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ABSTRACT

Selection and the Distribution of Female Hourly Wages in the U.S.*

We analyze the role of selection bias in generating the changes in the observed distribution of female hourly wages in the United States using CPS data for the years 1975 to 2020. We account for the selection bias from the employment decision by modeling the distribution of the number of working hours and estimating a nonseparable model of wages. We decompose changes in the wage distribution into composition, structural and selection effects. Composition effects have increased wages at all quantiles while the impact of the structural effects varies by time period and quantile. Changes in the role of selection only appear at the lower quantiles of the wage distribution. The evidence suggests that there is positive selection in the 1970s which diminishes until the later 1990s. This reduces wages at lower quantiles and increases wage inequality. Post 2000 there appears to be an increase in positive sorting which reduces the selection effects on wage inequality.

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1 Introduction

The dramatic increase in female wage inequality in the United States since the early 1980s (see, for example, Katz and Murphy 1992, Katz and Autor 1999, Lee 1999, Autor et al. 2008, Acemoglu and Autor 2011, Autor et al. 2016 and Murphy and Topel 2016) has been accompanied by substantial changes in both female employment rates and the distribution of their annual hours of work. Given the prominence that accounting for the selection bias (see Heckman 1974, 1979) from employment decisions has played in empirical studies of the determinants of female wages it seems natural to investigate its role in the evolution of wage inequality. This paper examines the sources of changes in the distribution of female hourly real wage rates in the United States from 1975 to 2020 while accounting for movements, and individuals' locations, in the annual hours of work distribution.

The inequality literature allocates wage changes to two sources. The first is the "structural effect" which captures the market value of an individual's characteristics. This includes skill premia, such as the returns to education (see, for example, Welch 2000 and Murphy and Topel 2016), cognitive and noncognitive skills (Heckman, Stixrud and Urzua 2006), declining minimum wages in real terms (see, for example, DiNardo et al. 1996 and Lee 1999), and the increasing use of non-compete clauses in employment contracts (Krueger and Posner 2018). The second source, referred to as the "composition effect", reflects differences across workers' observed characteristics. These include increases in educational attainment. Earlier papers (see, for example, Angrist et al. 2006, and Chernozhukov et al. 2013) have estimated these two effects under general conditions. However, as they focus on male wages they have ignored the role of selection.

Understanding the role of selection in the female wage inequality context is important. First, as the impact of selection is frequently interpreted as reflecting sorting patterns it is valuable from a policy perspective to understand how worker productivity has changed as an increasing proportion of women have entered the labor market. Second, the importance of accounting for selection bias in estimating the determinants of female wages suggests that an evaluation of the role of structural and composition effects requires an appropriate treatment of selection. Third, assessing the impact of selection on wages and inequality is particularly relevant

when the composition of the working and nonworking populations have evolved as drastically as has occurred in our sample period. Finally, understanding the impact of selection bias may provide policy makers with guidance as to which measures may be taken to reduce wage inequality.

Three important papers have investigated the role of selection in the United States over a period of increasing female wage inequality.[1] Mulligan and Rubinstein (2008), hereafter MR, correct for selection in the female mean wage in the United States for the years 1975-1999 and argue that the sharp increase of female wages partially reflected that the selected population of working females became increasingly more productive in terms of unobservables. They also find the pattern of sorting turned from negative to positive in the early 1990s. Evaluating the contribution of selection bias on the mean wage is straightforward in additive models as the selection component, under some assumptions, can be separated. The pattern of sorting is inferred from the coefficient for the selection correction. The nonseparable model required for estimating wage distributions has greater difficulties in isolating the selection component. Maasoumi and Wang (2019), hereafter MW, employ the copula based estimator for quantile selection models of Arellano and Bonhomme (2017), hereafter AB. AB and MW define the selection effect as the difference between the observed wage distribution and the counterfactual wage distribution simulated via their models' estimates assuming 100% participation. MW provide a similar conclusion regarding the pattern of sorting as MR for the overlapping years in their studies. Blau et al. (2021) follow Olivetti and Petrongolo (2008) who use a "selection on unobservables" approach to impute wages for nonworkers based on propensity scores for employment. They also compute the predicted wage distribution assuming 100% participation. In contrast to MR and MW, they find a more modest role for selection and that sorting did not change sign over the sample period. We define the selection effect as the difference in the observed wage distribution and the wage distribution that would result under the participation process associated with the year with the lowest participation rate. We find that the direction of sorting did not change during the years considered by MR.

We address several methodological and empirical issues regarding selection in the

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[1]Fernández-Val et al. (2022) study the impact of annual hours worked on inequality but their focus is on annual earnings and not wages.

female wage inequality context. Our methodological contributions are the following. First, we extend the Fernández-Val, Van Vuuren, and Vella (2021), hereafter FVV, estimator for nonseparable models with censored selection rules. The FVV estimator incorporates the number of working hours rather than the binary work decision as the selection variable and here we allow for different censoring points conditional on the individual's characteristics. This variation across censoring points captures differences in "fixed working costs" (see, for example, Cogan 1981). Second, we provide a procedure for decomposing changes in the wage distribution into structural, composition and selection effects in a nonseparable model which allows for selection from the choice of annual working hours. We contrast our decomposition approach with the corresponding exercise based on the Heckman (1979) selection model (HSM). Third, we extend our estimator to allow selection into annual hours to reflect two separate selection mechanisms. Namely, the choices of annual weeks and weekly hours. Fourth, we provide an estimator motivated by the ordered treatment model of Heckman and Vytlacil (2007) which allows for bunching in annual hours or annual weeks and apply it via our decomposition method.

Our following empirical contributions feature results based on the two most commonly employed Current Population Survey (CPS) data sets. First, unlike MR and MW who analyze wages for full-time full-year (FTFY) workers, we obtain a fuller picture of the evolution of the wage distribution by including all workers and accounting for selection from the hours of work decision. Second, we confirm previous findings, restricted to FTFY workers, regarding movements in the wage distribution. Female wage growth at lower quantiles is modest although the median wage has grown steadily. Gains at the upper quantiles are large and have produced an increase in female wage inequality. Finally, we provide new evidence regarding the role of selection. Changes in selection are especially important at the lower end of the wage distribution and have generally decreased wage growth and increased wage inequality. Although we are able to reproduce the estimated sorting pattern as MR and MW, we illustrate this reflects the employed identification assumptions. We show that exploiting the variation in hours worked as a form of identification produces results consistent with positive sorting for the whole sample period.

An important empirical result relates to the pattern of sorting and its implication

for the impact of selection on wages and inequality. We find clear evidence of positive sorting in the mid 1970s. The period 1975 to 2000 experiences a shift in the distribution of female annual hours of work, accompanied by a reduction in the level of positive sorting. These two forces decrease wages at lower quantiles and increase wage inequality. For the remainder of our sample period there appears to be a return to higher levels of positive sorting and a decrease in the impact of selection on wage inequality.

The rest of the paper is organized as follows. The next section discusses the data. Section 3 describes our empirical model and defines our decomposition exercise. It also provides alternative estimators employing ordered or multiple censored selection rules. This section concludes with a comparison of our decomposition approach with that associated with the HSM. Section 4 presents the empirical results. Section 5 reconciles the difference between our results with those of MR, while Section 6 investigates the impact of the changes in selection in wage inequality. Section 7 offers some concluding comments.

2   Data

We employ the two most commonly analyzed micro-level data sets, the Annual Social and Economic Supplement (ASEC) and the Merged Outgoing Rotation Groups (MORG), from the CPS. Appendix A of Lemieux (2006) provides a comparison of the two data sets. We employ both to contrast results and to allow comparisons with earlier studies.

2.1   Annual Social and Economic Supplement

We employ the ASEC for the 46 survey years from 1976 to 2021 reporting annual earnings for the previous calendar year.[2] Unless otherwise stated, we refer to the year for which the data are collected and not that of the survey. The 1976 survey is the first for which information on weeks worked and usual hours of work per week last year are available. To avoid issues related to retirement and ongoing educational

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[2]The data were obtained via the IPUMS-CPS website maintained by the Minnesota Population Center (Flood et al. 2015).

investment we restrict attention to those aged 24–65 years in the survey year. This produces an overall sample of 2,219,820 females. The annual sample sizes range from 33,924 in 1976 to 59,622 in 2001.

Annual hours worked are defined as the product of weeks worked and usual weekly hours of work last year. Those reporting zero hours usually respond that they are not in the labor force in the week of the March survey. We define hourly wages as the ratio of reported annual labor earnings in the year before the survey, converted to constant 2019 prices using the consumer price index for all urban consumers, and annual hours worked. Hourly wages are unavailable for those not in the labor force. For the self-employed, unpaid family workers and the Armed Forces annual earnings or annual hours tend to be poorly measured and we exclude these groups from our sample. This results in a deletion of 5.4, 0.4 and 0.07 percent of the observations for the self employed, unpaid family workers and the Armed Forces, respectively. The figures for self employed and the armed forces have trended upwards while those for family workers have trended downwards over the sample period. These groups do not show any cyclical variation. The only exception is the number of self employed during the Great Recession which, compared to the total employed, dropped considerably. We use observations with imputed wages for their values of working hours but do not use them in the wage sample. The restriction to civilian dependent employees with positive hourly wages and people out of the labor force last year results in a sample of 2,055,063 females. The subsample of civilian dependent employees with positive hourly wages comprises 1,190,928 observations. A benefit of the ASEC is its extensive family background variables.

2.2 Merged Outgoing Rotation Groups

We use the years 1979 to 2019 for the MORG using the CEPR extracts. The MORG contains information on hourly wages in the survey week for those paid by the hour and on weekly earnings from the primary job during the survey week for those not paid by the hour. Lemieux (2006) and Autor et al. (2008) argue that the ORG CPS data are preferable as they provide a point-in-time wage measure and workers paid by the hour, more than half of the U.S. workforce, may better recall their hourly wages. Based on comparable data restrictions as the ASEC, the MORG results in
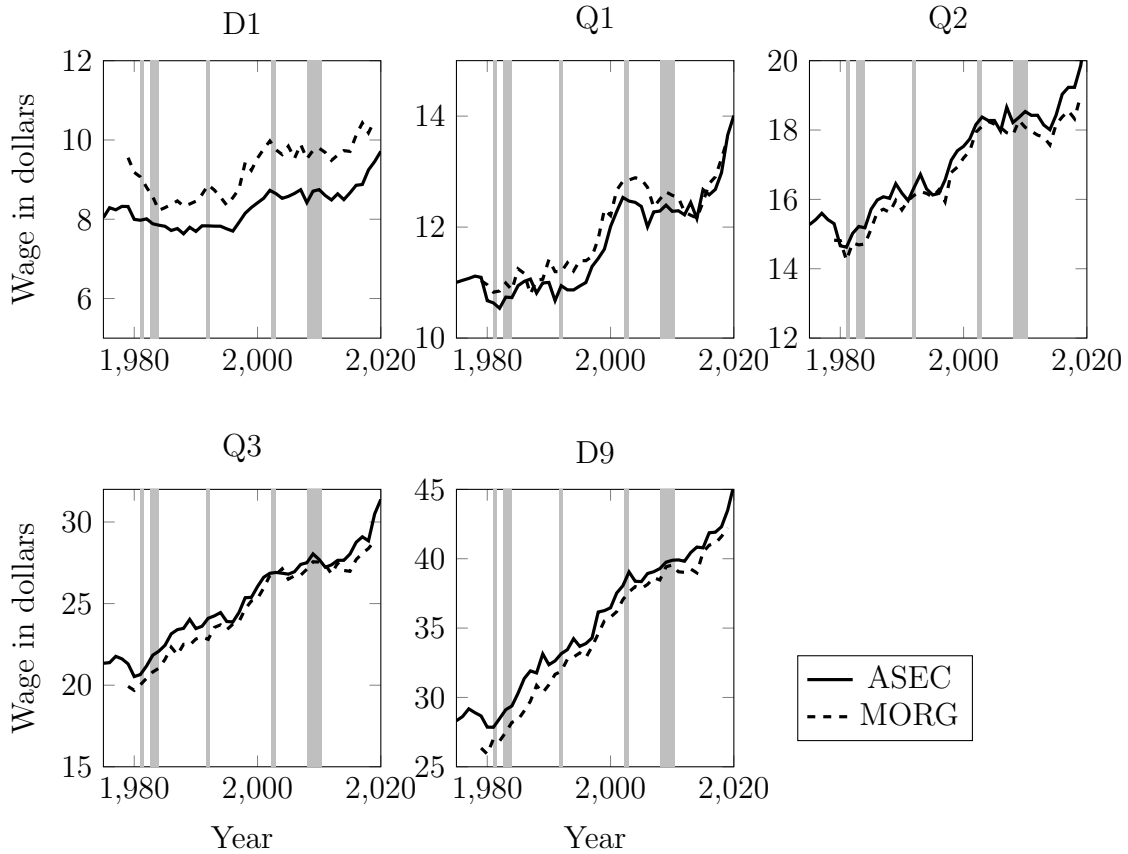
Figure 1: Real hourly wage at different quantiles (measured in 2019 dollars).

4,298,682 observations with the highest numbers of observations in 1980 (121,786) and the lowest in 2019 (91,647). The subsample of civilian dependent employees working in the reference week is 2,219,820 observations. This low figure, relative to the ASEC, is expected as employees who did not work in the reference week may have worked in another week. Family background variables are only available since 1984. This restricts the family background characteristics to family size.

2.3 Descriptive statistics

Figure 1 presents female real hourly wage rates at selected quantiles for both the ASEC and MORG. The shaded areas in Figure 1 indicate recessionary periods as dated by the National Bureau of Economic Research. Quartiles are denoted by Q and deciles by D. Figure 1 confirms two observations made by Lemieux (2006). First, median wages for the ASEC and MORG are very similar. The median wage in the ASEC, despite occasional dips, increases by 32.5 percentage points over the

period 1975-2020 while the MORG shows very similar patterns with growth of 25.9% for 1979-2019. Second, the ASEC features more wage dispersion with relatively lower wages at quantiles below Q2, and higher wages at quantiles above Q2. The profiles at D1 and Q1 are similar to that at Q2 with increases of 18.8% and 24.1% respectively. At Q3 and D9 there has been strong and steady growth since 1980 with an increasing gap between each and the median wage. Q3 increases by 40.2% and D9 by 48.4%. The MORG shows very similar patterns with growth of 25.9% for the median wage for 1979-2019, and increases of 22.5%, 37.1% and 45.8% at D1, Q3 and D9. The corresponding increase in the ASEC median wage for the same time period is 26.8%. The only remarkable difference across the figures are the wages at D1 which are clearly higher for the MORG than for the ASEC. This, as noted by Lemieux, may reflect measurement error. However, measurement error does not explain why the difference is greater at D1 than D9. Another cause, as we explain below, may be the relatively lower employment rate in the MORG. This implies that the MORG D1 is higher in the population distribution than the ASEC D1. The difference between the data sets decreases for the MORG in 1979-1981 with a corresponding smaller decrease for the ASEC. The ASEC and the MORG then show similar growth with the ASEC wage consistently below the MORG. As noted by Lemieux (2006), the period 1979 to 1984 displays a sharp increase in the residual variance in the MORG not found for the ASEC.

Figure 2 confirms the widening wage gap with an increase in the interquartile ratio for our sample period. For the ASEC there is an increase of 49.4%. For the shorter period the MORG data features an increase of 45.0% with the corresponding increase in the ASEC for that period of 44.3%. Both data sets confirm the drastic increases in female wage inequality. The pattern associated with the interdecile ratio is somewhat more complicated. Overall, the MORG has the larger increase but the difference reflects wage movements for the period 1979 to 1984. The increase in the interdecile ratio post 1984 is relatively lower for the MORG. This is consistent with Lemieux (2006) who notes that the ASEC not only has higher wage dispersion but also increases faster over time.

Figure 3 presents the female employment rates and average working weeks and hours of those employed. As explained above, the higher employment rate for ASEC
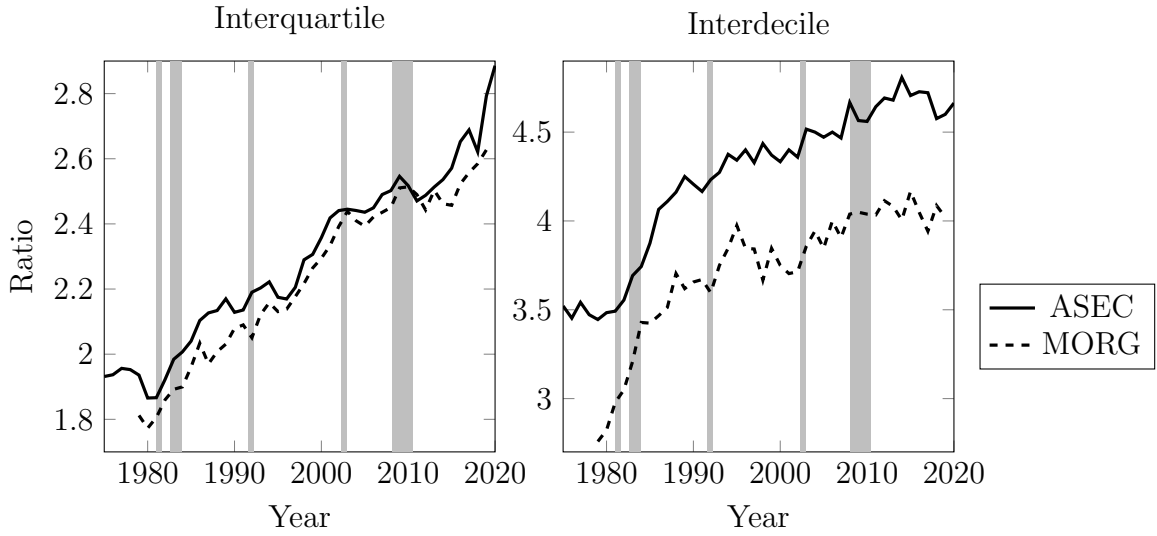
Figure 2: Measures of inequality.

is expected. The employment rates increased drastically during the last decades of the previous century before decreasing slightly in the 2000s and then slightly rebounding in the most recent years. The decreased employment since 2000 resulted in the US employment rate at the end of the Great Recession returning to the 1980 level (see Beaudry et al. 2016). The average number of working hours per week increased over the sample period from 35.3 to 38.3 for the ASEC and from 35.3 to 37.8 for the MORG. Hours are highly cyclical. There is not a large difference between the two samples although the increase in working hours has been slightly smaller in the 1990s for the MORG. The number of weeks also increased steeply from 42.0 to 46.2. The drop in 2020 during the COVID19 pandemic is noteworthy.

Figure 4 reveals that the wage levels of FTFY workers, defined as working 52 weeks per year for at least 40 hours, are somewhat higher than for all workers. Their fraction increased from 50 to 71%. The difference in wages diminishes at higher quantiles and, though not reported here, becomes negligible at D9. Contrasting full-year (FY) and full-time (FT) wages against non-FY and non-FT wages provides a similar finding. The differences at the bottom of the distribution in comparison to the top, may reflect that the choice of working FT and/or FY should be incorporated in the selection model. Moreover, as non-FTFY (or non-FT/non-FY) workers are potentially more vulnerable to interruptions it seems inappropriate to exclude them in an evaluation of wage inequality. The wage differences between these two working

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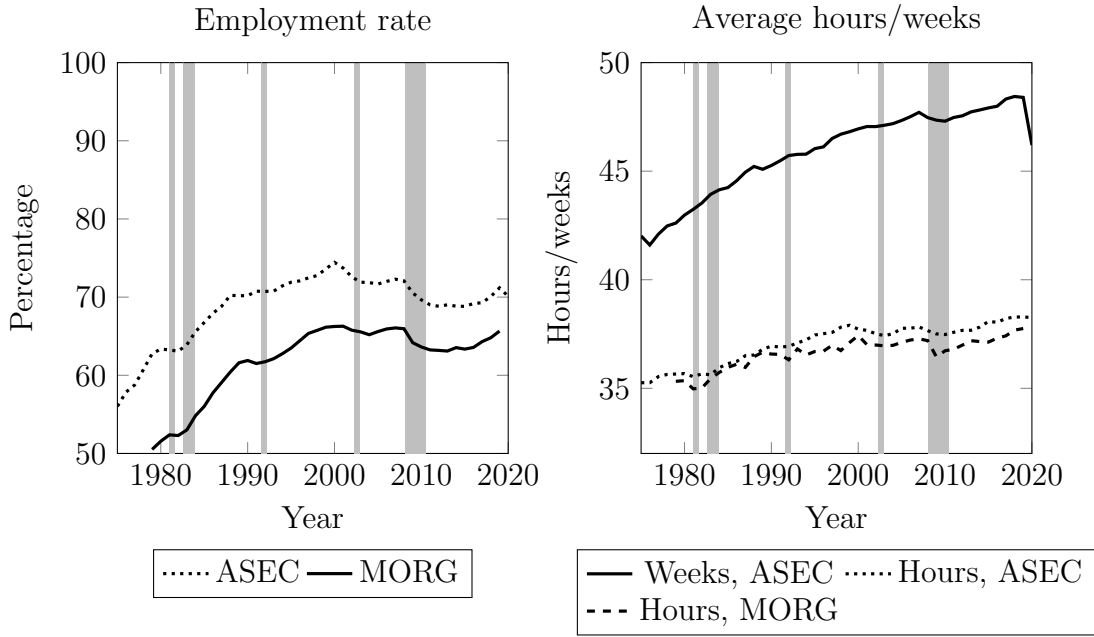
Figure 3: Employment rates and hours/weeks worked of employees.

categories might also reflect selection effects. The failure to include those who do not work FTFY means that the selection effects in earlier studies may reflect movement from the non-FTFY to FTFY, rather than from non-employment to FTFY.

3   Econometric analysis

3.1   Model with Censored Selection

We consider a version of the HSM where the censoring rule for the selection process incorporates the information on annual hours worked rather than the binary employment/non-employment decision. The model has the form:

$$Y = g(X, E), \quad \text{if } D = 1, \tag{1}$$

$$H = h(Z, V), \quad \text{if } D = 1, \tag{2}$$

$$D = \mathbf{1}\{h(Z, V) > \mu(Z)\}, \tag{3}$$

where Y is the logarithm of hourly wages, H is annual hours worked, D is a selection or employment indicator equal to 1 if H and Y are observed or equivalently if H

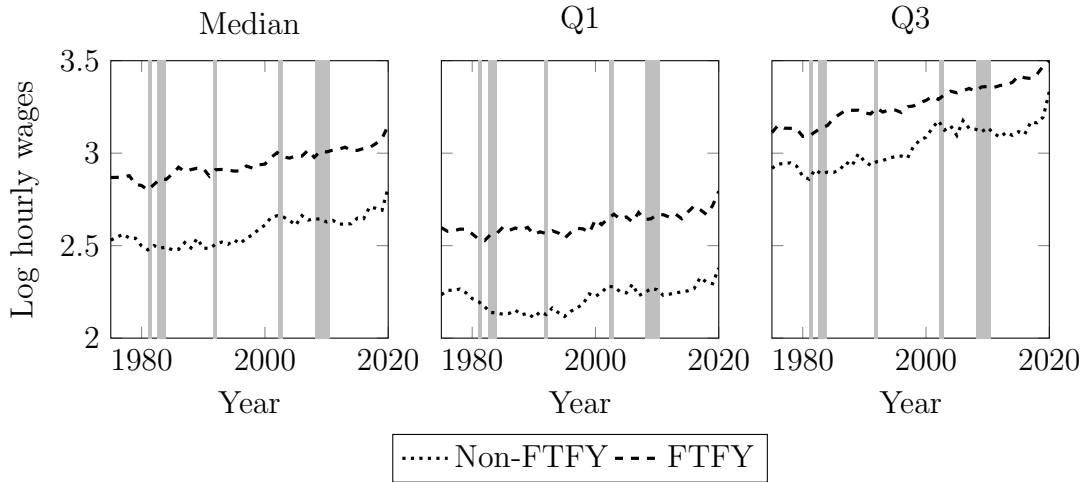Figure 4: Percentiles of real log hourly wages by full-time full-year (FTFY) employment status

is not censored at $\mu(Z)$, X and Z are vectors of observable conditioning variables, g, h and μ are unknown functions, and E and V are respectively a vector and a scalar of potentially dependent unobservable variables with cumulative distribution functions (CDFs) F_E and F_V. We assume that X is a, not necessarily strict, subset of Z, i.e. $X \subseteq Z$.

We refer to equation (3) as the selection rule. It corresponds to censored selection with an unobserved censoring point, that is we observe the censoring status, D, but not the censoring point, $\mu(Z)$. Equations (2)–(3) can be considered a reduced-form representation for hours worked. The model is a nonparametric and nonseparable version of the Tobit type-3 model considered by FVV, extended to incorporate an unknown censoring threshold which is a function of Z. This threshold is motivated by fixed labor costs measured in terms of hours. Individuals only work if the desired number of hours exceeds a minimum number given by $\mu(Z)$. Cogan (1981) shows that fixed labor costs reduce the number of individuals working very few hours. We allow the fixed labor costs to vary by individual and household characteristics.

Let $\perp\!\!\!\perp$ denote stochastic independence. We assume:

Assumption 1 (Control Function) $(E, V) \perp\!\!\!\perp Z$, V *is a continuously distributed random variable with strictly increasing CDF on the support of* V, *and* $v \mapsto h(Z, v)$ *is strictly increasing a.s.*

Without loss of generality, we can normalize V to be standard uniformly distributed. The potential dependence between E and V implies $Z's$ independence of E in the entire population does not exclude dependence in the selected population with $D = 1$. FVV showed that Z is independent of E conditional on V and $D = 1$ when $\mu(Z) = \mu$. Let $H^* = h(Z, V)$. Lemma 1 extends FVV's result to our model.

Lemma 1 (Existence of Control Function) *Under the model in* (1)–(3) *and Assumption 1:*

$$E \perp\!\!\!\perp Z \,|\, V, D = 1.$$

Moreover, $V = F_{H^* \,|\, Z}(H \,|\, Z)$ *for* $H^* = h(Z, V)$, *and* $F_{H^* \,|\, Z}$ *is identified by*

$$F_{H^* \,|\, Z}(h \,|\, z) = 1 - \pi(z)[1 - F_{H \,|\, Z, D}(h \,|\, z, 1)],$$

where $\pi(z) = \mathbb{P}(D = 1 \,|\, Z = z)$ *is the propensity score of selection.*

The proof of the first statement follows from the same argument as in Lemma 1 of FVV. Thus, conditioning on (Z, V) makes $D = 1$ or $h(Z, V) > \mu(Z)$ deterministic. The assumption that Z is independent of (E, V) then proves the result implying that V is an appropriate control function.[3] The result $V = F_{H^* \,|\, Z}(H \,|\, Z)$ follows directly from the assumption that h is strictly increasing in its second argument and the normalization on the distribution of V. Identification of $F_{H^* \,|\, Z}$ follows from Buchinsky and Hahn (1998). In the Appendix, we propose an estimator of $F_{H^* \,|\, Z}$ based on distribution regression. This estimator is an alternative to the estimators of Buchinsky and Hahn (1998) and Chernozhukov and Hong (2002), which are based on quantile regression.

The decompositions presented below require a wage distribution which incorporates the value of V and a statement regarding the region in which it is identified. To proceed, we denote the support of random variables and vectors by calligraphic letters while lower case letters in parentheses indicate that the support is conditional on a stochastic vector taking a particular value; e.g. $\mathcal{Z}(x)$ is the support of $Z \,|\, X = x$. We define the Local Average Structural Function (LASF) and Local

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[3]This result is closely related to that of Imbens and Newey (2009), who consider estimation and identification of a nonseparable model with a single continuous endogenous explanatory variable.

Distribution Structural Function (LDSF) as:

$$\mu(x,v) := \mathbb{E}[g(x,E)\,|\,V=v], \quad G(y,x,v) := \mathbb{P}[g(x,E) \leq y\,|\,V=v]. \qquad (4)$$

They represent the mean and distribution of Y if all individuals with control function equal to v had observable characteristics equal to x. An argument similar to FVV shows that:

$$\mu(x,v) = \mathbb{E}[Y\,|\,V=v, X=x, D=1], \quad G(y,x,v) = \mathbb{P}[Y \leq y\,|\,V=v, X=x, D=1],$$

provided $(x,v) \in \mathcal{XV}^*$, where:

$$\mathcal{XV}^* = \{(x,v) \in \mathcal{X} \times \mathcal{V}\,|\,\exists z \in \mathcal{Z}(x) : h(z,v) > \mu(z)\}.$$

This set, referred to as the identification set by FVV, is identical to the support of (X,V) among the selected population. Lemma 1 implies that the LASF and LDSF equal the mean and distribution of the observed Y conditional on (X,V) and that it is identified. This follows directly from $(E,V) \perp\!\!\!\perp Z$ and that $(x,v) \in \mathcal{XV}^*$ implies the ability to find a (z,v) combination for which $h(z,v) > \mu(z)$. We refer to FVV for a discussion on how the size of the identified set depends on the availability of exclusion restrictions on Z with respect to X.

There are different candidates for H in (2). As the ASEC provides both usual hours worked per week and annual hours, calculated as the product of weeks worked last year and the usual number of hours worked per week, we employ several alternatives. Although the usual hours per week may be the variable in the ASEC that is closest to the hours decision in labor supply models (Killingsworth, 1983), it may also reflect whether the job has pre-set hours. Therefore, we employ the annual measure which incorporates the weeks decision. As the extensive margin may capture whether an individual has worked a positive number of hours in the past year, we also investigate the use of the number of worked weeks. A theoretical motivation for this measure follows from search models in which the offered wages depend positively on the job offer arrival rate and negatively on the separation rate (Burdett and Mortensen, 1998). As these rates also determine the number of weeks

worked, it implies a relationship between weeks worked and wages. The appropriate censoring variable in the MORG is the number of hours worked in the reference week. Note that this variable solves some of the problems mentioned above.

3.2 Counterfactual distributions

We consider counterfactual CDFs constructed by integrating the LDSF with respect to different joint distributions of the conditioning variables and control function.[4] Counterfactual CDFs enable the construction of wage decompositions and facilitate counterfactual analyses similar to those in DiNardo et al. (1996), Ñopo (2008), Fortin et al. (2011), and Chernozhukov et al. (2013). We focus on CDFs for the selected population. To simplify notation, the superscript s denotes that we condition on $D = 1$. The decompositions are based on the following representation of the CDF of the observed Y:[5]

$$\mathbb{P}[Y \leq y \,|\, D = 1] =: G_Y^s(y) = \int G(y, x, v)\, dF_{Z,V}^s(z, v),$$

where:
$$F_{Z,V}^s(z, v) = \frac{\mathbf{1}\{h(z, v) > \mu(z)\}\, F_{Z,V}(z, v)}{\int \mathbf{1}\{h(z, v) > \mu(z)\}\, dF_{Z,V}(z, v)},$$

denotes the joint CDF of (Z, V) in the selected population and $F_{Z,V}$ denotes the joint CDF of Z and V in the entire population.

The counterfactual CDFs are constructed by combining the CDFs G and $F_{Z,V}$ with the selection rule (3) for different groups, each group corresponding to a different time period or a subpopulation defined by certain characteristics. Specifically, let G_t be the LDSF in group t, F_{Z_k, V_k} be the joint CDF of Z and V in group k, and let $\mathbf{1}\{h_r(z, v) > \mu_r(z)\}$ be the selection rule in group r. The counterfactual CDF of Y when G is as in group t, $F_{Z,V}$ is as in group k, and the selection rule is as in group r is defined as:

$$G_{Y_{\langle t,k,r \rangle}}^s(y) = \frac{\int G_t(y, x, v)\, \mathbf{1}\{h_r(z, v) > \mu_r(z)\}\, dF_{Z_k, V_k}(z, v)}{\int \mathbf{1}\{h_r(z, v) > \mu_r(z)\}\, dF_{Z_k, V_k}(z, v)}, \tag{5}$$

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[4]Counterfactual means can be constructed similarly using the LASF in place of the LDSF; see Section 3.5.

[5] We refer to FVV for details.

provided that the integrals are well-defined. Since the mapping $v \mapsto h(z, v)$ is strictly monotonic, the condition $h_r(z, v) > \mu_r(z)$ in (5) is equivalent to the condition:

$$v > F_{H_r^* \mid Z}(\mu_r(z) \mid z) = 1 - \pi_r(z), \qquad (6)$$

where $F_{H_r^* \mid Z}(\mu_r(z) \mid z)$ is the probability of working less hours than the censoring point conditional on $Z = z$ in group r and $\pi_r(z)$ is the propensity of working in that group. Given $G_{Y_{\langle t,k,r \rangle}}^s(y)$, the corresponding counterfactual quantile function (QF) is:

$$q_{Y_{\langle t,k,r \rangle}}^s(\tau) = \inf\{y \in \mathbb{R} \colon G_{Y_{\langle t,k,r \rangle}}^s(y) \geq \tau\}, \qquad 0 \leq \tau \leq 1. \qquad (7)$$

Under these definitions the observed CDF and QF of Y for the selected population in group t are $G_{Y_{\langle t,t,t \rangle}}^s$ and $q_{Y_{\langle t,t,t \rangle}}^s$ respectively.

Nonparametric identification of (5) and (7) depends on whether the integrals in (5) are well defined. They are when two conditions are met. First, if $\mathcal{Z}_k \subseteq \mathcal{Z}_r$, then π_r is identified over all z combinations in the integral. Second, when $(\mathcal{XV}_k \cap \mathcal{XV}_r^*) \subseteq \mathcal{XV}_t$, then the LDSF is identified for all combinations of z on which we integrate. Here, $\mathcal{XV}_r^*$ denotes the support of (X, V) for the selected population in group r. The identification conditions simplify when we consider two years for q, r, and t, such as 0 and 1, which is relevant for the decompositions. For example, we need $\mathcal{XV}_1 \cap \mathcal{XV}_0^* \subseteq \mathcal{XV}_1^*$ and $\mathcal{Z}_1 \subseteq \mathcal{Z}_0$ to identify $G_{\langle 1,1,0 \rangle}$ and $\mathcal{XV}_0^* \subseteq \mathcal{XV}_1^*$ to identify $G_{\langle 1,0,0 \rangle}$. A sufficient condition for $\mathcal{XV}_1 \cap \mathcal{XV}_0^* \subseteq \mathcal{XV}_1^*$ and $\mathcal{XV}_0^* \subseteq \mathcal{XV}_1^*$ is that the employment rates in year 0, conditional on X, are lower than those in year 1.

Using (7), we decompose the difference in the observed QF of Y for the selected population between any two groups, say group 1 and group 0, as:[6]

$$q_{Y_{\langle 1,1,1 \rangle}}^s - q_{Y_{\langle 0,0,0 \rangle}}^s = \underbrace{[q_{Y_{\langle 1,1,1 \rangle}}^s - q_{Y_{\langle 1,1,0 \rangle}}^s]}_{[1]} + \underbrace{[q_{Y_{\langle 1,1,0 \rangle}}^s - q_{Y_{\langle 1,0,0 \rangle}}^s]}_{[2]} + \underbrace{[q_{Y_{\langle 1,0,0 \rangle}}^s - q_{Y_{\langle 0,0,0 \rangle}}^s]}_{[3]}, \qquad (8)$$

where [1] is a selection effect that captures changes in the selection rule given the joint distribution of Z and V, [2] is a composition effect that reflects changes in the joint distribution of Z and V, and [3] is a structural effect that reflects changes

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[6]Note that alternative orders are possible. We investigated the impact of changes in the order on our empirical results and these were minor. Details are available upon request.

in the conditional distribution of Y given Z and V. These effects are relative to the base year. We stress that this definition of the selection effect differs from the standard definition. This is discussed in Section 3.5.

3.3 Double selection mechanism

The model can be extended to a multiple censored selection mechanism operating through both weeks and hours. The model has the form:

$$Y = g(X, E), \text{ if } h(Z, V_H) > \mu_H(Z) \text{ and } w(Z, V_W) > \mu_W(Z), \tag{9}$$

$$H = h(Z, V_H), \text{ if } h(Z, V_H) > \mu_H(Z), \tag{10}$$

$$W = w(Z, V_W), \text{ if } w(Z, V_W) > \mu_W(Z), \tag{11}$$

where the unobserved thresholds $\mu_H(Z)$ and $\mu_W(Z)$ are functions of the individual's characteristics.[7] The distributions of $H^* := h(Z, V_H)$ and $W^* := w(Z, V_W)$ conditional on Z, denoted V_H and V_W, respectively, are each required as control functions. The analysis of this model is similar to that above. However, it is necessary to employ both control functions to, for example, calculate the LDSF, *i.e.*

$$G(y, x, v_H, v_W) = \mathbb{P}(g(x, E) \leq y \,|\, V_H = v_H, V_W = v_W).$$

The identification conditions change to accommodate that the support condition is defined over two control functions. Using the same notation as Section 3.2, the support requirements for the counterfactuals are $\mathcal{Z}\mathcal{V}_H\mathcal{V}_{W,k} \subseteq \mathcal{Z}\mathcal{V}_H\mathcal{V}_{W,r}$ and $(\mathcal{X}\mathcal{V}_H\mathcal{V}_{W,k} \cap \mathcal{X}\mathcal{V}_H\mathcal{V}_{W,r}^*) \subseteq \mathcal{X}\mathcal{V}_H\mathcal{V}_{W,t}$. We acknowledge that there are circumstances under which this model will collapse to the single censoring mechanism case. However, as these are somewhat obvious we do not detail them here.

3.4 Model with ordered selection

The models above employ control functions which assume that the selection variable is continuous. However, both the numbers of weeks and hours worked feature

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[7]A further complication could be introduced by assuming that either the equation (10) or (11) is the sole source of selection.

bunching at specific values (*e.g.* 40 hours and 52 weeks). The following model with ordered selection incorporates bunching:

$$Y = g(X, E) \text{ if } H \geq 1,$$

$$H = \begin{cases} 0 & \text{if } V \leq \mu_0(Z), \\ 1 & \text{if } \mu_0(Z) < V \leq \mu_1(Z), \\ \vdots \\ K & \text{if } V > \mu_{K-1}(Z), \end{cases}$$

where the variables have similar interpretations as above. The main difference between this model and those above is that it allows a discrete distribution of H at the expense of requiring separability in the selection process. We assume $Z \perp\!\!\!\perp (E, V)$ and V follows a standard uniform distribution. This model is related to the ordered choice model of Heckman and Vytlacil (2007, p. 4980), but unlike their model $g(X, E)$ does not depend on H. It can also be interpreted as an extension of Newey (2007) to multiple ordered outcomes.

We define the identification set as:

$$\mathcal{XP}_K := \{(x, p) \in \mathcal{X} \times [0, 1) \,|\, \exists (h, z) \in \mathcal{HZ}(x) : \mu_{h'}(z) = p, h' \leq h, h > 0\}.$$

This set collects (x, p) combinations in the selected sample (i.e. $H = h > 0$) for which there is a (h, z) combination in $\mathcal{HZ}(x)$ such that $\mu_{h'}(Z) = p$ for the propensity score of a value of H smaller or equal than the observed value h. For example, if $H = 3$, this restriction is satisfied when $\mu_{h'}(z) = p$, for some $h' \in \{0, 1, 2, 3\}$. We define the LDSF as in (4). We prove the following lemma in the Appendix.

Lemma 2 *Suppose that $(x, p) \in \mathcal{XP}_K$ and $\mathcal{Z}(x, h) = \mathcal{Z}(x)$ for all $h > 0$. Then,*

$$\frac{1}{1-p} \int_p^1 G(y, x, v) dv = \mathbb{P}(Y \leq y \,|\, X = x, H > h, \mu_h(Z) = p),$$

and the probability in the RHS is identified.

Lemma 2 implies $(x, p) \in \mathcal{XP}_K$ is also a sufficient condition for identification as

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(see also Heckman and Vytlacil, 2007):

$$G(y, x, p) = -\frac{\partial}{\partial p} \int_p^1 G(y, x, v) dv = \frac{\partial}{\partial p} \left\{ (1 - p) \mathbb{P}(Y \leq y \mid X = x, H > h, \mu_h(Z) = p) \right\}.$$

We need additional assumptions to obtain counterfactual distributions. In the models with continuous censoring we hold the value of the control function constant and change the lowest value at which the individual is participating (see (6)). We cannot follow the same strategy here as V is not point identified. However, from the values of H and Z we know that the value of V is between $\mu_{H-1}(Z)$ and $\mu_H(Z)$. This implies that individuals with $H = 1$ have the lowest values of V. Therefore, if we increase $\mu_0(Z)$, while leaving V unchanged, some individuals with $H = 1$ would no longer participate although we do not know who. Hence, we integrate over the distribution of V and change the range of integration accordingly. We show in the appendix that:

$$G_Y^s(y) = \frac{\sum_{h=1}^K \int_{\mathcal{Z}} \int_{\mu_{h-1}(z)}^{\mu_h(z)} G(y, x, v) dv dF_Z(z)}{\int_{\mathcal{Z}} (1 - \mu_0(z)) dF_Z(z)}, \tag{12}$$

where $\mu_K(z) := 1$ for any z. This equation is comparable to equation (7.2) of Heckman and Vytlacil, 2007). Based on (12), the counterfactual distribution when G is as in group t, F_Z is as in group k, and the selection rule is as in group r is:

$$\begin{aligned} G_{Y_{\langle t,k,r \rangle}}^s(y) = {} & \frac{\int_{\mathcal{Z}^k} \int_{\mu_0^r(z)}^{\mu_1^k(z)} G_t(y, x, v) dv dF_{Z^k}(z)}{\int_{\mathcal{Z}^k} (1 - \mu_0^r(z)) dF_{Z^k}(z)} + \\ & \frac{\sum_{h=2}^K \int_{\mathcal{Z}^k} \int_{\mu_{h-1}^k(z)}^{\mu_h^k(z)} G_t(y, x, v) dv dF_{Z^k}(z)}{\int_{\mathcal{Z}^k} (1 - \mu_0^r(z)) dF_{Z^k}(z)}. \end{aligned} \tag{13}$$

The decompositions are identical to (8). The identification restrictions are related to the integrals in (13). The integral in the numerator of the second line of (13) can be written as

$$\int_{\mathcal{Z}^k} \left[\int_{\mu_{h-1}^k(z)}^1 G_t(y, x, v) dv - \int_{\mu_h^k(z)}^1 G_t(y, x, v) dv \right] dF_{Z^k}(z).$$

For both of these terms to be identified for any h, we need that $\mathcal{XP}_K^k \subseteq \mathcal{XP}_K^t$. For

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the identification of the integral in the numerator of the first line of (13), a similar argument gives $(\mathcal{XP}_K^r \cap \mathcal{XP}_K^k) \subseteq \mathcal{XP}_K^t$. We also need that $\mathcal{Z}^k \subseteq \mathcal{Z}^r$ otherwise $\mu_0^r(z)$ is not identified. The identification restrictions imply that, for example, to identify $G_{Y_{\langle 1,1,0 \rangle}}^s$, one needs that $\mathcal{XP}_K^0 \subseteq \mathcal{XP}_K^1$ and $\mathcal{Z}^1 \subseteq \mathcal{Z}^0$. The interpretation of these restrictions not only depends on the employment rates between year 0 and 1 but also on whether the propensity scores in year 1 overlap those of year 0.

Despite these requirements, there is a benefit of using an ordered rather than dichotomous selection rule. In the latter case, the restriction for $G_{Y_{\langle 1,1,0 \rangle}}^s$ would have been that the support of the propensity scores of employment for year 1, $\mu_0^1(Z)$, should overlap with those of year 0, $\mu_0^0(Z)$. For ordered selection it is only necessary that one of the propensity scores in year 1, *i.e.* $\mu_h^1(Z)$, $h = 1, \ldots, K$, overlaps with the propensity score of employment for year 0, *i.e.* $\mu_0^0(Z)$.

3.5 Comparison with the Heckman selection model

The decomposition (8) yields different effects to those derived by MR for a parametric version of the HSM. The MR selection effect excludes one component that we attribute to the selection effect and includes another that we assign to the composition effect. Two other components are sorting effects that cannot be separately identified from the structural effect in nonseparable models. A common measure of a selection effect is the difference between the average wage for the selected population and that for the entire population. The change in the selection effect is how this difference varies over time. We define the change in the selection effect as the difference between the average wage in the selected population with that which would result under a more restrictive selection rule.

To illustrate the difference with MR, suppose that the population model in period t is the following parametric version of the HSM:

$$Y_t = \alpha_t' X_t + E_t, \quad \text{if } H_t > 0, \tag{14}$$

$$H_t = \max\{\gamma_t' Z_t + V_t, 0\}, \tag{15}$$

where the first element of X_t is the constant term, and E_t and V_t are distributed independently of (X_t, Z_t) as bivariate normal with zero means, variances $\sigma_{E_t}^2$ and

$\sigma_{V_t}^2$, and correlation coefficient ρ_t.[8] The counterfactual mean of Y for the selected population when the LASF is as in group t, $F_{Z,V}$ is as in group k, and the selection rule is as in group r, is:

$$\mu_{Y_{\langle t,k,r\rangle}}^s = \frac{\int \mu_t(x,v)\,\mathbf{1}\{v > \Phi(-\gamma_r'z)\}dF_{Z_k,V_k}(z,v)}{\int \mathbf{1}\{v > \Phi(-\gamma_r'z)\}\,dF_{Z_k,V_k}(z,v)}, \quad (16)$$

where:

$$\mu_t(x,v) = \alpha_t'x + \rho_t\sigma_{E_t}\Phi^{-1}(v),$$

denotes the LASF in group t. The observed mean of Y_t in the selected population, integrating over Z_t, is:

$$\mu_{Y_{\langle t,t,t\rangle}}^s = \alpha_t'\,\mathbb{E}[X_t \mid H_t > 0] + \rho_t\sigma_{E_t}\mathbb{E}[\lambda(\gamma_t'Z_t/\sigma_{V_t}) \mid H_t > 0],$$

where λ denotes the inverse Mills ratio. We decompose the difference $\mu_{Y_{\langle 1,1,1\rangle}}^s - \mu_{Y_{\langle 0,0,0\rangle}}^s$ between two time periods, $t = 0$ and $t = 1$, into selection, composition and structural effects.

MR define the selection effect as:

$$\rho_1\sigma_{E_1}\mathbb{E}[\lambda(\gamma_1'Z_1/\sigma_{V_1}) \mid H_1 > 0] - \rho_0\sigma_{E_0}\mathbb{E}[\lambda(\gamma_0'Z_0/\sigma_{V_0}) \mid H_0 > 0]. \quad (17)$$

This comprises the following four elements:

$$\rho_1\sigma_{E_1}\int \left[\lambda(\gamma_1'z/\sigma_{V_1})\Phi_1(\gamma_1'z/\sigma_{V_1}) - \lambda(\gamma_0'z/\sigma_{V_0})\Phi_1(\gamma_0'z/\sigma_{V_0})\right]dF_{Z_1}(z) +$$

$$+ \rho_1\sigma_{E_1}\left[\int \lambda(\gamma_0'z/\sigma_{V_0})\Phi_1(\gamma_0'z/\sigma_{V_0})\,dF_{Z_1}(z) - \int \lambda(\gamma_0'z/\sigma_{V_0})\Phi_0(\gamma_0'z/\sigma_{V_0})\,dF_{Z_0}(z)\right] +$$

$$+ \rho_1(\sigma_{E_1} - \sigma_{E_0})\,\mathbb{E}\left[\lambda(\gamma_0'Z_1/\sigma_{V_0}) \mid H_0 > 0\right] +$$

$$+ \sigma_{E_0}(\rho_1 - \rho_0)\,\mathbb{E}\left[\lambda(\gamma_0'Z_1/\sigma_{V_0}) \mid H_0 > 0\right], \quad (18)$$

where $\Phi_k(\gamma_r'z/\sigma_{V_r}) = \Phi(\gamma_r'z/\sigma_{V_r})/\int \Phi(\gamma_r'z/\sigma_{V_r})\,dF_{Z_k}(z)$ is the counterfactual probability of selection in group k when the selection rule is as in group r and Φ denotes the standard normal CDF. The first two elements in (18) capture the effect of

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changes over time in the observable characteristics of the selected population. The first results from applying the selection rule from period 1 to period 0 holding the composition of period 1 fixed. The second captures changes in the distribution of characteristics from period 1 to period 0. The third element captures the effect of changes over time in the composition of the selected population in terms of unobservables through the variance in wages. The fourth element is a sorting effect that captures changes over time in the composition of the selected population in terms of unobservables through the correlation coefficient.[9] In our view the first element belongs to the selection effect while the second element belongs to the composition effect as it is driven by changes over time in the distribution of Z. It is also not clear why the change in the variance of wages should be interpreted as a selection effect as it captures factors potentially unrelated to selection. As the final element reflects changes how unobservables are valued it is arguably a component of the selection effect. However, it could be assigned the interpretation of a structural effect as it captures the market value of unobserved characteristics. We agree with the MR interpretation that the sign of ρ_t captures the nature of sorting.

We now present the selection, composition and structural effects for our decomposition. Plugging the expression for $\mu_t(x, v)$ into (16) gives, after some straightforward calculations:

$$\mu^s_{Y_{\langle t,k,r \rangle}} = \int \left[\alpha'_t x + \rho_t \sigma_{E_t} \lambda(\gamma'_r z / \sigma_{V_r}) \right] \Phi_k(\gamma'_r z / \sigma_{V_r}) \, dF_{Z_k}(z).$$

Our selection effect is:

$$\mu^s_{Y_{\langle 1,1,1 \rangle}} - \mu^s_{Y_{\langle 1,1,0 \rangle}} = \alpha'_1 \int x \left[\Phi_1(\gamma'_1 z / \sigma_{V_1}) - \Phi_1(\gamma'_0 z / \sigma_{V_0}) \right] dF_{Z_1}(z) +$$
$$+ \rho_1 \sigma_{E_1} \int \left[\lambda(\gamma'_1 z / \sigma_{V_1}) \Phi_1(\gamma'_1 z / \sigma_{V_1}) - \lambda(\gamma'_0 z / \sigma_{V_0}) \Phi_1(\gamma'_0 z / \sigma_{V_0}) \right] dF_{Z_1}(z). \quad (19)$$

The first element on the right-hand-side of (19) is the effect on the average wage from changes in the distribution of observable characteristics of the selected population, holding the population distribution constant, resulting from applying the selection equation from period 0 to period 1. It is positive if those entering the

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[9] MR make the strong assumption that the distribution of the covariates does not change over time, i.e., $F_{Z_0} = F_{Z_1}$, so the second component drops out.

selected population have characteristics associated with higher wages. This element is missing in the selection effect in (18). The second element is the corresponding effect for the unobservable characteristics and corresponds to the first in (18).

Our composition effect is:

$$\mu^s_{Y_{\langle 1,1,0 \rangle}} - \mu^s_{Y_{\langle 1,0,0 \rangle}} = \alpha'_1 \left[\int x\, \Phi_1(\gamma'_0 z/\sigma_{V_0})\, dF_{Z_1}(z) - \int x\, \Phi_0(\gamma'_0 z/\sigma_{V_0})\, dF_{Z_0}(z) \right] +$$
$$+ \rho_1 \sigma_{E_1} \left[\int \lambda(\gamma'_0 z/\sigma_{V_0}) \Phi_1(\gamma'_0 z/\sigma_{V_0})\, dF_{Z_1}(z) - \int \lambda(\gamma'_0 z/\sigma_{V_0}) \Phi_0(\gamma'_0 z/\sigma_{V_0})\, dF_{Z_0}(z) \right].$$
$$(20)$$

The first element on the right-hand side of (20) is the change in the average wage resulting directly from changes over time in the distribution of the observable characteristics while the second element is the same as the second term in (18).

Finally, our structural effect is:

$$\mu^s_{Y_{\langle 1,0,0 \rangle}} - \mu^s_{Y_{\langle 0,0,0 \rangle}} = (\alpha_1 - \alpha_0)' \, \mathbb{E}\left[X_0 | \, H_0 > 0 \right] + (\rho_1 \sigma_{E_1} - \rho_0 \sigma_{E_0}) \, \mathbb{E}\left[\lambda \left(\gamma'_0 Z_0/\sigma_{V_0} \right) | \, H_0 > 0 \right].$$
$$(21)$$

The first element on the right-hand side of (21) reflects the impact of changes over time in the returns to observable characteristics while the second captures the type and degree of sorting and is the same as the sum of the third and fourth elements in (18). As the expectation involving the inverse Mills ratio is positive, its contribution is positive when $\rho_1 \sigma_{E_1} > \rho_0 \sigma_{E_0}$.

Finally, consider a simple example illustrating that the two elements of the structural effect cannot generally be identified in nonseparable models.[10] Consider a multiplicative version of the parametric HSM obtained by replacing (14) with:

$$Y_t = \alpha'_t X_t \, E_t, \quad \text{if } H_t > 0,$$

and weaken the parametric assumption on the joint distribution of E_t and V_t by only requiring that $V_t \sim N(0,1)$ and $\mathbb{E}[E_t \,|\, Z_t, H_t > 0] = \rho_t \lambda(\gamma'_t Z_t)$. α_t and ρ_t cannot be

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[10]Chernozhukov et al. (2019) separate the structural effect from the sorting effect using an exclusion restriction in the copula for the unobservables of the employment and wage equation. They provide a wage decomposition with 4 components: structural, composition, selection and sorting. Compared to our decomposition, the composition and selection effects are the same, but our structural effect corresponds to the sum of their structural and sorting effects.

separately identified from the moment condition:

$$\mathbb{E}[Y_t \mid Z_t, H_t > 0] = \alpha_t' X_t \, \rho_t \lambda(\gamma_t' Z_t).$$

4 Empirical results

4.1 Hours equation

We start by describing the variables included in Z using the ASEC. Following MR, we include six indicator variables for the highest educational attainment reported. Namely, (i) 0-8 years of completed schooling, (ii) high school dropouts, (iii) high school graduates or 12 years of schooling, (iv) some college, (v) college, and (vi) advanced degree. We include a quartic polynomial in potential experience and interact this with the education levels.[11] We use 5 indicator variables for marital status: (1) married, (2) separated, (3) divorced, (4) widowed, and (5) never married. We include indicator variables for Black and Hispanic and four indicator variables for regions: northeast, midwest, south and west. Finally, we use linear terms for the number of children aged less than 5 years interacted with the indicator variables for marital status. For the MORG we use 5 levels of education as the two lower categories in the ASEC are merged. The variables Black, Hispanic, experience and region are the same. Only one indicator for marital status is used (married or not) and we employ household size, and its interaction with marital status, as the only household characteristic.

With the exception of the household size and composition variables, all of the conditioning variables appear as both determinants of annual hours and hourly wages. While one might argue that household size and composition may affect hourly wage rates, we regard these exclusion restrictions as reasonable and note that similar restrictions have been previously employed (see, for example, MR). However, given their potentially contentious use we explore the impact of not using them below. The assumption that annual hours of work do not affect the hourly wage rate means that the variation in hours across individuals is a source of identification.

Although our primary focus is the wage decomposition, we highlight the ma-

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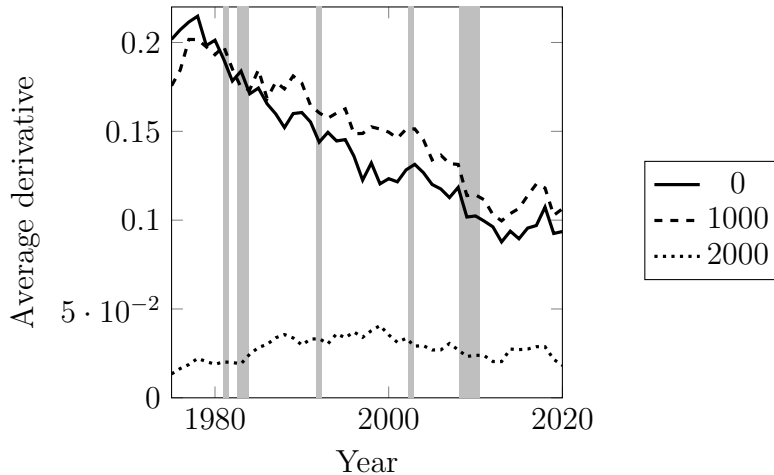[11]We employ the methodology described in MR for education and potential experience.

Figure 5: Average derivative of the exclusion restriction at different levels of hours.

jor features of the hours equation estimates. The hours equation is estimated by distribution regression and we report the marginal impact of being below a certain point in the annual hours distribution. We do this for education level, race, marital status and for one of the exclusion restrictions (having a child below the age of 5). We only report the result of the latter here; see Figure 5. The selected points in the hours distribution are 0, 1000 and 2000. We find that many of the individual characteristics have an impact on the level of annual hours worked. This is not particularly surprising given the large literature on labor supply documenting the roles of education and marital status on labor market participation. Perhaps what is more surprising is that the magnitude of the impact of these variables does not appear to change substantially over the sample period in either the ASEC or MORG data. The exception is with respect to the exclusion restrictions which became less important over time. This is consistent with Card and Hyslop (2021). Note that the level of education has drastically increased over the sample period and this has had a substantial impact on the hours distribution.

We also estimated models for annual weeks of work using distribution regression and ordered models for annual weeks and annual hours using the ASEC. The flavor of the results are very similar to those for the annual hours equations. The individual variables which also appear in the wage equations are statistically significant and their impact does not appear to change a great deal over the sample period. The variables employed as exclusion restrictions have a statistically significant impact

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on the level of reported work.

4.2 Decompositions

The wage equations are estimated for each year by distribution regression over the subsample with positive hourly wages. The conditioning variables are those in the hours equation with the exception of the household size and composition variables. We also include the appropriate control function, its square, and interactions between the other conditioning variables and the control function. The decompositions require a base year. The steadily increasing female participation rate suggests that 1975 is a reasonable choice as it has the lowest level of participation. It is reasonable to assume that those with a certain combination of x and v working in 1975 also have a positive probability of working in other years as required by the identification conditions. We present results for the censoring mechanism being either the number of annual hours or the number of annual weeks for the ASEC. These are shown in Figures 6 and 7 respectively. The results for the MORG, using numbers of hours in reference week as the censoring mechanism, are in Figure 8. Before presenting the decompositions we highlight that they reflect the change in the contributions of each component over time. For example, if the structural effects are zero this should be interpreted as the contribution of the structural effects not changing, not that there are no structural effects.

   We start with annual hours as the censoring mechanism and Figure 6A presents the decomposition for the mean, which increases by 25% over our sample period. The total effect is driven by the composition effect although in several instances the structural effect is contributing. It is generally negative and small relative to the composition effect. The contribution of the selection component is negative and small. A negative selection component implies that females are positively selected into employment and those who entered employment between 1975 and 2020 were less productive than those already employed.

   Figures 6B– 6F present the decompositions for the 10th, 25th, 50th, 75th and 90th percentiles. A number of important conclusions can be drawn. The large increases at each quantile are driven by the large changes in the composition effects. This reflects the increasing education levels of the female workforce. The large in-
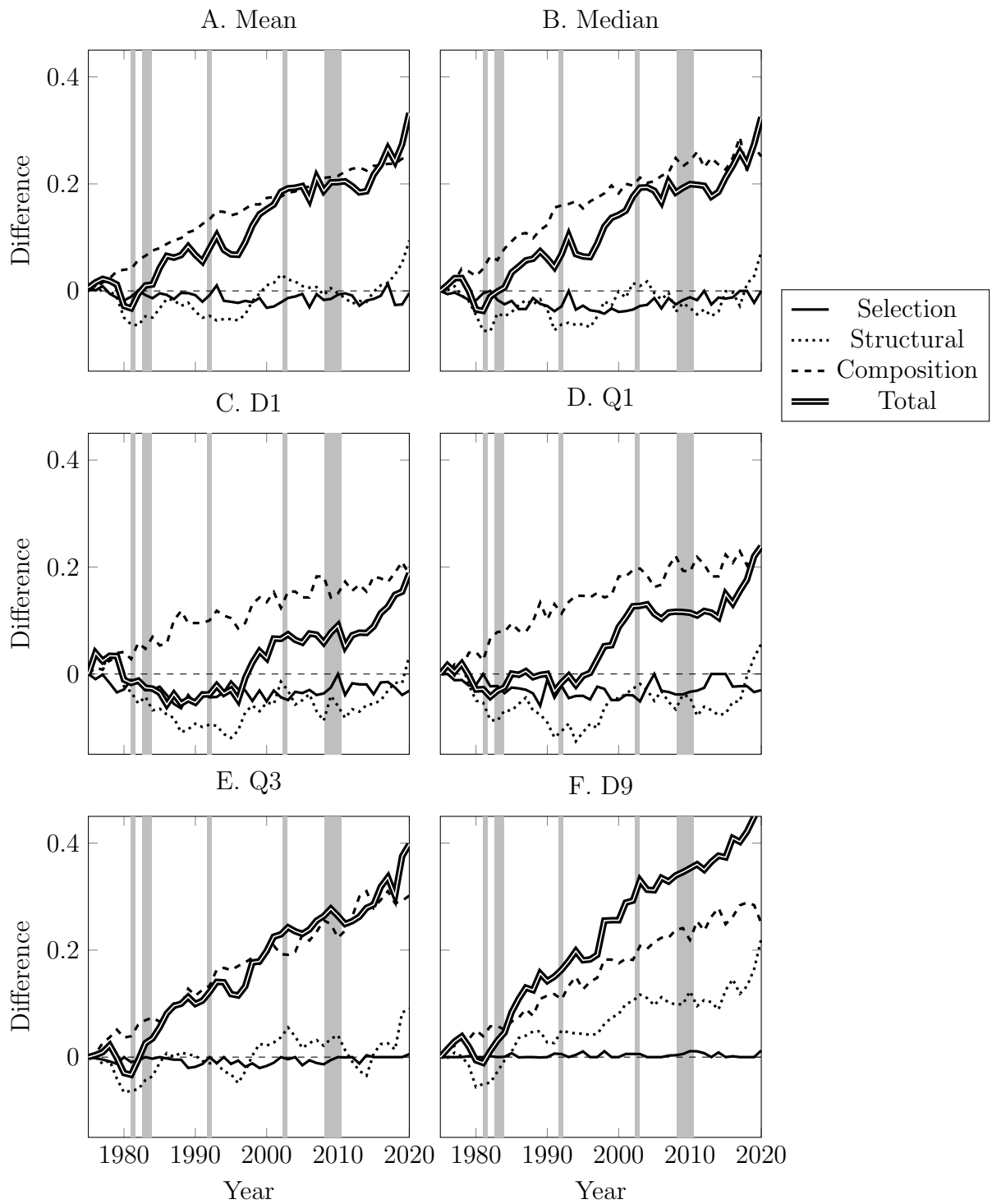
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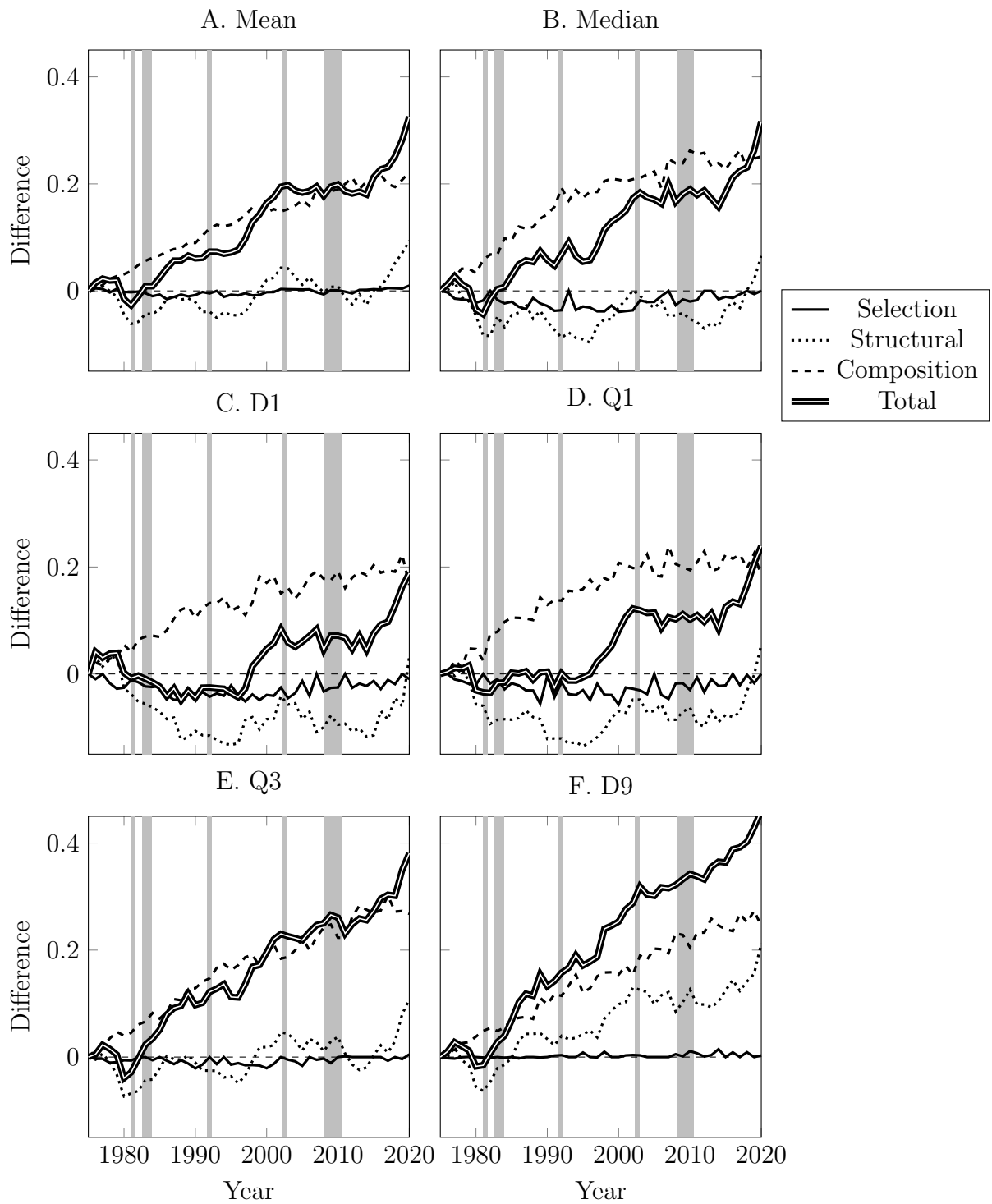Figure 6: Decompositions using the ASEC data and annual working hours.

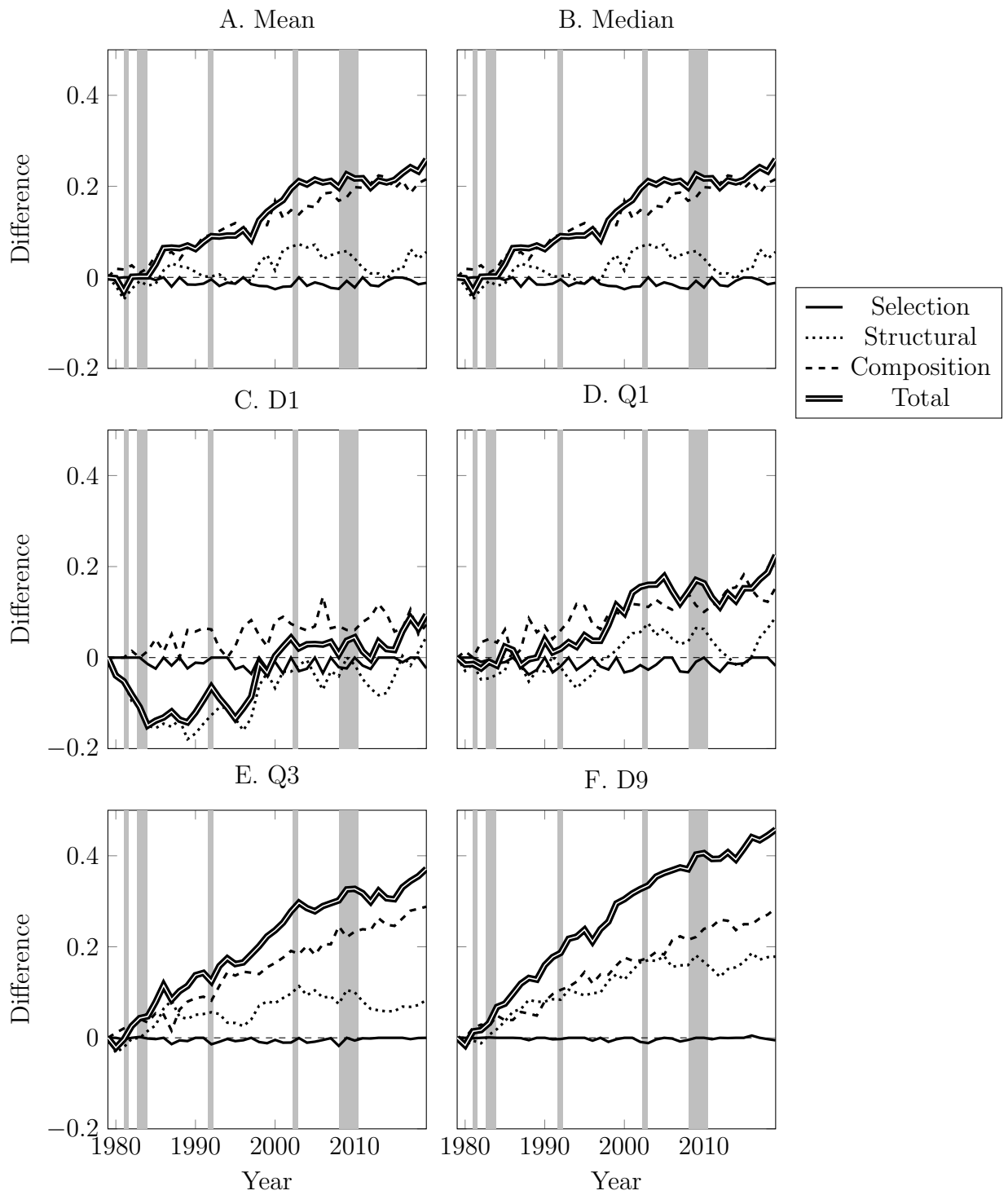Figure 7: Decompositions using the ASEC data and annual working weeks.

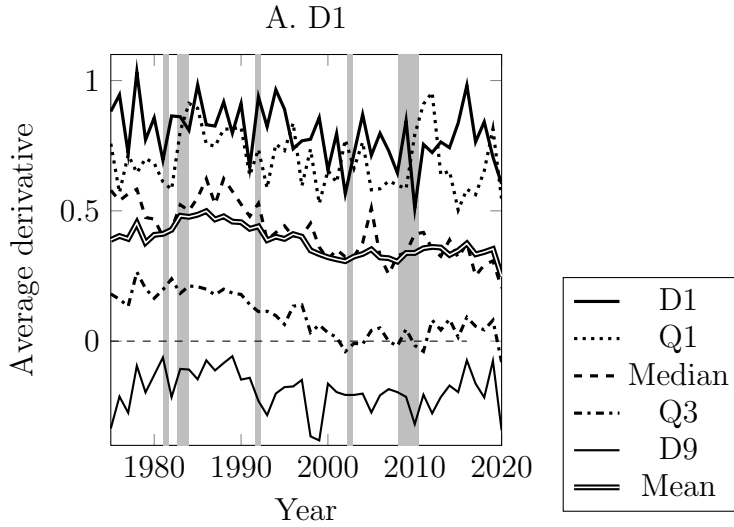Figure 8: Decompositions using the MORG data and annual working hours.

A. D1

Figure 9: Average derivatives at different quantiles

creases from the composition effects at each quantile are somewhat offset by the structural effects at lower quantiles and there is only clear evidence of an economically important and positive structural effect at D9. At this quantile almost half of the 44% increase in real wages experienced over the sample period is due to the structural effect. At lower quantiles the structural effects are somewhat cyclical while the sustained increase at D9 reflects the increasing skill premia.

Figures 6B– 6F reveal no indication of changes in the selection effects at Q3 and D9. Individuals located here are likely to have had a relatively strong commitment to employment in 1975 and there have been no substantial movements in their hours distribution. Moreover, these individuals are less likely to incur periods of unemployment. Lower down the wage distribution there is evidence of small negative changes in selection during some years in our sample. At D1 and Q1 the selection effects are negative and economically important. The confidence intervals for these selection effects are presented in Figure 10. They indicate that for several time periods selection effects are statistically significantly different from zero. At D1 the total real wage increase is less than 10% for the majority of the sample period and the negative selection effect is frequently of the order of around 3%.

We explore the form of sorting implied by these results. Given the non-separable nature of the model, this is not straightforward although one indication of the sorting pattern is how wages change at each quantile in response to a change in the

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control function. This corresponds to the local average derivative function of FVV evaluated at each quantile. This is somewhat comparable to the coefficient on the selection term in the HSM. A positive average derivative at a specific quantile suggests that the unobservables increasing hours are positively correlated with wages at that quantile. The results are shown in Figure 9. At D1, Q1 and Q2 these estimated derivatives are positive and large for the whole sample period. At Q3 the derivative is positive for the vast majority of the sample period although there are instances where it is close to zero or very slightly negative. At D9 it is negative but small in magnitude. The results clearly support the existence of a positive relationship between V and wages at the bottom of the distribution implying positive selection which is more important in the lower half of the wage distribution. This is similar to the findings of AB for female wages in the British labor market for 1978 to 2000.

Annual hours is an economically attractive censoring mechanism as it exploits the variation in annual hours induced both by hours and by weeks. However, it is possible that selection operates either through hours or weeks exclusively. We first address this issue by replacing annual hours with annual weeks as the selection mechanism. The results from these decompositions using the ASEC are in Figures 7A–7F. Their primary feature is their similarity to those for annual hours. This suggests that the control function from the annual hours censoring mechanism is highly correlated with that from annual weeks despite the differences in their respective distributions.

Now consider the decompositions for the MORG recalling that wages are measured differently than in the ASEC and the hours measure is based on the survey week. We implement our censored selection estimator using hours in the survey week as the censoring mechanism noting that only a subset of the exclusion restrictions used in the ASEC are available in the MORG. The results are shown in Figures 8A-8F. As the MORG covers a different sample period, the figures look slightly different from those for the ASEC. Nevertheless, the findings regarding the role of selection are almost identical. The similarity across figures is remarkable given the differences in measurement, data and exclusion restrictions.

We acknowledge that although also employed by MR and MW, the use of household composition variables as exclusion restrictions is controversial. Accordingly,
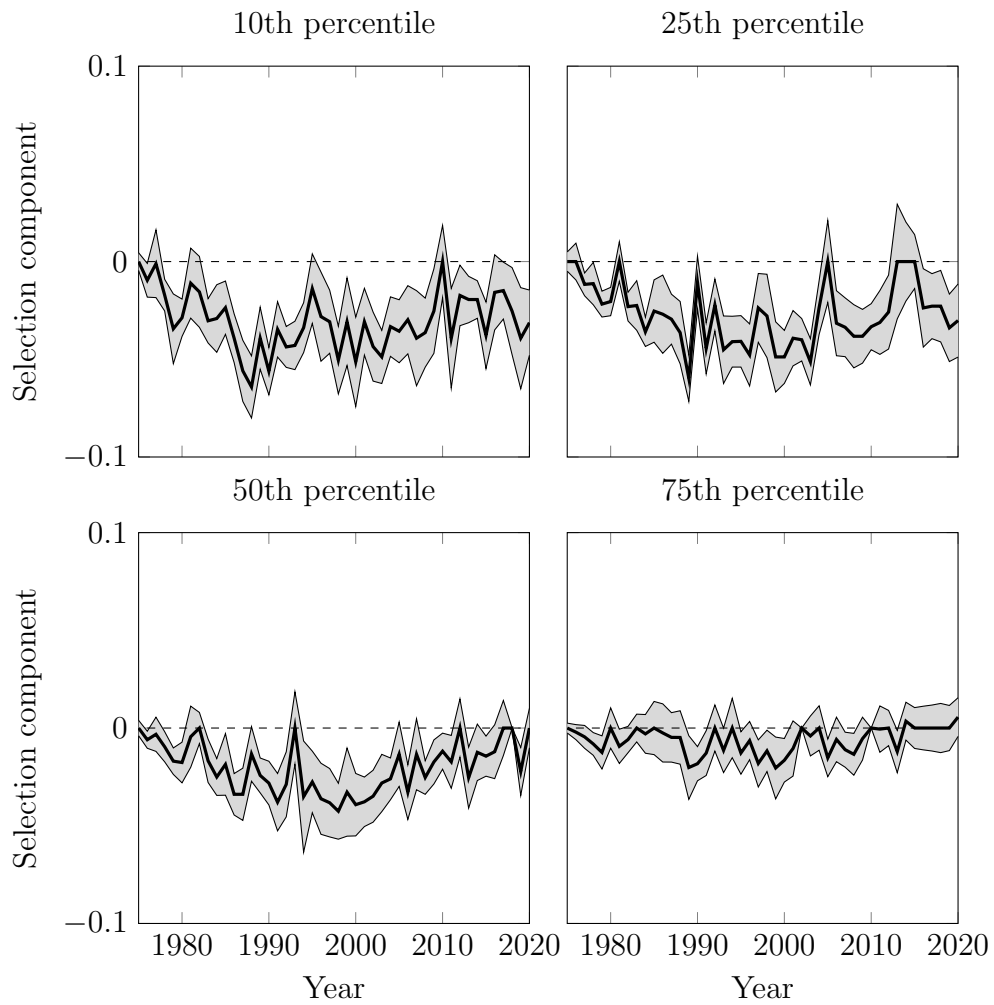
Figure 10: Selection components and associated 95% confidence intervals for females at various percentiles

we reproduced the decompositions from the censored selection mechanisms first excluding the household variables from both the hours and wage equations and then including them in both equations. The model is now only identified by the variation in the number of hours worked. We do not find any remarkable changes from either model, in comparison to the specification employed above, with respect to the presence or magnitude of selection effects. The only notable difference is the presence of occasionally larger negative selection effects at the bottom decile for the specification which excludes the family composition variables from both equations.

4.3   Results of the double selection model

Our results from Section 4.2 seem robust to the use of either hours or weeks as the selection variable in the censored selection model. Figures 11A–11C report the decomposition for the double selection mechanism. There continues to be no evidence of selection above the median so we report the decompositions for D1, Q1 and the median.

While there are some differences in these figures compared to those for selection using only annual hours or annual weeks they are relatively small. These results seem to suggest that the unobservables which increase participation on any margin, such as usual weeks, usual hours, hours in survey week, are all highly correlated. To pursue this possibility we estimate the average derivative with respect to each of the control functions evaluated at the mean wage as this provides some insight into the source of selection. The derivatives, reported in Figure E.1 in the appendix, indicate that at the mean wage both sources of selection are important. The derivative for the weeks' control function is the bigger of the two and the effect is relatively constant with the exception of a notable decrease at the time of the financial crisis. The hours' control function derivative is negative for the earliest years of the sample before turning and remaining positive for the remaining years. Prior to the financial crisis it increases in magnitude and for a very short period it is the larger of the two. The high correlation between the two control functions makes it difficult to interpret this figure but this evidence suggests that selection operates both through the weeks and hours decisions and the effect of each is similar.
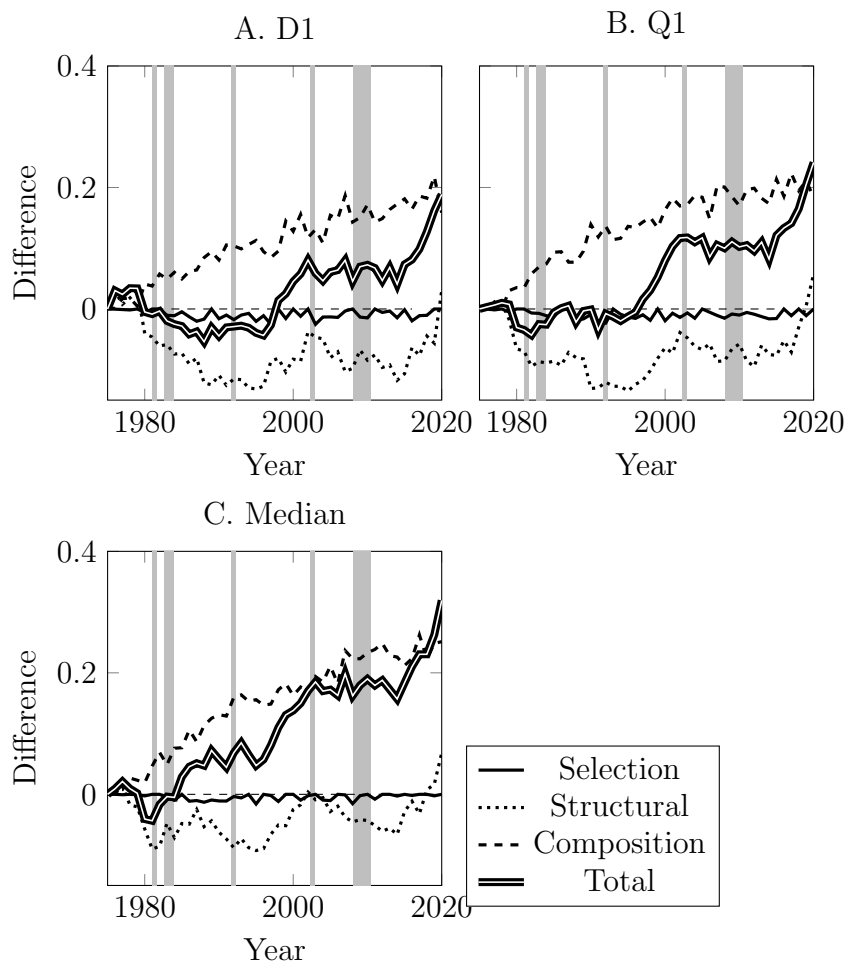
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Figure 11: Decompositions using a double selection mechanism based on ASEC data.

4.4 Results of the ordered selection model

Figure 12 reports the decomposition exercise using the model in Section 3.4 incorporating bunching in the selection variable. We first employ the ASEC with annual weeks as the ordered selection rule. The separation values are 0, 10, 20, 30, 40 and 50 weeks. The decompositions are very similar to those for the continuous selection rule. The most notable feature of the decompositions are the selection effects at D1 and Q3. These effects are slightly larger than those at the corresponding quantiles in the earlier figures and represent important economic effects. There is clear evidence that the changes in selection effects have decreased wages at this quantile in some time periods. We also employed annual hours as the ordered selection variable using 0, 500, 1000, 1500, 2000 and 2500 annual hours as the separation values. These results, available from the authors upon request, are very similar to those using ordered weeks. The selection effects are very similar to those in Figure 12 although the allocation of the changes across the structural and composition effects differs somewhat.

5   Comparison with Mulligan and Rubinstein (2008)

MR examined the mean female full time wage and find negative sorting effects in the 1970s. These become positive in the early 1980s and by the late 1990s they are economically large. The change from negative to positive sorting is inconsistent with our findings and we attempt to reconcile these contrasting conclusions. The appropriate figures are in Appendix E.

We first explore the term $\rho_t \sigma_{E_t}$ in (17). We estimate the HSM using the MR sample and exclusion restrictions to obtain the results in Figure E.2-A. To ensure the differences across results are not driven by their focus on FTFY white females, we repeat the exercise including non whites and all those reporting positive working hours. This is reported in Figure E.2-B. These figures confirm the MR results. For both samples $\rho_t \sigma_{E_t}$ is negative at the beginning of the sample but becomes positive and economically large during the sample period. The results imply that the changes in the selection effects, as defined by MR, are large.

Figure E.3 presents the selection effect based on our sample and the estimates
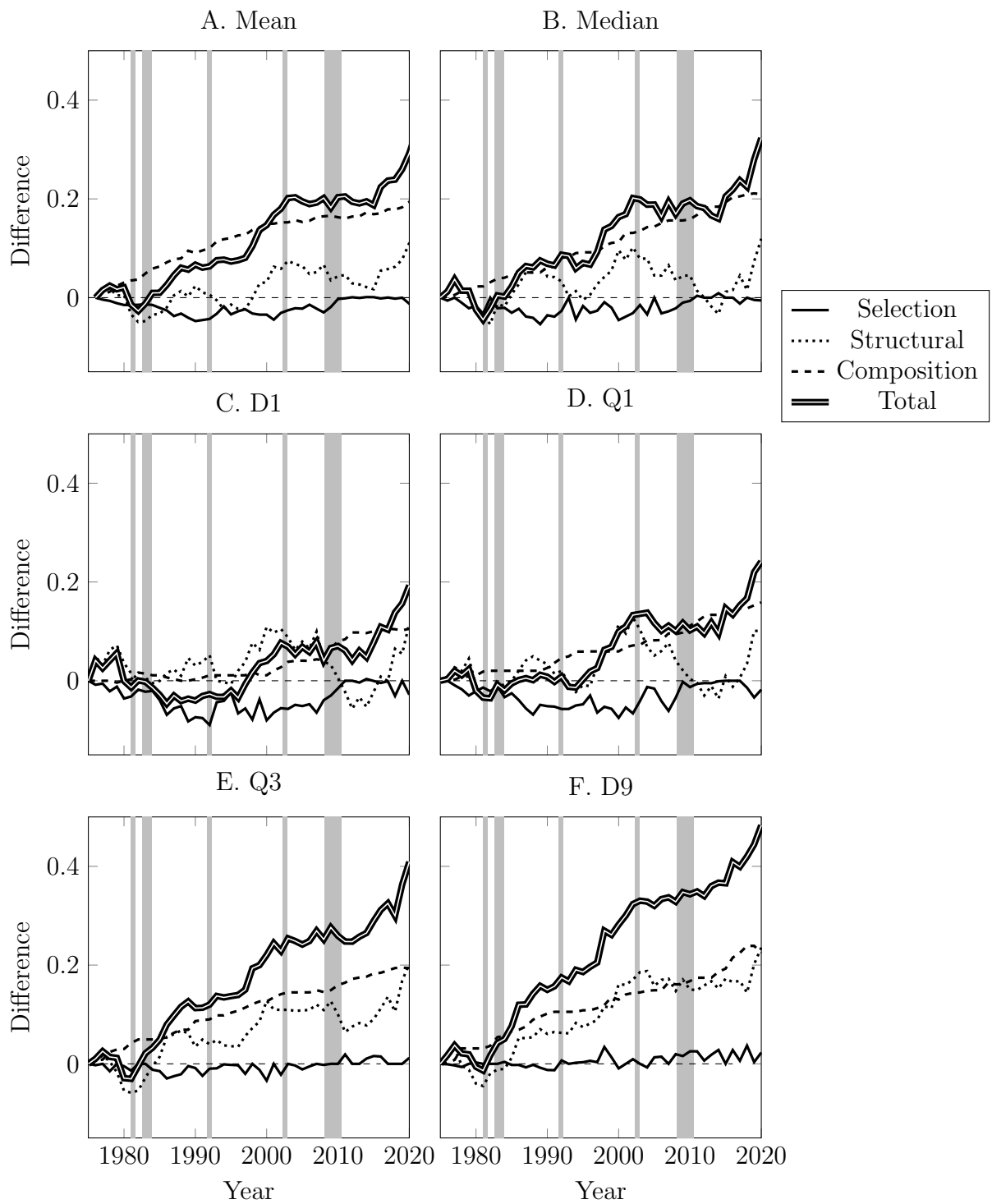
Figure 12: Decompositions using the ASEC data and annual working weeks using our alternative model taking account of bunching.

from the HSM. The figure decomposes the total selection effect into the four components shown in (18). It reveals that the movements in the selection effects are almost entirely due to the change in ρ_t. The impact of the unobservables from the selection equation on wages was initially negative but became positive and subsequently large. Figure E.4 presents our selection, structural and composition effect based on (19)-(21). We find that our selection effect is small and the majority of the wage change reflects composition effects. However, Figure E.4 is similar to Figure 6-A obtained via our approach. The reason for our smaller selection effect is clear from Figure E.3. The figure reveals that the second component, which from (18) is included in our selection effect, is small and similar in size to that of Figure 6-A and Figure E.4. This suggests that the difference is partially due to the difference in definitions recalling this reflects the inability to isolate the MR selection effects in nonseparable models. We now examine whether components of the remaining difference reflect the specific parametric form or the source of identification employed.

That ρ_t becomes large is not particularly controversial. However, its change of sign has implications for the nature of sorting into the labor market. As ρ_t captures the mapping from unobservables in the work equation to wages, we explore if our approach uncovers the same pattern by evaluating once more the average derivative as presented in Figure 9-A. While the derivative's value at the mean fluctuates over the sample period, it does not change sign and is always consistent with positive sorting.

This contrasts with MR. The three obvious causes are the use of the normality assumption in the HSM, the identifying power introduced through hours as a censoring variable in the selection equation, and the nonseparable nature of our model. To address these issues we first estimate the model using a parametric approach which relies on normality but which exploits the variation in hours for identification purposes. We employ the Vella (1993) procedure which estimates the hours equation by Tobit and computes the generalized residual, defined for the $H > 0$ observations as $H - Z'\widehat{\gamma}$ where $\widehat{\gamma}$ is estimated by Tobit, to include as the correction for selection in the wage equation. This assumes hours do not directly effect hourly wage rates.[12] To more closely correspond to the HSM we divide the generalized residual by the

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[12]Hirsch (2005) provides empirical evidence supporting this assumption.

estimated standard deviation of working hours, σ_{V_t}. The only difference with the HSM is the use of the Tobit generalized residual rather than the inverse Mills ratio. We plot the corresponding coefficient on the Tobit generalized residual in Figure 9-B. The coefficient on the Tobit generalized residual also estimates $\rho_t \sigma_{E_t}$.

Two striking features are revealed in Figure 9-B. First, under normality the estimates of $\rho_t \sigma_{E_t}$ and the coefficient on the generalized residual should be identical. However, the estimates are very different and most importantly the coefficient estimate is always positive. As there is no reason that departures from normality will bias the estimates of $\rho_t \sigma_{E_t}$ and the coefficient for the Tobit generalized residual in the same manner one could interpret the difference in the estimates as evidence of non-normality. However, recall that the Tobit generalized residual also exploits variation in the hours variable for identification purposes and this could contribute to the difference in the signs and the behavior of the two coefficients. Second, the pattern of movement in the coefficient on the generalized residual is almost identical to the average derivative of our control function despite the drastically different ways in which each is computed. The two procedures are very different but each exploits the variation in hours as a means of identification.

While it appears that the use of the variation in hours as the source of identification is the cause of the differences with MR, it is possible that the departures from normality may also be responsible. The final approach we explore is the use of the propensity score as the control function noting we allow it to enter the wage equation in a nonseparable manner (see, for example, Newey, 2007). The propensity score employs the exclusion restrictions as the sole source of identification. We estimated the model and computed the average derivative of wages with respect to the propensity score. The results are presented in Figure 9-C. This derivative also changes sign as we move through the sample period and shows behavior similar to ρ_t. We conclude that the differences in terms of the relationship between E_t and V_t between our results and MR are due to the use of the variation in hours which appear to identify a different pattern of sorting.

For reconciling the difference in the results associated with the two controls, it is necessary to examine their sources of variation. For the subsample of workers, the variation in their values of the inverse Mills ratio is due to the variation in

Z. In contrast, an individual's value of the Tobit generalized residual also exploits the value of H. Consider a case where ρ_t is positive and there are two working individuals with identical Z but one receives a much larger positive value of V_t. This produces a relatively larger positive value of E_t and that individual will have relatively higher hours and wages. In this setting the value of the inverse Mills ratio for both individuals will be the same while the Tobit generalized residual of the individual with the higher value of H will be greater than the other. This suggests that the Tobit generalized residual is capturing information regarding "sorting" into hours which is ignored by the inverse Mills ratio. Moreover, the inverse mills ratio, unlike the Tobit generalized residual, is unable to explain the variation in wages across these two individuals.

It is important to explore why ρ_t might change sign for the models identified solely by exclusion restrictions. A negative ρ_t implies that the working individuals with the lowest probabilities of participation should have the lowest observed wages among individuals with the same observed characteristics relevant for the wages, X. The reverse is true for a positive ρ_t. We explore this by estimating a wage regression identical to the second stage of MR while replacing the inverse Mills ratio by a dummy variable for a child below the age of 5 years. The impact of having a "young child" was negative until 1982 at which time it turned, and remained, positive. This corresponded to a period, also reported by Card and Hyslop (2021), in which the magnitude of the negative impact of a "young child" on the employment decision decreased. While we acknowledge the presence of other ongoing factors this change in the impact of "young child" could generate a change in the sign of ρ_t. For example, in the absence of other influences, the large positive influence of "young child" on the value of the inverse Mills ratio combined with negative correlation between "young child" and wages would produce a negative value of ρ_t. In contrast a decreasing effect of "young child" on participation would produce a smaller value for the inverse Mills ratio and that, combined with the positive correlation between "young child" and wages, would produce a positive ρ_t.

We highlight that we consider the above discussion as suggestive rather than conclusive. Our objective is to consider the possible causes of the differences in the results from the use of the two control functions. The evidence suggests that part
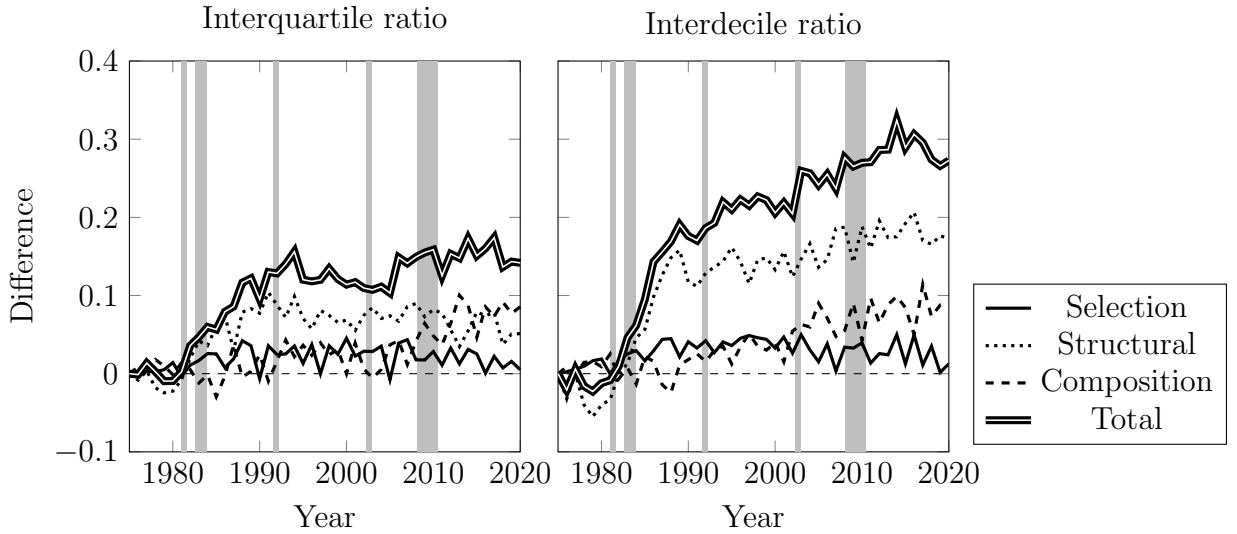
Figure 13: Decompositions of the interquartile and interdecile ratio using the ASEC.

of the difference is due to the use of variation in hours as a source of identification for ρ_t. However, it is clear that the effect of the exclusion variables on the hours and work decisions is also an important factor in identifying ρ_t, and this has changed over time. Related to this last issue is the validity of the exclusion restrictions employed and how this validity has changed over the sample period. While the use of either of the control functions should produce consistent estimates of the sorting parameters when the model is correctly specified, the impact under misspecification is unclear.

6   Wage Inequality

We noted above that despite the large literatures on the impact of selection on females' wages and the increase in female wage inequality, there are few papers which focus on both issues. The important exceptions are AB, MW and Blau et al. (2021), which evaluate the impact of selection by contrasting changes in the observed levels of wage inequality with the counterfactual levels associated with the total population working. The latter corresponds to a participation rate of 100%. We investigate the impact of selection by evaluating changes in the distributions, and also inequality, by holding the selection rule constant across time. While both approaches have merit we prefer ours as participation rates do not approach 100%
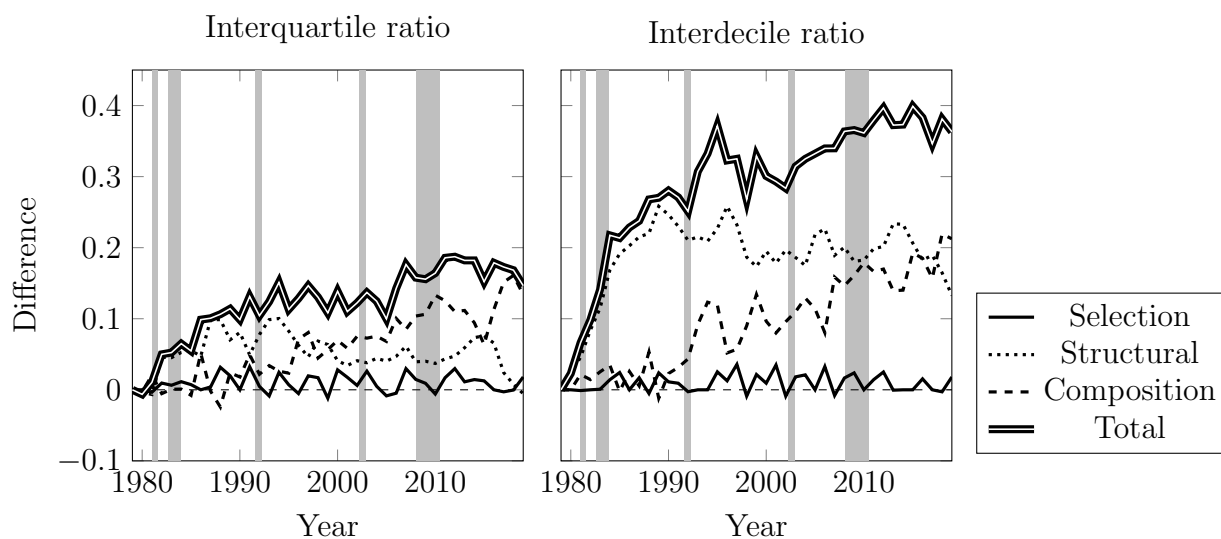
Figure 14: Decompositions of the interquartile and interdecile ratio using the MORG.

in the sample period.[13]

We provide the decompositions of changes in inequality using the annual hours as the censored selection variable for the ASEC data, hours in the survey week for the MORG and annual weeks as the ordered selection variable for the ASEC. For each of these models and selection rules we decompose the interquartile and interdecile ratios. Those for annual hours using the ASEC are reported in Figure 13 and those for the MORG in Figure 14. The interquartile ratio is driven by each of the components. Neither the composition or structural effect dominates throughout the sample period. The selection effect contributes throughout the period and clearly increases inequality. The interdecile ratio is driven primarily by the structural effect especially during the drastic increase at the beginning of the sample period. The selection effect is clearly important and frequently more important than the composition effect. For the MORG the conclusions regarding the structural and composition effects are similar to those for ASEC while the selection effects are slightly smaller. This reflects the smaller selection effects at lower quantiles in the wage decompositions (as presented in Figures 6 and 8). The evidence for both data sets support that selection has a modest but important impact on wage inequality that varies in magnitude over the sample period. As the wage decompositions based

[13]Chernozhukov et al. (2019) considered both approaches.

on the ordered selection rule suggested selection was more important than in the censored selection models (see Figure 12) we examine now whether this carries over to the inequality decompositions. We do not report the result but note that the evidence is similar to that for the censored selection rule.

Our results indicate that as an increasing number of females have entered the labor market they have reduced wages at the lower parts of the wage distribution while having no impact on wages above Q2. This increases measures of inequality based on ratios involving lower and upper quantiles. Potentially, there are two reasons why selection increases inequality based on whether either the observed or unobserved characteristics of those participating has changed over time. However, an examination of education levels, for example, suggests that observed characteristics have played a minor role. In particular, we find that those with education higher than high school degrees were more likely to participate over the whole sample period and that this did not change over time. This suggests that our results reflect changing unobserved characteristics. The selection effect captures the difference between the observed wage distribution and the counterfactual in which women participated as in 1975. Our decomposition method presented in (5) imposes that this difference captures the exit of females with lower levels of the control function. Figure E.5, reveals a strong and positive relationship between wages and the control function suggesting that selection effects reflect that women entering the labor market were less productive than observationally identical women participating in 1975.

Our evidence of positive sample selection over the whole period implies that the decision to work is largely based on economic motivations. However, as employment rates have increased this has seen a reduction in sorting on economic grounds. This is consistent with the explanation provided by AB for the U.K. labor market. This is also consistent with the results above that the conventional household background family characteristics have become less important in explaining participation and hours worked. The reduction in positive sorting describes the changes in the hours distribution from the mid 1980s to the end of the 1990s. Blau et al. (2021) argue that the booming economy and welfare reform may have played an important role in the 1990s. Our collective evidence suggests that post 2000 there was an increase in positive selection. This supports the evidence in Blau et al. (2021). Towards the

end of the sample period it appears that the impact of selection on inequality and, more generally, wages has returned to 1975 levels.

7 Conclusions

This paper documents the changes in female real wages over the period 1975 to 2020. We decompose these changes into structural, composition and selection components by estimating a nonseparable model with selection. Female wage growth at lower quantiles is modest although the median wage has grown steadily. The increases at the upper quantiles for females are substantial and reflect increasing skill premia. These changes have resulted in a substantial increase in female wage inequality. As our sample period is associated with large changes in the participation rates and the hours of work of females we explore the role of changes in "selection" in wage movements. We find that the impact of these changes is to decrease the wage growth of those at the lower quantiles with very little evidence of selection effects at other locations in the female wage distribution. The selection effects appear to increase wage inequality for the period 1975 to 2000 and this reflects a reduction in the level of positive sorting. However, post 2000 there appears to be an increase in positive sorting and the selection effects on wages and inequality appear to return to their 1975 levels by the end of our sample.

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Appendix

A   Control function with unknown censoring point

We provide an estimator of $F_{H^* \mid X, Z}$ based on distribution regression. This is an alternative to Buchinsky and Hahn (1998) and Chernozhukov and Hong (2002), which developed estimators based on quantile regression. Start with a distribution regression model for $F_{H^* \mid Z}$. That is:

$$F_{H^* \mid Z}(h \mid x) = \Lambda(r(z)'\gamma(h)),$$

where Λ is a known link function and $r(z)$ is a d_r-dimensional vector of transformations of z with good approximating properties. We also assume a binary response model for the propensity score of selection:

$$\pi(z) = \Lambda(p(z)'\delta),$$

where $p(z)$ is a d_p-dimensional vector of transformations with good approximation properties such that:

$$\Lambda\left(r(z)'\gamma(\mu(z))\right) \approx 1 - \Lambda(p(z)'\delta).$$

Let $\{Y_i D_i, H_i, D_i, Z_i\}_{i=1}^n$ be a random sample of (HY, H, D, Z), where HY denotes that Y is only observed when $D = 1$. The proposed estimator consists of 2 steps:

1. Estimation of $\pi(z)$ using binary regression in the entire sample: $\widehat{\pi}(z) = \Lambda(p(z)'\widehat{\delta})$ where:

$$\widehat{\delta} \in \arg \max_{\delta \in \mathbb{R}^{d_p}} \sum_{i=1}^n \left\{ D_i \log \Lambda(p(Z_i)'\delta) + (1 - D_i) \log[1 - \Lambda(p(Z_i)'\delta)] \right\}.$$

2. Estimation of $F_{H^* \mid Z}$ by distribution regression with sample selection correction in the selected sample: $\widehat{F}_{H^* \mid Z}(h \mid z) = \Lambda(r(z)'\widehat{\gamma}(h))$ where:

$$\widehat{\gamma}(h) \in \arg \max_{\gamma \in \mathbb{R}^{d_r}} \sum_{i=1}^n D_i \left\{ 1(H_i \le h) \log \left[\Lambda(r(Z_i)'\gamma) + \widehat{\pi}(Z_i) - 1 \right] \right.$$
$$\left. + 1(H_i > h) \log \left[1 - \Lambda(r(Z_i)'\gamma) \right] \right\}.$$

B   Estimation of wage equation

We estimate the LASF $\mu(x, v)$ and the LDSF $G(y, x, v)$ for each group using flexibly parametrized ordinary least squares (OLS) and distribution regressions, where the unknown control function is replaced by its estimator $\widehat{V}_i = \widehat{F}_{H^* \mid Z}(H_i \mid Z_i)$ from the previous step. For reasons explained in FVV, we estimate over a sample trimmed with respect to the censoring variable H. We employ the following trimming indicator among the selected sample:

$$T = \mathbf{1}\{0 < H \le \overline{h}\}$$

for some $\overline{h} \in (0, \infty)$ such that $\mathbb{P}(T = 1) > 0$. The estimator of the LASF is $\widehat{\mu}(x, v) = w(x, v)'\widehat{\beta}$, where $w(x, v)$ is a d_w-dimensional vector of transformations of (x, v) with good approximating properties, and $\widehat{\beta}$ is the OLS estimator:

$$\widehat{\beta} = \left[\sum_{i=1}^{n} T_i \, \widehat{W}_i \widehat{W}_i' \right]^{-1} \sum_{i=1}^{n} T_i \, \widehat{W}_i Y_i,$$

where $\widehat{W}_i = w(X_i, \widehat{V}_i)$. The estimator of the LDSF is $\widehat{G}(y, x, v) = \Lambda(w(x, v)'\widehat{\beta}(y))$, where $y \in \mathbb{R}$ and $\widehat{\beta}(y)$ is the logistic distribution regression estimator:

$$\widehat{\beta}(y) = \arg \max_{b \in \mathbb{R}^{d_w}} \sum_{i=1}^{n} T_i \left[\mathbf{1}\{Y_i \le y\} \log \Lambda(\widehat{W}_i'b)) + \mathbf{1}\{Y_i > y\} \log \Lambda(-\widehat{W}_i'b)) \right].$$

Finally, in the third step we use (6) to estimate the counterfactual CDF (5) by:

$$\widehat{G}^s_{Y_{\langle t,k,r \rangle}}(y) = \frac{1}{n^s_{kr}} \sum_{i=1}^{n} \Lambda(\widehat{W}_i'\widehat{\beta}_t(y)) \, \mathbf{1}\{\widehat{V}_i > 1 - \widehat{\pi}_r(Z_i)\},$$

where the average is taken over the sample values of $\widehat{V}_i$ and Z_i in group k, $n^s_{kr} = \sum_{i=1}^{n} \mathbf{1}\{\widehat{V}_i > 1 - \widehat{\pi}_r(Z_i)\}$, $\widehat{\beta}_t(y)$ is the logistic distribution regression estimator for group t from the second step, and $\widehat{\pi}_r(z)$ is the estimator of the propensity score of selection for group r from the first step. Given $\widehat{G}^s_{Y_{\langle t,k,r \rangle}}$, we estimate the counterfactual QF (7) by:

$$\widehat{q}^s_{Y_{\langle t,k,r \rangle}}(\tau) = \int_0^\infty \mathbf{1}\{\widehat{G}^s_{Y_{\langle t,k,r \rangle}}(y) \le \tau\} dy - \int_{-\infty}^0 \mathbf{1}\{\widehat{G}^s_{Y_{\langle t,k,r \rangle}}(y) > \tau\} dy.$$

Following FVV, inference is based on the weighted bootstrap (Praestgaard and Wellner 1993). This method obtains the bootstrap version of the estimator of interest by repeating all the estimation steps including sampling weights drawn from a nonnegative distribution with mean and variance equal to one (e.g., standard exponential).

C   Proof of Lemma 2

$$\frac{1}{1-p} \int_p^1 G(y, x, v) dv = \frac{1}{1-p} \int_p^1 \mathbb{P}(g(x, E) \le y \,|\, V = v) dv$$

$$= \int_p^1 \mathbb{P}(g(x, E) \le y \,|\, V = v) dF_{V \,|\, V > p}(v)$$

$$= \mathbb{P}(g(x, E) \le y \,|\, V > \mu_h(z), Z = z, \mu_h(Z) = p)$$

$$= \mathbb{P}(Y \le y \,|\, H > h, X = x, \mu_h(Z) = p),$$

The first equality is definition. The second equality uses $V \sim U(0, 1)$ and the third equality uses independence of (E, V) and Z. The final equality uses the definitions of Y and H and is identified because $(x, p) \in \mathcal{X}\mathcal{P}_K$.

D Derivations of equation (12)

Adapting the representation of the distribution of the observed Y in Section 3.2 to the ordered selection rules yields

$$
G_Y^s(y) = \frac{\int_{\mathcal{Z}^k} \int G(y, x, v) \mathbf{1}\{v > \mu_0(z)\} dv dF_Z(z)}{\int_{\mathcal{Z}^k} \int \mathbf{1}\{v > \mu_0(z)\} dv dF_Z(z)} =
$$

$$
\frac{\sum_{h=1}^{K} \int_{\mathcal{Z}^k} \int G(y, x, v) \mathbf{1}\{\mu_{h-1}(z) < v \le \mu_h(z)\} dv dF_Z(z)}{\int_{\mathcal{Z}^k} \int \mathbf{1}\{v > \mu_0(z)\} dv dF_Z(z)},
$$

where the second equality uses that the interval $(\mu_0(z), 1]$ is the union of the disjoint intervals $(\mu_{h-1}(z), \mu_h(z)]$, $h = 1, \ldots, K$.

E Figures
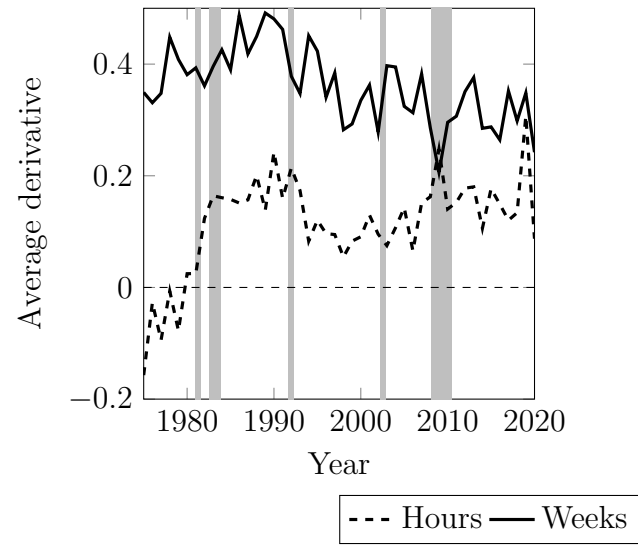
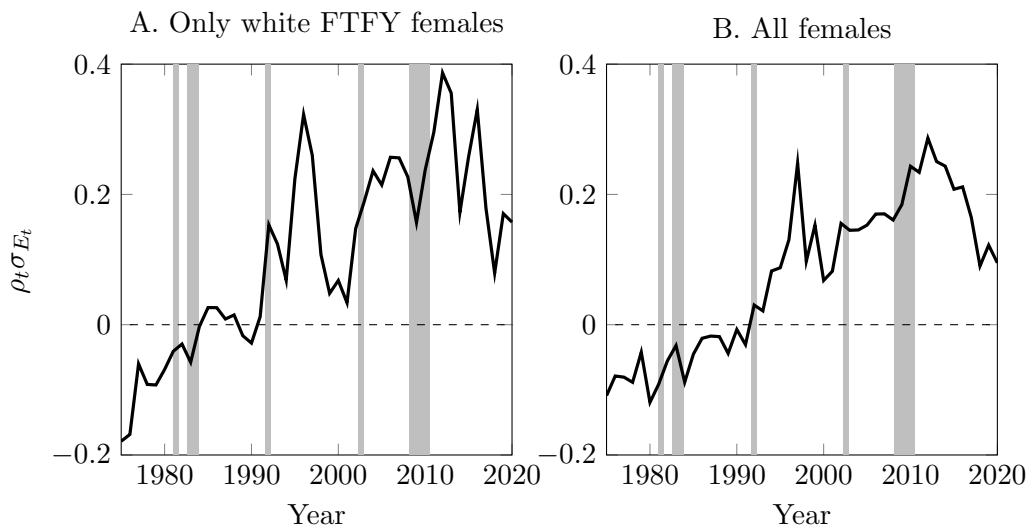Figure E.1: Average derivative

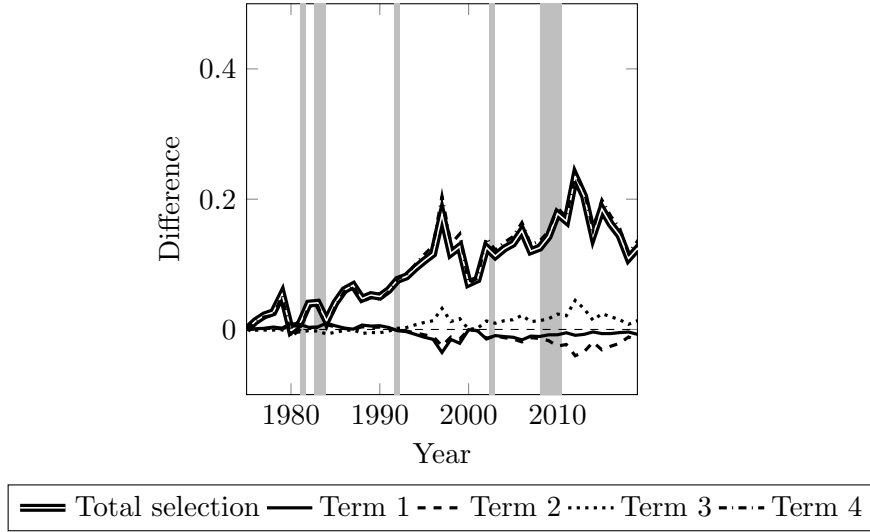Figure E.2: Estimation of $\rho_t \sigma_{E_t}$ over time.

Figure E.3: Selection effect using the Heckman-selection model and the 4 parts of the selection effect are represented by (18); total sample is full population between 24 and 65 and selected population is all workers working a positive number of hours.
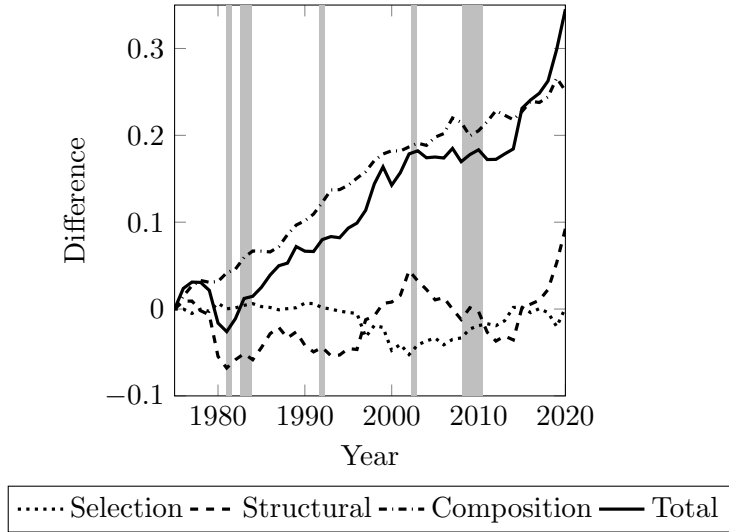

Figure E.4: Computation of the selection, composition and structural effect based on equation (19)-(21); total sample is full population between 24 and 65 and selected population is all workers working a positive number of hours.
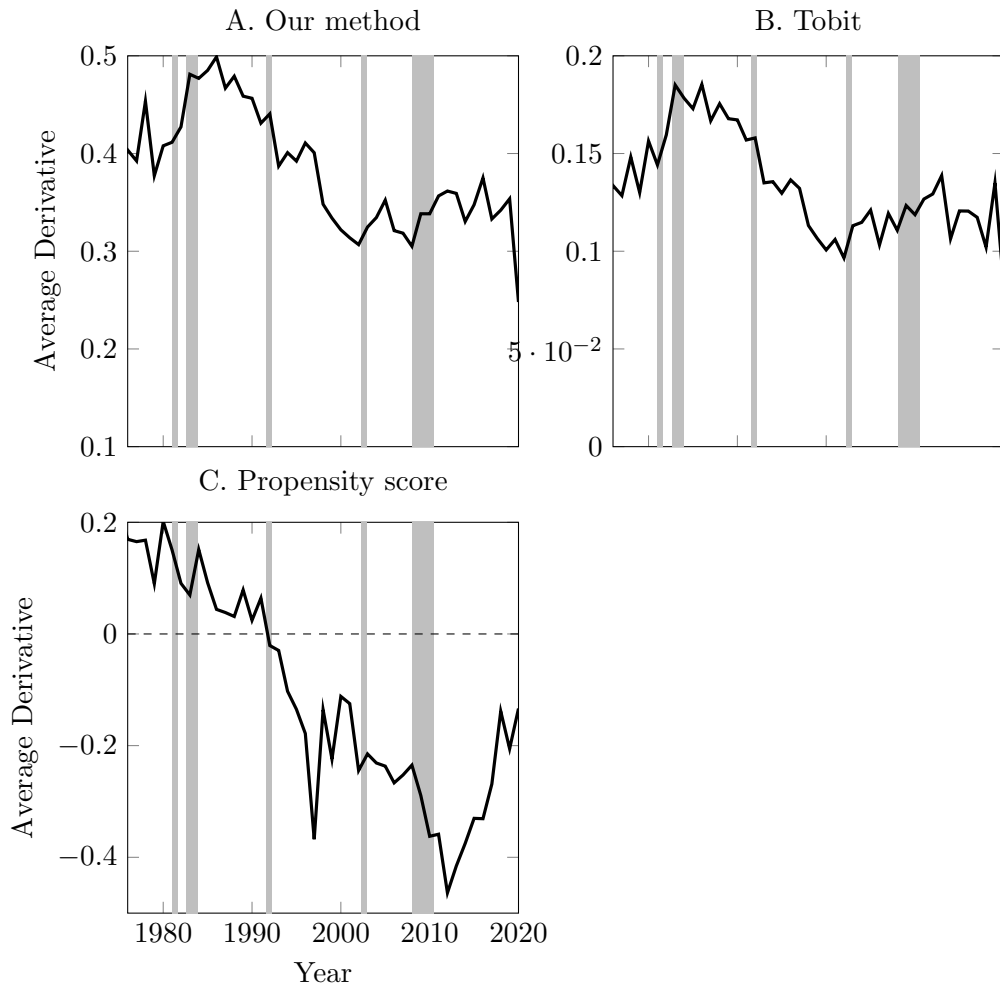
Figure E.5: Average derivative of the wage. A. is the derivative of our control function, B. is the average derivative using derivatives of a Tobit, C. uses a non-separable sample selection model.