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ABSTRACT

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We consider the collective model of labor supply with marketable domestic production. We first show that, if domestic production is mistakenly ignored, the "collective" indirect utilities that are retrieved from observed behavior will be unbiased if and only if the profit function is additive. Otherwise, in the non-additive case, the direction and the size of the bias will depend on the complementarity/substitutability of spouses' time inputs in the production process. We then show that, even if domestic labor supplies are not observed, valid welfare comparisons are possible. This identification result generalizes that in Chiappori (1992).

JEL Classification: D13, J22

Keywords: household, collective model, labor supply, home production, welfare analysis, identification

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1 Introduction

The collective model of labor supply, developed by Chiappori (1988, 1992), is by now a standard tool for analyzing household behavior, and its empirical success over the last ten years has been considerable. Specifically, the possibility of identifying the intra-household sharing of income from the sole observation of labor supplies has turned out to be very attractive. This allows us to carry out welfare comparisons at the individual level, instead of exclusively concentrating on the distribution of well-being across households, as is generally the case in traditional models.

The empirical study of Chiappori, Fortin and Lacroix (2001) is very representative of the potential of the collective approach. The authors estimate a simple model of household labor supply with a sample of couples extracted from the Panel Study of Income Dynamics. They then observe that a one dollar increase in household non-labor income will increase the wife's share of income by 70 cents and the husband's by 30 cents. They also remark that divorce legislations and the state of the marriage market affect the intra-household distribution of income. The relevance of these results for economic policies that target a particular person in the household is evident. Lise and Seitz (2004) go even farther in applying collective models of labor supply to welfare issues. They investigate recent changes in the distribution of resources in the United Kingdom and conclude that the rise in consumption inequality over time at the household level may overstate the degree of consumption inequality at the individual level by 40% between the late 1960s and the present.[1]

One of the most serious criticisms of the collective model of labor supply, however, concerns the treatment of domestic production. In the simplest form of this model, non-market time coincides with pure leisure and housework is ignored. Thus, a low level of market labor supply is automatically interpreted as greater consumption of leisure, whereas it may in fact reflect the specialization of one of the members in home production. Apps and Rees (1997) naturally conclude — even if it is not formally demonstrated in their paper — that the presence of homework may significantly distort welfare analyses based on Chiappori's initial identification results. Similar criticisms are addressed by feminist economists; see Grossbard-Schechtman (2000).

[1] Other empirical applications of the collective model of labor supply for various countries include Fortin and Lacroix (1997), Moreau and Donni (2002), Blundell, Chiappori, Magnac and Meghir (2004), Clark, Couprie and Sofer (2004) and Vermeulen (2004).

This problem is tackled by Chiappori (1997), who considers a collective model of domestic and market labor supplies. The household consists of a couple without children. Each spouse is characterized by egotistical preferences and, as usual, the decision process results in Pareto-efficient outcomes. There are two (private) consumption goods: a market good, as in the simple model of labor supply, and a domestic good which can be produced within the household from a technology using time inputs. The latter can be consumed by household members or exchanged on the market at a constant price.[2] Chiappori then shows that, if domestic and market labor supplies are both observed, spouses' preferences and the outcome of the decision process can be completely identified (up to a constant). In addition, testable restrictions on market and domestic labor supplies can be derived.[3]

In spite of this important theoretical contribution, empirical applications of collective models accounting for home production are surprisingly rare.[4] The most probable reason is that these models are very demanding in terms of data and turn out to be difficult to estimate. Time use surveys, even if they are quite broadly available, are generally fragmented and unreliable regarding wages and income. In this case, *the main question is to determine what the econometrician can say about the internal decision process without information on spouses' housework*. This is our objective in this paper. To do so, we adopt Chiappori's (1997) framework with marketable domestic production and make the following contributions.

Firstly, we establish the general conditions on preferences and technologies under which the restrictions derived from Chiappori's (1988, 1992) original framework continue to hold in the extended setting at stake here. If these general conditions are satisfied, there exist at least two 'structures' (depending on the econometrician's presupposition about domestic production) that equivalently rationalize household behavior. Hence, the econometrician is not able to reject empirically the simple model of labor supply in favor of a

[2] Apps and Rees (1988, 1997) and Chiappori (1997) also consider the case of a non-marketable domestic good. In that case, the price of this good is endogenously determined within the household. Identification raises further difficulties, though.

[3] Other theoretical generalizations of the collective model of labor supply are given by Fong and Zhang (2000), Donni (2003) and Chiappori, Blundell and Meghir (2004).

[4] Apps and Rees (1996) and Aronsson, Daunfeldt, Wikstrom (2001) are probably the most notable exceptions. These empirical investigations are based upon Apps and Rees's (1997) and Chiappori's (1997) set-up. Couprie (2004) estimates a model of domestic and market labor supplies but she supposes that spouses produce a public good (instead of a private one). The strategy of identification is then completely different.

more sophisticated model that incorporates domestic production. And, if the econometrician mistakenly ignores domestic production, she may well draw wrong conclusions about the welfare impact of economic policy.

Secondly, we examine this very case — in which the underlying structure is not unique — and determine the distortions in welfare analyses that may appear if the econometrician adopts the 'wrong' structure. Our result indicates that welfare analyses are unbiased if and only if the profit function is additive. Otherwise, in the non-additive case, the size and the direction of the bias depend on the complementarity/substitutability of spouses' time inputs in the production process. If spouses' labor are substitutes (resp. complements), the direct effect of wage on spouses' welfare is generally underestimated (resp. overestimated) and the cross effect overestimated (resp. underestimated). This result is illustrated by an application to Chiappori, Fortin and Lacroix (2001)'s estimates.

Thirdly, we prove that, even if domestic labor supplies are not observed by the econometrician, (i) market labor supplies have to satisfy testable restrictions under the form of partial differential equations, and (ii) the most important components of the decision process can be identified so that valid welfare comparisons are still possible. More precisely, we show that the 'collective' indirect utilities of spouses can be recovered up to composition by an increasing transform.[5] This information is generally sufficient to perform welfare analyses at the individual level. Importantly, the system of partial differential equations that defines these utilities in the present context generalizes Chiappori's (1992) system in that the solution of the former reduces to the solution of the latter if the profit function is additive. Since this result does not require more data than typically necessary for estimating a simple collective model of labor supply, we finally advocate a new, more general strategy for performing welfare comparisons with collective models.

The paper is structured as follows. In Section 2, the model that we study is described. In Section 3, the implications of the omission of domestic production are discussed. In particular, the form of preferences and technologies that generates non-unique structures is characterized. In Section 4, the new identification results, valid in the presence of domestic production, are derived. In Section 5, a conclusion and a summary are presented.

[5] Collective indirect utilities measure the level of welfare that individuals *actually* attain in the household when facing a given set of wages, incomes and other variables.

2 The model

The model is similar to Chiappori's (1997) and our description will be very brief. We consider a two-person household, consisting of a wife (f) and a husband (m), who make decisions about market and domestic labor supplies.[6] Spouse i's domestic and market labor supplies are respectively denoted by t_i and h_i ($i = f, m$). Each spouse is characterized by specific preferences which can be represented by utilities with the usual regularity properties:

$$u_i(T - L_i, C_i, Z_i), \tag{1}$$

where L_i ($= t_i + h_i$), C_i and Z_i respectively denote spouse i's total labor supply, her/his consumption of an aggregate marketable good and her/his consumption of an aggregate domestic good, and T denotes the total endowment of time. The domestic good is produced with the following technology:

$$Z = \phi(t_f, t_m),$$

where ϕ is a strictly concave function. This good can be bought and sold on the market at a constant price.[7] In consequence, we generally have: $Z \neq Z_f + Z_m$. We consider the case of cross-sectional data so that the price of both goods can be normalized to one. Spouse i's market wage is denoted by w_i and household non-labor income by y.

Following the basic idea of the collective approach, we assume that the decision process, whatever its true nature, always generates Pareto-efficient outcomes. There is one distribution factor, i.e., one exogenous variable that influences the decision process without affecting preferences or the budget constraint, denoted by s (the extension to several distribution factors is trivial). Then, from the Theorems of Welfare Economics, the allocation problem can be decentralized. In a first step, spouses determine domestic labor supply in order to maximize profit:

$$\pi(w_f, w_m) = \max_{t_f, t_m} \left\{ \phi(t_f, t_m) - w_f t_f - w_m t_m \right\},$$

[6] The couple is not necessarily married. The terminology is just for convenience.

[7] This assumption is introduced by Gronau (1977) who regards "work at home as a time use that generates services which have a close substitute in the market, while leisure has only poor market substitutes". See Chiappori (1997) for a discussion of its relevance.

where $\pi(w_f, w_m)$ is a normalized profit function. This function is assumed to be three times continuously differentiable. From Hotelling's Lemma, domestic labor supplies have the following form:

$$t_i = -\frac{\partial \pi(w_f, w_m)}{\partial w_i} = t_i(w_f, w_m).$$

These functions are twice continuously differentiable and satisfy a symmetry property. In a second step, spouses agree on the sharing of total income, defined by

$$\psi = y + \pi(w_f, w_m).$$

Each spouse i then receives a share ρ_i, with $\rho_m + \rho_f = \psi$, and independently maximizes her/his utility subject to a personal budget constraint:

$$\max_{L_i, C_i, Z_i} u_i(T - L_i, C_i, Z_i) \tag{2}$$

subject to

$$C_i + Z_i - w_i L_i = \rho_i(w_f, w_m, y, s).$$

In other words, the function ρ_i can be seen as the natural generalization of the sharing rule in the case of household production. The total labor supplies that result from this program have the following form:

$$L_i = L_i(w_i, \rho_i),$$

with $\rho_m + \rho_f = \psi$. The functions L_i and ρ_i are assumed to be three times continuously differentiable. Combining these expressions yields market labor supplies:

$$h_i = L_i(w_i, \rho_i) - t_i(w_f, w_m). \tag{3}$$

In what follows, we assume that only the market labor supplies are observed by the econometrician.

To conclude the description of the model, we have to introduce a last concept that turns out to be useful in welfare analysis. Since the price of both aggregate goods is normalized to one, the indirect utilities that correspond to

(1) can be written as $v_i(w_i, Tw_i + \rho_i)$. However, we follow Chiappori (1992) and also define 'collective' indirect utility as:

$$v_i^*(w_f, w_m, y, s) = v_i(w_i, Tw_i + \rho_i(w_f, w_m, y, s)). \tag{4}$$

This expression describes the level of welfare that spouse i attains in the household when she/he faces a wage-income bundle (w_f, w_m, y) and a distribution factor s. The function v_i^* is generally more convenient than v_i for making welfare comparisons because it directly yields the actual change in welfare due to a modification in the household environment.

3 A first look at the problem

3.1 Definition and characterization

We know that, if there is domestic production, market labor supply is described by (3). In principle, the econometrician will not make mistakes in carrying out welfare analysis since the theoretical restrictions listed by Chiappori (1988, 1992) and Chiappori, Fortin and Lacroix (2001)[8] for the simple model of labor supply do not need to hold in a more general context. Hence, if domestic production is mistakenly ignored, the collective approach should be almost always 'rejected'. This assertion has, however, to be nuanced in practice since data are necessarily imperfect and statistical tests are often misleading. We thus consider a particular class of 'observable' market labor supplies that, in spite of domestic production, can be confused with those resulting from Chiappori's (1988, 1992) original framework, and we investigate the nature of the bias that may result in welfare analysis. This class, denoted by $\mathcal{O}$, is defined as follows.

Definition 1 *Suppose that there is marketable domestic production. A system of market labor supplies belongs to class $\mathcal{O}$ if and only if there exist some twice continuously differentiable functions g_i and φ_i, with $\varphi_m + \varphi_f = y$, such that market labor supplies can be written as: $h_i = g_i(w_i, \varphi_i)$ for $i = m, f$.*

[8] To be precise, the restrictions of the collective model of labor supply with distribution factors were derived by Chiappori, Fortin and Lacroix (2001) but the collective model which initiated the research program is that of Chiappori (1988, 1992). Since the idea of decentralization is already contained in the latter, and for the sake of conciseness, we simply refer below to the restrictions of Chiappori's (1988, 1992) model.

In this definition, the relation $h_i = g_i(w_i, \varphi_i)$ describes the structure that characterizes market labor supplies in absence of domestic production. Hence, if the system of 'observed' market labor supplies belongs to class $\mathcal{O}$, the underlying structure is not uniquely defined. To be more explicit, we have the following identity:

$$L_i(w_i, \rho_i) - t_i(w_f, w_m) = g_i(w_i, \varphi_i). \tag{5}$$

The 'true' structure that accounts for domestic production is on the left-hand-side and the 'false' structure that ignores domestic production is on the right-hand-side. In this case, the econometrician is not able to empirically reject the simple model of labor supply in favor of a model with domestic production. And, if the econometrician mistakenly assumes that there is no domestic production, she may well draw wrong conclusions about the intra-household distribution of resources. The consequences of this possible misinterpretation thus need to be examined carefully.

The first step is to completely characterize the form of preferences and technologies such that market labor supplies belong to class $\mathcal{O}$, i.e., such that (5) holds. This is the objective of the following propositions.

Proposition 2 *A system of market labor supplies belongs to class $\mathcal{O}$ if the profit function is additive, i.e., if there exist some functions π_i such that $\pi = \pi_f(w_f) + \pi_m(w_m)$.*

Proof If the profit function is additive, then $L_i(w_i, \rho_i) - t_i(w_i) = g_i(w_i, \varphi_i)$, with $\rho_m + \rho_f = y + \pi_m(w_m) + \pi_f(w_f)$. We then define $\varphi_i = \rho_i - \pi_i(w_i)$ and $g_i(w_i, \varphi_i) = L_i(w_i, \pi_i(w_i) + \varphi_i) - t_i(w_i)$.‖

In words, this proposition states that, whatever spouses' preferences or the sharing of income, the assumption that spouses' time inputs are independent is sufficient for market labor supplies to satisfy the conditions listed in Chiappori (1988, 1992). Moreover, since there is a close relationship between profit and production functions, it can easily be shown that the additivity of the former is equivalent to the additivity of the latter, i.e.:

$$\phi(t_f, t_m) = \phi_f(t_f) + \phi_m(t_m).$$

A simple consequence of Proposition 2 is then that a test of the collective approach can be implemented without taking spouses' housework into account if the production function is additive. We will see below that the additivity assumption has other attractive implications.

The next proposition considers a more general family of production technologies and completes the characterization of class $\mathcal{O}$. In this case, Engel curves have to be linear. The following regularity condition is necessary.

Condition R Preferences and the sharing of income are such that $\partial L_i/\partial \rho_i \neq 0$ and $\partial \rho_i/\partial s \neq 0$ for any (ρ_i, s) in $\mathbb{R}^2$.

Proposition 3 *Suppose that (i) the profit function is non-additive and (ii) Condition R is satisfied. Then, a system of market labor supplies belongs to class $\mathcal{O}$ if and only if there exist some functions α_i, β_i, γ_i and G such that $L_i(w_i, \rho_i) = \alpha_i(w_i)\rho_i + \beta_i(w_i)$ and*

$$\pi(w_f, w_m) = \gamma_f(w_f) + \gamma_m(w_m) + \frac{G\left(\int \alpha_f(w_f)\,dw_f - \int \alpha_m(w_m)\,dw_m\right)}{\exp\left(\int \alpha_f(w_f)\,dw_f + \int \alpha_m(w_m)\,dw_m\right)}.$$

The proof of this proposition is postponed until the end of this section since it requires intermediate results that are given below. However, we may note at this stage that the linearity of Engel curves has well-known implications in terms of preferences. Gorman (1961) shows that the indirect utilities, in this case, are of the form:

$$v_i(w_i, Tw_i + \rho_i) = \int \frac{T - \beta_i(w_i)}{\exp\left(\int \alpha_i(w_i)\mathrm{d}w_i\right)}\mathrm{d}w_i + \frac{\rho_i}{\exp\left(\int \alpha_i(w_i)\mathrm{d}w_i\right)},$$

where α_i and β_i are defined as in Proposition 3; see also Pollak and Wales (1992, p. 27). This assumption may seem a priori restrictive but, in fact, many empirical studies estimate functional forms that are linear in income. Specifically, we will see below that Chiappori, Fortin and Lacroix's (2001) functional form has this property.

3.2 Identifying the wrong model

In this preliminary step, we suppose that the observed system of market labor supplies belongs to class $\mathcal{O}$ so that a misinterpretation of the observed model cannot be dismissed a priori. We also suppose that Condition R is satisfied. The reasoning then follows in three stages.

1. If we differentiate (5) with respect to y and s, and solve the resulting system of partial differential equations, we obtain:

$$\frac{\partial g_i}{\partial \varphi_i} = \frac{\partial L_i}{\partial \rho_i}, \tag{6}$$

and

$$\frac{\partial \varphi_i}{\partial y} = \frac{\partial \rho_i}{\partial y}, \tag{7}$$

$$\frac{\partial \varphi_i}{\partial s} = \frac{\partial \rho_i}{\partial s}. \tag{8}$$

2. If we differentiate (5) with respect to w_j ($j = f, m$ and $j \neq i$), use (6) and rearrange, we obtain:

$$\frac{\partial \varphi_i}{\partial w_j} = \frac{\partial \rho_i}{\partial w_j} - \nu_i. \tag{9}$$

where

$$\nu_i = \frac{\partial t_i}{\partial w_j} \left(\frac{\partial L_i}{\partial \rho_i} \right)^{-1}$$

represents an error in the estimated derivative of the sharing rule.

3. If we differentiate the adding-up restriction $\varphi_m + \varphi_f = y$ with respect to w_i and use (9), we obtain:

$$\frac{\partial \varphi_i}{\partial w_i} = -\frac{\partial \rho_j}{\partial w_i} + \nu_j.$$

We now differentiate the adding-up restriction $\rho_m + \rho_f = \psi = \pi + y$ with respect to w_i and use Hotelling's Lemma to obtain:

$$\frac{\partial \rho_j}{\partial w_i} = -t_i - \frac{\partial \rho_i}{\partial w_i}.$$

All in all, these relations give:

$$\frac{\partial \varphi_i}{\partial w_i} = \frac{\partial \rho_i}{\partial w_i} + (t_i + \nu_j). \tag{10}$$

Welfare distortions. The effect of the husband's (say) wage on his own share of income cannot be directly interpreted in terms of welfare variations. To infer the distortions that may result from ignoring housework, we have to use collective indirect utilities. If we then differentiate (4) with respect to y, s and w_j $(j \neq i)$, we simply obtain:

$$\frac{\partial v_i^*}{\partial y} = \lambda_i \frac{\partial \rho_i}{\partial y}, \quad \frac{\partial v_i^*}{\partial s} = \lambda_i \frac{\partial \rho_i}{\partial s}, \quad \frac{\partial v_i^*}{\partial w_j} = \lambda_i \frac{\partial \rho_i}{\partial w_j}, \tag{11}$$

where $\lambda_i = \partial v_i / \partial \left(T w_i + \rho_i\right)$ is the marginal utility of money. This indicates that the impact of these variables on spouse i's welfare coincides with the derivatives of the sharing rule up to a multiplicative term. Moreover, if we differentiate (4) with respect to w_i and use Roy's Identity, we have:

$$\frac{\partial v_i^*}{\partial w_i} = \lambda_i \left((h_i + t_i) + \frac{\partial \rho_i}{\partial w_i} \right). \tag{12}$$

We can now describe the distortions in welfare analysis resulting from the omission of domestic production. To do so, define $\hat{v}_i^*$ as the collective indirect utility that is obtained from the relations: $h_i = g_i(w_i, \varphi_i)$. That is, the collective indirect utility that is mistakenly retrieved if the econometrician ignores domestic production.

Proposition 4 *Suppose that Condition R is satisfied. If the system of market labor supplies belongs to class $\mathcal{O}$, then*

$$\begin{aligned}
\frac{\partial v_i^*}{\partial y} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial y}, \\
\frac{\partial v_i^*}{\partial s} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial s}, \\
\frac{\partial v_i^*}{\partial w_j} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial w_j} + \eta_i \nu_i, \\
\frac{\partial v_i^*}{\partial w_i} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial w_i} - \eta_i \nu_j,
\end{aligned}$$

for $i, j = m, f$ and $i \neq j$, where μ_i and η_i are positive functions.

Proof Let $\mu_i = \lambda_i / \hat{\lambda}_i$, where $\hat{\lambda}_i$ is the marginal utility of money for the model without domestic production, be the price of utility $\hat{v}_i^*$ in terms of utility v_i^*, and $\eta_i = \lambda_i$ be the price of money in terms of utility v_i^*. Then, the proof straightforwardly results from (7) to (12) and the system of equations which defines collective indirect utilities in Chiappori (1992, p. 451).‖

In words, the effect of non-labor income and distribution factors on welfare is correctly estimated by the simple model of labor supply. However, the effect of wages is generally biased and depends on the function ν_i. More precisely, the bias resulting from the omission of domestic production is related to the substitutability/complementarity relationship between spouses' time inputs in the production process. If leisure is a superior good — an uncontroversial assumption — and if spouses' labor are substitutes (resp. complements), and consequently $\nu_i < 0$ (resp. $\nu_i > 0$), the direct effect of the wage on spouses' welfare is underestimated (resp. overestimated) and the cross effect is overestimated (resp. underestimated). We also have the important corollary that follows.

Corollary 5 *The omission of domestic production will not invalidate welfare comparisons if and only if the profit function is additive.*

This corollary specifies the conditions that the underlying profit function has to satisfy to make valid welfare comparisons with Chiappori's (1988, 1992) initial model.[9] The empirical relevance of this condition is examined in the next subsection. However, another interesting application of this corollary is that welfare comparisons obtained from the simplest form of the collective model are valid if for unspecified reasons the husband's (say) domestic labor supply is perfectly inelastic — this extreme situation certainly represents a convenient approximation for many households in which the husband does not contribute to household chores.[10]

3.3 A numerical example

The complementarity/substitutability relationship between spouses' time is clearly an empirical issue. What can be learned from data? Let us consider

[9] Of course, the direct utilities $\hat{u}_i$ which are recovered from the relations: $h_i = g_i(w_i, \varphi_i)$ cannot be interpreted in the usual way, even if the production function is additive. They are a mixture of individual preferences u_i and individual technologies π_i.

[10] In a certain sense, the implications of the model are correct although the assumptions on which the model is based are inaccurate. This is reminiscent of the instrumentalist view of economics by Friedman (1953).

an activity which is valued primarily for its output rather than its inherent satisfaction — it includes cleaning and cooking but excludes child caring that, broadly speaking, can be assimilated to leisure. Research then shows that "relative wages of the couple appear to matter some, but that much of the division of labor is independent of wages" (Juster and Stafford, 1991, p. 498). In other words, the number of hours worked at home is relatively inflexible to variations in wages. Having said that, the majority of studies in the United States seem to support the hypothesis of a slight substitutability, even if conclusive evidence is lacking. In one of these rare investigations, based on the IXth wave of the Panel Study of Income Dynamics, Graham and Green (1984) show that the own-wage elasticity of the wife's domestic labor supply is equal to -0.169 (t-statistics = 3.593) and the cross-wage elasticity to 0.047 (t-statistics = 0.888). Hill and Juster (1985) draw a similar conclusion with time use data and a disaggregated framework. See also Gronau (1977) for more empirical results using United States data.[11]

Empirical evidence thus suggests that domestic labor supply is relatively insensitive to partner's wage. Now, the previous subsection has shown that the distortions in welfare analysis are small if the derivative of each domestic labor supply with respect to the partner's wage is close to zero. Should we *necessarily* conclude that the simple model of labor supply is sufficient to make precise welfare analysis? Actually, the answer is no. The size of the bias also depends on the sensitivity of leisure demands. And several empirical studies have unambiguously shown that the impact of income shares on leisure demands is very small. Consequently, the bias in welfare analysis may actually be substantial. The numerical example that follows illustrates this point.

To begin with, the previous theoretical discussion implies that any functional form for market labor supplies that entails a linear structure compatible with Chiappori's (1988, 1992) model, such as $h_i = \alpha_i(w_i)\varphi_i + \xi_i(w_i)$ for some function $\xi_i(w_i)$, is also compatible with a more sophisticated model accounting for non-additive profit functions. Then, let us consider Chiappori, Fortin and Lacroix's (2001) functional form for market labor supply:

$$h_i = a_i + b_i \ln w_f + c_i \ln w_m + d_i \ln w_f \ln w_m + e_i y + d_i fs, \qquad (13)$$

[11] The most natural application of the model of marketable domestic production concerns agricultural households. It appears that, in this case, the impact of wages on domestic labor supplies is generally larger. See Singh, Squire and Strauss (1986) and Taylor and Adelman (2004) for surveys.

where $a_i, \ldots, e_i$ and f are parameters. This form implies a structure with linear Engel curves whose slopes α_f and α_m are constants, uniquely defined by

$$\alpha_f = e_f - \frac{d_f}{d_m} e_m, \qquad \alpha_m = e_m - \frac{d_m}{d_f} e_f.$$

We now suppose, in contrast with Chiappori, Fortin and Lacroix (2001), that there is domestic production. For example, the underlying profit function may have the form:

$$\begin{aligned} \pi \;\; = \;\; & \kappa_0 - \kappa_f w_f - \kappa_m w_m - \frac{\kappa_{ff}}{2} w_f^2 - \frac{\kappa_{mm}}{2} w_m^2 \\ & -\lambda \exp\left(\alpha_f w_f + \alpha_m w_m\right)^{-1}, \end{aligned} \tag{14}$$

where $\kappa_0, \kappa_i, \kappa_{ii}$ and λ are parameters, which is consistent with the generic form given in Proposition 3. From Hotelling's Lemma, domestic labor supply derived from (14) is:

$$t_i = \kappa_i + \kappa_{ii} w_i + \alpha_i \lambda \times \exp\left(\alpha_f w_f + \alpha_m w_m\right)^{-1}. \tag{15}$$

Under these assumptions, the functional form of market labor supply given by (13) is compatible with domestic production.

In what follows, we use Chiappori, Fortin and Lacroix's estimation of equation (13) to evaluate the amplitude of the distortions due to the omission of domestic production. Note that the dependent variable is measured by the number of hours worked per year. Firstly, we follow Chiappori, Fortin and Lacroix's (2001, Tables 2 & 4) results and conclude that a one-thousand-dollar increase in the wife's (resp. husband's) share of yearly income implies, on average, a decrease in market labor supply by about 10 hours (resp. 20 hours) over the year. That is, $\alpha_f \simeq 0.01$ and $\alpha_m \simeq 0.02$. Secondly, we conjecture that a one-dollar increase in the wife's (resp. husband's) hourly wage implies an increase in the husband's (resp. wife's) domestic labor supply by about 5 hours.[12] This is, of course, a rough approximation. Hence, $\nu_f \simeq 500$ and $\nu_m \simeq 250$. Thirdly, we examine the wife's welfare. From

[12] This figure corresponds to an elasticity of 0.05 for a wage equal to \$10 and a number of domestic hours equal to 1000.

Chiappori, Fortin and Lacroix (2001, Table 4), we obtain:[13]

$$\begin{aligned}\partial \hat{v}_f^* / \partial w_f &\simeq 107 \times \hat{\lambda}_f, \\ \partial \hat{v}_f^* / \partial w_m &\simeq 600 \times \hat{\lambda}_f.\end{aligned}$$

The striking point is that the effect of the wife's wage on her welfare is smaller than the effect of the husband's wage. However, using (9) to (12), our correction gives:

$$\begin{aligned}\partial v_f^* / \partial w_f &\simeq (107 + \nu_m) \times \lambda_f = 357 \times \lambda_f, \\ \partial v_f^* / \partial w_m &\simeq (600 - \nu_f) \times \lambda_f = 100 \times \lambda_f.\end{aligned}$$

Wife's welfare is now more sensitive to variations in her own wage than to variations in her partner's wage. This is more in line with intuition.

The conclusion of this example is that, even if domestic labor supplies are quite inelastic, distortions in welfare comparisons may be important.

3.4 Proof of Proposition 3

Let us go back to the proof of Proposition 3. The reader who wishes to avoid technicalities will have no scruples in skipping this subsection.

a) (First Necessary Condition) The first step is to show that, under Condition R, Engel curves have to be linear. To do so, we differentiate (5) with respect to w_j ($j \neq i$). We obtain:

$$\frac{\partial L_i}{\partial \rho_i} \frac{\partial \rho_i}{\partial w_j} - \frac{\partial t_i}{\partial w_j} = \frac{\partial g_i}{\partial \varphi_i} \frac{\partial \varphi_i}{\partial w_j}.$$

We differentiate this expression again with respect to s. From (6) and (8) and the cross-derivative restrictions ($\partial^2 \rho_i / \partial w_j \partial s = \partial^2 \rho_i / \partial s \partial w_j$), we obtain:

$$\frac{\partial^2 L_i}{\partial \rho_i^2} \frac{\partial \rho_i}{\partial s} \left(\frac{\partial \rho_i}{\partial w_j} - \frac{\partial \varphi_i}{\partial w_j} \right) = 0.$$

[13] The second line is directly computed from the estimates of the derivatives of the sharing rule at the average point of the sample (600). The first line uses the estimates of the derivatives of the sharing rule (−1634) and the average number of market hours worked by women (1741). Then, 107 = 1741 − 1634. It is fair to say, however, that the standard deviations for these estimates are quite large.

The term in parentheses on the right-hand-side is different from zero because of the non-additivity of the profit function (and thus $\nu_i \neq 0$). Then, if $\partial\rho_i/\partial s \neq 0$, Engel curves are linear: $\partial^2 L_i/\partial\rho_i^2 = 0$. The linearity is thus a first necessary condition in the non-additive case.

b) (Second Necessary Condition) We now show that the specific form for the profit function given in Proposition 3 is also necessary. If Engel curves are linear, (5) becomes:

$$\alpha_i(w_i)\rho_i + \beta_i(w_i) - t_i(w_f, w_m) = \alpha_i(w_i)\varphi_i + \xi_i(w_i),$$

for some function $\xi_i(w_i)$. From this expression, we obtain:

$$\rho_i - \frac{t_i(w_f, w_m)}{\alpha_i(w_i)} + \delta_i(w_i) = \varphi_i, \tag{16}$$

where

$$\delta_i(w_i) = \frac{\beta_i(w_i) - \xi_i(w_i)}{\alpha_i(w_i)},$$

since $\alpha_i(w_i) \neq 0$ by Condition R. We sum up (16) for each spouse and use Hotelling's Lemma. We obtain:

$$\pi(w_f, w_m) + \sum\nolimits_i \frac{1}{\alpha_i(w_i)} \frac{\partial\pi(w_f, w_m)}{\partial w_i} + \sum\nolimits_i \delta_i(w_i) = 0. \tag{17}$$

The fact that the profit function can be seen as a solution to this partial differential equation for some function $\delta_i(w_i)$ (and thus some function $\xi_i(w_i)$) is a second necessary condition. The explicit solution of this type of equation is given in Lemma 7 in the appendix.

Since the partial differential equation (17) always has a solution, the two necessary conditions are sufficient as well.||

4 A new identification result

4.1 Result and interpretation

One of the main results presented in the previous section is that welfare analyses based on a theoretical framework that omits home production is valid if and only if the profit function is additive. The properties of the profit function is an empirical issue which should be left to the econometrician. However, evidence is not clear-cut. Moreover, we show in our numerical illustration that even small deviations from additivity may greatly distort welfare conclusions. In this case, the main question is: How can we carry out welfare analysis without observing domestic labor supplies or making strong assumptions about the profit function?

This important issue is addressed in the present section. Precisely, we show that the most important components of the model can be retrieved from the sole observation of market labor supplies. Hence, welfare analysis is possible without the observation of domestic labor supplies. To do so, we introduce the following definitions:

$$\Delta_i = \frac{\partial h_i}{\partial y} - \left(\frac{\partial h_i/\partial s}{\partial h_j/\partial s}\right)\frac{\partial h_j}{\partial y} \quad \text{with} \quad i,j = f,m \text{ and } j \neq i,$$

where the denominator is supposed to be different from zero, and we suppose the following regularity condition holds.

Condition R′ Market labor supplies are such that $\partial h_i/\partial s \neq 0$, $\Delta_i \neq 0$, $\partial \Delta_i/\partial y \neq 0$ for almost all (w_f, w_m, y, s) in $\mathbb{R}_+^2 \times \mathbb{R}^2$.

The main result is then formally stated in the next proposition.

Proposition 6 *Suppose collective rationality with marketable domestic production. A system of market labor supplies is observed. Then, if Condition R′ is satisfied,*

1. *Market labor supplies have to satisfy testable constraints under the form of partial differential equations;*

2. *The income share and the domestic labor supply of spouse f (resp. m) can be identified up to an additive function of w_f (resp. w_m);*

3. *The indirect collective utility of both spouses can be identified up to composition by an increasing transform.*

Before demonstrating this proposition in the next subsection, we should make several remarks.

1. The second statement can be interpreted as follows. If $\rho_i^*(w_f, w_m, y, s)$ is a particular solution for spouse i's share that is compatible with both market labor supplies, then the general solution is given by

$$\rho_i(w_f, w_m, y, s) = \rho_i^*(w_f, w_m, y, s) + k^i(w_i),$$

for some unknown function $k^i(w_i)$. The interpretation is similar for the identification of domestic labor supplies. Let us note that, if both domestic labor supplies were observed, the indeterminacy in income shares would consist merely of a constant, instead of a function.

2. The identification of income shares is thus incomplete. However, since the functions $v_i^*(w_f, w_m, y, s)$ — i.e., the most useful concept to carry out welfare comparisons — are identifiable up to composition by an increasing transform, it is possible to make policy recommendations in almost all circumstances.

3. Condition R′ is quite complicated. This condition excludes (i) Engel curves for labor supplies that are linear functions of spouses' shares *and* (ii) spouses' shares of income that are linear functions of non-labor income.[14] Thus, the functional form used by Chiappori, Fortin and Lacroix (2001) does not allow us to perform welfare analysis. The theory on which our numerical example is based is thus necessary to evaluate the bias resulting from the omission of domestic production in the linear context.

4. The definitions of the derivatives of the indirect collective utilities with respect to non-labor income or distribution factors are exactly the same as those in Chiappori (1992). The definitions of the derivatives of the collective indirect utilities with respect to wages differ but they reduce to Chiappori's definitions if the profit function is additive. Our identification result is thus a generalization of what is perhaps the most famous result regarding collective models.

[14] Condition R′ is unsurprisingly related to the conditions on preferences and technologies listed in Propositions 2 and 3: by definition, some identification problems — although they do not necessarily prevent welfare analyses from being performed — appear if the underlying structure is not uniquely defined. Condition R′ is stronger than Condition R.

5. Our recommended strategy is thus the following. To begin with, if she does not have prior information on the underlying domestic technology, the econometrician should use the general model which allows for domestic production. Then, a statistical test of the validity of the additivity hypothesis can be performed since the cross-derivatives of domestic labor supplies are identifiable. Finally, the econometrician is allowed to use Chiappori's (1992) formulae to retrieve collective indirect utilities, when she performs welfare comparisons, only if the additivity hypothesis is not rejected by the data.

4.2 Proof of Proposition 6

Proof of Statements 1 and 2. If we differentiate (3) with respect to y and s, we obtain a system of four partial differential equations of the form:

$$\frac{\partial h_i}{\partial y} = \frac{\partial L_i}{\partial \rho}\frac{\partial \rho_i}{\partial y}, \quad \frac{\partial h_i}{\partial s} = \frac{\partial L_i}{\partial \rho}\frac{\partial \rho_i}{\partial s},$$

with $\partial\rho_f/\partial y + \partial\rho_m/\partial y = 1$ and $\partial\rho_f/\partial s + \partial\rho_m/\partial s = 0$. Chiappori, Fortin and Lacroix (2001) show that, if Condition R$'$ is satisfied, this system can be solved with respect to the derivatives of income shares and total labor supplies. Using the notation defined above, the solutions are:

$$\frac{\partial L_i}{\partial \rho} = \Delta_i, \tag{18}$$

and

$$\frac{\partial \rho_i}{\partial y} = \frac{\partial h_i/\partial y}{\Delta_i}, \quad \frac{\partial \rho_i}{\partial s} = \frac{\partial h_i/\partial s}{\Delta_i}. \tag{19}$$

The cross-derivative restrictions applied to (19) imply that

C1: $\dfrac{\partial h_i}{\partial y}\dfrac{\partial \Delta_i}{\partial s} = \dfrac{\partial h_i}{\partial s}\dfrac{\partial \Delta_i}{\partial y}$ for $i = f, m$.

Let us now consider the derivatives of (3) with respect to w_j ($j = f, m$ and $j \neq i$). We obtain:

$$\frac{\partial h_i}{\partial w_j} = \frac{\partial L_i}{\partial \rho}\frac{\partial \rho_i}{\partial w_j} - \frac{\partial t_i}{\partial w_j}. \tag{20}$$

If we differentiate this expression with respect to y, and use (18), we obtain:

$$\frac{\partial \rho_i}{\partial w_j} = \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i / \partial y}{\Delta^i}. \tag{21}$$

The cross-derivative restrictions applied to (19) and (21) imply that

C2: $\frac{\partial^2 \Delta_i}{\partial y \partial w_j} \frac{\partial \Delta_i}{\partial y} \frac{\partial h_i}{\partial y} + \frac{\partial \Delta_i}{\partial w_j} \frac{\partial \Delta_i}{\partial y} \frac{\partial^2 h_i}{\partial y^2} - \frac{\partial \Delta_i}{\partial w_j} \frac{\partial^2 \Delta_i}{\partial y^2} \frac{\partial h_i}{\partial y} - \left(\frac{\partial \Delta_i}{\partial y} \right)^2 \frac{\partial^2 h_i}{\partial y \partial w_j} = 0$
for $i, j = f, m$ and $j \neq i$.

C3: $\frac{\partial^2 \Delta_i}{\partial s \partial w_j} \frac{\partial \Delta_i}{\partial s} \frac{\partial h_i}{\partial s} + \frac{\partial \Delta_i}{\partial w_j} \frac{\partial \Delta_i}{\partial s} \frac{\partial^2 h_i}{\partial s^2} - \frac{\partial \Delta_i}{\partial w_j} \frac{\partial^2 \Delta_i}{\partial s^2} \frac{\partial h_i}{\partial s} - \left(\frac{\partial \Delta_i}{\partial s} \right)^2 \frac{\partial^2 h_i}{\partial s \partial w_j} = 0$
for $i, j = f, m$ and $j \neq i$.

Substituting (21) in (20) yields the cross-derivatives of the domestic labor supplies:

$$\frac{\partial t_i}{\partial w_j} = \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i}{\partial y} - \frac{\partial h_i}{\partial w_j}. \tag{22}$$

Symmetry implies that

C4: $\frac{\partial h_f}{\partial w_m} - \frac{\partial h_f}{\partial y} \frac{\partial \Delta_f / \partial w_m}{\partial \Delta_f / \partial y} = \frac{\partial h_m}{\partial w_f} - \frac{\partial h_m}{\partial y} \frac{\partial \Delta_m / \partial w_f}{\partial \Delta_m / \partial y}.$

Let us note incidentally that, if the profit function is additive, i.e., $\partial t_i / \partial w_j = 0$, (22) gives:

$$\frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i}{\partial y} = \frac{\partial h_i}{\partial w_j}.$$

Substituting this expression in (21) yields the definition of the derivative of collective indirect utilities given by Chiappori, Fortin and Lacroix (2001).

Proof of Statement 3. If we differentiate (4) with respect to y and s, and use the definitions above, we obtain:

$$\frac{\partial v_i^*}{\partial y} = \lambda_i \frac{\partial h_i / \partial y}{\Delta_i}, \quad \frac{\partial v_i^*}{\partial s} = \lambda_i \frac{\partial h_i / \partial s}{\Delta_i}.$$

Similarly, if we differentiate (4) with respect to w_j ($j = f, m$ and $j \neq i$), we obtain:

$$\frac{\partial v_i^*}{\partial w_j} = \lambda_i \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i / \partial y}{\Delta_i}.$$

From the constraint $\rho_m + \rho_f = \psi$, and Hotelling's Lemma, we have:

$$\frac{\partial \rho_i}{\partial w_i} = -\left(t_i + \frac{\partial \Delta_j / \partial w_i}{\partial \Delta_j / \partial y} \frac{\partial h_j / \partial y}{\Delta_j} \right). \tag{23}$$

Differentiating (4) with respect to w_i, and using (23) and Roy's Identity, we obtain:

$$\frac{\partial v_i^*}{\partial w_i} = \lambda_i \left(h_i - \frac{\partial \Delta_j / \partial w_i}{\partial \Delta_j / \partial y} \frac{\partial h_j / \partial y}{\Delta_j} \right).$$

Since the partial derivatives are defined up to a multiplicative function λ_i, the collective indirect utilities are defined up to composition by an increasing transform.‖

5 Summary and conclusion

The present paper can be regarded as a toolbox for applied econometricians who are interested in performing welfare comparisons at the individual level. A synthesis of our main conclusions may be useful at this stage. Consider a sample of couples and assume there is domestic production. Then,

1. A simple model of market labor supplies, which does not allow for domestic production, may conveniently fit the data if and only if (i) the profit function is additive or (ii) Engel curves are linear and the profit function has a particular, not necessarily additive form.

2. If (i) or (ii) is satisfied, then the econometrician is not able to empirically reject the simple model of labor supply. Let us suppose the econometrician arbitrarily decides to ignore domestic production.

3. If (i) is satisfied, the welfare analyses the econometrician makes are valid. If (ii) is satisfied, the welfare analyses are biased: if spouses' labor supplies are substitutes (resp. complements), the direct effect of the wage on spouses' welfare is underestimated (resp. overestimated) and the cross effect is overestimated (resp. underestimated).

4. Finally, there exists a simple method for retrieving collective indirect utilities that is robust to household production. This method necessitates the observation of the sole market labor supplies, allows to carry out correct welfare analyses and, if the profit function is additive, reduces to the traditional method.

To sum up, the main — and probably the most unexpected — result in this paper is that the econometrician can generally get out of observing housework when performing welfare comparisons at the individual level. This opens up new horizons for empirical investigations with collective models. Quite importantly, however, the results that precede crucially depend on the assumption that domestic goods are marketable. Admittedly, the goods trade on outside markets are likely imperfect substitutes for goods produced within the household — except in agricultural households, for which domestic goods have a simple, natural interpretation. Our argument here is that the assumption of marketability is certainly less restrictive than the straight exclusion of domestic production as in the simple collective model of labor supply. Future research should, however, investigate the case of non-marketable domestic goods.

A Appendix — A Useful Lemma

Lemma 7 *Consider the following partial differential equation in f :*

$$f(x,y) + \frac{1}{a(x)}\frac{\partial f(x,y)}{\partial x} + \frac{1}{b(y)}\frac{\partial f(x,y)}{\partial y} + c(x) + d(y) = 0, \tag{24}$$

where functions $a(x)$, $b(y)$ are continuously differentiable in $\mathbb{R}$ and do not vanish simultaneously at any point of $\mathbb{R}$, and functions $c(x)$, $d(y)$ are continuous in $\mathbb{R}$. Then, the general solution of (24) on $\mathbb{R}^2$ is

$$f(x,y) = \frac{G\left(A(x) - B(y)\right)}{\exp\left(A(x) + B(y)\right)} + \frac{C(x)}{\exp A(x)} + \frac{D(y)}{\exp B(y)}$$

for some function G, where

$$\begin{aligned} A(x) &= \int a(x) \cdot dx, \\ B(y) &= \int b(y) \cdot dy, \\ C(x) &= \int a(x)c(x) \exp A(x) \cdot dx, \\ D(y) &= \int b(y)d(y) \exp B(y) \cdot dy. \end{aligned}$$

Proof. The idea of the proof is well-known in the theory of partial differential equations (see Zachmanoglou and Thoe, 1976, for instance). It follows in stages. We consider the solution of the homogenous partial differential equation that corresponds to (24), i.e.,

$$f(x,y) + \frac{1}{a(x)} \frac{\partial f(x,y)}{\partial x} + \frac{1}{b(y)} \frac{\partial f(x,y)}{\partial y} = 0,$$

and use a particular solution of (24) to solve the non-homogenous case.

Homogenous case. We have to introduce new coordinates, r and t, in terms of which (24) takes the form of an ordinary differential equation that can be easily solved. Let the new coordinates be related to the old ones by the equations: $r = r(x,y)$ and $t = t(x,y)$. We require that the functions $r(x,y)$ and $t(x,y)$ are continuously differentiable and their Jacobian is different from zero, i.e.,

$$J_{\text{pde}} \equiv \frac{\partial r}{\partial x} \frac{\partial t}{\partial y} - \frac{\partial r}{\partial y} \frac{\partial t}{\partial x} \neq 0.$$

If this condition is satisfied at the point (x_0, y_0), we also have in the neighborhood of (x_0, y_0) the inverse relations: $x = x(r,t)$ and $y = y(r,t)$. Substituting these expressions into (24) and using the Chain Rule, we obtain the following equation:

$$\zeta(r,t) + \alpha(r,t) \frac{\partial \zeta(r,t)}{\partial r} + \beta(r,t) \frac{\partial \zeta(r,t)}{\partial t} = 0,$$

where $\zeta(r,t) = f(x(r,t), y(r,t))$ and

$$\alpha = \frac{1}{a}\frac{\partial r}{\partial x} + \frac{1}{b}\frac{\partial r}{\partial y}, \qquad \beta = \frac{1}{a}\frac{\partial t}{\partial x} + \frac{1}{b}\frac{\partial t}{\partial y}.$$

We see that $\alpha = 0$ if r is a solution of the following partial differential equation:

$$\frac{1}{a}\frac{\partial r}{\partial x} + \frac{1}{b}\frac{\partial r}{\partial y} = 0. \tag{25}$$

This is the characteristic equation of the partial differential equation. There are infinitely many solutions. Supposing for example that $a(x) \neq 0$, the characteristic direction is given by:

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{b(y)}{a(x)}.$$

Then, for any change of variable such that this condition is satisfied, we have: $\alpha = 0$. The characteristic curves, which satisfy by definition this condition, can be obtained by solving the differential equation:

$$b(y) \cdot \mathrm{d}y = a(x) \cdot \mathrm{d}x.$$

That is:

$$\int b(y) \cdot \mathrm{d}y - \int a(x) \cdot \mathrm{d}x = k,$$

where k is a constant of integration. We choose thus the following change of variable:

$$r = \int b(y) \cdot \mathrm{d}y - \int a(x) \cdot \mathrm{d}x.$$

It is easy to show that (25) is satisfied for this change of variable. The second change of variable can be arbitrarily assigned such that $J_{\mathrm{pde}} \neq 0$. We choose:

$$t = y.$$

Using this change of variable gives a simple ordinary differential equation in ζ. That is:

$$\zeta(r,t) + \frac{1}{b(t)}\frac{\partial \zeta(r,t)}{\partial t} = 0.$$

The general solution to this ordinary differential equation is:

$$\zeta(r,t) = g(r) \times \exp\left(-\int b(t) \cdot \mathrm{d}t\right),$$

where g is any function. The general solution of the original (homogeneous) equation is:

$$f(x,y) = g\left(\int b(y) \cdot \mathrm{d}y - \int a(x) \cdot \mathrm{d}x\right) \times \exp\left(-\int b(y) \cdot \mathrm{d}y\right)$$

or

$$f(x,y) = \frac{G\left(A(x) - B(y)\right)}{\exp\left(A(x) + B(y)\right)}$$

for some function G.

Non-homogenous case. The second step is to obtain a solution in the non-homogeneous case. If we add a particular solution of the non-homogenous partial differential equation to the general solution which is derived above, we obtain the general solution of the non-homogeneous partial differential equation. In particular, a solution is:

$$\phi(x,y) = \frac{C(x)}{\exp\left(A(x)\right)} + \frac{D(y)}{\exp\left(B(y)\right)}.$$

This concludes the proof of the Lemma.‖

References

[1] Apps, Patricia F. and Ray Rees (1988), "Taxation and the household", *Journal of Public Economics*, vol. 35, pp. 355–369.

[2] Apps, Patricia F. and Ray Rees (1996), "Labour supply, household production and intra-family welfare distribution", *Journal of Public Economics*, vol. 60. pp. 199–219.

[3] Apps, Patricia F. and Ray Rees (1997), "Collective labor supply and household production", *Journal of Political Economy*, vol. 105, pp. 178–190.

[4] Aronsson, Thomas, Sven-Olov Daunfeldt, Magnus Wikstrom (2001), "Estimating intrahousehold allocation in a collective model with household production", *Journal of Population Economics*, vol. 14, pp. 569–584.

[5] Blundell, Richard, Pierre-André Chiappori, Thierry Magnac and Costas Meghir (2003), "Collective Labor Supply : Heterogeneity and Nonparticipation", Working Paper 01/19, Institute of Fiscal Studies.

[6] Chiappori, Pierre-André (1988), "Rational household labor supply", *Econometrica*, vol. 56, pp. 63–90.

[7] Chiappori, Pierre-André (1992), "Collective labor supply and welfare", *Journal of Political Economy*, vol. 100, pp. 437–467.

[8] Chiappori, Pierre-André (1997), "Introducing household production in collective models of labor supply", *Journal of Political Economy*, vol. 105, pp. 191–209.

[9] Chiappori Pierre-André, Richard Blundell and Costas Meghir (2004), "Collective Labour Supply with Children", Working Paper, Institute of Fiscal Studies.

[10] Chiappori, Pierre-André, Bernard Fortin and Guy Lacroix (2001), "Marriage market, divorce legislation, and household labor supply", *Journal of Political Economy*, vol. 110, pp. 37–72.

[11] Clark, Andrew, Hélène Couprie and Catherine Sofer (2004), "La modélisation collective de l'offre de travail. Mise en perspective et application aux données britanniques", *Revue économique*, vol. 55, pp. 767–789.

[12] Couprie, Hélène (2003), "Time Allocation within the Family : Welfare Implications of Life in a Couple", Working Paper, GREQAM, Université de la Méditerranée.

[13] Donni, Olivier (2003), "Collective household labor supply: non-participation and income taxation", *Journal of Public Economics*, vol. 87, pp. 1179–1198.

[14] Fong Y. and Junsen Zhang (2001), "The identification of unobservable independent and spousal leisure", *Journal of Political Economy*, vol. 109, pp. 191–202.

[15] Fortin, Bernard and Guy Lacroix (1997), "A Test of the Collective and UnitaryModel of Labour Supply", *Economic Journal*, vol. 107, pp. 933–955.

[16] Friedman M. (1953), "The methodology of positive economics". In: *Essays in Positive Economics*, Chicago: The University of Chicago Press.

[17] Gorman W.M. (1961), "On a class of preference fields", *Metroeconomica*, vol. 13, pp. 53–56.

[18] Graham, John W. and Carole A. Green (1984), "Estimating the parameters of a household production function with joint production", *Review of Economics and Statistics*, vol. 66, pp. 277–282.

[19] Gronau, Reuben (1977), "Leisure, home production and work — The theory of the allocation of time revisited", *Journal of Political Economy*, vol. 85, pp. 1099–1123.

[20] Grossbard-Schechtman Shoshana A. (2000), "Marriage market imbalances and the changing economic roles of women", Working Paper 00-02, San Diego State University.

[21] Hill, Martha S. and F. Thomas Juster (1985), "Constraints and complementarities in time use". In: F. Thomas Juster and Frank P. Stafford (eds), *Time, Goods, and Well-being*, Ann Arbor: Institute for Social Research, University of Michigan.

[22] Juster, F. Thomas and Frank P. Stafford (1991), "The allocation of time: empirical findings, behavioral models, and problems of measurement", *Journal of Economic Literature*, vol. 29, pp. 471–522.

[23] Lise, Jeremy and Shannon Seitz (2004), "Consumption inequality and intra-household allocations", Working Paper, Queen's University.

[24] Moreau Nicolas and Olivier Donni (2002), "Un modèle collectif d'offre de travail avec taxation", *Annales d'Économie et de Statistique*, vol. 65, pp. 55-81.

[25] Pollak Robert A. and Terence J. Wales (1992), *Demand System Specification and Estimation*, New-York: Oxford University Press.

[26] Singh, Inderjit, Lyn Squire and John Strauss (eds.) (1986), *Agricultural Household Models: Extensions, Applications, and Policy*, Baltimore: Johns Hopkins Press (for World Bank).

[27] Taylor J. Edward and Irma Adelman (2003), "Agricultural Household Models: Genesis, Evolution, and Extensions", *Review of Economics of the Household*, vol. 1, pp. 33–58.

[28] Vermeulen, Frederic (2004), "And the winner is... An empirical evaluation of two competing approaches to household labour supply", *Empirical Economics* (forthcoming).

[29] Zachmanoglou, E.C. and Dale W. Thoe (1976), "Introduction to Partial Differential Equations with Applications", New-York: Dover Publications.