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Relative Performance Pay, Bonuses, and Job-Promotion Tournaments

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ABSTRACT

Relative Performance Pay, Bonuses, and Job-Promotion Tournaments*

Several empirical studies have challenged tournament theory by pointing out that (1) there is considerable pay variation within hierarchy levels, (2) promotion premiums only in part explain hierarchical wage differences and (3) external recruitment is observable on nearly any hierarchy level. We explain these empirical puzzles by combining job-promotion tournaments with higher-level bonus payments in a two-tier hierarchy. Moreover, we show that under certain conditions the firm implements first-best effort on tier 2 although workers earn strictly positive rents. The reason is that the firm can use second-tier rents for creating incentives on tier 1. If workers are heterogeneous, the firm strictly improves the selection quality of a job-promotion tournament by employing a hybrid incentive scheme that includes bonus payments.

JEL Classification: D82, D86, J33

Keywords: bonuses, external recruitment, job promotion, limited liability, tournaments

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1 Introduction

Empirical literature on internal labor markets has documented stylized facts that are not in line with traditional models. In particular, Baker, Gibbs and Holmström (1994a, 1994b) and others[1] have emphasized that three empirical puzzles question the traditional theory of job-promotion tournaments: (1) there is considerable variation in pay on each hierarchy level, which contradicts the important prerequisite of tournaments that wages must be attached to jobs in order to generate incentives, (2) promotion premiums that are paid to workers when moving to higher levels in the hierarchy can explain only part of the hierarchical wage differences in firms, (3) we can observe external recruiting on almost any hierarchy level in diverse firms from different countries, which would erase incentives from internal job-promotion tournaments.

In our paper, we combine job-promotion tournaments with additional incentive schemes to explain these empirical puzzles. We consider a two-tier hierarchy where workers produce only ordinal performance information on the first level, but are individually visible after promotion to the second level. Here, they become responsible for certain (managerial) tasks that lead to individual and verifiable performance signals. The firm can use three different instruments to stimulate incentives. First, it can make use of relative performance pay on the first hierarchy level. Second, it can install a bonus scheme on hierarchy level 2 based on individual performance. Finally, it can combine both hierarchy levels by employing a job-promotion scheme that assigns the better performing worker of level 1 to the next hierarchy level. The adoption of such a promotion tournament implies that the prize for superior first-level performance is supplemented by expected rents from the second-level incentive contract.

Our results show that in many situations the firm prefers to combine all three incentive devices, thus explaining the three mentioned puzzles above: since promoted workers receive different bonuses depending on success and failure, we have a natural variation in pay on the second hierarchy level, which

[1] See Lazear (1992), Ariga, Ohkusa and Brunello (1999), Seltzer and Merrett (2000), Treble et al. (2001), Dohmen, Kriechel and Pfann (2004), Gibbs and Hendricks (2004) and Grund (2005).

explains puzzle (1). As a promoted worker earns both relative performance pay and bonuses, hierarchical wage increases are only in part determined by job promotion, hence explaining puzzle (2). In this context, one of the empirical findings by Dohmen, Kriechel and Pfann (2004) is interesting. Contrary to other firm studies, they are able to determine the exact point in time when a worker realizes a pay increase, and they find out that promotion and wage increase are often not simultaneous. This observation fits quite well to our model that combines job-promotion with bonus pay. Finally, our model points out that the combination of job-promotion tournament and bonus pay has one crucial disadvantage – it restricts the set of implementable effort pairs for the two hierarchy levels as both levels are interlinked. We show that sometimes the firm prefers external recruiting in order to partly get rid of this problem, which explains empirical puzzle (3).

The aim of this paper is twofold. On the one hand, it addresses empirical puzzles that cannot be explained by standard tournament models. In this sense, it follows the advice of Waldman (forthcoming) to develop a more sophisticated tournament model that is able to explain the empirical findings by Baker, Gibbs and Holmström (1994a, 1994b), which contradict traditional tournament models. On the other hand, we want to add to the theory of rank-order tournaments[2] by combining tournaments with further incentive schemes. In our model, the workers are protected by limited liability and earn strictly positive rents. By combining bonus pay on hierarchy level 2 with job-promotion, the rent earned by a promoted worker can be used to create incentives on level 1 as each worker wants to win the tournament and, hence, the rent on the next level. Interestingly, the use of level 2 rents for creating incentives on level 1 improves workers' performances only on level 2, but not on level 1. If the rent is not too large relative to the optimal tournament prize spread, the firm will implement first-best effort on the second hierarchy level. Recently, contract theorists as Schmitz (2005) have pointed out that optimal bonus payments that lead to positive rents can be reinterpreted as

[2]See the seminal papers by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) that discuss tournaments in a contract-theoretic context with application to labor economics.

efficiency wages. Since rents are strictly increasing in effort in single-agent hidden action models, the implemented effort level is inefficiently small in the case of continuous effort. However, in our model the firm implements first-best effort on level 2 although this effort is associated with a strictly positive rent that also monotonically increases in effort. Therefore, combining tournaments with bonuses allows for efficiency wages in a more literal sense.

Our analysis points to the following trade-off: on the one hand, the combined use of tournaments and bonuses (combined contract) leads to the advantage that the firm can use workers' rents for improving incentives. On the other hand, interlinking incentives on both hierarchy levels restricts the set of implementable efforts. If the rent on the second hierarchy level is only moderate, the firm will clearly profit from a combined contract, but otherwise either a combined contract or two separate contracts for the two respective hierarchy levels may be optimal. In a next step, we show that the firm will never prefer two separate contracts if, on hierarchy level 2, it can hire workers from outside. In this situation, the firm always benefits from a strictly positive rate of internal job-promotion within a combined contract. However, it may even improve on the combined contract by allowing external entry on hierarchy level 2. Such external recruitment is useful for the firm in light of the trade-off mentioned above: if the firm hires from outside on level 2, it can choose the optimal separate contract for hierarchy level 2 that does not impose any further restriction on effort implementation, besides the usual incentive, participation, and limited-liability constraints.

As a further extension, we introduce heterogeneity of workers. In particular, we assume symmetric uncertainty about the ability or talent of each individual worker (i.e., neither the workers nor the firm can observe individual talent). In our setting, a worker's talent and his effort are complements on each hierarchy level. We show that under heterogeneity the firm implements strictly larger efforts on both hierarchy levels when using a combined contract compared to optimal efforts implemented under separate contracts. The intuition for this result comes from the fact that high efforts are desirable for two distinct reasons within a combined contract: first, the higher workers' efforts on level 1 the higher will be the probability that the more

talented worker is promoted to the next level, thus improving the selection quality of a job-promotion tournament. Second, under a combined contract all players update their beliefs about the unknown talent of the promoted worker. Due to the selection properties of the tournament, the posterior expected talent of the promoted worker is higher than the workers' expected talent prior to the tournament. Since talent and effort are complements, the posterior efficient effort on hierarchy level 2 is also higher than the ex-ante efficient one.

Our paper is related to those two tournament models that also combine a rank-order tournament with an additional incentive scheme. Tsoulouhas, Knoeber, and Agrawal (2007) analyze optimal handicapping of internal and external candidates in a contest to become CEO. To do so, they also consider a promotion tournament where the prize is the incentive contract on the next hierarchy level. However, apart from addressing a quite different question, their model also differs from ours in several respects. First, they do not allow for relative performance pay on the first tier of the hierarchy. Second, they assume that the firm cannot commit to a second-period contract at the beginning of the game.[3] Furthermore, even though they are of limited liability, promoted agents do not earn rents because they are assumed to have a sufficiently high reservation utility. Schöttner and Thiele (2008) also investigate incentive contracting within a two-tier hierarchy, but consider a production environment where there is an individual and contractible performance signal on the first tier. They examine the optimal combination of piece rates for level 1 workers and a promotion tournament to the next tier.

Ohlendorf and Schmitz (2008) do not analyze tournaments, but combine two principal-agent contracts in successive periods. As in our model, the agent is wealth-constrained and earns a non-negative rent that can be used for incentive purposes. Compared to our paper, Ohlendorf and Schmitz consider a completely different scenario with a single agent. In their model, the principal is integrated in the production process and can invest in each of the two periods. Hence, the natural application of their model is a supplier-buyer relationship where the principal can terminate the joint project after

[3]However, the authors also discuss an extension where commitment is possible.

the first period. In the Ohlendorf-Schmitz paper, optimal second-period incentives serve as a carrot or a stick since they depend on first-period success.

Since we introduce the possibility of external recruiting in Section 4, our paper is also related to those tournament models that discuss external hiring versus internal promotion via tournaments. We show that, under certain circumstances, the firm recruits from the external labor market with some positive probability. The reason is that, by sometimes hiring an external candidate for the second level, the firm can mitigate first-level incentives. This is desirable when second-level rents are so high that level 1 workers tend to work too hard. The point that external recruitment diminishes internal incentives has also been addressed by Chan (1996). However, while he emphasizes that external candidates are handicapped to *strengthen* internal incentives, we point out the potential existence of a reverse relationship: in our model, higher-tier workers are recruited from the outside to *alleviate* excessive incentives. The existing literature highlights other beneficial aspects of external recruitment. For example, Chen (2005) shows that external recruitment may reduce sabotage and collusion within firms. Tsoulouhas, Knoeber and Agrawal (2007) demonstrate that firms handicap internal candidates if external ones are of sufficiently superior ability.

The remainder of the paper is organized as follows. In the next section, we introduce our basic model. Section 3 offers a solution to this model, comparing a combined contract with two separate contracts. In Section 4, we introduce the possibility of external recruiting. Section 5 extends the basic model by assuming heterogeneous workers. Section 6 concludes.

2 The Basic Model

We consider two representative periods in the life span of a firm that consists of two hierarchy levels. In the first period, the firm employs two homogeneous workers A and B at hierarchy level 1. Each worker i $(i = A, B)$ exerts effort $\hat{e}_i \geq 0$ that has the non-verifiable monetary value $\hat{v}(\hat{e}_i)$ to the firm with $\hat{v}'(\cdot) > 0$ and $\hat{v}''(\cdot) \leq 0$. The firm neither observes $\hat{e}_i$ nor $\hat{v}(\hat{e}_i)$, but receives an unverifiable, ordinal signal $\hat{s} \in \{\hat{s}_A, \hat{s}_B\}$ about the relative performance of

the two workers. The signal $\hat{s} = \hat{s}_A$ indicates that worker A has performed best, whereas $\hat{s} = \hat{s}_B$ means that worker B has performed better relative to his co-worker. The probability of the event $\hat{s} = \hat{s}_A$ is given by $\hat{p}(\hat{e}_A, \hat{e}_B)$ and that of $\hat{s} = \hat{s}_B$ by $1 - \hat{p}(\hat{e}_A, \hat{e}_B)$.

We assume that the probability function $\hat{p}(\hat{e}_A, \hat{e}_B)$ exhibits the properties of the well-known contest-success function introduced by Dixit (1987):[4]

(i) $\hat{p}(\cdot, \cdot)$ is symmetric, i.e. $\hat{p}(\hat{e}_i, \hat{e}_j) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i)$,

(ii) $\hat{p}_1 > 0$, $\hat{p}_{11} < 0$, $\hat{p}_2 < 0$, $\hat{p}_{22} > 0$,

(iii) $\hat{p}_{12} > 0 \Leftrightarrow \hat{p} > 0.5$.

According to (ii), exerting effort has positive but decreasing marginal returns. Property (iii) means that if, initially, player A has chosen higher effort than B, a marginal increase in $\hat{e}_B$ will make it more attractive to A to increase $\hat{e}_A$ as well, due to the more intense competition the increase of $\hat{e}_B$ has caused.

Spending effort $\hat{e}_i$ leads to costs $\hat{c}(\hat{e}_i)$ for worker i $(i = A, B)$ with $\hat{c}(0) = \hat{c}'(0) = 0$ and $\hat{c}'(\hat{e}_i) > 0$, $\hat{c}''(\hat{e}_i) > 0$ for all $\hat{e}_i > 0$. Furthermore, to guarantee some regularity conditions, we make the following technical assumptions. To ensure concavity of the firm's objective function, we assume that $\hat{c}'''(\hat{e}_i) \geq 0$ and $\frac{\partial^2}{\partial \hat{e}^2}\hat{p}_1(\hat{e}, \hat{e}) \leq 0$. Finally, to obtain an interior solution, we assume that $\hat{c}''(0) = 0$.

In the second period, the firm needs to hire one worker for hierarchy level 2. Here, in contrast to level 1, a worker's effort generates an individual and verifiable performance signal. For example, we can think of a two-tier hierarchy where, at level 1, workers fulfill tasks that do not lead to individually attributable outputs. However, at level 2, we have a managerial task accompanied by personal responsibility generating a publicly observable performance measure. The position on level 2 may be head of a department or a division, for example.

Following the binary-signal model by Demougin and Garvie (1991) and Demougin and Fluet (2001), we assume that the worker on level 2 chooses effort $e \geq 0$ leading to an observable and contractible signal $s \in \{s^L, s^H\}$ on the worker's performance with $s^H > s^L$. The observation $s = s^H$ is

[4] Subscripts of $p(\cdot, \cdot)$ denote partial derivatives.

favorable information about the worker's effort choice in the sense of Milgrom (1981). Let the probability of this favorable outcome be $p(e)$ with $p'(e) > 0$ (strict monotone likelihood ratio property) and $p''(e) < 0$ (convexity of the distribution function condition). Moreover, we assume that effort choice e has the monetary value $v(e)$ to the firm with $v'(\cdot) > 0$ and $v''(\cdot) \leq 0$. Again, neither e nor $v(e)$ is observable by the firm.[5] Exerting effort e entails costs $c(e)$ to the worker on level 2 with $c(0) = c'(0) = 0$ and $c'(e) > 0$, $c''(e) > 0$ for all $e > 0$. Furthermore, analogous to the technical assumptions for the first hierarchy level, we assume that $c'''(e) \geq 0$, $p'''(e) \leq 0$, and $c''(0) = 0$.

We assume that all players are risk-neutral. Workers are protected by limited liability, i.e. they cannot make payments to the firm. On both tiers of the hierarchy, workers have zero reservation values. For simplicity, we neglect discounting.

In the given setting, the firm can use three different instruments to provide incentives: First, it can employ *relative performance pay* (i.e. a rank-order tournament) at hierarchy level 1. Under relative performance pay, the better performing worker receives a high wage w_H whereas the other worker obtains a low wage w_L. Due to limited liability, both wages must be non-negative ($w_L, w_H \geq 0$). Note that, even though the signal $\hat{s}$ is unverifiable, relative performance pay is still feasible due to the self-commitment property of the fixed tournament prizes w_L and w_H.[6] Second, the firm can install a *bonus scheme* at hierarchy level 2. In case of a favorable signal ($s = s^H$) the worker gets a high bonus b_H, whereas he receives a low bonus b_L if the signal is bad news ($s = s^L$). Again, payments must be non-negative due to limited liability ($b_L, b_H \geq 0$). Finally, the firm can design a *job-promotion scheme* by announcing that the better performing worker from level 1 will be promoted to level 2 at the end of the first period. This creates indirect incentives for level 1 if promotion is attractive to a worker.

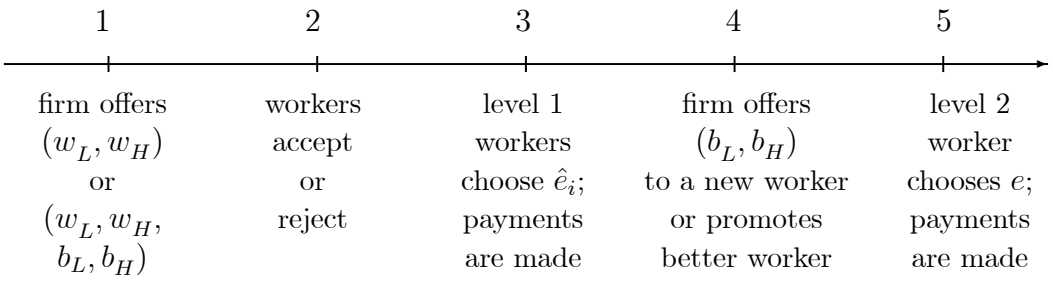
According to these incentive devices, at the beginning of the first period, the firm can offer one of the following two types of contracts. Under the

[5] Note that $v(\cdot)$ measures the worker's contribution to total firm profits and is not identical with department or division profits.

[6] See Malcomson (1984, 1986) on this important property of tournaments.

first type, the firm fills the positions on both hierarchy levels independent of each other. At the beginning of the first period, the firm offers two workers a contract (w_L, w_H). At the end of the first period, both workers leave the firm. The firm then offers the contract (b_L, b_H) to a new worker who is to be employed at level 2. We call this scheme *separate contracts*. The second type of contract is called a *combined contract*. In this case, at the start of the first period, the firm offers two workers a contract (w_L, w_H, b_L, b_H), which includes the promise to promote the better worker at the end of the first period. Then, in the second period, the promoted worker will be rewarded according to the pre-specified bonus scheme.[7] The worker who did not achieve promotion is dismissed. Furthermore, we assume that the worker selected for promotion can quit and realize his zero reservation value in the second period.

The time-schedule of the game can be summarized as follows.



First, the firm either offers a separate contract for the first tier or a combined contract to two workers. Then, the workers decide on acceptance of the contract. At stage 3, the workers exert efforts $\hat{e}_A$ and $\hat{e}_B$ on level 1; workers get w_L or w_H, respectively, whereas the firm receives $\hat{v}(\hat{e}_A) + \hat{v}(\hat{e}_B)$. Thereafter, under separate contracts, the position on hierarchy level 2 is filled with a worker that accepts the contract (b_L, b_H). Under the combined

[7] We assume that the firm can commit to such a bonus contract at the beginning of the first period. As will become clear later, this is without loss of generality because under the optimal contract there is no scope for mutual beneficial renegotiation: ex post, the firm would like to lower the bonus, but the agent is always better off under the original contract, which pays him a larger rent.

contract, the firm promotes the better level 1 worker to the next tier. Finally, the level 2 worker chooses effort yielding either a low or a high bonus payment while the firm earns $v(e)$.

In the following, we will analyze incentives and worker behavior under both kinds of contracts and discuss whether a combined contract (that includes a job-promotion scheme) or two separate contracts will be optimal from the viewpoint of the firm.

3 Worker Behavior and the Optimal Contract

3.1 Separate Contracts

In this section, we investigate the case of separate contracts (w_L, w_H) and (b_L, b_H). First, we analyze hierarchy level 1. Here, the two workers compete in a tournament for relative performance pay w_H and w_L. To analyze the firm's problem, we first characterize the workers' effort choices. Given the wages w_H and w_L, worker A chooses his effort level to solve

$$\max_{\hat{e}_A} w_L + \hat{p}(\hat{e}_A, \hat{e}_B) \cdot [w_H - w_L] - \hat{c}(\hat{e}_A) \tag{1}$$

whereas worker B solves

$$\max_{\hat{e}_B} w_L + [1 - \hat{p}(\hat{e}_A, \hat{e}_B)] \cdot [w_H - w_L] - \hat{c}(\hat{e}_B). \tag{2}$$

The equilibrium effort levels must satisfy the first-order conditions

$$(w_H - w_L)\,\hat{p}_1(\hat{e}_A, \hat{e}_B) = \hat{c}'(\hat{e}_A) \quad \text{and} \quad -(w_H - w_L)\,\hat{p}_2(\hat{e}_A, \hat{e}_B) = \hat{c}'(\hat{e}_B).$$

Recall that, due to the symmetry property (i) of the probability function $\hat{p}(\cdot,\cdot)$ we have $\hat{p}(\hat{e}_B, \hat{e}_A) = 1 - \hat{p}(\hat{e}_A, \hat{e}_B)$. Differentiating both sides with respect to $\hat{e}_B$ yields $\hat{p}_1(\hat{e}_B, \hat{e}_A) = -\hat{p}_2(\hat{e}_A, \hat{e}_B)$ so that the first-order conditions can be rewritten as

$$w_H - w_L = \frac{\hat{c}'(\hat{e}_A)}{\hat{p}_1(\hat{e}_A, \hat{e}_B)} = \frac{\hat{c}'(\hat{e}_B)}{\hat{p}_1(\hat{e}_B, \hat{e}_A)}.$$

Thus, we have a unique symmetric equilibrium $(\hat{e}_A, \hat{e}_B) = (\hat{e}, \hat{e})$ given by

$$w_H - w_L = \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})}. \tag{3}$$

Our assumptions do not rule out the existence of additional asymmetric equilibria. However, in the following, we restrict attention to the symmetric equilibrium, which seems plausible in the given setting with homogeneous contestants.[8] Condition (3) shows that equilibrium efforts increase in the tournament prize spread $w_H - w_L$.[9] To simplify notation, we denote by $\Delta w(\hat{e})$ the prize spread that induces the effort level $\hat{e}$, i.e.,

$$\Delta w(\hat{e}) := \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})} \tag{4}$$

The firm maximizes $2\hat{v}(\hat{e}) - w_L - w_H$ subject to the incentive constraint (3), the participation constraint[10]

$$w_L + \frac{1}{2}(w_H - w_L) - c(\hat{e}) \geq 0, \tag{5}$$

and the limited-liability constraints

$$w_L, w_H \geq 0. \tag{6}$$

Note that, under the equilibrium effort $\hat{e}$, a worker must obtain at least the same expected payment as if he exerted zero effort, i.e.,

$$w_L + \frac{1}{2}\Delta w(\hat{e}) - c(\hat{e}) \geq w_L + \hat{p}(0, \hat{e})\Delta w(\hat{e}) - c(0). \tag{7}$$

Hence, $\frac{1}{2}\Delta w(\hat{e}) - c(\hat{e}) \geq 0$, implying that the firm optimally chooses $w_L^s = 0$.

[8] For example, asymmetric equilibria do not exist if the probability function is described by the well-known Tullock or logit-form contest-success function. Also, in case of the Lazear-Rosen or probit contest-success function (which does not have the properties (ii) and (iii), however), pure-strategy equilibria are unique and symmetric.

[9] Note that $\frac{\partial}{\partial \hat{e}}\hat{p}_1(\hat{e}, \hat{e}) = \hat{p}_{11}(\hat{e}, \hat{e}) + \hat{p}_{12}(\hat{e}, \hat{e}) < 0$ due to properties (ii) and (iii) of the probability function.

[10] Note that, in the symmetric equilibrium, each worker's winning probability is 1/2.

Together with (3), it follows that $w_H^s = \Delta w(\hat{e})$ is optimal. Thus, the firm implements the effort level $\hat{e}^s > 0$ given by[11]

$$\hat{e}^s = \arg\max_{\hat{e}} 2\hat{v}\left(\hat{e}\right) - \Delta w(\hat{e}).$$

Now we turn to hierarchy level 2. For this tier, the firm solves the optimization problem

$$\begin{aligned}
&\max_{b_L, b_H, e} v\left(e\right) - b_L - p\left(e\right) \cdot \left(b_H - b_L\right) \\
\text{s.t.} \quad & e = \arg\max_{z} \left\{b_L + p\left(z\right) \cdot \left(b_H - b_L\right) - c\left(z\right)\right\} && (8) \\
& b_L + p\left(e\right) \cdot \left(b_H - b_L\right) - c\left(e\right) \geq 0 && (9) \\
& b_L, b_H \geq 0 && (10)
\end{aligned}$$

The firm maximizes its profit net of wage payments taking into account the incentive compatibility constraint (8), the participation constraint (9), and the limited liability constraints (10). Due to the monotone likelihood ratio property and the convexity of the distribution function condition, the incentive constraint (8) is equivalent to its first-order condition

$$b_H - b_L = \frac{c'\left(e\right)}{p'\left(e\right)}. \tag{11}$$

Using this relationship, the firm's problem can be transformed to

$$\begin{aligned}
&\max_{b_L, e} v\left(e\right) - b_L - p\left(e\right) \cdot \frac{c'\left(e\right)}{p'\left(e\right)} \\
\text{s.t.} \quad & b_L + p\left(e\right) \cdot \frac{c'\left(e\right)}{p'\left(e\right)} - c\left(e\right) \geq 0 && (12) \\
& b_L \geq 0.
\end{aligned}$$

Regarding the participation constraint, we can make the following observation, which is important for our further analysis.

[11] The second-order condition $2\hat{v}''\left(\hat{e}\right) - \Delta w''(\hat{e}) < 0$ is satisfied due to our technical assumptions $\hat{c}'''(\hat{e}) \geq 0$ and $\frac{\partial^2}{\partial \hat{e}^2}\hat{p}_1\left(\hat{e}, \hat{e}\right) \leq 0$. An interior solution is guaranteed by the assumption $\hat{c}''(0) = 0$. For $\Delta w''(\hat{e}) > 0$ see the additional pages for the referees.

Lemma 1 *The term*

$$r(e) := p(e)\frac{c'(e)}{p'(e)} - c(e) \tag{13}$$

is strictly positive and monotonically increasing for all $e > 0$.

Proof. $r(e) > 0$ can be rewritten as $c(e) - c'(e)\frac{p(e)}{p'(e)} < 0$. Note that $\frac{p(e)}{p'(e)} > e \Leftrightarrow p(e) - ep'(e) > 0$ is true since $p(\cdot)$ is strictly concave. But then we also have $c(e) - c'(e)\frac{p(e)}{p'(e)} < c(e) - ec'(e) < 0$ from the strict convexity of $c(\cdot)$. The derivative $r'(e) = p(e)\left[\frac{c''(e)p'(e) - p''(e)c'(e)}{[p'(e)]^2}\right]$ is positive for all $e > 0$ by strict concavity of $p(e)$ and strict convexity of $c(e)$. ■

Hence, given e, the transformed participation constraint (12) is satisfied for all $b_L \geq 0$. Therefore, the firm optimally sets $b_L^s = 0$. Intuitively, under each bonus spread $b_H - b_L$ that induces e, the firm chooses the one that minimizes expected wage costs, which is the case if $b_L^s = 0$. After substituting b_L^s into the firm's objective function, we obtain that the firm induces the effort level $e^s > 0$ given by[12]

$$e^s = \arg\max_e v(e) - r(e) - c(e).$$

The results of this section are summarized in the following proposition.

Proposition 1 *Under separate contracts, the firm implements the effort levels*

$$\hat{e}^s = \arg\max_{\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e}), \tag{14}$$

$$e^s = \arg\max_e v(e) - r(e) - c(e). \tag{15}$$

The optimal contract elements are

$$w_L^s = 0,\ w_H^s = \Delta w(\hat{e}^s),\ b_L^s = 0,\ b_H^s = \frac{c'(e^s)}{p'(e^s)}, \tag{16}$$

where $\Delta w(\hat{e})$ *and* $r(e)$ *are given by (4) and (13), respectively.*

[12] Due to our technical assumptions, the objective function is strictly concave. Furthermore, the assumption $c''(0) = 0$ ensures an interior solution. For $r''(e) > 0$ see the additional pages for the referees.

From Lemma 1, it follows that the worker on level 2 earns a strictly positive rent $r(e^s)$. This suggests that the firm may be able to benefit from a job-promotion scheme where the better performing level 1 worker is promoted to the next hierarchy level. Then, the rent provides additional effort incentive at the first hierarchy level. This is the case under a combined contract, which we analyze in the following section.

3.2 Combined Contract

Under a combined contract (w_L, w_H, b_L, b_H), the firm specifies w_L, w_H, b_L, and b_H at the beginning of the first period. At the end of the first period, the better performing level 1 worker will be promoted to level 2. We solve the game by backwards induction. In the second period, all payments and costs from hierarchy level 1 are sunk. Thus, given the bonus payments b_L and b_H, the promoted worker faces the same kind of decision problem as under separate contracts. Provided that his participation constraint (9) is satisfied, he chooses the effort level characterized by (8). In the first period, however, workers' optimization problems fundamentally differ from the case of two separate contracts. Now, increasing effort also raises the chance of being promoted and, consequently, earning a rent under the bonus contract. Hence, worker A's and B's optimization problem, respectively, is

$$\max_{\hat{e}_A} \; w_L + \hat{p}\left(\hat{e}_A, \hat{e}_B\right) \cdot \left[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)\right] - \hat{c}\left(\hat{e}_A\right) \quad (17)$$

$$\max_{\hat{e}_B} \; w_L + \left[1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)\right] \cdot \left[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)\right] - \hat{c}\left(\hat{e}_B\right). \quad (18)$$

Comparing the workers' objective functions with those under separate contracts, (1) and (2), we can see that, under combined contracts, the "prize" of performing better at level 1 increases by the expected payment of the promoted worker, $b_L + p(e)(b_H - b_L) - c(e)$. Analogous to the case of separate contracts, one can show that there is a unique symmetric equilibrium given by

$$\hat{p}_1\left(\hat{e}, \hat{e}\right)\left[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)\right] = \hat{c}'\left(\hat{e}\right). \quad (19)$$

The first-period participation constraint thus is

$$w_L + \frac{1}{2}(w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)) - \hat{c}(\hat{e}) \geq 0. \tag{20}$$

We can now state the firm's optimization problem as

$$\max_{e,\hat{e},w_L,w_H,b_H,b_L} [2\hat{v}(\hat{e}) - w_L - w_H] + [v(e) - b_L - p(e)(b_H - b_L)] \tag{21}$$

$$\text{subject to (8), (9), (19), (20),} \tag{22}$$

$$w_L, w_H, b_L, b_H \geq 0. \tag{23}$$

By solving this problem, we obtain the following result.

Proposition 2 *Under a combined contract, the firm implements the effort levels*

$$(\hat{e}^c, e^c) \in \arg\max_{\hat{e},e} \{2\hat{v}(\hat{e}) - \Delta w(\hat{e}) + v(e) - c(e)\} \tag{24}$$

$$\textit{subject to } \Delta w(\hat{e}) - r(e) \geq 0. \tag{25}$$

Furthermore, the optimal contract elements are

$$w_L^c = 0,\ w_H^c = \Delta w(\hat{e}^c) - r(e^c),\ b_L^c = 0,\ b_H^c = \frac{c'(e^c)}{p'(e^c)}, \tag{26}$$

where $\Delta w(\hat{e})$ and $r(e)$ are given by (4) and (13), respectively.

Proof. See Appendix. ■

3.3 Comparison of the Two Contracts

We can now compare the optimal separate contracts, as given in Proposition 1, with the optimal combined contract, which we have just derived in Proposition 2. Our conjecture was that the combined contract may have the advantage of partially substituting direct first-level incentives $w_H - w_L$ for indirect incentives which arise due to the expected second-period rent $r(e)$. By comparing the optimal contract elements (16) and (26), we can see that this

is indeed the case. If, under separate contracts, the firm wanted to induce the same effort levels $(\hat{e}^c, e^c)$ as under the combined contract, it would have to pay the same second-level high bonus $b_H = b_H^c$, but a higher reward for better relative performance at the first stage, i.e., $w_H = \Delta w(\hat{e}^c)$. By contrast, under combined contracts, the firm can reduce w_H by the second-period rent $r(e^c)$.

However, interlinking first- and second-period incentives may also have a detrimental effect. To see this, assume that the firm wishes to induce a certain second-period effort e. Then, to induce a relatively low first-period effort $\hat{e}$ with $\Delta w(\hat{e}) < r(e)$, the firm must offer "adverse" relative performance pay at level 1, i.e., the firm must reward the worker who performed *worse*, $w_L > w_H$. The proof of Proposition 2 shows that such a relative performance scheme cannot be optimal from the firm's point of view. Intuitively, $w_L > w_H$ means that the firm *pays* for reducing first-level incentives. However, the firm would be better off by setting $w_H = w_L = 0$, thereby increasing first-level effort and decreasing first-level rents. Consequently, under a combined contract, the firm induces only those effort levels that satisfy constraint (25), whereas such a restriction is not present under separate contracts.

To decide under which circumstances a combined contract dominates a separate contract, we have to distinguish whether (25) is binding under the optimal combined contract or not. First, assume the constraint is *non-binding.* Then, a comparison of the objective functions in (14), (15), and (24) immediately reveals that the firm prefers the combined contract. For each combination of efforts $(\hat{e}, e)$, implementation costs under a combined contract, $\Delta w(\hat{e}) + c(e)$, are lower than implementation costs under separate contracts, $\Delta w(\hat{e}) + r(e) + c(e)$. This is because, by Lemma 1, $r(e) > 0$ for $e > 0$. Also note that, under the combined contract, effort on the second hierarchy level corresponds to the first-best effort level, i.e.,

$$e^c = e^{FB} = \arg\max_e \{v(e) - c(e)\}.$$

Concerning the first hierarchy level, however, the comparison of (14) and (24) points out that $\hat{e}^c = \hat{e}^s$. Interestingly, the use of the rent $r(e)$ for

incentive purposes on hierarchy level 1 does *not* lead to improved incentives on that hierarchy level. Instead, the indirect incentives are completely used to improve incentives at the second level. This result is due to the fact that raising incentives on the second hierarchy level increases efforts on both levels, but level 1 efforts are then decreased again by a pure substitution of direct incentives $w_H - w_L$ for indirect ones.

In the last decade, contract theorists have reconsidered the concept of *efficiency wages*. According to Tirole (1999, p. 745), Laffont and Martimort (2002, p. 174) and Schmitz (2005) efficiency wages occur if workers are protected by limited liability and earn positive rents under the optimal contract. Of course, in their models the implemented effort level is inefficiently small. Interestingly, in our setting the firm implements the efficient effort level e^{FB} although implementation is associated with a strictly positive rent that is monotonically increasing in effort. Hence, combining both hierarchy levels for creating optimal incentives allows for efficiency wages in a more literal sense. As a crucial condition, the expected rent $r(e^{FB})$ is not allowed to become arbitrarily large. Then, restriction (25) will be binding at the optimal solution, which leads us to the second case.

If restriction (25) is *binding,* the firm still profits from using the second-period rent for creating incentives on level 1. In fact, the firm solely relies on indirect incentives via the second-period rent (since $\Delta w(\hat{e}^c) = r(e^c)$, we must have $w_H^c = 0$). However, this also means that second-period incentives tend to be too strong, so that the previously discussed detrimental effect from interlinking the two levels arises. Then, separate contracts may become optimal. Overall, we have the following results:

Proposition 3 *(i) If restriction (25) is non-binding, the combined contract strictly dominates separate contracts. Effort levels under the two contracts compare as follows:*

$$\hat{e}^c = \hat{e}^s \qquad \textit{and} \qquad e^c = e^{FB} > e^s$$

(ii) If restriction (25) is binding, the firm implements

$$\hat{e}^c > \hat{e}^s \quad \text{and} \quad e^{FB} > e^c > e^s.$$

The firm prefers a combined contract to separate contracts if and only if

$$2\hat{v}\left(\hat{e}^c\right) + v\left(e^c\right) - r(e^c) - c(e^c) > 2\hat{v}\left(\hat{e}^s\right) + v\left(e^s\right) - \Delta w(\hat{e}^s) - r(e^s) - c(e^s).$$

Proof. See Appendix. ■

The proposition shows that if the rent for implementing first-best effort e^{FB} on hierarchy level 2 is not too large (i.e. (25) is non-binding), it is optimal for the firm to use *all three incentive schemes* via a combined contract: first, the firm makes use of moderate relative performance pay on the first tier of the hierarchy by choosing a tournament winner prize $w_H^c = \Delta w(\hat{e}^c) - r(e^{FB}) > 0$, which is smaller than the winner prize under separate contracts, $w_H^s = \Delta w(\hat{e}^c)$ (with $\hat{e}^s = \hat{e}^c$). Second, it installs high-powered incentives via a bonus system on level 2 of the hierarchy; whereas the optimal bonus is zero in case of an unfavorable performance signal ($b_L^c = 0$), the worker receives a high bonus $b_H^c = \frac{c'\left(e^{FB}\right)}{p'(e^{FB})}$ in case of a favorable signal, which is larger than that under two separate contracts ($b_H^s = \frac{c'(e^s)}{p'(e^s)}$). Third, since the firm prefers the combined contract to separate contracts, it also uses a job-promotion scheme, which creates indirect incentives by the expected rent on the second level of the hierarchy.

If the rent for implementing e^{FB} is too large (i.e. (25) is binding), the firm faces the following trade-off: on the one hand, the combined contract leads to larger effort levels than the separate contracts, yielding a higher monetary value to the firm, $2\hat{v}\left(\hat{e}^c\right) + v\left(e^c\right) > 2\hat{v}\left(\hat{e}^s\right) + v\left(e^s\right)$. On the other hand, high efforts lead to rather high implementation costs, so that it is not clear whether the use of indirect incentives under the combined contract is accompanied by lower labor costs. If the combined contract is still dominating, the firm utilizes a bonus scheme and a job-promotion scheme but foregoes relative performance pay (i.e. $w_L^c = w_H^c = 0$). If, however, separate contracts are advantageous to the firm because of lower implementation costs, it will make

use of relative performance pay and a bonus system but renounce a job-promotion scheme.

Interestingly, our results nicely explain those empirical findings of Baker, Gibbs and Holmström (1994a, 1994b) that seem to be puzzling in the light of standard tournament models.[13] First, Baker, Gibbs and Holmström find that there is considerable variation in pay on each hierarchy level (see, for example, Figure VI in Baker, Gibbs and Holmström 1994a, p. 906). This finding contradicts the important prerequisite of tournament models that wages must be attached to jobs and, therefore, to hierarchy levels in order to generate incentives. However, if a combined contract dominates separate contracts, we will have a job-promotion scheme *with* pay variation because the promoted worker may earn different bonus payments on hierarchy level 2. Second, according to standard tournament theory, hierarchical wage differences should be completely explained by promotion premiums paid to workers when moving to higher levels in the hierarchy. Unfortunately, Baker, Gibbs and Holmström (1994a) find that "promotion premiums explain only part of the differences in pay between levels" (p. 909). In fact, often hierarchical wage differences are even five times higher or more than the corresponding promotion premiums. This puzzle can be explained using the optimal combined contract of our model. Here, a promoted worker does not only earn the promotion premium $w_H - w_L$ but also possible bonus payments. In particular, the higher the expected rent on hierarchy level 2 the smaller will be the promotion premium in our model since direct relative incentives are replaced with indirect incentives. If, in general, exerting effort on higher hierarchy levels is more valuable to firms than effort choices on lower levels, we will have considerable rents on higher tiers, thus reducing corresponding promotion premiums.

[13] Note that the same two puzzles are also found by Treble et al. (2001), who analyzed a British firm and not a US corporation. Considerable wage variation within job levels is also documented by the empirical studies of Seltzer and Merrett (2000), Dohmen, Kriechel and Pfann (2004), Gibbs and Hendricks (2004) and Grund (2005). Moreover, Dohmen, Kriechel and Pfann (2004) show that promotion and wage increase are often not simultaneous, which gives further evidence that salaries are also determined by bonuses and not solely by promotion premiums.

4 External Recruitment

In the last section, we have seen that a combined contract dominates separate contracts when the second-level rent $r(e^{FB})$ is smaller than $\Delta w(\hat{e}^s)$, the winner prize on level 1 under separate contracts. If this is not the case, however, combined contracts may be inferior because they make the implementation of low first-level effort expensive. To counteract this problem, the firm could sometimes recruit level 2 workers from the external labor market. Then, level 1 workers do not always "win" the second-level rent. Consequently, first-level incentives and effort decreases. A further advantage of this approach is that, to an external candidate, the firm can offer the contract (b_L^s, b_H^s), which *ex post* dominates the bonus scheme (b_L^c, b_H^c) of an internally promoted worker.

To allow for the possibility of external recruitment, we assume that the firm commits to promote the better performing level 1 worker with probability $\alpha \in [0,1]$. With probability $1-\alpha$, the firm hires a level 2 worker from outside. Thus, the contract offered in stage 1 of the game is now given by $(w_L, w_H, b_L, b_H, \alpha)$.[14] Note that this specification includes the two previously analyzed contracts: $\alpha = 0$ is equivalent to the case of separate contracts (Section 3.1) and $\alpha = 1$ corresponds to the combined contract without external recruitment (Section 3.2). We assume that the firm is able to commit to a certain value of α at the beginning of the first period. Note, however, that this assumption implies that the firm cares about its reputation in future employment relationships. If there are no such future relationships, the firm would ex post always want to hire an external candidate, since it can offer him the ex post optimal bonus contract. In practice, we can think of $1-\alpha$ as the proportion of times when the firm fills the second-level position with an external candidate.

The possibility of external promotion modifies the incentive structure on level 1. On level 2, however, the structure of an internally promoted worker's decision problem remains unchanged. Assume that e satisfies the second-level incentive compatibility constraint (8) and, moreover, the second-level

[14] As an alternative to external recruitment, the firm could sometimes neglect first-level performance when making the promotion decision. Then, the promotion decision is made purely randomly. We discuss this case at the end of this section.

participation constraint (9) holds. Then, at the first level, worker A chooses $\hat{e}_A$ to maximize

$$w_L + \hat{p}\left(\hat{e}_A, \hat{e}_B\right)\left(w_H - w_L\right) + \alpha \cdot \hat{p}\left(\hat{e}_A, \hat{e}_B\right) \cdot \left[b_L + p(e)(b_H - b_L) - c(e)\right] - \hat{c}\left(\hat{e}_A\right),$$

whereas, by choosing $\hat{e}_B$, worker B maximizes

$$w_L + \left(1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)\right)\left(w_H - w_L\right) + \alpha \cdot \left(1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)\right) \cdot \left[b_L + p(e)(b_H - b_L) - c(e)\right] - \hat{c}\left(\hat{e}_B\right).$$

Thus, the symmetric equilibrium on level 1 is implicitly given by

$$\hat{p}_1\left(\hat{e}, \hat{e}\right)\left(w_H - w_L + \alpha\left[b_L + p(e)(b_H - b_L) - c(e)\right]\right) = \hat{c}'\left(\hat{e}\right). \tag{27}$$

The first-level participation constraint is

$$w_L + \frac{1}{2}(w_H - w_L) + \frac{1}{2}\alpha\left[b_L + p(e)(b_H - b_L) - c(e)\right] - \hat{c}(\hat{e}) \geq 0. \tag{28}$$

Hence, the firm has to solve

$$\max_{\substack{\alpha \in [0,1], e, \hat{e}, \\ w_L, w_H, b_H, b_L}} 2\hat{v}(\hat{e}) - w_L - w_H + \alpha\left[v\left(e\right) - b_L - p\left(e\right)\left(b_H - b_L\right)\right] + (1 - \alpha)E$$

$$\text{subject to (8), (9), (27), (28), (23).}$$

Here, E denotes the firm's profit from hiring an external candidate for the job on the second hierarchy tier. To such a candidate, the firm offers the optimal separate contract. Consequently, we have $E = v(e^s) - r(e^s) - c(e^s)$. The restrictions (8) and (9) are the incentive compatibility and participation constraints, respectively, for level 2. Conditions (27) and (28) are the new incentive compatibility constraint and participation constraint, respectively, for level 1. Finally, (23) are the limited liability constraints.

By solving this problem, we obtain the following results.[15]

Proposition 4 *(i) The optimal contract* $(w_L^r, w_H^r, b_L^r, b_H^r, \alpha^r)$ *comprises a*

[15] The superscript "r" indicates that the optimal contract considered in this section includes a rule for recruiting from outside.

strictly positive probability of internal promotion, i.e., $\alpha^r > 0$.
(ii) External recruitment is excluded, i.e. $\alpha^r = 1$, if $r(e^{FB}) \leq \Delta w(\hat{e}^s)$. Then, the combined contract $(w_L^c, w_H^c, b_L^c, b_H^c)$ is implemented.
(iii) If $r(e^{FB}) > \Delta w(\hat{e}^s)$ and

$$E + [v'(e^c) - c'(e^c)] \frac{r(e^c)}{r'(e^c)} > [v(e^c) - c(e^c)], \tag{29}$$

the probability of recruiting from the external labor market will be strictly positive, i.e. $\alpha^r < 1$. If $\alpha^r < 1$, we have $\Delta w(\hat{e}^r) < r(e^r)$. Moreover, $\hat{e}^s < \hat{e}^r \leq \hat{e}^c$ and $e^c \leq e^r < e^{FB}$.

Proof. See Appendix. ■

The proposition shows that the firm always integrates the opportunity of internal job-promotion in the optimal contract. In other words, it is never beneficial to exclusively hire from the external labor market or, equivalently, to have a separate contract as analyzed in Section 3.1. Intuitively, at least to some extent, the firm should use the second-level rent to provide incentives for first-level workers. If $r(e^{FB}) \leq \Delta w(\hat{e}^s)$, we have $\alpha^r = 1$, so that the combined contract without external recruitment as specified in Section 3.2 is still optimal. If $r(e^{FB}) > \Delta w(\hat{e}^s)$, however, external recruitment may occur. Then, the firm implements effort levels that it would never want to induce under a combined contract without external recruitment, i.e. $\Delta w(\hat{e}^r) < r(e^r)$. More specifically, the firm induces lower effort on level 1 and higher effort on level 2 relative to the combined contract, which leads to strict improvement of the combined contract.

Condition (29) points out that deviating from $\alpha = 1$ to a strictly positive probability of external recruitment (i.e., $\alpha < 1$) has two advantages and one disadvantage:[16] first, the firm benefits from choosing the optimal separate contract for level 2 in case of external recruitment, leading to profits E. This contract is beneficial since it maximizes level 2 profits. Second, the profit associated with level 2 under a combined contract is realized less often which is detrimental, as indicated by the right-hand side of (29). Third, in

[16] In the Appendix we show for a quadratic cost function, a linear value function and a Tullock or logit-form contests-success function that this condition can indeed be satisfied.

cases where this profit is realized it will be larger than under $\alpha = 1$. This positive effect is shown by the second term on the left-hand side of (29). Here, $[v'(e^c) - c'(e^c)]$ denotes the increase in level 2 profits under a combined contract by marginally raising e at $e = e^c$. Recall that the strictly concave function $v(e) - c(e)$ has its maximum at $e = e^{FB}$ so that raising effort from $e^c < e^{FB}$ to e^r, $e^c < e^r < e^{FB}$, is beneficial for the firm. The term $\frac{r(e^c)}{r'(e^c)}$ describes how much level 2 effort e rises due to a marginal decrease in α at $\alpha = 1$. Technically, the proof in the Appendix shows that

$$-\frac{\partial e}{\partial \alpha}\bigg|_{\alpha=1, e=e^c} = \frac{r\left(e^c\right)}{r'\left(e^c\right)}.$$

Altogether, if the first and the third effect together dominate the second effect, a strictly positive probability of recruiting from outside will be optimal for the firm.[17] In the proof of Proposition 4 we show that, if $\alpha^r < 1$, it is given by $\alpha^r = \Delta w(\hat{e}^r)/r(e^r)$. Consequently, external recruitment occurs more often if $\Delta w(\hat{e}^r)$ is small relative to $r(e^r)$, implying that first-level incentives are low-powered compared to second-level ones. This is what constitutes the benefit of supplementing a combined contract with external recruitment: it allows to implement lower effort on the first tier ($\hat{e}^r \leq \hat{e}^c$) while further increasing effort on the second tier ($e^c \leq e^r$).

Our findings on the optimality of external recruitment fits quite well with another puzzle raised by the empirical literature on internal labor markets: several empirical studies document that there exist ports of entry on many hierarchy levels in diverse firms from different countries.[18] This finding is puzzling in the light of standard tournament theory because generating incentives by a job-promotion tournament requires a strict ban on external recruiting.[19] However, the results of Proposition 4 show that including bonus

[17] Note that condition (29) is sufficient and may be too strong. Indeed, since $\hat{e}^r \leq \hat{e}^c$ and $e^r \geq e^c$ the firm may not only benefit from external recruiting by increasing level 2 effort, but also by decreasing level 1 effort. Recall that $\hat{e}^c$ is larger than effort level $\hat{e}^s$ that maximizes level 1 profits, characterized by (14) (see Proposition 3).

[18] See Lazear (1992), Baker, Gibbs and Holmström (1994a, 1994b), Ariga, Ohkusa and Brunello (1999), Seltzer and Merrett (2000), Dohmen, Kriechel and Pfann (2004), Gibbs and Hendricks (2004) and Grund (2005).

[19] Standard tournament theory suggests a strict ban as long as one exclusively focuses

schemes at higher hierarchy levels can lead to a completely different situation compared to the standard tournament model, which analyzes job-promotion in isolation. External recruiting now allows the firm to fine-tune incentives, thereby improving combined contracts.

As an alternative to external recruitment, the firm could employ a promotion policy where the first-period performance signal is sometimes neglected. Then, the promotion decision is made randomly, e.g. by tossing a coin. However, such a procedure introduces new commitment problems on the side of the firm. To see this, assume that the bonus contract is contingent on the nature of the promotion decision, i.e. whether it had been based on first-level performance or random selection. Then, there is a potential source of conflict between the promoted worker and the firm: the worker prefers the contract with the higher bonus, which makes him work harder but also earns him a higher rent. By contrast, ex post, the firm will usually favor the contract with the lower rent. Thus, since in general a third party is not able to observe the true nature of the promotion decision, it is difficult to make contracts contingent on it, even if the firm cares about its reputation in future relationships. Thus, it might be more realistic to assume that, under internal random promotion, the bonus contract is independent of the nature of promotion. In this case, however, external recruitment dominates a random internal promotion procedure since, under the former, different bonus contracts are feasible.

Nevertheless, under certain circumstances, external recruitment is not desirable, e.g. if level 1 workers acquire firm-specific human capital that significantly raises their productivity on level 2. Then, it can be shown that, under random internal promotion with non-contingent bonus contracts, the probability of basing promotion on first-level performance is always strictly positive. Hence, analogous to the case of external recruitment, purely separate contracts are never optimal.[20]

on the provision of incentives (as we have done so far) and excludes harmful activities, such as sabotage, or selection aspects.

[20] Formal proofs of the arguments from this paragraph are available from the authors upon request.

5 Heterogeneous Workers

Typically, real workers are not homogeneous as assumed in our basic model. In this section, we skip the homogeneity assumption and introduce workers that differ in their talents or abilities. Thereafter, we will analyze which type of contract – separate contracts or a combined contract – will be advantageous for the firm in this more realistic setting.

5.1 Modifications of the Basic Model

We assume that either worker may have high talent t_1 or low talent t_0 with $t_1 > t_0 > 0$, and that neither the workers nor the firm observe the workers' individual talents during the whole game. In other words, we introduce symmetric uncertainty about the quality of the workers.[21] Let all players (i.e. the workers and the firm) have the same prior distribution about worker talent. For simplicity, let each talent be equally likely so that unknown talent can be described by a random variable t that takes values t_0 and t_1 with probability $\frac{1}{2}$, respectively, and has mean $E[t] = (t_0 + t_1)/2$.

On each hierarchy level, a worker's talent influences both the value of effort for the firm and the probability of generating a favorable signal. Let the value of worker i $(i = A, B)$ to the firm when exerting effort $\hat{e}_i$ on level 1 be $t \cdot \hat{v}(\hat{e}_i)$, and that on level 2 when choosing effort e_i be $t \cdot v(e_i)$. In analogy, the probability of a favorable signal on level 2 is now given by $t \cdot p(e)$, with $t_1 \cdot p(e) \leq 1$, $\forall e$. For a relative performance signal on level 1 we have to differentiate four possible situations. If both workers have equal talents, A's probability of winning the tournament will again be described by the function $\hat{p}(\hat{e}_A, \hat{e}_B)$. In addition, now we also have two possible asymmetric pairings. If worker A has high talent t_1 and worker B low talent t_0, A's probability of getting the better evaluation will be described by $\hat{p}(\hat{e}_A, \hat{e}_B; t_1)$ whereas B's one is given by $1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_1)$. In the opposite asymmetric case with B being more talented than A, worker A wins the tournament with probability

[21] The assumption of symmetric talent uncertainty is widespread in labor economics. See, among many others, Harris and Holmström (1982), Murphy (1986), Holmström (1999) and Gibbons and Waldman (1999).

$\hat{p}(\hat{e}_A, \hat{e}_B; t_0)$ and B with probability $1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_0)$.

We assume that the new probability functions have analogous properties (i)–(iii) as the function $\hat{p}(\cdot, \cdot)$ (see Section 2). For example, in the basic model we have $\hat{p}_1(\hat{e}_j, \hat{e}_i) = -\hat{p}_2(\hat{e}_i, \hat{e}_j)$, which follows from the symmetry assumption (i). In analogy, we assume that also in heterogeneous pairings the specific identity of a certain worker does not have any influence on his (marginal) winning probability, that is whether a worker acts on the first or on the second position in $\hat{p}(\cdot, \cdot; t)$ does not influence the (marginal) returns of his effort choice for a given asymmetric pairing. Technically, this means that $\hat{p}(\hat{e}_i, \hat{e}_j; t_1) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i; t_0)$, implying

$$\hat{p}_1(\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_2(\hat{e}_j, \hat{e}_i; t_0) \quad \text{and} \quad \hat{p}_2(\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_1(\hat{e}_j, \hat{e}_i; t_0) \tag{30}$$

for $i, j = A, B$; $i \neq j$. Of course, talent should have an impact on a worker's absolute winning probability and his marginal one. In particular, we assume that, for given effort levels, the more talented worker has a higher winning probability than the less talented one:

$$\hat{p}(\hat{e}_i, \hat{e}_j; t_1) > \hat{p}(\hat{e}_i, \hat{e}_j; t_0). \tag{31}$$

Furthermore, let effort and talent be complements in the sense of

$$\hat{p}_1(\hat{e}_i, \hat{e}_j; t_1) > \hat{p}_1(\hat{e}_i, \hat{e}_j; t_0) \quad \text{and} \quad -\hat{p}_2(\hat{e}_i, \hat{e}_j; t_0) > -\hat{p}_2(\hat{e}_i, \hat{e}_j; t_1), \tag{32}$$

that is marginally increasing effort is more effective under high talent than under low one. Properties (ii) and (iii) from the basic model should also hold analogously for heterogeneous workers. Note that property (iii) together with symmetry here implies that $\hat{p}_{12}(\hat{e}, \hat{e}; t_1) = -\hat{p}_{12}(\hat{e}, \hat{e}; t_0)$: if workers choose identical efforts the more able one has a higher winning probability; if now the other worker increases his effort, competition becomes more intense so that the more able worker raises his effort, too. Again, this effect should be independent of whether a worker acts on the first or on the second position in $\hat{p}(\cdot, \cdot; t)$. Finally, we assume analogous regularity conditions to hold as in the basic model with homogeneous workers.

In the following, we will investigate how the comparison between separate contracts and a combined contract will change when workers are heterogeneous.

5.2 Separate Contracts

Under worker heterogeneity, the equilibrium on hierarchy level 1 is characterized by the first-order conditions

$$(w_H - w_L)\frac{1}{4}\left(\hat{p}_1\left(\hat{e}_A, \hat{e}_B; t_1\right) + \hat{p}_1\left(\hat{e}_A, \hat{e}_B; t_0\right) + 2\hat{p}_1\left(\hat{e}_A, \hat{e}_B\right)\right) = \hat{c}'\left(\hat{e}_A\right),$$
$$(w_H - w_L)\frac{1}{4}\left(-\hat{p}_2\left(\hat{e}_A, \hat{e}_B; t_1\right) - \hat{p}_2\left(\hat{e}_A, \hat{e}_B; t_0\right) - 2\hat{p}_2\left(\hat{e}_A, \hat{e}_B\right)\right) = \hat{c}'\left(\hat{e}_B\right).$$

Using $\hat{p}_1\left(\hat{e}_B, \hat{e}_A\right) = -\hat{p}_2\left(\hat{e}_A, \hat{e}_B\right)$ and (30) shows that there exists a symmetric equilibrium in which each worker chooses $\hat{e}$ characterized by

$$w_H - w_L = \Delta\tilde{w}(\hat{e}) \tag{33}$$

$$\text{with} \quad \Delta\tilde{w}(\hat{e}) := \frac{4\hat{c}'\left(\hat{e}\right)}{\hat{p}_1\left(\hat{e}, \hat{e}; t_1\right) + \hat{p}_1\left(\hat{e}, \hat{e}; t_0\right) + 2\hat{p}_1\left(\hat{e}, \hat{e}\right)} \tag{34}$$

and $\Delta\tilde{w}'(\hat{e}) > 0$.[22] The firm maximizes $2E\left[t\right]\hat{v}\left(\hat{e}\right) - w_L - w_H$ subject to the participation constraint (5),[23] the limited-liability constraints (6) and the incentive constraint (33). The optimal tournament prizes are, therefore, given by $w_L^s = 0$ and $w_H^s = \Delta\tilde{w}(\hat{e})$, and the firm implements the effort level[24] $\hat{e}_h^s$ that solves

$$\max_{\hat{e}} 2E\left[t\right]\hat{v}\left(\hat{e}\right) - \Delta\tilde{w}(\hat{e}). \tag{35}$$

[22] Note that $\frac{\partial}{\partial\hat{e}}\left(\hat{p}_1\left(\hat{e}, \hat{e}; t_1\right) + \hat{p}_1\left(\hat{e}, \hat{e}; t_0\right) + 2\hat{p}_1\left(\hat{e}, \hat{e}\right)\right) = \hat{p}_{11}\left(\hat{e}, \hat{e}; t_1\right) + \hat{p}_{12}\left(\hat{e}, \hat{e}; t_1\right) + \hat{p}_{11}\left(\hat{e}, \hat{e}; t_0\right) + \hat{p}_{12}\left(\hat{e}, \hat{e}; t_0\right) + 2\hat{p}_{11}\left(\hat{e}, \hat{e}\right) + 2\hat{p}_{12}\left(\hat{e}, \hat{e}\right) < 0$.

[23] Note that, due to the symmetric equilibrium, the participation constraint will be the same as in the basic model.

[24] Here and in the following, the subscript "h" for optimal efforts indicates heterogeneity of workers.

On hierarchy level 2, the firm's optimization problem now reads as

$$\begin{gathered}\max_{b_L,b_H,e} E[t]\,v(e) - b_L - E[t]\,p(e)\,(b_H - b_L) \\ \text{subject to } \; e = \arg\max_z \{b_L + E[t]\,p(z)\,(b_H - b_L) - c(z)\} \\ b_L + E[t]\,p(e)\,(b_H - b_L) - c(e) \geq 0 \\ b_L, b_H \geq 0.\end{gathered}$$

In analogy to the basic model, the incentive constraint can be replaced with the first-order condition $b_H - b_L = \frac{c'(e)}{E[t]p'(e)}$. It is straightforward to show that, under the optimal bonus contract, $b_L^s = 0$, the participation constraint is identical to (12) and the firm implements effort e_h^s with

$$e_h^s = \arg\max_e E[t]\,v(e) - r(e) - c(e) \tag{36}$$

and $r(e)$ being defined in (13). Altogether, the comparison of (35) and (36) with (14) and (15) from the basic model shows that introducing heterogeneity leads to changes in the expected values of the workers' effort choices and in the optimal winner prize w_H^*, but leaves the implementation costs on level 2 unchanged for a given effort level e.

5.3 Combined Contract

Solving the game by backwards induction, we first consider the actions on hierarchy level 2. Here, all players update their beliefs about the unknown talent of the promoted worker. Let $E[t|\hat{s}]$ denote the expected talent of the promoted worker, that is each player calculates a new expectation depending on the realization of the relative performance signal $\hat{s}$. Note that at any prior point in time the workers as well as the firm already know that they have to update their beliefs in light of the promotion decision and that they will not receive further information. Hence, when designing the optimal combined contract, the firm has to include the incentive constraint

$$b_H - b_L = \frac{c'(e)}{E[t|\hat{s}]\,p'(e)} \tag{37}$$

and the participation constraint

$$b_L + E\left[t|\hat{s}\right] p\left(e\right)\left(b_H - b_L\right) - c\left(e\right) \geq 0 \Leftrightarrow b_L + r\left(e\right) \geq 0, \tag{38}$$

where the last inequality follows from (13) and (37).

At level 1, worker A and worker B maximize

$$\begin{aligned}
&w_L + \left(w_H - w_L + b_L + E\left[t|\hat{s}\right] p\left(e\right)\left(b_H - b_L\right) - c\left(e\right)\right) \\
&\times \frac{1}{4}\left(\hat{p}\left(\hat{e}_A, \hat{e}_B; t_1\right) + \hat{p}\left(\hat{e}_A, \hat{e}_B; t_0\right) + 2\hat{p}\left(\hat{e}_A, \hat{e}_B\right)\right) - \hat{c}\left(\hat{e}_A\right) \qquad \text{and} \\
&w_L + \left(w_H - w_L + b_L + E\left[t|\hat{s}\right] p\left(e\right)\left(b_H - b_L\right) - c\left(e\right)\right) \\
&\times \frac{1}{4}\left(\left(1 - \hat{p}\left(\hat{e}_A, \hat{e}_B; t_1\right)\right) + \left(1 - \hat{p}\left(\hat{e}_A, \hat{e}_B; t_0\right)\right) + 2\left(1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)\right)\right) - \hat{c}\left(\hat{e}_B\right),
\end{aligned}$$

respectively. Equations (37) and (13) together with the first-order conditions, $\hat{p}_1\left(\hat{e}_B, \hat{e}_A\right) = -\hat{p}_2\left(\hat{e}_A, \hat{e}_B\right)$ and (30) yield

$$\begin{aligned}
\left(w_H - w_L + b_L + r\left(e\right)\right) \frac{\hat{p}_1\left(\hat{e}_A, \hat{e}_B; t_1\right) + \hat{p}_1\left(\hat{e}_A, \hat{e}_B; t_0\right) + 2\hat{p}_1\left(\hat{e}_A, \hat{e}_B\right)}{4} &= \hat{c}'\left(\hat{e}_A\right) \\
\left(w_H - w_L + b_L + r\left(e\right)\right) \frac{\hat{p}_1\left(\hat{e}_B, \hat{e}_A; t_0\right) + \hat{p}_1\left(\hat{e}_B, \hat{e}_A; t_1\right) + 2\hat{p}\left(\hat{e}_B, \hat{e}_A\right)}{4} &= \hat{c}'\left(\hat{e}_B\right).
\end{aligned}$$

Thus, in the symmetric equilibrium each worker exerts $\hat{e}$ described by

$$w_H - w_L + b_L + r\left(e\right) = \Delta\tilde{w}(\hat{e}) \tag{39}$$

with $\Delta\tilde{w}(\hat{e})$ being defined in (34).

Now we can summarize the firm's problem. It maximizes

$$\begin{aligned}
&2E\left[t\right]\hat{v}(\hat{e}) - 2w_L - \left(w_H - w_L\right) + E\left[t|\hat{s}\right] v\left(e\right) - b_L - E\left[t|\hat{s}\right] p\left(e\right)\left(b_H - b_L\right) \\
&\overset{(13),(37),(39)}{=} 2E\left[t\right]\hat{v}(\hat{e}) - \Delta\tilde{w}(\hat{e}) + E\left[t|\hat{s}\right] v\left(e\right) - 2w_L - c\left(e\right)
\end{aligned}$$

subject to the limited-liability constraints (23), the incentive compatibility constraints (37) and (39), the participation constraint for the second hierar-

chy level (38) and the participation constraint for the first level,

$$w_L + \frac{1}{2}\left(w_H - w_L + b_L + E\left[t|\hat{s}\right] p\left(e\right)\left(b_H - b_L\right) - c\left(e\right)\right) - \hat{c}\left(\hat{e}\right) \geq 0$$
$$\overset{(13),(37),(39)}{\Leftrightarrow} w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}\left(\hat{e}\right) \geq 0.$$

Moreover, the firm has to note that $E\left[t|\hat{s}\right]$ depends on the workers' equilibrium efforts chosen on hierarchy level 1:

$$\begin{aligned} E\left[t|\hat{s}\right] &= \frac{1}{4}t_1 + \frac{1}{4}t_0 + \frac{1}{4}\left(\hat{p}\left(\hat{e},\hat{e};t_1\right)t_1 + \left(1 - \hat{p}\left(\hat{e},\hat{e};t_1\right)\right)t_0\right) \\ &\quad + \frac{1}{4}\left(\hat{p}\left(\hat{e},\hat{e};t_0\right)t_0 + \left(1 - \hat{p}\left(\hat{e},\hat{e};t_0\right)\right)t_1\right) \\ &= E\left[t\right] + \frac{\Delta t\left(\hat{p}\left(\hat{e},\hat{e};t_1\right) - \hat{p}\left(\hat{e},\hat{e};t_0\right)\right)}{4} \overset{(31)}{>} E\left[t\right] \end{aligned} \tag{40}$$

with $\Delta t := t_1 - t_0$. Thus, the posterior expectation is larger than the prior one because the more talented worker is promoted with higher probability in case of an asymmetric pairing in the tournament. Furthermore, the posterior mean strictly increases in level 1 equilibrium efforts as talent and effort are complements:

$$\begin{aligned} \frac{\partial E\left[t|\hat{s}\right]}{\partial\hat{e}} &= \frac{\Delta t}{4}\left(\hat{p}_1\left(\hat{e},\hat{e};t_1\right) + \hat{p}_2\left(\hat{e},\hat{e};t_1\right) - \hat{p}_1\left(\hat{e},\hat{e};t_0\right) - \hat{p}_2\left(\hat{e},\hat{e};t_0\right)\right) \\ &\overset{(30)}{=} \frac{\Delta t}{2}\left(\hat{p}_1\left(\hat{e},\hat{e};t_1\right) - \hat{p}_1\left(\hat{e},\hat{e};t_0\right)\right) \overset{(32)}{>} 0. \end{aligned} \tag{41}$$

Applying the same two-step procedure as in the basic model yields that the firm implements the effort pair $(\hat{e}_h^c, e_h^c)$ with[25]

$$(\hat{e}_h^c, e_h^c) \in \arg\max_{\hat{e},e}\left\{2E\left[t\right]\hat{v}(\hat{e}) + E\left[t|\hat{s}\right]v\left(e\right) - \Delta\tilde{w}(\hat{e}) - c\left(e\right)\right\} \tag{42}$$

$$\text{subject to } \Delta\tilde{w}(\hat{e}) - r\left(e\right) \geq 0. \tag{43}$$

When comparing optimal efforts under the combined contract with those under two separate contracts, we have to distinguish whether the restriction (43) is binding or not at the optimum. In case of a *non-binding restriction*,

[25] See the additional pages for the referees.

optimal efforts $(\hat{e}_h^c, e_h^c)$ are described by the first-order conditions

$$2E\left[t\right]\hat{v}'(\hat{e}) + \frac{\partial E\left[t|\hat{s}\right]}{\partial \hat{e}} v\left(e\right) = \Delta\tilde{w}'(\hat{e}) \quad \text{and} \quad E\left[t|\hat{s}\right] v'\left(e\right) = c'\left(e\right). \tag{44}$$

Comparing the first equation with (35) clearly shows that $\hat{e}_h^c > \hat{e}_h^s$ as $\partial E\left[t|\hat{s}\right] / \partial \hat{e} > 0$. The comparison of the second equation with (36) points out that $e_h^c > e_h^s$, due to Lemma 1 and the fact that $E\left[t|\hat{s}\right] > E\left[t\right]$. Finally, we have to consider the case of a *binding restriction* (43). Using this restriction, we can express level 2 effort as a function of level 1 effort, $e\left(\hat{e}\right)$, with $\frac{\partial e}{\partial \hat{e}} = \frac{\Delta\tilde{w}'(\hat{e})}{r'(e)} > 0$. Now, the firm's objective function under a combined contract can be rewritten as

$$2E\left[t\right]\hat{v}(\hat{e}) + E\left[t|\hat{s}\right] v\left(e\left(\hat{e}\right)\right) - \Delta\tilde{w}(\hat{e}) - c\left(e\left(\hat{e}\right)\right).$$

The first-order condition yields

$$2E\left[t\right]\hat{v}'(\hat{e}) + \frac{\partial E\left[t|\hat{s}\right]}{\partial \hat{e}} v\left(e\left(\hat{e}\right)\right) - \Delta\tilde{w}'(\hat{e}) + \left[E\left[t|\hat{s}\right] v'\left(e\left(\hat{e}\right)\right) - c'\left(e\left(\hat{e}\right)\right)\right] \frac{\partial e}{\partial \hat{e}} = 0.$$

Inserting for $\partial e / \partial \hat{e}$ leads to

$$2E\left[t\right]\hat{v}'(\hat{e}) + \frac{\partial E\left[t|\hat{s}\right]}{\partial \hat{e}} v\left(e\left(\hat{e}\right)\right) + \frac{E\left[t|\hat{s}\right] v'\left(e\left(\hat{e}\right)\right) - c'\left(e\left(\hat{e}\right)\right) - r'\left(e\left(\hat{e}\right)\right)}{r'\left(e\left(\hat{e}\right)\right)} \Delta\tilde{w}'(\hat{e}) = 0.$$

Since the first two expressions as well as $r'\left(e\left(\hat{e}\right)\right)$ and $\Delta\tilde{w}'(\hat{e})$ are positive, we must have that the numerator of the last expression is negative. As this numerator is a strictly concave function of $e\left(\hat{e}\right)$ and since $E\left[t|\hat{s}\right] > E\left[t\right]$, we obtain from the comparison with (36) that $e_h^c > e_h^s$. Finally, we have to consider optimal effort implementation on hierarchy level 1. Since (43) is binding, the effort $\hat{e}$ that would maximize level 1 profit corresponds to a level 2 effort that is below the effort e that maximizes level 2 profit $E\left[t|\hat{s}\right] v\left(e\right) - c\left(e\right)$. Hence, the firm may be interested in further raising $\hat{e}$. As both profit functions are strictly concave, we can apply the same argument as in the proof of Proposition 3: the firm would, thus, never implement a smaller $\hat{e}$ than the optimal effort under a non-binding restriction. Since that

effort was larger than the optimal level 1 effort under separate contracts, we have proved that $\hat{e}_h^c > \hat{e}_h^s$ also holds under a binding restriction.

Proposition 5 *Irrespective of whether restriction (43) is binding or not at the optimum, we have* $\hat{e}_h^c > \hat{e}_h^s$ *and* $e_h^c > e_h^s$.

Proposition (5) points out that under a combined contract the firm implements strictly larger efforts on hierarchy level 1 than under separate contracts. This result sharply contrasts with our findings in Proposition (3) on homogeneous workers. The intuition comes from the fact that in case of heterogeneous workers the firm has an additional motive of implementing large efforts on hierarchy level 1: the larger $\hat{e}$ the higher will be the probability that the more talented worker is promoted to level 2 in case of a heterogeneous pairing, that is $\hat{p}_1(\hat{e}, \hat{e}; t_1) > 0$. This, in turn, increases the posterior expected talent of the promoted worker: $\partial E[t|\hat{s}]/\partial \hat{e} > 0$ according to (41) since $E[t|\hat{s}]$ monotonically increases in $\hat{p}(\hat{e}, \hat{e}; t_1)$. In other words, if workers are heterogeneous, then the tournament scheme has to fulfill two purposes – creating incentives and achieving efficient selection. By inducing higher incentives on level 1 the firm improves better worker selection for level 2, because both incentives and selection are strictly interlinked.

If the restriction (43) is non-binding at the optimum, again the firm will be strictly better off by choosing a combined contract than two separate contracts since a combined contract leads to first-best implementation on hierarchy level 2, i.e. $e_h^c = \arg\max_e \{E[t|\hat{s}]\, v(e) - c(e)\}$. However, there is a crucial difference in comparison to the basic model with homogeneous workers. Under heterogeneity, we have the additional effect that combining both hierarchy levels via a job-promotion scheme even improves on first-best implementation under uncertainty as $E[t|\hat{s}] > E[t]$. By inducing large efforts $\hat{e}$ on level 1 the firm raises the posterior expected talent of the promoted worker (i.e. $\partial E[t|\hat{s}]/\partial \hat{e} > 0$) which, in turn, increases the *efficient* effort level e_h^c on level 2 that maximizes $E[t|\hat{s}]\, v(e) - c(e)$.

Finally, we can compare the selection properties of a combined contract with those of separate contracts and those of a job-promotion tournament that is used without bonus scheme at the next level. The first comparison

shows that the probability of promoting the better worker is strictly larger under the combined contract than under two separate contracts where the worker for level 2 is chosen by random; technically, we have $\hat{p}\left(\hat{e}_h^c, \hat{e}_h^c; t_1\right) > 1/2$ due to $\hat{p}\left(\hat{e}_i, \hat{e}_j; t_1\right) = 1 - \hat{p}\left(\hat{e}_j, \hat{e}_i; t_0\right)$ and (31). The second comparison seems to be even more interesting since it contrasts promotion under the combined solution to promotion within a standard job-promotion tournament with wages attached to jobs (i.e. fixed prizes). Note that the latter one is described in our model by the solution for hierarchy level 1 under two separate contracts. Since $\hat{e}_h^c > \hat{e}_h^s$, we obtain the following interesting result.

Corollary 1 *Combining job-promotion with incentive pay on the next hierarchy level always improves the selection quality of a job-promotion tournament.*

Proof. $\hat{p}\left(\hat{e}_h^c, \hat{e}_h^c; t_1\right) > \hat{p}\left(\hat{e}_h^s, \hat{e}_h^s; t_1\right)$ since $\frac{\partial}{\partial \hat{e}} \hat{p}\left(\hat{e}, \hat{e}; t_1\right) = \hat{p}_1\left(\hat{e}, \hat{e}; t_1\right) + \hat{p}_2\left(\hat{e}, \hat{e}; t_1\right)$ $\overset{(30)}{=} \hat{p}_1\left(\hat{e}, \hat{e}; t_1\right) - \hat{p}_1\left(\hat{e}, \hat{e}; t_0\right) \overset{(32)}{>} 0$. ■

At the end of Section 3, we mentioned empirical puzzles that contradict standard tournament theory but can be explained by combining job-promotion tournaments with bonuses as in our model. One of these puzzles was that wages are not attached to jobs and, therefore, to hierarchy levels. As has been shown in this section, the selection quality of standard job-promotion tournaments can be significantly improved by replacing wages that are attached to jobs by incentive pay such as a bonus scheme. Hence, missing wages-attached-to-jobs in the empirical literature on firms' wage policies can be nicely explained by the existence of heterogeneous workers that requires both appropriate incentives and efficient selection.

6 Conclusion

We analyzed a two-tier hierarchy where workers compete in a rank-order tournament on level 1. On the second tier, a worker is hired from outside or promoted from the first tier to carry out a managerial task that leads to an individual performance signal. Workers are protected by limited liability

on either hierarchy level. From a theoretical perspective, combining a job-promotion tournament on level 1 with bonus payment on level 2 generates two possible advantages: if workers are homogeneous, rents from level 2 can be used to create incentives on level 1. The firm may even implement first-best effort on the second hierarchy level although the worker earns a strictly positive rent on this level. If workers are heterogeneous, the firm additionally benefits from a complementary bonus scheme, which strictly improves the tournament's selection quality in finding out the most talented worker.

Probably, the combination of tournament and bonus scheme may lead to further advantages if workers are heterogeneous. For example, Münster (2007) shows that more able workers may be deterred from participating in a tournament in case of sabotage among the contestants. Then, the advantage of higher talent is completely erased since more able workers are sabotaged more heavily than less able ones, thus equalizing the winning probabilities of the heterogeneous workers. If a tournament is combined with a bonus scheme at the next level and more able workers earn higher rents at this level, the problem of adverse participation may be mitigated.

In a different setting, the combination of tournament and bonus scheme may be useful to make the competition between heterogeneous contestants more even. As is known in the tournament literature, the more uneven the competition the less effort will be chosen in equilibrium. Imagine that talent and effort are substitutes on each hierarchy level and not complements as in our paper. Then workers' rents on the second hierarchy level may be decreasing in ability. In this situation, adding a bonus scheme to the tournament would have the direct consequence that the uneven competition between heterogeneous workers on level 1 becomes less uneven as more able workers have lower expected rents from winning the tournament than less able ones. If the firm cannot rely on handicaps (e.g., due to only ordinal information) to counterbalance ability differences, such decreasing rents would be an appropriate instrument for regulating competition.

7 Appendix

7.1 Proof of Proposition 2

We can solve problem (21)-(23) in two steps: *First*, we derive the firm's minimum cost for inducing a given pair of effort levels $(\hat{e}, e)$. *Then*, we use the optimal cost function to solve the profit maximization problem and determine the optimal effort pair $(\hat{e}^c, e^c)$. The cost minimization problem for a given effort pair $(\hat{e}, e)$ reads as

$$\begin{aligned} &\min_{w_L, w_H, b_L, b_H} 2w_L + (w_H - w_L) + b_L + p(e)(b_H - b_L) \\ &\text{subject to (8), (9), (19), (20), } w_L, w_H, b_L, b_H \geq 0. \end{aligned}$$

By the incentive compatibility constraint (8), $b_H - b_L = \frac{c'(e)}{p'(e)}$. Thus, in combination with the incentive compatibility constraint (19), we obtain

$$w_H - w_L = \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})} - b_L - p(e)\frac{c'(e)}{p'(e)} + c(e) = \Delta w(\hat{e}) - b_L - r(e), \quad (45)$$

where $\Delta w(\hat{e})$ is given by (4) and $r(e)$ by (13).[26]

Using (45), the first-level participation constraint (20) boils down to

$$w_L + \frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0. \quad (46)$$

Furthermore, the second-level participation constraint (9) becomes

$$b_L + p(e)\frac{c'(e)}{p'(e)} - c(e) = b_L + r(e) \geq 0. \quad (47)$$

Thus, substituting for the tournament prize spread $w_H - w_L$ and the bonus

[26] Recall that $\Delta w(\hat{e})$ is the prize spread necessary to induce $\hat{e}$ under *separate* contracts. However, note that $\Delta w(\hat{e})$ will usually be different from $w_H^c - w_L^c$.

spread $b_H - b_L$, the cost minimization problem can be simplified to[27]

$$\min_{w_L, b_L} 2w_L + \Delta w\left(\hat{e}\right) + c(e) \quad \text{subject to (46), (47) and}$$
$$\Delta w\left(\hat{e}\right) - b_L - r(e) + w_L,\ w_L,\ b_L \geq 0. \tag{48}$$

By Lemma 1, we obtain $b_L^c = 0$ for the optimal low bonus: this satisfies the participation constraint for the second hierarchy level (47) and is also best for ensuring that $w_H = \Delta w\left(\hat{e}\right) - b_L - r(e) + w_L \geq 0$. Hence, we can skip constraint (47) and obtain

$$\min_{w_L} 2w_L + \Delta w\left(\hat{e}\right) + c(e) \quad \text{subject to (46) and}$$
$$\Delta w\left(\hat{e}\right) - r(e) + w_L,\ w_L \geq 0.$$

Hence, the cost-minimizing w_L is given by

$$w_L = \max\left\{0,\ \hat{c}(\hat{e}) - \frac{1}{2}\Delta w\left(\hat{e}\right),\ r(e) - \Delta w\left(\hat{e}\right)\right\}.$$

From (7), we know that $\frac{1}{2}\Delta w\left(\hat{e}\right) - \hat{c}(\hat{e}) \geq 0$. Hence,

$$w_L = \max\left\{0,\ r(e) - \Delta w\left(\hat{e}\right)\right\}.$$

We now have to distinguish two cases. The first case is

$$w_H - w_L = \Delta w\left(\hat{e}\right) - r(e) \geq 0.$$

Then, $w_L = 0$ and $w_H = \Delta w\left(\hat{e}\right) - r(e)$. In the second case,

$$w_H - w_L = \Delta w\left(\hat{e}\right) - r(e) < 0.$$

Hence, $w_L = r(e) - \Delta w\left(\hat{e}\right)$ and $w_H = 0$. In the first case, the firm's expected labor costs are

$$2w_L + \Delta w\left(\hat{e}\right) + c(e) = \Delta w\left(\hat{e}\right) + c(e),$$

[27] Note that the optimal high bonus, $b_H = \frac{c'(e)}{p'(e)} + b_L$, is non-negative due to $b_L \geq 0$.

and in the second scenario the firm's costs amount to

$$2w_L + \Delta w\left(\hat{e}\right) + c(e) = 2r(e) - \Delta w\left(\hat{e}\right) + c(e).$$

We can now turn to the second step of the solution procedure, the solution of the firm's profit maximization problem. The optimal effort pair $(\hat{e}^c, e^c)$ solves

$$\max_{e,\hat{e}} \begin{cases} 2\hat{v}(\hat{e}) + v(e) - \Delta w\left(\hat{e}\right) - c(e) & \text{if} \quad \Delta w\left(\hat{e}\right) - r(e) \geq 0 \\ 2\hat{v}(\hat{e}) + v\left(e\right) - \left[2r(e) - \Delta w\left(\hat{e}\right) + c(e)\right] & \text{otherwise.} \end{cases}$$

We can see that in case 2 (i.e., the second line of the maximization problem) the firm's objective function is monotonically increasing in $\hat{e}$. Hence, for each e, the firm chooses the maximum possible $\hat{e}$, which makes the given restriction just binding, i.e., $\Delta w\left(\hat{e}\right) = r(e)$. This implies that case 2 becomes a special case of case 1. Thus, the firm never wants to induce effort levels $(\hat{e}, e)$ such that $\Delta w\left(\hat{e}\right) < r(e)$. Doing so would imply that $0 = w_H^c < w_L^c$. Intuitively, this means that, by implementing an adverse relative performance scheme, the firm pays for reducing first-level incentives that stem from the second-level rent $r(e)$. Such a contract cannot be optimal. The firm would be better off by setting $0 = w_H^c = w_L^c$, thereby increasing first-level effort and reducing workers' first-period rents.

Hence, we are always in the first case. Consequently, $w_L^c = 0$ and the results of the proposition follow.

7.2 Proof of Proposition 3

(i) $\hat{e}^c = \hat{e}^s$ immediately follows from examining the objective functions (14) and (24). $e^c > e^s$ follows from $r'(e) > 0$, which we have proven in Lemma 1, and $r''(e) > 0$, which follows from our regularity assumptions and is straightforward to check.[28]

It remains to prove result (ii). Due to the binding restriction, we can

[28] See the additional pages for the referees.

consider e as an implicitly defined function of e, i.e., $e(\hat{e})$ with

$$\frac{\partial e}{\partial \hat{e}} = \frac{\Delta w'(\hat{e})}{r'(e)} > 0.$$

Moreover, the firm's objective function (24) becomes

$$2\hat{v}(\hat{e}) + v(e(\hat{e})) - \Delta w(\hat{e}) - c(e(\hat{e})).$$

The respective first-order condition is

$$2\hat{v}'(\hat{e}) - \Delta w'(\hat{e}) + [v'(e(\hat{e})) - c'(e(\hat{e}))] \frac{\partial e}{\partial \hat{e}} = 0. \tag{49}$$

Hence, compared to the case where the restriction is non-binding, we either have higher effort at hierarchy level 1 and lower effort at level 2, or vice versa. Inserting $\partial e/\partial \hat{e}$ in (49) yields

$$2\hat{v}'(\hat{e}) + \frac{v'(e(\hat{e})) - c'(e(\hat{e})) - r'(e(\hat{e}))}{r'(e(\hat{e}))} \Delta w'(\hat{e}) = 0.$$

Recall that $\Delta w'(\hat{e}) > 0$ and $r'(e) > 0$. The optimal effort, e^c, must therefore satisfy $v'(e^c) - c'(e^c) - r'(e^c) < 0$. Under separate contracts, we have $v'(e^s) - c'(e^s) - r'(e^s) = 0$. Thus, since $v(e) - c(e) - r(e)$ is strictly concave, it follows that $e^c > e^s$.

Now consider the effort choice on hierarchy level 1 under a binding restriction (25). Suppose that the firm wants to implement the same effort level as under a non-binding restriction, i.e., $\hat{e}^s = \arg\max_{\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e})$. However, since (25) is binding in this situation, the corresponding level 2 effort is below the optimal one, e^{FB}. Of course, the firm can raise e to increase $v(e) - c(e)$, but then it has to increase $\hat{e}$ as well because of $\partial e/\partial \hat{e} > 0$. Whether such an adjustment is beneficial to the firm or not depends on the functional forms. In any case, since both functions $2\hat{v}(\hat{e}) - \Delta w(\hat{e})$ and $v(e) - c(e)$ are strictly concave, the firm will never raise e above e^{FB}. This is because, if $e > e^{FB}$ and $\hat{e} > \hat{e}^s$, the firm can increase profits by decreasing both effort levels, while keeping (25) binding. This proves $e^c < e^{FB}$.

Since $e^c < e^{FB}$ implies $v'(e^c) - c'(e^c) > 0$, from (49) we obtain that the corresponding optimal effort on hierarchy level 1 must satisfy $2\hat{v}'(\hat{e}) - \Delta w'(\hat{e}) < 0$. Thus, this effort must be larger than the optimal level 1 effort under a non-binding restriction (25). Since that effort was identical with the optimal level 1 effort under separate contracts, $\hat{e}^s$, we have $\hat{e}^c > \hat{e}^s$ under the binding restriction.

Finally, the last inequality of result (ii) directly follows from a comparison of the firm's overall net profits under the two contractual forms. Note that, under a combined contract with binding restriction (25), expected labor costs are $\Delta w(\hat{e}^c) + c(e^c) \overset{(25)}{=} r(e^c) + c(e^c)$. We obtain overall net profits under separate contracts by summing up (15) and (14).

7.3 Proof of Proposition 4

The solution procedure is analogous to the one in Proposition 2. First, we consider the firm's problem of minimizing implementation costs for a given pair of effort levels $(\hat{e}, e)$.

$$\min_{\substack{\alpha\in[0,1],\\ w_L,w_H,b_H,b_L}} \quad w_L + w_H + \alpha\left[b_L + p\left(e\right)\left(b_H - b_L\right)\right]$$
$$\text{subject to (8), (9), (27), (28), (23).}$$

By the incentive compatibility constraint (8), $b_H - b_L = \frac{c'(e)}{p'(e)}$. Thus, in combination with the incentive compatibility constraint (27), we obtain

$$w_H - w_L = \frac{\hat{c}'\left(\hat{e}\right)}{\hat{p}_1\left(\hat{e},\hat{e}\right)} - \alpha\left[b_L + p(e)\frac{c'\left(e\right)}{p'\left(e\right)} - c(e)\right] = \Delta w\left(\hat{e}\right) - \alpha\left[b_L + r(e)\right], \tag{50}$$

where $r(e)$ is given by (13) and $\Delta w\left(\hat{e}\right)$ by (4).

Using (50), the first-level participation constraint (28) boils down to

$$w_L + \frac{1}{2}\Delta w\left(\hat{e}\right) - \hat{c}(\hat{e}) \geq 0. \tag{51}$$

Furthermore, the second-level participation constraint (9) becomes

$$b_L + p(e)\frac{c'(e)}{p'(e)} - c(e) = b_L + r(e) \geq 0. \tag{52}$$

Thus, substituting for the tournament prize spread $w_H - w_L$ and the bonus spread $b_H - b_L$, the cost minimization problem can be simplified to[29]

$$\min_{\alpha\in[0,1],w_L,b_L} 2w_L + \Delta w\left(\hat{e}\right) + \alpha c(e) \quad \text{subject to (51), (52) and}$$
$$\Delta w\left(\hat{e}\right) - \alpha\left[b_L + r(e)\right] + w_L,\ w_L,\ b_L \geq 0.$$

By Lemma 1, we obtain $b_L^r = 0$ for the optimal low bonus: this satisfies the participation constraint for the second hierarchy level (52) and is also best for ensuring that $w_H = \Delta w\left(\hat{e}\right) - \alpha\left[b_L + r(e)\right] + w_L \geq 0$. Hence, we can skip constraint (52) and obtain

$$\min_{\alpha\in[0,1],w_L} 2w_L + \Delta w\left(\hat{e}\right) + \alpha c(e) \quad \text{subject to (51) and}$$
$$\Delta w\left(\hat{e}\right) - \alpha r(e) + w_L,\ w_L \geq 0.$$

The cost-minimizing w_L is thus given by

$$w_L = \max\left\{0,\ \hat{c}(\hat{e}) - \frac{1}{2}\Delta w\left(\hat{e}\right),\ \alpha r(e) - \Delta w\left(\hat{e}\right)\right\}.$$

From (7), we know that $\frac{1}{2}\Delta w\left(\hat{e}\right) - \hat{c}(\hat{e}) \geq 0$. Hence,

$$w_L = \max\left\{0,\ \alpha r(e) - \Delta w\left(\hat{e}\right)\right\}.$$

We now have to distinguish two cases. The first case is

$$w_H - w_L = \Delta w\left(\hat{e}\right) - \alpha r(e) \geq 0.$$

[29] Note that the optimal high bonus, $b_H = \frac{c'(e)}{p'(e)} + b_L$, is non-negative due to $b_L \geq 0$.

Then, $w_L = 0$ and $w_H = \Delta w\left(\hat{e}\right) - \alpha r(e)$. In the second case,

$$w_H - w_L = \Delta w\left(\hat{e}\right) - \alpha r(e) < 0.$$

Hence, $w_L = \alpha r(e) - \Delta w\left(\hat{e}\right)$ and $w_H = 0$. In the first case, the firm's expected labor costs are

$$2w_L + \Delta w\left(\hat{e}\right) + \alpha c(e) = \Delta w\left(\hat{e}\right) + \alpha c(e),$$

and in the second scenario the firm's costs amount to

$$2w_L + \Delta w\left(\hat{e}\right) + \alpha c(e) = 2\alpha r(e) - \Delta w\left(\hat{e}\right) + \alpha c(e).$$

We can now turn to the second step of the solution procedure, the solution of the firm's profit maximization problem. The optimal combination $(\alpha^r, \hat{e}^r, e^r)$ solves

$$\max_{\alpha\in[0,1],e,\hat{e}} \begin{cases} 2\hat{v}(\hat{e}) + \alpha v(e) - \Delta w\left(\hat{e}\right) - \alpha c(e) + (1-\alpha)E & \text{if } \Delta w\left(\hat{e}\right) - \alpha r(e) \geq 0 \\ 2\hat{v}(\hat{e}) + \alpha v\left(e\right) - \left[2\alpha r(e) - \Delta w\left(\hat{e}\right) + \alpha c(e)\right] + (1-\alpha)E & \text{otherwise.} \end{cases}$$

With the same argumentation as in the proof of Proposition 2, it follows that the firm will never implement effort levels $(\hat{e}, e)$ such that $\Delta w\left(\hat{e}\right) - \alpha r(e) < 0$. Thus, $w_L^r = 0$ and the firm's optimization problem is

$$\max_{\alpha\in[0,1],e,\hat{e}} 2\hat{v}(\hat{e}) - \Delta w\left(\hat{e}\right) + \alpha[v(e) - c(e) - E] + E \quad \text{s.t.} \quad \Delta w\left(\hat{e}\right) - \alpha r(e) \geq 0. \tag{53}$$

First, assume that the restriction is not binding at the optimal solution. Then, we have $\hat{e}^r = \hat{e}^s$, $e^r = e^{FB}$. Since $v(e^{FB}) - c(e^{FB}) - E > 0$, it follows that $\alpha^r = 1$. Hence, this case occurs if and only if $\Delta w\left(\hat{e}^s\right) \geq r(e^{FB})$, and the solution is then identical to the optimal combined contract specified in Proposition 2.

For the remainder of this proof, assume the constraint is binding at the optimal solution, i.e. $\alpha = \frac{\Delta w(\hat{e})}{r(e)}$. We first show that $\alpha^r > 0$, which is

equivalent to $\Delta w(\hat{e}) > 0$ or $\hat{e} > 0$. To do so, we simplify problem (53) to

$$\begin{aligned} &\max_{e,\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e}) + \frac{\Delta w(\hat{e})}{r(e)}[v(e) - c(e) - E] + E \quad \text{s.t.} \quad \frac{\Delta w(\hat{e})}{r(e)} \leq 1 \\ \Leftrightarrow &\max_{e,\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e}) + \frac{\Delta w(\hat{e})}{r(e)}[v(e) - c(e) - E] + E \quad \text{s.t.} \quad \Delta w(\hat{e}) - r(e) \leq 0. \end{aligned} \tag{54}$$

If the restriction in (54) is binding, then $\alpha^r = 1$ and we are back to the case of combined contracts without external recruitment. Now assume that the restriction in (54) is not binding. The first derivative of the objective function with respect to $\hat{e}$ is

$$2\hat{v}'(\hat{e}) - \Delta w'(\hat{e}) \left[1 - \frac{v(e) - c(e) - E}{r(e)}\right].$$

Thus, at $\hat{e} = 0$, because $\Delta w'(0) = 0$, the objective function is increasing in $\hat{e}$. Furthermore, since the restriction is not binding, it is feasible to increase $\hat{e}$. Therefore, $\hat{e} = 0$ cannot be optimal. From $\hat{e} > 0$, it then follows that $\alpha^r = \frac{\Delta w(\hat{e})}{r(e)} > 0$.

Now consider the case $\alpha^r < 1$. By the constraint in (54), we then have $\Delta w(\hat{e}^r) < r(e^r)$. We now derive a sufficient condition for $\alpha^r < 1$. To do so, consider again problem (53). Due to the binding constraint $\Delta w(\hat{e}) = \alpha r(e)$ we can rewrite the firm's objective function as

$$\begin{aligned} &2\hat{v}(\hat{e}) - \alpha r(e) + \alpha[v(e) - c(e) - E] + E \\ &= 2\hat{v}(\hat{e}) + \alpha[v(e) - c(e) - r(e) - E] + E \\ &= 2\hat{v}(\hat{e}) - \alpha[V(e^s) - V(e)] + V(e^s) \end{aligned}$$

with $V(\cdot) := v(\cdot) - c(\cdot) - r(\cdot)$ being strictly concave with maximum at e^s. Furthermore, because of the binding constraint we can write e as a function of α with

$$\frac{\partial e}{\partial \alpha} = -\frac{r(e)}{\alpha r'(e)} < 0.$$

Problem (53) can then be restated as

$$\max_{\alpha\in[0,1],\hat{e}} 2\hat{v}(\hat{e}) - \alpha[V(e^s) - V(e(\alpha))] + V(e^s).$$

We will have an interior solution $\alpha^r < 1$ if and only if marginally decreasing α at $\alpha = 1$ raises firm's profits, that is

$$\frac{\partial}{\partial\alpha}\left[2\hat{v}(\hat{e}) - \alpha[V(e^s) - V(e(\alpha))] + V(e^s)\right]\Big|_{\alpha=1} < 0.$$

Using that $\alpha = 1$ corresponds to the solution under a combined contract ($\hat{e} = \hat{e}^c$, $e = e^c$), we obtain[30]

$$\begin{aligned}
&\frac{\partial}{\partial\alpha}\left[2\hat{v}(\hat{e}) - \alpha[V(e^s) - V(e(\alpha))] + V(e^s)\right]\Big|_{\alpha=1} \\
&= -[V(e^s) - V(e)] - \alpha\left[V'(e)\frac{r(e)}{\alpha r'(e)}\right]\Big|_{\alpha=1} \\
&= -[V(e^s) - V(e^c)] - V'(e^c)\frac{r(e^c)}{r'(e^c)} \\
&= -\left(V(e^s) - V(e^c) + V'(e^c)\frac{r(e^c)}{r'(e^c)}\right).
\end{aligned}$$

Hence, we will have an interior solution $\alpha^r < 1$ if

$$V(e^s) - V(e^c) + V'(e^c)\frac{r(e^c)}{r'(e^c)} > 0. \tag{55}$$

Since e^s maximizes $V(e)$, it holds that $V(e^s) > V(e^c)$. However, due to $e^c > e^s$, we also have $V'(e^c) < 0$. Thus, whether (55) is satisfied or not depends on the specific functional forms. Resubstitution for $V(\cdot)$ yields

$$E + [v'(e^c) - c'(e^c)]\frac{r(e^c)}{r'(e^c)} > [v(e^c) - c(e^c)].$$

We now show that $\alpha^r < 1$ may indeed incur. This is the case if the constraint (54) is non-binding at the optimal efforts $(e^r, \hat{e}^r)$. First note that

[30] Using $\hat{e}$ as a function of α yields the same condition.

e^r is then independent of $\hat{e}^r$ and given by

$$e^r = \arg\max_e \frac{v(e) - c(e) - E}{r(e)}.$$

Denote $M := \frac{v(e^r)-c(e^r)-E}{r(e^r)}$. Then,

$$\hat{e}^r = \arg\max_e 2\hat{v}(\hat{e}) - (1-M)\Delta w\left(\hat{e}\right).$$

Assume that $M < 1$, i.e., we have an interior solution with $\hat{e}^r > 0$. For example, consider $\hat{v}(\hat{e}) = a\hat{e}$, $a > 0$, $\hat{c}(\hat{e}) = \hat{e}^2/2$ and $\hat{p}(\hat{e}_A, \hat{e}_B) = \frac{\hat{e}_A}{\hat{e}_A+\hat{e}_B}$. The parameter a does not appear in any other function. Then, we obtain

$$\begin{aligned}\hat{e}^r &= \arg\max_e 2a\hat{e} - (1-M)4\hat{e}^2 \\ \Leftrightarrow \hat{e}^r &= \frac{a}{4(1-M)} \Leftrightarrow \Delta w(\hat{e}^r) = \frac{a^2}{4(1-M)^2}.\end{aligned}$$

We then have $\alpha^r < 1$ if $\Delta w(\hat{e}^r) = \frac{a^2}{4(1-M)^2} < r(e^r)$. Since M and $r(e^r)$ are independent of a, this inequality is satisfied if a is sufficiently small.

Furthermore, from the binding constraint in (53), we obtain

$$\frac{\partial e}{\partial \alpha} = -\frac{r\left(e\right)}{\alpha r'\left(e\right)} < 0 \quad \text{and} \quad \frac{\partial \hat{e}}{\partial \alpha} = \frac{r\left(e\right)}{\Delta w'\left(\hat{e}\right)} > 0.$$

As a result, since we must have $\alpha \leq 1$ and $\alpha = 1$ corresponds to the combined contract (with a binding constraint (25)), we have $e^r \geq e^c$ and $\hat{e}^r \leq \hat{e}^c$. It remains to show that $\hat{e}^s < \hat{e}^r$ and $e^r < e^{FB}$. To do so, we denote by μ the Lagrange multiplier for the constraint in (53) and assume that the constraint is binding, i.e., $\mu > 0$. Then, the first-order conditions w.r.t. $\hat{e}$ and e are

$$2\hat{v}'(\hat{e}^r) - (1-\mu)\Delta w'\left(\hat{e}^r\right) = 0 \tag{56}$$

$$\alpha[v'(e^r) - c'(e^r) - \mu r'(e^r)] = 0. \tag{57}$$

Since $\mu > 0$, we obtain from the second condition that $e^r < e^{FB}$. Combining

both equations yields

$$\mu = \frac{2\hat{v}'(\hat{e}^r) - \Delta w'\left(\hat{e}^r\right)}{-\Delta w'\left(\hat{e}^r\right)} = \frac{v'(e^r) - c'(e^r)}{r'(e^r)} > 0. \tag{58}$$

Hence, it must hold that $2\hat{v}'(\hat{e}^r) - \Delta w'\left(\hat{e}^r\right) < 0$ and, therefore, $\hat{e}^r > \hat{e}^s$.

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8 Appendix for Referees

8.1 Separate Contracts with Homogeneous Workers

Second-order condition for the firm's objective function on the first hierarchy level, $\hat{v}(\hat{e}) - \Delta w(\hat{e})$.

$$\Delta w(\hat{e}) = \frac{\hat{c}'}{\hat{p}_1}$$
$$\Delta w'(\hat{e}) = \frac{\hat{c}''\hat{p}_1 - \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^2} = \frac{\hat{c}''}{\hat{p}_1} - \frac{\frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^2}$$
$$\Delta w''(\hat{e}) = \frac{\hat{c}'''\hat{p}_1 - \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}''}{[\hat{p}_1]^2} - \frac{\left[\frac{\partial^2 \hat{p}_1}{\partial \hat{e}^2}\hat{c}' + \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}''\right][\hat{p}_1]^2 - 2\hat{p}_1\frac{\partial \hat{p}_1}{\partial \hat{e}}\frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^4} > 0$$

The last inequality follows since $\hat{c}''' \geq 0$, $\frac{\partial \hat{p}_1}{\partial \hat{e}} < 0$, $\frac{\partial^2 \hat{p}_1}{\partial \hat{e}^2} \leq 0$.

Second-order condition for the firm's objective function on the second hierarchy level, $v(e) - r(e) - c(e)$.

$$r(e) = p\frac{c'}{p'} - c$$
$$r'(e) = \frac{c''p' - p''c'}{[p']^2} = \frac{c''}{p'} - \frac{p''c'}{[p']^2}$$
$$r''(e) = \frac{c'''p' - p''c''}{[p']^2} - \frac{[p'''c' + p''c'']\,[p']^2 - 2p'p''p''c'}{[p']^4} > 0.$$

The last inequality follows since $c''' \geq 0$, $p'' < 0$, $p''' \leq 0$.

8.2 Combined Contract with Heterogeneous Workers

Step 1: Minimizing costs

Since $b_H \geq 0$ is ensured by the incentive constraint for hierarchy level 2 in combination with $b_L \geq 0$ the problem of minimizing implementation costs

reduces to

$$\begin{gathered}\min_{w_L,w_H,b_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c\,(e) \\ \text{subject to} \;\; b_L + r\,(e) \geq 0 \\ w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}\,(\hat{e}) \geq 0 \\ w_H - w_L + b_L + r\,(e) = \Delta\tilde{w}(\hat{e}) \\ w_H, w_L, b_L \geq 0.\end{gathered}$$

Replacing w_H yields:

$$\begin{gathered}\min_{w_L,b_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c\,(e) \\ \text{subject to} \;\; b_L + r\,(e) \geq 0 \\ w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}\,(\hat{e}) \geq 0 \\ \Delta\tilde{w}(\hat{e}) - b_L - r\,(e) + w_L, \;\; w_L, \;\; b_L \geq 0.\end{gathered}$$

From Lemma 1 we know that $r\,(e) \geq 0$ so that $b_L^c = 0$ and the minimization problem further reduces to

$$\begin{gathered}\min_{w_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c\,(e) \\ \text{s.t.} \;\; w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}\,(\hat{e}) \geq 0 \\ \Delta\tilde{w}(\hat{e}) - r\,(e) + w_L, \;\; w_L \geq 0.\end{gathered}$$

Hence,

$$w_L = \max\left\{0, \;\; \hat{c}\,(\hat{e}) - \frac{1}{2}\Delta\tilde{w}(\hat{e}), \;\; r\,(e) - \Delta\tilde{w}(\hat{e})\right\}.$$

We know that $\frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}\,(\hat{e}) \geq 0$; otherwise, $\hat{e}$ would not be an equilibrium strategy. Thus,

$$w_L = \max\left\{0, \;\; r\,(e) - \Delta\tilde{w}(\hat{e})\right\}.$$

We have to distinguish two cases. First, $w_H - w_L = \Delta\tilde{w}(\hat{e}) - r(e) \geq 0$. Then,

$$w_L = 0 \quad \text{and} \quad w_H = \Delta\tilde{w}(\hat{e}) - r(e).$$

Second, $w_H - w_L = \Delta\tilde{w}(\hat{e}) - r(e) < 0$. Then,

$$w_L = r(e) - \Delta\tilde{w}(\hat{e}) \quad \text{and} \quad w_H = 0.$$

In the first case, the firm's expected labor costs are

$$\Delta\tilde{w}(\hat{e}) + 2w_L + c(e) = \Delta\tilde{w}(\hat{e}) + c(e)$$

and in the second they amount to

$$\Delta\tilde{w}(\hat{e}) + 2w_L + c(e) = 2r(e) - \Delta\tilde{w}(\hat{e}) + c(e).$$

Step 2: Maximizing expected profits
Therefore the optimal effort pair $(\hat{e}^c, e^c)$ solves

$$\max_{\hat{e},e} \begin{cases} 2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - \Delta\tilde{w}(\hat{e}) - c(e) & \text{if } \Delta\tilde{w}(\hat{e}) - r(e) \geq 0 \\ 2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - 2r(e) + \Delta\tilde{w}(\hat{e}) - c(e) & \text{otherwise.} \end{cases}$$

In analogy to the basic model, again the firm's objective function in the second line is monotonically increasing in $\hat{e}$ (recall that $\partial E[t|\hat{s}]/\partial\hat{e} > 0$ according to (41)). Hence, for each e the firm chooses the maximum possible $\hat{e}$ that makes the given restriction just bind so that the second line becomes a special case of the problem in line 1. The firm chooses $w_L^c = 0$ and implements the effort pair $(\hat{e}_h^c, e_h^c)$ with

$$(\hat{e}_h^c, e_h^c) \in \arg\max_{\hat{e},e} \{2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - \Delta\tilde{w}(\hat{e}) - c(e)\}$$
$$\text{subject to } \Delta\tilde{w}(\hat{e}) - r(e) \geq 0.$$