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David de la Croix
Olivier Pierrard
Henri R. Sneessens

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David de la Croix
CORE and IRES, Université catholique de Louvain

Olivier Pierrard
Banque centrale du Luxembourg
and IRES, Université catholique de Louvain

Henri R. Sneessens
CREA, Université du Luxembourg,
IRES, Université catholique de Louvain and IZA

ABSTRACT

Aging and Pensions in General Equilibrium: Labor Market Imperfections Matter[*]

This paper re-examines the effects of population aging and pension reforms in an OLG model with labor market frictions. The most important feature brought about by labor market frictions is the connection between the interest rate and the unemployment rate. Exogenous shocks (such as aging) leading to lower interest rates also imply lower equilibrium unemployment rates, because lower capital costs stimulate labor demand and induce firms to advertize more vacancies. These effects may be reinforced by increases in the participation rate of older workers, induced by the higher wage rates and the larger probability of finding a job. These results imply that neglecting labor market frictions and employment rate changes may seriously bias the evaluation of pension reforms when they have an impact on the equilibrium interest rate.

Corresponding author:

Henri Sneessens
CREA, Université du Luxembourg
Campus Limpertsberg, BC 3.17
162 A, avenue de la Faïencerie
L-1511 Luxembourg
E-mail: henri.sneessens@uni.lu

1   Introduction

Declining fertility, increasing longevity, and the aging of the baby-boom generation will all contribute to substantial increases of the old-age dependency ratio, especially in Western countries. Most of the increase in the old-age dependency ratio is expected to take place between now and 2050. This has raised concern about the sustainability of existing pay-as-you-go (PAYG) public pension schemes (see for instance Gruber and Wise (2004, 2005)) and has motivated pension reform plans.

It is now well understood that a correct appraisal of population aging and pension reform effects calls for a general equilibrium approach. OLG models such as those developed by Auerbach and Kotlikoff (1987) (deterministic setting) and Ríos-Rull (1996) (stochastic set-up) provide a useful analytical framework, exploited in many papers to obtain quantitative evaluations (see for instance Börsch-Supan et al. (2002, 2006) and the series of papers presented at the Carnegie-Rochester Conference (2007)). All these studies point out that population aging will make labor relatively scarcer and change equilibrium factor prices. Wages are expected to rise and interest rates to fall. Because aging is a worldwide but asynchronized phenomenon, savings rates are expected to evolve differently in different countries, leading to significant international capital flows. Börsch-Supan et al. (2006) and Attanasio et al. (2007) both emphasized that a proper evaluation of the impact of aging on equilibrium prices and quantities calls for a multi-country approach. However, Attanasio et al. (2007) suggested that accounting for international capital flows might have little impact on the fiscal variables themselves, because of the counteracting effects of interest and wage rate changes (for instance, lower capital tax revenues compensated for by higher labor tax revenues following an increase in payroll or consumption taxes).

Most existing models rely though on quite streamlined representations of the labor market and assume perfect competition. The focus is on individual hours of work rather than employment. In this context, considering endogenous rather than exogenous labor supply decisions barely changes the estimated quantitative impact of population aging (see for instance Attanasio et al. (2007), Krüger and Ludwig (2007)). As noted by Lucas (2007), neglecting labor adjustments along the extensive margin may be quite misleading. Such adjustments can play a significant role, especially in countries (like many European ones) where labor market imperfections and institutions can have a strong, age-specific impact on job creation and destruction.

The aim of this paper is to allow for such effects and evaluate their quantitative contribution to the macroeconomic effects of population aging. To that end, we combine two strands of research, life-cycle and search/matching models. Labor market imperfections are modeled as in Pissarides (2000), with matching and wage bargaining. To concentrate on the extensive margin effects, we shut off the intensive margin channels by assuming that each worker supplies inelastically one unit of labor. Changes in total hours of work are solely due to changes in the

number of employed workers[1]. We furthermore allow for early-retirement effects.

It is well documented that participation rates vary over the life-cycle. Participation rates of middle-aged workers (25-55) are high and similar in both the US and the EU. The participation rates of older (>55) workers are however lower in the EU. The literature stresses that differences in older participation rates reflect cross-country differences in labor market frictions and in the generosity of early-retirement provisions. Several empirical studies have shown that financial incentives do have a significant impact on early-retirement decisions (see for instance Gruber and Wise (1999) or Duval (2003) and the references therein). Although the official retirement age is exogenous in most countries, the actual average retirement age is endogenous and contributes to explaining the lower employment rate of older workers. Like Heijdra and Romp (2007), we thus distinguish the normal (compulsory) retirement age (65 years) and the early eligibility age (55). Workers may choose to retire between 55 and 65, and receive then an early-retirement compensation. Like Börsch-Supan et al. (2006), we consider a stylized two-tier pension system, so that after 65 the revenue of pensioners is made up of two components, a public PAYG pension and a private pre-funded pension (the revenue from accumulated savings).

A few studies have already examined some of the consequences of introducing life-cycle features into frictional labor markets; none of them has however evaluated the effects of population aging. Hairault et al. (2010) examined early-retirement decisions and job search intensities in a search model à la McCall (1970), with exogenous wage distribution extended to include stochastic aging. The emphasis is on the relationship between the time to the retirement age and the financial incentives to invest in search activities. Chéron et al. (2008a,2008b) developed this modeling strategy further. They introduced a matching process similar to that used by Mortensen and Pissarides (1994) adapted to include finite lifetimes. They showed that in some circumstances (in particular at fixed, age-independent worker productivity) the job destruction rate is higher for older workers, because the time horizon relevant for the employment decision decreases with age. In these models, there is no post-retirement consumption and no savings; the interest rate is exogenous.

Our model has three main building blocks (i) demographics; (ii) labor market frictions and institutions (including early-retirement provisions); (iii) life-cycle consumption and savings. Ríos-Rull (2001) emphasized the importance of a careful description of the demographic process, which can vary a lot from country to country. To fit actual data, we use French mortality, fertility and migration rates, so that the size of each cohort in every period coincides with the most recent demographic projections. We assume no aggregate uncertainty. In line with most of the literature, we also assume perfect risk sharing across individuals of a given generation,

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[1]Börsch-Supan and Ludwig (2010) distinguish labor supply adjustments along both the extensive and the intensive margins and discuss the connections/interactions between pension reforms and labor market reforms. Changes along the extensive margin however remain exogenous to the model.

so that individuals are perfectly insured against life and income uncertainty.[2] There are no intended bequests. All workers are perfect substitutes, although the worker's productivity is age-dependent. There is no age discrimination, so that firms posting a vacancy may end up hiring a worker of any age category. We examine in this setup the effects of population aging and pension reforms on equilibrium unemployment and activity rates, and on the cost of public PAYG pensions as a percentage of GDP. In order to focus on labor market imperfections and their consequences, we do not at this stage build a multi-country model. We consider a single country, and illustrate the potential impact of international capital flows by comparing two polar cases, the closed and the small open economy case.

We use numerical simulations to examine the properties of our model and the consequences of aging and pension reforms. The most important feature brought about by labor market frictions is the connection between the interest rate and the unemployment rate. Exogenous shocks (like aging or pension reforms) leading to lower interest rates also imply lower equilibrium unemployment rates. This is because lower capital costs stimulate labor demand and induce firms to post more vacancies. The consequence is a higher employment rate and higher bargained wages. When participation of older workers is endogenous, these employment effects are reinforced by the increased participation rate of older workers, induced both by the higher wage rate and the larger probability of finding a job. It is shown in that context that neglecting labor market frictions and employment rate changes may lead to the underestimation of pension reform effects when they affect the equilibrium interest rate.

The remainder of this paper is organized as follows. The model is described in Section 2. The calibration is detailed in Section 3. Simulation results are presented and discussed in Section 4. in Section 5 we present our conclusions.

2   The Model

We develop a dynamic general equilibrium model with overlapping generations and frictions on the labor market. Frictions are modeled following Diamond, Mortensen, and Pissarides (DMP), with (exogenous) job destruction and a matching function. We assume perfect substitutability between all workers, although labor productivity is age-dependent. Perfect substitutability means that there is a single matching function (all vacancies can be filled by any worker, whatever his or her age). Age-directed search is not a credible strategy in our setup. Because the value of an unfilled vacancy is zero at equilibrium (free entry condition), a firm which wanted to target at, say, young workers, would eventually hire the first worker who applied,

---

[2]There are a few exceptions. Kruger and Ludwig (2007) and Hairault and Langot (2008), for instance, investigated the consequences of policy reforms (increased taxes or lower benefits) on inequality when financial markets are incomplete and individuals face idiosyncratic productivity shocks.

provided the surplus to be shared was positive. Bargained wages will reflect differences in work efficiency.

2.1 Demographics

We do not model education and human capital accumulation and focus on the behavior of people between 25 and 104 (the maximum life duration). One period of time lasts five years. Each member of a given generation can thus live for up to sixteen five-year periods (from age 25 till 104), indexed from 0 to 15. Let $Z_{a,t}$ denote the size of the generation reaching age a at period t. The size of new generations changes over time at an exogenous rate x_t:

$$Z_{0,t} = (1 + x_t) Z_{0,t-1}, \quad \forall t > 0, \tag{1}$$

where x_t includes both fertility and migration shocks at age zero. As in de la Croix et al. (2007), we assume that migrants of each generation have the same characteristics as natives. Abstracting from later migration shocks, the size of a given generation t declines deterministically through time. This size is determined by a cumulative survival probability $\beta_{a,t+a}$ so that:

$$Z_{a,t+a} = \beta_{a,t+a} Z_{0,t} + X_{a,t+a}, \quad \forall a \in \{1, 2, ...15\}, \tag{2}$$

where $0 \leq \beta_{a,t+a} \leq 1$ is decreasing in a, with $\beta_{0,t} = 1$. Migration flows after age 25 are taken into account through $X_{a,t+a}$. Total (adult) population at time t is equal to $Z_t = \sum_{a=0}^{15} Z_{a,t}$. The demographic growth and survival probability vector can vary over time. These changes are assumed to be exogenous.

We use the dummy variable $z_{a,t+a}$ to define the population of working age:

$$P_{a,t+a} = z_{a,t+a} Z_{a,t+a}, \tag{3}$$

where $z_{a,t+a} = 1$ for $0 \leq a \leq 7$, $z_{a,t+a} = 0$ otherwise. We assume a mandatory retirement age of 65, so that all people above 64 ($8 \leq a \leq 15$) are inactive. We further assume that the participation rate between 25 and 54 is exogenous and normalized to unity. Between 55 and 64, workers can choose to take early-retirement. People of working age are thus either employed (N), unemployed (U), or on an early-retirement scheme (E). Thus:

$$
\begin{aligned}
P_{a,t} &= N_{a,t} + U_{a,t} + E_{a,t}, \\
&= \left[n_{a,t} + u_{a,t} + e_{a,t} \right] P_{a,t}, \qquad 0 \leq a \leq 7 \\
\Leftrightarrow \quad 1 &= n_{a,t} + u_{a,t} + e_{a,t},
\end{aligned}
\tag{4}
$$

where lower-case n, u and e denote the proportion of individuals in each state. The early-retirement rate before 55 is zero ($e_{a,t} = 0$ for $a < 6$). Let $\lambda_{6,t}$ denote the fraction of people who

choose to retire early and leave the labor market at age $a = 6$ (between 55 and 60), so that the number of early retired workers of that age group is $E_{6,t} = \lambda_{6,t} P_{6,t}$. Similarly, let $\lambda_{7,t}$ denote the fraction of active workers who decide to leave the labor market at age $a = 7$ (between 60 and 64). The total number of workers on early-retirement at time t is then equal to:

$$
\begin{aligned}
E_{6,t} + E_{7,t} \;&=\; e_{6,t}\, P_{6,t} + e_{7,t}\, P_{7,t}\,, \\
\text{with} \qquad e_{6,t} &= \lambda_{6,t}\,, \\
e_{7,t} &= \lambda_{6,t-1} + \lambda_{7,t}\,(1 - \lambda_{6,t-1})\,.
\end{aligned}
\tag{5}
$$

2.2   Labor Market Flows

We assume a constant returns to scale matching function:

$$
M_t = M(V_t, \Omega_t)
\tag{6}
$$

where V_t and Ω_t stand respectively for the total number of vacancies and job seekers at the beginning of period t. Ω_t is the sum across all age categories:

$$
\begin{aligned}
\Omega_t &= \sum_{a=0}^{7} \Omega_{a,t}\,, \\
&= P_{0,t} + \sum_{a=1}^{5} \left[1 - (1-\chi)\,n_{a-1,t-1}\right] P_{a,t} \\
&\quad + \left[1 - (1-\chi)\,n_{5,t-1}\right](1 - \lambda_{6,t})\, P_{6,t} \\
&\quad + \left[(1 - \lambda_{6,t-1}) - (1-\chi)\,n_{6,t-1}\right](1 - \lambda_{7,t})\, P_{7,t}.
\end{aligned}
\tag{7}
$$

At the beginning of period t, all new entrants $P_{0,t}$ are job seekers. Except for early-retirement decisions, job separations are determined by an exogenous job destruction rate χ. Parameters $\lambda_{6,t}$ and $\lambda_{7,t}$ introduce the effects of early-retirement.

The probabilities of finding a job and of filling a vacancy are defined as follows:

$$
p_t = \frac{M_t}{\Omega_t} \qquad \text{and} \qquad q_t = \frac{M_t}{V_t}\,.
\tag{8}
$$

The number of employed workers in age group a is determined by the sum of non-destroyed jobs (when $a > 0$) and new hires:

$$
\begin{aligned}
n_{a,t} \;&=\; p_t\, \frac{\Omega_{a,t}}{P_{a,t}} & \text{for } a = 0\,; \\
&=\; (1-\chi)\,n_{a-1,t-1} + p_t\, \frac{\Omega_{a,t}}{P_{a,t}} & \text{for } 1 \le a \le 5\,; \\
&=\; (1 - \lambda_{a,t})(1-\chi)\,n_{a-1,t-1} + p_t\, \frac{\Omega_{a,t}}{P_{a,t}} & \text{for } 6 \le a \le 7\,.
\end{aligned}
$$

After substituting for $\Omega_{a,t}$, this equation becomes:

$$
\begin{aligned}
n_{a,t} &= p_t\,, & \text{for } a = 0\,; \\
&= (1 - p_t)(1 - \chi)\, n_{a-1,t-1} + p_t & \text{for } 1 \le a \le 5\,; \\
&= (1 - p_t)(1 - \lambda_{a,t})\,(1 - \chi)\, n_{a-1,t-1} + p_t(1 - \lambda_{a,t}) & \text{for } a = 6\,; \\
&= (1 - p_t)(1 - \lambda_{a,t})\,(1 - \chi)\, n_{a-1,t-1} + p_t(1 - \lambda_{a,t})(1 - \lambda_{a-1,t-1}) & \text{for } a = 7\,.
\end{aligned}
\tag{9}
$$

The same equation can be written in terms of the probability of filling a vacancy q_t by using $p_t = q_t\, V_t / \Omega_t$. Total employment will then be equal to:

$$
N_t = \sum_{a=0}^{7} N_{a,t}\,, \quad \text{with } N_{a,t} = n_{a,t}\, P_{a,t}\,.
$$

2.3 Households

It is assumed that each individual belongs to a representative household, one for each age category. There is no aggregate uncertainty, and all households have perfect foresight. There is perfect insurance against the adverse effects of individual lifetime uncertainty. There are no intended bequests. Participation rates of workers below 55 are assumed to be exogenous and normalized to unity. The household's decision variables are consumption, savings and early-retirement rates, subject to the lifetime budget constraint.

Participation Decisions and early-retirement

Participation decisions have been investigated separately in the OLG and the search/matching literature. Various contributions in the OLG literature have focused on the determinants of the optimal retirement age. Hu (1979) assumed that agents live for two periods. The optimal retirement age is determined by the optimal fraction of time devoted to leisure in the second period of life. He examined the effects of social security on capital accumulation and labor supply in a PAYG case. The short-run effects of pensions depended on the elasticities of demand and supply of labor; long-run effects were influenced by the elasticities of savings and bequests. Michel and Pestieau (1999) modeled the labor participation choice in a similar fashion. The fraction of time spent working in the second period declined when the size of the social security payment increase. For the first-best optimum (golden rule steady state) to be decentralized as a competitive equilibrium, both PAYG social security tax and the retirement age must be set optimally. De la Croix, Mahieu, and Rillaers (2004) also used a two-period lifetime setup to analyze retirement decisions. They focused on optimal dynamic adjustments following a fertility drop and showed that the optimal adjustment of the retirement age (the fraction of the second period devoted to leisure) depends on preference and technology parameters. Boucekkine et al.

(2002) and Heijdra and Romp (2009) have endogenous retirement in a continuous time OLG model with finite lifetimes and indivisible labor (participation is either 1 or 0). Heijdra and Romp (2009) focused on the effects of demographic shocks and pension benefits on individual early-retirement decisions. Their results suggest that only drastic reforms have a significant impact on individual retirement decisions.

None of the previous papers takes into account labor market frictions. Most contributions dealing with participation decisions in the search and friction literature have used models without savings and capital accumulation. Recently though, Ravn (2008) has used a stochastic growth model with infinitely lived consumers to investigate the role of endogenous participation decision on the transmission of shocks at business cycle frequencies. Labor is indivisible; individual workers are perfectly insured against idiosyncratic consumption risk, as in the lottery model developed by Rogerson (1988). This leads to a utility function linear in the participation rate, which implies a strong relationship between consumption and labor market tightness, the so-called consumption-tightness puzzle. Because of the linearity of the utility function, participation rates react too strongly to labor market tightness, which makes unemployment procyclical. To avoid this unrealistic implication, Den Haan and Kaltenbrunner (2009) assumed perfect risk sharing of both consumption and leisure. All consumers are part of a representative household and share their revenues. The household decides the per capita consumption level and the participation rate. This in effect amounts to assuming that labor is divisible, with each individual choosing the fraction of time spent on the labor market (as employed or unemployed). See Tripier (2003) for a similar approach.

Our model combines life-cycle features and labor market frictions. In order the avoid the consumption-tightness puzzle, we shall rely on the Den Haan and Kaltenbrunner (2009) specification and introduce it into our OLG structure. Without frictions, our specification would be similar to that used by Börsch-Supan et al. (2006) (i.e. in a perfect competition setup, with elderly workers characterized by stronger leisure preferences and choosing a smaller working time).

Optimization Program of the Representative Household

We can write the objective function of the household (effectively one cohort) as:

$$
W_t^H = \max_{c_{a,t+a}, \lambda_{6,t+6}, \lambda_{7,t+7}} \sum_{a=0}^{15} \beta^a \, \beta_{a,t+a} \left\{ \mathcal{U}(c_{a,t+a}) - d^n \, n_{a,t+a} \, z_{a,t+a} + d_a^e \, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi} \, z_{a,t+a} \right\} Z_{0,t} \,,
\tag{10}
$$

where β is a subjective discount factor[3], $\beta_{a,t+a}$ is a cumulative survival probability, $z_{a,t+a}$ is the working age dummy variable ($z_{a,t+a} = 1$ for $0 \leq a \leq 7$, 0 otherwise), and $Z_{0,t}$ is the initial size of the cohort. Instantaneous utility is assumed to be separable. The utility of per capita consumption c is represented by a concave function $\mathcal{U}$ (we shall use a logarithmic function). The marginal disutility of working is assumed to be constant[4] and equal to d^n. The extra utility derived from early-retirement is represented by a concave function of the early-retirement rate e ($0 < \phi < 1$). The decision variables are c, λ_6 and λ_7. These last two variables refer to the fraction of agents in the corresponding age groups who decide to take early-retirement and leave the labor market, at ages 55 and 60 respectively. Employment rates $n_{a,t+a}$ and early-retirement rates $e_{a,t+a}$ (and their the connection to the λ's) are given by Equations (5) and (9).

The household's flow budget constraint at time $t + a$ takes the form:

$$
\left[(1 - \tau_{t+a}^w) w_{a,t+a} \cdot n_{a,t+a} + b_{a,t+a}^u \cdot u_{a,t+a} + b_{a,t+a}^e \cdot e_{a,t+a} \right] \cdot z_{a,t+a} + b_{a,t+a}^i \cdot (1 - z_{a,t+a})
$$
$$
+ \frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} R_{t+a} s_{a-1,t+a-1} = (1 + \tau_{t+a}^c) c_{a,t+a} + s_{a,t+a} \tag{11}
$$

Wage and consumption tax rates are given by τ^w and τ^c respectively; $b_{a,t+a}^u, b_{a,t+a}^e, b_{a,t+a}^i$ are the replacement benefits received respectively by the unemployed worker, the early retiree and the pensioner; $s_{a,t+a}$ is the financial wealth accumulated at time $t + a$, in per capita terms. This financial wealth is held in the form of either shares or physical capital. Because there is perfect insurance against individual lifetime uncertainty (as if there were a perfect annuity market), the total return to savings is equal to the gross risk-free interest rate R_{t+a} divided by the survival probability $\beta_{a,t}/\beta_{a-1,t-1}$.

The optimal consumption plan must satisfy the usual Euler equation:

$$
\frac{u'_{c_{a,t+a}}}{1 + \tau_{t+a}^c} = \beta R_{t+a+1} \frac{u'_{c_{a+1,t+a+1}}}{1 + \tau_{t+a+1}^c} .
$$

After substitution and rearrangements, the condition determining the optimal proportion of early retirees aged 60 can be shown to be:

$$
\frac{b_{7,t+7}^e}{(1 + \tau_{t+7}^c) c_{7,t+7}} + d_7^e (e_{7,t+7})^{-\phi} = \pi_{7,t+7} \left[\frac{(1 - \tau_{t+7}^w) w_{7,t+7}}{(1 + \tau_{t+7}^c) c_{7,t+7}} - d^n \right]
$$
$$
+ (1 - \pi_{7,t+7}) \left[\frac{b_{7,t+7}^u}{(1 + \tau_{t+7}^c) c_{7,t+7}} \right] , \tag{12}
$$

where π is the unconditional probability of being employed (i.e., the probability that an active worker chosen at random is actually employed). A similar condition holds for early-retirement

---

[3]As stressed by Ríos-Rull(2001), β can represent both pure time preference and the effect of family size changes (implying that consumption is enjoyed differently at different ages).

[4]Our formulation normalizes the disutility of search activities of the unemployed to zero. Setting $d^n \geq 0$ amounts to assuming that the disutility of working can be larger than that of searching.

at age 55 (see details in the appendix). Equation (12) says that the household's optimal early-retirement rate is such that the marginal utility of early-retirement (early-retirement income plus leisure utility) is equal to the expected marginal utility of remaining active on the job market (wage income net of labor disutility and unemployment benefit, each weighted by their respective probability). Which member of the household will actually take early-retirement does not depend on the initial employment status. Both employed and unemployed workers may become early retirees. This follows from our specification of labor market flows (Equation (9)). Assuming that only previously unemployed workers can shift to early-retirement would be much too restrictive and unrealistic. Firms do take advantage of the generosity of early-retirement schemes to adjust the number of their employees, while elderly workers do agree to quit jobs earlier if the early-retirement compensation is appropriate.

For later use, we also note that the value of an additional job for a household of age a is given by:

$$
\frac{1}{u'_{c_{a,t}}} \frac{\partial W_t^H}{\partial N_{a,t}} = \frac{1}{u'_{c_{a,t}}} \frac{1}{z_{a,t} Z_{a,t}} \frac{\partial W_t^H}{\partial n_{a,t}}
$$

$$
= \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}} \beta^j \frac{u'_{c_{a+j,t+j}}}{u'_{c_{a,t}}} \left\{ \frac{(1 - \tau_{t+j}^w) w_{a+j,t+j} - b_{a+j,t+j}^u}{(1 + \tau_{t+j}^c)} - \frac{d^n}{u'_{c_{a+j,t+j}}} \right\} \frac{\partial n_{a+j,t+j}}{\partial n_{a,t}}
$$

$$
\tag{13}
$$

where $\partial n_{a+j,t+j}/\partial n_{a,t}$ can be obtained from Equation (9).

2.4 Firms

There are two productive factors, labor and capital. Labor is measured in efficiency units. Efficiency may vary across age (because of experience) and across generations (because of education). We define the total labor input as follows:

$$
H_t = \sum_{a=0}^{7} h_{a,t}.N_{a,t}.
$$

We assume a constant-return-to-scale production function in labor and capital:

$$
Y_t = A_t F(K_t, H_t), \tag{14}
$$

where A_t stands for total factor productivity. Firms rent capital at cost $v_t = R_t + \delta - 1$ and pay a gross wage $w_{a,t}$ to workers of age a. We denote by ζ the employer wage tax. The representative

firm maximizes the discounted value of all the dividends (profits) that will be distributed to its shareholders. Profits at time t are given by:

$$\Pi_t = F(K_t, H_t) - v_t K_t - \sum_{a=0}^{7} (1 + \zeta_t) w_{a,t} N_{a,t} - \mathsf{a} V_t \tag{15}$$

where "a" stands for the cost of posting a vacancy. The value of the firm can thus be written as:[5]

$$W_t^F = \max_{K_t, V_t} \left\{ F(K_t, H_t) - v_t K_t - \sum_{a=0}^{7} (1 + \zeta_t) w_{a,t} N_{a,t} - \mathsf{a} V_t \right\} + R_{t+1}^{-1} W_{t+1}^F \tag{16}$$

subject to Equation (9) and $p_t = q_t V_t / \Omega_t$. The first-order optimality conditions are

$$v_t = F_{K_t}, \tag{17}$$

$$\mathsf{a} = q_t \sum_{a=0}^{7} \frac{\Omega_{a,t}}{\Omega_t} \frac{\partial W_t^F}{\partial N_{a,t}}, \tag{18}$$

where $\frac{\partial W_t^F}{\partial N_{a,t}}$ is the value at time t of an additional worker of age a. With a job destruction rate χ, this value is equal to:

$$\begin{aligned}
\frac{\partial W_t^F}{\partial N_{a,t}} &= \frac{1}{z_{a,t} Z_{a,t}} \frac{\partial W_t^F}{\partial n_{a,t}} \\
&= \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}} R_{t,t+j}^{-1} (1 - \lambda_{a+j-1,t+j-1}) (1 - \lambda_{a+j,t+j}) (1 - \chi)^j \\
&\qquad \cdot \left\{ h_{a+j,t+j} F_{H_{t+j}} - (1 + \zeta_{t+j}) w_{a+j,t+j} \right\},
\end{aligned} \tag{19}$$

where $\lambda_{a+i,t+i} \equiv 0$ for $a + i < 6$.

2.5 Government

We assume that unemployment and (early or standard) retirement benefits are determined by an exogenous fraction of the relevant gross wage, so that

$$\begin{aligned}
b_{a,t}^u &= \rho_t^u w_{a,t} &&\text{for} \quad 0 \leq a \leq 7 \,; \\
b_{a,t}^e &= \rho_t^e w_{a,t} &&\text{for} \quad 6 \leq a \leq 7 \,; \\
b_{a,t}^i &= \rho_t^i \sum_{i=0}^{3} \frac{w_{a-i,t-i}}{4} &&\text{for} \quad 8 \leq a \leq 15 \,.
\end{aligned} \tag{20}$$

---

[5]Shareholders may belong to different age groups and have different consumption levels. Still, they all have the same discount factor given by $\tilde{\beta}_{t+1} = \beta \frac{u'_{c_{a+1,t+1}}}{u'_{c_{a,t}}} = R_{t+1}^{-1}$, $\forall a \in \{0, 14\}$.

11

The retirement benefit is computed from the average wage during the last four periods. Total transfer expenditures are then equal to:

$$T_t = \rho_t^u \sum_{a=0}^{7} w_{a,t}\, u_{a,t}\, Z_{a,t} + \rho_t^e \sum_{a=6}^{7} w_{a,t}\, e_{a,t}\, Z_{a,t} + \rho_t^i \sum_{i=0}^{3} \frac{w_{a-i,t-i}}{4} \sum_{a=8}^{15} Z_{a,t}. \tag{21}$$

Public consumption is assumed to be a fraction of output, net of vacancy costs, i.e.

$$G_t = \bar{g}\,(Y_t - \mathsf{a}\,V_t). \tag{22}$$

We further assume that the "government" balances its budget in every (five-year) period by adjusting consumption taxes (i.e., τ_t^c is the adjusting variable):[6]

$$\tau_t^c\, C_t + (\tau_t^w + \zeta_t)\left(\sum_a w_{a,t}\, n_{a,t}\, P_{a,t}\right) = G_t + T_t\,, \tag{23}$$

where aggregate consumption $C_t = \sum_a c_{a,t}\, Z_{a,t}$. For convenience, we assume no public debt. Public debt could be introduced by postulating an exogenous path of the debt, and assuming that the deficit adjusts (via τ^c) to match that path.

2.6 Wages

Wages are renegotiated in every period. They are determined by a standard Nash bargaining rule:

$$\max_{w_{a,t}} \left(\frac{\partial W_t^F}{\partial N_{a,t}}\right)^{1-\eta} \left(\frac{1}{u'_{c_{a,t}}} \frac{\partial W_t^H}{\partial N_{a,t}}\right)^{\eta}. \tag{24}$$

where $\partial W_t^H/\partial N_{a,t}$ is given by Equation (13) and $\partial W_t^F/\partial N_{a,t}$ by Equation (19).

The first-order optimality condition can then be written:

$$(1-\eta)\frac{1}{u'_{c_{a,t}}}\frac{\partial W_t^H}{\partial N_{a,t}} = \eta\,\frac{1-\tau_t^w}{(1+\zeta_t)(1+\tau_t^c)}\frac{\partial W_t^F}{\partial N_{a,t}}. \tag{25}$$

2.7 Capital Market Equilibrium

Let Q_t denote the total financial value of firms at time t. In our deterministic setup, the return on equities must be equal to the market interest rate. In other words, the value of equities must be such that, for all $t \geq 0$:

$$\frac{Q_{t+1} + \Pi_{t+1}}{Q_t} = R_{t+1}\,. \tag{26}$$

The left-hand side is the return of one unit of savings investment in equities while the right-hand side is the return if invested in firms' bonds.

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[6]Changes in τ_t^c affect all incomes in the same way, while changes in τ_t^w, for instance, would change net replacement rates.

Equilibrium on the capital market then implies that:

$$K_{t+1} + Q_t = \sum_{a=0}^{14} s_{a,t} Z_{a,t} . \tag{27}$$

2.8   Intertemporal General Equilibrium

We shall consider two versions of the model, one with frictions and the other with perfect competition on the labor market. The two corresponding intertemporal general equilibria are formally defined In Definitions 1 and 2 below.

Definition 1 *Given the following exogenous processes and initial conditions:*

- *demographic variables $\{x_t\}_{t=0..+\infty}$ (fertility), $\{\beta_{a,t}\}_{t=0..+\infty}^{a=0..15}$ (mortality), $\{X_{a,t}\}_{t=0..+\infty}^{a=0..15}$ (migration),*

- *policy variables $\{\rho_t^u, \rho_t^e, \rho_t^i\}_{t=0..+\infty}$ (replacement rates) and $\{\tau_t^w, \zeta_t\}_{t=0..+\infty}$ (tax rates),*

- *initial population $\{Z_{a,-1}\}^{a=0..15}$, assets $\{s_{a,-1}\}^{a=0..14}$ and capital stock $\bar{K}_0 < \sum_{a=0}^{14} s_{a,-1} Z_{a,-1}$,*

an inter-temporal equilibrium with perfect foresight and labor market frictions *is such that:*

1. *saving $\{s_{a,t}\}_{t=0..+\infty}^{a=0..14}$, consumption $\{c_{a,t}\}_{t=0..+\infty}^{a=0..15}$ and early-retirement decisions $\{e_{a,t}, \lambda_{a,t}\}_{t=0..+\infty}^{a=6,7}$ maximize households' utility (10) subject to budget constraint (11) and to (5);*

2. *capital input $\{K_t\}_{t=0..+\infty}$, posted vacancies $\{V_t\}_{t=0..+\infty}$ and output $\{Y_t\}_{t=0..+\infty}$ maximize firms' profits (16) subject to (8), (9), (14), and $K_0 = \bar{K}_0$;*

3. *the number of new hires $\{M_t\}_{t=0..+\infty}$, the probability of finding a job $\{p_t\}_{t=0..+\infty}$ and of filling a vacancy $\{q_t\}_{t=0..+\infty}$, and the employment rates $\{n_{a,t}\}_{t=0..+\infty}^{a=0..7}$ satisfy the matching technology (6), (8) and (9);*

4. *total population and the population of working age $\{Z_{a,t}, P_{a,t}\}_{t=0..+\infty}^{a=0..7}$, and the number of job seekers $\{\Omega_t\}_{t=0..+\infty}$ satisfy the population dynamics (1), (2), (3) and (7);*

5. *unemployment $\{u_{a,t}\}_{t=0..+\infty}^{a=0..7}$ is such that the time constraint (4) holds;*

6. *wages $\{w_{a,t}\}_{t=0..+\infty}^{a=0..7}$ are negotiated following the Nash bargaining rule (24);*

7. *government benefits $\{b_t^u \, b_t^e, b_t^i\}_{t=0..+\infty}$ follow the rules defined by (20), and government spending $\{G_t\}_{t=0..+\infty}$ follow (22);*

8. consumption taxes $\{\tau_t^c\}_{t=0..+\infty}$ are set by the government to balance its budget (23);

9. stock market prices and interest rate $\{Q_t, R_t\}_{t=0..+\infty}$ satisfy the arbitrage condition (26) and the financial market clearing condition (27).

Definition 2 *Given the same exogenous processes and initial conditions as in Definition 1,* **an inter-temporal equilibrium with perfect foresight with frictionless labor market** *is such that:*

1. *saving* $\{s_{a,t}\}_{t=0..+\infty}^{a=0..14}$, *consumption* $\{c_{a,t}\}_{t=0..+\infty}^{a=0..15}$ *and early-retirement decisions* $\{e_{a,t}, \lambda_{a,t}\}_{t=0..+\infty}^{a=6,7}$ *maximize households' utility (10) subject to budget constraint (11) and to (5);*

2. *capital input* $\{K_t\}_{t=0..+\infty}$, *posted vacancies* $\{V_t\}_{t=0..+\infty}$ *and output* $\{Y_t\}_{t=0..+\infty}$ *maximize firms' profits (16) subject to* $q_t = 1$, *(9), (14),* $a = 0$ *and* $K_0 = \bar{K}_0$;

3. *the number of new hires* $\{M_t\}_{t=0..+\infty}$ *satisfies* $M_t = V_t$, *the probability of finding a job* $\{p_t\}_{t=0..+\infty}$ *and of filling a vacancy* $\{q_t\}_{t=0..+\infty}$ *satisfy* $q_t = p_1 = 1$, *and the employment rates* $\{n_{a,t}\}_{t=0..+\infty, a=0..7}$ *satisfy (9);*

4. *total population and the population of working age* $\{Z_{a,t}, P_{a,t}\}_{t=0..+\infty}^{a=0..7}$, *and the number of job seekers* $\{\Omega_t\}_{t=0..+\infty}$ *satisfy the population dynamics (1), (2), (3) and (7);*

5. *wages* $\{w_{a,t}\}_{t=0..+\infty}^{a=0..7}$ *are such that* $n_{a,t} + e_{a,t} = 1$;

6. *government benefits* $\{b_t^u \, b_t^e, b_t^i\}_{t=0..+\infty}$ *follow the rules defined by (20);*

7. *consumption taxes* $\{\tau_t^c\}_{t=0..+\infty}$ *are set by the government to balance its budget (23);*

8. *the interest rate* $\{R_t\}_{t=0..+\infty}$ *satisfies the financial market clearing condition (27).*

3   Calibration

Our objective is to reexamine the channels by which population aging affects the economy in an OLG setup with labor market frictions. More specifically we want to evaluate the role of labor market imperfections and the interactions between pension reforms and the labor market. Our calibration is meant to represent a European situation such as France with fairly generous unemployment benefits and public pensions, and substantial early-retirement benefits inducing low participation rates among elderly workers. The parameters of the model fall into four categories: (i) general technological and preference parameters; (ii) parameters specific to the frictional labor market; (iii) parameters determining taxes and transfers; (iv) parameters determining demographic changes (fertility, mortality, migration).

Technological and Preference Parameters

We assume a constant-returns-to-scale Cobb-Douglas production function. The elasticity of output with respect to capital is set at $\alpha = 0.33$. In order to focus on the effects of demographic changes, we leave aside technological progress and assume constant values of the TFP and age-specific human capital parameters (A_t and $h_{a,t}$ respectively). To reproduce the life cycle profile of wages, we assume that a worker's productivity increases with age until 50, and then decreases, but only very slowly so, as suggested by empirical findings. Bargained wages broadly follow the same pattern, with a negative age effect (due the shorter expected job duration of older workers) on top of the productivity effect.

We assume that the instantaneous utility is logarithmic in consumption, so that the wealth and substitution effects of a change in the interest rate cancel each other out. The subjective rate of time preference is given the customary value of 1% per quarter. The depreciation rate of capital is then set at 2.5% per quarter, so as to obtain a capital-output ratio equal to 2.50 on a yearly basis. There is no bequest motive. Given the assumed demographic structure (see below), the resulting steady state real interest rate is 5.27% p.a. Individual consumption rises over the life cycle and savings are negative during the first two periods of life.

We set the labor disutility parameter d^n at 0.25 (which, at the steady state consumption level, represents about 15% of the wage income). The values of the two leisure (early-retirement) parameters $d_6^e = 0.031$ and $d_7^e = 0.049$ are chosen to give early-retirement rates for workers aged 50-59 and 60-64 respectively equal to 15% and 50% in the year 2000 (7.4% and 19% in the final steady state). Parameter ϕ is set to 0.80, which implies a Frisch elasticity of around 0.6, in line with estimated values (see Den Haan and Kaltenbrunner (2009))[7].

Labor Market Parameters

The matching process is represented by a constant-returns-to-scale Cobb-Douglas function. The elasticity of matches with respect to vacancies ν and the bargaining power of workers η are both fixed at 0.50. The job separation rate χ is set at 2% per quarter. The matching efficiency parameter μ and the vacancy cost a are chosen to be 0.90 and 43.5 respectively, so that the steady state probabilities (over a five-year period) of finding a job p and of filling a vacancy q are both equal to 90%. This value of the vacancy cost a represents 33% of the gross wage of a 45-year-old worker. With these parameter values, the steady state unemployment rates are 10% for young workers (25-29), around 3.8% for mature adults (35-54), and around 3.3% for workers of early-retirement age.

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[7]The Frisch elasticity is $\frac{e}{1-e} \frac{1}{\phi}$.

Symbol	Value	Symbol	Value
Production function			
A	20.000	α	0.33
δ (quarterly)	0.025		
Age-dependent productivity			
h_0	2.70	h_1	3.30
h_2	3.85	h_3	4.45
h_4	5.00	h_5	5.80
h_6	5.60	h_7	5.50
Preferences			
β (quarterly)	0.990	d^n	0.250
d_6^e	0.031	d_7^e	0.049
ϕ	0.800		
Labor Market			
μ	0.90	ν	0.50
χ (quarterly)	0.02	a	43.50
η	0.50		
Policy variables			
ζ	0.35	τ^w	0.150
$\bar{g}$	0.20	ρ^u	0.400
ρ^e	0.50	ρ_t^i (year 2000)	0.550
ρ_t^i (year 2050)	0.35	ρ_t^i (year 2100)	0.325

Table 1: Parameter values

Taxes and Transfers

Government consumption is a constant fraction of GDP ($\bar{g}$ =20%). The gross replacement rate for unemployed workers is fixed at $\rho^u = 0.40$, implying a net replacement rate of about 47%. Actual net replacement rates vary a lot across countries and unemployment durations. The chosen value is close to the OECD median value for a two-year unemployment spell (see OECD (2009), Table 1.6). The replacement rate for early-retirement is assumed to remain constant, at $\rho^e = 0.50$. The reference wage used to compute pension benefits is typically an average of wages in the best years of activity. For simplicity, we define the reference wage as the average wage of workers aged 45-64. At a given replacement rate, our formulation implies that pensions are indexed on current wages (rather than prices). We calibrate the pension replacement rate ρ_t^i on French data, as reported in EC (2009). The 2000 value is $\rho^i = 0.55$. The French pension reforms of 1993 and 2003 are however expected to progressively decrease this replacement rate through three main channels: (i) reference wage calculated over a longer period; (ii) indexation to prices rather than wages; (iii) increase in the number of years of activity required to have full pension rights[8]. The effects of these reforms will appear progressively in future years and should mean a 20 percentage points decrease in the value of the replacement rate by 2035 (30 percentage points by the year 2050). These changes are included in our base scenario (see Table 1).

Demographic Variables

Expected population growth rates vary across countries. We chose to calibrate the main demographic variables on French data. Survival probabilities from 1900 till 2100 are taken from Vallin and Meslé (2001). The expected decrease in mortality rates is illustrated in the left panel of figure 1. These changes imply that life expectancies at birth would increase from 77.9 years in 2000 to 86.5 in 2050 and 90.8 in 2095. Fertility and migration shocks (variables x_t and $X_{a,t+a}$) are calibrated so as to reproduce the population changes (by age category) observed or expected to take place from 1900 till 2100. The projections till 2050 are the latest ones published by INSEE (see Robert-Bobée (2006), updated in April 2007)[9]. We assume that the population stabilizes progressively after 2050. These projections are illustrated in the right panel of Figure 1. The reported population changes (total and adults over 24) imply a quasi-constant population of age 25-64. The dependency ratio (defined as the ratio between the population over 64 and that aged 25-64) increased around 10 percentage points between 1950 and 2000; it is expected to double during the next sixty years, reaching 60% in 2060 and stabilizing at 69% after 2100.

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[8]See Bozio (2008) for an *ex post* evaluation of the effects of this reform.

[9]Available at http://www.insee.fr/fr/themes/document.asp?ref_id=PROJPOPACT0650#a1
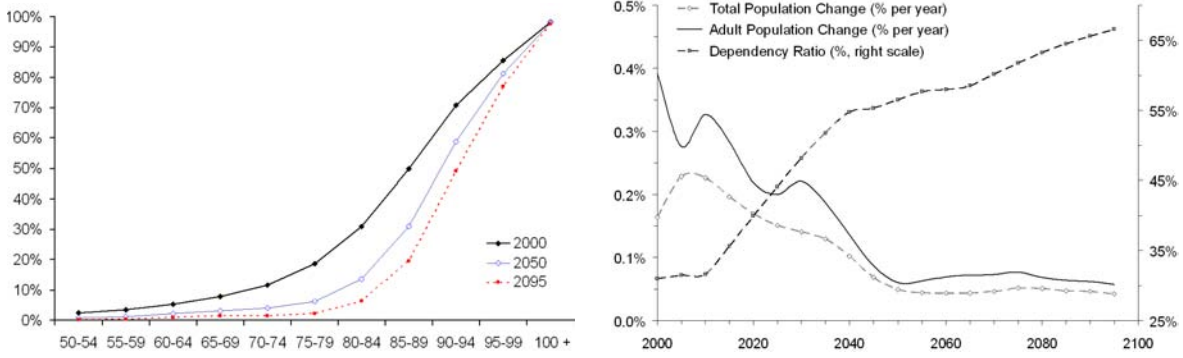
Figure 1: Projected age-specific death rates (left), (adult) population growth and dependency ratio (right)

Eigenvalues

Once the model has been calibrated, we can assess the convergence speed by computing the eigenvalues of the Jacobian matrix of the linearized system around the steady state. It is generally found that such eigenvalues are complex numbers which generate non-monotonic dynamics (see Azariadis, Bullard and Ohanian, 2004). Figure 2 represents the stable eigenvalues of the two models. Considering only the largest eigenvalues, which are those that matter as far as sluggishness is concerned, we observe that those of the model with frictions are larger, but the difference is very small. We conclude from this that labor market frictions do not mean more sluggishness compared to a Walrasian set-up, at least in an OLG model with five-year periods.

4 Simulation Results

We first present the simulation results obtained in the base scenario and illustrate the specific impact of migration and fertility shocks. We then highlight the role of labor market imperfections by contrasting the results of the base scenario with those obtained under the assumption of a competitive labor market. To illustrate the properties and quantitative implications of our model, we next examine two hypothetical policy scenarios assumed to take place after 2000, the elimination of early-retirement payments and the shift to a fully-funded pension scheme. We finally discuss the sensitivity of our results, especially with respect to international capital flows.
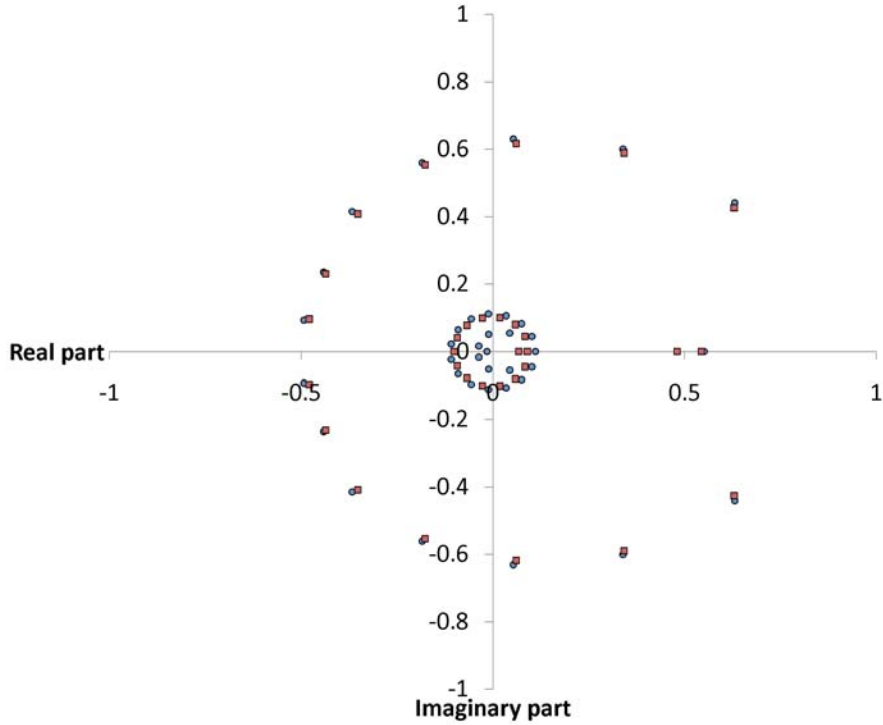
Figure 2: Eigenvalues of the model with (circles) and without (squares) frictions

4.1   Base Scenario

The base scenario incorporates the following exogenous shocks, described in the previous section: (i) decreasing mortality rates; (ii) fertility and migration shocks; (iii) a decreasing replacement rate ρ_t^i after 2005, as a consequence of the 1993 and 2003 pension reforms. We simulate the model under the assumption that the year 1900 was a steady state. After 2100, all exogenous variables keep the same value and the economy progressively converges to a new, final steady state. The period of interest is the transition taking place during the 21st century (2000-2100). Values simulated for that sub-period are unlikely to be affected by the initial conditions (1900 values).

Main Results

Figure 3 shows the change in some key variables in the base scenario (solid line). Disregard temporarily the dotted line. Population aging implies increased savings and capital accumulation (not shown in the figure). Because of diminishing returns, the interest rate decreases. Capital accumulation also leads to a substantial increase in average labor productivity and wages, and to an increase in the aggregate employment rate (about 6.4 percentage points). This higher
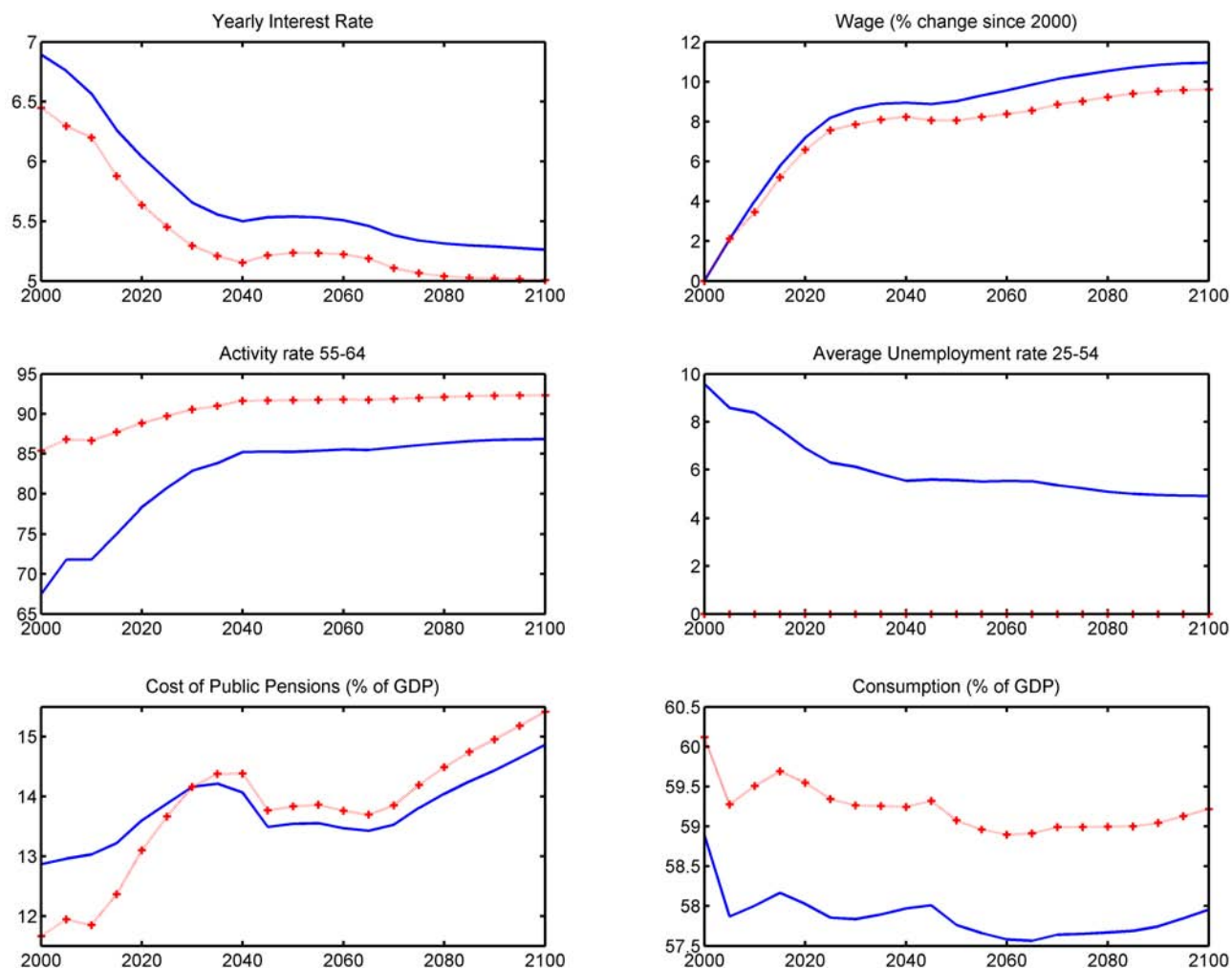
19

Figure 3: The base scenario in frictional (solid line) and perfectly competitive (dotted line) economies

employment rate comes from both a decrease in unemployment for young and mature workers (25-54) and from an increase in the participation rate of elderly workers (55-64). These effects are related to capital accumulation and lower interest rates, which increase profitability and the number of job openings. They imply a higher optimal average retirement age (less early-retirement), in an unchanged institutional environment. At given replacement rate, the higher participation rate of elderly workers is due mainly to the increased probability of having a job (see Equation (12)), which we can interpret as a lower "discouragement" effect.

The estimated cost of public pensions (including early-retirement benefits, and measured in % of GDP) follows quite closely the projected values for France given in EC (2009), for the period 2000-2060, under the assumption of an (approximately) 5 percentage points (exogenous) increase in the aggregate employment rate. Thanks to the pension reforms decided in 1993-
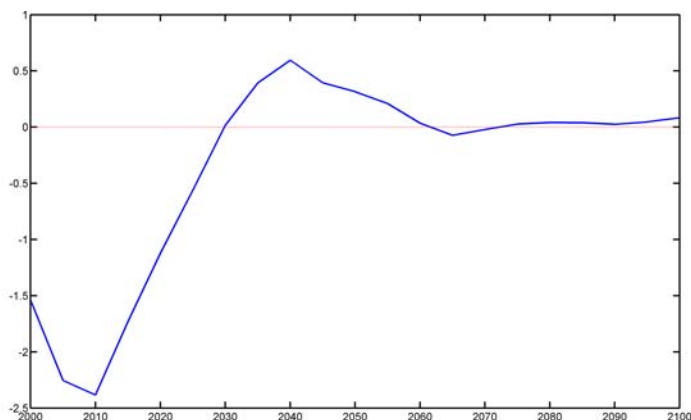
Figure 4: Contributions of fertility and migration shocks to the cost of public pensions (changes in % of GDP)

2003, the increase in public pension costs over the period 2000-2060 is no more than 1-1.5 percentage point (instead of 7 percentage points in a simulation without pension reform). We find however that, after 2070, the continuing aging of the population may lead to additional and substantial increases in the cost of public pensions.

The Contribution of Fertility and Migration Shocks

The cost of a given public pensions scheme in percent of GDP depends both on the expected increases in longevity and the size of the migration and fertility shocks. To separate these two effects, we simulated the model again with all fertility and migration variables set at their 2100 values. The results are summarized in Figure 4, which indicates by how much the cost of public pensions is changed by current and past fertility and migration shocks. During the first two decades, the effect is strongly negative. This is the continuation of the baby boom effect, which maintains the dependency ratio at low levels. After 2025, the effect becomes positive: baby-boomers retire, and the dependency ratio remains above normal for about five periods. After 2060, the effect of fertility and migration shocks becomes negligible, ie, the increase in the cost of public pensions after 2060 (see Figure 3) is solely driven by longevity effects.

4.2   Competitive vs Frictional Labor Markets

Increased longevity changes the savings behavior of households and pushes the equilibrium interest rate downwards. Firms use more capital, which pushes labor productivity and labor demand upwards. The final impact on employment depends on the working of the labor market. In models with search frictions, labor demand changes may affect the equilibrium

21

(un-)employment rate. To evaluate the quantitative importance of such adjustments along the extensive margin, we simulate the equilibrium with a frictionless labor market (Definition 2). The participation decision of elderly workers remains endogenous and determined in exactly the same fashion as before (see Equation (12) for workers aged 60-64), where the employment probability for participating workers is now equal to one ($\pi_{7,t+7} = 1$).

The comparison between the two economies is summarized in Figure 3. Values for the perfectly competitive economy are represented by a dotted line. In the competitive setup, employment rate changes can only arise from changes in the participation rate of elderly workers. Because there is no unemployment risk and the market wage is higher, there is little early-retirement in the competitive economy, although the generosity of the early-retirement scheme is the same as in the frictional labor market economy. Despite increased labor productivity and wages, the participation rate of elderly workers thus barely changes and has a negligible impact on the aggregate employment rate. The comparison with the frictional labor market economy is striking. In the frictional market, the increased labor demand leads to higher employment probabilities for all age groups. A higher employment probability has a significant impact on the participation of elderly workers. Overall, the employment rate increases significantly, and stimulates further capital accumulation and investment. Altogether, the aggregate employment rate increases by more than 6%.

These differences explain why the two economies react differently to the pension reforms and the ensuing decrease in compensation rates. The cost of public pensions is initially smaller in the competitive economy (because there is less early-retirement). In the next 30 years, it increases by 2.7 percentage points and becomes larger than in the frictional economy (where the increase is 1.3 percentage points). This difference reflects the different impact of pension reforms on employment rates, and the failure of the competitive model to account for adjustments along the extensive margin.[10]

4.3 Policy Scenarios

We focus on two policy scenarios and compare their implications in the frictional and the competitive economies. The first scenario is a complete elimination of all early-retirement payments after 2000. The second is a shift to a fully-funded system, announced in 2005 and taking place in 2015. These two scenarios are admittedly extreme ones. They facilitate the

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[10]The comparison between the competitive and the frictional economies does not change much if we keep the activity rates of older workers fixed at their 2000 values. Employment rates then change only as a result of unemployment rate changes. In that scenario, the increase in pension costs between 2000 and 2035 is 1.11 percentage points larger in the competitive economy than in the frictional economy.
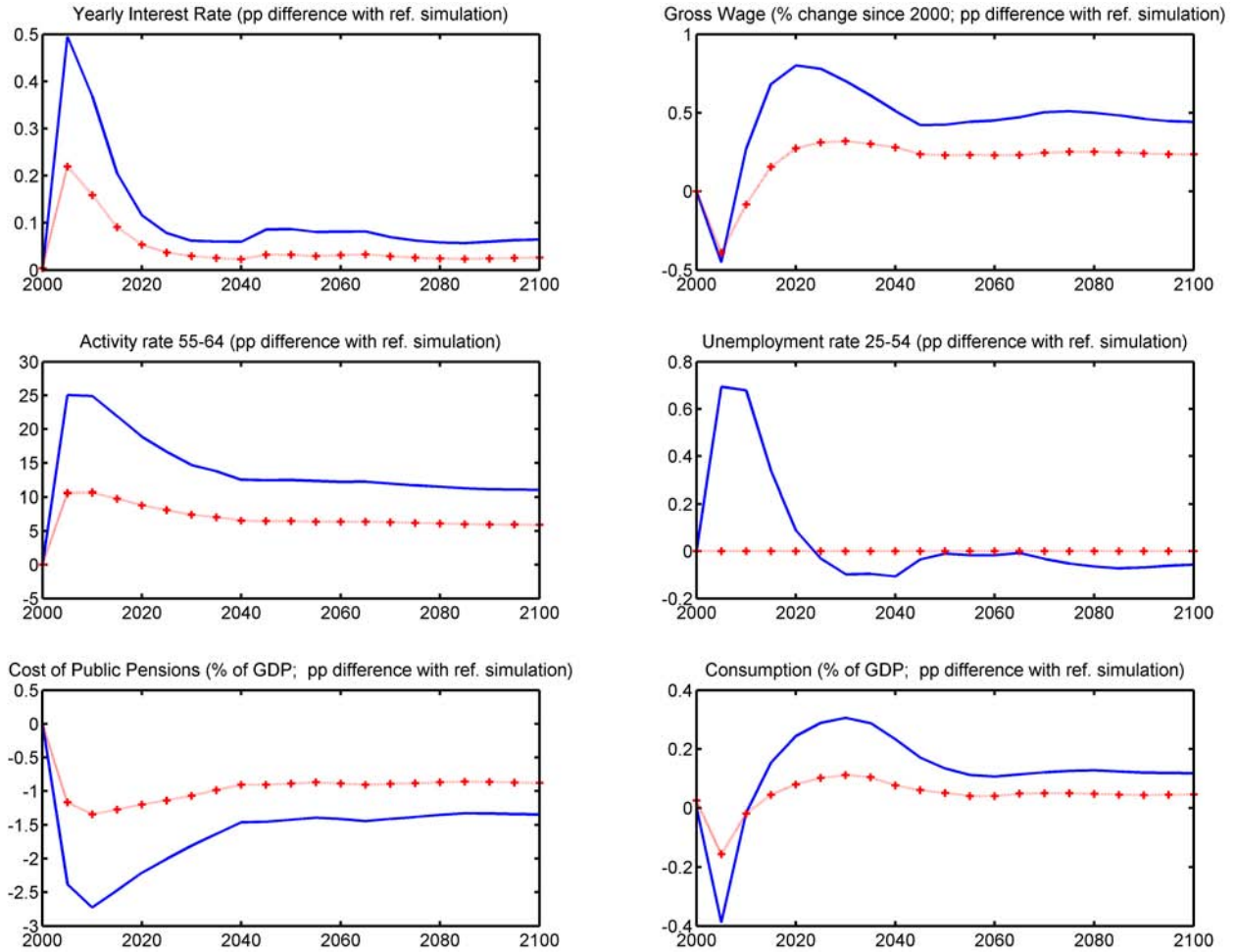
Figure 5: The consequences of eliminating early-retirement benefits after 2000, in the frictional (solid line) and the competitive (dotted line) economy

comparisons between the two economies (frictional and competitive) and the interpretation of the mechanisms at work.

Early-retirement reform

The consequences of the elimination of all early-retirement benefits are illustrated in Figure 5. This labor market policy change stimulates the participation of elderly workers, both in the frictional and in the competitive economy. All effects are qualitatively similar, but of a totally different magnitude, and much stronger in the frictional economy. In the competitive economy, because there is no unemployment and the equilibrium wage is higher, the initial participation rate is already high, and increases by 10 percentage points (from 85 to 95% at the time of the shock), compared to 25 percentage points (from 68 to 93%) in the frictional economy.

The induced impact effects on employment, capital productivity (the interest rate), wages and consumption (both decline at impact) are thus much more pronounced in the frictional economy. There is also (at impact) an increase in the unemployment rate of younger workers. Eventually, the unemployment rate returns to its initial value, but employment, wages and output increase more in the frictional economy. The cost of public pensions (in % of GDP) decreases in the long run by 1.5 percentage points in the frictional economy (1 percentage point in the competitive economy). Short run effects are larger, because the activity rates of older workers also react in the longer run to changes in the unemployment rate and other variables engineered by capital accumulation and the falling interest rate.

Shift from a PAYG to a FF Pension Scheme

Figure 6 illustrates the effects of announcing in 2005 that a shift to a purely fully-funded system will take place in 2015. We first consider the frictional economy. At impact, aggregate consumption falls, and savings and capital accumulation rise, which leads (with a one-period lag) to a lower interest rate. The accumulation of capital and its positive effect on labor productivity stimulates labor demand and job creation (the unemployment rate starts falling) as well as wage growth. The activity rate of elderly workers increases, by 12 percentage points at impact (when consumption has fallen) and 5 percentage points in the long run (when consumption has increased). The equilibrium unemployment rate is about 2.5 percentage points lower in the long run, and implies a substantially larger output (and consumption) level (not shown in the figure).

In the competitive economy, the unemployment rate cannot vary; the participation rate of elderly workers increases by 3 percentage points. Aggregate employment barely changes. Capital accumulation leads to a larger output level, but this effect is not reinforced by job creation, as in the frictional economy.

4.4   Sensitivity Analysis: International Capital Flows

The objective of this section is to check to what extent our results might depend on specific assumptions. We repeated the previous simulation experiments for different values of a few key parameters: the subjective rate of time preference ($\beta = 1$ instead of 0.99), bargaining power ($\eta = 0.70$ instead of 0.50), concavity of leisure utility ($\phi = 0.90$ or more, instead of 0.80). Such changes did not alter our main conclusions and are not reproduced here.[11] Instead we focus on the role of international capital flows and their interactions with labor market imperfections.

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[11]As discussed above, assuming linear preferences in leisure ($\phi = 0$) would imply an infinitely large Frisch elasticity and unrealistically large swings in participation rates.
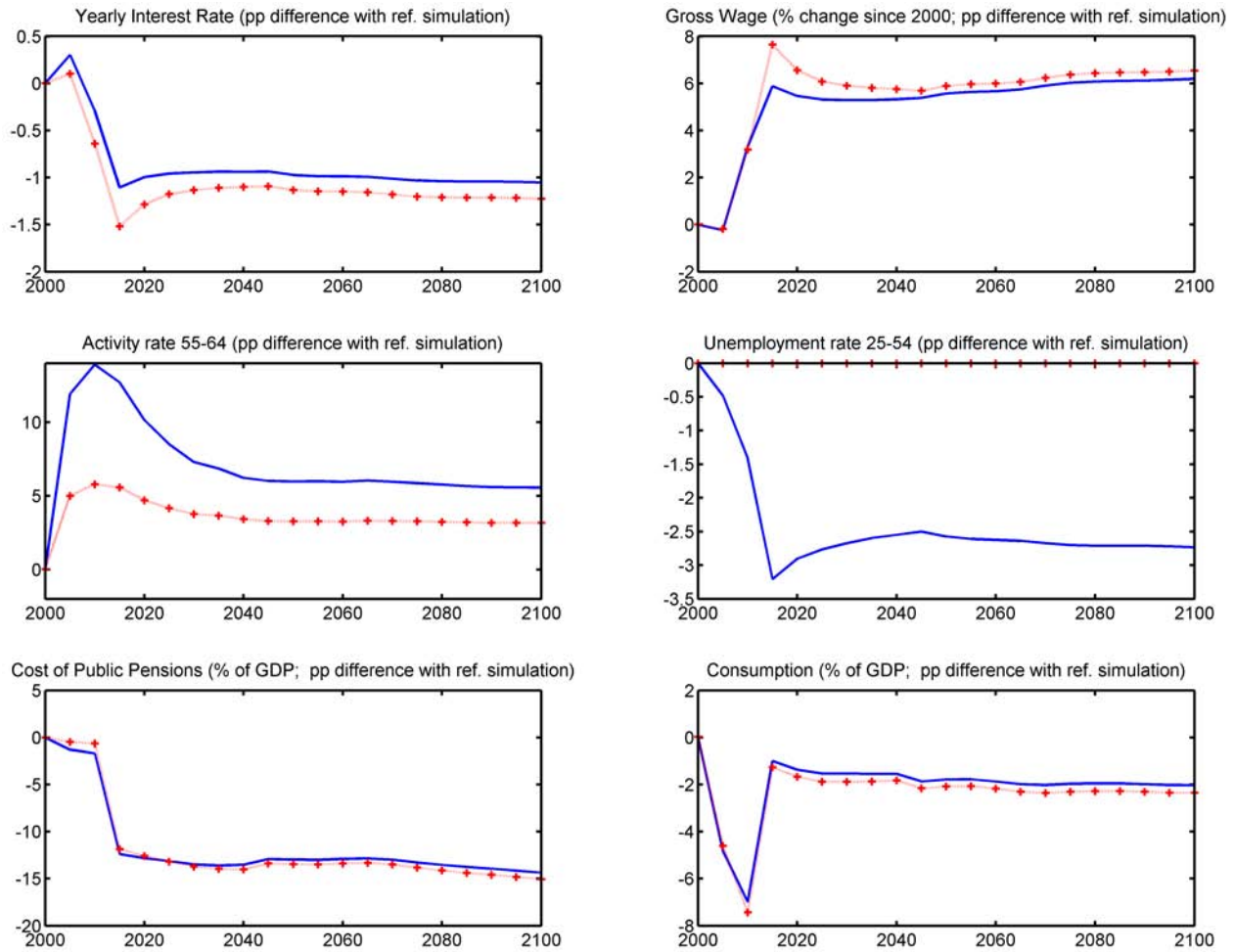
Figure 6: The consequences of announcing in 2005 a shift to a fully-funded system in 2015, in the frictional (solid line) and the competitive (dotted line) economy

We do so by contrasting the closed economy scenario with the small open economy one. In the latter, the interest rate is exogenous. Most actual economies are in between these two extremes.

Figure 7 shows the development of the key variables already considered in the base scenario (see Figure 3) and compares them to the values obtained with the small open economy model.

In both the closed and the small open economy, population aging implies increased savings and asset accumulation. In the small open economy, agents can borrow or lend freely from the rest of the world, at given interest rate, assumed to remain constant. In the closed economy, agents can only save by accumulating domestic capital; because of diminishing returns, the interest rate decreases. This difference in interest rates explains all the other differences. With constant returns to savings in the small open economy, per capita consumption remains almost constant during the demographic transition, while it substantially decreases in the closed economy. In the closed economy, domestic capital accumulation leads to a substantial increase in average
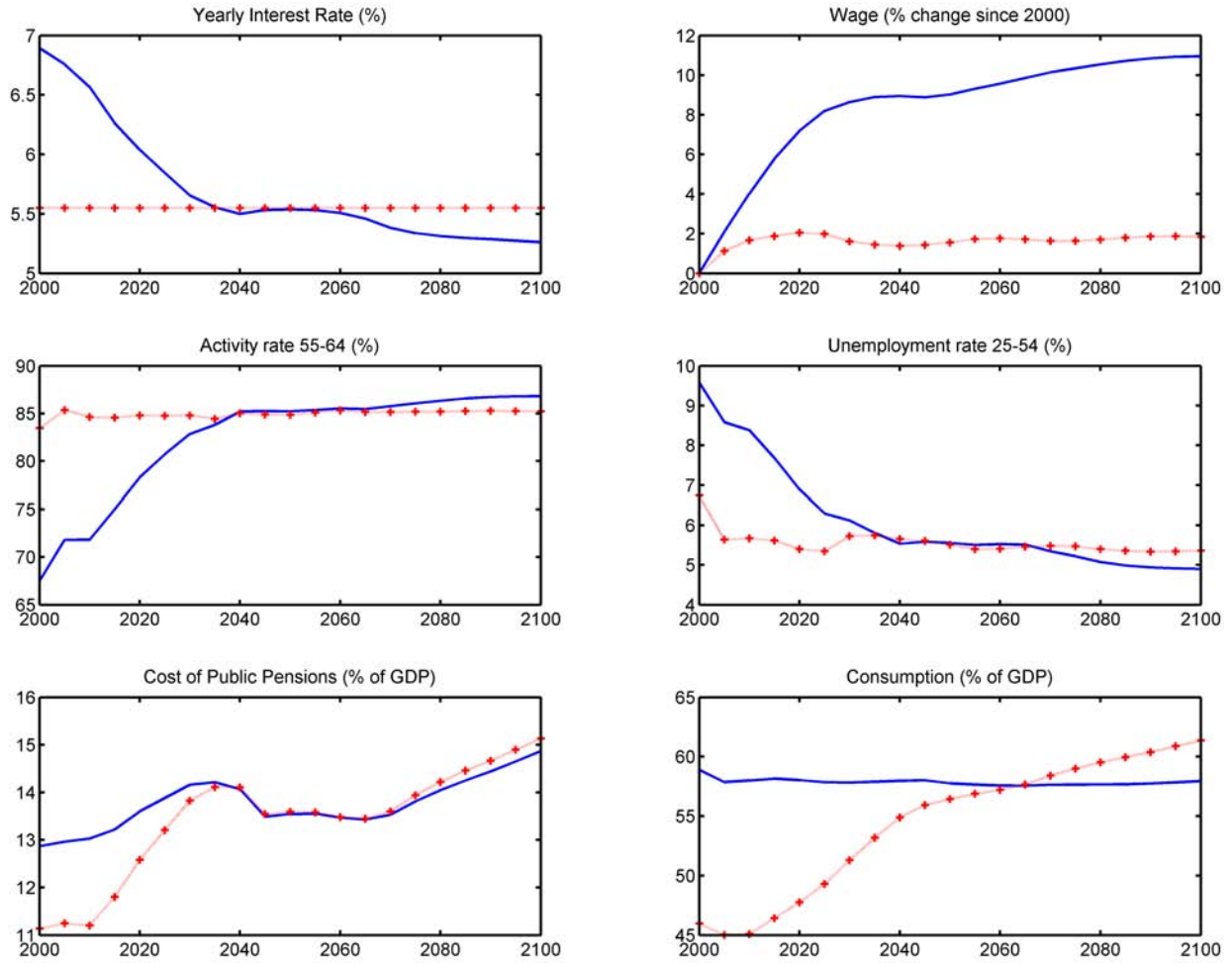
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Figure 7: Comparing the closed (solid line) and the small open economy (dotted line) scenarios

labor productivity (hence in gross wages), and also an increase in aggregate employment. This higher employment level comes from both a decrease in the unemployment rate of young and mature workers (25-54) and from an increase in the participation rate of elderly workers (55-64). These effects are related to capital accumulation and lower interest rates, which increase profitability and the number of job openings. These changes imply a higher optimal average retirement age (less early-retirement), in an unchanged institutional environment. These effects are non-existent in the small open economy with fixed interest rates. This contrast has large implications for the cost of public pensions. In the small open economy, population aging implies a 2 percentage point increase in the cost of public pensions between 2000 and 2060, compared to 0.5% in the closed economy. It is worth stressing that pension costs (in % of GDP) evolve in similar ways in the small open economy with frictions and in the closed economy with perfect competition. This is because in both cases the employment level does not vary much and does not amplify the effects of policy changes.

Both Börsch-Supan et al. (2006) and Attanasio et al. (2007) have already stressed the role of international capital flows in determining how an economy will react to population aging. We obtain a similar conclusion and emphasize that in economies with a frictional labor market, changes in the equilibrium unemployment rate provide an additional mechanism by which international capital flows may affect the adjustment process and the future cost of public pensions (as a % of GDP). With frictional labor markets, even the fiscal variables are affected by capital flows.

5   Conclusions

We have introduced labor frictions and early-retirement decisions into an otherwise standard OLG setup with population aging. In this setup, we show that population aging reduces the equilibrium unemployment rate, through its effect on capital accumulation, the interest rate and job openings. We have also shown that these effects can be substantially reinforced by endogenous participation rate changes (lower early-retirement rates).

Introducing frictions also changes the quantitative effects of policy changes (for instance lower retirement benefits), again via the effect of these policies on savings, the interest rate and equilibrium unemployment.

By contrasting the results obtained in the case of a small open economy with those of a closed economy, we have illustrated the importance of capital flows. When the interest rate is exogenously determined on world financial markets, population aging and pension reforms have similar effects on employment rates and the fiscal variables (public pension costs), although the implications for per capita consumption and the saving rate may be quite different.

In a two-country model, the interest rate would again be endogenous, and the new transmission mechanisms introduced by labor market frictions would work in much the same way as in the closed economy model, at least qualitatively. Future research should focus on interest rate determination and the impact of asymmetric labor market institutions in a multi-country setup.

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Appendix: The Household Optimization Problem

With initial and final financial wealth equal to zero (no bequests), the household's intertemporal budget constraint can be written as: [12]

$$\sum_{a=0}^{15} R_{t,t+a}^{-1} \beta_{a,t+a} \Bigg\{ \Big[(1 - \tau_{t+a}^w)\, w_{a,t+a}\, n_{a,t+a} + b_{a,t+a}^u\, u_{a,t+a} + b_{a,t+a}^e\, e_{a,t+a} $$
$$+ b_{a,t+a}^i\, i_{a,t+a} \Big] \cdot z_{a,t+a} - (1 + \tau_{t+a}^c)\, c_{a,t+a} \Bigg\} = 0 \,. \tag{28}$$

The discount factor $R_{t,t+a}$ is defined by $R_{t,t} = 1$ and $R_{t,t+j} = \Pi_{j=1}^a R_{t+j}$ for $j \geq 1$.

The values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ maximizing the household objective function (10) subject to (5) and (9) and the intertemporal budget constraint (28) can thus be obtained from the maximization of the Lagrangean function

$$\frac{W_t^H}{Z_{0,t}} = \max_{c_{a,t+a}, \lambda_{6,t+6}, \lambda_{7,t+7}} \sum_{a=0}^{15} \beta_{a,t+a} \Bigg\{ \beta^a \left(u(c_{a,t+a}) - d^n\, n_{a,t+a} \cdot z_{a,t+a} + d_a^e\, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi}\, z_{a,t+a} \right)$$
$$+ \mu_t\, R_{t,t+a}^{-1} \Bigg(\Big[b_{a,t+a}^u + \big((1 - \tau_{t+a}^w)\, w_{a,t+a} - b_{a,t+a}^u \big)\, n_{a,t+a} $$
$$+ (b_{a,t+a}^e - b_{a,t+a}^u)\, e_{a,t+a} \Big] \cdot z_{a,t+a} - (1 + \tau_{t+a}^c)\, c_{a,t+a} \Bigg) \Bigg\},$$

---

[12]It is clear from Equation (28) that consumption taxes have the same effect as income taxes.

where μ_t is the Lagrange multiplier associated with the intertemporal budget constraint. The optimal values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ must satisfy the first-order optimality conditions:

$$\beta^a \, \frac{u'_{c_{a,t+a}}}{1 + \tau^c_{t+a}} = \mu_t \, R^{-1}_{t,t+a} \, ; \tag{29}$$

$$\left[d^n \, \frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} - d^e_6 \, (e_{6,t+6})^{-\phi} \, \frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} \right] + \beta \frac{\beta_{7,t+7}}{\beta_{6,t+6}} \left[d^n \, \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} - d^e_7 \, (e_{7,t+7})^{-\phi} \, \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} \right]$$
$$= \frac{u'_{c_{6,t+6}}}{1 + \tau^c_{t+6}} \left[\left((1 - \tau^w_{t+6}) \, w_{6,t+6} - b^u_{6,t+6} \right) \frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} + \left(b^e_{6,t+6} - b^u_{6,t+6} \right) \frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} \right] \tag{30}$$
$$+ \beta \frac{\beta_{7,t+7}}{\beta_{6,t+6}} \frac{u'_{c_{7,t+7}}}{1 + \tau^c_{t+7}} \left[\left((1 - \tau^w_{t+7}) \, w_{7,t+7} - b^u_{7,t+7} \right) \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} + \left(b^e_{7,t+7} - b^u_{7,t+7} \right) \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} \right] \, ;$$

$$\left[d^n \, \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} - d^e_7 \, (e_{7,t+7})^{-\phi} \, \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} \right]$$
$$= \frac{u'_{c_{7,t+7}}}{1 + \tau^c_{t+7}} \left[\left((1 - \tau^w_{t+7}) \, w_{7,t+7} - b^u_{7,t+7} \right) \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} + \left(b^e_{7,t+7} - b^u_{7,t+7} \right) \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} \right] \, . \tag{31}$$

In these expressions,

$$\frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} = 1 \, ; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} = (1 - \lambda_{7,t+7}) \, ; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} = (1 - \lambda_{6,t+6}) \, ;$$

$$\frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+6}}{1 - \lambda_{t+6}} \, ; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+7}}{1 - \lambda_{t+6}} \, ; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} = -\frac{n_{t+7}}{1 - \lambda_{t+7}} \, .$$

The first optimality condition (29) is the usual Euler condition. It implies that

$$\frac{u'_{c_{a,t+a}}}{1 + \tau^c_{t+a}} = \beta \, R_{t+a+1} \, \frac{u'_{c_{a+1,t+a+1}}}{1 + \tau^c_{t+a+1}} \, .$$

The other two optimality conditions are specific to this model and determine the activity rate of older workers. After substitution and rearrangement, and with the assumption that $u(c_{a,t+a})$ is logarithmic, these optimality conditions can be recast as follows:

$$\left[\frac{b^e_{6,t+6} - b^u_{6,t+6}}{(1 + \tau^c_{t+6}) \, c_{6,t+6}} + d^e_6 \, (e_{6,t+6})^{-\phi} \right] (1 - e_{6,t+6})$$
$$+ \beta \frac{\beta_{7,t+7}}{\beta_{6,t+6}} \left[\frac{b^e_{7,t+7} - b^u_{7,t+7}}{(1 + \tau^c_{t+7}) \, c_{7,t+7}} + d^e_7 \, (e_{7,t+7})^{-\phi} \right] (1 - e_{7,t+7})$$
$$= \left[\frac{(1 - \tau^w_{t+6}) \, w_{6,t+6} - b^u_{6,t+6}}{(1 + \tau^c_{t+6}) \, c_{6,t+6}} - d^n \right] n_{6,t+6} \tag{32}$$
$$+ \beta \frac{\beta_{7,t+7}}{\beta_{6,t+6}} \left[\frac{(1 - \tau^w_{t+7}) \, w_{7,t+7} - b^u_{7,t+7}}{(1 + \tau^c_{t+7}) \, c_{7,t+7}} - d^n \right] n_{7,t+7} \, ,$$

31

and

$$\left[\frac{b_{7,t+7}^e - b_{7,t+7}^u}{(1 + \tau_{t+7}^c)c_{7,t+7}} + d_7^e \, (e_{7,t+7})^{-\phi} \right] (1 - e_{7,t+7}) = \left[\frac{(1 - \tau_{t+7}^w)w_{7,t+7} - b_{7,t+7}^u}{(1 + \tau_{t+7}^c)c_{7,t+7}} - d^n \right] n_{7,t+7}. \quad (33)$$

The economic interpretation of these optimality conditions becomes easier if we note that the unconditional probability of having a job is given by

$$\pi_{a,t+a} = \frac{n_{a,t+a}}{n_{a,t+a} + u_{a,t+a}} = \frac{n_{a,t+a}}{1 - e_{a,t+a}} \ ,$$

so that the last optimality condition can be written as

$$\frac{b_{7,t+7}^e}{(1 + \tau_{t+7}^c)\, c_{7,t+7}} + d_7^e \, (e_{7,t+7})^{-\phi} = \pi_{7,t+7} \left[\frac{(1 - \tau_{t+7}^w)\, w_{7,t+7}}{(1 + \tau_{t+7}^c)\, c_{7,t+7}} - d^n \right] + (1 - \pi_{7,t+7}) \left[\frac{b_{7,t+7}^u}{(1 + \tau_{t+7}^c)\, c_{7,t+7}} \right] \ ,$$

and similarly for the other optimality condition.