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DISCUSSION PAPER SERIES

IZA DP No. 15149

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ABSTRACT

Multidimensional Equality of Opportunity in the United States*

Are the United States still a land of opportunity? We provide new insights on this question by invoking a novel measurement approach that allows us to target the joint distribution of income and wealth. We show that inequality of opportunity has increased by 77% over the time period 1983-2016. Increases are driven by two distinct forces: (i) a less opportunity-egalitarian distribution of income until 2000, and (ii) a less opportunity-egalitarian distribution of wealth after the financial crisis in 2008. In sum, our findings suggest that the US have consistently moved further away from a level playing field in recent decades.

JEL Classification: D31, D63, J62

Keywords: fairness, intergenerational mobility, time trends, measurement

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1 INTRODUCTION

In a fair economy, people act on a level playing field to acquire monetary resources. This idea—oftentimes labeled as *equality of opportunity*—is widely reflected in fairness conceptions of academic philosophers and the general public alike (Alesina et al., 2018; Almås et al., 2020; Arneson, 2018; Cappelen et al., 2007; Cohen, 1989; Fong, 2001; Rawls, 1971; Roemer, 1998). As a consequence, there is an active literature in economics that assesses the satisfaction of the opportunity-egalitarian ideal in different countries at different points in time.[1] We contribute to this literature by providing the first analysis of the association between family background characteristics and the joint distribution of income and wealth in the US.

Existing studies on inequality of opportunity and intergenerational mobility predominantly focus on income to measure monetary resources.[2] However, there are at least two important reasons for why income and wealth are imperfect substitutes when analyzing the degree to which the monetary resources of individuals depend on their family background characteristics. First, well-off parents directly transmit monetary resources to the next generation through bequests and inter vivo gifts (Boserup et al., 2016; Elinder et al., 2018; Wolff, 2002). In turn, expected wealth transfers distort the education and labor supply decisions of children (Kindermann et al., 2020; Koeniger and Prat, 2018; Kopczuk, 2013). Such behavioral responses may therefore create a wedge between the relative positions of individuals in the income and wealth distribution, respectively: individuals that receive a lot of wealth from their parents are not necessarily those who generate a lot of income for themselves. This observation is particularly relevant for the analysis of time trends as the quantitative importance of inheritances has grown in many Western societies in recent decades (Piketty and Zucman, 2015). Second, changes in wealth are a function of savings and changes in asset prices. While the savings channel depends on income, the price channel depends on portfolio compositions. Therefore, changes in asset prices are another force that drives a wedge between the relative positions of individuals in the income and wealth distribution, respectively. Again, this observation is particularly relevant for the analysis of

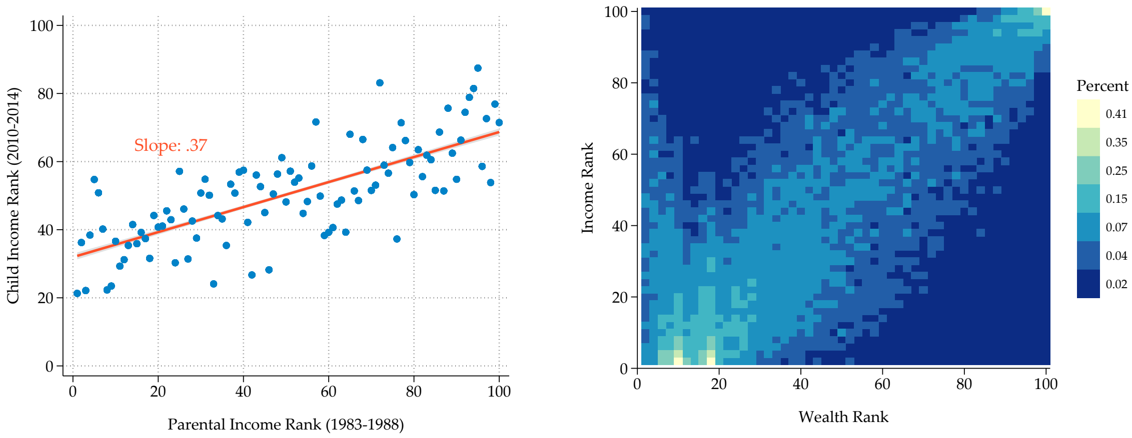
[1]For the United States, an extensive literature investigates intergenerational income mobility (among others Chetty et al., 2014a; Davis and Mazumder, 2017; Solon, 1992). Studies aiming at more comprehensive conceptions of inequality of opportunity include Hufe et al. (2021), Niehues and Peichl (2014), and Pistolesi (2009).

[2]Exceptions include Adermon et al. (2018), Charles and Hurst (2003), Fagereng et al. (2021), and Pfeffer and Killewald (2018). While these studies focus on wealth, they also invoke a unidimensional conception of monetary resources.

time trends as wealth-to-income ratios—and therefore the sensitivity of wealth to asset price fluctuations—has grown in recent decades (Kuhn et al., 2020).

In Figure 1, we use data from the Panel Study of Income Dynamics (PSID) to show that these concerns are relevant for the analysis of equal opportunities in the United States. In Panel (A), we replicate the well-known finding that child incomes increase with the income of their parents during childhood: an increase of parental income by 10 percentile ranks is associated with an average increase of 3.7 percentile ranks in child income. Albeit being more noisy, this slope estimate is very similar to the slope estimate of 0.34 by Chetty et al. (2014a). In Panel (B), a heatmap of income and wealth ranks demonstrates that income and wealth are far from perfect correlates. Taken to-

FIGURE 1. Intergenerational Income Mobility and the Distribution of Monetary Resources in the United States



(A) Income Transmission

(B) Joint Distribution of Income and Wealth

Data: PSID.
Note: Panel (A) shows a non-parametric binned scatter plot of average child income ranks in the years 2010-2016 by income rank of their parents in the years 1983-1988. All individuals are aged 25-60. Panel (B) shows a heatmap of year-specific income and wealth ranks for the pooled sample of individuals aged 25-60 in the time period 1983-2016. Each data point shows the share of individuals in a fixed two-percentile income (wealth) bin that belong to a particular two-percentile wealth (income) bin. See Section 3 for detailed definitions of income and wealth.

gether, these patterns suggest that unidimensional analyses of equality of opportunity and intergenerational mobility may miss important information when analyzing the playing field for the acquisition of monetary resources in the US.

In this paper, we address these shortcomings by analyzing the association between family background characteristics and the joint distribution of income and wealth. In particular, we use the PSID to implement a novel measure of multidimensional equality of opportunity (Kobus et al., 2020). Our analysis proceeds in two steps. First, we construct an *intergenerational sample* in which we measure equality of opportunity in monetary resources by using parental income rank as the sole proxy variable for socio-economic background characteristics. This practice is consistent with the literature on intergenerational mobility; however, the sparsity of data links across generations prevents meaningful analyses of time trends. Second, we construct an *individual sample* in which we substitute parental income rank by a vector of alternative socio-economic background characteristics. These data are available on an annual basis and allow us to assess equality of opportunity for the acquisition of income and wealth over the period 1983-2016.

Our findings can be summarized as follows. First, inequality of opportunity is consistently higher when accounting for the multidimensionality of monetary resources. This finding entails that unidimensional analyses that focus on income only underestimate the extent to which monetary resources are associated with family background characteristics. Second, inequality of opportunity in 2016 is 77% higher than in 1983; hence, the playing field in the US has become more tilted in recent decades. Time trends are markedly different when accounting for the multidimensionality of monetary resources. For example, an exclusive focus on income suggests only moderate increases in unequal opportunities after the year 2000. This relative stability, however, is accompanied by strong increases in the wealth dimension. As a consequence, when accounting for the multidimensionality of monetary resources, it is much harder to reject the hypothesis that opportunities for the acquisition of monetary resources have become more unequal in recent years.

The contribution of this paper is twofold. First, we complement recent literature that characterizes the joint distribution of income and wealth in the US (Berman and Milanovic, 2020; Kuhn et al., 2020). This literature focuses on inequalities in outcomes but remains silent on opportunities and intergenerational transmission processes. Second, we extend the literature on equality of opportunity by a multidimensional perspective on monetary resources. In particular, we provide novel insights regarding the development of equality of opportunity in the United States. While existing literature documented relative stability of equality of opportunity in terms of income after 2000 (Chetty et al., 2014b; Davis and Mazumder, 2017), we show that decreases come to the fore once we account for the wealth dimension.

The remainder of the paper is organized as follows. Section 2 introduces the measurement framework. Section 3 describes the data. We present baseline results in Section 4 and conduct sensitivity analyses in Section 5. Section 6 concludes the paper.

2 MEASUREMENT

Consider a population $\mathcal{N} := \{1, ..., N\}$ and a set of outcomes $\mathcal{K} := \{1, \ldots, K\}$ that capture monetary resources. Individuals $i \in \mathcal{N}$ receive utility from $q \in \mathcal{K}$. Hence, we can summarize the distribution of monetary resources in the economy by a multidimensional vector of outcomes $X := \{x_{1,1}, ..., x_{N,K}\}$ that has a matrix of population means X^{μ}. Outcomes are determined by two sets of factors: a set Ω that captures family background characteristics such as parental income ranks, and a set Θ that captures individual choices. We define $\omega_i \in \Omega$ ($\theta_i \in \Theta$) as a comprehensive description of the family background characteristics (choices) of $i \in \mathcal{N}$. For each outcome q, there is an outcome generating function defined as follows:

$$x_q = f_q(\omega_i, \theta_i),\ \forall i \in \mathcal{N}. \tag{1}$$

In an equal-opportunity society, outcome differences are determined by individual choices θ_i but are invariant to family background characteristics ω_i (Roemer, 1998). There are different ways of translating this idea into measures. Most empirical literature relies on an *ex-ante* approach, which broadly consists of two steps. First, one partitions the population into a set of types $T = \{t_1, ..., t_M\}$. Individuals belong to one type if they share the same set of family background characteristics: $i, j \in t_m \Leftrightarrow \omega_i = \omega_j$. For example, in rank-rank measures of intergenerational income mobility, types are defined by parental income ranks. Second, one summarizes differences in average outcomes across types by regressing child outcomes on a measure of family background characteristics:

$$x_{iq} = \alpha_q + \beta_q \omega_i + \epsilon_{iq}. \tag{2}$$

In existing literature, there are two prominent ways of summarizing the resulting information to obtain measures of inequality of opportunity: (i) β_q, which is the standard statistic in the literature on *intergenerational mobility* (Black and Devereux, 2011). (ii) $I(\mathbb{E}[x_{iq}|\omega_i] = X_{\mu})$, where $I()$ is any inequality index. This is the standard statistic in the literature on *equality of opportunity* (Roemer and Trannoy, 2016). Clearly, both measures are isomorphic and capture the opportunity-egalitarian idea: the higher β_q, the more are life outcomes x_q predicted by family background characteristics ω_i, and the higher the corresponding measure of inequality of opportunity.

In this paper, we follow the tradition of the equality of opportunity literature and summarize outcome differences across types with an inequality index. In particular, we use the measure developed in Kobus et al. (2020), which allows us to account for the multidimensionality of monetary resources. It is the only multidimensional measure satisfying *inequality aversion between types* and *within-type transfer insensitivity*—which are standard principles for equal-opportunity measures, as well as *monotonicity*, *utilitarian aggregation*, and *ratio scale invariance*—which are standard properties of inequality indexes.[3] Furthermore, it can be decomposed by the different outcome dimensions—a property that is non-standard for multidimensional measures and which we exploit in our empirical analysis of section 4. Formally, the measure is given by

$$I(X) = 1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \frac{U^t[(X_\mu)_1^t]}{U^t[(X^\mu)_1^t]} \right)^{\frac{1}{\sum_{q=1}^{K} r_q}}. \tag{3}$$

There are three elements that require further explanation for an intuitive understanding of the measure.[4] First, U^t represents type-specific utility from obtaining income and wealth, respectively. Utility functions are concave, submodular, and of the form $U^t = \prod_{q=1}^{K} a_t (X_{iq}^t)^{r_q}$. Due to concavity, the measure exhibits inequality aversion between types. As a consequence of submodularity, the measure exhibits sensitivity to correlation-increasing transfers. That is, inequality of opportunity increases with the cross-type association of income and wealth. Note that the specific form of the utility function is non-arbitrary and derived from first principles—see Kobus et al. (2020) for details. Second, $r_q < 0$ are dimension weights that determine aversion to between-type inequalities in outcome dimension q. The lower r_q, the higher the concavity in U^t, and the more sensitive is the measure to between-type differences in outcome dimension q.[5] Third, $a_t < 0$ are type weights that determine how much the social planner values type-specific outcomes. The lower a_t, the higher the weight attached to type t. Note that r_q and a_t are parameters chosen by the researcher. In our empirical application, we therefore show that our main conclusions are insensitive to a wide range of plausible parameter choices.

[3] See Kobus et al. (2020) for a derivation of the measure from first principles. Note that we obtain *within-type transfer insensitivity* by using a simplified version of their measure with $\delta_t = 1$.

[4] In Supplementary Material A, we furthermore provide some simple examples to illustrate its mechanics and properties.

[5] r_q is related to the degree of inequality aversion via $r_q = 1 - \epsilon_q$. Existing literature usually chooses $\epsilon_q \in [1, 2]$ (Young, 1990).

Inequality of opportunity is minimized if there is perfect equality between types, i.e. $X_\mu = X^\mu \Leftrightarrow I(X) = 0$. Furthermore, the measure is bounded in the interval $[0, 1)$. It follows the tradition of normative inequality measures and therefore has an intuitive interpretation (Atkinson, 1970; Kolm, 1969; Sen, 1973). For example, a value of 0.25 (0.5) means that society is giving up 25% (50%) of its resources in every dimension $q \in \mathcal{K}$ due to existing inequality of opportunity.

3 DATA

Data Source. In this paper, we assess the extent of equal opportunities in the US while accounting for the multidimensionality of monetary resources and paying attention to changes over time. Therefore, we require data with detailed information on income, wealth, and family background characteristics that are available for a long period of time. In the US, the Panel Study of Income Dynamics (PSID) is the only publicly available data source that satisfies these criteria. For example, while the Survey of Consumer Finances (SCF) offers a long time series on household income and wealth, it does not contain detailed information on the family background characteristics of its respondents.

The PSID is the world's longest running household panel survey, tracking a nationally representative sample of US households from 1968 until today (PSID, 2021). Since its inception, the PSID collects rich information on income and family background characteristics. Since 1984 it also collects data on wealth.[6] Children who leave the parental household become independent units in the PSID sampling frame. As a consequence, it is possible to link data across generations. In its most recent waves, the PSID comprises more than 9,000 US households.

Information on income is collected for the year predating the survey year. Hence, we use information over the income reference (survey) period 1983-2016 (1984-2017). We now turn to a description of relevant variables.[7]

[6]Until 1999 wealth information was collected every five years. Since then, the wealth questionnaire is a regular part of every PSID wave.

[7]Data preparation follows the protocol outlined in Hufe et al. (2021). For example, we re-weight data to match the Current Population Survey and correct for under-reporting in both government benefits and labor income. Furthermore, we follow their coding protocol for income and family background variables. Please see their data appendix for detailed descriptions.

Monetary Resources. We consider two dimensions of monetary resources: income and wealth. We measure income as annual disposable household income. It comprises total household income from labor, asset flows, windfall gains, private transfers, public transfers, private retirement income and social security pensions net of total household taxes. We scale all household incomes by the modified OECD equivalence scale and express income in 2015 USD.

We measure wealth as household net worth. It comprises the sum of home equity, other real estate, private businesses, vehicles, transaction accounts, corporate equities, annuities/IRAs and other savings net of any debt. In analogy to income, we scale household wealth by the modified OECD equivalence scale and express it in 2015 USD.

Wealth data in the PSID is often considered inferior to wealth data in the SCF. Therefore, we compare PSID and SCF with respect to time trends in household net worth in Supplementary Figure S.1. Due to oversampling of wealthy households, the SCF provides better coverage at the top and assigns a larger share of total net worth to the top 10% of the wealth distribution. Yet, in our analysis level differences at the top are the only notable difference between PSID and SCF. Importantly, time trends in household net worth are consistent across both data sources.[8]

Family Background Variables and Types. We consider two alternative ways to measure family socio-economic status and to partition the population into types. First, we use parental income ranks with respect to total incomes of mothers and fathers averaged over the years 1983-1988. Second, we use a vector of alternative socio-economic background variables. This vector includes parental education (3 categories), parental occupation (3 categories), race (2 categories), and Census region of upbringing (2 categories). We partition the population into 36 types based on the combination of these family background variables.

Estimation Samples. We base our estimates on two different samples. First, we construct an *intergenerational sample*. Leveraging the panel dimension of the PSID, we match all respondents to their biological or adoptive parents. In turn, we drop observations with (i) missing links between parents and children, (ii) missing information on parental income, (iii) missing information on individual income and wealth, and (iv) missing information on parental education, parental occupation, race and region

[8] See also Pfeffer et al. (2016), for a detailed comparison of wealth definitions in PSID and SCF.

of upbringing. Lastly, we restrict observations to children aged 25-60 in the time period 2010-2016 and parents aged 25-60 in the time period 1983-1988.[9] As a consequence, we obtain a sample of $1,366$ individuals. The *intergenerational sample* allows us to proxy ω with parental income rank, which is common practice in the literature on intergenerational mobility. However, the *intergenerational sample* imposes severe restrictions on the analysis of time trends since one requires information on both parental and child outcomes while allowing for a sufficient time span between these observations.

Second, to investigate time trends, we construct an *individual sample*. In contrast to the previous sample, we drop requirements (i) and (ii). Again, we limit the sample to individuals aged between 25-60. As a consequence, we obtain a sample of at least $4,000$ observations in every year over the period 1983-2016. The *individual sample* allows us to monitor the development of equality of opportunity in the US over a 33-year period while proxying ω with a vector of family background characteristics.

Descriptive statistics for all estimation samples are disclosed in Table S.1.

4 RESULTS

Our analysis proceeds in two steps. First, we use the *intergenerational sample* to measure equality of opportunity. Thereby, we either use parental income ranks or the vector of socio-economic background variables to proxy for family background characteristics. We will show that both approaches yield very similar results. Second, having validated the use of alternative socio-economic background variables, we use the *individual sample* to analyze time trends in equality of opportunity for the acquisition of monetary resources.

Empirical implementation of our measure requires choices for r_q, a parameter governing the aversion to between-type inequality in dimension q, and a_t, a parameter governing sensitivity to between-type inequalities in the tails of the distribution. For our baseline estimates, we choose dimension weights $r_{Income} = r_{Wealth} = -0.2$. Furthermore, we choose a_t to be inversely proportional to type ranks in monetary resources. In

[9] Existing literature documents life-cycle bias in intergenerational mobility estimates (Haider and Solon, 2006; Nybom and Stuhler, 2016). Due to heterogeneity in life cycle earnings profiles, estimates obtained when children are young (old) tend to be downward (upward) biased. This bias is typically minimized by measuring income in midlife. In Supplementary Figure S.3 we show that restricting the child generation to narrower age ranges does not affect our estimates in a systematic way.

Section 5, however, we show that our main conclusions are robust to different choices of both r_q and a_t.

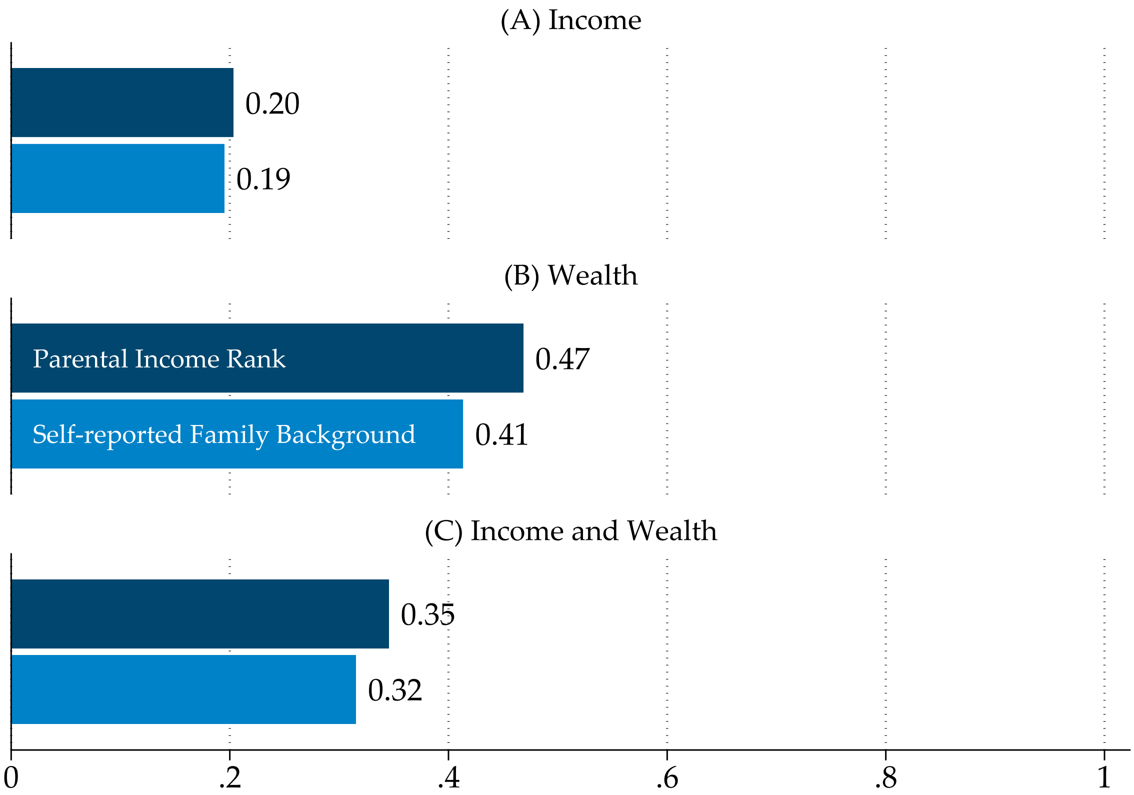
Intergenerational Estimates. Figure 2 shows estimates for inequality of opportunity in the *intergenerational sample* for different combinations of outcomes and family background variables.

First, we focus on the dark-blue bars that show estimates based on parental income ranks. Therefore, they provide a close analogue to measures of intergenerational mobility. In Panel (A), we measure monetary resources by income only—which is prevalent practice in extant literature. Inequality of opportunity amounts to 0.20. In Panel (B), we measure monetary resources by wealth. As a consequence, inequality of opportunity more than doubles to a level of 0.47. Finally, in Panel (C) we account for the multidimensionality of monetary resources by considering both income and wealth. Inequality of opportunity in monetary resources amounts to 0.35. These results suggest that we tend to underestimate tilt in the playing field when relying on income as the sole proxy for monetary resources.

It is well-known that PSID subsamples with intergenerational links are positively selected with respect to their socio-economic status (Ward, 2021). Therefore, we re-weight the *intergenerational sample* to match the broader population characteristics with respect to parental education, parental occupation, race, Census region of upbringing, and age. Indeed, the re-weighted sample is less likely to be white, more likely to have been raised in the South, and has parents of lower educational and occupational status. Furthermore, the re-weighted sample is characterized by lower levels of both income and wealth (Table S.1). However, we note that our conclusions about inequality of opportunity in the intergenerational sample are robust to re-weighting and the associated changes in sample characteristics (Figure S.2).

Second, we focus on a comparison between dark-blue bars and light-blue bars. To estimate the latter, we replace parental income ranks with socio-economic background characteristics. Results remain unchanged by this alternation. Hence, we conclude that it is unimportant whether we proxy family background characteristics by parental income ranks or a vector of socio-economic background characteristics. In general, this is an encouraging message as data sets including intergenerational links are much scarcer than data sets including retrospective information on various socio-economic

FIGURE 2. Inequality of Opportunity in the US Intergenerational Sample



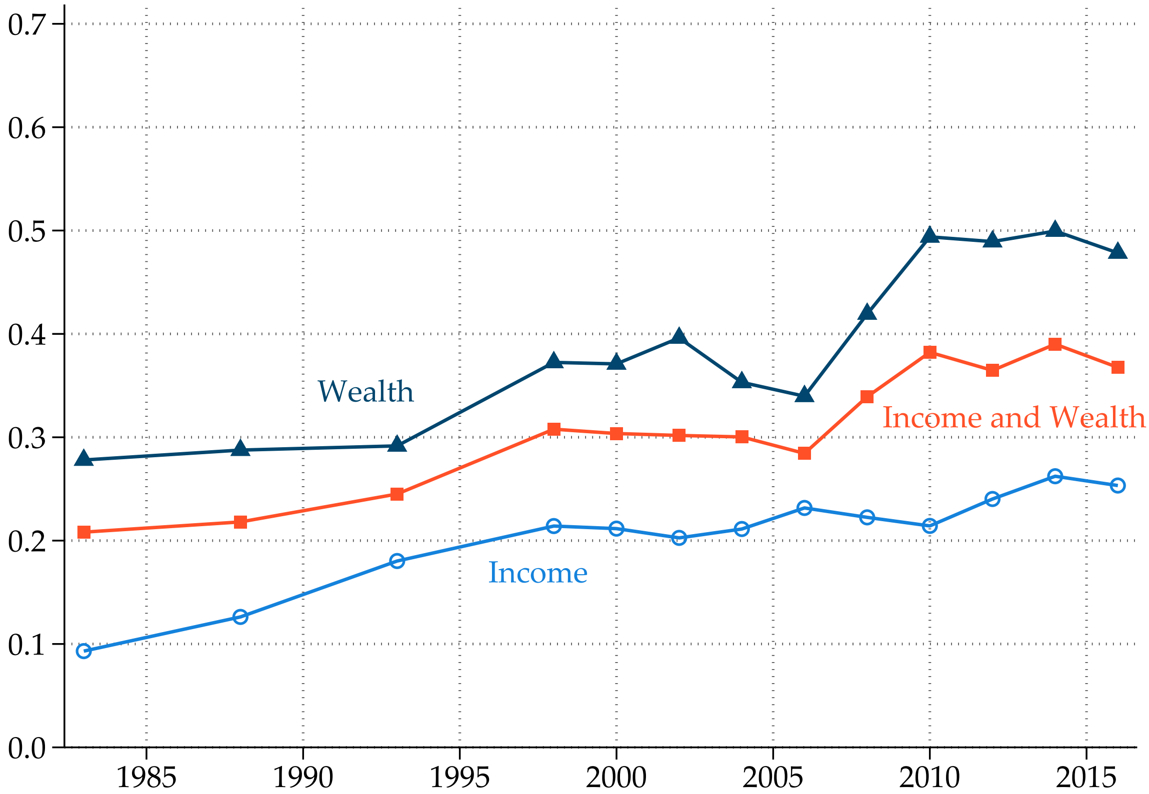
Data: PSID.
Note: This figure shows estimates of inequality of opportunity in the US for the *intergenerational sample*. Panel (A) (Panel [B]) shows results for a unidimensional definition of monetary resources based on income (wealth). Panel (C) shows results for a multidimensional definition of monetary resources based on income and wealth. In each panel, inequality of opportunity estimates are based on 36 types according to alternative definitions (Panel [A]: parental income rank; Panel [B]: parental education, parental occupation, race, region of upbringing). Estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

background variables. In the particular case of this paper, it allows us to assess time trends in the statistics of interest.[10]

Time Trend (1983-2016). Figure 3 shows the development of inequality of opportunity in the US over the period 1983-2016. The following patterns emerge.

[10] A similar strategy is applied by Jácome et al. (2021), who estimate historical trends in US intergenerational income mobility by approximating parental income with race, region of upbringing, and the occupation of fathers.

FIGURE 3. Inequality of Opportunity in the US, 1983-2016
Baseline Estimates



Data: PSID.
Note: This figure shows estimates of inequality of opportunity in the US for the *individual sample* over the time period 1983-2016. Inequality of opportunity estimates are based on 36 types according to the following socio-economic background characteristics: parental education, parental occupation, race, region of upbringing. Estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

First, inequality of opportunity in income increased from 0.09 to 0.25—and therefore almost tripled—over time. Related to this general trend, we can distinguish two distinct time periods. On the one hand, we observe marked increases from 1983 to 1998. On the other hand, there are only moderate increases after the year 2000. This twopartite pattern is consistent with findings from the literature on intergenerational mobility (Aaronson and Mazumder, 2008; Davis and Mazumder, 2017; Hartley et al., 2017). According to these studies, equality of opportunity has decreased in the 1980s and 1990s and remained rather constant after 2000.

Second, inequality of opportunity in wealth increased by 72% from 0.28 to 0.48 over time. Again, we can distinguish two distinct time periods driving this trend. On the one hand, we observe moderate increases from 1983 to 2006. In this time period, increases in the stock market were accompanied by a robust housing market (Kuhn et al., 2020; Wolff, 2017). Owner-occupied housing has higher weight in the portfolios of individuals from lower socio-economic background. Therefore, increasing house prices attenuated the tendency towards a less opportunity-egalitarian distribution of wealth. On the other hand, differences in portfolio compositions across socio-economic backgrounds started working in the opposite direction with the financial crisis in 2008. While the stock market experienced a quick recovery, house prices did not catch up to their pre-crisis level. As a consequence, the wealth distribution has become less opportunity-egalitarian with the crisis in 2008—a trend that has not reverted ever since.

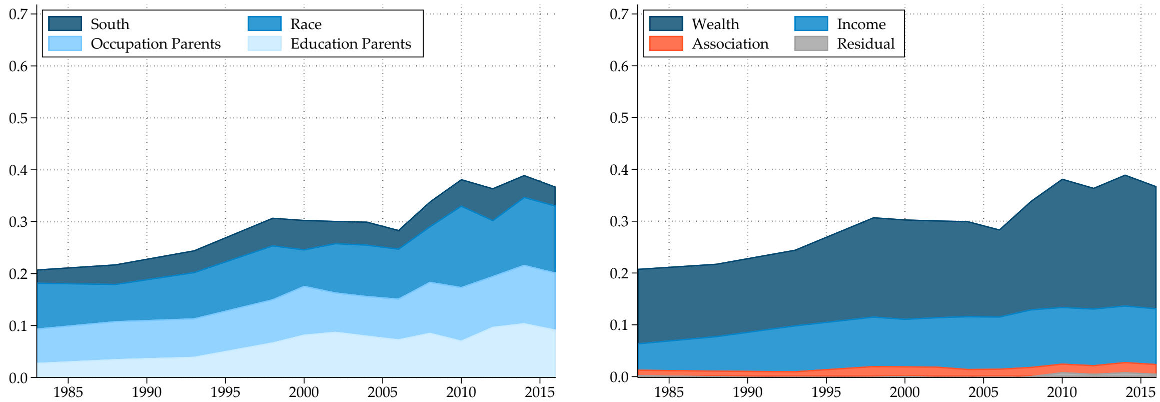
Taken together, the playing field for the acquisition of monetary resources has become more tilted over time.[11] Starting at a level of 0.21 in 1983, inequality of opportunity in the joint distribution of income and wealth has attained a level of 0.37 in the latest period of observation. This shift corresponds to an increase of 77%. Importantly, the trend towards decreasing opportunities to acquire monetary resources clearly continues after the year 2000. This finding can be related to extant literature invoking intergenerational mobility estimates to conclude relative stability in equality of opportunity in recent years (Aaronson and Mazumder, 2008; Chetty et al., 2014b; Davis and Mazumder, 2017; Hartley et al., 2017; Hertz, 2007; Lee and Solon, 2009). To the extent that these works aim to proxy financial opportunities more generally, they miss important information by focusing on income only. Indeed, when accounting for the multidimensionality of monetary resources, it is much harder to reject claims that opportunities in the US have declined after the year 2000.

Decomposition. To develop a better understanding of these trends, we conduct a Shapley value decomposition based on Shorrocks (2013), i.e. we decompose the trend in equality of opportunity into the contributions from different family background characteristics: parental education, parental occupation, race, and the region of upbringing. The results of this decomposition are shown in Panel (A) of Figure 4.

The increase in unequal opportunities over time is especially driven by parental background characteristics and race. First, 68% of the overall increase in inequality of opportunity can be explained by parental education and occupation. This finding is con-

[11] We note that the multidimensional measure is not just a weighted average of unidimensional measures. It is sensitive to correlation increasing transfers, i.e. even if between-type inequality in income and wealth remain constant, the measure increases (decreases) if the association in between-type income and wealth distributions increases (decreases). See also our illustration based on simple examples in Supplementary Material A as well as the decomposition in Figure 4, Panel (B).

FIGURE 4. Inequality of Opportunity in the US, 1983-2016
Decomposition by Background Characteristic and Outcome Dimension



(A) Shapley Value Decomposition

(B) Attribute Decomposition

Data: PSID.
Note: This figure shows a decomposition of inequality of opportunity in the US for the *individual sample* over the time period 1983-2016. Inequality of opportunity estimates are based on 36 types according to the following socio-economic background characteristics: parental education, parental occupation, race, region of upbringing. Estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$. The decomposition in Panel (A) is based on the Shapley value procedure proposed in Shorrocks (2013). The decomposition in Panel (B) is based on the attribute decomposition derived in Supplementary Material B.

sistent with Hufe et al. (2021) who identify these components as the strongest drivers of increasing inequality of opportunity for income acquisition in the US. Second, 26% of the overall increase in inequality of opportunity can be explained by race. At first glance, this finding appears at odds with the stagnation of racial income gaps since the civil rights era (Bayer and Charles, 2018; Derenoncourt and Montialoux, 2021). However, we note that the importance of race increases only after the 2008 financial crisis. Therefore, decreased opportunities to acquire monetary resources are most likely driven by the sustained effect of the financial crisis on the housing wealth of Black Americans (Kuhn et al., 2020; Wolff, 2017).

We also conduct an attribute decomposition, i.e. we decompose the time trend into the contributions of (i) inequality of opportunity in income, (ii) inequality of opportunity in wealth, as well as (iii) the cross-type association in both outcomes. The last dimension is of particular interest as it cannot be detected in unidimensional measures of inequality of opportunity. In Supplementary Material B, we show that $I(X)$ can be decomposed as follows:

$$
\begin{aligned}
I(X) = {} & \frac{r_1}{r_1+r_2} \underbrace{\left(1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1} \right)^{r_1} \right)^{\frac{1}{r_1}} \right)}_{=I_1(\text{Income})} \\
& + \frac{r_2}{r_1+r_2} \underbrace{\left(1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2} \right)^{r_2} \right)^{\frac{1}{r_2}} \right)}_{=I_2(\text{Wealth})} \\
& + \frac{1}{r_1+r_2} \underbrace{\left(1 - \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}} \right)}_{=\kappa_I(\text{Association})} \\
& + R,
\end{aligned} \tag{4}
$$

where μ_q (μ_q^t) define population (type) means in outcome dimension q, I_q is a unidimensional index of inequality of opportunity in outcome dimension q, κ_I is a measure of cross-type association in outcomes, and R is a residual resulting from linear approximation.[12]

The results of this decomposition are shown in Panel (B) of Figure 4. Inequality of opportunity in both income and wealth are increasing over time. 36% and 58% of the overall increase in inequality of opportunity can be explained by trends in unidimensional inequality of opportunity in income and wealth, respectively. Recent research points to an increasing correlation of income and wealth in the US (Berman and Milanovic, 2020; Kuhn and Ríos-Rull, 2016). However, this increase in association at the individual level is only weakly reflected in the association of these outcomes across family background types. The cross-type association of income and wealth explains 5% of the overall increase in unequal opportunities.

5 SENSITIVITY ANALYSIS

Parameter Choices. We assess the sensitivity of our main conclusions to changes in the measurement parameters, i.e. dimension weights r_q and type weights a_t.

[12] In fact, I_q are unidimensional inequality of opportunity measures based on the Atkinson (1970) index of inequality.

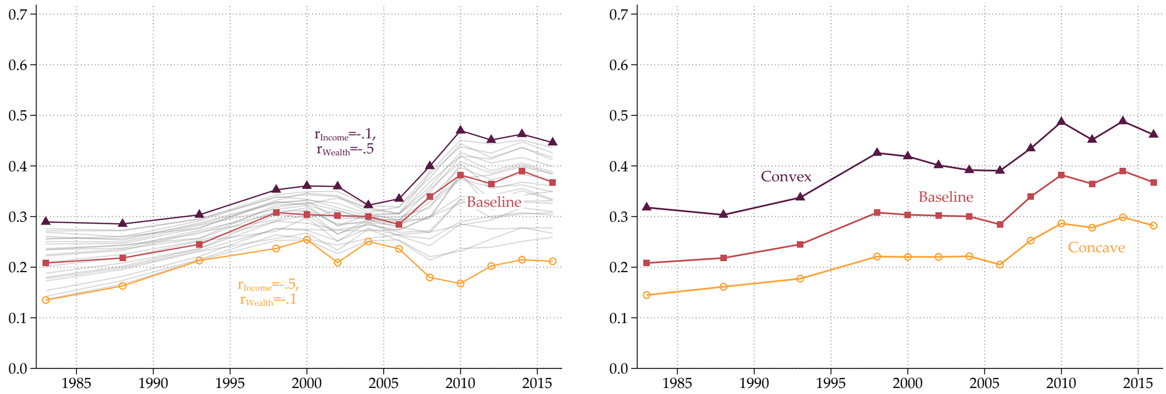
First, dimension weights r_q determine inequality aversion in income and wealth, respectively. In our baseline estimates, we give both dimensions equal weight and choose $r_{Income} = r_{Wealth} = -0.2$. Panel (A) of Figure 5 shows alternative results for all pairwise combinations over the parameter grid $r_q \in (-0.1, -0.2, -0.3, -0.4, -0.5)$. Lowest estimates of inequality of opportunity are obtained for $r_{Income} = -0.5$ and $r_{Wealth} = -0.1$; that is, in the case where we place little weight on the less opportunity-egalitarian wealth dimension, and more weight on the income dimension. We note that such income-focused parameterization yields a flat trend after the year 2000. This result is expected and consistent with existing work on intergenerational income mobility (Aaronson and Mazumder, 2008; Davis and Mazumder, 2017; Hartley et al., 2017). Conversely, highest estimates of inequality of opportunity are obtained for $r_{Income} = -0.1$ and $r_{Wealth} = -0.5$; that is, in the case where we place more weight on the less opportunity-egalitarian wealth dimension, and little weight on the income dimension. We note that even small increases in the wealth focus lead to upward corrections in inequality of opportunity estimates and overturn the conclusion of flat time trends after the year 2000.

Second, type weights a_t determine inequality aversion in the bottom and the top of the type-specific distribution, respectively. In our baseline estimates, we choose linear a_t that are inversely related to type ranks in monetary resources. Panel (B) of Figure 5 shows alternative results for convex (a_t^2) and concave type weights ($a_t^{0.5}$). Lowest estimates of inequality of opportunity are obtained for concave type weights, where we place relatively less weight on inequality in the lower tail of the distribution. Conversely, highest estimates of inequality of opportunity are obtained for convex type weights, where we place relatively more weight on inequality in the upper tail of the distribution. In spite of changes in levels, our conclusions with respect to time trends are insensitive to parameter choices in a_t.

Data Choices. In Supplementary Figure S.4, we furthermore document that our main conclusions are robust to different choices in the definition of relevant variables.

First, we recompute inequality of opportunity for different ways of dealing with non-positive income and wealth. For our baseline estimates, we drop observations with negative income/wealth and set observations with zero income/wealth to 1 USD, respectively. Alternatively, we (i) drop all observations with negative and zero income/wealth, or (ii) retain all observations with negative and zero income/wealth in the sample. In spite of slight level changes, time trends are very similar regardless of alternations in these specification choices.

FIGURE 5. Inequality of Opportunity in the US, 1983-2016
Sensitivity to Parameter Choices



(A) Alternations in r_q (B) Alternations in a_t

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US for the *individual sample* over the time period 1983-2016 under different parameter choices. Panel (A) shows the sensitivity to alternations in r_q. We display are all pairwise combinations of $r_{Income} \in (-0.1, -0.2, -0.3, -0.4, -0.5)$ and $r_{Wealth} \in (-0.1, -0.2, -0.3, -0.4, -0.5)$. The central line replicates our baseline estimates from Figure 3 where we use linear $r_{Income} = r_{Wealth} = -0.2$. Panel (B) shows the sensitivity to alternations in a_t. We construct convex (concave) weights as a_t^2 ($a_t^{0.5}$). The central line replicates our baseline estimates from Figure 3 where we use linear a_t.

Second, we recompute inequality of opportunity for different partitions into types. To this end, we code three additional variables and add them to the vector of family background characteristics: the number of siblings (11 categories), a dummy indicating whether at least one parent is foreign born, and a dummy indicating whether the respondent grew up with both parents. In turn, we follow Brunori et al. (2021) and let a regression tree algorithm decide on the optimal type partition. We re-estimate the optimal type partition in each year of our analysis. Again, time trends are very similar to our baseline estimates, suggesting that our conclusions are robust to alternations in type partitions.

Third, we recompute inequality of opportunity while smoothing transitory changes in income and wealth. In particular, we replace annual values of income and wealth with their 5-year averages. As a consequence, outcome variables provide better proxies for the long-term income and wealth potential of individuals (Solon, 1992). However, time trends are again very close to our baseline estimates and do not overturn our main conclusions.

Fourth, we recompute inequality of opportunity using alternative definitions of income and wealth. One may argue that our baseline definitions creates a mechanical relationship between income and wealth. Wealth enters household income through capital returns; reversely, savings from household income increase wealth in a given time period. To address this concern, we divorce both concepts as follows: first, we replace household disposable income with household labor market earnings; i.e. we use an income concept that is not mechanically related to asset returns. Second, we adjust household net worth by deducting active saving in a given time period; i.e. we use a wealth concept that is not mechanically related to the contemporaneous saving decisions of the households.[13] We note that our time series are not sensitive to these adjustments, suggesting that mechanical relationships between income and wealth are not the main driver of our results.[14]

We conclude: while the level of inequality of opportunity and the magnitude of its increase vary with different measurement choices, all main conclusions from our baseline estimates remain in place.

6 CONCLUSION

In this paper, we study inequality of opportunity for the acquisition of monetary resources in the US over the time period 1983-2016. In contrast to existing work, we account for the multidimensionality of monetary resources by targeting the joint distribution of income and wealth. This extension provides important new insights about the economic playing field in the US: first, we document a more unequal distribution of opportunities when complementing income with the wealth dimension. Second, there are strong and consistent increases in inequality of opportunity over time. This trend is driven by both income and wealth to varying extents depending on the time period of interest.

We look forward to future research that extends the multidimensional approach taken in this paper beyond the domain of material resources by focusing on other dimensions of individual well-being, including health and social participation.

[13]We largely follow the definitions and imputation protocol of Bosworth and Anders (2008) to derive an aggregate measure for active savings. We thank the authors for generously sharing the details of their imputation method with us.

[14]Furthermore, this finding is consistent with the small contribution of cross-type associations of income and wealth in the attribute decomposition of Figure 4, Panel (B).

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Multidimensional Equality of Opportunity in the United States

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Supplementary Material
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A MEASUREMENT—SOME SIMPLE EXAMPLES

Consider a society with two types that are of equal size. Both income and wealth are unequally distributed. Since (i) inequality in both dimensions is exactly the same, and (ii) both dimensions are perfectly correlated across types, inequality of opportunity in monetary resources is exactly the same (0.17) regardless of whether we focus on income (I_{Income}) or wealth (I_{Wealth}) in isolation, or whether we focus on the joint distribution of income and wealth ($I_{Income,Wealth}$).

We now consider three alternative societies in which unidimensional and multidimensional measures of inequality of opportunity diverge.

	Income	Wealth
Type 1	50	500
Type 2	100	1000

$$I_{Income} = 0.17 \quad I_{Wealth} = 0.17 \quad I_{Income,Wealth} = 0.17$$

(a)

	Income	Wealth
Type 1	75	500
Type 2	75	1000

$$I_{Income} = 0.00 \quad I_{Wealth} = 0.17 \quad I_{Income,Wealth} = 0.09$$

(b)

	Income	Wealth
Type 1	50	1000
Type 2	100	500

$$I_{Income} = 0.17 \quad I_{Wealth} = 0.17 \quad I_{Income,Wealth} = 0.06$$

(c)

	Income	Wealth
Type 1	40	1100
Type 2	110	400

$$I_{Income} = 0.27 \quad I_{Wealth} = 0.27 \quad I_{Income,Wealth} = 0.12$$

(a) We equalize outcomes across types in the income dimension. Therefore, I_{Income} decreases, and I_{Wealth} stays the same. The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrate the measure's *inequality aversion between types*.

(b) We maintain inequality across types but reverse the cross-type association of income and wealth. Therefore, I_{Income} stays the same, and I_{Wealth} stays the same.

The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrate the measure's *sensitivity to correlation-increasing transfers*.

(c) We increase inequality across types in both dimensions and reverse the cross-type association of income and wealth. Therefore, I_{Income} increases, and I_{Wealth} increases. The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrates the existence of cases where unidimensional and multidimensional measures lead to opposing conclusions. While to former would detect an increase of inequality of opportunity in comparison to the baseline, the latter would detect a decrease in unequal opportunities.

B ATTRIBUTE DECOMPOSITION

In this appendix, we derive and prove the attribute decomposability of $I(X)$ as defined in equation (3). Our derivation is based on results presented in Abul Naga and Geoffard (2006). For the following exposition, we focus on the case of two outcome dimensions and let $K = 2$ and $X := \{X_1 \quad X_2\}$; however, we note this restriction can be easily relaxed.

Attribute Decomposability. In general, $I(X) = 1 - \delta(X)$, where $\delta(X) \in [0, 1)$. $I(X)$ is attribute decomposable if and only if

$$\delta(X) = f_1(\gamma_1(X_1)) + f_2(\gamma_2(X_2)) + f_3(\kappa(X)), \tag{5}$$

where f_1, f_2, f_3 are increasing functions ($\mathbb{R}_+ \mapsto \mathbb{R}_+$), γ_1 and γ_2 are unidimensional equality indices, and κ is a measure of association between X_1 and X_2.

Proposition 1. *Let μ_q (μ_q^t) define population (type) means in outcome dimension q. Then, $\delta(X)$ is attribute decomposable as follows:*

$$\ln \delta(X) = \frac{r_1}{r_1 + r_2} \ln \gamma_1(X_1) + \frac{r_2}{r_1 + r_2} \ln \gamma_2(X_2) + \frac{1}{r_1 + r_2} \ln \kappa(X), \tag{6}$$

where

$$\gamma_1(X_1) = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1} \right)^{r_1} \right)^{\frac{1}{r_1}},$$

$$\gamma_2(X_2) = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2} \right)^{r_2} \right)^{\frac{1}{r_2}},$$

$$\kappa(X) = \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}}.$$

Proof. First, δ is the proportion of μ_q that is necessary to achieve the same level of welfare if all attributes were distributed equally across types, see Kobus et al. (2020). Formally, let $w_0 = \sum_{t=1}^{M} N_t U^t(\delta\mu_1, \delta\mu_2)$ denote the welfare level associated with X_μ. Second, let ρ_1 be the proportion of μ_1 that is necessary to attain w_0, if (i) the first attribute was equally distributed across types, and (ii) the distribution of the second attribute across types remained as is. Formally, $w_0 = \sum_{t=1}^{M} N_t U^t(\rho_1\mu_1, \mu_2^t)$. Third, let γ_1 be the proportion of μ_1 that is necessary to attain w_0, if (i) the first attribute was equally distributed across types, and (ii) the distribution of the second attribute was equally distributed across types. Formally, $w_0 = \sum_{t=1}^{M} N_t U^t(\gamma_1\mu_1, \rho_2\mu_2)$.

It follows that

$$w_0 = \sum_{t=1}^{M} N_t a_t (\delta\mu_1)^{r_1} (\delta\mu_2)^{r_2} = \sum_{t=1}^{M} N_t a_t (\gamma_1\mu_1)^{r_1} (\rho_2\mu_2)^{r_2}.$$

After modification, we get $\delta^{r_1+r_2} = (\gamma_1)^{r_1}(\rho_2)^{r_2}$, and we obtain

$$\ln(\delta) = \frac{r_1}{r_1+r_2}\ln(\gamma_1) + \frac{r_2}{r_1+r_2}\ln(\gamma_2) + \frac{1}{r_1+r_2}\ln(\rho_2/\gamma_2)^{r_2}, \tag{7}$$

which is the desired decomposition with $\kappa := (\rho_2/\gamma_2)^{r_2}$.

We now need to derive functional forms of γ_1, γ_2 and κ.

Note that $w_0 = \sum_{t=1}^{M} N_t a_t (\gamma_1\mu_1)^{r_1} (\rho_2\mu_2)^{r_2} = \sum_{h=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\rho_2\mu_2)^{r_2}$. Solving for γ_1 yields:

$$\gamma_1 = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1} \right)^{r_1} \right)^{\frac{1}{r_1}}.$$

Proceeding in analogy, for γ_2 we get:

$$\gamma_2 = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2} \right)^{r_2} \right)^{\frac{1}{r_2}} .$$

Furthermore, we use $w_0 = \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\rho_2 \mu_2)^{r_2} = \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}$ to obtain

$$\rho_2 = \left(\frac{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2)^{r_2}} \right)^{\frac{1}{r_2}} .$$

Finally, substituting the expressions for γ_2 and ρ_2 into $\kappa := (\rho_2/\gamma_2)^{r_2}$ we get:

$$\kappa = \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}}.$$

$\square$

Linear Approximation. Collecting terms and reversing the log-linearization of $\delta(X)$, we obtain the attribute decomposition of $I(X)$ (see equation (5)):

$$I(X) = 1 - (\gamma_1)^{\frac{r_1}{r_1+r_2}} (\gamma_2)^{\frac{r_2}{r_1+r_2}} (\kappa)^{\frac{1}{r_1+r_2}} . \tag{8}$$

Applying a linear approximation around the point of perfect equality (i.e. $\gamma_1 = \gamma_2 = \kappa = 1$), we get the linear decomposition displayed in equation (4):

$$\begin{aligned} I(X) &= \tfrac{r_1}{r_1+r_2}(1-\gamma_1) + \tfrac{r_2}{r_1+r_2}(1-\gamma_2) + \tfrac{1}{r_1+r_2}(1-\kappa) + R, \\ &= \tfrac{r_1}{r_1+r_2} I_1 + \tfrac{r_2}{r_1+r_2} I_2 + \tfrac{1}{r_1+r_2} \kappa_I + R. \end{aligned} \tag{9}$$

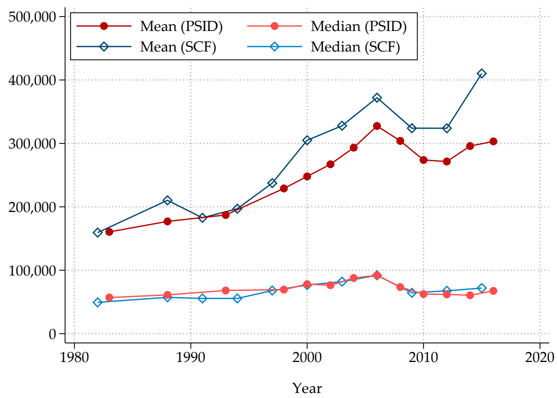
C ADDITIONAL FIGURES AND TABLES

TABLE S.1. Descriptive Statistics

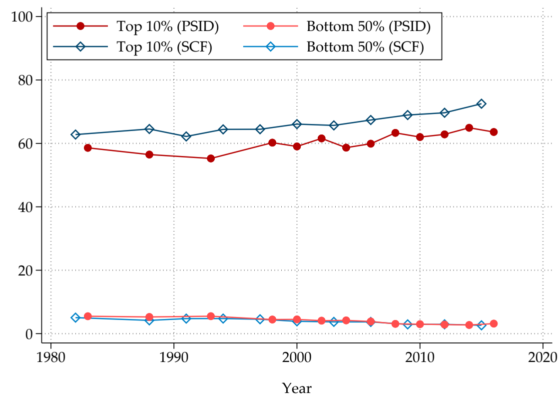
	Income	Wealth	Family Background				N
			Educ.	Occ.	Race	Region	
Panel (A): Intergenerational Sample							
	55,745	279,509	2.26	2.32	0.87	0.26	1,366
Panel (B): Re-weighted Intergenerational Sample							
	49,150	205,852	2.20	2.28	0.75	0.34	1,366
Panel (C): Individual Sample							
1983	34,299	131,661	1.75	1.87	0.84	0.32	5,368
1988	40,999	159,464	1.87	1.94	0.82	0.31	5,357
1993	39,941	154,854	1.95	2.00	0.81	0.31	5,070
1998	44,123	162,827	2.04	2.09	0.79	0.37	4,213
2000	45,750	179,103	2.06	2.12	0.78	0.37	4,106
2002	44,960	181,600	2.05	2.13	0.77	0.38	4,238
2004	47,273	217,515	2.05	2.14	0.77	0.36	5,197
2006	47,905	231,474	2.07	2.15	0.76	0.36	5,250
2008	47,078	198,579	2.08	2.16	0.76	0.36	5,079
2010	44,722	175,371	2.09	2.18	0.73	0.35	5,039
2012	44,820	157,919	2.10	2.19	0.72	0.36	5,047
2014	45,767	162,720	2.12	2.20	0.71	0.35	5,013
2016	46,002	175,061	2.12	2.21	0.70	0.35	4,957

Data: PSID.
Note: This table displays summary statistics for the *intergenerational sample* (Panel [A]), the re-weighted *intergenerational sample* (Panel [B]) and the *individual sample* (Panel [C]). Income is defined as the annual disposable household income, wealth as household net worth. Both income and wealth are scaled by the modified OECD equivalence scale and expressed in constant 2015 USD. We furthermore drop observations with negative income/wealth and set zero amounts to 1 USD. The family background variables Educ. (Occ.) show the average education (occupation) level of the parent with the highest education (occupation) status measured on a 3-point scale. Race displays the share of whites; region the share of respondents who grew up in the US Census region South. The last column shows the number of observations.

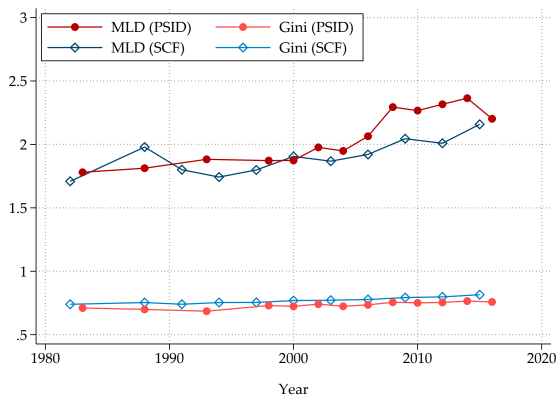
FIGURE S.1. Wealth in PSID and SCF, 1983-2016



(A) Mean and Median (in USD)



(B) Income Shares at Bottom and Top (in %)



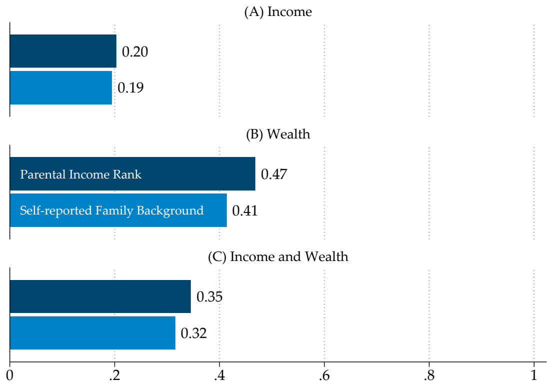
(C) Inequality

Data: PSID, SCF+ (Kuhn et al., 2020).
Note: This figure compares the wealth distributions between the PSID and the Survey of Consumer Finances (SCF). In both data sources, wealth is defined as equivalized household net worth (see Section 3); we drop negative values, replace zero values with 1 USD, and winsorize from above at the 99.9 percentile. Samples are restricted to household heads. All figures are expressed in constant 2015 USD.

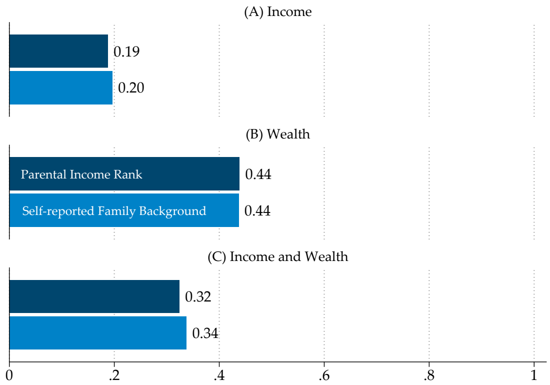
FIGURE S.2. Inequality of Opportunity in the US
Re-weighted Sample

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US when accounting for selective sample attrition in the *intergenerational sample*. In particular, we re-weight the *intergenerational* sample to match the *individual sample* in observation period 2010-2016 with respect to age, parental education, parental occupation, race, and region of upbringing. All estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$ and use parental income rank as a proxy for family background.

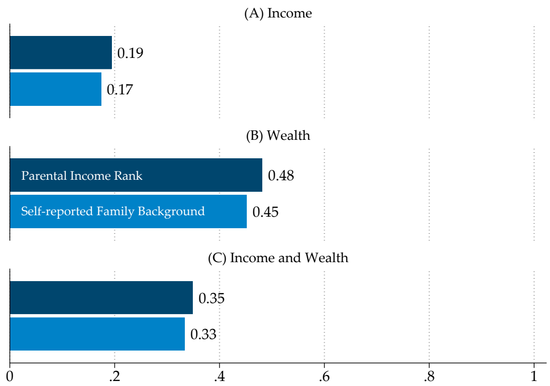
FIGURE S.3. Equality of Opportunity in the US
Varying Age Restrictions



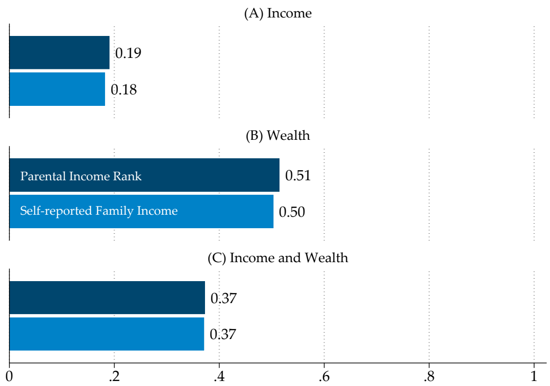
(A) Child Age 25-60 (Baseline)



(B) Child Age 30-55



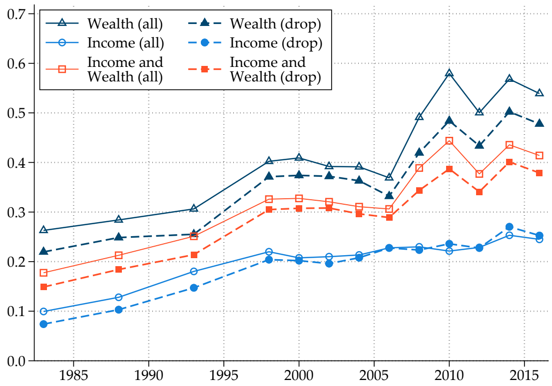
(C) Child Age 35-50



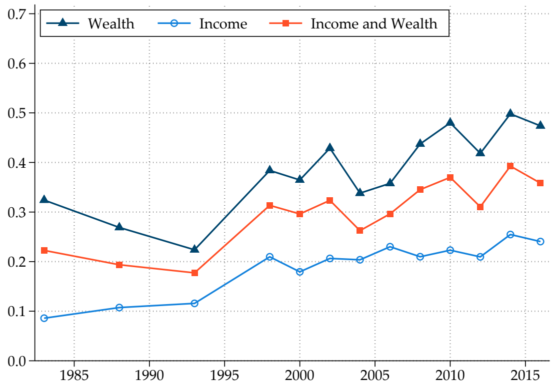
(D) Child Age 40-45

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US to different sample restrictions regarding the age of children. Panel (A) replicates our baseline estimates from Figure 2. In Panels (B)-(D) we sequentially narrow the age restriction to 30-55, 35-50, and 40-45. All estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

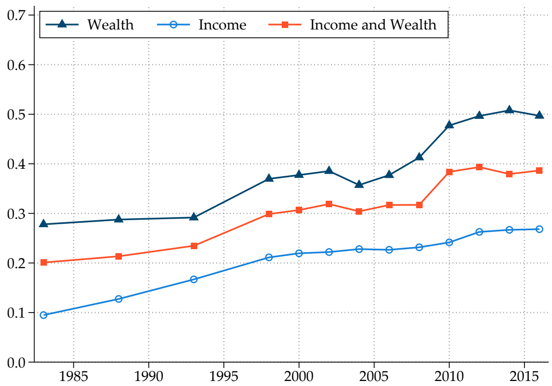
FIGURE S.4. Inequality of Opportunity in the US, 1983-2016
Sensitivity to Data Choices



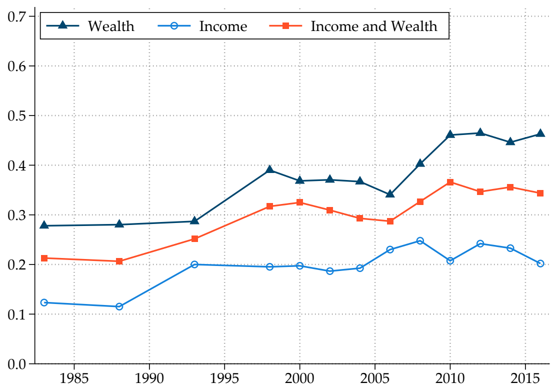
(A) Accounting for Negative Values



(B) Alternative Type Partition



(C) 5-Year Averages



(D) Labor Income and Wealth Net of Active Savings

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US for the *individual sample* over the time period 1983-2016. In Panel (A) we keep zero income and wealth without adjustment (solid line) or drop individuals with zero income or wealth (dashed line). In Panels (B) we let a regression tree determine the underlying type partition. In Panel (C), we take a 5-year moving average of income and wealth. Panel (D) displays our estimates for the sub-components of labor income and wealth net of active savings in the period of interest. Estimates are computed based on equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

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