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ABSTRACT

Technical Change, Task Allocation, and Labor Unions*

We propose a novel framework that integrates the "task approach" for a more precise production modeling into the search-and-matching model with low- and high-skilled workers, and wage setting by labor unions. We establish the relationship between task reallocation and changes in wage pressure, and examine how skill- biased technical change (SBTC) affects the task composition, wages of both skill groups, and unemployment. In contrast to the canonical model with a fixed task allocation, low-skilled workers may be harmed in terms of either lower wages or higher unemployment depending on the relative task-related productivity profile of both worker types. We calibrate the model to the US and German data for the periods 1995-2005 and 2010-2017. The simulated effects of SBTC on low-skilled unemployment are largely consistent with observed developments. For example, US low-skilled unemployment increases due to SBTC in the earlier period and decreases after 2010.

JEL Classification: J64, J51, E23, E24, O33

Keywords: task approach, search and matching, labor unions, skill-biased technical change, labor demand, wage setting

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1 Introduction

In conventional production theory the production process is usually considered to be a black box that is left largely unspecified – except for some assumptions regarding marginal products of production factors, returns to scale, and the elasticity of substitution. For many economic applications such a coarse modeling of production has proven to be sufficient. However, when it comes to a discussion of technical change, the conventional approach may make us blind to some negative implications on labor market outcomes.

To get a fuller picture of the consequences of technical change, it has been suggested to interpret the production process as a set of tasks that are combined to produce output. The tasks are then assigned to production factors based on the principles of comparative advantage and the theory of optimum assignment; see Autor (2013) for an overview. Applying the task approach to production modeling allows to explain how skill-biased technical change (SBTC) leads to task reallocation between low- and high-skilled workers. Acemoglu and Autor (2011), Acemoglu and Restrepo (2018a, 2018b, 2021), and Hémous and Olsen (2022) develop task-based approaches for the analysis of SBTC and automation. Assuming perfect competition in the labor market, these papers focus on the impact of technical change on wages and the labor share but do not consider the effects on unemployment.[1]

This paper contributes to the literature by proposing a new modeling framework for the analysis of SBTC that combines the task approach, wage setting by labor unions, as well as search and matching frictions.[2] Our model involves two important channels through which SBTC and the resulting task reallocation affect the labor market, and which are absent in task-based perfect competition models. First, in a world with matching frictions, firms intending to change their task composition will quite likely have to adjust their posted vacancies, thereby affecting aggregate labor market tightness. This in turn affects the outside option of labor unions and the extent of wage pressure in the economy as well as labor market flows. Second, changes in task assignments trigger changes in the

[1] The task-based approach has also been used in the literature on the labor-market effects of offshoring, see, e.g., Grossman and Rossi-Hansberg (2008).

[2] Some papers already combine search and matching frictions with labor union wage setting for other research questions, albeit without a task-based production modeling, see, e.g., Delacroix (2006), Krusell and Rudanko (2016), and Morin (2017).

wage-setting behavior of labor unions through the effect on the wage elasticity of labor demand. This channel has not been explored in the literature so far and deserves more attention since it decides to a large extent on whether low-skilled workers are ultimately harmed by SBTC.

Whereas it is well known that wage claims of labor unions negatively depend on the labor demand elasticity, the relationship between labor demand elasticity and task allocation is less obvious. In general equilibrium models with union wage setting the analysis is often simplified by assuming a Cobb-Douglas production function because it leads to a constant labor demand elasticity and a constant union wage markup; see, e.g., Layard et al. (1991). However, we show that even with a simple Cobb-Douglas structure at the task level, the labor demand elasticity is in general not constant but depends on the task threshold that divides the range of tasks assigned to low-skilled and high-skilled workers. How changes in the task threshold affect the wage elasticity of labor demand, is determined by the shape of the relative task productivity schedule that describes the comparative advantage of the two skill groups in performing the various tasks. Since we allow for both, concave and convex shapes of the relative task productivity schedule, the impact of changes in the task threshold on the labor demand elasticity and union wage setting is in general ambiguous and can be explained by different degrees of substitutability of high- and low-skilled workers.

Another distinctive feature of our model is that, in contrast to the canonical model, SBTC can even harm low-skilled workers either in terms of higher unemployment or lower real wages. To establish the general equilibrium effects of SBTC, in analogy to standard search-and-matching models, we derive from the model equations a two-equations system describing the job creation of firms and the wage-setting behavior of labor unions. Whereas the job creation curve has the usual negative slope, the wage curve can be either upward or downward sloping. In the case of a positively sloped wage curve low-skilled workers benefit from an increase in the productivity of high-skilled workers because of higher employment as well as higher real wages. However, with a downward-sloping wage curve low-skilled workers may be harmed either by an increase in unemployment or a reduction of real wages. The key factor governing the slope of the wage curve is the response of the labor demand elasticity to changes in the task threshold. The slope gets negative if the labor demand elasticity increases with increasing task threshold and this reaction is strong thus implying strong upward wage pressure if the task threshold decreases. This discussion highlights that the SBTC channel working through task allocation to wage setting of labor unions is relevant *(i)* per se as it could provide a technological explanation for different degrees of wage rigidity across sectors and countries as reflected in different slopes of the wage curve, as well as *(ii)* because of the consequences of SBTC for the labor market outcomes for low-skilled workers.

To demonstrate the applicability of our framework, we calibrate the model to the data of the US and Germany for two time periods: 1995 to 2005 and 2010 to 2017. The simulated effects of SBTC on unemployment and the skill premium are largely consistent with observed developments. For example, in the first period Germany experiences a stronger increase in low-skilled unemployment than the US, while the US experiences a stronger increase in the skill premium. In the second period the impact of SBTC on employment reverses its sign in the US, which is in line with the observed decline in US low-skilled unemployment. For Germany the model predicts a further increase in low-skilled unemployment in the second period due to SBTC. However, this negative effect is overcompensated by a reduction in unemployment benefits in the course of the Hartz IV reform in 2005 which helped to increase low-skilled employment in Germany.

The rest of the paper is organized as follows: Section 2 presents the general model and discusses the implications of changes in the task threshold on the wage elasticity of labor demand for low-skilled workers. Section 3 performs a comparative-static analysis of the labor market consequences of SBTC. Section 4 calibrates the model to data of the US and Germany. Section 5 contains a summary of the results and some conclusions.

2 The Model

2.1 Firms

There is a mass one of identical firms in the economy. Timing is discrete and will be explained in more details below. At the end of period t the representative firm produces the final good Y_t by using the services of a continuum of tasks $y_t(i)$, measured on the unit interval, according to the Cobb-Douglas-function

$$Y_t = \exp\left[\int_0^1 \ln y_t(i)\mathrm{d}i\right]. \tag{1}$$

The firm assigns L_t low-skilled workers and H_t high-skilled workers to the different tasks according to the task-specific production function

$$y_t(i) = A_{Lt}\,\alpha_L(i)\,l_t(i) + A_{Ht}\,\alpha_H(i)\,h_t(i), \tag{2}$$

where $l_t(i)$ and $h_t(i)$ denote the low-skilled and high-skilled labor input assigned to task i in period t, respectively, and

$$L_t = \int_0^1 l_t(i)\mathrm{d}i \qquad \text{and} \qquad H_t = \int_0^1 h_t(i)\mathrm{d}i. \tag{3}$$

A_{Lt} and A_{Ht} denote factor-augmenting technology, whereas the functions $\alpha_H(i)$ and $\alpha_L(i)$ describe the productivity of high- and low-skilled workers in task i, respectively. It is assumed that the ratio $\alpha_H(i)/\alpha_L(i)$ of these productivities, hereafter also referred to as the relative task productivity schedule, is continuously differentiable and strictly increasing, implying that the comparative advantage of high-skilled (low-skilled) workers in performing the different tasks is increasing (decreasing) in the task index i.

The goods market and the labor market for high-skilled workers are competitive, hence high-skilled workers are always fully employed. In contrast, the low-skilled labor market is characterized by matching frictions and monopoly unions at the firm level. The matching frictions are described by the linear homogeneous matching function $M_{Lt} = M(V_{Lt}, U_{Lt})$, where V_{Lt} denotes vacant jobs for low-skilled workers and U_{Lt} the low-skilled unemployed persons.

The timing is as follows. At the beginning of period t there are L_{t-1} employed low-skilled workers and U_{Lt} unemployed workers. The total labor force is normalized to one, hence $U_{Lt} = 1 - L_{t-1} - H_{t-1}$. The representative labor union chooses a wage w_{Lt} anticipating that the respective firm may adjust the employment level by posting vacancies accordingly. This timing contrasts with that used in standard search and matching models but is in line with studies incorporating trade unions into the search and matching framework, such as Delacroix (2006) and Morin (2017). For simplicity, the inflow of unemployed workers into jobs and exogenous job separations happen simultaneously in such a way that a further change of a worker's employment/unemployment status within the same period is not possible. At the end of the period, production takes place as outlined above.

If the representative firm wants to increase the number of low-skilled workers it has to post vacant jobs first and bear the (constant) search costs s_L for each vacant job. With rate $M_{Lt}/V_{Lt} \equiv m(\theta_{Lt})$ job vacancies are filled, where $\theta_{Lt} \equiv V_{Lt}/U_{Lt}$ describes labor market tightness in the low-skilled labor market in period t. The single firm considers labor market tightness and thus the job filling rate as given. With the exogenous rate q_L low skilled jobs are destroyed. The dynamics for low-skilled employment is therefore described by

$$L_t = (1 - q_L)L_{t-1} + m(\theta_{Lt})V_{Lt}. \tag{4}$$

To simplify the analysis we follow Pissarides (2000, p. 68) in assuming that each firm is large enough to eliminate all uncertainty about the flow of labor. Moreover, the final good is chosen as the numeraire. The representative firm maximizes profits

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} [Y_t - w_{Lt}L_t - w_{Ht}H_t - s_L V_{Lt}] \tag{5}$$

s.t. eqs. (1) – (4) and the conditions

$$l_t(i) \geq 0 \qquad \text{and} \qquad h_t(i) \geq 0, \tag{6}$$

where r is the constant real interest rate, and $l_0(i)$ and $h_0(i)$ are given. There are no productivity differences within the group of high- or low-skilled workers. Due to perfect competition in the high-skilled labor market and the fact that no high-skilled worker would supply labor to tasks paying lower wages, all high-skilled workers obtain the same real wage w_{Ht}. Similarly, the representative labor union sets a uniform real wage w_{Lt} for low-skilled workers.

The first-order conditions of this optimization problem are derived in Appendix A.1. The analysis focuses on the steady state in which $A_{Lt} = A_L$, $A_{Ht} = A_H$, $L_{t-1} = L_t = L$ and the time index on all variables can be omitted. Similar to the perfect competition model of Acemoglu and Autor (2011), there exists a task threshold $0 < I < 1$ such that unit labor costs of low-skilled workers are equal to those of high-skilled workers at I:

$$\frac{\widetilde{w}_L}{A_L \alpha_L(I)} = \frac{w_H}{A_H \alpha_H(I)}, \qquad \text{with} \quad \widetilde{w}_L \equiv w_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}. \tag{7}$$

The modified wage $\widetilde{w}_L$ represents the low-skilled labor costs relevant to the representative firm, i.e. the low-skilled real wage w_L plus the labor adjustment costs. In the tasks $i < I$ only low-skilled workers are employed, i.e. $h(i) = 0$, whereas in tasks $i > I$ only high-skilled workers are employed, i.e. $l(i) = 0$. Eq. (7) can be written as:

$$\bar{\alpha}(I) = \frac{\widetilde{\omega}}{\bar{A}}, \quad \text{with} \quad \bar{\alpha}(I) \equiv \frac{\alpha_H(I)}{\alpha_L(I)}, \quad \widetilde{\omega} \equiv \frac{w_H}{\widetilde{w}_L}, \quad \text{and} \quad \bar{A} \equiv \frac{A_H}{A_L}. \tag{8}$$

This leads to:

$$I = I\left(\widetilde{\omega}, \bar{A}\right), \qquad \text{with} \qquad \frac{\partial I}{\partial \widetilde{\omega}} > 0 \quad \text{and} \quad \frac{\partial I}{\partial \bar{A}} < 0. \tag{9}$$

As shown in Appendix A.1, from the first-order conditions it follows that $\widetilde{w}_L l(i) = Y = w_H h(i)$. This has two important implications. First, the same labor input is used in all low-skilled and high-skilled tasks, respectively, i.e. $l(i) = l = L/I$ for $i < I$ and $h(i) = h = H/(1-I)$ for $i > I$. Second, it holds that $I = \widetilde{w}_L L/Y$ and $1 - I = w_H H/Y$ so that I represents the modified labor share of the low-skilled workers (as it refers to labor costs $\widetilde{w}_L$ and not w_L), and $1 - I$ corresponds to the high-skilled labor share. It follows that:

$$L = \frac{I}{1-I} \widetilde{\omega} H. \tag{10}$$

Wage changes have two effects on L: a direct effect at a given threshold I and an indirect effect due to a change of this threshold. Eq. (10) in combination with eq. (9) can be interpreted as the labor demand function for low-skilled workers for given H:

$$L = L\left(\widetilde{\omega}, I\left(\widetilde{\omega}, \bar{A}\right), H\right) \equiv L^d\left(\widetilde{\omega}, \bar{A}, H\right). \tag{11}$$

Moreover, taking into account the optimality conditions, the production function for the final good takes the following Cobb-Douglas form:

$$\begin{aligned} Y &= B\left(A_L \frac{L}{I}\right)^I \left(A_H \frac{H}{1-I}\right)^{1-I}, \\ B &\equiv e^{\xi(I)}, \quad \xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i. \end{aligned} \tag{12}$$

2.2 Labor Unions for Low-Skilled Workers

With the timing assumption outlined in the last section, the present discounted utility of a low-skilled worker being employed at the end of period t is

$$\Psi_{EL,t} = w_{Lt} + \frac{1}{1+r}\left[q_L\, \Psi_{UL,t+1} + (1-q_L)\, \Psi_{EL,t+1}\right], \tag{13}$$

where $\Psi_{UL,t+1}$ denotes the present discounted utility of a low-skilled worker being unemployed at the end of period $t+1$. The job separation rate q_L refers to period $t+1$ but since it is assumed to be constant the time index is omitted. A low-skilled worker being unemployed at the end of period t has the present discounted utility

$$\Psi_{UL,t} = z_{Lt} + \frac{1}{1+r}\left[p_{L,t+1}\, \Psi_{EL,t+1} + (1-p_{L,t+1})\, \Psi_{UL,t+1}\right], \tag{14}$$

where z_{Lt} denotes net unemployment benefits and p_{Lt} is the exit rate out of unemployment which positively depends on labor market tightness, i.e. $p_{Lt} \equiv M_{Lt}/U_{Lt} = \theta_{Lt} m(\theta_{Lt})$. As we are not interested in the implications of different tax systems on the wage-setting process, we assume for simplicity that unemployment benefits are financed by lump-sum taxes.

The low-skilled wage w_{Lt} is determined by firm-level monopoly unions.[3] Similar to

[3]Assuming Nash bargaining between firms and unions would not change the qualitative results. We choose the present setup as it is our intention to explain the mechanisms of the model in the simplest possible way.

Manning (1991) we assume as a starting point that the single union sets the wage for n periods. The union thereby considers the aggregate labor market tightness to be given and constant which is consistent with an analysis of steady-state wage pressure. If the wage w_{Lt} is set in period t for n periods, it will affect the utility difference $\Psi_{EL} - \Psi_{UL}$ from period t until period $t+n-1$, but not from period $t+n$ onwards. Running forward $\Psi_{EL,t} - \Psi_{UL,t}$ for n periods leads to

$$\Psi_{EL,t} - \Psi_{UL,t} = \left(\frac{1-\delta^n}{1-\delta}\right)(w_{Lt} - z_{Lt}) + \delta^n(\Psi_{EL,t+n} - \Psi_{UL,t+n}), \tag{15}$$

where $\delta \equiv (1 - q_L - p_L)/(1 + r) < 1$. The representative labor union at the firm level maximizes the rent of the employed low-skilled workers:

$$\max_{w_{Lt}} (\Psi_{EL,t} - \Psi_{UL,t}) \, L_t \tag{16}$$

subject to the labor demand equation. In line with, among others, Pissarides (1985), Layard and Nickell (1990), and Beissinger and Egger (2004), we restrict our steady state analysis to the case $n \to \infty$. Omitting time indices for steady-state values, the rent-maximizing wage w_L implies the following wage costs $\widetilde{w}_L$ relevant to the firm (see Appendix A.2):

$$\widetilde{w}_L = \kappa_L \, \widetilde{z}_L, \quad \text{with} \quad \kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1} \quad \text{and } \widetilde{z}_L \equiv z_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}, \tag{17}$$

where κ_L denotes the wage markup on $\widetilde{z}_L$ consisting of unemployment benefits and labor adjustment costs. The wage markup is negatively related to the wage elasticity of the demand for low-skilled labor (in absolute values), $\varepsilon_{L,\widetilde{w}_L}$. The next subsection takes a closer look at this elasticity and shows that $\varepsilon_{L,\widetilde{w}_L} > 1$, implying $\kappa_L > 1$.

2.3 Task Reallocation and the Elasticity of Labor Demand

The wage elasticity of the demand for low-skilled labor (in absolute values) can be written as

$$\varepsilon_{L,\widetilde{w}_L} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_L} \right| = 1 + \frac{1}{1-I} \frac{\partial \ln I}{\partial \ln \widetilde{\omega}} = 1 + \frac{1}{(1-I) \cdot \varepsilon_{\bar{\alpha},I}(I)} > 1, \tag{18}$$

where

$$\varepsilon_{\bar{\alpha},I}(I) \equiv \frac{\mathrm{d} \ln \bar{\alpha}(I)}{\mathrm{d} \ln I} > 0$$

denotes the elasticity of the relative task productivity schedule at the task threshold with respect to a one-percent change in I.

The wage elasticity of the demand for low-skilled labor is the sum of a direct wage effect (equal to one) for a given task allocation, and a task reallocation effect caused by the change in the task threshold I due to a change in relative labor costs $\widetilde{\omega}$. The task reallocation effect implies that with an increase in $\widetilde{w}_L$ fewer tasks are allocated to low-skilled labor. The *strength* of this effect depends on the task threshold I in two ways. The more tasks are allocated to low-skilled labor the larger is $1/(1-I)$ which *cet. par.* increases the task reallocation effect and thereby $\varepsilon_{L,\widetilde{w}_L}$. However, the size of the task reallocation effect also negatively depends on $\varepsilon_{\bar{\alpha},I}$. In general, $\varepsilon_{\bar{\alpha},I}$ is a function of I with the sign of $\mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I$ depending on the functional form of $\bar{\alpha}(I)$, i.e. $\mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I \lesseqgtr 0$ is possible.[4] This leads to

Proposition 1. *An increase in the task threshold I leads to the following change in the wage elasticity of the demand for low-skilled labor:*

$$\frac{\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d} \ln I} \begin{cases} > 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I < I/(1-I) \\ = 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I = I/(1-I) \\ < 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I > I/(1-I). \end{cases}$$

[4] As shown in Appendix A.3, the second-order condition for the optimization problem of the representative labor union puts a restriction on $\mathrm{d} \ln \bar{\alpha} / \mathrm{d} \ln I$ equivalent to $\bar{\alpha}''(I)\, I / \bar{\alpha}'(I) > -2$ so that $\bar{\alpha}(I)$ must not be "too concave".

Proof. Taking into account eq. (18) and the definition of κ_L in eq. (17), $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ can be written as:

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{19}$$

Since $\kappa_L > 0$, Proposition 1 immediately follows from eq. (19). □

Since the wage markup κ_L is negatively related to $\varepsilon_{L,\widetilde{w}_L}$, Proposition 1 can be directly applied to establish the effect of the threshold I on κ_L:

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}, \quad \text{with} \quad \mathrm{sgn}(\varepsilon_{\kappa_L,I}) = -\mathrm{sgn}\left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right). \tag{20}$$

The important insight from this analysis is that changes in the task allocation have an impact on the effective wage-setting power of labor unions. The way in which a change in the task allocation affects the labor demand elasticity and thus the labor union's wage markup crucially depends on the shape of the relative task productivity schedule.

To provide some intuition, consider, for example, a negative $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$. This implies a concave shape of $\bar{\alpha}(I)$ suggesting that at lower thresholds I the low- and high-skilled workers are more apart in term of their productivities than at higher I. At a low I, a reduction of the task threshold leads to a pronounced drop in the relative high-skilled productivity $\bar{\alpha}(I)$, thereby making the substitution of low-skilled by those with higher skills harder. Therefore, an increase in $\widetilde{w}_L$ requires only a small decline in I to induce such an increase in the relative productivity of low-skilled workers $1/\bar{\alpha}(I)$ that the unit labor costs of both skill groups are equal again in optimum. As a result, labor demand decreases only slightly with increasing $\widetilde{w}_L$ at a low I. In contrast, with low-skilled workers being more substitutable at a high I, the response of labor demand to increasing $\widetilde{w}_L$ is more pronounced. This explains why a concave $\bar{\alpha}(I)$ function leads to a rise in the labor demand elasticity with an increase in the task threshold I. The opposite applies if $\bar{\alpha}(I)$ is convex and $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$ is sufficiently large. These results can be relevant when comparing the outcomes of the wage-setting process in different sectors of the economy. Some sectors may encompass a range of tasks which rapidly increase in their complexity so that the relative productivity of high-skilled workers increases in a more exponential manner. Other sectors may display a task complexity profile that is prone to stronger substitution of different skills.

One special case of Proposition 1 is especially worth mentioning – the case of a constant wage elasticity of labor demand. The corresponding functional form of $\bar{\alpha}(I)$ is specified in

Lemma 1. *The wage elasticity of labor demand for unskilled workers $\varepsilon_{L,\widetilde{w}_L}$ is constant if and only if the relative task productivity schedule of high- and low-skilled workers is given by*

$$\bar{\alpha}(I) = b\left(\frac{I}{1-I}\right)^{\eta}, \quad \textit{with} \quad \eta > 0 \quad \textit{and} \quad b > 0, \tag{21}$$

leading to $\varepsilon_{L,\widetilde{w}_L} = 1 + 1/\eta$. For the special case $\eta = 1$, we have $\varepsilon_{L,\widetilde{w}_L} = 2$.

Proof. See Appendix A.4. □

A more generic function $\bar{\alpha}(I)$ that nests function (21) as a special case and also allows for different responses of the labor demand elasticity summarized in Proposition 1 is:

$$\bar{\alpha}(I) = b\,\frac{I^{\eta_H}}{(1-I)^{\eta_L}}, \quad \text{with} \quad \eta_H \geq 0, \eta_L \geq 0, \eta_H + \eta_L > 0. \tag{22}$$

In Appendix A.5 we outline the properties of the above function as well as its three special cases.

2.4 Solution of the Model in the Steady State

In the steady state the inflows into low-skilled employment are equal to the outflows from low-skilled employment. From eq. (4) follows $m(\theta_L)V_L = q_L L$. Equivalently, the inflows into low-skilled jobs are equal to the flows out of unemployment, i.e. $m(\theta_L)V_L = p_L(\theta_L)U_L$, where again $p_L(\theta_L) \equiv m(\theta_L)\theta_L$. With a mass one of individuals, $L + U_L + H = 1$. Hence,

$$q_L L = p_L(\theta_L)\,(1 - H - L). \tag{23}$$

Since the final good is chosen as numeraire, its price equals one. This implies

$$\int_0^I \ln\left(\frac{\widetilde{w}_L}{A_L \alpha_L(i)}\right) \mathrm{d}i + \int_I^1 \ln\left(\frac{w_H}{A_H \alpha_H(i)}\right) \mathrm{d}i = 0, \tag{24}$$

which closes the model. The solution of the model is described in

Proposition 2. *The general equilibrium values of the task threshold I, low-skilled employment L, labor market tightness θ_L, and the firm's wage costs $\widetilde{w}_L$ and w_H are determined by eqs. (7), (10), (17), (23) and (24). From the definition of $\widetilde{w}_L$ in eq. (7) the low-skilled wage w_L is obtained. The solution for output follows from eq. (12).*

3 Comparative Statics: Technical Change

In the following, we will analyze the implications of SBTC in the task-based matching model. To keep things as simple as possible, we consider a one-time increase in A_H while A_L remains constant ($\mathrm{d}\ln \bar{A} = \mathrm{d}\ln A_H$). The high-skilled and low-skilled labor force is assumed to be given. With perfect competition in the high-skilled labor market this assumption implies that high-skilled employment remains constant, i.e. $\mathrm{d}\ln H = 0$.

To ease the exposition, the matching function is assumed to be of the Cobb-Douglas type:

$$M_L = M(V_L, U_L) = V_L^{1-\beta_L}\, U_L^{\beta_L}, \qquad \text{with} \quad 0 < \beta_L < 1, \tag{25}$$

which implies $m(\theta_L) = \theta_L^{-\beta_L}$. Therefore, β_L is the (constant) elasticity of the job filling rate $m(\theta_L)$ with respect to labor market tightness θ_L (in absolute values). The elasticity of the job finding rate p_L with respect to θ_L is $(1-\beta_L)$.

It is useful to write the model equations in log differences. They can be condensed into a three-equations system for θ_L, $\widetilde{w}_L$ and I summarized in

Proposition 3 (Comparative Statics)**.** *Let u_L denote the low-skilled unemployment rate, and let $\varepsilon_{\widetilde{z}_L,\theta_L}$ be the elasticity of $\widetilde{z}_L$ with respect to θ_L, where $\widetilde{z}_L$ is defined in eq. (17) and $0 < \varepsilon_{\widetilde{z}_L,\theta_L} < \beta_L$. Moreover, $\varepsilon_{\kappa_L,I}$ denotes the elasticity of the wage markup κ_L with respect to the task threshold I, as defined in eq. (20). Then*

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\, u_L}\left[\mathrm{d}\ln A_H - \frac{1}{1-I}\mathrm{d}\ln\widetilde{w}_L + \frac{1}{1-I}\mathrm{d}\ln I\right], \tag{26}$$

$$\mathrm{d}\ln\widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I, \tag{27}$$

$$\mathrm{d}\ln I = -(\varepsilon_{L,\widetilde{w}_L} - 1)\,\mathrm{d}\ln\widetilde{w}_L. \tag{28}$$

Proof. See Appendix A.6.

Eq. (26) represents the job creation condition, eq. (27) is the wage equation for low-skilled workers, and eq. (28) can be interpreted as the "task allocation" equation (respectively in log differences). It is evident that both, job creation and wage setting, are influenced by changes in the task threshold I. The equations in Proposition 3 represent general equilibrium relationships in which the adjustment of high-skilled wages necessary for full-employment of high-skilled workers has already been taken into account.[5]

According to the job creation equation, an increase in A_H *cet. par.* leads to higher labor market tightness. As can be seen from the wage equation, an increase in labor market tightness leads to higher wage pressure and *cet. par.* increases $\widetilde{w}_L$. However, this increase in labor costs induces firms to reduce the range of tasks allocated to low-skilled labor, which reduces labor market tightness and has ambiguous effects on wage setting as explained above. Inserting the task allocation equation in the other two equations leads to

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[\mathrm{d}\ln A_H - \frac{\varepsilon_{L,\widetilde{w}_L}}{1-I}\,\mathrm{d}\ln\widetilde{w}_L\right], \tag{29}$$

$$\mathrm{d}\ln\widetilde{w}_L = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\,\mathrm{d}\ln\theta_L, \tag{30}$$

where for the latter expression eq. (20) and the definition of κ_L in eq. (17) have been used, and $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I \neq 1$ must hold. In this version of the job creation and wage setting equation the adjustment of the task threshold I due to a change in firm's low-skilled labor costs is already taken into account. This two-equations system can be graphically represented by a job creation curve (JC) and wage curve (WC) in $\theta_L - \widetilde{w}_L$ space. As can be seen from eq. (29), increases in A_H lead to rightward shift of the JC. Moreover, the JC is downward sloping, i.e.

$$\Phi \equiv \left.\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\theta_L}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{\varepsilon_{L,\widetilde{w}_L}} < 0. \tag{31}$$

As regards the WC described by eq. (30), the relationship between θ_L and $\widetilde{w}_L$ is not

[5] This is, for instance, the reason why changes in A_H have been "netted out" of the task allocation equation.

unambiguous. The slope of the WC is

$$\Gamma \equiv \left.\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \theta_L}\right|_{\mathrm{WC}} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} \gtrless 0. \tag{32}$$

Quite similar to standard matching models, the slope of the WC positively depends on $\varepsilon_{\widetilde{z}_L,\theta_L}$ which is a function of r, q_L, β_L, and s_L, as shown in eq. (A.19) in Appendix A.6. In addition to these parameters, the slope of the WC in the task-based model also depends on $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$, i.e. on how changes in the task allocation affect the wage elasticity of labor demand.

In a conventional matching model an increase in labor market tightness leads to higher wage claims of workers, implying an upward–sloping WC in θ_L–$\widetilde{w}_L$ space. In eq. (30) this situation arises if $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$. However, as is evident from Proposition 1, the case $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ is also possible, implying that the corresponding WC would be downward-sloping. In that case, two situations can be distinguished depending on whether the JC is steeper or flatter than the WC, i.e. depending on whether $|\Phi| \gtrless |\Gamma|$. In Appendix A.7 we first demonstrate that, irrespectively of the slope of the WC, a steady state equilibrium exists in all situations. Moreover, we show that all steady state equilibria can be in principle (saddle-path) stable so that we cannot rule out the possibility of a downward-sloping WC in a general comparative-static analysis. The results of this analysis are summarized in

Proposition 4 (Comparative-Static Results)**.** *High-skilled labor-augmenting technical change has the following effects on the labor market equilibrium:*

(i) Low-skilled labor market tightness:

$$\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln A_H} \begin{cases} > 0, & \textit{if} \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \ \vee \ \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \ \wedge \ |\Phi| > |\Gamma|\right) \\ < 0, & \textit{otherwise.} \end{cases}$$

(ii) Low-skilled labor costs:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H}\begin{cases} > 0, & \textit{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \ \vee \ \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \ \wedge \ |\Phi| < |\Gamma|\right) \\ < 0, & \textit{otherwise.} \end{cases}$$

(iii) Task threshold:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H}\begin{cases} < 0, & \textit{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \ \vee \ \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \ \wedge \ |\Phi| < |\Gamma|\right) \\ > 0, & \textit{otherwise.} \end{cases}$$

Proof. Solving eqs. (29) and (30) leads to:

$$\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln A_H} = \frac{1-I}{\varepsilon_{L,\widetilde{w}_L}}\frac{1}{|\Phi|+\Gamma},$$
$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln A_H} = \frac{(1-I)\,\Gamma}{\varepsilon_{L,\widetilde{w}_L}}\frac{1}{|\Phi|+\Gamma}.$$

Because of eq. (28) it holds:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H} = -(\varepsilon_{L,\widetilde{w}_L}-1)\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H} = -\frac{(1-I)\,\Gamma}{\kappa_L}\frac{1}{|\Phi|+\Gamma}.$$

□

Figure 1 illustrates the comparative-static results. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, the WC is upward-sloping (see Figure 1a). In this case an increase in A_H leads to an increase in labor market tightness θ_L and in firms' labor costs $\widetilde{w}_L$. Figure 1b depicts the situation where both curves are downward-sloping and the WC is steeper than the JC. In that case an increase in A_H still leads to an increase in $\widetilde{w}_L$, but θ_L is declining. If the JC is steeper than the WC, as depicted in Figure 1c, the opposite results are obtained, i.e. θ_L increases whereas $\widetilde{w}_L$ decreases.

To give some intuition to the the comparative-static results, it is useful to distinguish between the decisions at the firm level and the general equilibrium results. At the firm level, the high-skilled wage and labor market tightness are considered as given. An increase in A_H lowers the unit labor costs for high-skilled workers relative to low-skilled workers at the old task threshold I. Hence, the firm has an incentive to reduce the range of tasks performed by low-skilled workers. The firm's labor union chooses a wage w_L that implies $\widetilde{w}_L = \kappa_L \widetilde{z}_L$, where $\widetilde{z}_L$ is taken as given. Depending on the curvature of the task productivity schedule $\bar{\alpha}$, the decline in I affects the labor demand elasticity as outlined in Proposition 1. In response to that, the labor union's wage claims may rise, remain unchanged or fall.

In the general equilibrium, the increase in the firms' demand for high-skilled workers *cet. par.* leads to a rise in high-skilled wages and in the firms' relative wage costs $\widetilde{\omega} = w_H/\widetilde{w}_L$.[6] The increase in $\widetilde{\omega}$ *cet. par.* increases labor demand for low-skilled workers. As shown in Appendix A.6,

$$\mathrm{d}\ln L = \frac{1}{1-I}\mathrm{d}\ln I + \mathrm{d}\ln\widetilde{\omega}, \tag{33}$$

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L. \tag{34}$$

Hence, whether L and therefore θ_L increase relative to the initial equilibrium depends on whether the positive effect on labor demand caused by the increase in $\widetilde{\omega}$ is larger than the negative effect caused by the decline in the task threshold I. Of course, changes in θ_L lead to changes in $\widetilde{z}_L$ which lead to further adjustments in labor unions' wage claims for low-skilled workers.

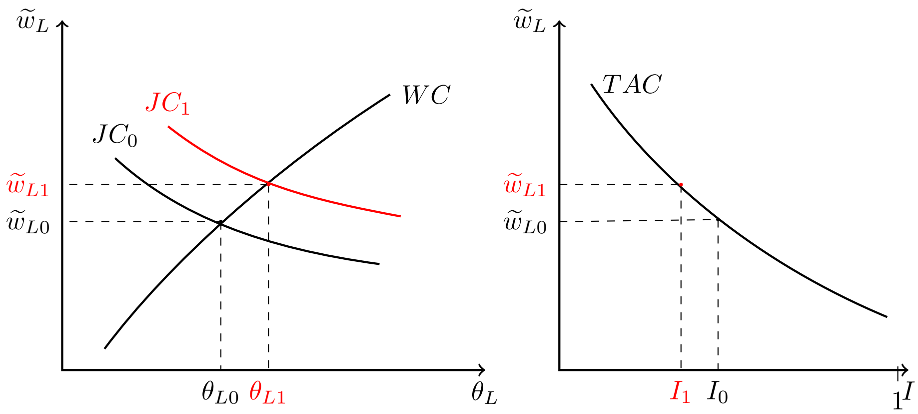
In Figure 1a the increase in $\mathrm{d}\ln\widetilde{\omega}$ dominates, and θ_L and $\widetilde{w}_L$ increase. Despite the decline in I, labor demand for low-skilled workers is higher in the new equilibrium because more workers are employed in each of the remaining low-skilled tasks. The WC is relatively steep for $0 < \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$ because the decline in I then leads to a lower labor demand elasticity and hence higher wage pressure. Vice versa, for $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 0$ the WC is relatively flat because the increase in labor unions' wage claims (due to higher θ_L) is dampened by the increase in the labor demand elasticity. For $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$ the slope of the resulting WC lies in between the other two cases. Since the slope of the WC is related to the concept of real wage rigidity, this analysis offers additional

[6] By how much w_H and $\widetilde{\omega}$ rise also depends on the change in low-skilled wage costs $\widetilde{w}_L$.

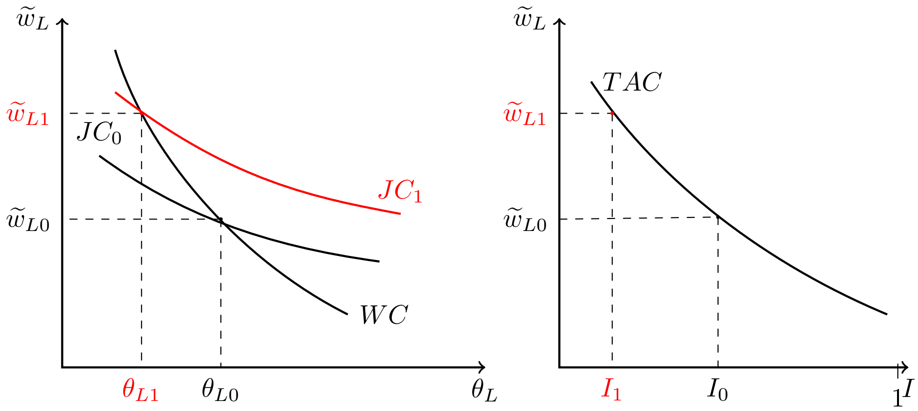
explanations for different degrees of real wage rigidity between countries or industries. In the literature, the degree of real wage rigidity is often explained by institutional factors such as the unemployment compensation system or the degree of centralization of wage bargaining; see e.g. Layard et al. (1991), Chapter 9. According to our analysis, changes in the task composition also affect the real wage response to changes in labor market tightness depending on the curvature of the task productivity schedule. In that sense, the production technology may also influence the extent of real wage rigidity in an industry or country.

In Figure 1b, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the slope of the JC is smaller than the slope of WC (in absolute values). Because of eq. (31), a relatively small $|\Phi|$ arises if the low-skilled unemployment rate u_L is relatively low and I is relatively high, i.e. many tasks are allocated to low-skilled workers, implying that the labor share of low-skilled workers is relatively high. The firm's reduction in I leads to a strong decline in the labor demand elasticity and thus to a strong increase in wage pressure. Since the labor share of low-skilled workers is high, the increase in w_L raises each firm's labor costs significantly, implying a relatively small increase in output, the labor demand for high-skilled workers, in w_H and $\widetilde{\omega}$. As a consequence, in eq. (33) the effect on L due to a decline in I dominates. The resulting decline in θ_L would *cet. par.* lead to lower wage pressure. However, this effect is overcompensated by the decline in the labor demand elasticity and thus the rising wage markup.

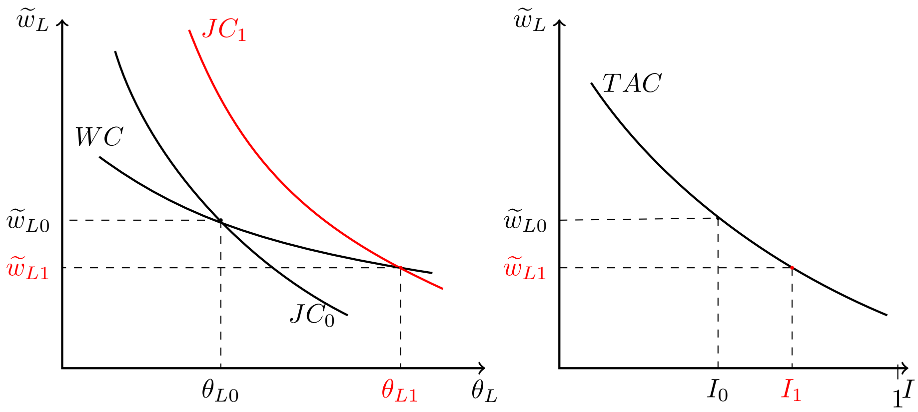
In Figure 1c, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the JC is steeper than the WC. In comparison to the previous figure, the situation is now reversed. The slope $|\Phi|$ is the larger, the lower I, i.e. the lower the low-skilled labor share in the initial equilibrium. Despite the initial increase in $\widetilde{w}_L$ caused by the decline in I, the rise of the firm's labor cost is this time comparably small, leading to a relatively strong increase in production, the demand for high-skilled workers, in w_H and $\widetilde{\omega}$. These general equilibrium effects lead to a strong increase in L and θ_L and even to an increase in I that reduces wage pressure. The rising wage pressure due to higher θ_L is overcompensated by the strongly declining wage pressure as the response of the labor demand elasticity and thus the markup is very strong. As a consequence, $\widetilde{w}_L$ is lower than in the initial equilibrium.



(a) Case 1: $\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L} / \mathrm{d} \ln I < 1$



(b) Case 2: $\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L} / \mathrm{d} \ln I > 1$; WC steeper than JC



(c) Case 3: $\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L} / \mathrm{d} \ln I > 1$; JC steeper than WC

Figure 1: Effects of skill-biased technological progress (increase in the productivity of high-skilled workers A_H) on labor market outcomes and task allocation

Notes: The effects depend on the size of $\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L} / \mathrm{d} \ln I$ and the relative slopes of the job creation curve (JC) and the wage curve (WC). Graphical illustration of the JC, WC and TAC follows from the formal analysis of the slopes and curvatures of these curves based on the general function $\bar{\alpha}(I) = bI^{\eta_H}/(1-I)^{\eta_L}$; see Appendix A.8. The diagrams should nevertheless be interpreted as a sketch. The axes scale is allowed to differ across cases and may encompass different ranges of values for $\widetilde{w}_L$, θ_L, and I.

Appendix A.9 summarizes how an increase in A_H affects other variables of the model. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, SBTC will unambiguously increase $\widetilde{\omega}$, w_H, L, and Y, and decrease u_L. In most of the cases, for example always when $1 > \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 0$, low-skilled workers will benefit from an increase in A_H in terms of their wages w_L as well. However, for negative responses of the labor demand elasticity it is even possible that w_L will decline implying a pronounced increase in wage inequality. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ the relative sizes of the slopes of the JC and WC again play a key role. In majority of the cases, the effects for w_L match qualitatively those for $\widetilde{w}_L$ discussed above. Therefore, if the low-skilled workers experience higher unemployment rates due to SBTC the rise in inequality is less pronounced than in a situation of declining unemployment rates.

The analysis of this section shows that, other than in a canonical model with a conventional modeling of a production process, SBTC can in our model harm low-skilled workers. This is always the case if the labor demand elasticity increases with higher I and this response is strong. Whether low-skilled workers lose in such a case in terms of their wages or their labor market tightness is to a large extent governed by the initial task composition. This insight could be relevant in sectoral context when different sectors may face similarly strong responses to increasing task thresholds but different division of tasks between low- and high-skilled workers.

4 Calibration

To quantify the effects of SBTC in our framework, we calibrate the model to the data of the US and Germany for two time periods: 1995 to 2005 and 2010 to 2017. The first period is characterized by an increasing unemployment rate for low-skilled workers in both countries, especially in Germany. In the second period the low-skilled unemployment rate in both countries decreases over the whole period. The years between 2006 and 2009 are excluded from the analysis to avoid biased results due to the financial crisis. We then simulate the effects of an increase in the productivity of high-skilled workers A_H by 10% to examine whether SBTC could partly explain the development of labor market variables in both countries. Nevertheless, we would like to point out that our primary goal in this calibration exercise is to illustrate the workings of the model rather than to precisely reproduce the observed developments in the US and German labor markets. For that we would have to take into account many other country-specific labor market characteristics and events, such as the introduction of the minimum wage in Germany, offshoring effects and changes in the welfare state.

To better fit the model to the data, we introduce a scale parameter ζ in the Cobb-Douglas matching function, $M(V_L, U_L) = \zeta \, V_L^{1-\beta_L} \, U_L^{\beta_L}$, which indexes the efficiency of the matching process. Moreover, we choose the concave relative task productivity schedule $\bar{\alpha}(I) = bI^{\eta_H}$, with $\eta_H < 1$ and $b = 1$; see Appendix A.5 for the properties of this function. In principle, this function allows to generate any of the cases described by Figures 1a–1c. Which case is obtained then depends on the parameterization of the model. The model is characterized by 18 exogenous parameters: $\{\beta_L,\, q_L,\, \zeta_c,\, r_c,\, z_{L,c},\, s_{L,c},\, H_c,\, A_{H,c},\, A_{L,c},\, \eta_{H,c}\}$, with $c \in \{US, DE\}$. We take nine parameters from the data or the literature, see Table 1, and calculate nine parameters to match the US and German data during the period 1995 to 2005 and 2010 to 2017, see Tables 2 and 3. One period in the model corresponds to one quarter, so all parameters are interpreted quarterly.

The first two parameters are without country and time variation. We set the matching elasticity β_L to 0.5, which is within the range of estimates reported in Petrongolo and Pissarides (2001). The quarterly separation rate q_L is assumed to be 0.0873 based on the monthly separation rate of 0.03 calibrated by Battisti et al. (2018). The last parameter taken from the literature is unemployment benefits z_L. We follow Cords and Prettner (2022) and assume a time-invariant value of 0.4 and 0.6 for the US and Germany, respectively. The real interest rate r and the share of high-skilled workers H are calculated from the data and vary over time and country. Lastly, we calculate the time-dependent skill bias A_H/A_L for the US. For detailed description of the data and the corresponding calculations see Appendix A.10.

We jointly calibrate the remaining nine parameters by matching nine targets obtained from US and German data over the two periods. The targets are summarized in Table 2 and the parameters that are obtained by matching these targets are shown in Table 3. The most important target is the task threshold I which is calculated as the relative

Table 1: Parameter values

Parameter	**Country**	**Value**		**Source**
		95-05	10-17	
Parameters without country variation				
Matching elasticity: β_L	–	0.5	0.5	Petrongolo and Pissarides (2001)
Separation rate: q_L	–	0.0873	0.0873	Battisti et al. (2018)
Parameters with country variation				
Real interest rate: r	US	0.0093	0.0042	FRED
	DE	0.0117	0.0016	Deutsche Bundesbank, FRED
Unemployment benefits: z_L	US	0.4	0.4	Cords and Prettner (2022)
	DE	0.6	0.6	
Share of high-skilled: H	US	0.305	0.372	CPS
	DE	0.24	0.285	EU-LFS
Skill bias: $\frac{A_H}{A_L}$	US	1.89	1.99	Own calculations based on data from Bowlus et al. (2021)

Notes: For detailed data description see Appendix A.10.

share of low-skilled and middle-skilled labor compensation in total labor compensation. The comparative-static results depend on whether $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ is below or above one, i.e. whether the task threshold I is smaller or larger than the specific boundary value $I_b = 1 - (\sqrt{1+\eta_H})/\eta_H$. The value for I_b for each country and time period is estimated from the calibrated η_H. To allow for model calibration for both countries jointly, we exploit the information on the real total factor productivity (RTFP) in Germany relative to the US taking into account that RTFP is in our model given by $A_L^I A_H^{1-I} B$, with B defined in (12).

The simulated results of an increase in A_H by 10% on different labor market variables are summarized in Table 4. In the period 1995–2005, the effect on θ_L is negative in both countries. This is due to (i) $I > I_b$ and (ii) such values for model parameters and targets that $|\Phi| < |\Gamma|$ (WC steeper than JC; see also Figure 1b). The decrease in θ_L is notably weaker in the US than in Germany which is in line with a stronger increase in the German low-skilled unemployment rate observed in the data for this period.

In the US, SBTC increases θ_L by less than 1% in the more recent time period. The direction of this effect results from $I < I_b$ and it corresponds to an observed decrease in the US unemployment rate for low-skilled workers. In Germany, the effect on θ_L remains

Table 2: Matched targets

Target	Country	Value		Source
		95-05	10-17	
Low-skilled unemployment rate: u_L	US	0.047	0.070	CPS
	DE	0.106	0.062	EU-LFS
Low-skilled labor market tightness: θ_L	US	0.4345	0.3643	JOLTS, CPS, IAB-JVS
	DE	0.204	0.4707	IAB-JVS, EU-LFS, BA
Skill premium: $\frac{w_H}{w_L}$	US	1.81	1.89	CPS
	DE	1.29	1.46	EU-SILC
Task threshold: I	US	0.5622	0.5308	WIOD SEA Release 2013, EU Klems Release 2017
	DE	0.6532	0.6229	
Relative real total factor productivity (RTPF): $\frac{RTFP_{DE}}{RTFP_{US}}$	–	1.0664	1.0465	Penn World Tables 10.0 by Feenstra et al. (2015)

Notes: For detailed data description see Appendix A.10.

Table 3: Calibrated parameters

Parameter	United States		Germany	
	95-05	10-17	95-05	10-17
Matching efficiency parameter: ζ	0.659	0.604	0.452	0.686
Search cost: s_L	2.248	3.540	1.223	1.727
High-skill biased technology: A_H	–	–	1.663	1.937
Low-skill biased technology: A_L	0.818	0.892	1.000	1.000
Parameter of function $\bar{\alpha}(I)$: η_H	0.614	0.717	0.950	0.950

Notes: Calibration has been done by matching the targets in Table 2. The model parameters satisfy the stability conditions described in Appendix A.7.

negative and amounts to 2.13% in absolute values. However, the strong decrease in the German low-skilled unemployment rate between 2010 and 2017 in the data would instead suggest an increase in θ_L. The difference between the real development in u_L and that implied by the simulation may be explained by the fact that in the simulation we have not so far accounted for factors other than A_H affecting the labor market. One of the most important interventions was the Hartz IV reform introduced in 2005 leading to a sharp decline in unemployment benefits in the subsequent years. If we account for this reform by simulating a decrease in German unemployment benefits in the second period in addition to the increase in A_H, we find that a very small reduction in z_L by 0.27% would be sufficient to compensate the negative effect of A_H on θ_L.

As regards the effect of an increase in A_H on other outcomes, there is a stronger reallocation of tasks towards high-skilled workers in the US than in Germany in both periods. The increase in wages of low-skilled workers is moderate in both countries and periods compared to the significant increase in wages of high-skilled workers, with the increase in the US being stronger than in Germany. This ultimately leads to a stronger rise in the skill premium in the US compared to Germany in both periods.

Table 4: Labor market effects of an increase in high-skill productivity A_H by 10% (changes in variables expressed in percent).

Variable	**United States**		**Germany**	
	95-05	10-17	95-05	10-17
Low-skilled labor market tightness: θ_L	–0.06%	0.81%	–4.41%	–2.13%
Task threshold: I	–3.45%	–3.50%	–2.67%	–2.79%
Low-skilled wage: w_L	1.25%	1.55%	1.29%	1.35%
High-skilled wage: w_H	8.81%	8.67%	8.34%	8.35%
Skill premium: $\frac{w_H}{w_L}$	7.56%	7.11%	7.05%	7.00%

5 Summary and Conclusions

This paper demonstrates how the task approach and the matching framework with labor unions can be combined into one consistent general equilibrium model. Our model goes beyond perfect competition task models that analyze the wage developments for different groups of workers, and additionally offers insights on how changes in the task allocation affect labor market tightness, unemployment, and the wage-setting power of labor unions.

In labor union models the wage elasticity of labor demand plays a crucial role for the extent of wage pressure in the economy. We show that this elasticity is influenced by the task threshold that divides the range of tasks performed by low- and high-skilled workers, respectively. More specifically, we demonstrate that the labor demand elasticity for low-skilled workers consists of a direct wage effect and a task reallocation effect. The latter implies that with an increase in low-skilled labor costs fewer tasks are allocated to low-skilled labor. The strength of the task reallocation effect depends on the intensity with which low-skilled workers are used in the production process and on the shape of the relative task productivity schedule that reflects the substitutability of high- and low-skilled workers. Since both convex and concave shapes of the relative task productivity schedule are theoretically possible, the effect of a change in the task allocation on the labor demand elasticity remains ambiguous.

This ambiguity carries over to the general equilibrium that is condensed into a system of two equations reflecting job creation by firms and wage claims of labor unions. Whereas in standard matching models an increase in labor market tightness leads to higher wage pressure along a positively sloped wage curve, in our model the wage curve can also be downward sloping. This has consequences for the effects of skill-biased technological change (SBTC). In contrast to the standard result that an increase in the productivity of high-skilled workers has a positive impact on employment and wages of low-skilled workers, in our model it is also possible that low-skilled workers may instead either experience higher unemployment or lower wages.

In the calibration of the model to US and German data for the two time periods, 1995 to 2005 and 2010 to 2017, we find that the impact of SBTC may even change its sign over time. According to the simulation results for the US, SBTC slightly increases low-skilled unemployment in the first period, while decreasing it in the second period. On the other hand, simulation results for Germany show that SBTC has a negative effect on low-skilled employment in both periods. The fact that in Germany the observed low-skilled unemployment declines in the second period is not due to technical change but to other factors such as the Hartz reforms.

In further research the model could be extended in different directions. For example, one could incorporate capital, especially automation capital, into the model and in this way analyze the impact of automation on unemployment and the wage-setting power of labor unions. Moreover, one could also address sector heterogeneity in the relative task productivity schedules. Some sectors may, for example, encompass a range of tasks which rapidly increase in their complexity so that the relative productivity of high-skilled workers increases in an exponential manner. Other sectors may display a task complexity profile that facilitates stronger substitution of different skills. The insights of our model could thus help to explain differing real wage developments for workers who exhibit the same skill level and face the same extent of SBTC but are employed in different sectors.

A Appendix

A.1 The Firm's Optimization Problem

Combining eqs. (1)–(5), and considering the restrictions (6) for $l_t(i)$ and $h_t(i)$, the firm's optimization problem can be written as

$$\max_{\{l_t(i),h_t(i),V_{Lt},\mu_{Lt}\}} \mathcal{L} = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} \left\{ \exp\left[\int_0^1 \ln\left(A_{Lt}\alpha_L(i)l_t(i) + A_{Ht}\alpha_H(i)h_t(i)\right) \mathrm{d}i\right] - w_{Lt}\int_0^1 l_t(i)\mathrm{d}i - w_{Ht}\int_0^1 h_t(i)\mathrm{d}i - s_L V_{Lt} \right\}$$
$$+ \sum_{t=0}^{\infty} \mu_{Lt} \left(\frac{1}{1+r}\right)^{t-1} \left[m(\theta_{Lt})V_{Lt} + (1-q_L)\int_0^1 l_{t-1}(i)\mathrm{d}i - \int_0^1 l_t(i)\mathrm{d}i\right]$$
$$\text{s.t.} \quad l_t(i) \geq 0, \qquad h_t(i) \geq 0, \qquad \text{and} \qquad l_0(i), h_0(i) \text{ given,}$$

where μ_{Lt} denotes the shadow price of L_t in period t. The single firm takes aggregate labor market tightness θ_{Lt} as given. The first-order conditions are $\partial\mathcal{L}/\partial\mu_{Lt} = 0$, $\partial\mathcal{L}/\partial V_{Lt} = 0$ (which gives $\mu_{Lt} = s_L/m(\theta_{Lt})$), and the complementary slackness conditions

$$\frac{\partial\mathcal{L}}{\partial h_t(i)} \leq 0, \qquad h_t(i) \geq 0, \qquad \frac{\partial\mathcal{L}}{\partial h_t(i)} h_t(i) = 0,$$
$$\frac{\partial\mathcal{L}}{\partial l_t(i)} \leq 0, \qquad l_t(i) \geq 0, \qquad \frac{\partial\mathcal{L}}{\partial l_t(i)} l_t(i) = 0.$$

This leads to

$$\frac{Y_t}{y_t(i)} A_{Ht}\alpha_H(i) \leq w_{Ht}, \qquad h_t(i) \geq 0, \tag{A.1}$$
$$\frac{Y_t}{y_t(i)} A_{Lt}\alpha_L(i) \leq \widetilde{w}_{Lt} \equiv w_{Lt} + \frac{s_L}{m(\theta_{Lt})} - \frac{s_L}{m(\theta_{L,t+1})}\frac{(1-q_L)}{(1+r)}, \qquad l_t(i) \geq 0. \tag{A.2}$$

Due to complementary slackness in each equation only one inequality can hold at the same time. As can be seen from eq. (A.2), the low-skilled labor costs relevant to the firm, $\widetilde{w}_{Lt}$, consist of the wage w_{Lt} plus the search costs incurred in period t, which are reduced by the vacancy posting costs that are saved in period $t+1$ if the employment relationship continues. For the discussion of the different cases we focus on the steady state in which

$\theta_{L,t+1} = \theta_{Lt} = \theta_L$. In that case, $\widetilde{w}_{Lt} = \widetilde{w}_L$, where

$$\widetilde{w}_L \equiv w_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}. \tag{A.3}$$

Case 1: $l(i) > 0$ and $h(i) = 0$. Due to eq. (2) in the main text $y(i) = A_L\alpha_L(i)l(i)$, implying $Y/l(i) = \widetilde{w}_L$ in eq. (A.2). The marginal product of unskilled labor in task i with respect to output Y equals the low-skilled labor costs relevant to the firm. It follows that $l(i) = l$, i.e. the same labor input l is chosen in all low-skilled tasks. From eq. (A.1) follows

$$\frac{\widetilde{w}_L}{A_L\alpha_L(i)} < \frac{w_H}{A_H\alpha_H(i)},$$

if the constraint on $h(i)$ is binding. Hence, low-skilled workers are employed in those tasks in which their unit labor costs are lower than those of high-skilled workers. At the margin where $\partial\mathcal{L}/\partial h(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L\alpha_L(I)) = w_H/(A_H\alpha_H(I))$.

Case 2: $h(i) > 0$ and $l(i) = 0$. From eq. (2) follows $y(i) = A_H\alpha_H(i)h(i)$, implying $Y/h(i) = w_H$ in eq. (A.1) which is interpreted analogously. It follows that $h(i) = h$, i.e. the same labor input h is chosen in all high-skilled tasks. From eq. (A.2) follows

$$\frac{\widetilde{w}_L}{A_L\alpha_L(i)} > \frac{w_H}{A_H\alpha_H(i)},$$

if the constraint on $l(i)$ is binding. Hence, high-skilled workers are employed in those tasks in which their unit labor costs are lower than those of low-skilled workers. At the margin where $\partial\mathcal{L}/\partial l(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L\alpha_L(I)) = w_H/(A_H\alpha_H(I))$.

Case 3: $h(i) > 0$ and $l(i) > 0$. Because of eq. (2) $y(i) = A_L\alpha_L(i)l(i) + A_H\alpha_H(i)h(i)$. In eq. (A.2) it holds that $(Y/y(i))A_L\alpha_L(i) = \widetilde{w}_L$. In eq. (A.1) it holds that $(Y/y(i))A_H\alpha_H(i) = w_H$. Hence,

$$\frac{\widetilde{w}_L}{A_L\alpha_L(I)} = \frac{w_H}{A_H\alpha_H(I)}.$$

Case 4: $h(i) = 0$ and $l(i) = 0$. In that case $y(i) = 0$ which due to the production function in eq. (1) is not possible.

In cases 1-3 the task threshold I is defined as the task where unit labor costs for high- and low-skilled workers are equal. This condition can be written as

$$\bar{\alpha}(I) \equiv \frac{\alpha_H(I)}{\alpha_L(I)} = \frac{A_L\, w_H}{A_H \widetilde{w}_L}. \tag{A.4}$$

Since $\bar{\alpha}'(i) > 0$, there is only one task $i = I$ where unit labor costs of both worker types are equal. It must hold that $I < 1$, because values $I \geq 1$ would imply that no high-skilled workers are used in the production process, in contradiction to our assumption that high-skilled workers are fully employed. Moreover, if unemployment benefits are not too high, it is never optimal for labor unions to demand such high wages that no unskilled workers are employed. In that case it must also hold that $I > 0$. As a consequence, $0 < I < 1$.

A.2 Wage Setting of Labor Unions

To simplify the notation, we define $R_{Lt} \equiv \Psi_{EL,t} - \Psi_{UL,t}$. With the wage w_{Lt} being set for n periods and unemployment benefits being equal to z_{Lt} in all periods,

$$R_{Lt} = \left(\frac{1-\delta^n}{1-\delta}\right)(w_{Lt} - z_{Lt}) + \delta^n R_{L,t+n},$$

where $\delta \equiv (1 - q_L - p_L)/(1 + r) < 1$. The representative labor union maximizes

$$\max_{w_{Lt}} V_{Lt} = R_{Lt}\, L_t \tag{A.5}$$

s.t. to the labor demand equation (11)

$$L_t = L^d\left(\widetilde{\omega}_t, \cdot\right), \qquad \text{with} \qquad \widetilde{\omega}_t \equiv \frac{w_{Ht}}{\widetilde{w}_{Lt}}.$$

The labor union considers aggregate labor market tightness to be given and constant, in line with steady-state considerations. Therefore, $\widetilde{w}_{Lt}$ corresponds to the expression in eq. (A.3). The first-order condition $\mathrm{d}V_{Lt}/\mathrm{d}w_{Lt} = 0$ gives

$$\frac{\partial R_{Lt}}{\partial w_{Lt}} L_t + R_{Lt} \frac{\partial L^d}{\partial \widetilde{\omega}_t} \frac{\partial \widetilde{\omega}_t}{\partial w_{Lt}} = 0. \tag{A.6}$$

Multiplying by $\widetilde{w}_{Lt}/L_t$ and defining

$$\varepsilon_{L\widetilde{w}_L,t} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_{Lt}} \right| = \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{\omega}_t} \tag{A.7}$$

leads to

$$\left(\frac{1-\delta^n}{1-\delta} \right) \widetilde{w}_{Lt} - R_{Lt}\, \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Defining

$$\widetilde{z}_{Lt} \equiv z_{Lt} + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}$$

and noting that $w_{Lt} - z_{Lt} = \widetilde{w}_{Lt} - \widetilde{z}_{Lt}$ gives

$$\frac{(1-\delta^n)}{1-\delta} \widetilde{w}_{Lt} - \left[\frac{(1-\delta^n)}{1-\delta} (\widetilde{w}_{Lt} - \widetilde{z}_{Lt}) + \delta^n R_{L,t+n} \right] \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Therefore, the wage w_{Lt} set in period t for n periods implies the following wage costs $\widetilde{w}_{Lt}$ in period t:

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} - 1} \left(\widetilde{z}_{Lt} - \frac{(1-\delta)\delta^n}{1-\delta^n} R_{L,t+n} \right). \tag{A.8}$$

In the steady state $R_{L,t+n} = (\widetilde{w}_{Lt} - \widetilde{z}_{Lt})/(1-\delta)$. Hence,

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} + \delta^n - 1} \widetilde{z}_{Lt}. \tag{A.9}$$

This result is in line with the result in Manning (1991) that wage pressure is the higher the longer the duration of the wage contract. We focus on the situation in which $n \to \infty$ and therefore $\delta^n \to 0$. Omitting the time index, this leads to

$$\widetilde{w}_L = \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1} \widetilde{z}_L. \tag{A.10}$$

A.3 Second-Order Condition for the Optimal Wage

The following exposition builds on Appendix A.2. With $n \to \infty$ and the definition of δ, $\mathrm{d}V_L/\mathrm{d}w_L$ can be written as

$$\frac{\mathrm{d}V_L}{\mathrm{d}w_L} = \frac{(1+r)L^d(\cdot)}{r+q_L+p_L}\left[1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)}{\widetilde{w}_L}\varepsilon_{L,\widetilde{w}_L}\right],$$

where $\varepsilon_{L,\widetilde{w}_L}$ is defined in eq. (A.7), and the time index has been omitted. Therefore,

$$\frac{\mathrm{d}^2V_L}{\mathrm{d}w_L^2} = -\frac{(1+r)\varepsilon_{L,\widetilde{w}_L}L^d(\cdot)}{(r+q_L+p_L)\widetilde{w}_L}\left[1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)\varepsilon_{L,\widetilde{w}_L}}{\widetilde{w}_L} + \frac{\widetilde{z}_L}{\widetilde{w}_L}\right.$$
$$\left. +(\widetilde{w}_L - \widetilde{z}_L)\frac{\mathrm{d}\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}I}\frac{\partial I}{\partial \widetilde{w}_L}\frac{1}{\varepsilon_{L,\widetilde{w}_L}}\right].$$

The last expression takes into account that $\varepsilon_{L,\widetilde{w}_L}$ is a function of the task threshold I which in turn depends on $\widetilde{w}_L$, as is evident from eq. (18). For a maximum, next to $\mathrm{d}V_L/\mathrm{d}w_L = 0$, the condition $\mathrm{d}^2V_L/\mathrm{d}w_L^2 < 0$ has to be satisfied. This implies that the term in brackets must be positive. Hence,

$$1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)\varepsilon_{L,\widetilde{w}_L}}{\widetilde{w}_L} + 1 - \frac{\widetilde{w}_L - \widetilde{z}_L}{\widetilde{w}_L} + \frac{(\widetilde{w}_L - \widetilde{z}_L)}{\widetilde{w}_L}\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{\partial \ln I}{\partial \ln \widetilde{w}_L} > 0$$

must hold. Since

$$\frac{\partial \ln I}{\partial \ln \widetilde{w}_L} = -\frac{\partial \ln I}{\partial \ln \widetilde{\omega}} = -\frac{1}{\varepsilon_{\bar{\alpha},I}},$$

this condition is equivalent to

$$1 + \varepsilon_{L,\widetilde{w}_L} + \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{1}{\varepsilon_{\bar{\alpha},I}} < 2\frac{\widetilde{w}_L}{\widetilde{w}_L - \widetilde{z}_L}.$$

From $\mathrm{d}V_L/\mathrm{d}w_L = 0$ follows $\varepsilon_{L,\widetilde{w}_L} = \widetilde{w}_L/(\widetilde{w}_L - \widetilde{z}_L) > 1$. Therefore,

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < (\varepsilon_{L,\widetilde{w}_L} - 1)\varepsilon_{\bar{\alpha},I} \ .$$

Taking account of eq. (18), the second-order condition for an optimum therefore requires

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{1}{1-I} \ . \tag{A.11}$$

Because of eq. (18) and the definition of κ_L in eq. (17)

$$\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln \varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right),$$

which will play an important role in Proposition 1. Moreover, it holds

$$\kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L}-1} = 1 + (1-I)\varepsilon_{\bar{\alpha},I} \ .$$

Therefore, the condition (A.11) can be alternatively written as

$$\frac{\mathrm{d}\ln \varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} > -(1+\varepsilon_{\bar{\alpha},I}). \tag{A.12}$$

Since $\mathrm{d}\ln \varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I = 1 - \varepsilon_{\bar{\alpha},I} + I\bar{\alpha}''/\bar{\alpha}'$, (A.12) implies

$$\bar{\alpha}''\frac{I}{\bar{\alpha}'(I)} > -2. \tag{A.13}$$

A.4 Proof of Lemma 1

Since $\varepsilon_{\bar{\alpha},I} \equiv \bar{\alpha}'(I)\, I/\bar{\alpha}(I)$, eq. (18) for the wage elasticity of labor demand for low-skilled workers can be written as

$$(\varepsilon_{L,\widetilde{w}_L}-1)\,\frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)} = \frac{1}{I\,(1-I)},$$

where $\varepsilon_{L,\widetilde{w}_L} > 1$ is (in this case) constant by assumption. Therefore,

$$(\varepsilon_{L,\widetilde{w}_L}-1)\int \frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)}\,\mathrm{d}I = \int \frac{1}{1-I}\,\mathrm{d}\ln I.$$

Hence,

$$(\varepsilon_{L,\widetilde{w}_L}-1)\ln\bar{\alpha}(I) + c_1 = \ln\frac{I}{1-I} + c_2,$$

where c_1 and c_2 are integration constants. Applying the exponential function, one arrives at

$$\bar{\alpha}(I) = b\left(\frac{I}{1-I}\right)^{\eta}, \tag{A.14}$$

where $\eta \equiv 1/(\varepsilon_{L,\widetilde{w}_L} - 1) > 0$ and $b \equiv e^{\eta\,(c_2 - c_1)}$ which must be positive for $\bar{\alpha}(I) > 0$. With this specific functional form for $\bar{\alpha}(I)$, it holds that

$$\varepsilon_{\bar{\alpha},I} = \eta\,\frac{1}{1-I} \quad \text{and} \quad \frac{\mathrm{d}\ln \varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = \frac{I}{1-I},$$

where the second equation is in line with Proposition 1 for the case of a constant $\varepsilon_{L,\widetilde{w}_L}$. Inserting the expression for $\varepsilon_{\bar{\alpha},I}$ in eq. (18) leads to $\varepsilon_{L,\widetilde{w}_L} = 1 + 1/\eta$ which concludes the proof of Lemma 1. □

A.5 Special Cases of the Task Productivity Schedule $\bar{\alpha}(I)$

Starting from Lemma 1, a more general function for $\bar{\alpha}(I)$ that allows to consider all three cases of Proposition 1 is given by eq. (22) that is repeated here:

$$\bar{\alpha}(I) = b\,\frac{I^{\eta_H}}{(1-I)^{\eta_L}},$$

where $\bar{\alpha}'(I) > 0$ requires $\eta_H \geq 0$, $\eta_L \geq 0$, and $\eta_H + \eta_L > 0$.
Then $\varepsilon_{L,\widetilde{w}_L} = 1 + 1/[I\,\eta_L + (1-I)\,\eta_H]$, and

$$\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} \begin{cases} > 0, & \text{if} \quad \eta_H > \eta_L \\ = 0, & \text{if} \quad \eta_H = \eta_L \\ < 0, & \text{if} \quad \eta_H < \eta_L. \end{cases}$$

Proof. From eq. (22) follows

$$\frac{\mathrm{d}\ln \varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = \frac{I}{1-I} - \frac{(\eta_H - \eta_L)I}{I\eta_L + (1-I)\eta_H}.$$

Inserting this expression into eq. (19) leads to:

$$\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\,\frac{(\eta_H - \eta_L)I}{I\eta_L + (1-I)\eta_H}.$$

With κ_L and I being positive, and with the restrictions for η_L and η_H, the above result for $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ holds, which completes the proof. $\square$

Moreover, if $\eta_L > \eta_H$, it holds that $|(\eta_H - \eta_L)I| < I\eta_L + (1-I)\eta_H$, and since $1/\kappa_L < 1$, this implies $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > -1$ in the case of the above general task productivity schedule $\bar{\alpha}(I)$.

We can consider three special cases of the above general function $\bar{\alpha}(I)$:

Case 1: $\eta_H = \eta_L = \eta$. This leads to the $\bar{\alpha}$ function in eq. (21) implying a constant labor demand elasticity.

Case 2: $\bar{\alpha}(I) = b\, I^{\eta_H}, \quad \eta_H < 1$

In this case $\bar{\alpha}(I)$ is isoelastic and concave. We have:

$$\varepsilon_{L,\widetilde{w}_L} = 1 + \frac{1}{\eta_H(1-I)}, \quad \kappa_L = 1 + \eta_H(1-I), \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{1+\eta_H(1-I)}\,\frac{I}{1-I} > 0.$$

Moreover, there exists a value $I_b = 1 - (\sqrt{1+\eta_H} - 1)/\eta_H$ such that:

$$\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} \lesseqgtr 1, \quad \text{if} \quad I \lesseqgtr I_b.$$

Case 3: $\bar{\alpha}(I) = b\,(1-I)^{-\eta_L}$

In this case $\bar{\alpha}(I)$ is convex. We have:

$$\varepsilon_{L,\widetilde{w}_L} = 1 + \frac{1}{\eta_L I}, \quad \kappa_L = 1 + \eta_L I, \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = -\frac{1}{1+\eta_L I}.$$

A.6 Proof of Proposition 3

From eqs. (7) and (10) follows

$$\mathrm{d}\ln I = \frac{1}{\varepsilon_{\bar{\alpha},I}}\left(\mathrm{d}\ln \widetilde{\omega} - \mathrm{d}\ln \bar{A}\right) \tag{A.15}$$

and

$$\mathrm{d}\ln L = \frac{1}{1-I}\,\mathrm{d}\ln I + \mathrm{d}\ln \widetilde{\omega}, \tag{A.16}$$

where it has been taken into account that $\mathrm{d}\ln H = 0$, and

$$\mathrm{d}\ln\widetilde{\omega} = \mathrm{d}\ln w_H - \mathrm{d}\ln\widetilde{w}_L. \tag{A.17}$$

From eq. (17) follows

$$\mathrm{d}\ln\widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I, \tag{A.18}$$

where

$$\varepsilon_{\widetilde{z}_L,\theta_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\theta_L} = \beta_L\frac{\widetilde{z}_L - z_L}{\widetilde{z}_L} = \beta_L\kappa_L\frac{\widetilde{w}_L - w_L}{\widetilde{w}_L} = \beta_L\frac{\frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}}{z_L + \frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}} < \beta_L \tag{A.19}$$

and

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = -\frac{\kappa_L - 1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{A.20}$$

Because of eq. (23)

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L, \tag{A.21}$$

where $u_L \equiv (1-H-L)/(1-H)$ denotes the low-skilled unemployment rate.

The price index equation (24) can be written as

$$I(\ln\widetilde{w}_L - \ln A_L) + (1-I)(\ln w_H - \ln A_H) - \xi(I) = 0,$$

where

$$\xi(I) \equiv \int_0^I \ln\alpha_L(i)\mathrm{d}i + \int_I^1 \ln\alpha_H(i)\mathrm{d}i.$$

The total differential of this equation is

$$\begin{aligned} &I(\mathrm{d}\ln\widetilde{w}_L - \mathrm{d}\ln A_L) + (1-I)(\mathrm{d}\ln w_H - \mathrm{d}\ln A_H) \\ &- [(\ln w_H - \ln\widetilde{w}_L) - (\ln A_H - \ln A_L) - \ln\bar{\alpha}(I)]\,\mathrm{d}I = 0, \end{aligned} \tag{A.22}$$

where it has been taken into account that $\xi'(I) = -\ln\bar{\alpha}(I)$. Since the task threshold is endogenously determined from profit maximization, eq. (7) must hold, implying $\ln \bar{\alpha}(I) = (\ln w_H - \ln \widetilde{w}_L) - (\ln A_H - \ln A_L)$. Hence, the term in brackets in the second line is zero, leading to

$$\mathrm{d}\ln w_H = \mathrm{d}\ln A_H + \frac{I}{1-I}\mathrm{d}\ln A_L - \frac{I}{1-I}\mathrm{d}\ln \widetilde{w}_L. \tag{A.23}$$

The job creation equation (26) in Proposition 3 is obtained by combining eqs. (A.23), (A.17), (A.16), and (A.21), and by assuming $\mathrm{d}\ln A_L = 0$. The wage-setting equation corresponds to eq. (A.18) and the task allocation equation follows from eqs. (A.15), (A.17) and (A.23), where $\mathrm{d}\ln \bar{A} = \mathrm{d}\ln A_H$ if $\mathrm{d}\ln A_L = 0$. This concludes the proof of Proposition 3. □

A.7 Stability Analysis

A.7.1 Model Dynamics and Saddle Path

Starting point for the stability analysis is the set of dynamic model equations:

$$\theta_{Lt} = \frac{V_{Lt}}{U_{Lt}}, \quad U_{Lt} = 1 - L_{t-1} - H, \quad M_{Lt} = V_{Lt}^{1-\beta_L} U_{Lt}^{\beta_L}, \quad L_t = (1-q_L)L_{t-1} + M_t,$$
$$\frac{\widetilde{w}_{Lt}}{A_L \alpha_L(I_t)} = \frac{w_{Ht}}{A_H \alpha_H(I_t)}, \quad L_t = \frac{w_{Ht}}{\widetilde{w}_{Lt}} \frac{I_t}{1-I_t} H,$$
$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} - 1}, \quad \widetilde{z}_{Lt} \equiv z_{Lt} + \frac{s_L}{m(\theta_{Lt})} - \frac{1-q_L}{1+r} \frac{s_L}{m(\theta_{L,t+1})},$$
$$\int_0^{I_t} \ln\left(\frac{\widetilde{w}_{Lt}}{A_L \alpha_L(i)}\right) \mathrm{d}i + \int_{I_t}^1 \ln\left(\frac{w_{Ht}}{A_H \alpha_H(i)}\right) \mathrm{d}i = 0.$$

Letting $\widehat{x}_t$ denote the percentage deviation of x_t from its steady-state level ($\widehat{x}_t \equiv \ln x_t - \ln x$), the linearized equation system is:

$$\widehat{\theta}_{Lt} = \widehat{V}_{Lt} - \widehat{U}_{Lt}, \; \widehat{U}_{Lt} = \frac{u_L - 1}{u_L}\widehat{L}_{t-1}, \; \widehat{M_{Lt}} = (1-\beta_L)\widehat{V}_{Lt} + \beta_L \widehat{U}_{Lt}, \; \widehat{L}_t = (1-q_L)\widehat{L}_{t-1} + q_L \widehat{M_t},$$
$$\widehat{I}_t = \frac{1}{\varepsilon_{\bar{\alpha},I}} \left(\widehat{w}_{Ht} - \widehat{\widetilde{w}}_{Lt}\right), \quad \widehat{L}_t = \widehat{w}_{Ht} - \widehat{\widetilde{w}}_{Lt} + \frac{1}{1-I} \widehat{I}_t,$$
$$\widehat{\widetilde{w}}_{Lt} = \varepsilon_{\kappa_L,I} \widehat{I}_t + \frac{s_L \beta_L \theta_L^{\beta_L}}{\widetilde{z}_L} \left(\widehat{\theta}_{Lt} - \frac{1-q_L}{1+r} \widehat{\theta}_{L,t+1}\right),$$
$$\widehat{w}_{Ht} = -\frac{I}{1-I} \widehat{\widetilde{w}}_{Lt}.$$

After appropriate substitutions, we can eliminate $\widehat{V}_{Lt}$, $\widehat{U}_{Lt}$, $\widehat{L}_t$, $\widehat{M_t}$, and $\widehat{w}_{Ht}$, and we obtain a dynamic version of the job creation (26) and wage curve (27):

$$\widehat{\theta}_{Lt} = -\frac{\varepsilon_{L,\widetilde{w}_L}}{(1-\beta_L)(1-I)q_L} \left(\widehat{\widetilde{w}}_{Lt} - \frac{u_L - q_L}{u_L} \widehat{\widetilde{w}}_{L,t-1} \right), \tag{A.24}$$

$$\widehat{\widetilde{w}}_{Lt} = \frac{1+r}{r+q_L} \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} \left(\widehat{\theta}_{Lt} - \frac{1-q_L}{1+r} \widehat{\theta}_{L,t+1} \right). \tag{A.25}$$

In this version, A_H is assumed to be constant as our focus is to examine stability of the model and not the effects of A_H. Finally, the above equation system in two endogenous variables $\widehat{\theta}_{Lt}$ and $\widehat{\widetilde{w}}_{Lt}$ can be reduced to a second-oder difference equation in $\widehat{\theta}_{Lt}$:

$$\begin{aligned}
&\widehat{\theta}_{Lt} = \phi_1 \widehat{\theta}_{L,t-1} + \phi_2 \widehat{\theta}_{L,t-2}, \quad \text{where} \\
&\phi_1 \equiv \frac{q_L(r+q_L)}{(1-q_L)u_L} \frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} \frac{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}{\varepsilon_{\widetilde{z}_L,\theta_L}} + \frac{1+r}{1-q_L} + u_L - q_L u_L, \\
&\phi_2 \equiv -\frac{1+r}{1-q_L} \frac{u_L - q_L}{u_L}.
\end{aligned} \tag{A.26}$$

The existence of the steady state and the dynamic properties of the model can then be studied based on the coefficients ϕ_1 and ϕ_2. In particular, for the steady state to exist it must hold $1 - \phi_1 - \phi_2 \neq 0$. To ensure that the model solution (path for θ_{Lt}) is unique, the model has to be saddle-path stable. See Krause and Lubik (2010) for a detailed discussion of stability and determinacy aspects in matching models. We could have also derived a first-order equation system for $\widehat{\theta}_{Lt}$ (jump variable) and $\widehat{L}_t$ (state variable), instead of a second-order difference equation for $\widehat{\theta}_{Lt}$. However, we are not interested in the solution for, e.g., θ_{Lt} or L_t, but rather in stability properties.

Saddle-path stability is fulfilled if for the eigenvalues of the system (A.26), λ_1 and λ_2, with $\lambda_{1,2} = (\phi_1 \pm \sqrt{\phi_1^2 + 4\phi 2})/2$, holds:

$$(|\lambda_1| < 1 \wedge |\lambda_2| > 1) \quad \vee \quad (|\lambda_1| > 1 \wedge |\lambda_2| < 1). \tag{A.27}$$

In the case of a second-order difference equation, saddle-path stability requires real eigenvalues only, i.e. $\Delta = \phi_1^2 + 4\phi_2 \geq 0$, as complex eigenvalues are complex conjugates with identical modulus. The following theorem summarizes conditions equivalent to (A.27) which do not demand explicit computation of eigenvalues.

Theorem 1. *The difference equation* $\widehat{\theta}_{Lt} = \phi_1 \widehat{\theta}_{L,t-1} + \phi_2 \widehat{\theta}_{L,t-2}$ *has real solutions and is saddle-path stable, if* $\Delta \geq 0$ *and one of the following two sets of conditions is fulfilled:*

(i) $1 - \phi_1 - \phi_2 > 0 \quad \wedge \quad 1 + \phi_1 - \phi_2 < 0,$

(ii) $1 - \phi_1 - \phi_2 < 0 \quad \wedge \quad 1 + \phi_1 - \phi_2 > 0.$

An interested reader can find the proof of these conditions in the next subsection.

In the next step, we apply the previously discussed conditions in our model. We begin by examining the existence of the steady state. Condition $1 - \phi_1 - \phi_2 \neq 0$ can be represented in terms of the slopes of the job creation and the wage curve as follows:

$$-\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} \neq \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}, \text{ if } \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1,$$

$$\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} \neq \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \text{ if } \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1.$$

In the presence of an upward-sloping wage curve, i.e., if $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, a steady state always exists, whereas for a downward-sloping wage curve, a steady state exists if the slopes of both curves differ. As regards saddle-path stability, we can distinguish between two cases. In the first case, the wage curve is upward-sloping or it is downward-sloping and steeper than the job creation curve:

$$-\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} < \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}, \text{ if } \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1,$$

$$\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} < \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \text{ if } \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1.$$

These inequalities are equivalent to $1 - \phi_1 - \phi_2 < 0$, which is the first part of condition (ii) of Theorem 1. Both parts of condition (ii) imply a restriction for u_L:

$$\begin{aligned}
&u_L > \left[\frac{2}{q_L} + \frac{r+q_L}{r+q_L+2(1-q_L)\,\Upsilon}\right]^{-1}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1,\\
&\frac{1}{\Upsilon} > u_L > \left[\frac{2}{q_L} + \frac{r+q_L}{r+q_L+2(1-q_L)\,\Upsilon}\right]^{-1}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1, \quad \text{with}\\
&\Upsilon \equiv \frac{(1-\beta_L)(1-I)}{\varepsilon_{L,\widetilde{w}_L}}\,\frac{|1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}{\varepsilon_{\widetilde{z}_L,\theta_L}}.
\end{aligned}$$

The second case corresponds to a downward-sloping wage curve being flatter than the job creation curve and represents the first inequality in condition (i) of Theorem 1:

$$\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} > \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \text{ with } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1.$$

According to both conditions (i), the resulting restriction for u_L is:

$$\frac{1}{\Upsilon} < u_L < \left[\frac{2}{q_L} + \frac{r+q_L}{r+q_L+2(1-q_L)\,\Upsilon}\right]^{-1} < \frac{q_L}{2}.$$

All these results can be summarized as follows. The model is saddle-path stable if:

(i) for an upward-sloping wage curve: $u_L > q_L/2$ (sufficient condition),

(ii) for a downward-sloping wage curve: either $1/\Upsilon > u_L > q_L/2$ (sufficient condition) or $1/\Upsilon < u_L < q_L/2$ (necessary condition).

A.7.2 Conditions for Saddle-Path Stability

Proof of Theorem 1. For $\Delta \geq 0$ to be satisfied, ϕ_2 must be either nonnegative or, if $\phi_2 < 0$, $|\phi_2| < \phi_1^2/4$. In the following, these two cases will be distinguished.

Case 1: $\phi_2 \geq 0$

We have $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$, with $\lambda_1 + \lambda_2 \neq 0$. If condition (i) is satisfied, then $\phi_1 < 1 - \phi_2 \leq 1$.

$$\lambda_1 < \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|) = 1$$

$$\lambda_2 < \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1+2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|)$$
$$= \begin{cases} -1, & \text{if} \quad \phi_1 > -2 \\ \phi_1 + 1 < -1, & \text{if} \quad \phi_1 < -2 \end{cases}$$

If condition (ii) is satisfied, then $\phi_1 > \phi_2 - 1 \geq -1$.

$$\lambda_1 > \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|)$$
$$= \begin{cases} \phi_1 - 1 > 1, & \text{if} \quad \phi_1 > 2 \\ 1, & \text{if} \quad \phi_1 < 2 \end{cases}$$

$$\lambda_2 > \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1+2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|) = -1$$

Case 2: $\phi_2 < 0$

We have $\lambda_1\lambda_2 > 0$. If condition (i) is satisfied, then $\phi_1 < \phi_2 - 1 < -1$.

$$\lambda_1 > \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1+2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|)$$
$$= \begin{cases} \phi_1 + 1 > -1, & \text{if} \quad \phi_1 > -2 \\ -1, & \text{if} \quad \phi_1 < -2 \end{cases}$$

$$\lambda_2 < \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1+2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|)$$
$$= \begin{cases} -1, & \text{if} \quad \phi_1 > -2 \\ \phi_1 + 1 < -1, & \text{if} \quad \phi_1 < -2 \end{cases}$$

If condition (ii) is satisfied, then $\phi_1 > 1 - \phi_2 > 1$.

$$\lambda_1 > \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|)$$
$$= \begin{cases} \phi_1 - 1 > 1, & \text{if} \quad \phi_1 > 2 \\ 1, & \text{if} \quad \phi_1 < 2 \end{cases}$$

$$\lambda_2 < \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 - |\phi_1 - 2|)$$
$$= \begin{cases} 1, & \text{if} \quad \phi_1 > 2 \\ \phi_1 - 1 < 1, & \text{if} \quad \phi_1 < 2 \end{cases}$$

□

A.8 Slopes and Curvatures of the JC, WC, and TAC

The following analysis is based on the general function $\bar{\alpha}(I) = bI^{\eta_H}/(1-I)^{\eta_L}$ introduced in Section 2.3 and discussed in more detail in Appendix A.5.

The slope of the JC in the $\theta_L - \widetilde{w}_L$ space is given by:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{JC}} = \Phi\,\frac{\widetilde{w}_L}{\theta_L} < 0.$$

It can be shown that the JC is convex in the $\theta_L - \widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{JC}} = \Phi\frac{\widetilde{w}_L}{\theta_L^2}\left(1 + \Phi - \frac{\mathrm{d}\ln\Phi}{\mathrm{d}\ \ln\theta_L}\right)$$
$$= \Phi\frac{\widetilde{w}_L}{\theta_L^2}\left[1 + (1-\beta_L)\left(u_L(1-I)\left(\frac{1}{\kappa_L}\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} + \frac{1}{\varepsilon_{L,\widetilde{w}_L}}\right) + (1-u_L)\left(1 + \frac{I}{\kappa_L}\right)\right)\right] > 0.$$

The positive sign is due to the fact that $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > -1$.

The slope of the WC in the $\theta_L - \widetilde{w}_L$ space is given by:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{WC}} = \Gamma\,\frac{\widetilde{w}_L}{\theta_L}\begin{cases} > 0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \\ < 0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1. \end{cases}$$

The curvature of the WC is:

$$\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{WC}} = -\Gamma\frac{\widetilde{w}_L}{\theta_L^2}\left(1 - \Gamma - \frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ \ln\theta_L}\right)$$
$$= -\Gamma\frac{\widetilde{w}_L}{\theta_L^2}\left[1 - \beta_L + \Gamma\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\left(\frac{1}{1 - \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\frac{\mathrm{d}\ln\left|\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right|}{\mathrm{d}\ln I}(\varepsilon_{L,\widetilde{w}_L} - 1) - 1\right)\right].$$

If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$ (upward-sloping WC), for the WC to be concave the expression in square brackets has to be positive.

(i) If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$, this condition is satisfied.

(ii) If $0 < \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, it is sufficient if the expression in round brackets is positive, which is satisfied in the special case of a concave isoelastic task productivity schedule $\bar{\alpha}(I) = bI^{\eta_H}$ introduced in Appendix A.5.

(iii) If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 0$, it is sufficient if the expression in round brackets is negative, which is satisfied in the special case of a convex task productivity schedule $\bar{\alpha}(I) = b(1-I)^{-\eta_L}$; see Appendix A.5.

If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$, on the other hand, the WC is downward-sloping and convex.

The TAC is downward-sloping and convex in the $I - \widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}I}\right|_{\mathrm{TAC}} = -\frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I} < 0,$$

$$\begin{aligned}\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}I^2}\right|_{\mathrm{TAC}} &= \frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I^2}\left(1+\frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}+\frac{\mathrm{d}\ln(\varepsilon_{L,\widetilde{w}_L}-1)}{\mathrm{d}\ln I}\right)\\ &= \frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I^2}\kappa_L\left(1+\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right) > 0.\end{aligned}$$

A.9 Comparative Statics: Effects on Other Variables

$$\frac{\mathrm{d}\ln \widetilde{\omega}}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{1}{\kappa_L}\,\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \;\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{1}{\kappa_L}\,|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{\varepsilon_{L,\widetilde{w}_L} - I}{\varepsilon_{L,\widetilde{w}_L}}\,\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \;\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{\varepsilon_{L,\widetilde{w}_L} - I}{\varepsilon_{L,\widetilde{w}_L}}\,|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln A_H} = \left(\frac{\kappa_L}{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} - 1\right)\frac{\varepsilon_{\widetilde{z}_L,\theta_L}\,\widetilde{z}_L}{\varepsilon_{L,\widetilde{w}_L}\,w_L}\,(1-I)\,\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \;1 > \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 - \kappa_L \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln L}{\mathrm{d}\ln A_H} = \frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \;\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln A_H} = -\frac{1-u_L}{u_L}\,\frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} < 0, & \text{if } \;\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ > 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln A_H} = \Big(|\Phi| + (1-I)\Gamma\Big)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \;\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < (1-I)\,|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

A.10 Data Description

In our quantitative analysis we use different data sets to calculate some of the parameters of the model and the targets for calibration of the remaining parameters. In the case of any skill-related quantity, we follow Battisti et al. (2018), Chassamboulli and Palivos (2014), and Krusell et al. (2000), and define high-skilled workers as workers with at least a Bachelor degree. This corresponds to levels 5-8 of the International Standard Classification of Education (ISCED 2011). Workers with less than a Bachelor degree (levels 0-4 of ISCED) belong, in fact, to the group of low- plus medium-skilled workers but they are classified as low-skilled workers for the purpose of calibration of our model that distinguishes between only two skills.

Real interest rate r. We follow Chassamboulli and Palivos (2014) to obtain real interest rates for both countries. First, we average over the government bond rates at 30-year constant maturity in the periods 1995-2005 and 2010-2017. Then, we translate the resulting annualized average bond rates into quarterly interest rates. In the next step, we calculate the quarterly GDP deflator as a ratio of the nominal GDP and the real GDP for the periods 1994-2005 and 2009-2017. Based on the GDP deflator series, we generate quarterly inflation rates. Finally, for both periods we subtract the average quarterly inflation rate from the quarterly interest rate to obtain quarterly real interest rates. Data on the market yield on US Treasury Securities at 30-year constant maturity as well as on the quarterly GDP series for both countries have been retrieved from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. Data source for the German government bond rates at 30-year constant maturity is Deutsche Bundesbank (series BBK01.WT3030).

Share of high-skilled workers H. We restrict our sample to individuals who are 25 years and older. For the US, we calculate the share of high-skilled workers in periods 1995-2005 and 2010-2017 as the average ratio of high-skilled workers (with at least a Bachelor degree) to the total labor force in the respective periods using data from the Current Population Survey (CPS). For Germany, we proceed analogously using data from the EU

Labor Force Statistic (EU-LFS) with education information derived from ISCED 2011.

Skill bias A_H/A_L. We follow Bowlus et al. (2021) to calculate the implied skill-biased technical change in the US. In particular, we assume an elasticity of substitution between high- and low-skilled workers of 5.2 and we use the time-series data on the relative skill prices and skill supplies for the US kindly provided by Audra Bowlus to obtain the path of skill bias. In the last step, we calculate the average skill bias over the periods 1995-2005 and 2010-2017.

Low-skilled unemployment rate u_L. We resort in this case to the same data sources as in the calculation of the high-skilled share: CPS for the US and EU-LFS for Germany. For the US, we compute the average ratio of unemployed low-skilled workers (with less than a Bachelor degree) to the low-skilled labor force in the periods 1995-2005 and 2010-2017. For Germany, however, data on unemployment levels by education are available only since 2005. Before 2005, only unemployment rates by education are available. Since we need the unemployment rate jointly for education levels 0-4, we weigh the unemployment rates for levels 0-2 and 3-4 by the share of persons in the corresponding education level in 2005. Finally, we compute the averages of the low-skilled unemployment rates in periods 1995 to 2005 and 2010 to 2017.

Low-skilled labor market tightness θ_L. To estimate the low-skilled labor market tightness in the US, we use data on job openings from the Job Openings and Labor Turnover Survey (JOLTS), which are available since 2001, and data on unemployment levels by education from the CPS. The data are given on a monthly basis, hence, we transform them into quarterly data. Due to the fact that the job openings are not disaggregated by education, we multiply aggregate vacancies in the US with the share of low-skilled vacancies (0-4 ISCED levels) in total vacancies in Germany between 2011 and 2017 from the IAB Job Vacancy Survey (IAB-JVS). Next, we calculate the low-skilled labor market tightness as the ratio of the estimated low-skilled vacancies to unemployed low-skilled persons (with less than a Bachelor degree). As a last step, we calculate the average low-skilled labor market tightness in the US in the periods 2001-2005 and 2010-2017.

For Germany, we use quarterly data on job vacancies by education level from the IAB-JVS, which start only in 2011, and quarterly data on unemployment levels by education from the EU-LFS. We calculate the quarterly low-skilled labor market tightness as the ratio of low-skilled vacancies (0-4 ISCED levels) to unemployed low-skilled persons, and then average over the values from 2011 to 2017. As regards the earlier time period, we use monthly data on registered vacancies from the Federal Employment Agency (BA). Due to the break in the classification in 2000 (before 2000: "gemeldete Stellen"; after 2000: "gemeldete Arbeitsstellen"), we restrict the sample to 2000-2005. Since the numbers for registered vacancies provided by the BA are much lower than the number of vacancies according to the IAB-JVS, the registered vacancies in the period 2000-2005 are adjusted using the average ratio of the IAB-JVS vacancies to the registered vacancies in the period 2011 to 2017. Moreover, since the registered vacancies are not disaggregated by education levels, the adjusted vacancies from the previous step are weighted with the average share of low-skilled vacancies (0-4 ISCED levels) in total vacancies in Germany between 2011 and 2017 from the IAB-JVS dataset. Finally, based on the obtained data we construct the average low-skilled labor market tightness between 2000 and 2005.

Skill premium w_H/w_L. The US skill premium is obtained based on the median usual weekly earnings by education provided by the CPS. We restrict the sample to wage and salary workers, excluding incorporated self-employed, who are 25 years or older. We calculate then the yearly skill premium as earnings of high-skilled workers (with at least Bachelor degree) relative to earnings of low-skilled workers (with less than Bachelor degree). Next, we average over the values of skill premia in periods 1995-2005 and 2010-2017. For Germany, we employ data from the European Union Statistics on Income and Living Conditions (EU-SILC). The data relevant in this context is the median income of individuals between 18 and 64 years by educational level. We calculate the German skill premium for each year as the median income of high-skilled (5-8 ISCED levels) relative to the median income of low-skilled (0-4 ISCED levels). The underlying data start only in 2005, so we consider the skill premium in 2005 as the skill premium for the first time period. As for the second period, we calculate the average skill premium between 2010 and 2017.

Task threshold I. According to our model, the task threshold can be obtained as the ratio of labor costs of low-skilled workers to the aggregate output, with aggregate output being equal to total labor costs. By assuming that labor costs are equal to labor compensation, the task threshold is calculated from the data as the average ratio of low-skilled and middle-skilled labor compensation to total labor compensation over the corresponding periods. For the period 1995 to 2005 the data for both countries are obtained from the Socio-Economic Accounts (SEA) which is a part of the World Input-Output Database (WIOD) Release 2013; see Timmer et al. (2015). Since the SEA data are available until 2009, for the second period we use data on labor compensation for Germany from EU Klems Release 2017 (available until 2015) to calculate the average task threshold from 2010 to 2015 for Germany. Since there is no corresponding EU Klems data for the US, we assume that the development of the share of low- and middle-skilled labor compensation in the US has been similar to Germany. Therefore, we estimate the task threshold in the US from 2010 to 2015 as:

$$I_{\text{US,10-15}} = I_{\text{US,95-05}} * \frac{I_{\text{DE,10-15}}}{I_{\text{DE,95-09}}},$$

where $I_{\text{US,95-05}}$ denotes the average task threshold in the US for the period 1995 to 2005, calculated from SEA, and $I_{\text{DE,10-15}}/I_{\text{DE,95-09}}$ is the average task threshold in Germany between 2010 and 2015 (calculated from EU Klems) relative to the average task threshold between 1995 and 2009 (calculated from SEA).

Relative real total factor productivity (RTFP). We calculate the time series of RTFP of Germany relative to the US using Penn World Tables 10.0 provided by Feenstra et al. (2015) as follows:

$$\frac{RTFP_{t,DE}}{RTFP_{t,US}} = relTFP_{2017} * \frac{RTFPInd_{t,DE}}{RTFPInd_{t,US}},$$

where $relTFP_{2017}$ denotes the value of the TFP in current prices (expressed in US dollars) in Germany relative to the US in the base year 2017, and $RTFPInd_{t,c}$ is the RTFP index series for country $c \in \{US, DE\}$. The RTFP index series are based on national currencies. Therefore, before we apply the above formula, we first transform the index series for Germany into constant US dollars using the EUR/USD exchange rate in 2017. As the final step, we calculate the average RTFP of Germany relative to the US over the years 1995 to 2005 and 2010 to 2017, respectively.

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