have been selected in round $1,\ldots,i-1$, (iii) the worker gets selected in round i itself and (iv) he is qualified. The probability that this occurs equals

$$\phi\rho^{i-1}\frac{n}{m+n+1}\cdot\frac{n-1}{m+n}\cdot\ldots\cdot\frac{n+2-i}{m+n+3-i}\cdot\frac{1}{m+n+2-i}=\phi\rho^{i-1}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!},$$

as long as $i\leq n+1$ and zero otherwise. Hence, the total probability that the worker gets a job offer equals

$$\hat{\psi}(m,n|r)=\phi\sum_{i=1}^{n+1}\rho^{i-1}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!},$$

which goes to $\frac{1}{m+1}$ for $r\to\infty$.

Actually, the number of competitors that a worker faces is a random variable. Both m and n follow Poisson distributions with respective parameters μ and $\nu=\lambda-\mu$. Hence, the probability to compete with exactly m direct and n indirect competitors equals

$$f_{m,n}(m,n|\phi\mu,\nu)=e^{-\mu}\frac{\mu^{m}}{m!}e^{-\nu}\frac{\nu^{n}}{n!}.$$

The ex ante probability to get a job offer therefore equals

$$\psi=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\hat{\psi}(n,m|r)\,f_{m,n}(m,n|\mu,\lambda-\mu).$$

Substituting the expression for $\hat{\psi}(m,n|r)$ and changing the order of summation yields

$$\psi=\phi\sum_{i=1}^{\infty}\rho^{i-1}\sum_{m=0}^{\infty}\sum_{n=i-1}^{\infty}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!}f_{m,n}(m,n|\mu,\lambda-\mu).$$

The last part of the right hand side can be rewritten as follows

$$\phi\sum_{m=0}^{\infty}\sum_{n=i-1}^{\infty}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!}f_{m,n}(m,n|\mu,\lambda-\mu)=\frac{\phi\,(\lambda-\mu)^{i-1}}{\lambda^{i}}\left(1-e^{-\lambda}\sum_{j=0}^{i-1}\frac{\lambda^{j}}{j!}\right).$$

Substituting this into the expression for ψ and applying a change in the order of summation gives

$$\psi=\phi\sum_{i=1}^{\infty}\frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^{i}}-\phi e^{-\lambda}\sum_{j=0}^{\infty}\frac{\lambda^{j}}{j!}\sum_{i=j+1}^{\infty}\frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^{i}}.$$

Since $\sum_{i=j+1}^{\infty} \frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^i} = \frac{1}{\kappa}\left(\frac{\rho(\lambda-\mu)}{\lambda}\right)^j$ with $\kappa = \frac{1}{r}\lambda + \frac{r-1}{r}\mu$, this implies

$$\begin{aligned}\psi &= \frac{\phi}{\kappa}\left(1 - e^{-\lambda}\sum_{j=0}^{\infty}\frac{\rho^j(\lambda-\mu)^j}{j!}\right) \\ &= \frac{\phi}{\kappa}\left(1-e^{-\kappa}\right).\end{aligned}$$

An expression for the firm's matching probability η can be derived in a similar way. Consider a firm with interview capacity R. Suppose that it has m qualified applicants who would accept the job and n other applicants. The number of workers from the former group selected from the former group follows a hypergeometric distribution and the firm will match if this number is at least one. In order to derive a closed-form expression for the matching probability, I follow the same approach as above. The firm hires in round i with probability

$$\rho^{i-1}\frac{n}{m+n}\cdot\frac{n-1}{m+n-1}\cdot\ldots\cdot\frac{n+2-i}{m+n+2-i}\cdot\frac{m}{m+n+1-i} = \rho^{i-1}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!},$$

as long as $i \leq n+1$ and zero otherwise. Hence, given m and n the total hiring probability equals

$$\hat{\eta}(m,n|r) = \sum_{i=1}^{n+1}\rho^{i-1}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!},$$

which goes to 1 for $r\to\infty$.

Both m and n follow Poisson distributions with respective parameters μ and $\nu = \lambda-\mu$. Hence, the ex ante matching probability is equal to

$$\begin{aligned}\eta &= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\hat{\eta}(n,m|r)\, f_{m,n}(m,n|\mu,\lambda-\mu) \\ &= \sum_{i=1}^{\infty}\rho^{i-1}\sum_{m=1}^{\infty}\sum_{n=i-1}^{\infty}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!}f_{m,n}(m,n|\mu,\lambda-\mu).\end{aligned}$$

The last part of the right hand side can be rewritten as follows

$$\sum_{m=1}^{\infty}\sum_{n=i-1}^{\infty}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!}f_{m,n}(m,n|\mu,\lambda-\mu) = \frac{\mu(\lambda-\mu)^{i-1}}{\lambda^i}\left(1-e^{-\lambda}\sum_{j=0}^{i-1}\frac{\lambda^j}{j!}\right),$$

which implies

$$\begin{aligned}\eta &= \mu\psi \\ &= \frac{\mu}{\kappa}\left(1-e^{-\kappa}\right).\end{aligned}$$

□

A.3 Proof of Lemma 2

Proof. The proof is a generalization of the proof provided by Galenianos & Kircher (2009). Consider application $i > 1$ and suppose it is not sent to a contract offering a wage in the interval $[\bar{w}_{i-1}, \bar{w}_i]$. Then there are two possibilities: 1) the application is sent to a contract offering a wage $w > \bar{w}_i$ or 2) the application is sent to a contract offering a wage $w < \bar{w}_{i-1}$. By the definition of $\bar{w}_i$, the first case would give the worker a payoff that is strictly lower than V_i, which cannot be optimal. In the second case, the worker could do better by deviating and sending the application to $\bar{w}_{i-1}$. Let $\bar{c}_i$ denote the contract $(r, \bar{w}_i)$, then the utility gain from deviating equals

$$\begin{aligned}
&(\psi(\bar{c}_{i-1}) V_E(\bar{w}_{i-1}) + (1-\psi(\bar{c}_{i-1})) V_{i-1}) - (\psi(c) V_E(w) + (1-\psi(c)) V_{i-1}) = \\
&(\psi(\bar{c}_{i-1})(V_E(\bar{w}_{i-1}) - V_{i-2}) - \psi(c)(V_E(w) - V_{i-2})) + (\psi(c) - \psi(\bar{c}_{i-1}))(V_{i-1} - V_{i-2}) > 0.
\end{aligned}$$

The first term is non-negative, since $\bar{w}_{i-1}$ maximizes $\psi(\bar{c}_{i-1})(V_E(\bar{w}_{i-1}) - V_{i-2})$ by definition. The second term is positive, since $\bar{w}_{i-1} > w$ implies that $\psi(\bar{c}_{i-1}) < \psi(c)$. □

A.4 Proof of Lemma 3

Proof. The proof is a generalization of the proof provided by Kircher (2009). First, note that for wages w below $\bar{w}_0 = V_E^{-1}(V_1)$, the market utility cannot be obtained. As a result, $\kappa(c) = 0$ and $\psi(c) = \phi$. At $w = V_E^{-1}(V_1)$, the market utility can only be obtained if $\kappa(c) = 0$ and $\psi(c) = \phi$. Lemma 2 establishes that the first application is sent to a wage in the interval $\left(V_E^{-1}(V_1), \bar{w}_1\right]$. As a result, $\psi(c) V_E(w) + (1-\psi(c)) V_0 = V_1$ needs to hold for all wages in this interval in order to satisfy the market utility condition. A similar argument applies to the other interval. Application $i > 1$ is sent to the interval $[\bar{w}_{i-1}, \bar{w}_i]$, which implies that the effective queue length $\kappa(c)$ and the job offer probability $\psi(c)$ are governed by equation (15). The effective queue length has to be continuous in the wage. This implies that $\bar{w}_i$ is determined as the wage at which equation (15) holds for both i and $i+1$. Solving the two equations yields

$$V_E(\bar{w}_i) = \frac{V_i^2 - V_{i-1} V_{i+1}}{2V_i - V_{i-1} - V_{i+1}}.$$

□

A.5 Proof of Lemma 4

Proof. Suppose that the lemma does not hold and that firms of type i offer the same contract (r, w) as firms of type $i+1$. By lemma 3, we must then have that $w = \bar{w}_i$. Let ψ_i denote the associated job offer probability. If the worker gets a job offer from both firms, but no higher one, if will accept the offer from firm $i+1$ by construction. As a result, the acceptance probabilities for both firms are related as follows

$$1-\Psi_i = (1-\psi_i)(1-\Psi_{i+1}) < 1-\Psi_{i+1}.$$

Consider first $\bar{w}_i \in [0,1)$. If the firms of type i offer a marginally higher wage $w' \downarrow \bar{w}_i$, their acceptance probability jumps up to $1-\Psi_{i+1}$. Clearly, such a deviation is profitable. Next, consider $\bar{w}_i = 1$. In that case, the firms make zero profits. At wages slightly below one, the queue length is lower but still positive. This deviation therefore yields higher profits to the firm. □

A.6 Proof of Lemma 5

Proof. Firms solve

$$\max_{\kappa\in\mathcal{K}_i, r\in\mathcal{R}} \frac{r\phi(1-\Psi_i)\left((1-e^{-\kappa})(V_E(1)-V_{i-1})-\frac{\kappa}{\phi}(V_i-V_{i-1})\right)}{1+(r-1)\phi(1-\Psi_i)} - k_V - k_R(r-1).$$

Start with the optimization over the effective queue length. The second derivative of the objective function with respect to κ equals

$$-\frac{r\phi(1-\Psi_i)e^{-\kappa}(V_E(1)-V_{i-1})}{1+(r-1)\phi(1-\Psi_i)},$$

which is strictly negative for all $\kappa \in \mathcal{K}_i$. Consequently, the objective function is strictly concave in the effective queue length.

The first order condition is given by

$$\frac{r\phi(1-\Psi_i)\left(e^{-\hat{\kappa}_i}(V_E(1)-V_{i-1})-\frac{1}{\phi}(V_i-V_{i-1})\right)}{1+(r-1)\phi(1-\Psi_i)} = 0.$$

Solving for $\hat{\kappa}_i$ yields

$$\hat{\kappa}_i = \log\frac{\phi(V_E(1)-V_{i-1})}{V_i-V_{i-1}}$$

If $\hat{\kappa}_i \in \mathcal{K}_i$, the optimal effective queue length is defined by this expression, i.e. $\kappa_i^* = \hat{\kappa}_i$. On the other hand, if $\hat{\kappa}_i < \bar{\kappa}_{i-1}$, then $\kappa_i^* = \bar{\kappa}_{i-1}$.

Next, consider the optimization over the recruitment technology r. In order to simplify notation, define

$$\widetilde{V}_{F,i} \equiv \left(1-e^{-\kappa_i^*}\right)(V_E(1)-V_{i-1}) - \frac{\kappa_i^*}{\phi}(V_i-V_{i-1}) = \max_{\kappa\in\mathcal{K}_i}\left(1-e^{-\kappa}\right)V_F(w(\kappa)),$$

The optimization problem can then be written as

$$\max_{r \in \mathcal{R}} \frac{r\phi\left(1-\Psi_i\right)\widetilde{V}_{F,i}}{1+\left(r-1\right)\phi\left(1-\Psi_i\right)} - k_V - k_R\left(r-1\right).$$

The first derivative of the objective function with respect to r equals

$$\frac{\phi\left(1-\Psi_i\right)\left(1-\phi\left(1-\Psi_i\right)\right)}{\left(1+\left(r-1\right)\phi\left(1-\Psi_i\right)\right)^2}\widetilde{V}_{F,i} - k_R,$$

whereas the second derivative is equal to

$$-2\frac{\phi^2\left(1-\Psi_i\right)^2\left(1-\phi\left(1-\Psi_i\right)\right)}{\left(1+\left(r-1\right)\phi\left(1-\Psi_i\right)\right)^3}\widetilde{V}_{F,i}.$$

The latter is strictly negative for all $r \in \mathcal{R}$, which implies that the objective function is strictly concave. Solving the first order condition yields

$$\widehat{r}_i = \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}}\sqrt{\frac{\widetilde{V}_{F,i}}{k_R}} - \frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}.$$

Substituting this into the objective function gives after some manipulation the following relationship

$$\sqrt{\frac{k_V-k_R}{k_R}} = \sqrt{\frac{\widetilde{V}_{F,i}}{k_R}} - \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}},$$

which implies that

$$\widehat{r}_i = \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}\frac{k_V-k_R}{k_R}}.$$

Hence, the optimal recruitment technology r_i^* equals $\widehat{r}_i$ if $\widehat{r}_i > 1$, and 1 otherwise. □

A.7 Proof of Proposition 2

Proof. The proof closely follows Galenianos & Kircher (2005, 2009). □

A.8 Proof of Proposition 3

Proof. The proof closely follows Galenianos & Kircher (2005). □

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