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ABSTRACT

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I develop an equilibrium, two-sided search model of marriage with endogenous population growth to study the interaction between fertility, the age structure of the population and the age at first marriage of men and women. Within a simple two-period overlapping generation model I show that, given an increase of the desired number of children, age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population produces a thicker market for young people, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, women tend to marry younger and men older, with single men outnumbering single women in equilibrium. Using an extended version of the model to a finite number of periods and fertility data, I show that two mechanisms described above may have acted as persistence mechanisms after the U.S "baby boom". I show that demographic transitional dynamics after the baby boom may account for approximately a 23% of the increase in men's age of marriage between 1985 and 2009, albeit the impact on women's age is small.

JEL Classification: J12, D83

Keywords: marriage, population structure, search

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1. Introduction

In this paper, I investigate the relationship between fertility, the age structure of the population and the timing of marriage of men and women. Looking at the evolution of those variables in the U.S., we observe several facts that suggest the existence of such a relationship. First, the median age of marriage of men and women appears to be negatively correlated with the total fertility rate (TFR), although with a delay.[1] Observe in Panel (a) of Figure 1 that fertility rates sharply fall from the early sixties to the mid seventies, while the age of marriage of men and women starts to unambiguously increase only when fertility rates become stable.

The relationship between marriage timing and fertility can be reconciled if we take a closer look at the demographic implications of the U.S. "Baby Boom". Panel (b) of Figure 1 shows the evolution of the share of population aged 15-29 years-old of those aged 15-44, together with the TFR; this measure of the demographic structure mimics fertility rate with a lag of about 20 years. Specifically, this share starts increasing in the early sixties, when the early "boomers" became 15 year-olds and peaks in the mid-seventies, when they turned 30, then decrease until the mid-nineties. Panel (c) shows the comparison between demographic structure and the median age of marriage of men and women. Notice that both measures of marriage timing start increasing approximately at the same time the population starts to become older again, around 1975.

Second, fertility and marriage timing are related to the age gap at marriage between men and women. As shown in Panel (d) of Figure 1, the difference between the average age at first marriage for men and appears to show a positive correlation with the total fertility rate, increasing from the forties to around the end of the baby boom, in 1963.[2] Notice also in Panel (e) the positive correlation between the sex ratio and the age gap in the time series, and in Panel (f) that the evolution in the sex ratio does not appear to follow changes in the relative total quantity of men and women, at least since 1940.[3] It is common in the empirical marriage literature to use the single sex ratio (or "availability ratio") as an exogenous measure of one sex scarcity that affects the relative timing of marriage of men and women.[4] As I will show later, the sex ratio and the age gap are jointly determined and their relationsip not necessarily depends on sex imbalances.

In this paper, I will argue that the timing of marriage for men and women is affected by two endogenous sources of frictions. The first one is the age structure of the population, which is a function of past fertility and produces a "thicker market externality" that induces men and women to marry earlier when the population becomes younger. The second is the ratio of single men and single women, which produces asymmetries in marriage opportunities for men and women. I will show in this paper that the age gap and the sex imbalances are jointly determined by the behavior of the agents.

In Section 2 I present the basic, steady state model. Specifically, I develop an overlapping generations general equilibrium model with two-sided search in which agents only differ in their

[1] Throughout this paper I use the median age at first marriage as a measure of marriage timing, because population structure is a key factor in this paper. The U.S. Census Bureau has annually published the estimated median age at first marriage since 1947, using an indirect method based on the proportion of people who were ever married within the single age cohort. This methodology prevents the age of marriage from being affected mechanically by population structure. Since the mean age of marriage is conditional on marriage, this measure is subject to those mechanical effects. However, I use the mean of the age at first marriage in order to calculate the age gap.

[2] In contrast to the previous marriage literature, which assumes that the age difference is constant, Ni Brolcháin (2001), has shown that the age gap changes according to the size of the cohorts.

[3] However, total and single sex ratios appear to be correlated between 1900 and 1940.

[4] See, for example, Fitzgerald (1991), McLaughlin et al. (1993), and Parrado and Centeno (2002).

age and are able to marry once in their lifetimes. I assume that they live as adults for two periods and that they obtain utility from the (idiosyncratic) quality of their marriage and from the joy of having children. I also assume that both men and women desire to have a given number of children, which I keep as exogenous. Given desired fertility, population growth and therefore the age structure of the population are endogenous. In this model, as people get older they become less attractive to their prospective younger mates. In the case of both sexes, this is because younger spouses know that they will lose the utility of being married relatively early in life when their older spouse dies. In addition, for women, the disutility of aging increases because they lose their fecundity relatively early.

Given the above assumptions, I show that, given an increase of the desired number of children, age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population becomes "younger" which reduces the frictions that men and women face in order to meet other younger counterparts, and therefore increases their likelihood to marry younger. The second channel comes from differential fecundity: if the biological constraints for older women are binding, young men become more reluctant to marry older women. Consequently, young women anticipate their shorter fecundity horizon and reduce their reservation match quality for both younger and older men. Therefore, women increase their likelihood of marrying older men, who have a greater willingness to marry than younger men. In the steady state equilibrium, as young women are more likely to marry older men, young men are (relatively) more likely to wait and single men will outnumber single women.

To study the empirical implications of the results above, in Section 3 I extend the basic model to a full life-cycle model with people living a finite numbers of yearly periods and where men and women differ not only in fecundity but also in mortality. In order to use this extended model to study potential impact that the "baby boom" may have had on marriage dynamics, I proceed in two steps, detailed in Section 4. First, I assume that the economy in its steady state and calibrate the model parameters to U.S. 1930 demographics, to obtain a steady state Then, since desired fertility is exogenous in this model, I take those parameters from the U.S. time series data and assume that desired fertility reaches a new steady state value in 1977. From that year onwards, marriage timing is exclusively affected by a persistence mechanism that is caused by the change of the age structure of the population due to the "baby boom".

The main findings of the calibration exercise are shown in Section 5. I find that the transitional dynamics of the model causes the age of marriage to slowly adjust to the steady state, particularly in the case of men. Specifically, I find that the decline in fertility in the period 1957-1976 may account for 23% of the increase in men's median age of marriage occurring between 1985 and 2009. In the benchmark model, men's age at first marriage increases until the age structure of the population reaches its steady state level (around the year 2040 in the model). The reason for such slow adjustment resides in the impact that demographic structure has on the sex ratio, increasing men's matching rates with respect to women's at the beginning of the transition period, that is, when a large share of people on marriageable age are young "baby boomers". For the same reason, the impact on women's marriage timing is small.

In order to understand the how much the demographic structure may have influenced men's slow convergence to the steady state, I perform two counterfactual experiments. In the first experiment, I assume that fertility rates converge slowly from the 1940 levels to the replacement rate, as if the "baby boom" never occurred. In the second experiment, I assume that even if there is an increase in desired fertility from the 1945 to 1965, it has no impact on in the demographic structure, so I only keep the partial equilibrium effects. What I find in either case is that men would have reached their steady state median age at first marriage in the mid eighties, around fifty years earlier than in the benchmark model.

Review of the literature.

The earliest work in economics devoted to the explanation of the age gap between men and women is based on gender roles of traditional societies. Bergstrom and Bagnoli (1993) present a model with incomplete information where women are valued as marriage partners for their ability to bear children and to manage a household, and men are valued for their ability to make money. Women's ability is observable earlier in life than men's ability, which is private knowledge. Therefore, men who expect to be successful delay marriage until they are able to give a signal that allows them to attract more desirable women.

Siow (1998) introduced the issue of the shorter fecundity period of women in the economics literature. In a two-period model with uncertainty about human capital accumulation, and where utility comes exclusively from having children, old and young men (all fertile) compete for young, fertile women. Consequently, some old men, those who successfully obtain a higher wage, are able to marry, displacing some of the young men in the competition over scarce fertile women.

This paper shares several features of the works mentioned. For example, as in Siow (1998), women have a shorter fecundity period than men. However, in this paper, population growth is endogenous, which allows me to explore the role of the demographic structure in marriage timing, not only by comparing two steady states but also by exploring the transitional dynamics from one steady state to the other.[5] The observed relationship between sex imbalances and the age gap makes search models particularly useful to analyze this problem.[6]

Section 2 of the paper builds on Giolito (2004), who models the age of the individuals both as a state variable and as a source of agents' heterogeneity, in a framework with ex-ante heterogeneous agents and non-transferrable utility (NTU) focused exclusively on the steady state with no population growth.[7] More recently, Díaz-Giménez and Giolito (2009) have developed a full life-cycle setting that accounts for differences in fecundity, earnings and mortality profiles between men and women. Focusing also in the steady state with no population growth, they find that differential fecundity is all that matter to account for age differences at first marriage, and that the distribution of age differences along the life cycle, which is increasing in age for men, and decreasing (in absolute value) in the age of women is also more consistent with a partially random matching search model that with a model where people exclusively meet within their own age group. Differentt form that work, the model on Section 3 is developed to study the transitional dynamics of the model after a demographic shock and the factors causing a slow adjustment in the age of marriage.

The present work is also related to previous studies on marriage timing, for example, Caucutt, Guner and Knowles (2002). They study the roles played by wage inequality, human capital accumulation, and returns to experience for the timing of marriage and fertility. They show that highly productive women marry and have children later in life than their less productive

[5] Another related paper is Tertilt (2005), who also allows for endogenous population growth to study the effects of polygyny in certain African countries. Also in the development literature, Edlund (1999) discusses possible links between son preference in certain Asian countries and marriage patterns (for example spousal age gap, as here).

[6] Seitz (2009) studies the impact of imbalances of the total sex ratio (like mortality or immigration) on differences in the marriage timing between whites and blacks. Differently from this paper, she does not allow for marriage across cohorts, and therefore the sex ratio is only affected by exogenous shocks on the total population of men and women.

[7] Following Burdett and Coles (1997) and Smith (2006), there is extensive literature in NTU equilibrium search models of marriage. See Burdett and Coles (1999) for a survey. More recent examples include Bloch and Ryder (2000), Cornelius (2003), Burdett et al. (2004), Wong (2003).and Coles and Francesconi (2007).

colleagues. With a richer structure than this model, they abstract from cross-age marriages and from population growth.

Recently, Coles and Francesconi (2007) analyzed a two-sided search model where men and women differ in their earnings (which evolve stochastically) and in their fitness, which decays deterministically with age. They show that when market opportunities are similar for both sexes, then the equilibrium exhibits "toy-boy marriages" (old women with young men) and "toy-girl marriages" (old men with young women). When labor market opportunities are better for men than for women, toy-boy marriages disappear. The main difference with the basic model of this paper is that Coles and Francesconi abstract from population growth and also assume that matching rates are independent of demographic structure.[8] Therefore, they abstract from the relationship between the sex ratio and the age gap, which is crucial to this model.

2. A two-period model of marriage

2.1. Demographics

The economy is inhabited by a continuum of men and women who live for three periods, one as children and two as adults: young (age 1) and old (age 2). Adult men and women derive utility from being married and from having children, do not discount future, and their main economic activity is searching for a mate. In the steady state, the population grows every period at a constant rate n, which is endogenous. Therefore, in any period t, the economy is populated by N_t young and N_{t-1} old agents, where $N_t = N_{t-1}(1+n) = N_0(1+n)^t$. Since half of the agents are male and half are female, I assume for simplicity that $N_0 = 2$.

The population dynamics in this economy determines the steady state age structure of the population. The fraction of young people in this economy is given by

$$q_t = \frac{N_t}{N_t + N_{t-1}} = \frac{N_0(1+n)^t}{N_0(1+n)^{t-1}(2+n)} = \frac{1+n}{2+n}, \tag{1}$$

which is increasing in the population growth rate.

Each period t, equal measures $s_{1,t}^w$ of young single women and $s_{1,t}^m$ of young single men enter the economy, where $s_{1,t}^m = s_{1,t}^w = (1+n)^t$. Consequently, every period there is a measure, $S_t^w = (1+n)^t + s_{2,t}^w$, of single women in the economy, of which $(1+n)^t$ are young and $s_{2,t}^w$ are old, with n and $s_{2,t}^w$ endogenous. Similarly, there is a measure, $S_t^m = (1+n)^t + s_{2,t}^m$, of single men, of which $(1+n)^t$ are young and $s_{2,t}^m$ are old. Therefore, the ratio of single men to single women (henceforth, the sex ratio), $\phi_t = \frac{S_t^m}{S_t^w}$ is also endogenous.

Since men and women marry in pairs, the total number of brides, $w_{1,t}$ young plus $w_{2,t}$ old, must be equal to the total of grooms, $h_{1.t}$ young plus $h_{2,t}$ old. Since in the steady state the sex ratio is constant, $\phi_t = \phi$, the fact that every period an equal number of young men and women enter the market requires the number of men who exit the marriage market either because they marry, or because they die single also to be equal to the number of women who leave the market. Consequently,

$$w_{1,t} + w_{2,t} + u_t^m = h_{1,t} + h_{2,t} + u_t^w \tag{2}$$

where u_t^m and u_t^w denote the number of men and women who die unmarried, respectively. Notice that condition (2) and the fact that people marry in pairs imply that $u_t^m = u_t^w$.

[8] They assume that the population of unmarried men is always equal to the population of unmarried women.

2.2. Matching technology

I assume that singles of both ages meet randomly at most once every period and decide whether or not to marry. Once married, both men and women remain so until the end of their lives and people whose spouse dies are not allowed to search for a new spouse.[9] Whenever a man and a woman meet, either one can propose marriage to the other. If they both accept, they marry. Otherwise they both stay single.

The total number of meetings between singles is represented, in the spirit of Pissarides (1990), by the following constant returns to scale matching function,

$$\eta_t = \rho \left(S_t^m\right)^{\theta} \left(S_t^w\right)^{1-\theta} \tag{3}$$

where $\theta \in (0,1)$ and $\rho \in (0, \frac{1}{2}]$.[10] As is standard in the search and matching literature, the probabilities of being matched depend on the sex ratio ϕ. Therefore, a single man meets a single woman with probability

$$\lambda^m = \frac{\eta_t}{S_t^m} = \rho \left(\phi\right)^{\theta-1} \tag{4}$$

Similarly, a single woman meets a single man with probability $\lambda^w = \rho \left(\phi\right)^{\theta}$. Since the sex ratio is fully endogenous in this economy, here the matching function also captures the way in which the aggregate effects of the marriage decision feed back into the individual decision problem.

The probability that an individual meets either a young or and old agent of the opposite sex depends on the age structure of the single population. For example, the probability that a single man meets a single young woman is $\lambda^m p_1^w$ and the probability of meeting an old single women is given by $\lambda^m p_2^w = \lambda^m \left(1 - p_1^w\right)$, where $p_a^w = \frac{s_{a,t}^w}{S_t^w}$ is the fraction of single women of age a, which is constant in the steady state. Similarly, from a single women point of view, the probabilities of meeting a young and an old man are given by $\lambda^w p_1^m$ and $\lambda^w \left(1 - p_1^m\right)$, respectively.[11]

2.3. Payoffs

The table below describes the utility that people derive from their marital status. I assume that married people derive utility both from the match quality and from the number of children they are able to have with a given spouse, and the utility of being single is normalized to zero. I assume that men and women desire to have $k \in \left[\frac{1}{2}, \overline{k}\right]$ children during their lifetime, where each "child" is a pair of twins, one girl and one boy. I also assume that women bear all their children in the same period they marry. Finally, men and women differ in their fecundity horizons. On one hand, while young women are able to bear k children, old women are only capable of bearing at most one child. On the other hand, men of age 1 and of age 2 are able to have k children.

The utility that a woman derives from being married to a specific man depends also on the man's type, assumed to be a random variable with cumulative distribution $G^m(y)$, support in $[0, \overline{y}]$, and mean μ_y. Similarly, I assume that the utility that a man derives from being married to a specific woman depends on the woman's type, which is a random variable with cumulative

[9] For a model of marriage with endogenous separations, see Brien, Lillard and Stern (2002) and Cornelius (2003). Moreover, Chiappori and Weiss (2006) examine equilibrium divorce incentives where there is a thick market externality in the remarriage market.

[10] Given that the sex ratio can be unbalanced in equilibrium, I assume that ρ is small enough to ensure that the matching probabilities are not higher than one. The interval assumed for ρ ensures the uniqueness of a symmetric equilibrium in a two-period economy.

[11] Given that the matching technology is CRS and that $s_{1,t}^m = s_{1,t}^w$, it is straightforward that $\lambda^w p_1^m = \frac{\eta}{S_t^w} \frac{(1+n)^t}{S_t^m} = \lambda^m . p_1^w$

distribution $G^w(x)$, support in $[0, \overline{x}]$, and mean μ_x. I further assume that both cumulative distributions are continuous and differentiable, and that their probability density functions are $g_m(y)$ and $g_w(x)$, respectively. Since I assume that singles are homogeneous in their quality, the distributions G^m and G^w are identical across singles, and the realizations that people draw at each meeting are independent.[12]

For example, if a young woman marries a young man, their utilities are $2ky$ and $2kx$, because the marriage will last for two periods. If a young woman marries an old man, they will also receive ky and kx but only for one period. On the other hand, if a young man marries an old woman, their utilities are only $\min\{k, 1\}x$ and $\min\{k, 1\}y$ for one period, respectively. Since old women are less fecund and only able to bear one child, they receive the same utility regardless of the age of their spouse. Furthermore, since in the model economy people who do not marry miss out on the joys of both companionship and childbearing, I normalize to zero the utility of being single.

Payoffs for Men and Women		
	Young Spouse	*Old Spouse*
Women		
Marry young	$2ky$	ky
Marry old	$\min\{k, 1\}y$	$\min\{k, 1\}y$
Never marry	0	0
Men		
Marry young	$2kx$	$\min\{k, 1\}x$
Marry old	kx	$\min\{k, 1\}x$
Never marry	0	0

In order to ensure existence of an equilibrium, I assume that the maximum value of k is such that the utility of marrying an average spouse and having k children with them is not higher than the joy of finding a perfect match and having only one child. That is, $k\mu_x \leq \overline{x}$ and $k\mu_y \leq \overline{y}$, which implies

$$\overline{k} = \min\left\{\frac{\overline{x}}{\mu_x}, \frac{\overline{y}}{\mu_y}\right\}. \tag{5}$$

2.4. The decision problem of singles

Since people match randomly in our economy, the single men's problem is to choose reservation values for young and old single women. Let $R^m_{a,b}$ denote the steady state reservation value that single men of age a set for single women of age b, and, similarly, let $R^w_{b,a}$ be the reservation values that age b single women set for age a single men. Since I assume that people meet only once per period, and that they derive zero utility from being single, the reservation values of old single men for both young and old single women is zero, trivially. That is, $R^m_{2,1} = R^m_{2,2} = 0$. Similarly, the reservation values of old single women for both old and young single men are also zero, that is, $R^w_{2,1} = R^w_{2,2} = 0$.[13]

[12] The assumption about the two separate distributions for men and women, as in Burdett and Coles (1997) is made exclusively for analytical tractability and does not affect the results.

[13] This rules out the trivial equilibrium where all women reject all marriages because they know that all men will reject them and vice versa.

2.4.1. Expected utility of marriage for old single people

Denote as $\gamma^j_{a,b}$ the steady state probability that a single agent of age a and gender $j \in \{m, w\}$ marries someone of age b. This probability depends first, on the probability of meeting someone of the opposite sex and age b, $\lambda^j p_b^{-j}$. Given the meeting probability, the probability of marriage depends on mutual acceptance. That is, for example for men,

$$\gamma^m_{a,b} = \lambda^m p^w_b [1 - G^m(R^w_{b,a})][1 - G^w\left(R^m_{a,b}\right)] \tag{6}$$

Let us observe the case of old single people. Since I assume that old people who do not marry obtain zero utility, they accept any marriage offer. For example, if an old single man (woman) meets a young single woman (man) they will marry provided the youngest agent accepts. Therefore,

$$\gamma^m_{2,1} = \lambda^m p^w_1 [1 - G^m(R^w_{1,2})] \text{ and } \gamma^w_{2,1} = \lambda^w p^m_1 [1 - G^w\left(R^m_{1,2}\right)] \tag{7}$$

On the other hand, if an old single man meets an old single woman, they marry with certainty. Without loss of generality, I assume that everybody eventually marries at some point in life. Consequently, those old men and women that either were unmatched or rejected by younger counterparts will face a frictionless technology that automatically matches them among themselves.[14] Therefore $\gamma^m_{2,2} = [1 - \gamma^m_{2,1}]$ and $\gamma^w_{2,2} = [1 - \gamma^w_{2,1}]$.

Given the payoffs described above, the expected utility of marriage for old single men is

$$\begin{aligned} U^m_2 &= \gamma^m_{2,1} k\mu_x + \left(1 - \gamma^m_{2,1}\right) \min\{k, 1\}\mu_x \\ &= \lambda^m p^w_1 [1 - G^m(R^w_{1,2})] k\mu_x + \left(1 - \lambda^m p^w_1 [1 - G^m(R^w_{1,2})]\right) \min\{k, 1\}\mu_x \end{aligned} \tag{8}$$

For old single women, since they will enjoy the same utility regardless of whom they marry, we have,

$$\begin{aligned} U^w_2 &= \gamma^w_{2,1} \min\{k, 1\}\mu_y + \left(1 - \gamma^w_{2,1}\right) \min\{k, 1\}\mu_y \\ &= \min\{k, 1\}\mu_y. \end{aligned} \tag{9}$$

2.4.2. Expected Utility of Marriage for Young Single People

Since young single men and women have a chance to marry in the future, they choose their reservation values taking into account the next period's prospects. For example, provided that a single young man meets a single young woman (with probability $\lambda^m p^w_1 = \lambda^w p^m_1$), they will marry if both of them find each other mutually acceptable. A single young woman will accept a single young man with probability $[1 - G^m(R^w_{1,1})]$, and he will accept her with probability $[1 - G^w(R^m_{1,1})]$. Therefore, the probability that a young man marries a young woman (or vice-versa) is

$$\gamma^m_{1,1} = \gamma^w_{1,1} = \lambda^m p^w_1 [1 - G^w\left(R^m_{1,1}\right)][1 - G^m(R^w_{1,2})] \tag{10}$$

On the other hand, if young man or woman meets an old single agent of the opposite sex (with probability $\lambda^m p^w_2$ or $\lambda^w p^m_2$, respectively), the acceptance of the young agent will be sufficient for marriage to occur. Consequently,

$$\gamma^m_{1,2} = \lambda^m p^w_2 [1 - G^w\left(R^m_{1,2}\right)] \tag{11}$$

[14] This assumption is for convenience, and will be relaxed in the next section, when I study the transitional dynamics of the model. Recall that by condition (2), steady state implies that an equal number of old men and women would remain unmatched after period 2.

and

$$\gamma_{1,2}^{w}=\lambda^{w}p_{2}^{m}[1-G^{m}\left(R_{1,2}^{w}\right)], \tag{12}$$

for men and women, respectively.

The expected utility of marriage for young single men, U_1^m, is then,

$$U_{1}^{m}=\gamma_{1,1}^{m}2kE\left[x\mid x\geq R_{1,1}^{m}\right]+\gamma_{1,2}^{m}\min\{k,1\}E\left[x\mid x\geq R_{1,2}^{m}\right], \tag{13}$$

and the one for women, U_1^w,

$$U_{1}^{w}=\gamma_{1,1}^{w}2kE\left[y\mid y\geq R_{1,1}^{w}\right]+\gamma_{1,2}^{w}kE\left[y\mid y\geq R_{1,2}^{w}\right] \tag{14}$$

2.4.3. The young single people's optimization problem

Young single people of gender $j\in\{m,w\}$ choose the reservation values for young and old single agents of the opposite sex, $R_{1,1}^{j}$ and $R_{1,2}^{j}$, which maximize their expected lifetime utility. In order to do this, they solve the following problem:

$$V_{1}^{j}=\underset{R_{1,1}^{j},R_{1,2}^{j}}{max}\left[U_{1}^{j}+\left(1-\Gamma^{j}\right)U_{2}^{j}\right] \tag{15}$$

subject to

$$\Gamma^{j}=\gamma_{1,1}^{j}+\gamma_{1,2}^{j} \tag{16}$$

where Γ_1^j stands for the steady state probability that a single individual of gender j marries at age 1.

The solutions to this problem require a young single agent to be indifferent between marrying when young or resampling again the following period. If a young single agent meets an old individual of the opposite sex he/she must compare the utility of a one period marriage with the value of marrying when old. In the case of a young man who meets an old women, this type of marriage becomes less valuable if their desired number of children is greater than the one that an old woman is able to conceive $(k>1)$. Therefore,

$$\begin{aligned}\min\{k,1\}R_{1,2}^{m} &= U_{2}^{m} \\ R_{1,2}^{m} &= \frac{U_{2}^{m}}{\min\{k,1\}}\end{aligned} \tag{17}$$

The case of a young woman who meets an old man is different. Since men of all ages are able to have k children, she must compare the utility of marrying young with the value of waiting until the next period and facing the limited fecundity which will be binding in the case where $k>1$. Then we have

$$kR_{1,2}^{w}=U_{2}^{w}=\min\{k,1\}\mu_{y},$$

hence

$$R_{1,2}^{w}=\frac{\min\{k,1\}\mu_{y}}{k}. \tag{18}$$

Now we focus on the reservation values that young singles set for people of their same cohort. Given that a young single man meets a young single woman (or vice-versa), they have to compare between the value of waiting and the utility of a two-period marriage with k children. Therefore,

$$2kR_{1,1}^{j}=U_{2}^{j},$$

or

$$R_{1,1}^j = \frac{U_2^j}{2k} \tag{19}$$

for $j \in \{m, w\}$.

To obtain the reservation values chosen by the young single men, substituting expression (8) into Equations (17) and (19), respectively, we get

$$R_{1,1}^m = \frac{\mu_x}{2k} \left\{ \min\{k,1\} + \lambda^m p_1^w \left[k - \min\{k,1\}\right] \left[1 - G^m \left(R_{1,2}^w\right)\right] \right\}, \tag{20}$$

and

$$\begin{aligned} R_{1,2}^m &= \mu_x \left\{ 1 + \lambda^m p_1^w \left[k - \min\{k,1\}\right] \left[1 - G^m \left(R_{1,2}^w\right)\right] \right\} \\ &= \frac{2k}{\min\{k,1\}} R_{1,1}^m. \end{aligned} \tag{21}$$

Notice that the young single men's reservation values depend positively on the average match quality of single women, μ_x, and on the difference between their desired number of children and the number that old women are capable to bear $(k - \min\{k,1\})$. When the desired number of children is not biologically binding for old women $(k \leq 1)$, we have that $R_{1,1}^m = \frac{\mu_x}{2}$ and $R_{1,2}^m = \mu_x$. On the other hand, when $k > 1$, we have that

$$R_{1,1}^m = \frac{\mu_x}{2k} \left\{ 1 + \lambda^m p_1^w \left[k - 1\right] \left[1 - G^m \left(R_{1,2}^w\right)\right] \right\}, \tag{22}$$

and

$$R_{1,2}^m = \mu_x \left\{ 1 + \lambda^m p_1^w \left[k - 1\right] \left[1 - G^m \left(R_{1,2}^w\right)\right] \right\}, \tag{23}$$

which are increasing on the probability of meeting a single young woman, $\lambda^m p_1^w$, and decreasing on $R_{1,2}^w$, i.e., the reservation value of young single women for old single men. Naturally, when $R_{1,2}^w$ decreases, that is, when it becomes more likely that young single women accept the proposals of old single men, young men are more willing to wait and therefore become choosier. Finally, notice that while $R_{1,1}^m$ is decreasing in the desired number of children, k, $R_{1,2}^m$ is increasing in k. That is, as the desired number of children increases, young men become choosier with respect to old women and less choosy with young women.

On the other hand, reservation values chosen by young single women are simply,

$$R_{1,1}^w = \frac{\min\{k,1\}}{2k} \mu_y, \tag{24}$$

and

$$R_{1,2}^w = \frac{\min\{k,1\}}{k} \mu_y. \tag{25}$$

Notice that, in this case, when fecundity is binding $(k > 1)$, the reservation values that young women set for all men become decreasing functions of their desired number of children.[15]

[15] Notice that the reservation values of men and women are all *continuous* in the value of the parameter k, which allows us to find an equilibrium for each value of k.

2.5. Steady state demographics

2.5.1. The number of single men and women

This section presents the endogenous number of single people in the steady state economy as a function of the probabilities of marriage at age 1, Γ_i^j for $j \in \{m, w\}$ and the rate of population growth, n.

Since the population of old single men and women is composed of the unmarried young singles of the previous period, and given that the population of young singles is $s_{1,t}^m = s_{1,t}^w = (1+n)^t$, we have that

$$s_{2,t}^j = s_{1,t-1}^j \left(1 - \Gamma^j\right) = (1+n)^{t-1} \left(1 - \Gamma^j\right), \tag{26}$$

Therefore, the total number of singles is

$$S_t^j = s_{1,t}^j + s_{2,t}^j = (1+n)^{t-1} \left(2 + n - \Gamma^j\right) \tag{27}$$

The age distributions of singles —that is the fraction of young single people of gender $j \in \{m, w\}$, is

$$p_1^j = \frac{s_{1,t}^j}{S_t^j} = \frac{(1+n)}{2+n-\Gamma^j}, \tag{28}$$

Finally, the sex ratio is

$$\phi = \frac{S_t^m}{S_t^w} = \frac{2+n-\Gamma^m}{2+n-\Gamma^w}. \tag{29}$$

Note that in the steady state the sex ratio and the fractions of young single people are constant. Moreover, the fractions of young singles depend positively on the population growth rate.

2.5.2. Fertility and population growth

To obtain the rate of population growth, recall that a woman that marries at age 1 is capable of having k pairs of twins, and that a woman who does so at age 2 is able to bear only one pair of twins. Given those assumptions, we have that the number of young men and women who enter the market each period are

$$s_{1,t}^m = s_{1,t}^w = k s_{1,t-1}^w \Gamma^w + \min\{k, 1\} s_{2,t-1}^w. \tag{30}$$

Given that $s_{1,t}^w = (1+n)^t$, substituting (26) in (30) ,and manipulating we have

$$(1+n)^2 = k(1+n)\Gamma^w + \min\{k, 1\}(1 - \Gamma^w). \tag{31}$$

Solving the quadratic equation (31) and taking the positive root we have that the rate of population growth in this economy is

$$n = \frac{k\Gamma^w}{2} - 1 + \frac{\sqrt{(k\Gamma^w)^2 + 4\min\{k,1\}(1-\Gamma^w)}}{2}. \tag{32}$$

Notice that women's fecundity is normalized to obtain $n = 0$ when $k = 1$. The population growth reaches its minimum when $k = 0.5$ and $\Gamma^w = 1$ $(n = -0.5)$ and its maximum when $k = \overline{k}$ and $\Gamma^w = 1$ $\left(n = \overline{k} - 1\right)$.

2.6. Steady State Equilibrium

2.6.1. Marriage market clearing

Since men and women marry in pairs, the total number of brides must be equal to the total of grooms in each type of marriage. Therefore, for men of age a and women of age b, for $a, b \in \{1, 2\}$, we have

$$s^m_{a,t}\gamma^m_{a,b} = s^w_{b,t}\gamma^w_{b,a}. \tag{33}$$

People marrying within their cohort. The condition above is trivially satisfied when $a = b = 1$, since $s^m_{1,t} = s^w_{1,t} = (1+n)^t$ and, by (10), $\gamma^m_{1,1} = \gamma^w_{1,1}$.

People marrying across cohorts

1. *Young men with old women*

$$\begin{aligned} s^m_{1,t}\gamma^m_{1,2} &= s^w_{2,t}\gamma^w_{2,1} \\ s^m_{1,t}\lambda^m_2[1 - G^w\left(R^m_{1,2}\right)] &= s^w_{2,t}\lambda^w p^m_1[1 - G^w\left(R^m_{1,2}\right)] \\ s^m_{1,t}\frac{\eta_t}{S^m_t}\frac{s^w_{2,t}}{S^w_t} &= s^w_{2,t}\frac{\eta_t}{S^w_t}\frac{s^m_{1,t}}{S^m_t}. \end{aligned} \tag{34}$$

2. *Old men with young women.*

$$\begin{aligned} s^m_{2,t}\gamma^m_{2,1} &= s^w_{1,t}\gamma^w_{1,2} \\ s^m_{2,t}\lambda^m p^w_1[1 - G^m\left(R^w_{1,2}\right)] &= s^w_{1,t}\lambda^w p^m_1[1 - G^m\left(R^w_{1,2}\right)] \\ s^m_{2,t}\frac{\eta_t}{S^m_t}\frac{s^w_{1,t}}{S^w_t} &= s^w_{1,t}\frac{\eta_t}{S^w_t}\frac{s^m_{2,t}}{S^m_t}. \end{aligned} \tag{35}$$

2.6.2. Definition

Given a desired number of children, k, a steady state equilibrium for this economy is a vector of reservation values for young single men, $R^m_1 = (R^m_{1,1}, R^m_{1,2})$, a vector of reservation values for young single women, $R^w_1 = (R^w_{1,1}, R^w_{1,2})$, a probability of marriage for young single men, $\Gamma^m = \gamma^m_{1,1} + \gamma^m_{1,2}$, a probability of marriage for young single women, $\Gamma^w = \gamma^w_{1,1} + \gamma^w_{1,2}$, and a population growth rate n, such that:

(i) Given R^w_1, Γ^m, Γ^w and n, R^m_1 solves the young single men's decision problem described in expression (15), and is determined by equations (20) and (21).

(ii) Given R^m_1, Γ^m, Γ^w and n, R^w_1 solves the young single women's decision problem described in expression (15), and is determined by equations (24) and (25).

(iii) The young men's probability of marriage, Γ^m, determined by (10) and (11), is consistent with the reservation values chosen both by the young single men and the young single women, the probability of marriage of young single women and the rate of population growth.

(iv) The young women's probability of marriage, Γ^w, determined by (10) and (12), is consistent with the reservation values chosen both by the young single men and women, the probability of marriage of young single men and the rate of population growth.

(v) The rate of population growth, n, determined by (32), is consistent with the reservation values and marriage probabilities of single young men and women.[16]

(vi) Marriage market clears, by (34) and (35).

2.6.3. Existence

Here I show the existence of a steady state equilibrium for this economy. Notice that, by expression (20), $R^m_{1,1}$ is uniquely determined by $R^m_{1,2}$. In addition, the reservation values of women are completely determined by k and μ_y (equations (24) and (25)). Therefore, I have to show the existence of an interior solution of a system of four equations in four unknowns: $R^m_{1,2}$, Γ^m, Γ^w and n.

Theorem 1 (Existence). *A steady state equilibrium exists for this economy.*

Proof. To proof existence I use Brouwer's Fixed Point Theorem. See Appendix A.1 ■

The following result rules out the existence of corner solutions and establishes that in equilibrium marriages across cohorts occur with positive probability.

Corollary 2. *The equilibrium is interior,*

$$\begin{aligned} R^m_{1,1}, R^m_{1,2} &\in (0, \overline{x}), \\ \Gamma^m &\in (0,1), \\ \Gamma^w &\in (0,1), \end{aligned}$$

and

$$n \in \left(-0.5, \overline{k} - 1\right).$$

Furthermore, in equilibrium all type of marriages occur with positive probability,

$$\begin{aligned} \gamma^m_{1,1} = \gamma^w_{1,1} &> 0 \\ \gamma^m_{1,2}, \gamma^w_{2,1} &> 0 \\ \gamma^w_{1,2}, \gamma^m_{2,1} &> 0. \end{aligned}$$

Proof. See Appendix A.1 ■

2.7. Equilibrium characterization

In this section I characterize the equilibrium of the economy, assuming that the distribution of types is identical for men and women, that is $G^m = G^w = G$, denoting its mean by μ. The analysis is divided into two parts. Next, I analyze the case where old women's biological restriction is not binding $(k \leq 1)$. This case is called "symmetric" because the decision problems faced by the single men and the single women are identical. Then, in Section 2.7.2 I analyze the case of differential fecundity, that is, when the desired fertility is higher than the number of children that old women are capable of bearing.

[16] Notice that, strictly speaking, the reservation values for old single men, $R^m_2 = (R^m_{2,1}, R^m_{2,2})$, and old single women, $R^w_2 = (R^w_{2,1}, R^w_{2,2})$, are also part of the steady state equilibrium of this economy. However, as shown in Section 2.4, they are trivially zero, and I have omitted them from the definition of the equilibrium for the sake of simplicity.

2.7.1. Symmetric case

Consider the case were people desire to have $k \in [0.5, 1]$ children.[17] Consequently, differential fecundity plays no role because the decision problems faced by the single men and the single women are identical. Moreover, in this case the reservation values of single men and single women are identical, and so are the numbers of single men and single women in the economy. Notice that in this case the reservation values of men and women are independent of their desired fertility.

Assumption 1. *$G^m = G^w = G$ with mean μ and density g.*

Assumption 2. *The probability density function g is log-concave.*

Under Assumption 1, by (21) , (20) , (24) and (25) we trivially have

$$R^m_{1,1} = R^w_{1,1} = \frac{\mu}{2}, \tag{36}$$

and

$$R^m_{1,2} = R^w_{1,2} = \mu. \tag{37}$$

Proposition 3. *Under Assumption 1, if people only want to have a number of children that is biologically feasible for old women, $k \in [\frac{1}{2}, 1]$, there is always a steady state equilibrium where men and women marry at age 1 with equal probability, $\Gamma^m = \Gamma^w = \Gamma^s$, and where population is not increasing over time, $n \leq 0$. This implies that the number of single men is equal to the number of single women, $S^m_t = S^w_t$, and therefore the sex ratio is equal to one, $\phi = 1$. Furthermore, the economy is mostly populated by old agents, $q \leq \frac{1}{2}$.*
Proof. *See Appendix A.2* ■

Proposition 4. *Under Assumptions 1 and 2 ,there is a unique steady state equilibrium for $k \in [\frac{1}{2}, 1]$.*
Proof. *See Appendix A.3* ■

Given that men and women face identical problems the existence of an unique equilibrium where men and women marry at the same age seems trivial. However, the interesting feature of the symmetric case comes from comparative statics. As we show below, when men and women are identical, an increase in the desired number of children leads to men and women marrying earlier as population growth increases, despite the constant reservation rules derived above.

Proposition 5. *Under Assumptions 1 and 2, if $k \in [\frac{1}{2}, 1)$, given an increase in the desired number of children:*

1. *Population growth rate increases: $\frac{\partial n}{\partial k} > 0$. This implies that the fraction of young people in the economy increases: $\frac{q}{\partial k} > 0$.*
2. *Men and women marry at age 1 with higher probability: $\frac{\partial \Gamma^s}{\partial k} > 0$.*

Proof. *See Appendix A.4.* ■

[17] Note that the results depend exclusively on symmetry and not on the negative population growth rates implied by the assumption of $k \leq 1$. This assumption is made merely for expositional reasons and will be relaxed in the next section.

The result above comes from the interaction between young people's preferences for marrying within their cohort and the effect of desired fertility on the age structure of the population. When k increases, population growth increases and therefore the age structure of the economy is affected, with a larger share of young people $q = \frac{1+n}{2+n}$. Therefore young people meet people of the same cohort with higher probability. Since young agents obtain higher utility from marrying other young people (and therefore set lower reservation values for them), they tend to marry younger as age the structure of population is also becoming younger.

2.7.2. Differential fecundity

Now consider the case where people desire to have $k \in (1, \overline{k}]$ children. If $k > 1$, old women are not able to bear their desired number of children. As shown in Section 2.4.1, if men wait until age 2 they still can marry a young woman and have k children. Recall from (8) that the expected utility that a man obtains from marrying at age 2 is $U_2^m = \gamma_{2,1}^m k\mu + \left(1 - \gamma_{2,1}^m\right)\mu$. On the other hand, if women wait until age 2, they will get the same utility regardless of whom they marry, $U_2^w = \mu$, (9). Since in equilibrium $\gamma_{2,1}^m > 0$ (from Theorem 1), this implies that differential fecundity increases the value of waiting for men. Therefore, men become relatively choosier than women.

Proposition 6. *Under Assumption 1, if people want to have more children than those that old women are able to bear $k \in (1, \overline{k}]$, there is always a steady state equilibrium where young men are choosier than young women, that is $R_{1,1}^m > R_{1,1}^w$ and $R_{1,2}^m > R_{1,2}^w$.*
Proof. *When $k > 1$ we have from (24) and (25) that $R_{1,1}^w = \frac{\mu}{2k}$ and $R_{1,2}^w = \frac{\mu}{k}$. Substituting $R_{1,2}^w$ in (20) and (21) we have,*

$$R_{1,1}^m = \frac{\mu}{2k}\left\{1 + \lambda^m p_1^w [k-1]\left[1 - G^m\left(\frac{\mu}{k}\right)\right]\right\} > R_{1,1}^w,$$

and

$$R_{1,2}^m = \mu\left\{1 + \lambda^m p_1^w [k-1]\left[1 - G^m\left(\frac{\mu}{k}\right)\right]\right\} > R_{1,2}^w,$$

since $\lambda^m p_1^w > 0$. ■

From simple observation of (24) and (25) it is clear that women's reservation values both for young and old men old men are decreasing in k. From Proposition 6 we know that if differential fecundity plays a role, men are choosier than women. The previous result does not imply that as desired fertility increases men's reservation values will necessarily increase. In fact, as I show below, if desired fertility increases men also reduce their reservation values for young women, but to a lesser extent than women do. However, young men will increase their reservation values for older women, which is the main driving force behind the difference in the timing of marriage.

Proposition 7. *Under Assumptions 1 and 2, then in the neighborhood of the unique steady state equilibrium where $k = 1$, given a small increase in the desired number of children:*

1. *Young men become choosier with old women and less choosy with young women: $\frac{\partial R_{12}^m}{\partial k} > 0$ and $\frac{\partial R_{11}^m}{\partial k} < 0$. In the latter case, the decrease in the reservation value is lower than the decrease in young women's reservation value: $\left|\frac{\partial R_{11}^m}{\partial k}\right| < \left|\frac{\partial R_{11}^w}{\partial k}\right|$.*

2. *Women marry young with higher probability:* $\frac{\partial \Gamma^w}{\partial k} > 0$*. In addition, the difference between the probability of marriage at age 1 of women and men,* $\Gamma^w - \Gamma^m$*, is positive and increasing in* k*.*

3. *The ratio of single men to single women,* ϕ*, is greater than 1 and increasing in* k*.*

4. *Similarly to Proposition 5, population growth increases, therefore the fraction of young people in the economy,* q *increases.*

Proof. *See Appendix A.5* ■

Proposition 7 establishes that when people desire to have more children than the number that a woman who marries at age 2 is able to bear, differential fecundity plays a role and single men become choosier than single women. Consequently, single women tend to marry younger than men, and the age gap is an increasing function of the desired number of children. Moreover, it establishes that, as the desired number of children increases, single men increasingly outnumber single women.

The mechanism behind Proposition 7 is the following. If people in the economy want to have a number of children that is feasible for women that marry at age 2, the biological differences between men and women play no role in the marriage market. However, biological gender asymmetries do appear if society want to have more children than those that women of age 2 are able to bear ($k > 1$). Therefore, young men are less willing than young women to accept an older spouse and the value of waiting until age 2 increases for men relative to women. Notice that men also decrease their reservation values for young women (albeit in a lesser extent) because if they wait they may not be able to marry a young woman when old.

The increase in the sex ratio is the necessary counterpart of the increase in the age gap. As young women are more likely to marry old men, in equilibrium more young men have to wait until period 2 in order to marry. Therefore, it must be the case that single men outnumber single women. [18]

Notice that while an increase in k makes women unambiguously marry younger, it has ambiguous effects on men's timing of marriage. In the case of men there are two countervailing effects: because of differential fecundity, they tend to marry older as k increases, but because of the change in the demographic structure they would marry younger (Proposition 5). These two effects will be analyzed in more detail in the next section when we study the model dynamics. As we will see later, the transitional dynamics of the model is particularly affected by the level of the sex ratio, ϕ_t, which acts as a persistence factor in determining the age gap, and by the evolution of the age structure of the population, q_t, which affects the marriage timing.

3. A generalized model

In this section I extend the model developed in Section 2 to a full life cycle model with a finite number of yearly periods. The objective of this extension is to study the empirical implications of the model developed above, specifically the dynamics of marriage behavior in the U.S. after the period known as the "baby boom". Since in this stylized model desired fertility is considered

[18] As stated in the Introduction, this result has implications in light of the empirical literature on marriage timing. It is a common practice in this literature to use the single sex ratio as a measure of relative availability of men and women, ignoring the endogeneity problem caused by the relationship between the sex ratio and the age gap. See, for example, Fitzgerald (1991), McLaughlin et al. (1993), and Parrado and Centeno (2002).

exogenous, the "baby boom" will be modeled as a shock on desired fertility. However, our main focus of interest will not be this period itself, but mainly the marriage behavior of the generations born during and after the "baby boom", affected by the demographic changes caused by this large temporary increase in fertility rates.

In this section, I study a model economy populated by a continuum of men and a continuum of women. Men and women live for at most J years, which we denote with subindex $j = 15, 16, \ldots, J$. Men and women in this model economy differ in their fecundity, and in their longevity. And they derive utility from being married and having children. From now on I relax the assumption of the constant population growth rate maintained thorough Section 2.

3.1. Mortality

In our model economy each period every person faces an exogenous probability of dying until the following period. We denote these probabilities by $\delta^i_{j,t}$, for $i \in \{m, w\}$ and we assume that they are gender, age and time dependent. Since people live at most for J periods, $\delta^m_{J,t} = \delta^w_{J,t} = 1$ for any period $t = 1, 2, \ldots$. These assumptions also imply that the probability that a person of gender i who is a year-old in period t survives until age b is

$$\Psi^i_t(a, b) = \prod_{j=a}^{b-1} \left(1 - \delta^i_{,j,t+j-a}\right) \tag{38}$$

3.2. Matching technology

Costly double sided search for spouses is a distinguishing feature of our model economies. The probabilities of being matched depend on an exogenous parameter that measures the search frictions, and on the ratio of available singles. Let $s^i_{j,t}$ denote the number of j year-old singles of gender i in period t and let $S^i_t = \sum_{j=15}^{J} s^i_{j,t}$ denote the total number of singles of gender i. Then the probability that a single man meets a single b year-old woman is

$$\lambda^m_{b,t} = \rho \left[\min\left(\frac{S^w_t}{S^m_t}, 1\right)\right] \frac{s^w_{b,t}}{S^w_t} \tag{39}$$

where parameter $0 < \rho \leq 1$ measures the search frictions.[19]

In this model economy the decision problems of single men and women are exactly identical. For notational convenience I describe the problem and variables that pertain to single men only. To obtain the corresponding variables for single women simply substitute the m's for w's and the b's for a's.

3.3. Fecundity

The expected number of children depends on an exogenous, time varying parameter that desired fertility k_t and biological constraints based on men's and women's age. (determined at the time of marriage) and biological constraints. Even though this is not an easy matter, I will make the

[19]Notice that this matching function is different from the one in expression (3). While the model of Section 2 required differentiability in the matching function, keeping the same function would add one more free parameter to the calibration exercise. This change does not affect the results at all.

following assumptions that are roughly consistent with the literature on the subject.[20, 21] Define as $c_t^m(a,b)$ the number of children that a man of age a expects to have if he marries at time t a woman of age b. Then,

$$c_t^m(a,b) = \begin{cases} k_t & \text{if } a \leq 55 \text{ and } b \leq 30 \\ \min[k_t, 3] & \text{if } a \leq 55 \text{ and } 30 < b \leq 35 \\ \min[k_t, 2.5] & \text{if } a \leq 55 \text{ and } 35 < b \leq 38 \\ \min[k_t, 2] & \text{if } a \leq 55 \text{ and } 38 < b \leq 40 \\ \min[k_t, 1] & \text{if } a \leq 55 \text{ and } 41 < b \leq 44 \\ \min[k_t, 0.5] & \text{if } a \leq 55 \text{ and } 44 < b \leq 49 \\ 0 & \text{if } a > 55 \text{ or } b > 49. \end{cases} \tag{40}$$

In a similar way $c_t^w(a,b)$ is defined.[22] Therefore, I assume that men are fecund until age 55 and women married after age 30 do not have enough time to have more than three children.[23]

3.4. Payoffs from marriage

The period utility of marriage that a single men a obtains from marrying in period t with someone of age b and a given marriage quality is $zc_t^m(a,b)$, where z is an independent and identically distributed realization from $G(z)$, the same for men and women (Assumption 1). The period utility of single people is constant endowment of A.[24]

The expected values of marriage. The value that an a year-old groom expects to obtain from marrying a b year-old bride who has drawn realization z and has proposed to him is

$$u_t^m(a,b,z) = zc_t^m(a,b) \sum_{\ell=0}^{D} \beta^\ell \Psi_t^m(a, a+\ell) \Psi_t^w(b, b+\ell) \tag{41}$$

where $D = \min\{J-a-15, J-b-15\}$ and $\beta \in (0,1)$ is the discount factor.

3.5. The decision problem of singles

The probabilities of marriage. The probability that an a year-old bachelor marries a b year-old single woman is

$$\gamma_t^m(a,b) = \lambda_{b,t}^m \{1 - G[R_t^m(a,b)]\} \{1 - G[R_t^w(b,a)]\} \tag{42}$$

[20] According to Wood and Weinstein (1988), women's probability of any conception changes rapidly after age 40, as a result of significant changes in the ovarian function. Between ages 25 and 40, this probability for women is remarkably constant. This finding suggests that any reduction in the physiological capacity to bear children between ages 25 and 40 is attributable to an elevation in intra-uterine mortality rather than to a decline in the ability to conceive. But, even accounting for intra-uterine loss, the pattern of effective fecundability remains fairly flat between ages 20 and 35.

[21] According to Hassan and Killick (2003), the effect of men's age on fecundity remains uncertain. Experiments that study the effect of age on male fecundity have been criticized on methodological grounds for using age at conception and for not taking into account confounding factors such as the age of the female or coital frequency. Hassan and Killick study male infecundity by comparing the time to pregnancy —measured from the onset of the attempts to achieve pregnancy— for men of different age groups. They found that male aging leads to a significant increase in the time to pregnancy, especially after ages 45 to 50.

[22] Here "2.5 children" should be interpreted as "two children and a 50% probability of a third child".

[23] This restriction will be only binding during the peak of the baby boom.

[24] In the two-period model of Section 2, for simplicity this endowment was normalized to 0.

where $R_t^m(a,b)$ denotes the reservation value that a single man who is a year-old in period t set for b year-old single women, and $R_t^w(a,b)$ is the reservation value that b year-old single women set for a year-old bachelors.

Consequently, the probability that a single a year-old bachelor marries a woman of any age is

$$\Gamma_{a,t}^m = \sum_{b=15}^{J} \gamma_t^m(a,b) = \sum_{b=15}^{J} \lambda_{b,t}^m \{1 - G[R_t^m(a,b)]\}\{1 - G[R_t^w(b,a)]\} \tag{43}$$

which naturally depends on the reservation values of both men and women.

The expected values of marrying in the current period. The value that an a year-old single expects to obtain for marrying in the current period t, before any matches have taken place is

$$EU_{a,t}^m = \sum_{b=15}^{J} \gamma_t^m(a,b)\, Eu_t^m[a,b,z \mid z \geq R_t^m(a,b)] \tag{44}$$

The expected values of remaining single. Therefore, the expected value of remaining single one more period for a single a year-old man is then,

$$V_{a,t}^m = EU_{a,t}^m + \left(1 - \Gamma_{a,t}^m\right)\left\{A + \left(1 - \delta_{a,t}^m\right)\beta E_t\left[V_{a+1,t+1}^m\right]\right\} \tag{45}$$

where the first term in the curly brackets shows the expected value of getting married before a match takes place and the second term the value of remaining single for one more period.

Reservation values. The optimal reservation values that a year-old men and b year-old women set for each other in each period can be found solving the system of $2J^2 \times T$ equations in $2J^2 \times T$ unknowns that result from equating the J^2 expressions (41) and (45) for the men and the corresponding J^2 women's equations for periods $t = 1, 2, 3 \ldots T$.. Formally, the $\{R_t^i(a,b))\}$ are the values of z that solve

$$u_t^m(a,b,R_t^m(a,b)) = \left\{A + \left(1 - \delta_{a,t}^m\right)\beta E_t\left[V_{a+1,t+1}^m\right]\right\} \tag{46}$$

and

$$u_t^w(a,b,R_t^w(a,b)) = \left\{A + \left(1 - \delta_{a,t}^w\right)\beta E_t\left[V_{a+1,t+1}^w\right]\right\}. \tag{47}$$

3.6. Fertility and Population Dynamics

Every period, a number of 15 year-old single men and women, $s_{15,t}^m = N_{15,t}^m$ and $s_{15,t}^w = N_{15,t}^m$ enter the market. Since now agents live 15 years as children, the population entering the market at period t is born at period $t-15$. Therefore equation (30) is replaced by

$$N_{15,t}^i = \varpi_{t-15}^i \Psi_{t-15}^i(0,15)\left\{\sum_{a=1}^{J} s_{a,t-15}^w \sum_{b=15}^{J} \gamma_{t-15}^w(a,b)\, c_{t-15}^w(a,b)\right\}, \tag{48}$$

where the expression between curly brackets is the number of children born in period $t-15$, ϖ_{t-15}^i is the fraction of children of gender $i \in \{m,w\}$ and $\Psi_{t-15}^i(0,15)$ is their survival probability up to age 15.[25] The stock of agents of older ages $\{N_{,j,t}^i\}$, is given by

$$N_{j,t}^i = \left(1 - \delta_{,j,t-1}^i\right) N_{j-1,t-1}^i \text{ for } 16 \geq j \leq J. \tag{49}$$

[25] For the easiness of exposition, Equation (48) implies that all children are born in the first year of marriage. Given that this assumption may affect results significantly in periods of rapidly changing fertility, when solving the problem I actually restricted women to having one children per period in up to four consecutive periods.

3.7. Equilibrium Definition

Given a vector of desired number of children over time, k_t , an equilibrium for this economy is a vector of measures of 15 year-olds, $\{N^i_{15,t}\}$, a matrix of measures of single people, $\{s^i_{j,t}\}$ for gender $i \in \{m, w\}$, and matrices of reservation values that single males set for single females and vice versa, $\{R^m_t(a, b), R^w_t(a, b)\}$, for ages $a, b, j \in \{15, 16, \ldots, J\}$ and period $t \in \{1, , \ldots, T\}$ such that,

(i) Given $R^i_t(a, b)$ and $s^i_{j,t}$, measures $\{N^i_{15,t}\}$ satisfy expression (48) and are consistent with the reservation values and the measures of singles.

(ii) Given $R^i_t(a, b)$ and $N^i_{15,t}$, measures $\{s^i_{j,t}\}$ satisfy

$$s^i_{15,t} = N^i_{15,t} \text{ and} \tag{50}$$

$$s^i_{j+1,t} = s^i_{j,t-1} \left(1 - \delta_{i,j}\right) \left(1 - \Gamma^i_{j,t}\right) \text{ for } j < 15 \leq J, \tag{51}$$

where the $\Gamma^i_{j,t}$are defined in expression (43), being also consistent with the reservation values and the measures of 15 year-old people.

(iii) Given $N^i_{15,t}$ and $s^i_{j,t}$, the reservation values $\{R^m_t(a, b), R^w_t(a, b)\}$ solve the decision problems of singles described in expressions (46) and (47) and are consistent with the measures of singles and 15 year-old people.

(iv) Marriage market clears every period, that is

$$s^m_{a,t}\gamma^m_t\left(a, b\right) = s^w_{b,t}\gamma^w_t\left(b, a\right)$$

where $\gamma^m_t\left(a, b\right)$ is defined in expression(42).

4. Calibration

To calibrate the model economy we must choose the duration of the model period an initial period, a functional form for the distribution function of the match values, $G(z)$, and a value for every parameter in the model economy. These parameters are the maximum life-time, J; the fecundity profiles; the mortality probabilities, $\delta^i_{j,t}$; the fraction of people born of gender i, ϖ^i_t, the search friction parameter ρ, the discount factor, β; and the parameters that characterize the payoffs, the desired number of children per period, k_t , and the period utility of remaining single, A.

The model period. I assume the period in the model is yearly. The model starts in 1930 but the first period where the stocks of men and women entering the market are actually "produced" by the model is 1945. Between 1930 and 1945 the initial stocks of men and women are taken from fertility data.

Expectations. Given that in this model the baby boom is exogenous, I will make some assumptions on the formation of expectations during that period. Specifically I assume that

$$E_t\left[V^m_{a+1,t+1}\right] = \pi\left[V^m_{a+1,t+1}\right] + (1 - \pi)\left[V^m_{a+1,t}\right], \tag{52}$$

I assume that the parameter π in expression (52) is equal to zero for all years before 1965. In other words, I assume that before and during the baby boom people form their expectations for the next period by observing the environment faced by singles who are one year older in the current period. Between 1965 and 1973, I assume that π increases linearly and from 1974 onwards I assume $\pi = 1$, relaxing the previous assumption.[26] This assumption is made to model the "baby boom" as an unanticipated shock in desired fertility. Therefore I am ruling out the possibility of, for example, an agent trying to anticipate (delay) marry in 1940 (1960) because she is foreseeing that ten years later her desired number of children will be considerably higher (lower). Remember also that the main object of this study is to analyze marriage behavior once the baby boom is over.[27]

The time discount factor. I choose $\beta = 0.96$, as is standard in the literature.

The distribution of the match values. I assume that the distribution of match values, $G(z)$, is the logarithm of normally distributed function with mean 0 and standard deviation σ.

The maximum life-time. I assume that $J = 75$.

The mortality probabilities. The mortality probabilities are death rates taken from the Human Mortality Database for the years 1933-2006.[28,29]

The fraction of people born by gender. I use the sex ratios at birth for 1940-2002 from Mathews and Hamilton (2005).

The fecundity profiles. The assumptions on fecundity profiles are detailed in expression (40). These values are chosen so that the fecundity profiles in the model economy are roughly consistent with the findings of the literature on the subject.[30]

To complete the calibration of the benchmark model economy we are left with three free parameters and a vector of the number of desired children per period, k_t. The free parameters are the standard deviation of the distribution of match values, σ; the matching function parameter, ρ and the period utility of remaining single, A. I will proceed in two steps. First, to obtain an initial state for this economy, I assume that the economy is in the steady state and choose the numerical values of these three free parameters using three targets from the 1930 U.S. census. These targets are the fraction of ever married men, the fraction of ever married women and the ratio 15-29 / 15-44 year-olds. For this initial state, I choose $k_1 = 3.1$, the average total fertility rate during the period 1920-29. Therefore, I choose the values of σ, ρ, and A that minimize the sum of the squared differences between outcomes in the model economy and our United States targets. The values the parameters that deliver this result are $\sigma = 0.495$, $\rho = 0.38$, and $A = 1.31$. Observe in Table 1, in addition to the model targets, the values of the median age at first marriage for men and women, the mean of age differences and the sex ratio for never married people.[31] All these model's measures are very close to the U.S data except for women marrying around one year later than in the data. This difference will remain when whe compare the model dynamics with the time series.

The desired number of children. Once I obtained the parameters for the steady state, the next step is to construct the vector of desired number of children per period, k_t in order to analyze the dynamic of agents' behavior. As stated before, I use the Total Fertility Rates (TFR) for the period 1940- 2000, and for crude birth rates from 1920-1939 all provided by the NCHS.[33]

[26] The results are basically the same if π is either a concave or convex function of time between 1965 and 1973.

[27] One alternative assumption would have been considering that only some specifc cohorts were affected by the boom. However, data shows that during the complete period age-specific fertility increased for all age groups.

[28] The Human Mortality Database is compiled by the University of California, Berkeley (USA) and the Max Planck Institute for Demographic Research (Germany). This dataset is available at www.mortality.org.

[29] I use the 2006 mortality rates for 2007 onwards.

[30] For example Hassan and Killick (2003) and Wood and Weinstein (1988), discussed above.

[31] Note that the mean age difference is not the difference between the median age art first marriage. As stated above the median age at first marriage is that age when 50% of the poulation has already married and is the standard measure of marriage timing.

[33] The total fertility rate computed by the National Center of Health Statistics is the sum of the birth rates

	1930 Census[32]	*Model*
Median age at first marriage for men	24.27	24.33
Median age at first marriage for women	21.41	22.57
Mean Age difference at first marriage (years)	2.86	2.81
Fraction ever married men *	0.66	0.66
Fraction ever married women *	0.73	0.72
Sex ratio for never married people	1.32	1.23
Ratio 15-29 /15-44 years-old *	0.54	0.54

* Calibration targets.

Table 1: The Initial (steady state) Economy and the 1930 U.S. Census

Specifically I run the following OLS regression, for the period 1930-65,

$$TFR_t = \mu_0 + \sum_{i=1}^{6} \mu_i (t)^i + \varepsilon_t, \tag{53}$$

where $\mu's$ are the estimated parameters of the regression. In addition, I will assume that the economy reaches eventually a steady state where population growth is equal to zero. Therefore, using the parameters chosen previously obtain the values of k_t that, in the steady state produces the replacement fertility rate, $k_T = 2.3$. Given the previous choices, the values of k_t are the following:

$$k_t = \begin{cases} \alpha_0 + \sum_{i=1}^{6} \mu_i (t)^i & \text{if } 1930 \leq t \leq 1965 \\ k_T + (k_{1965} - k_T) e^{-0.3t} & \text{if } 1966 \leq t \leq 1976 \\ k_T & \text{if } t \geq 1977 \end{cases} \tag{54}$$

where the expression in the second line of (54) ensures that desired fertility converges smoothly to their steady state value (in 1977). Observe finally that the parameter α_0 in expression (54) is different from the constant term μ_0 in (53). Since the fertility rate in the model depends on marriage probabilities, α_0 becomes a fourth free parameter chosen to minimize the sum square differences between the TFR in the data and those produced by the model.

Given the previous choices, the evolution of the TFR in the data and the model is displayed in Panel (a) of Figure 2. Two vertical dashed lines are shown in the figure: the first one shows the peak on the total fertility rate, in 1957, while the second is located in 1985, which will be the starting point of our analysis. Notice that by 1985 U.S. fertility rates are in the process of recovery after having reached their minimum level in 1977. Observe that the model's fertility is almost reaching its steady state level, given the parameters were set at a constant level since 1977 (see expression(54)). Therefore, I chose a "conservative" starting point to ensure that all changes in model's variables come from model dynamics.

The key force affecting model dynamics is the age structure of the population. As a measure of age structure, I choose the fraction of 15-29 over 15-44 years old (from now on, "the fraction

of mothers in 5-year age groups multiplied by five. The birth rates are the numbers of live births per 1,000 women in a given age group. Beginning in 1970, the total fertility rate excludes the children born by nonresidents. The National Center of Health Statistics data is available at www.cdc.gov/nchs/data/statab/t991x07.pdf.Data on birth rates for the period 1909-2000 is available from www.cdc.gov/nchs/data/statab/t001x01.pdf.

Year	*Men* (1) Data	*Men* (2) Model	*Women* (3) Data	*Women* (4) Model
1957	22.6	23.3	20.3	21.4
1985	25.5	25.1	23.3	24.2
2009	28.1	25.7	25.9	24.2
2040		26.2		24.5
△ 1957-1985	2.9	1.8	3.0	2.8
△ 1985-2009	2.6	0.6	2.6	0.0
△ 2009-2040		0.5		0.3

Table 2: Median age at first marriage: Benchmark and U.S. data

of young people") whose evolution compared with U.S. data is shown in Panel (b) of Figure 2.[34] Notice that this demographic measure reaches its maximum in 1975 (at a time when the first baby boomers, born starting in 1946, are about to turn 30 years old) and then minimum in 1995 (when those born at the end of the baby boom turn 30). Notice that the model economy shows a new maximum before 2020, which indicates that the cohort of children of early baby boomers (born around 1980), will turn 30. Observe finally that the fraction of young people would reach its own steady state value (0.5) between 2030 and 2040, which implies that fertility rates reach their steady state around 1990 despite the fact that all model parameters have been constant since 1977.

5. Findings

5.1. Median Age at First Marriage

As stated above, the objective of the extended model developed in the previous section is to study a potential causal effect from demographic changes to marital behavior. Specifically, I study if the demographic change originated by the baby boom that took place mainly in the 1950's and early 1960's had any impact in the marriage timing of the generations born during and after the baby boom.

Table 2 and Panels (c) and (d) of Figure 2 show the evolution of the median age at first marriage in the model and U.S. data, for men and women respectively. Panel (c) shows that men's age in the model closely follows the evolution of the data from 1940 to the early 1980's. Then, while in the data men's age continues increasing up to around 28.1 years old in 2009, in our benchmark economy it increases at a slower pace. Since the inverse relationship between fertility and age of marriage is well-known by demographers, it seems particularly interesting to study the increase in the age of marriage that occurred after the fertility rate reached its historical minimum, during the late seventies. As shown in Column 1 of Table 2, men's median age at first marriage increased by 2.6 years between 1985 and 2009, while, during the same period the model shows an increase of 0.6 years, 23% of the variation in the data. Notice that men's age only reaches its steady state level around 2040 (with an additional increase of 0.4 years), so the transitional dynamics of the model are able to account for a one year increase in men's median age of marriage since 1985. The comparison for women is shown in Panel (d) of

[34] Data of the age structure of the population is taken from the Interim State Population Projections from the U.S. Census Bureau.

Figure 2 and Column 2 of Table 2. Note that women in the model are more than one year older than in the data in the initial state (see Table 1 above), and of course that initial divergence make the evolution of women's age at first marriage in the model follow the data less closely than in the case of men. Moreover, the increase in the period 1985-2009 is negligible and only accounts for an increase of 0.3 years from 1985 to the steady state.

The last two panels of Figure 2 display comparisons of important variables that merit being shown despite their data limitations. The evolution of the average age gap at first marriage is shown in Panel (e) from 1940 to 1995.[35,36] In this case the decrease in the benchmark model's age gap appears to be greater than what the data shows. Notice that since the early 1990's the model gap starts to recover, but unfortunately NCHS did not collect data after 1995. Finally, Panel (f) displays the evolution of the ratio single men/ single women), in the model and in the U.S. Census (only decennial data is available until 2000). Note in this case that in both cases the sex ratio increases during the baby boom and then decreases. However, the model sex ratio recovers as of the late1980's before reaching its steady state level, different from the pattern observed in decennial data, which shows a decrease in the sex ratio between the 1990 and 2000 censuses.

5.2. The Benchmark Model Economy

In this section I explore the benchmark economy in more detail, in order to better understand the previous findings of a slow convergence of men's age at marriage and the age gap to the steady state. Before starting, I would like to focus briefly on what happened during the baby boom. The first four Panels of Figure 3 are a clear example of Propositions 5 and 7: the population become younger (Panel (b)), men and women marry younger (Panel (c) and there is an increase in the sex ratio and the age gap (Panel (d)). However, different from the model developed in Section 2, this is not a steady state economy and the changes on the age structure of the population, with a lag of around two decades from the fertility boom, are only temporary. Here I argue that the changes in demographic structure caused by the "baby boom" are behind the persistence mechanism that generates a slow adjustment in the age of marriage after the boom is over.

Panels (a) and (b) of Figure 3 show the comparative evolution of the fraction of young people with the TFR and the single sex ratio, respectively. Observe first that the evolution age structure of the population virtually mirrors the TFR with a 20-year lag, falling below its steady state level during the 1990's and the early 2000's. In other words, the minimum in the ratio of young people occurs when the generation of younger people born at times of the lowest fertility rates (late 1970's to early 1980's) are in their late teens or their twenties. These younger people are the children of baby boomers, a large cohort in absolute value, but small if we compare it with the previous cohorts, which explain the drop in the fraction of young people. Second, when comparing the single sex ratio, which is a key determinant of meeting rates, with the fraction of young people, we can observe a negative correlation between the two demographic measures, especially after the peak of the fertility rates in 1957 and until the early 1980's. After the imbalance produced in the sex ratio during the baby boom, the arrival of larger cohorts of younger people to the marriage market helped to balance the sex ratio relatively quickly (late

[35] In order to calculate the average age gap we need data on actual marriages. Although highly correlated, the age gap is not an equivalent measure to the difference in median age at marriage (taken from the fraction of ever married population from either Census data or the Current population Survey.), which are relative measures of marriage timing within a cohort.

[36] Unfortunately, there are not consistent series of the age at first marriage for the whole period under consideration. See Footnote 1 (pp. 2).

1960's and 1970's). Beginning in the early 1980's, the single sex ratio recovers, mainly during the period where the fraction of young people was also below its steady state level.

In Panel (c) I compare the median age at first marriage with the TFR, while Panel (d) shows that the high correlation between the age difference and the single sex ratio, which, despite the data limitations, seems to be consistent with the evolution shown in Panel (e) of Figure 1. Notice that, after reaching a minimum in 1957 (the peak of fertility rates), the median age at first marriage of both men and women started to recover, with women's recovery beginning earlier and at a much more rapid pace than men's. Observe also that if we either look at the differences in timing (median ages at first marriage Panel (c)) or to the mean of age gap at marriage directly (Panel (d)), the gap seems to shrink from the peak of the baby boom to the early eighties and then to increase again, to reach a steady state that is lower than the initial state. Moreover, Panel (d) shows that the evolution of the single sex ratio and that of the age gap seem to mirror each other.[37]

The last two panels of Figure 3 show the evolution of two model variables that may be helpful in order to understand the mechanism behind the results. Panel (e) shows the expected value of being in the marriage market at ages 25 and 30, which is $V_{a,t}^{i}$ from expression (**??**) , with $i \in \{m, w\}$ and $a \in \{25, 35\}$. As may be expected, the value of marriage increases for both 25 year-old men and women during the baby boom (reaching a peak in 1957) and then stabilizes in the early nineties. Notice that the case of 35 year-olds is different: while for men it replicates the pattern of 25 year-olds at a lower level, the search value for 35 year-old women sharply decreases during the baby boom and then recover during the seventies, as fertility decreases. The narrowing of the gap between the value of search of 25 and 35 year-old women can be seen as a relative increase in the value of waiting and helps to explain why women's median age at first marriage recovers more rapidly than men's.

Finally, in Panel (f) of Figure 3, I show the evolution of the probability of receiving a proposal at ages 25 and 35 for men and women. We define the probability that a man of age a receives a marriage proposal from a woman of age b as the probability of meeting such a woman ($\lambda_{b,t}^{m}$ from expression (39)) times the probability that she is willing to marry a man of age a, $\{1 - G[R_t^w(b, a)]\}$. Therefore the probability that a man of age a receives a marriage proposal is

$$\Lambda_{a,t}^{m} = \sum_{b=15}^{J} \lambda_{b,t}^{m} \{1 - G[R_t^w(b, 25)]\} \tag{55}$$

Since proposals are determined by the other gender's reservation value, here we can see the asymmetric market faced by men and women during and after the baby boom. Notice again that during the baby boom, 35 year-old women are less likely to receive a marriage proposal than men of their same cohort. At the end of the baby boom, proposals for older women increase sharply, explaining the pattern of the value of search in Panel (e).

So far I have presented arguments to justify why women's age at marriage increased more rapidly than men's after the baby boom. However, the reason for the slow adjustment in men's age remains to be explained. In the next sections I perform two counterfactual experiments to explain that slow adjustment. In the first experiment, I assume that there is not such an increase in desired fertility from the mid 1940's to the 1960's, as if the "baby boom" had never occurred. In the second experiment, the economy is just like in the benchmark but there is no population growth. Therefore, I "shut down" the general equilibrium effect caused by the changes in the demographic structure. Therefore, the goal of this second experiment is to isolate the persistence

[37] See the interpretation of Proposition 7 on page 16.

mechanism that may have affected the marriage behavior of the cohorts born during the baby boom.

5.3. Counterfactual Experiment I: "No baby boom"

In the first counterfactual experiment, I assume that k_t converges slowly from the initial level (before 1940) to the steady state (replacement rate). The results of this experiment are shown in Figure 4 and in columns (2) and (5) of Table 3. Panel (a) of Figure 4 shows the nature of the experiment. Notice that fertility rates fall slowly from 1940 to reach their steady state values in 1990. By "eliminating" the baby boom, the structure of the population is only slightly affected and reaches its steady state value by the year 2000, with the population slightly "older" than in the initial state. Panel (b) shows the evolution of the median age at first marriage for men and women. As shown in the figure, if the baby boom had not occurred, the age at first marriage for men and women would have slowly risen to a new state level.[38]

Notice, however, that the convergence in marriage timing with the benchmark model occurs much later than the convergence in fertility rates (around 1990). As shown in Table 3, for example in 2005 men's median age at marriage was 0.5 years higher than in the benchmark model, while women's age was 0.3 years higher.

As fertility rates decrease from 1940's to early 1980, we observe an "older" age structure of the population (Panel (c)), together with a joint decrease in the sex ratio and in the age gap (Panels (d) and (e))). However, when fertility rates stabilize, the sex ratio is below its steady state level, as in the benchmark model. Therefore, as of the early 1980's the age gap starts recovering, and so does the sex ratio, to reach their new steady state. Observe also in Panel (f) that proposals received for both men and women are below the benchmark.

The connection between the slow convergence in the age at marriage and demographic structure is suggested by the evolution of marriage proposals shown in Panel (f). Notice that if the baby boom had not occurred, the period probability of receiving a marriage proposal would have been lower than in the benchmark model for both men and women, to converge as the demographic structure is getting closer to its steady state value.

In order to investigate further the connection between the demographic structure and the age of marriage, in the next experiment the economy will be like that in the benchmark model during the baby boom period. However, I will assume that the baby boom does not affect the age structure of the population, in order to isolate the general equilibrium effect caused by the demographic structure.

5.4. Counterfactual Experiment II: Constant Inflow of 15 year-olds

In this experiment I assume that, every period, the inflow rate of 15 year-olds to the economy is constant, regardless of what happens with fertility rates. As shown in Panel (a) of Figure 5, in this case, the evolution of fertility rates mimics those in the benchmark model, but the age structure of the population is constant. As a consequence, men's age at first marriage recovers much more rapidly than in the benchmark economy, starting to increase in 1957 reaching levels

[38] If we look at these findings in light of Proposition 7 , as a result of a permanent decrease in desired fertility we should expect women marrying unambiguously older, with the impact on men's age at marriage being ambiguous. The two countervailing effects on men's age at marriage are, on the one hand, a decrease in search friction because they will be more eager to marry older women. On the other hand, the "aging" structure of the population will induce them to marry older (see the interpretation of the analytical results on pp. 16). In this case it appears that the second factor is playing a more important role.

Year	*Men*			*Women*		
	(1)	(2)	(3)	(4)	(5)	(6)
	Benchmark	No boom	Constant	Benchmark	No boom	Constant
1957	23.3	25.0	23.3	21.4	23.9	21.4
1985	25.1	25.7	25.9	24.2	24.4	24.2
2009	25.7	26.0	26.0	24.2	24.4	24.4
2040	26.2	26.2	26.2	24.5	24.5	24.5
△ *1957-1985*	1.8	0.7	2.6	2.8	0.4	2.8
△ *1985-2009*	0.6	0.3	0.1	0.0	0.1	0.2
△ *2009-2040*	0.5	0.2	0.2	0.3	0.1	0.1

Table 3: Median age at first marriage: Benchmark and Counterfactual Experiments

close to their steady state in the early 1980's (see Panel (b)). For example, Table 3 shows that in 1985 the age of marriage of men is 0.8 years higher with constant inflow of the population compared to the benchmark economy. In the case of women, the evolution is much closer to the one in the benchmark economy, but there is also a slow adjustment to the steady state, albeit small.

If we look at the evolution of the sex ratio on Panel (d), we see that in both the benchmark and the counterfactual economy, the sex ratio starts rising with the baby boom, because of the incentives driven by differential fecundity analyzed in Section 2. In the partial equilibrium case the sex ratio continues increasing until around 1965, and only starts decreasing when the fertility rates plummeted during the late 1960's, to reach its steady state level in the late 1990's, almost simultaneously with the TFR. As desired fertility remains high, keeping everything else constant, the sex ratio would mechanically continue growing, because it is already unbalanced (due to the initial shock) and any new marriage takes one man and one woman out of the market. However, in the benchmark economy, the sex ratio starts decreasing in the late 1950's, falling below its steady state levels around the mid eighties and then recovering to reach the steady state only around the year 2040. The evolution of the age gap, shown in Panel (e) is a mirror of what happen with the sex ratio. While in the partial equilibrium experiment the age difference decreases smoothly towards the steady state once the baby boom is over, in the benchmark economy it decreases sharply below the steady state level and recovers to reach the steady state only forty years later.

The factor behind the slower adjustment patterns in the benchmark model with respect to this partial equilibrium experiment is the entrance of the first baby boomers in the economy before the baby boom is over (i.e. between 1960 and 1965, see Panel (a)). Even though the asymmetries in the incentives about marriage timing between men and women remain high until the mid 1960's, the fact that larger, sex balanced cohorts enter the economy prevents the sex ratio from continuing to grow. Therefore, when the baby boom is over, the sex ratio and the age gap will decrease sharply, not only because women's biological constraints become less binding, but also because the demographic structure is temporarily "younger" due to the baby boomers cohorts.

As the temporary change in the age structure of the population delays the adjustment of the sex ratio to the steady state, it also alters the matching process and therefore the timing of marriage. As shown in Panel (f), although both men and women receives more proposals than in the steady state, because the larger cohorts of young people create "a thicker market externality", the impact is sizable for men, since they face relatively higher meeting rates than women. As men keep receiving relatively more proposals due to the impact of age structure of the population on the sex ratio, their median age at first marriage will slowly adjust toward its steady state as the demographic structure finds its own steady state level.

6. Conclusion

In this paper, I develop an equilibrium, two-sided search model of marriage with endogenous population growth to study the interaction between fertility, the age structure of the population and the age at first marriage of men and women. I show that, given an increase in the desired number of children (which is an exogenous parameter in this model), age at marriage is affected through two different channels. First, as population growth, the age structure of the population acts as a "thicker market" externality, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, young women become relatively less choosy than young men. In equilibrium, women are more likely to marry older men and single men outnumber single women.

The results above have dynamic implications that are consistent with observed patterns in the U.S. data, specifically after the "baby boom" period, which in the context of the model can be seen as a temporary increase of desired fertility. The main finding that the change in the demographic structure caused by the entrance of "baby boomers" in the marriage markets acts as a persistence mechanism, affecting marriage timing. Specifically, I found that the change in demographic structure may have delayed the adjustment in the sex ratio after the baby boom, increasing matching rates of men relative to those of women.

Even though that this stylized model is not able to account for all the factors that may have influenced the continuous increase in the age of marriage since the mid-seventies, it provides a link, demographic composition, to reconcile the evolution of fertility rates with time series of the age of marriage. However, as fertility rates reached the replacement rates in the U.S. decades ago, the biological time frame for women to have two children in their lifetime is so wide that additional structure would be necessary to study further the causes of the changes in marriage timing.

The marriage market model developed in this article can be easily extended to a richer environment that includes human capital accumulation, labor supply and fertility choice in order to make a quantitative analysis of the dynamics of marriage in the last few decades. The results above suggest that the baby boom may have affected the marriage behavior of baby boomers themselves through identifiable, persistence factors, whose study requires going beyond standard, comparative static analysis.

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A. Technical Appendix

A.1. Existence

To proof existence, I will make use of Brouwer's Fixed Point Theorem to show that the following functional relationship is a fixed point

$$\Psi\left(R^m_{1,2},\Gamma^m,\Gamma^w,n\right) \longrightarrow \begin{pmatrix} R^m_{1,2}=\Psi_1\left(R^m_{1,2},\Gamma^m,\Gamma^w,n\right)\\ \Gamma^m=\Psi_2\left(R^m_{1,2},\Gamma^m,\Gamma^w,n\right)\\ \Gamma^w=\Psi_3\left(R^m_{1,2},\Gamma^m,\Gamma^w,n\right)\\ n=\Psi_4\left(R^m_{1,2},\Gamma^m,\Gamma^w,n\right)\end{pmatrix} \tag{56}$$

where

$$\Psi_1=\mu_x+\left\{\rho\left(\frac{2+n-\Gamma^m}{2+n-\Gamma^w}\right)^{\theta-1}\left(\frac{1+n}{2+n-\Gamma^w}\right)\left[1-G^m\left(\frac{\min\{k,1\}}{k}\mu_y\right)\right][k-\min\{k,1\}]\right\}\mu_x \tag{57}$$

$$\begin{aligned}\Psi_2=\rho\left(\frac{2+n-\Gamma^m}{2+n-\Gamma^w}\right)^{\theta-1}&\left\{\left(\frac{1+n}{2+n-\Gamma^w}\right)\left[1-G^w\left(\frac{\min\{k,1\}R^m_{1,2}}{2k}\right)\right]\left[1-G^m\left(\frac{\min\{k,1\}\mu_y}{2k}\right)\right]\right.\\ &\left.+\left(\frac{1-\Gamma^w}{2+n-\Gamma^w}\right)\left[1-G^w\left(R^m_{1,2}\right)\right]\right\}\end{aligned} \tag{58}$$

$$\begin{aligned}\Psi_3=\rho\left(\frac{2+n-\Gamma^m}{2+n-\Gamma^w}\right)^{\theta}&\left\{\left(\frac{1+n}{2+n-\Gamma^m}\right)\left[1-G^w\left(\frac{\min\{k,1\}R^m_{1,2}}{2k}\right)\right]\left[1-G^m\left(\frac{\min\{k,1\}\mu_y}{2k}\right)\right]\right.\\ &\left.+\left(\frac{1-\Gamma^m}{2+n-\Gamma^m}\right)\left[1-G^m\left(\frac{\min\{k,1\}}{k}\mu_y\right)\right]\right\}\end{aligned} \tag{59}$$

$$\Psi_4=\frac{k\Gamma^w}{2}-1+\frac{\sqrt{\left(k\Gamma^w\right)^2+4\min\{k,1\}\left(1-\Gamma^w\right)}}{2} \tag{60}$$

Define a set $\Omega=[0,\overline{x}]\times[0,1]\times[0,1]\times\left[-0.5,(\overline{k}-1)\right]$. The first step is to show that $\Psi(.)$ is continuous. The continuity of $\Psi(.)$ is ensured because of the continuity of $G^w(.)$and $G^m(.)$ and the boundaries of the ratio single men/single women. The next step is to show that $\Psi(.)$ maps from Ω to itself. In order to show this, I pick a point $\chi_0=(R^m_0,\Gamma^m_0,\Gamma^w_0,n_0)$, where $R^m_0\in[0,\overline{x}]$, $\Gamma^m_0\in[0,1]$, $\Gamma^w\in[0,1]$ and $n_0\in[-0.5,(k-1)]$. I have to show that $\Psi_1(\chi_0)\in[0,\overline{x}]$, $\Psi_2(\chi_0)\in[0,1]$, $\Psi_3(\chi_0)\in[0,1]$ and $\Psi_4(\chi_0)\in\left[-0.5,(\overline{k}-1)\right]$.

Consider first the case of Ψ_1. By simply observation of equation (57), we know that $\Psi_1(\chi_0)\geq\mu_x$, so we only have to look for its upper bound, which occurs at $k=\overline{k}$. By definition, $\lambda^m_1(\chi_0)=\rho\left(\frac{2+n_0-\Gamma^m_0}{2+n_0-\Gamma^w_0}\right)^{\theta-1}\left(\frac{1+n_0}{2+n-\Gamma^w_0}\right)\leq 1$ and $G^m\left(\frac{\mu_y}{k}\right)\in(0,1)$. Therefore, evaluating Ψ_1 at $\overline{k}=\frac{\overline{x}}{\mu_x}$ (by condition (5)), we have

$$\begin{aligned}\Psi_1(\chi_0)&=\mu_x+\left\{\lambda^m_1(\chi_0)\left[1-G^m\left(\frac{\mu_y}{\overline{k}}\right)\right]\left[\overline{k}-1\right]\right\}\mu_x\\ &=\mu_x+\left\{\lambda^m_1(\chi_0)\left[1-G^m\left(\frac{\mu_y}{\overline{k}}\right)\right]\left[\overline{x}-\mu_x\right]\right\}<\overline{x},\end{aligned}$$

which implies that $\Psi_1 = R^m_{1,2} \in [\mu_x, \overline{x})$.

Consider now Ψ_2. Denote $p^w_0 = \left(\frac{1+n_0}{2+n-\Gamma^w_0}\right) > 0$. Given that by definition $\rho\left(\frac{2+n_0-\Gamma^m_0}{2+n_0-\Gamma^w_0}\right)^{\theta-1} \leq 1$, that $G^m\left(\frac{\min\{k,1\}\mu_y}{2k}\right) \in (0,1)$ and also that $G^m\left(\frac{\min\{k,1\}\mu_y}{2k}\right) \in (0,1)$, whe have,

$$\Psi_2(\chi_0) = \rho\left(\frac{2+n_0-\Gamma^m_0}{2+n_0-\Gamma^w_0}\right)^{\theta-1}\left\{p^w_0\left[1-G^w\left(\frac{\min\{k,1\}R^m_0}{2k}\right)\right]\left[1-G^m\left(\frac{\min\{k,1\}\mu_y}{2k}\right)\right]\right.$$
$$\left. + \left(1-p^w_0\right)\left[1-G^w\left(R^m_0\right)\right]\right\} \in (0,1). \tag{61}$$

Therefore $\Psi_2 = \Gamma^m \in (0,1)$. This trivially implies $\gamma^m_{1,1} = \gamma^w_{1,1} > 0$.

A similar argument can be used to show that $\Psi_3 = \Gamma^w \in (0,1)$. Notice that the fact that Γ^m and Γ^w are both less than one imply that $\left(\frac{1-\Gamma^m}{2+n-\Gamma^m}\right) > 0$ and $\left(\frac{1-\Gamma^w}{2+n-\Gamma^w}\right) > 0$. In addition, $R^m_{1,2} < \overline{x}$, as shown above. Therefore, we have that $\gamma^m_{1,2} > 0$ and $\gamma^w_{1,2} > 0$, which, by (34) and (35), imply $\gamma^w_{2,1} > 0$ and $\gamma^m_{2,1} > 0$.

Finally, in Section 2.5.2 I have already shown that $n \in \left[-0.5, \overline{k}-1\right]$. Given the interior solutions of Γ^m and Γ^w we have that $\Psi_4 \in \left(-0.5, \overline{k}-1\right)$. Therefore an interior steady state equilibrium exists for this economy

A.2. Proof of Proposition 3 (symmetric equilibrium)

I have to show that when $G^m = G^w = G$ and $k \in [\frac{1}{2}, 1)$, the point $\chi_s = (\mu, \Gamma^s, \Gamma^s, n^s)$ is a solution of $\Psi(.)$.[39] Plugging χ_s into $\Psi(.)$, making $k < 1$ and manipulating we get

$$\begin{aligned}
\Psi_2(\chi_s) &= \rho\left(\frac{2+n^s-\Gamma^s}{2+n^s-\Gamma^s}\right)^{\theta-1}\left\{\left(\frac{1+n^s}{2+n^s-\Gamma^s}\right)\left[1-G\left(\frac{\mu}{2}\right)\right]^2 + \left(\frac{1-\Gamma^s}{2+n^s-\Gamma^s}\right)\left[1-G(\mu)\right]\right\} \\
&= \left(\frac{1+n^s}{2+n^s-\Gamma^s}\right)\left[1-G\left(\frac{\mu}{2}\right)\right]^2 + \left(\frac{1-\Gamma^s}{2+n^s-\Gamma^s}\right)\left[1-G(\mu)\right] \\
&= \Psi_3(\mu, \Gamma^s, \Gamma^s, n^s) \\
\Psi_4(\chi_s) &= \frac{k\Gamma^s}{2} - 1 + \sqrt{\frac{(k\Gamma^s)^2}{4} + k(1-\Gamma^s)}.
\end{aligned}$$

Therefore χ_s is an equilibrium of $\Psi(.)$ and satisfies

$$\begin{aligned}
\Gamma^m &= \Gamma^w \in (0,1) \\
n &\in (-0.5, 0).
\end{aligned}$$

Given that $\Gamma^m = \Gamma^w$, it is straightforward that the number of single old men equal the number of old single women, $s^m_{2,t} = s^w_{2,t} = (1+n)^{t-1}(1-\Gamma^s)$ (by (26)), which makes $S^m_t = S^w_t$. Furthermore, since $n < 0$, the fraction of young people in the population, $q_1 = \frac{1+n}{2+n} < \frac{1}{2}$ (by (1)).

[39] Recall that, when $k \leq 1$, $R^m_{1,2} = R^w_{1,2} = \mu$, by (36). Similarly, $R^m_{1,1} = R^w_{1,1} = \frac{\mu}{2}$.

A.3. Proof of Proposition 4 (uniqueness of the symmetric equilibrium)

Now I have to show that the equilibrium found in Proposition 3 is unique for each value of $k \in \left[\frac{1}{2}, 1\right]$. Define

$$\digamma\left(R^m_{1,1}, \Gamma^m, \Gamma^w, n\right) = \begin{pmatrix} \digamma_1\left(R^m_{1,1}, \Gamma^m, \Gamma^w, n\right) \\ \digamma_2\left(R^m_{1,1}, \Gamma^m, \Gamma^w, n\right) \\ \digamma_3\left(R^m_{1,1}, \Gamma^m, \Gamma^w, n\right) \\ \digamma_4\left(R^m_{1,1}, \Gamma^m, \Gamma^w, n\right) \end{pmatrix} \tag{62}$$

where $\digamma_1 = \left[\frac{\min\{k,1\}}{2k}\Psi_1 - R^m_{1,1}\right]$, $\digamma_2 = [\Psi_2 - \Gamma^m]$, $\digamma_3 = [\Psi_3 - \Gamma^w]$ and $\digamma_4$ is given by equation (31).[40] In Proposition 3 whe found that the point $\chi_s = (\Gamma^s, \Gamma^s, n^s)$ is a solution of $\Psi(.)$.To show the uniqueness of χ_s using the Implicit Function Theorem, we need continuity in all the parcial derivatives of $\digamma(.)$. We also need that Jacobian determinant of $\digamma(.)$ with respect to the endogenous variables is nonzero when evaluated at χ_s. We know that the system continuous and differentiable for $k \in [\frac{1}{2}, 1)$, being their partial derivatives also continuous because g is differentiable.[41]

Given that under symmetry $R^m_{1,1} = \frac{\mu}{2}$ (by (36)), we can eliminate one equation $(\digamma_1)$ and one unknown $\left(R^m_{1,1}\right)$ from the system $\digamma(.)$.Therefore, evaluating our system at point χ_s we have,

$$\begin{pmatrix} \digamma_2(\Gamma^s, \Gamma^s, n^s) \\ \digamma_3(\Gamma^s, \Gamma^s, n^s) \\ \digamma_4(\Gamma^s, \Gamma^s, n^s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

and we need that,

$$|J| = Det\begin{bmatrix} \digamma_{22} & \digamma_{23} & \digamma_{24} \\ \digamma_{32} & \digamma_{32} & \digamma_{33} \\ \digamma_{42} & \digamma_{43} & \digamma_{44} \end{bmatrix} \neq 0. \tag{63}$$

Calculating the partial derivatives of $\digamma(.)$ and manipulating, we get

$$|J| = \frac{1}{(2 + n^s - \Gamma^s)^3}(2 + n^s - \rho[1 - G(\mu)])\Big\{(2 + n^s - \Gamma^s)^2(2 + 2n - k\Gamma^s) + (2 + n^s(4 + k + 2n) - k(1 + 2n)\Gamma^s)\rho\left(\left[2 - G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right) + G(\mu)\right)\Big\}.$$

Given that by assumption $\rho \leq \frac{1}{2}$ and that always $n^s \geq -\frac{1}{2}$, we have that $(2 + n^s - \rho[1 - G(\mu)]) > 0$. Therefore $|J|$ will be positive if the expresion under curly brackets, $\{x_1 + x_2 * x_3\} > 0$, where $x_1 = (2 + n^s - \Gamma^s)^2(2 + 2n - k\Gamma^s)$, $x_2 = (2 + n^s(4 + k + 2n) - k(1 + 2n)\Gamma^s)\rho$ and $x_3 = \left(\left[2 - G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right) + G(\mu)\right)$.We clearly have that $x_1 > 0$ $x_2 > 0$. In the case of x_3, the log-concavity of g implies

$$\left[2 - G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right) \leq G(\mu) \tag{64}$$

and therefore $x_3 \leq 0$ and $x_2 * x_3 \leq 0$.[42] However, given that $|x_3| \leq G(\mu)$. it is sufficient that $\rho \leq \frac{1}{2}$ to have $|x_1| > |x_2| * |x_3|$. Therefore $|J| > 0$ which implies that the equilibrium is unique.

[40] Notice that, in order to make the exposition of the following results easier, now the system is expressed in terms of $R^m_{1,1}$.

[41] The system $\digamma(.)$ is not differentiable at $k = 1$, that is, the right hand side and the left hand side derivatives with respect to k are not equal. However, $\digamma(.)$ is continuous in k and differentiable for the intervals $k \in [\frac{1}{2}, 1)$ and $k \in (1, \overline{k}]$.

[42] Log-concave probability densisities have the "New is better than used" (NBU) property: $[1 - F(x + y)] \leq [1 - F(x)][1 - F(y)]$. Making $x = y = \frac{\mu}{2}$, this implies that $\left(\left[2 - F\left(\frac{\mu}{2}\right)\right]F\left(\frac{\mu}{2}\right)\right) \leq F(\mu)$. See An (2003).

A.4. Proof of Proposition 5 (comparative statics of the symmetric equilibrium).

In Proposition 4 we found that $|J| > 0$, which allows us to define Γ^s and n^s as implicit functions of k in a neighborhood of any point χ_s that is a solution of the system $F(.)$ defined in (62). Given that $F(.)$ has a unique solution for each value of $k \in [\frac{1}{2}, 1)$, the following results hold for the whole interval.

Now I turn to comparative statics. Applying Cramer's rule, the derivatives of the endogenous variables in $F(.)$ with respect to k are $\frac{\partial \Gamma^m}{\partial k} = \frac{\partial \Gamma^w}{\partial k} = \frac{|J_2|}{|J|}$, and $\frac{\partial n}{\partial k} = \frac{|J_4|}{|J|}$, where where $|J|$ is defined in (63) and $|J_2|$ and $|J_4|$ are obtained by replacing $-F_{ik} = -\frac{\partial F_i}{\partial k}$ for $i = \{2,3,4\}$ in the respective column of $|J|$. Therefore we have,

$$|J_2| = -\frac{(1-\Gamma^s)}{(2+n^s-\Gamma^s)^3}(1+n\Gamma^s)(2+n^s-\rho[1-G(\mu)])\rho\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu)\right)$$

Given that $\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu)\right) \leq 0$ (by (64)), we have that $|J_1| > 0$. Therefore

$$\frac{\partial \Gamma^s}{\partial k} > 0, \text{ for } k \in [\frac{1}{2}, 1).$$

Similarly,

$$|J_4| = \frac{(2+n^s-\rho[1-G(\mu)])}{(2+n^s-\Gamma^s)^3}\left\{(2+n^s-\Gamma^s)^2+(1+n^s)\rho\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu)\right)\right\}$$

We know that $(2+n^s-\Gamma^s) > 1+n^s \geq 0.5$ (because of $\Gamma^s < 1$ and the boundaries of n) and that $\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu) \leq G(\mu)$. Given that $\rho \leq \frac{1}{2}$, it must follow that $|J_4| > 0$. Therefore

$$\frac{\partial n}{\partial k} > 0, \text{ for } k \in [\frac{1}{2}, 1).$$

Finally, since $q_1 = \frac{1+n}{2+n}$, it must follow that $\frac{\partial q_1}{\partial k} > 0$.

A.5. Proof of Proposition 7 (comparative statics of the differential fecundity equilibrium).

To show the comparative statics with differential fecundity we use the system defined in (62). Given than now $k \in (1, \bar{k}]$, F_1 is defined by (20) and we work with the full system of four equations in four unknowns. For tractability I pick a point s' defined by the unique solution of the system at $k = 1$. Since the right-hand side derivatives of $F(.)$ are defined and are continuous for $k = 1$, we are able to make local comparative statics for the neighborhood of χ'_s (for $k = 1+\epsilon$, for ϵ small). First of all, we have to show that the Jacobian of the full system, $|H|$ is nonzero when evaluated at χ'_s. Calculating the partial derivatives of $F(.)$ and manipulating, we get

$$|H| = \frac{(2-\Gamma^s-\rho[1-G(\mu)])\left\{(2-\Gamma^s)^2+\rho\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu)\right)\right\}}{(2-\Gamma^s)^2} > 0$$

Therefore, we can define $R^m_{1,1}$, Γ^m, Γ^w and n as implicit functions of k in a neighborhood of χ'_s.Applying Cramer's rule, the derivatives of the endogenous variables in $F(.)$ with respect to k are $\frac{\partial R^m_{1,1}}{\partial k} = \frac{|H_1|}{|H|}$, $\frac{\partial \Gamma^m}{\partial k} = \frac{|H_2|}{|H|}$, $\frac{\partial \Gamma^w}{\partial k} = \frac{|H_3|}{|H|}$ and $\frac{\partial n}{\partial k} = \frac{|H_4|}{|H|}$, where $|H_1|$, $|H_2|$, $|H_3|$ and $|H_4|$ are obtained by replacing $-F_{ik} = -\frac{\partial F_i}{\partial k}$ for $i = \{1,2,3,4\}$ in the respective column of $|H|$.

Now I will show that in a neighborhood of χ'_s we have that, given an increase in k:

1. *Young men are less choosy with young women and choosier with old women:* $\frac{\partial R_{11}^m}{\partial k} < 0$ and $\frac{\partial R_{12}^m}{\partial k} > 0$.

   To show that men's reservation values of men for young women are decreasing in k we need that,

$$\frac{|H_1|}{|H|} = -\frac{\mu}{2}\frac{(2-\Gamma^s-\rho[1-G(\mu)])}{(2-\Gamma^s)} < 0$$

   In addition, since $\frac{(2-\Gamma^s-\rho[1-G(\mu)])}{(2-\Gamma^s)} < 1$, we also have $\left|\frac{\partial R_{11}^m}{\partial k}\right| < \left|\frac{\partial R_{11}^w}{\partial k}\right| = \frac{\mu}{2}$ (evaluated at $k = 1$). On the other hand, we have that young men become choosier with respect to old women. By (21), $R_{12}^m = 2kR_{11}^m$ when $k \geq 1$. Therefore,

$$\begin{aligned}\frac{\partial R_{12}^m}{\partial k} &= 2\left[R_{1,1}^m + \frac{\partial R_{11}^m}{\partial k}\right] \\ &= 2\left[\frac{\mu}{2}\left(1 - \frac{(2-\Gamma^s-\rho[1-G(\mu)])}{(2-\Gamma^s)}\right)\right] > 0\end{aligned}$$

2. *Probability of marriage*

   (a) *Women marry young with higher probability.*
   To show that $\frac{\partial \Gamma^w}{\partial k} > 0$ is enough to show that $|H_3| > 0$. Therefore,

$$\begin{aligned}|H_3| = \frac{1}{(2-\Gamma^s)^3}&\Big\{-(2-\Gamma^s-\rho[1-G(\mu)])(1-\Gamma^s)\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)-G(\mu)\right) \\ &+\mu(2-\Gamma^s)^2\left[1-G\left(\frac{\mu}{2}\right)\right](2-\Gamma^s-\rho[1-G(\mu)])(4-2\Gamma^s-\rho[1-G(\mu)])\,g\left(\frac{\mu}{2}\right) \\ &+2(2-\Gamma^s)(1-\Gamma^s)\mu f(\mu)\Big\{\left[(2-\Gamma^s)^2\left[2-\Gamma^s-(1-\theta)\rho+\rho^2\theta\right]-(1+\theta(3-2\Gamma^s))\rho^2\right] \\ &+\left[(2-\Gamma^s)(1-\theta)\rho F(\mu)+\rho^2 G(\mu)^2(1+\theta(1-\Gamma^s))\right] \\ &+(2-\Gamma^s-\rho[1-G(\mu)])\left[2-G\left(\frac{\mu}{2}\right)\right](1-\theta)G\left(\frac{\mu}{2}\right)\Big\}\Big\}\end{aligned}$$

   We know by (64) that the first line is positive. Recalling that $\rho \leq \frac{1}{2}$, it is also clear that the second, fourth and fifth lines are also positive. In the the third line we have $-1 < (1+\theta(3-2\Gamma^s))\rho^2 < 0$. However, at $k = 1$ we have $\phi = 1$ which implies $\Gamma^s \leq \rho$ and $\left[2-\Gamma^s-(1-\theta)\rho+\rho^2\theta\right] \geq 1$. Therefore, the third line is also positive and $|H_3| > 0$, which implies $\frac{\partial \Gamma^w}{\partial k} > 0$ given that $|H| > 0$.

   (b) *The difference between the probability of marriage at age 1 of women and men,* $\Gamma^w - \Gamma^m$*, is positive and increasing in k:* $\frac{\partial \Gamma^w}{\partial k} > \frac{\partial \Gamma^m}{\partial k}$.
   Here we need $\frac{|H_3|-|H_2|}{|H|} > 0$

$$\frac{|H_3|-|H_2|}{|H|} = \frac{(1-\Gamma^s)\rho\mu(2-\Gamma^s+\rho[1-G(\mu)])\,g(\mu)}{(2-\Gamma^s)(2-\Gamma^s-\rho[1-G(\mu)])} > 0$$

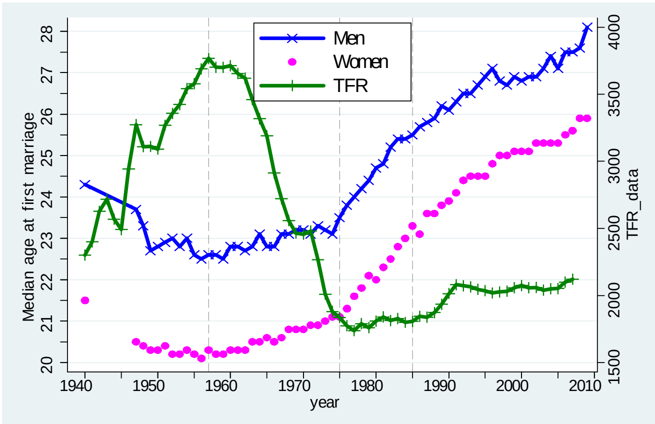
3. *The ratio of single men to single women,* ϕ*, is greater than 1 and increasing in k.*

   From (26) we know that $s_{2,t}^m = s_{1,t-1}^m(1-\Gamma^m)$ and $s_{2,t}^w = s_{1,t-1}^w(1-\Gamma^w)$. Since $s_{1,t}^m = s_{1,t}^w$, $s_{2,t}^m > s_{2,t}^w$ because $\Gamma^w > \Gamma^m$. Therefore $S_t^m > S_t^w$ and $\phi > 1$. Furthermore, since $\frac{\partial \Gamma^w}{\partial k} > \frac{\partial \Gamma^m}{\partial k}$, we have $\frac{\partial \phi}{\partial k} > 0$.

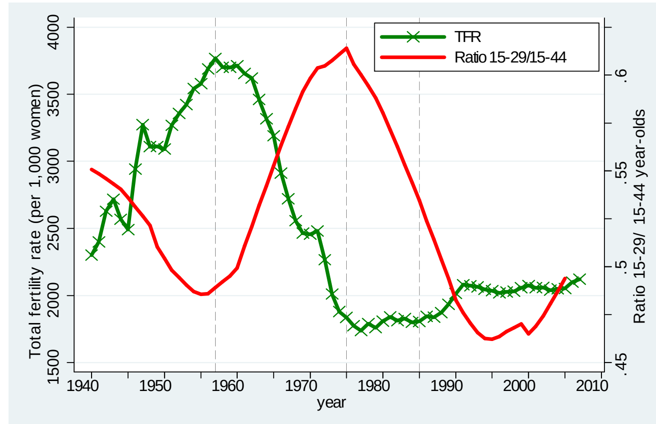
4. *Population growth and the fraction of young people in the economy increases.*For $\frac{\partial n}{\partial k} > 0$ we need

$$\frac{|H_4|}{|H|} = \frac{\Gamma^s}{(2 - \Gamma^s)} > 0$$

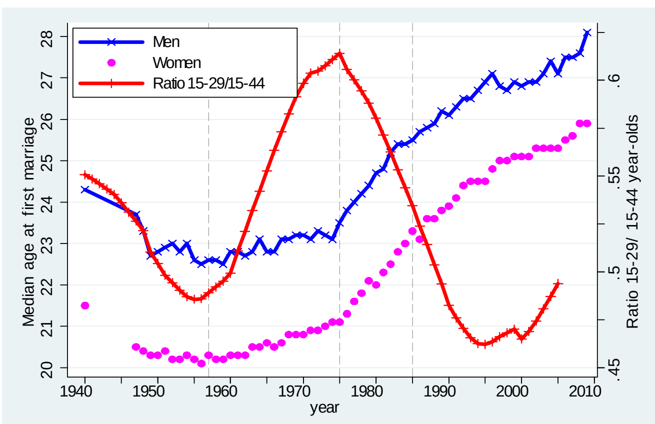
It is straightforward that here we also have $\frac{\partial q_1}{\partial k} > 0$, as in Proposition 5.



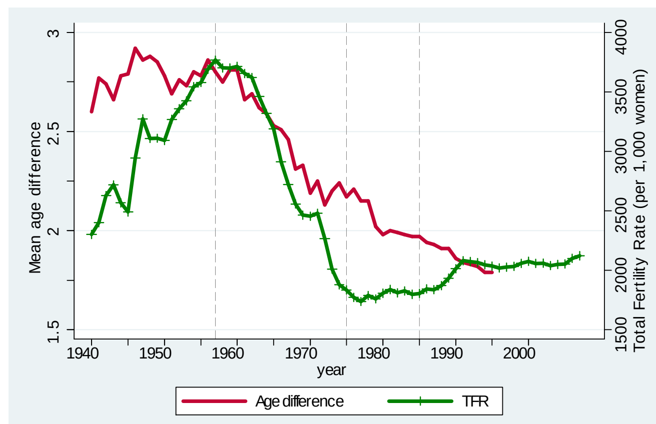
(a) Median age at first marriage and TFR.



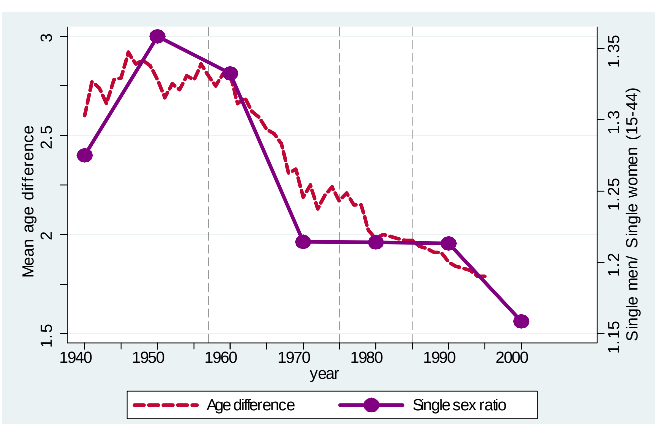
(b) TFR and age structure of the population.



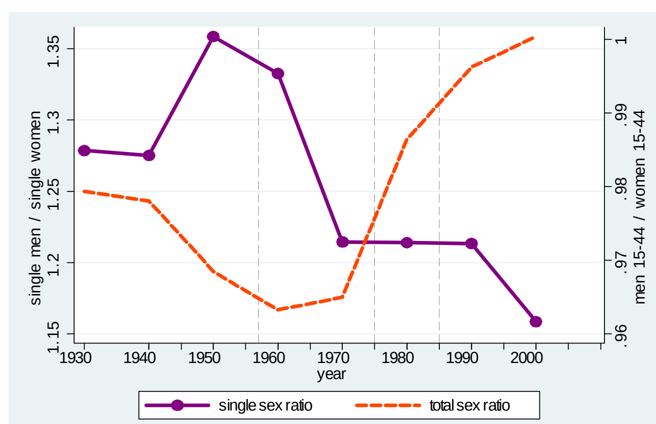
(c) Median age at first marriage and population structure.



(d) Mean age difference and TFR.

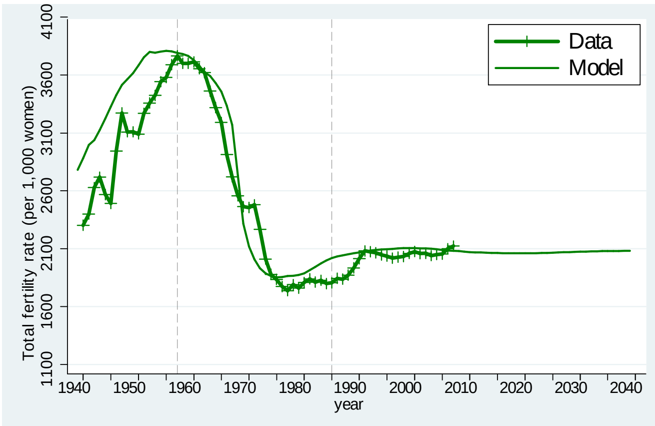


(e) Mean age difference and sex ratio.

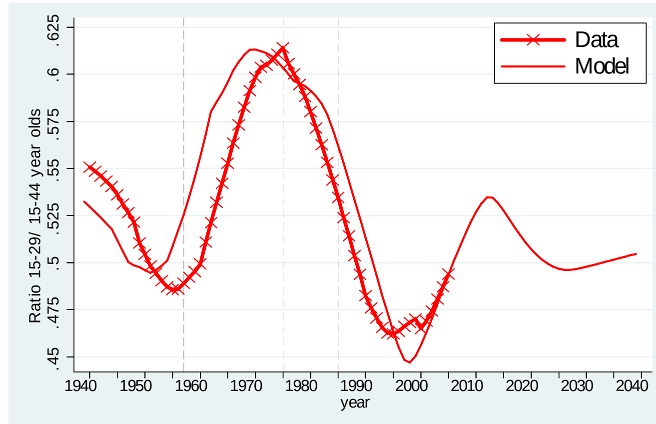


(f) Single and total sex ratio (ages 15-44).

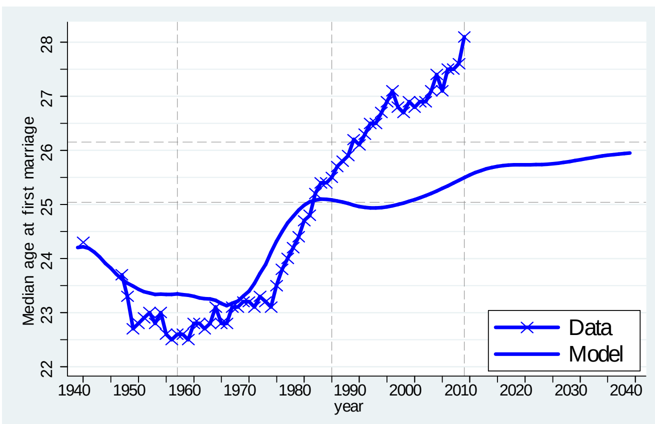
Figure 1: Marriage and fertility indicators in the U.S.



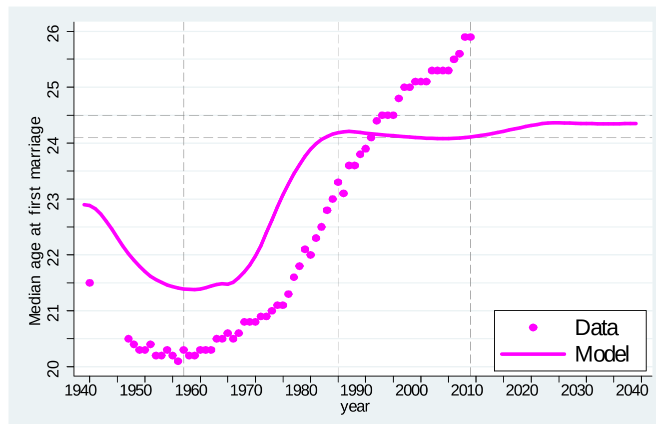
(a) Total fertility rate.



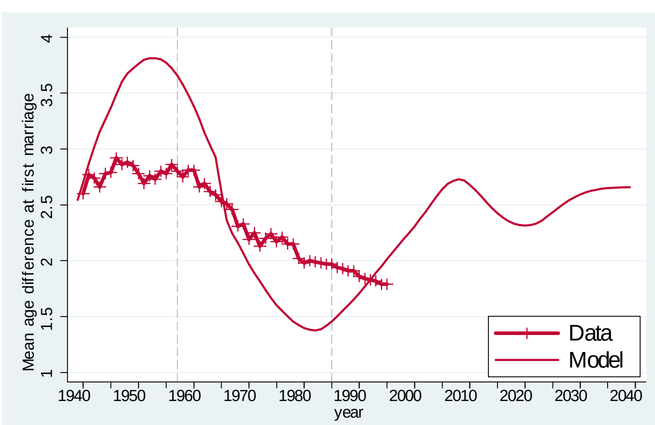
(b) Age structure of the population (15-29 /15-44).



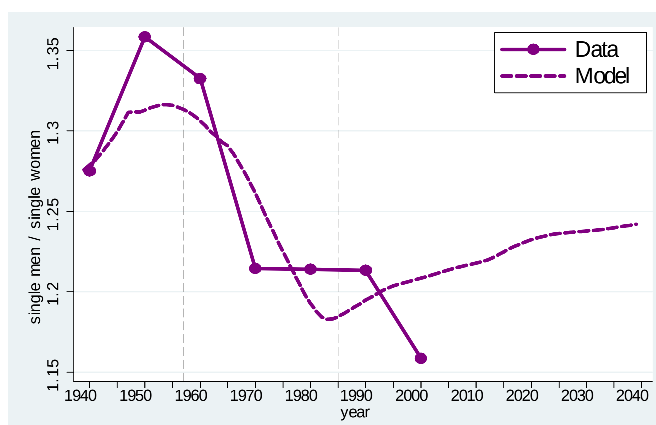
(c) Median age at first marriage (men).



(d) Median age at first marriage (women).

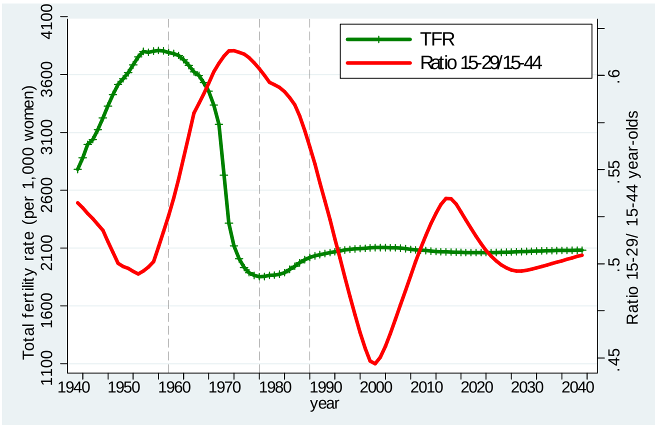


(e) Mean age difference at first marriage .

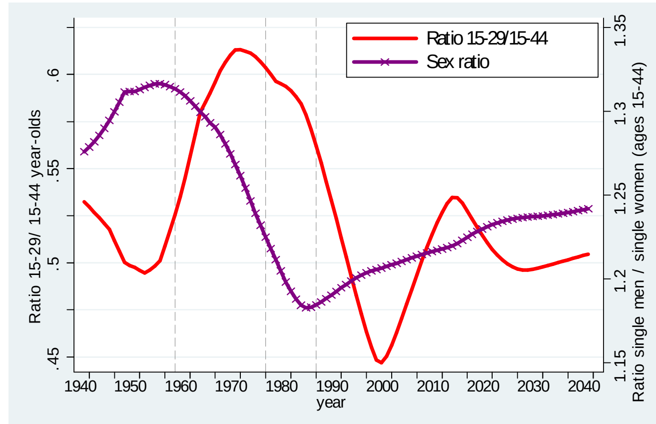


(f) Ratio single men / single women (15-44).

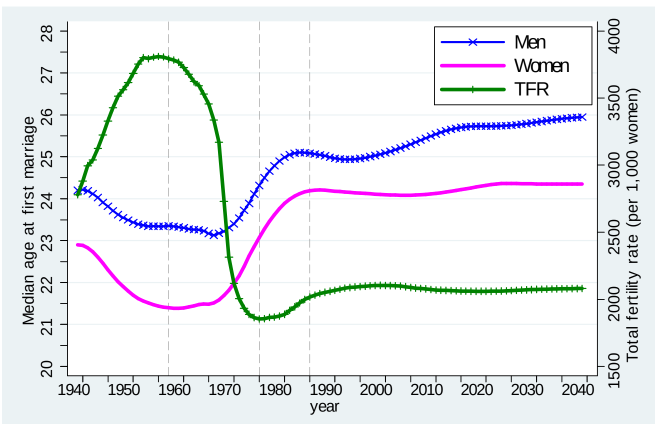
Figure 2: The Benchmark Economy and U.S. data.



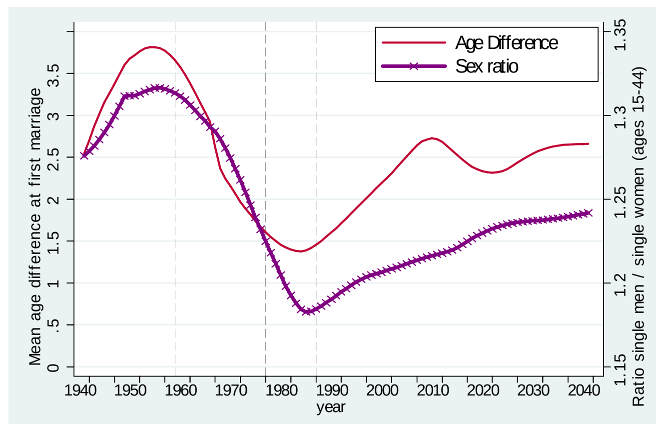
(a) TFR and age structure of the population.



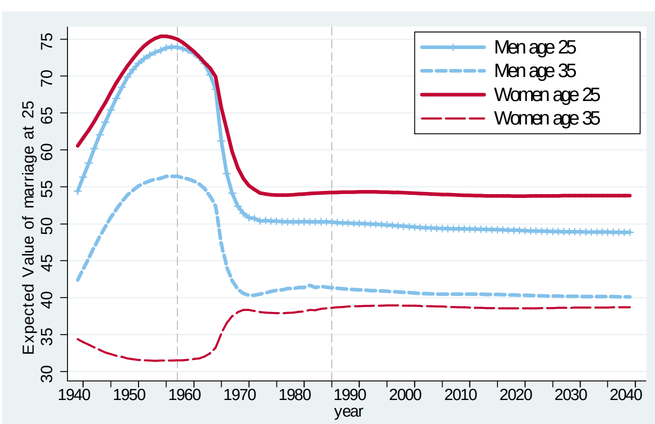
(b) Sex ratio and age structure of the population.



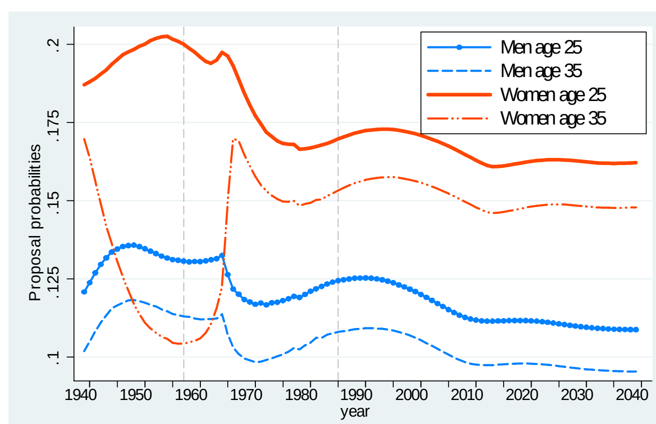
(c) Median age at first marriage and TFR.



(d) Mean age difference and sex ratio.

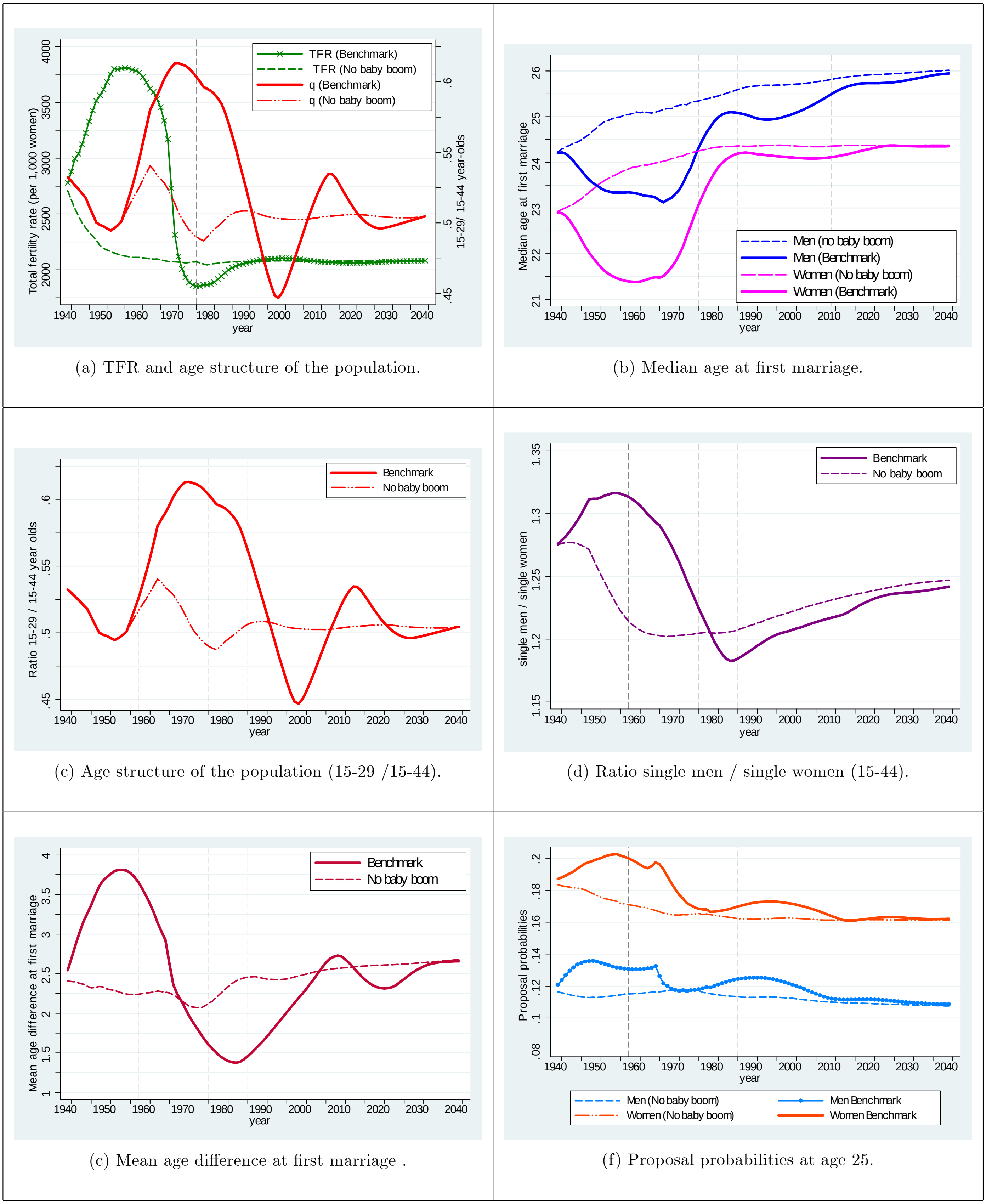


(e) Value of search at ages 25 and 35.



(f) Proposal probabilities at ages 25 and 35.

Figure 3: The Benchmark Economy.



(a) TFR and age structure of the population.

(b) Median age at first marriage.

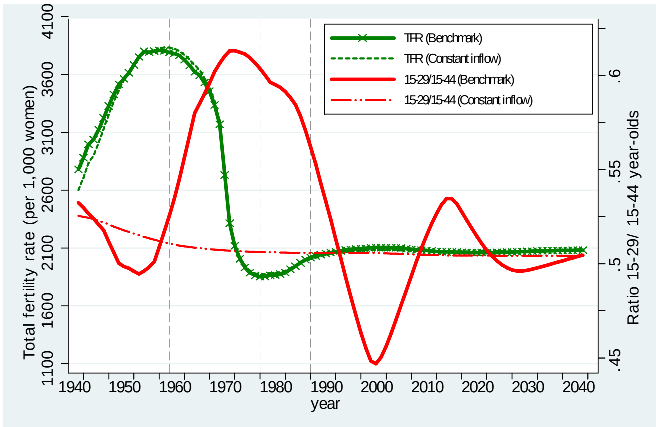
(c) Age structure of the population (15-29 /15-44).

(d) Ratio single men / single women (15-44).

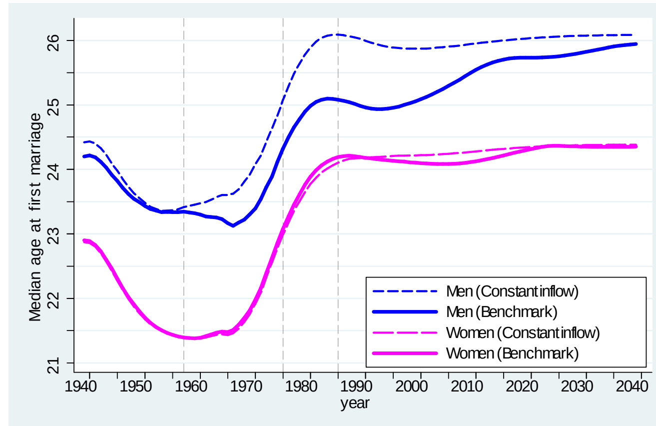
(c) Mean age difference at first marriage .

(f) Proposal probabilities at age 25.

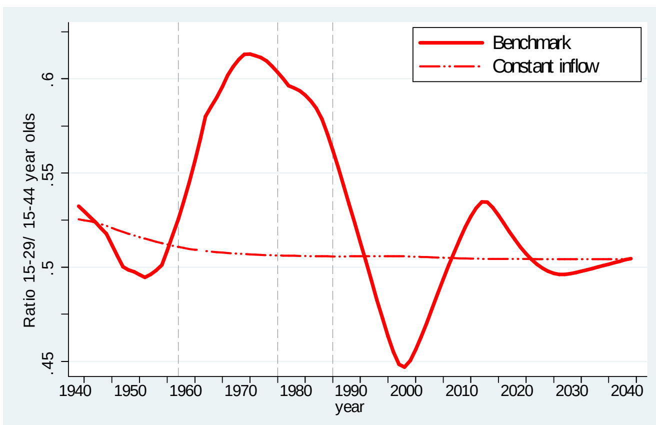
Figure 4: Counterfactual Economy I (No baby boom).



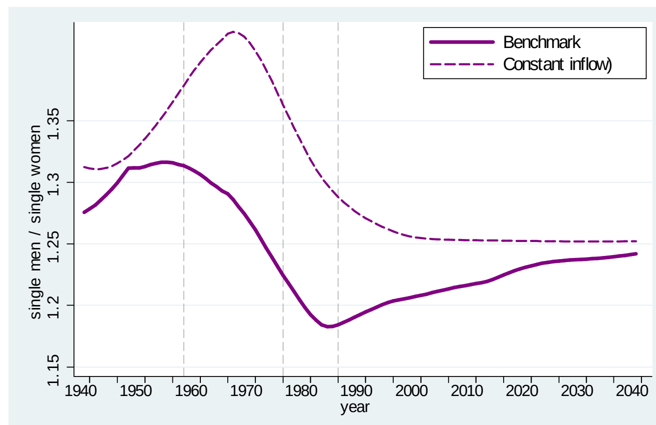
(a) TFR and age structure of the population.



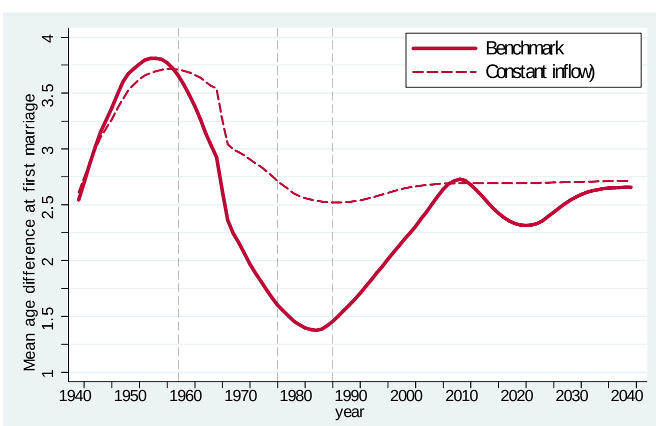
(b) Median age at first marriage.



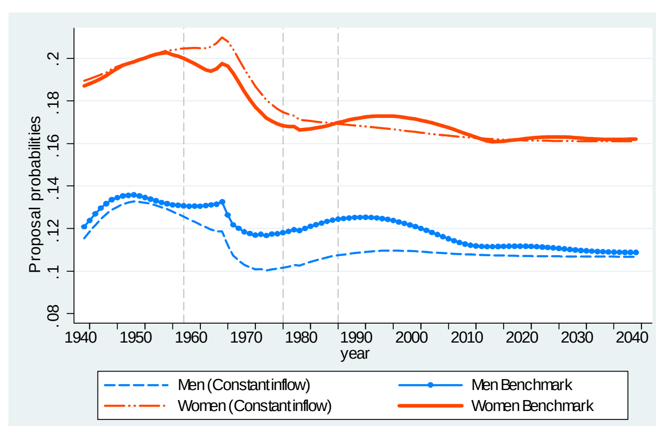
(c) Age structure of the population (15-29 /15-44).



(d) Ratio single men / single women (15-44).



(e) Mean age difference at first marriage .



(f) Proposal probabilities at age 25.

Figure 5: Counterfactual Economy II (Constant population inflow).