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DISCUSSION PAPER SERIES

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ABSTRACT

Intra-Industry Trade, Involuntary Unemployment and Macroeconomic Stability*

We study the impact of intra-industry trade and capital mobility on steady state welfare and on the stability properties of two countries with identical technologies and preferences. We consider a two-factor overlapping generations model, featuring one-sector of differentiated goods with taste for variety. There is imperfect competition in the output market and increasing returns to scale in production (fixed costs and externalities). In one country there is full employment and saddle path stability in autarky, whereas in the other there are efficiency wages, and the autarkic equilibrium may be locally indeterminate. After opening the borders, the rigid wage country may export indeterminacy to the full employment country, particularly if it is big enough. In contrast, when the full employment country is sufficiently big, local indeterminacy, and therefore expectations driven fluctuations may be eliminated in the world. In any case, stochastic and deterministic fluctuations (associated with local indeterminacy and bifurcations) are possible with smaller externalities, whatever the relative size of the two countries. Steady state welfare improves in the full employment country with free trade and capital mobility, while unemployment increases in the country with labor market rigidities, reducing welfare. We also find that *taste for variety* (and therefore intra-industry trade), reduces the likelihood of local indeterminacy, but leads to flip bifurcations under more plausible values of the model parameters.

JEL Classification: C62, E32, F12, F43, F44, O41

Keywords: bifurcations, indeterminacy, intra-industry trade, taste for variety, involuntary unemployment

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1 Introduction

The present work investigates whether free-trade in differentiated goods driven by *taste for variety*, and the liberalization of capital movements, may bring welfare gains at the country level and decrease unemployment in countries with rigid wages, and whether it stabilizes (or destabilizes) the economies with respect to endogenous deterministic and stochastic fluctuations driven by autonomous volatile changes in expectations. Although several studies have analyzed the link between macroeconomic volatility driven by beliefs and inter-industry trade, the macroeconomic literature has not yet addressed the implications of *intra-industry trade*, i.e. trade where countries exchange differentiated goods of the same industry. Our work fills this gap, analyzing also the effects of this type of trade on steady state production, unemployment, wages and welfare in both countries. Nowadays *intra-industry* trade represents a significant and increasing percentage of the trade between developed countries. Indeed, according to a recent OECD study, more than 60% of U.S. trade, and 65% of European trade, is *intra-industry* trade.[1] This further justifies the need for addressing the impact of this type of trade.

We consider a two-country, overlapping generations model featuring a sector of differentiated goods, produced out of labor and capital under increasing returns to scale due to a fixed cost and labor externalities. In this sector there is imperfect competition à la Dixit and Stiglitz (1977), and equilibrium is characterized by a constant markup with an endogenous and proyclical number of varieties. We introduce *taste for variety*, according to which an increase of product diversity decreases the aggregate price at the symmetric equilibrium (Bénassy (1996)). The model combines and extends the closed economy framework with *taste for variety* developed in Seegmuller (2008), and the two-country model with international capital mobility and productive labor externalities considered in Aloi and Lloyd-Braga (2010). As in the latter, we assume that the two countries only differ in their labor market structure: efficiency wages and involuntary unemployment prevail in one country,[2] whereas in the other there is perfect competition in the labor market and full employment. This framework is particularly well suited to investigate whether the employment and welfare effects of *intra-industry* trade depend on the existing labor market structure. This is an important issue as developed countries, where *intra-industry* trade is particularly relevant, are characterized by different labor market institutions, and it is frequently conjectured that labor market rigidities may hinder the expected benefits of trade.

The novelty of our work relatively to Aloi and Lloyd-Braga (2010) is the introduction of *taste for variety*, and therefore of monopolistic competition in the output market with product differentiation, allowing for the existence of *intra-industry* trade. Indeed, in Aloi and Lloyd-Braga (2010), without *taste for variety*, there is no trade, the current account balance reflecting only international payments of capital income.[3] Also, while Aloi and Lloyd-Braga (2010) only analyze how international capital movements affect the likelihood of expectations-driven cycles associated with

[1] See http://oecdobserver.org/news/archivestory.php/aid/850/The_global_business _.html

[2] Goette et al. (2007) document the pervasiveness of real wage rigidity due to efficiency wages or bargaining power of workers, namely in Germany, Italy and the UK.

[3] The two countries only exchange capital services, the respective capital return being paid with the homogenous output.

local indeterminacy, we study how globalization (i.e. free trade and capital mobility) affects also the emergence of bifurcations and the corresponding deterministic and stochastic fluctuations. We show that equilibrium volatility driven by changes in expectations induce the existence of net imports or net exports leading to fluctuations in the balance of trade, even in the absence of shocks on fundamentals.

We start by analyzing autarkic equilibria in each country. We then consider equilibria with free-trade and capital mobility between the two countries, studying the effects of opening the borders on steady state welfare and on the stability properties of the world economy. As in Aloi and Lloyd-Braga (2010), in autarky, local indeterminacy, and therefore local sunspots fluctuations, emerge around the unique steady state in the country with efficiency wages and involuntary unemployment, for intermediate values of both the propensity to consume and the degree of externalities. In the country with a perfectly competitive labor market and full employment, the unique steady state is saddle path determinate in autarky. Given the asymmetry across the two countries in terms of local stability properties it is relevant to understand which type of dynamics will prevail under globalization. Will saddle path stability emerge in the globalized world, leading to the absence of expectations driven fluctuations as in the closed full employment country? Or will world indeterminacy and sunspot fluctuations occur as in the rigid wage country under autarky? Will endogenous fluctuations due to supercritical bifurcations occur due to globalization even if the world equilibrium is determinate? These are questions analyzed in this paper.

We find that the effects on stability of opening the borders to both free *intra-industry* trade and capital mobility depend on the existing degree of labor externalities and on the relative size of the two countries. Globalization may bring local saddle path stability to the world, eliminating local indeterminacy in the rigid wage country, in particular if the full employment country is sufficiently big in relative terms. In this case, the local stability properties of the (big) full employment country are exported to the other one, and the world economy is insulated from belief driven local fluctuations. In contrast, if the rigid wage country is big enough, local indeterminacy prevails at the world level, being exported from the country with rigid wages to the world economy. In this case, the full employment country, which was stable in autarky, will also face local expectation driven fluctuations with origin in the other country. Furthermore, for any country size, indeterminacy, and therefore local sunspots fluctuations at the world level, require a smaller degree of externalities, more likely to be compatible with empirical evidence. Also, bounded deterministic and stochastic fluctuations associated with supercritical Hopf and *flip* bifurcations, which did not exist in autarky, become possible after opening the borders for empirically plausible values of the parameters. Moreover, we show that a higher*taste for variety* shrinks the local indeterminacy region (in terms of the parameters of the model) under free *intra-industry* trade with capital mobility, promoting saddle-path stability. As *taste for variety* is the feature responsible for the existence of trade of differentiated goods of the same industry, we conclude that *intra-industry* trade reduces the likelihood of local sunspot fluctuations associated with local indeterminacy. However, we also find that, with *taste for variety*, obtaining a *flip* bifurcation, and its associated persistent deterministic and stochastic fluctuations which emerge when the steady state is saddle stable, becomes more plausible. Hence, the effects of the degree of *taste for variety*, and therefore of *intra-industry* trade, on the macroeconomic stability of the world are ambiguous. Previous studies on the effects of trade on the stability properties of trading countries obtained distinct results, depending on the framework considered. See, for instance, Nishimura and Shimomura (2002), Iwasa and Nishimura (2014) and Iwasa and Nishimura (2019) and Sim and Ho (2007).[4] When international capital mobility is also introduced the results obtained tend to support the view that trade is destabilizing, as in Nishimura et al. (2010) and Nishimura et al. (2014) in a two-good, two-factor model with infinitely-lived agents and Le Riche (2017) and Le Riche (2020) in an overlapping generations framework. However, all these papers assumed inter-industry trade and an inelastic labor supply. In contrast, in this paper we consider *intra-industry* trade with *taste for variety* and unemployment.

In terms of steady state effects of globalization, several natural questions arise. Will unemployment increase in the rigid wage country? Will we observe a displacement of industries and jobs from the rigid to the flexible wage country? What happens to real wages, to the number of varieties, to output and to welfare in the two countries? We show that employment decreases in the country with efficiency wages, i.e. globalization exacerbates unemployment in the rigid wage country. The number of firms, and therefore activity, is also reduced in that country. On the contrary, the number of varieties produced locally and the capital stock increase in the country with a perfectly competitive labor market and full employment. Finally, the steady state welfare of those employed in the country with efficiency wages and involuntary unemployment remains the same, while in the country with a perfectly competitive labor market and full employment citizens are better off. These steady state results operate through the interaction between differences in labor market rigidities and *intra-industry* trade with *taste for variety*.[5] We conclude that *intra-industry* trade may not bring benefits for all countries involved, hurting in particular those with labor market distortions.[6] Helpman and Itskhoki (2010), considering a static model with no capital, find, like us, an asymmetric impact of trade: the country with lower frictions in the labor market gains proportionately more.

We conclude that in the full employment country steady-state welfare increases with free (*intra-industry*) trade and capital movements, but macroeconomic fluctuations may become more prevalent, i.e. there is a trade off between steady state welfare gains and (de)stabilization. On the other hand, for the rigid wage country, opening the borders reduces unambiguously steady state welfare, *intra-industry* trade amplifying the effects of labor market rigidities on unemployment, without necessarily diminishing the likelihood of macroeconomic instability. Our work improves therefore

[4] Nishimura and Shimomura (2002), considering a two-factor, two-sector, two-country model with infinitely-lived agents, where countries only differ with respect to their initial factor endowments, show that inter-industry trade has no effect on the stability properties of both countries. However, Iwasa and Nishimura (2014) and Iwasa and Nishimura (2019), extending Nishimura and Shimomura (2002) by introducing a consumable capital good, find that endogenous fluctuations may emerge in the world economy. In contrast Sim and Ho (2007), introducing different technologies across countries (different degrees of productive externalities), find that saddle-path stability prevails in the world economy, even if before trade one country exhibits sunspot fluctuations.

[5] Indeed, Aloi and Lloyd-Braga (2010) with international capital mobility, but no trade, find no changes in stationary welfare. Their autarkic steady state and the steady state with international capital mobility are identical.

[6] Rodríguez-Clare et al. (2020), considering a quantitative trade model for the US economy, also find welfare losses in response to trade shocks for states characterized by downward nominal wage rigidity.

our understanding of the interaction between globalization and (un)employment, in the presence of *taste for variety* and labor market distortions. This is also a novel feature of our work, which to the best of our knowledge has not been addressed when assessing the effects of *intra-industry trade* on both stationary welfare and macroeconomic stability. New results in terms of endogenous fluctuations are highlighted, namely that globalization in economies with different degrees of rigidity in the labor market may stabilize or destabilize depending on their relative size, and that the emergence of world bounded deterministic fluctuations, affecting both countries, are plausible.

The rest of the paper is organized as follows. In Section 2 we present the model and obtain the perfect foresight equilibrium for the two economies in autarky, discussing local dynamics. Section 3 provides the analysis of the two-country model. In Section 4 we prove the existence of a unique steady state in the two country model and we discuss the changes in steady state activity, employment and welfare resulting from opening the economies. We analyze local dynamics of the two-country model and present the effects of *intra-industry* trade on stability in Section 5. Finally, section 6 concludes. Proofs are gathered in the Appendix.

2 Autarky

We consider two infinite horizon discrete time economies, country A and country B, that share the same production structure. Both countries have monopolistic competition in the output market and perfect competition in the capital services market, only differing in the functioning of the labor market. In country A there is involuntary unemployment with efficiency wages, while full employment and perfectly competitive wages prevail in country B. Households in both countries live for two periods, work when young and consume at each period a composite good. This composite good is an aggregate of all the differentiated goods (varieties) produced by firms, exhibiting *taste for variety.*

2.1 The model

In both countries population is constant over time and individuals live for two periods. In each period H^j individuals are born in country $j \in \{A, B\}$. In the first period of life, a young employed agent that does not shirk, offers a unit of effort, receiving a wage income, w_t. He uses this income to purchase the composite consumption good, C_t, and to save in the form of capital, $\tilde{K}_{t+1}$, which he rents to firms in the following period. In the second period of life, old retired agents use the rents received to finance consumption, D_{t+1}. As usually done in the literature, the composite good (C_t and D_{t+1}) is defined as an aggregate of the quantities consumed of all varieties i (respectively, c_{it} and d_{it+1}):

$$C_t = N_t^{1+\beta} \left[\frac{1}{N_t} \sum_{i=1}^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, D_{t+1} = N_{t+1}^{1+\beta} \left[\frac{1}{N_{t+1}} \sum_{i=1}^{N_t} d_{it+1}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where $\beta \geq 0$, N is the number of varieties, and $\varepsilon > 1$ is the intratemporal elasticity of substitution between varieties.[7]

Following Bénassy (1996), we define the function $t(N)$, which represents the gain from consuming one unit of N different varieties instead of consuming N units of a single variety. It follows from the definition of the composite good given in (1) that $t(N) = N^{\beta}$. Accordingly, the degree of *taste for variety* is given by the elasticity of $t(N)$:

$$\tau(N) \equiv \tfrac{Nt'(N)}{t(N)} = \beta. \tag{2}$$

If β is zero, households have no *taste for variety* while if β is higher than zero, there is *taste for variety*.[8]

Agents have preferences defined over consumption in the first period of life, C_t, consumption in the second period of life, D_{t+1}, and young age effort, e_t. A young agent born at period t solves the following dynamic program, taking (1) into consideration:

$$\begin{aligned} \max_{C_t, D_{t+1}, \tilde{K}_{t+1}} \quad & C_t^{\alpha} D_{t+1}^{1-\alpha} - \nu e_t \\ s.t. \quad & P_t C_t + P_t \tilde{K}_{t+1} = w_t, \\ & P_{t+1} D_{t+1} = r_{t+1} \tilde{K}_{t+1}, \\ & C_t, D_{t+1}, \tilde{K}_{t+1} \geq 0, \end{aligned} \tag{3}$$

where $\nu > 0$ is the disutility of effort, $\alpha \in (0, 1)$ is the propensity to consume when young, $e_t \in \{0, 1\}$ represents the effort supplied, r_{t+1} denotes the nominal rental rate of capital, w_t is the nominal wage and P_t represents the price of the aggregate consumption good.[9]

We adopt a two-stage maximization procedure. First, given a fixed amount of the composite good, C_t and D_{t+1} as defined in (1), a young agent born at period t chooses c_{it} and d_{it}, in order to minimize respective spending:

$$\sum_{i=1}^{N_t} p_{it} c_{it} = P_t C_t \quad \text{and} \quad \sum_{i=1}^{N_t} p_{it+1} d_{it+1} = P_{t+1} D_{t+1}$$

where p_{it} is the price of a variety i. We obtain:

$$c_{it} = N_t^{\beta(\varepsilon-1)-1} \left(\tfrac{p_{it}}{P_t}\right)^{-\varepsilon} C_t, d_{it+1} = N_{t+1}^{\beta(\varepsilon-1)-1} \left(\tfrac{p_{it+1}}{P_{t+1}}\right)^{-\varepsilon} D_{t+1} \tag{4}$$

[7]When $\varepsilon > 1$, the differentiated goods are substitutes.

[8]Ardelean (2009) estimates consumer's love for variety and suggests that variety matters for both imported and domestically produced goods while Drescher et al. (2008) present evidence on consumers' preferences for variety in food consumption.

[9]The reader may note that our results would be the same if we had considered instead a perfectly competitive market of a final good (with price P_t) produced out of the differentiated intermediate products according to (1).

so that:

$$P_t = \frac{1}{N_t^{\beta}} \left[\frac{1}{N_t} \sum_{i=1}^{N_t} p_{it}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{5}$$

Second, he determines his intertemporal choice between consumption when young and old and young age effort. Defining the real wage $\omega_t \equiv \frac{w_t}{P_t}$, and the real interest rate $\rho_t \equiv \frac{r_t}{P_t}$ the first-order conditions can be written as:

$$C_t = \alpha\omega_t, D_{t+1} = (1-\alpha)\omega_t\rho_{t+1} \text{ and } \tilde{K}_{t+1} = (1-\alpha)\omega_t. \tag{6}$$

Plugging now the demands for the composite goods C_t and D_{t+1} into the utility function defined in (3), we obtain the lifetime indirect utility function of a young agent that supplies a given effort e_t:

$$V(\omega_t, \rho_{t+1}, e_t) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}\omega_t - \nu e_t. \tag{7}$$

In the case of a young employed worker supplying $e_t = 1$, the utility becomes

$$V(\omega_t, \rho_{t+1}, 1) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}(\omega_t - \bar{\omega}_t) \tag{8}$$

where

$$\bar{\omega}_t \equiv \frac{\nu}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}} \tag{9}$$

represents the real reservation wage. Note that, as the indirect utility of an unemployed worker is zero, all youngsters are willing to work, supplying $e_t = 1$, when $\omega_t > \bar{\omega}_t$, while for $\omega_t < \bar{\omega}_t$ the labor supply is zero.

As in Dixit and Stiglitz (1977), we assume monopolistic competition in the goods market. In each period $t = 1, ..., \infty$, entry and exit are free and the zero profit condition determines the number of firms. Furthermore, we consider the existence of labor externalities in production, as in Aloi and Lloyd-Braga (2010) and in Lloyd-Braga et al. (2007). Each firm produces one variety $i \in \{1, ..., N\}$ of output using the following technology:

$$y_{it} = \Theta \left[a_{it}^{s} l_{it} \overline{L}_t^{\gamma} - \phi \right] \tag{10}$$

where s is the share of capital in total income, Θ is the total factor productivity, $a_{it} = k_{it}/l_{it}$ the capital-labor ratio used by firm i, $\overline{L}_t$ is aggregate employment which firms take as given, $\gamma > 0$ represents the degree of the labor externality and $\phi > 0$ a fixed cost.

Aggregate production in period t, P_tY_t, is shared between consumption of young agents, consumption of old agents and investment:

$$P_tY_t = \sum_{i=1}^{N_t} p_{it}y_{it} = [L_tC_t + L_tI_t + L_{t-1}D_t]\, P_t. \tag{11}$$

We assume full depreciation of capital so that $I_t = \tilde{K}_{t+1}$ and we consider that I_t is defined by an index of varieties similar to the one used for consumption:

$$I_t = N_t^{1+\beta}\left[\frac{1}{N_t}\sum_{i=1}^{N_t} i_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where i_{it} is optimally determined by

$$i_{it} = N_t^{\beta(\varepsilon-1)-1}\left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} I_t. \tag{12}$$

It follows that the demand for variety i, υ_{it}, is given by $\upsilon_{it} = c_{it} + d_{it} + i_{it}$, so that using (4) and (12) we obtain:

$$\upsilon_{it} = N_t^{\beta(\varepsilon-1)-1}\left(\frac{p_{it}}{P_t}\right)^{-\varepsilon}\left[L_t\left(C_t + I_t\right) + L_{t-1}D_t\right]. \tag{13}$$

Country A exhibits labor market rigidity due to the existence of efficiency wages.[10] As in Shapiro and Stiglitz (1984), workers may shirk and in that case the level of effort supplied is zero, i.e. $e_t^A = 0$. A worker who shirks is caught with (ex-ante) probability $0 < \lambda < 1$. Firms fire immediately workers who are caught shirking, without any wage, so that they get zero utility. A young agent faces therefore three possibilities: (i) being unemployed, (ii) being employed and not shirking or (iii) being employed and shirking ($e_t^A = 0$, $w_t^A > 0$). Using the indirect utility function given in (7), the utility of an employed worker who shirks is $V^{e,s} = (1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}(\rho_{t+1}^A)^{1-\alpha}\omega_t^A$. Then, using (8), it is easy to see that employed workers will not shirk ($e_t^A = 1$) if wages are such that $\omega_t^A \geq \frac{\bar{\omega}_t^A}{\lambda}$, i.e. if wages satisfy the No Shirking Condition (hereafter NSC).

Since the output of a worker who shirks is zero, firms, in order to maximize profits, take into account the NSC, so that the wage chosen induces all works to exert effort. Hence, the problem solved by firms in country A is the following:

$$\begin{aligned} \max_{w_t^A, l_{it}^A, k_{it}^A \in \Re_{++}^3} \quad & p_{it}^A y_{it}^A - w_t^A l_{it}^A - r_t^A k_{it}^A \\ s.t. \quad & w_t^A \geq \frac{\bar{\omega}_t^A}{\lambda} P_t^A, \end{aligned} \tag{14}$$

where y_{it}^A is given by (10), and p_{it}^A is such that $y_{it}^A = \upsilon_{it}^A$, with the the demand function υ_{it}^A given in (13).

The first-order conditions are then given by (10), (13) and

$$\begin{aligned} r_t^A &= \Theta p_{it}^A\left(\frac{\varepsilon-1}{\varepsilon}\right) s(a_{it}^A)^{s-1}\left(\overline{L}_t^A\right)^{\gamma}, \\ w_t^A &= \Theta p_{it}^A\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)(a_{it}^A)^{s}\left(\overline{L}_t^A\right)^{\gamma}, \\ w_t^A &= \frac{\bar{\omega}_t^A}{\lambda} P_t^A. \end{aligned} \tag{15}$$

[10] As already noticed in Aloi and Lloyd-Braga (2010), similar results would apply if we had instead considered monopoly unions or search generated unemployment.

At the symmetric equilibrium $l_{it}^A = l_t^A$, $k_{it}^A = k_t^A$, $a_{it}^A = a_t^A$ and $p_{it}^A = p_t^A$ for all firms in country A and $\overline{L}_t^A = L_t^A = N_t^A l_t^A$. Then, using (5), the real interest rate, ρ_t^A, the real wage, ω_t^A, and the aggregate price, P_t^A, in country A are given by:

$$\begin{aligned} \rho_t^A &= \Theta (N_t^A)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s (a_t^A)^{s-1} \left(L_t^A\right)^{\gamma}, \\ \omega_t^A &= \Theta (N_t^A)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) (1-s)(a_t^A)^{s} \left(L_t^A\right)^{\gamma}, \\ P_t^A &= (N_t^A)^{-\beta} p_t^A. \end{aligned} \tag{16}$$

Moreover real wages are set as mark up over the real reservation wage $\omega_t^A = \frac{\bar{\omega}_t^A}{\lambda}$.

On the contrary, in country B, we consider a perfectly competitive labor market with full employment. Therefore, labor demand of firm i is determined by the equality $w_t^B = \Theta p_{it}^B \left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)(a_{it}^B)^s \left(H_t^B\right)^{\gamma}$, where, at full employment equilibria, $w_t^B > \bar{\omega}_t^B P_t^B$, with $\bar{\omega}_t^B$ given by (9), and employment satisfies $\sum_{i=1}^{N_t^B} l_{it}^B = H^B$. We also have that the rental rate of capital is given by the equality $r_t^B = \Theta p_{it}^B \left(\frac{\varepsilon-1}{\varepsilon}\right) s (a_{it}^B)^{s-1} \left(H_t^B\right)^{\gamma}$. Note that the markup factor in the differentiated goods market is constant and given by $\varepsilon/(\varepsilon-1)$ in both countries.

At a symmetric equilibrium, $l_{it}^B = l_t^B = \frac{H^B}{N_t^B}$, $k_{it}^B = k_t^B$, $a_{it}^B = a_t^B$ and $p_{it}^B = p_t^B$ for all firms in country B. Then, using (5), the real interest rate, ρ_t^B, the real wage, ω_t^B, and the aggregate price, P_t^B, in country B are given by:

$$\begin{aligned} \rho_t^B &= \Theta (N_t^B)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s (a_t^B)^{s-1} \left(H^B\right)^{\gamma}, \\ \omega_t^B &= \Theta (N_t^B)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) (1-s)(a_t^B)^{s} \left(H^B\right)^{\gamma}, \\ P_t^B &= (N_t^B)^{-\beta} p_t^B. \end{aligned} \tag{17}$$

From (16) and (17) the aggregate price P_t^j is equal to p_t^j at the symmetric equilibrium when $\beta = 0$. However, if $\beta > 0$ the aggregate price decreases with the number of varieties, as shown in (5). Moreover, the real interest rate, ρ_t^j, and the real wage, ω_t^j, increase with the number of varieties N_t^j for $j \in \{A, B\}$.

In both countries the free-entry condition is determined by the zero profit condition, $p_t^j y_t^j - k_t^j r_t^j - l_t^j w_t^j = 0$. Hence, using (10), and the expressions obtained above for w and r, in (15) for A and in a similar way for B, we obtain that:

$$\frac{(a_t^j)^s l_t^j \left(\overline{L}_t^j\right)^{\gamma}}{\varepsilon} = \phi. \tag{18}$$

From (10) and (18), we derive that the production level of each firm is constant at equilibrium and identical in both countries:

$$y_t^j = \Theta[(a_t^j)^s l_t^j \left(\overline{L}_t^j\right)^{\gamma} - \phi] = \Theta\phi(\varepsilon - 1). \tag{19}$$

2.2 Country A - Equilibrium with involuntary unemployment

In country A, at a symmetric equilibrium the aggregate demand for capital services is given by $K_t^A = N_t^A k_t^A$. Using the free-entry condition given in (18), and the fact that at the symmetric equilibrium $L_t^A = K_t^A / a_t^A$, we obtain the number of varieties (firms) in country A, N_t^A:[11]

$$N_t^A = \frac{(a_t^A)^{s-1-\gamma}(K_t^A)^{1+\gamma}}{\varepsilon\phi} \equiv N^A(a_t^A, K_t^A). \tag{20}$$

Using (16) and (20), the real interest rate and the real wage can also be written as function of K_t^A and of a_t^A

$$\begin{array}{l} \rho_t^A = \Theta N^A(a_t^A, K_t^A)^\beta \left(\frac{\varepsilon-1}{\varepsilon}\right) s \left(a_t^A\right)^{s-1-\gamma} \left(K_t^A\right)^\gamma \equiv \rho(a_t^A, K_t^A), \\ \omega_t^A = \Theta N^A(a_t^A, K_t^A)^\beta \left(\frac{\varepsilon-1}{\varepsilon}\right) (1-s) \left(a_t^A\right)^{s-\gamma} \left(K_t^A\right)^\gamma \equiv \omega(a_t^A, K_t^A). \end{array} \tag{21}$$

Employment is determined by the equality $\frac{\bar{\omega}_t^A}{\lambda} = \omega(a_t^A, K_t^A)$, with the real reservation wage given by (9). We assume that the level of employment satisfying this condition verifies $L_t^A = N_t^A l_t^A < H^A$, so that we obtain an equilibrium with unemployment. Indeed, in contrast with perfect competition, wages are set as a mark up over the reservation wage, so that involuntary unemployment emerges. It is worth noting that expectations influence equilibrium through the labor market, since the reservation wage and employment level at period t depend on ρ_{t+1}^A, i.e. on the expectations for the future real interest rate which, under perfect foresight, coincide with its realized value.

In the capital services market, at equilibrium, aggregate demand $K_t^A = N_t^A k_t^A$, must equal aggregate supply, $L_{t-1}^A \tilde{K}_t^A$ so that using (6) we obtain $K_t^A = (1-\alpha) L_{t-1}^A \omega_{t-1}^A$.

The dynamics of the economy are given by the labor market equilibrium condition, $\omega_t^A = \frac{\bar{\omega}_t^A}{\lambda}$, and by the evolution of the capital stock, K_t^A. We then define:

Definition 1. *An intertemporal equilibrium with perfect foresight under autarky for the rigid wages country A is a sequence $\left\{a_t^A, K_t^A\right\}_{t=0}^\infty$ which, given the initial capital stock $K_{t=0}^A > 0$, satisfies the capital accumulation equation and the labor market equilibrium condition:*

$$K_{t+1}^A = (1-\alpha)\, \omega(a_t^A, K_t^A) \frac{K_t^A}{a_t^A}, \tag{22}$$

$$(1-\alpha)^{1-\alpha} \alpha^\alpha \omega(a_t^A, K_t^A) \rho(a_{t+1}^A, K_{t+1}^A)^{1-\alpha} = \frac{\nu}{\lambda}. \tag{23}$$

where $\omega(a_t^A, K_t^A)$ and $\rho(a_{t+1}^A, K_{t+1}^A)$ are given by (21).

Equations (22)-(23) rule the dynamics of country A in autarky, and define a two-dimensional dynamic system with one predetermined variable, aggregate capital, which is given by past savings. In contrast, employment in t, and therefore a_t^A, are affected by expectations about the future real interest rate, opening the way for expectations driven fluctuations.

[11] As usually done in the literature, we refrain from considering the condition that N should be an integer number. However, by choosing a sufficiently small ϕ, we can ensure that the number of varieties is higher than one in both countries.

2.2.1 Steady State

A steady state of the dynamic system (22)-(23) is a solution $(a^A, K^A) = (a_t^A, K_t^A)$ for all t, such that

$$a^A = (1-\alpha)\,\omega(a^A, K^A), \tag{24}$$

$$(1-\alpha)^{1-\alpha}\alpha^{\alpha}\omega(a^A, K^A)\rho(a^A, K^A)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{25}$$

This system only has one solution (a^A, K^A) as claimed in the following Proposition.[12]

Proposition 1. *There exists a unique stationary solution (a^A, K^A) of the dynamic system (22)-(23) given by*

$$a^A = \left(\tfrac{\nu}{\lambda}\right) \tfrac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^{\alpha} s^{1-\alpha}}, \tag{26}$$

$$K^A = \left[\tfrac{\varepsilon^{1+\beta}\phi^{\beta}\left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(1-\alpha)(1-s)(\varepsilon-1)}\right]^{\frac{1}{\gamma+\beta(1+\gamma)}}. \tag{27}$$

Using (21), we can express the real interest rate as $\rho_t^A = s\omega_t^A/[(1-s)a_t^A]$. Substituting now (24) in this last expression evaluated at the steady state we obtain:

$$\rho^A = \frac{s}{(1-s)(1-\alpha)}. \tag{28}$$

We shall consider that α is high enough so that the real interest rate at equilibrium is positive, i.e. $\rho^A > 1$. In the following Assumption we summarize the restrictions on the parameters' values we will consider from now on.

Assumption 1. $s \in (1/4, 1/2)$, $0 \leq \beta < s/(1-s)$ *and* $1 > \alpha > \max\ \{(1-2s)/(1-s), 1/2\} \equiv \underline{\alpha}$.

Under this Assumption, the conditions on s, i.e. on the capital share of output in the economy, ensure that it takes an empirically plausible value. See for example Cecchi and Garcia-Peñalosa (2010). The restriction on β stipulates that *taste for variety* is not too high, in accordance with empirical findings. See Ardelean (2009). Moreover, this restriction allows the equilibrium labor (capital) demand curve to be downward sloping, by guaranteeing that this is the case when $\gamma = 0$. As stated above we suppose that the real interest rate is higher than one, i.e. that $\alpha > (1-2s)/(1-s)$. Finally, we also assume, in accordance with most empirical values obtained from national accounts of OECD countries, that the propensity to consume when young is higher than 1/2.

Before proceeding to analyze the local dynamics, it is interesting to discuss the effects of β - a novel feature of our work - on steady state outcomes. Remark first that β does not influence output per firm, which is constant (see (18) and (19)), nor a^A (see (22)), the real interest rate (see (28)) or

[12] Existence of equilibrium unemployment at the steady state is ensured by assuming that H^A is high enough, so that $L^A = K^A/a^A < H^A$. Then, trajectories that stay close to the steady state also exhibit unemployment.

the reservation wage (see (9)). Therefore it neither influences the real wage $\omega^A = \frac{\bar{\omega}^A}{\lambda}$. Using now (16) and (18) we conclude that $(N^A)^\beta (L^A)^\gamma$ and $l^A (L^A)^\gamma \equiv \frac{(L^A)^{1+\gamma}}{N^A}$ can not change with β. From the last expression we conclude that changes in L^A and N^A must go in the same direction. Since $(N^A)^\beta$ increases with β for a given N^A higher than one, the product $(N^A)^\beta (L^A)^\gamma$ will not remain constant if both N^A and L^A increase. We conclude therefore that N^A and L^A decrease with β. As *taste for variety* does not influence a^A it holds that the steady state capital stock, $K^A = L^A a^A$, also decreases with β. One would expect that a higher *taste for variety* would result in the production of a higher number of varieties, and therefore in a higher aggregate employment. However, we obtain the opposite result. This happens because of the existence of real wage rigidity: in country A the real wage is only influenced by the variables and parameters that determine indirect utility and the mark-up (and therefore the real reservation wage) and does not respond to changes in any other variables or parameters.

2.2.2 Local dynamics and (in)determinacy

In this subsection we characterize the local dynamics of system (22)-(23). We analyze the role of the propensity to consume when young, α, of the degree of *taste for variety*, β, and of the degree of increasing returns, γ, on the emergence of local indeterminacy and expectation driven fluctuations. Remark that since this system is loglinear, bifurcations are not possible. Denoting percentage deviations from the steady state respectively by $\hat{K}_t^A \equiv \left(K_t^A - K^A\right)/K^A$ and $\widehat{a}_t^A \equiv \left(a_t^A - a^A\right)/a^A$ and loglinearizing (22)-(23) we obtain:

$$\begin{pmatrix} \hat{K}_{t+1}^A \\ \widehat{a}_{t+1}^A \end{pmatrix} = J \begin{pmatrix} \hat{K}_t^A \\ \widehat{a}_t^A \end{pmatrix} \tag{29}$$

where J, given in Appendix 7.1, is the Jacobian matrix of the dynamic system. Then, the following Proposition holds.

Proposition 2. *The characteristic polynomial of system (22)-(23) is defined by* $P(\lambda) = \lambda^2 - \lambda T + D$*, where the trace,* T*, and the determinant,* D*, are given by:*[13]

$$T = \frac{1-\alpha(1+\beta)(1+\gamma-s)}{(1-\alpha)(1+\gamma-s)(1+\beta)}, \quad D = \frac{s}{(1-\alpha)(1+\gamma-s)} > 0. \tag{30}$$

Proof. See Appendix 7.1. □

[13] Note that while the propensity to consume when young, α, the degree of increasing returns, γ, and the capital share, s, influence both the trace and the determinant, β, the degree of *taste for variety* only affects the trace.

Following Grandmont et al. (1998), we study the local stability properties of our model, which are determined by the eigenvalues of the characteristic polynomial $P(\lambda) = \lambda^2 - \lambda T + D$,[14] by referring to the diagram represented in Figure 1.

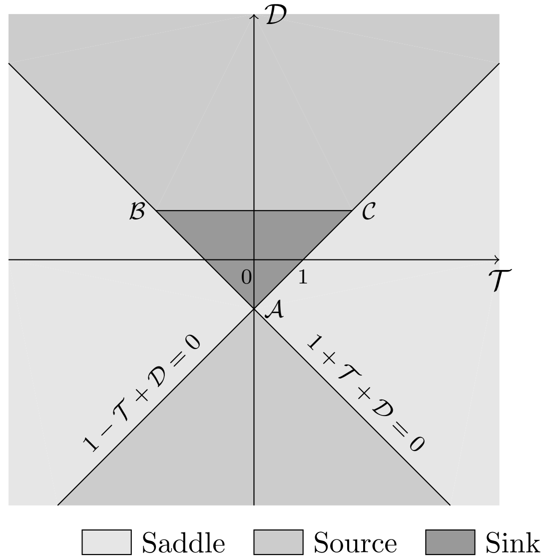

Figure 1: Stability triangle.

One eigenvalue is equal to 1 on the line AC ($D = T - 1$). On the line AB ($D = -T - 1$) one eigenvalue is equal to -1. On the segment BC the two eigenvalues are complex conjugates with modulus equal to 1. Therefore the steady state is a sink (both eigenvalues with modulus lower than one) when (T, D) is inside the triangle ABC. Since only capital is a predetermined variable, when the steady state is a sink, it is locally indeterminate[15] and there are infinitely many stochastic endogenous fluctuations (sunspots) arbitrarily close to the steady state.[16] The steady state is a source (both eigenvalues with modulus higher than one) if (T, D) is above AB, AC and BC or below AB and AC. It is saddle stable (one eigenvalue with modulus higher than one and one eigenvalue with modulus lower than one) in the remaining cases. In the Proposition below we present conditions on the parameters under which the steady state is a sink, a saddle or a source.

Proposition 3. *Consider Assumption 1 satisfied and define* $\gamma_1^{aut} \equiv \frac{\alpha(1-s)-(1-2s)}{(1-\alpha)}$, $\gamma_2^{aut} \equiv \frac{2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]}{(1+\beta)(2\alpha-1)}$ *and* $\tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$. *Then, in the country with rigid wages (country A), the following generically holds in autarky:*[17]

$\lfloor i \rfloor$ *When* $\underline{\alpha} < \alpha < \tilde{\alpha}$, *for* $\gamma < \gamma_1^{aut}$ *the steady state is a source, becomes a sink (locally*

[14] Note that T and D correspond respectively to the product and sum of the two roots (eigenvalues) of the associated characteristic polynomial.

[15] Indeterminacy occurs when the number of eigenvalues strictly lower than one in absolute value is larger than the number of predetermined variables.

[16] See also Woodford (1986).

[17] Note that when $\beta = 0$ our critical values for α and γ are identical to the ones of Aloi and Lloyd-Braga (2010).

indeterminate) for $\gamma_1^{aut} < \gamma < \gamma_2^{aut}$, becoming a saddle for $\gamma > \gamma_2^{aut}$.

$\lfloor ii \rfloor$ *When $\underline{\tilde{\alpha}} < \alpha < 1$, for $\gamma < \gamma_2^{aut}$ the steady state is a source, becoming a saddle for $\gamma > \gamma_2^{aut}$.*

Proof. See Appendix 7.2. □

This Proposition shows that when α is not too low, nor too high, local indeterminacy is possible. However, a minimal degree of labor externalities, $\gamma > \gamma_1^{aut} > 0$, is also necessary for indeterminacy, as in Farmer and Guo (1994) with infinitely lived agents, and Aloi and Lloyd-Braga (2010) or Lloyd-Braga et al. (2007), with an overlapping generations framework.[18] Note however that, in the presence of empirically plausible values for α which exceed 0.5, local indeterminacy remains possible, in the country with labor market imperfections, for small increasing returns.[19] Proposition 3 also tells us that labor externalities can not be too high for indeterminacy to occur, i.e. $\gamma < \gamma_2^{aut}$.

In order to understand why indeterminacy requires a lower bound and an upper bound on the labor externality, consider that the economy is at the steady state in period t and suppose that agents anticipate a raise in the future interest rate. According to (9), the increase of the expected future interest rate, ρ_{t+1}^A, will decrease the current reservation wage, $\bar{\omega}_t^A$, and the current wage, ω_t^A, so that, considering the labor demand curve is downward sloping, the current level of employment, L_t^A, will increase. Accordingly, current savings also rise implying an increase in the future capital stock, K_{t+1}^A. When $\beta = \gamma = 0$ this increase will unambiguously reduce the future interest rate, see (21), so that expectations can not be fulfilled. However, in the presence of *taste for variety* , $\beta > 0$, and/or labor externalities, $\gamma > 0$, this increase in K_{t+1}^A will shift the labor demand curve to the right, increasing the future level of employment. Note that this increase will be higher for bigger values of γ. In turn, this increase in future employment will increase the future real interest. Therefore, if the positive effect on ρ_{t+1}^A is sufficiently high, i.e. if γ is sufficiently big ($\gamma > \gamma_1^{aut}$), it may overcome the negative effect due to the increase in capital in $t+1$. As a result, in this last case, expectations can be self-fulfilling. Local indeterminacy also requires a future reversal in the trajectory so that, in the absence of further shocks to expectations, the system will return to the steady state. This implies that we must observe a decrease in the future capital stock, that is the future wage bill must decrease. For this to happen L_{t+1}^A must not increase too much, i.e. γ can not be too big ($\gamma < \gamma_2^{aut}$).

It is worth mentioning the role of *taste for variety*, β, on the local dynamics of the autarkic system. Looking first at the critical bounds of the labor externality, we can see that γ_1^{aut} does not depend on β, while γ_2^{aut} is a decreasing function of β. Second, note also that $\tilde{\alpha}$, the upper bound on α above which indeterminacy does not emerge, decreases with β. Therefore, a higher *taste for variety* reduces the likelihood of local indeterminacy, by shrinking the interval of parameters'

[18] As emphasized in Lloyd-Braga et al. (2007), the conditions for indeterminacy are similar in an overlapping generation model with a propensity to current consumption compatible with what is observed in data, and in a model of infinitely lived agents as the one explored in Farmer and Guo (1994). In both set ups considering a more elastic labor supply curve, which in our case is is infinitely elastic due to efficiency wages, reduces the lower bound for γ required for indeterminacy.

[19] For example, when the share of capital in total income is 0.3 we have $\underline{\alpha} = 0.571$, and considering $\alpha = 0.6$ we have $\gamma_1^{aut} = 0.05$. Considering a slightly higher $s = 1/3$, and still considering $\alpha = 0.6$, we have $\gamma_1^{aut} = 0.167$.

values, under which indeterminacy emerges, enlarging the set of parameter values for which the steady state is saddle stable. Our results contrast with those obtained by Seegmuller (2008) who does not consider current consumption nor labor externalities, and finds that *taste for variety* facilitates the emergence of local indeterminacy in a closed economy.

2.3 Country B - Equilibrium with full employment

In country B the labor market is perfectly competitive and full employment exists, so that $L^B = H^B$. It follows, that the number of varieties is only a function of the current capital stock K_t^B:

$$N_t^B = \frac{\left(K_t^B\right)^s \left(H^B\right)^{1+\gamma-s}}{\varepsilon\phi} \equiv N^B(K_t^B). \tag{31}$$

The real interest rate and the real wage can also be written as functions of K_t^B:

$$\begin{aligned} \rho_t^B &= \Theta N^B(K_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s \left(K_t^B\right)^{s-1} \left(H^B\right)^{1+\gamma-s} \equiv \rho^B(K_t^B), \\ \omega_t^B &= \Theta N^B(K_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) (1-s) \left(K_t^B\right)^{s} \left(H^B\right)^{\gamma-s} \equiv \omega^B(K_t^B). \end{aligned} \tag{32}$$

Therefore, equilibrium dynamics in country B are totally determined by the evolution of capital.

Definition 2. *An intertemporal equilibrium with perfect foresight under autarky for the full employment country B is a sequence $\left\{K_t^B\right\}_{t=0}^\infty$, which given the initial capital stock $K_{t=0}^B > 0$, satisfies the capital accumulation equation:*

$$K_{t+1}^B = (1-\alpha)\,\omega^B(K_t^B)H^B. \tag{33}$$

with $\omega^B(K_t^B)$ given by (32).

This equation defines a one-dimensional system which characterizes the dynamics of country B in autarky.

2.3.1 Steady State

A steady state of the dynamic system (33) is a solution $K^B = K_t^B = K_{t+1}^B$ for all t, such that

$$K^B = (1-\alpha)\,\omega^B(K^B)H^B. \tag{34}$$

We can easily prove that:[20]

[20] To guarantee that full employment exists at the steady state we ensure that $\omega^B > \bar{\omega}^B$ at the steady state by choosing a sufficiently small ν. Therefore along trajectories sufficiently close to the steady state $\omega_t^B > \bar{\omega}_t^B$.

Proposition 4. *K^B is a unique stationary solution of the dynamic system (33). The value of K^B is given by*

$$K^B = \left[\frac{\Theta(1-\alpha)(1-s)(\varepsilon-1)(H^B)^{(1+\gamma-s)(1+\beta)}}{\varepsilon^{1+\beta}\phi^{\beta}}\right]^{\frac{1}{1-s(1+\beta)}}. \tag{35}$$

The number of varieties evaluated at the steady state is given by

$$N^B = \left[\frac{\Theta(1-\alpha)(1-s)(\varepsilon-1)\left(H^B\right)^{\frac{1+\gamma-s}{s}}}{\varepsilon^{\frac{1}{s}}\phi^{\frac{1-s}{s}}}\right]^{\frac{s}{1-s(1+\beta)}}. \tag{36}$$

From this last expression we can see that, as expected, the number of varieties in country B increases with *taste for variety*. This contrasts with what happens in country A, where the number of varieties (firms) decreases with β due to wage rigidity as explained before.

Note also that the steady state real interest rate in country B is identical to the steady state real interest rate of country A given in (28). Using (32), we can express country B's real interest rate as $\rho_t^B = s\omega_t^B H^B/[(1-s)K_t^B]$. Substituting now (34) in this last expression evaluated at the steady state we obtain:

$$\rho^B = \rho^A = \frac{s}{(1-s)(1-\alpha)}. \tag{37}$$

2.3.2 Local dynamics and (in)determinacy

Differentiating the capital accumulation given in (33) we obtain:

$$\frac{dK_{t+1}^B}{K_{t+1}^B} = s\left(1+\beta\right)\frac{dK_t^B}{K_t^B}. \tag{38}$$

We can immediately see that, as the dynamic system is loglinear, bifurcations are not possible. Moreover, under Assumption 1, we can state the following.

Proposition 5. *Consider Assumption 1 satisfied. Then, the steady state of the full employment country B is stable as $s\left(1+\beta\right) < 1$.*

3 The Two-Country Model

We consider a world economy with two countries, A and B, which differ only in the functioning of their labor markets. We suppose that capital is mobile across countries, which implies that the nominal interest rates are equalized, i.e. $r_t^A = r_t^B$, while labor is internationally immobile. Furthermore, goods are freely traded, so that households, both from country A and country B, have access to all N_t^W varieties existing in the world, some produced in country A and others in country B , i.e. $N_t^W = N_t^A + N_t^B$. Hence, in the world economy, the composite goods C_t^j and D_{t+1}^j

and investment I_t^j in country $j \in \{A, B\}$ are defined as:

$$C_t^j = (N_t^W)^{1+\beta} \left[\frac{1}{N_t^W} \sum_{i=1}^{N_t^W} (c_{it}^j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad D_{t+1}^j = (N_{t+1}^W)^{1+\beta} \left[\frac{1}{N_{t+1}^W} \sum_{i=1}^{N_{t+1}^W} (d_{it+1}^j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{39}$$

$$I_t^j = (N_t^W)^{1+\beta} \left[\frac{1}{N_t^W} \sum_{i=1}^{N_t^W} (i_{it}^j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{40}$$

It follows that the demand for a variety i at the world level is now given by

$$v_{it}^W = \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} \sum_{j=A,B} (N_t^W)^{\beta(\varepsilon-1)-1} \left[L_t^j \left(C_t^j + I_t^j \right) + L_{t-1}^j D_t^j \right]. \tag{41}$$

Therefore, the solution of the problems faced by producers of each differentiated good, in country A and B, is similar to that under autarky. The price of the composite good, given in (5) is now defined over $i = 1, ..., N_t^W$, and is identical for all households. Since we have $p_{it}^B = p_{it}^A = p_t$ at a symmetric equilibrium in the world output market, we obtain $P_t^W = (N_t^W)^{-\beta} p_t$. Taking into account this same world price of the composite good in both countries, together with international capital mobility, real interest rates have to be identical across countries. Adapting (16) and (17), the following relation holds at equilibrium:

$$\left(a_t^A\right)^{s-\gamma-1} \left(K_t^A\right)^{\gamma} = \left(K_t^B\right)^{s-1} \left(H^B\right)^{1+\gamma-s}. \tag{42}$$

We denote by K_t^W the world capital stock $K_t^W = K_t^A + K_t^B$. Then, using (42) with $K_t^B = K_t^W - K_t^A$, it is possible to express a_t^A as a function of K_t^A and of K_t^W :

$$a_t^A = \frac{(K_t^W - K_t^A)^{\frac{1-s}{1+\gamma-s}} \left(K_t^A\right)^{\frac{\gamma}{1+\gamma-s}}}{H^B} \equiv a^A(K_t^A, K_t^W). \tag{43}$$

The free-entry condition in each country is given by (18) and the production at the firm level in both countries is still constant and given by (19), so that (20) and (31) still apply. Taking into account that $N_t^W = N_t^A + N_t^B$, the number of varieties in the world can be written as a function of K_t^A and of K_t^W:

$$N_t^W = \frac{a^A(K_t^A, K_t^W)^{s-1-\gamma}(K_t^A)^{1+\gamma} + \left(H^B\right)^{1+\gamma-s}(K_t^W - K_t^A)^s}{\varepsilon\phi} \equiv N^W(K_t^A, K_t^W). \tag{44}$$

As the aggregate price at the world level is defined over all varieties, using (16), (17) and (42) we have that the real interest rate is the same across countries:

$$\begin{aligned} \rho^W(K_t^A, K_t^W) &= \Theta N_t^W(K_t^A, K_t^W)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s a^A(K_t^A, K_t^W)^{s-1-\gamma}(K_t^A)^{\gamma} \\ &= \Theta N_t^W(K_t^A, K_t^W)^{\beta} \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s(K_t^W - K_t^A)^{s-1} \left(H^B\right)^{1+\gamma-s} \end{aligned} \tag{45}$$

However, real wages are different across countries:

$$\begin{aligned}\omega^{A,W}(K_t^A,K_t^W) &= \Theta N_t^W(K_t^A,K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)a^A(K_t^A,K_t^W)^{s-\gamma}(K_t^A)^{\gamma},\\ \omega^{B,W}(K_t^A,K_t^W) &= \Theta N_t^W(K_t^A,K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)(K_t^W-K_t^A)^s\left(H^B\right)^{\gamma-s}\end{aligned} \tag{46}$$

although they are linked by the following relationship:

$$\omega^{B,W}(K_t^A,K_t^W) = \tfrac{K_t^W-K_t^A}{a^A(K_t^A,K_t^W)H^B}\omega^{A,W}(K_t^A,K_t^W). \tag{47}$$

3.1 Equilibrium

The world equilibrium is given by two dynamic equations describing respectively the world capital accumulation that prevails under international capital mobility and the labor market equilibrium in country A.

Capital accumulation in the world is driven by the sum of savings in both countries, i.e.:

$$K_{t+1}^W = (1-\alpha)\left[\omega^{A,W}\left(K_t^A,K_t^W\right)\tfrac{K^A}{a_t^A\left(K_t^A,K_t^W\right)} + \omega^{B,W}\left(K_t^A,K_t^W\right)H^B\right] \tag{48}$$

and the labor market equilibrium in country A is given by

$$(1-\alpha)^{1-\alpha}\alpha^{\alpha}\omega^{A,W}(K_t^A,K_t^W)\rho^W(K_{t+1}^A,K_{t+1}^W)^{1-\alpha} = \tfrac{\nu}{\lambda} \tag{49}$$

with $a_t^A\left(K_t^A,K_t^W\right)$, $\rho^W(K_t^A,K_t^W)$, $\omega^{A,W}\left(K_t^A,K_t^W\right)$ and $\omega^{B,W}\left(K_t^A,K_t^W\right)$ given in (43), (45) and (46).

Definition 3. *An intertemporal equilibrium with perfect foresight of the world economy is a sequence* $\left\{K_t^A,K_t^W\right\}_{t=0}^{\infty}$ *which satisfies (48)-(49), given the initial world capital stock* $K_{t=0}^W>0$.

Equations (48)-(49) define a two-dimensional dynamic system with one predetermined variable, the world aggregate capital which is given by past savings. However, capital used in production in each country is a non predetermined variable as capital moves freely across countries.

3.2 Trade Balance and Capital Flows

The trade balance of country $j\in\{A,B\}$, $\mathcal{TB}_t^j$, is defined as the country's excess supply of goods. From the aggregate production given in (11) we get that:

$$\mathcal{TB}_t^j = P_tY_t^j - P_t\left(L_t^jC_t^j + L_t^j\tilde{K}_{t+1}^j + L_{t-1}^jD_t^j\right).$$

When the trade balance of country $j\in\{A,B\}$, $\mathcal{TB}_t^j$, is positive in equilibrium, the country $j\in\{A,B\}$ is a net exporter of goods as output is higher than domestic demand. Of course, in equilibrium, the sum of the two countries' trade balances must be zero, i.e. $\mathcal{TB}_t^A+\mathcal{TB}_t^B=0$.

Using the budget constraint of households we can rewrite the balance of trade as:

$$\mathcal{TB}_t^j = P_t \left[Y_t^j - L_t^j \omega_t^{j,W} - \rho_t^W L_{t-1}^j \tilde{K}_t^j \right].$$

Recall that firms are subject to a free-entry condition so that $P_t Y_t^j = P_t(\rho_t^W K_t^j + \omega_t^{j,W} L_t^j)$. It follows that:

$$\mathcal{TB}_t^j = P_t \rho_t^W (K_t^j - L_{t-1}^j \tilde{K}_t^j).$$

Denoting by $\eta_t^j = K_t^j - L_{t-1}^j \tilde{K}_t^j$ the inflows of capital services to country $j \in \{A, B\}$ in period t, we obtain

$$\mathcal{TB}_t^j = P_t \rho_t^W \eta_t^j. \tag{50}$$

4 Steady state of the world economy

A steady state of the world economy is a sequence $(K_t^A, K_t^W) = (K^A, K^W)$ for all t satisfying

$$K^W = (1-\alpha)\left[\omega^{A,W}\left(K^A, K^W\right) \tfrac{K^A}{a^A(K^A, K^W)} + \omega^{B,W}\left(K^A, K^W\right) H^B\right] \tag{51}$$

$$(1-\alpha)^{1-\alpha}\alpha^\alpha \omega^{A,W}(K^A, K^W)\rho^W(K^A, K^W)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{52}$$

The following Proposition establishes existence and uniqueness of the steady state with open borders.

Proposition 6. *Consider that Assumption 1 is satisfied. Then, there exists a unique stationary solution (K^A, K^W) of the dynamic system of the world economy (48)-(49) given by*

$$K^W - K^A = \left[\frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^\beta}(H^B)^{(1+\beta)(1+\gamma-s)}\right]^{\frac{1}{(1+\beta)(1-s)}} (K^W)^{\frac{\beta}{(1+\beta)(1-s)}} \tag{53}$$

where K^A is the unique solution of

$$\frac{\varepsilon^{1+\beta}\phi^\beta \left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)} = \left[\left(K^A\right)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + \left(H^B a^A\right)^{\frac{1+\gamma-s}{1-s}} \left(K^A\right)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^\beta \tag{54}$$

with

$$a^A = \left(\tfrac{\nu}{\lambda}\right) \tfrac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}}. \tag{55}$$

Moreover, the steady state world real interest rate is identical to the autarkic ones in both countries:

$$\rho^W = \rho^A = \rho^B = \frac{s}{(1-s)(1-\alpha)}. \tag{56}$$

Proof. See Appendix 7.4 □

As before opening the borders the steady state real interest rates were already identical in both countries, there will be no incentives for international capital movements once they are liberalized. Naturally, the absence of capital movements at the steady state implies that the world interest rate will not adjust, keeping a value identical to the one observed under autarky. It also means that, at the steady state, capital used in production equals savings in both countries, so that capital inflows are zero, $\eta^j = K^j - L^j\tilde{K}^j = 0$, $j \in \{A, B\}$. Using (50) we also have that at the stationary state the trade account is balanced. Accordingly we can state:

Proposition 7. *Consider that Assumption 1 is satisfied. Then, at the world steady state:*

$\lfloor i \rfloor$ *Capital used in production in each country equals savings, i.e. there are no net capital movements.*

$\lfloor ii \rfloor$ *Both countries have a balanced trade account.*

Proof. See Appendix 7.5 □

Since each variety is only produced in one country (in one firm) but consumed in both countries, trade exists, due to the existence of *taste for variety*, even at the steady state. However, according to Proposition 7 at the steady state the value of imports is identical to the value of exports so that the steady state is characterized by no net *intra-industry* trade. However, as discussed in the next section, non-steady state equilibria exhibit net trade. In the presence of indeterminacy, autonomous changes in self fulfilling expectations influence equilibria in country A and will also affect economy B, not only through capital mobility but also through the number of varieties A produces that are also consumed in B. Which country becomes a net importer or a net exporter depends on how expectations change, and countries may change from being net importers to net exporters (and vice-versa) due to changes in expectations.

It is also interesting to analyze the effects of the degree of *taste for variety*, β, on steady state equilibrium. Using (55) it is easy to see that a^A is not affected by β. In Appendix 7.6 we prove that steady state capital stock and employment in country A after opening the borders are smaller when β is higher. The same happens with the number of varieties produced in country A at the steady state with trade and capital mobility, which is a decreasing function of β.[21] As in the autarkic equilibrium this is due to the existence of wage rigidity. In contrast, in country B, after opening the borders, the steady state capital stock and varieties produced increase with β. Indeed from (42) we can see that as a^A is not affected by β and as K^A decreases, K^B must increase with β.

4.1 Steady state effects of opening the borders

In this section we analyze the steady state effects of opening the borders to *intra-industry* trade and to capital flows in both countries. Our main results are summarized in Proposition 8 below.

[21] Using (20), we can see that if capital decreases with β the same happens with N^A, as a^A does not vary with β.

Proposition 8. *Consider that Assumption 1 is satisfied and $\beta > 0$. Then, the following results hold:*

$\lfloor i \rfloor$ *In country A at the free-trade steady state with capital mobility, the capital stock, the number of varieties produced and employment are lower than their respective values at the autarkic steady-state. The steady state real wage rate and the real interest rate are identical at both steady-states. The world share of varieties produced in country A, the world share of the capital stock of country A and the world share of savings of country A are smaller than their respective values at the autarkic steady state;*

$\lfloor ii \rfloor$ *In country B at the free-trade steady state with capital mobility, the capital stock, the number of varieties produced, and the real wage are higher than their respective values at the autarkic steady-state. The real interest rate is identical at both steady-states. The world share of varieties produced in country B, the world share of the capital stock of country B and the world share of savings of country B are higher than their respective values at the autarkic steady state;*

$\lfloor iii \rfloor$ *The total number of varieties consumed in each country at the free-trade steady state with capital mobility is higher than the number of the varieties consumed in autarky in each country.*

Proof. See Appendix 7.7 and the paragraphs below. □

Aloi and Lloyd-Braga (2010), considering a similar framework, but without *taste for variety* ($\beta = 0$) and no trade, find that the autarkic steady state and the steady state with perfect capital movements coincide. It follows that all the steady state effects stated in Proposition 8 above are due to the presence of *intra-industry trade* with *taste for variety*, $\beta > 0$, and not associated with capital mobility. Indeed when $\beta = 0$, since the number of varieties no longer plays a role, all these effects vanish. The intuition is as follows. The steady state world real interest rate is identical to the steady state autarkic real interest rates in both countries, i.e. $\rho^W = \rho^A = \rho^B$. See Proposition 6. Then, since the labor market equilibrium condition in the rigid wage country is the same before and after opening the borders, comparing (25) with (52), it is easy to see that the steady state real wage in country A after opening the borders, $\omega^{A,W}$, is identical to the autarkic real wage in country A, ω^A. Comparing now (21) with (46), both evaluated at the steady state, as a^A is the same before and after opening the borders and the number of varieties consumed in country A with trade, N^W, is higher than the number of varieties consumed in A in autarky (see Appendix 7.7), we conclude that with *taste for variety*, $\beta > 0$, in country A at the free-trade steady state with capital mobility the capital stock is lower than the capital stock at the autarkic steady state. Moreover, since a^A is the same before and after opening the borders, L^A also decreases with trade. Finally, as according to (20) the number of varieties produced in country A at the steady state is an increasing function of the steady state capital stock of country A , we also conclude that the number of varieties produced in country A at the steady state is smaller after opening to trade.

In country B, as $\rho^W = \rho^B$ and the number of varieties consumed in country B with trade, N^W, is higher than the number of varieties consumed in B in autarky (see Appendix 7.7), comparing (32) with (45) both evaluated at the steady state, we conclude that in presence of *taste for variety* $\beta > 0$, the steady state capital stock in B increases with free trade. Finally, since both the number of varieties and the capital stock increase after opening the borders, using (46) we conclude that the steady state real wage also increases in country B.

Although our model is quite simple, it is able to capture some fears commonly associated with globalization/free-trade agreements, that are based on the belief that opening the borders will displace industries and jobs abroad, increasing unemployment. Indeed, in our framework this happens in the rigid wage country. The reverse implication of this mechanism, is that a country without significant labor market rigidities, will suffer drastic losses by reverting to an autarkic regime if most of the trade is *intra-industry*. Egger et al. (2011), in a static model, analyze how differences in labor market imperfections influence the share of *intra-industry* trade. They show that free trade and capital mobility lead to a higher number of varieties produced abroad when labor market rigidities increase in the home country, a result consistent with ours, according to which globalization leads to an increase in the number of varieties produced in the flexible full employment economy.

It is also interesting to analyze the effects of trade on capital intensity at the firm level in both countries. In country B, as aggregate employment is constant and the number of firms increases, employment at the firm level decreases. However, as production per firm is constant, (see (19)) capital per firm increases. Therefore, firms in country B become more capital intensive. Indeed, since in this country, ω/ρ, the ratio between real rages and the real interest rate increases, firms will substitute labor for capital. In contrast in country A, as ω/ρ does not change, capital intensity at the steady state remains unchanged after opening the borders.

4.1.1 Stationary Welfare

We now compare steady state welfare in the two countries before and after the opening of the borders. In country A, as the real interest rate and the real wage are identical before and after trade, the utility of a worker that keeps its job when there is trade is the same in the two steady states. However, as employment is smaller in the steady state with free trade and perfect capital mobility, and the utility of an unemployed worker is zero, it follows that, under an utilitarian social welfare function, aggregate utility decreases. In contrast, in country B, there is full employment before and after opening the borders. As the real wage is higher in the world steady state and the real interest rate does not change, we conclude that individual and aggregate utility increase with trade. The following Proposition summarizes these results.

Proposition 9. *Consider that Assumption 1 is satisfied. Then, the following results hold at the steady state:*

$\lfloor i \rfloor$ *In country A, the utility level of a worker that keeps its job when there is trade is the same* *as in autarky. However, those workers that lose their jobs are worse off with free trade.*

$\lfloor ii \rfloor$ *All agents in country B gain from trade.*

The full employment country is the one that unambiguously benefits in terms of steady state welfare from free *intra-industry* trade. In contrast, in country A we observe, due to the existence of labor market distortions, an aggregate reduction in steady state welfare after opening to trade. In economies with distortions such as ours, benefits from trade are not guaranteed for all countries (see Helpman and Krugman, 1985) so that this result should not surprise us. However, Helpman and Itskhoki (2010), considering a static model with no capital, but with both intra and inter industry trade between two countries that also differ in the degree of labor market rigidities, find that both countries gain from trade in welfare terms. Nevertheless, like us, they also find an asymmetric impact of trade: the country with lower frictions in the labor market gains proportionately more.

As it is well known *intra-industry* trade influences welfare through two channels: the scale effect and the variety effect. The first one emerges because trade, increasing market size, allows firm to produce more, benefitting from scale economies. Moreover, with trade, each country gains access to a larger number of varieties which increases utility in the presence of *taste for variety*. In our framework, the scale of production at the firm level is constant (see (19)), so that the scale effect is absent. Hence, we are able to ensure that all the effects of *intra-industry* trade on welfare operate via *taste for variety*.

Previous works that have analyzed the effects of opening the borders on welfare in an two-country overlapping generations framework did not consider *intra-industry* trade nor the existence of labor market imperfections. An exception is Aloi and Lloyd-Braga (2010) that, like us, introduce labor market rigidities in an overlapping generations structure with capital mobility, but no trade. When labor mobility is also admitted, they find that unemployment decreases and world output expands, if workers migrate to the country with the competitive labor market. In the opposite case there is an increase in unemployment and a contraction in world output, the direction of the labor flows being determined by the relative size of the countries. On the contrary, when only international capital mobility is allowed, opening the borders does not affect stationary welfare as referred above. In our paper, without labor mobility but with *intra-industry* trade, the welfare and the output of the country with the competitive labor market always increase, regardless of the relative size of each country.

5 Dynamics in the Two-Country model

We start by providing a full characterization of the local stability properties around the unique steady state equilibrium. We first loglinearize system (48)-(49). Denoting percentage deviations from the steady state respectively by $\widehat{K}_t^W \equiv \left(K_t^W - K^W\right)/K^W$ and $\widehat{K}_t^A \equiv \left(K_t^A - K^A\right)/K^A$ we have that

$$\begin{pmatrix} \widehat{K}_{t+1}^W \\ \widehat{K}_{t+1}^A \end{pmatrix} = J^W \begin{pmatrix} \widehat{K}_t^W \\ \widehat{K}_t^A \end{pmatrix} \tag{57}$$

where J^W, given in Appendix 7.8, is the Jacobian matrix of the dynamic system. The following Proposition gives the characteristic polynomial.

Proposition 10. *The trace, T^W, and determinant, D^W, of matrix J^W, given below, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P^W(\lambda^W) \equiv (\lambda^W)^2 - \lambda^W T^W + D^W$:*

$$T^W = 1 - \frac{\gamma - n^A(1+\gamma-s)[(1+\beta)s-\beta]}{(1-\alpha)n^A(1+\beta)(1+\gamma-s)(1-s)}, D^W = -\frac{s[\gamma - n^A(1+\gamma-s)]}{(1-\alpha)n^A(1+\gamma-s)(1-s)} \tag{58}$$

where $n^A \equiv N^A/N^W$ denotes the share of varieties produced in country A at the steady state.[22]

Proof. See Appendix 7.8. □

As in autarky, we refer to Grandmont et al. (1998) in order to appraise the local stability properties of the dynamic system defined by (48)-(49). Note that, in contrast to what happened in autarky, the dynamic system with trade and capital mobility is not loglinear. Therefore, bifurcations are now possible. We can use Figure 1 to study local bifurcations. When a (bifurcation) parameter is made to vary continuously in its admissible range, if the values of T^W and D^W cross the interior of the segment BC, a pair of complex conjugate eigenvalues crosses the unit circle and a Hopf bifurcation generically occurs. In this case there are deterministic cycles describing orbits that lie over an invariant closed curve, surrounding the steady state, in the state space. If the Hopf bifurcation is subcritical, this curve emerges when the steady state is a sink. When the Hopf bifurcation is supercritical the invariant closed curve appears when the steady state is determinate, a source, and although sunspot equilibria that stay arbitrarily close to the steady state do not exist, there are nevertheless infinitely many equilibria exhibiting bounded stochastic fluctuations around the invariant closed curve. Moreover, when T^W and D^W cross the AB line, a *flip* bifurcation (supercritical or subcritical) generically occurs, leading to the appearance of deterministic cycles of period two. Moreover, a cascade of period doubling cycles is expected to occur as the bifurcation parameter moves further away from its bifurcation value, eventually leading to the appearance of bounded aperiodic equilibrium trajectories.

In Proposition 11 we present our results, considering γ as our bifurcation parameter. As usually done in the literature, we consider the normalized steady state in country A with $a^A = 1 = K^A$, by fixing the scaling parameters λ and Θ at the appropriate level. Then we use the scaling parameter H^B to ensure that n^A does not vary with the other parameters that influence directly the trace and the determinant given in (58).[23]

[22] As in autarky β does not influence the determinant.

[23] Using (55) and $a^A = 1$, we get $\lambda = \nu \frac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}} \in (0,1)$ provided $\nu < \frac{\alpha^\alpha s^{1-\alpha}}{(1-\alpha)(1-s)^{1-\alpha}}$. Using (54) together with $K^A = 1$ and $a^A = 1$ we obtain $\Theta = \left\{\varepsilon^{1+\beta}\phi^\beta\right\} / \left\{(\varepsilon - 1)(1-\alpha)(1-s)\left[1 + \left(H^B\right)^{(1+\gamma-s)/(1-s)}\right]^\beta\right\}$. Then, using (20), (42) and (44), the value of the parameter $n^A \equiv N^A/N^W$ can be exogenously controled by fixing H^B as $H^B = \left[(1-n^A)/n^A\right]^{\frac{1-s}{1+\gamma-s}}$ at the normalized steady state.

Proposition 11. *Consider that Assumption 1 is satisfied and define* $\gamma_1^W \equiv \frac{n^A(1-s)[\alpha(1-s)-(1-2s)]}{s-n^A[\alpha(1-s)-(1-2s)]}$, $\gamma_2^W \equiv \frac{n^A(1-s)[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}{1+s(1+\beta)-n^A[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}$ *and* $\tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$. *Then, the following generically holds at the world level:*[24]

$\lfloor i \rfloor$ *When* $\underline{\alpha} < \alpha < \tilde{\alpha}$, *for* $\gamma < \gamma_1^W$ *the steady state is a source, undergoes a Hopf bifurcation when* γ *crosses the critical threshold* γ_1^W, *becomes a sink (locally indeterminate) for* $\gamma_1^W < \gamma < \gamma_2^W$, *undergoes a flip bifurcation when* γ *crosses the critical threshold* γ_2^W, *becoming a saddle for* $\gamma > \gamma_2^W$.

$\lfloor ii \rfloor$ *When* $\underline{\tilde{\alpha}} < \alpha < 1$, *for* $\gamma < \gamma_2^W$ *the steady state is a source, undergoes a flip bifurcation when* γ *crosses the critical threshold* γ_2^W, *becoming a saddle for* $\gamma > \gamma_2^W$.

Proof. See Appendix 7.9. □

This Proposition shows that, in the presence of *intra-industry* trade and free international capital flows, the world economy, i.e. not only country A, but also country B, can exhibit local fluctuations driven by changes in expectations.[25] This will occur through bifurcations, that were not possible in autarky, and/or when the world equilibrium is locally indeterminate (a sink). Indeterminacy, as in autarky, requires intermediate values of the propensity to consume of a young agent, $\underline{\alpha} < \alpha < \tilde{\alpha}$ and a lower and an upper bound for the labor externality, γ_1^W and γ_2^W respectively. Although the bounds on the propensity to consume are the same as the ones in autarky, the bounds on the labor externality are different, depending on the value of n^A. It follows that the effects of opening the economies on local stability, can be studied by comparing the critical values for γ, γ_1^W and γ_2^W, with the relevant critical values in autarky, γ_1^{aut} and γ_2^{aut}. Concentrating in the case $\underline{\alpha} < \alpha < \tilde{\alpha}$, under Assumption 1, $\gamma_1^W > 0$ is an increasing function of n^A, becoming identical to γ_1^{aut} for $n^A = 1$. Therefore we have $\gamma_1^W < \gamma_1^{aut}$. Similarly $\gamma_2^W > 0$ is an increasing function of n^A, becoming identical to γ_2^{aut} for $n^A = 1$. Therefore we have $\gamma_2^W < \gamma_2^{aut}$. However, γ_2^W can be higher or lower than γ_1^{aut}, depending on the value of n^A. Accordingly we have the following Lemma.

Lemma 1. *Assume that* $\underline{\alpha} < \alpha < \tilde{\alpha}$ *and consider Assumption 1 satisfied. Then, defining* $n_*^A \in (0,1)$

$$n_*^A \equiv \frac{[1+s(1+\beta)][\alpha(1-s)-(1-2s)]}{s\,[2(1+\beta)[(1-\alpha)(1-s)+s]-\beta]}$$

we have:

$\lfloor i \rfloor$ *For* $n^A < n_*^A$, $0 < \gamma_1^W < \gamma_2^W < \gamma_1^{aut} < \gamma_2^{aut}$;

$\lfloor ii \rfloor$ *For* $n^A > n_*^A$, $0 < \gamma_1^W < \gamma_1^{aut} < \gamma_2^W < \gamma_2^{aut}$.

[24] As in autarky, when $\beta = 0$ our critical values for α and γ are identical to the ones of Aloi and Lloyd-Braga (2010).

[25] Indeed, as discussed below, changes in expectations in country A will trigger changes in the number of varieties produced and in the capital used by firms in A, which through trade and capital mobility will also influence economic activity in country B.

5.1 Effects of opening the economies on stability

From Proposition 11 and Lemma 1, it follows that the relative size of the two countries will influence the results. To facilitate the analysis we present in Figure 2, in the space (n^A, γ), the critical values of γ delimiting the regions where the steady state is locally a source, sink and saddle, both under autarky and after opening the borders, considering $\underline{\alpha} < \alpha < \tilde{\alpha}$. To further ease the discussion we provide a numerical illustration. In accordance with Assumption 1, we consider $s = 1/3$ a sufficiently small value for $\beta = 0.01$, and $\alpha = 0.6 \in (\underline{\alpha} = 0.5, \tilde{\alpha} = 0.833)$, so that $\gamma_1^{aut} = 0.167$, $\gamma_2^{aut} = 5.95$ and $n_*^A = 0.222$. In order to concentrate the discussion on empirically plausible values for the parameters, in Figure 2 we will only consider values for γ below $s = 1/3$,[26] i.e. γ_2^{aut} will not be depicted. Moreover, we denote by $n_{**}^A \in (n_*^A, 1)$ the value of n^A such that $\gamma_2^W = s$. With our parametrization we have $n_{**}^A = 0.371$.

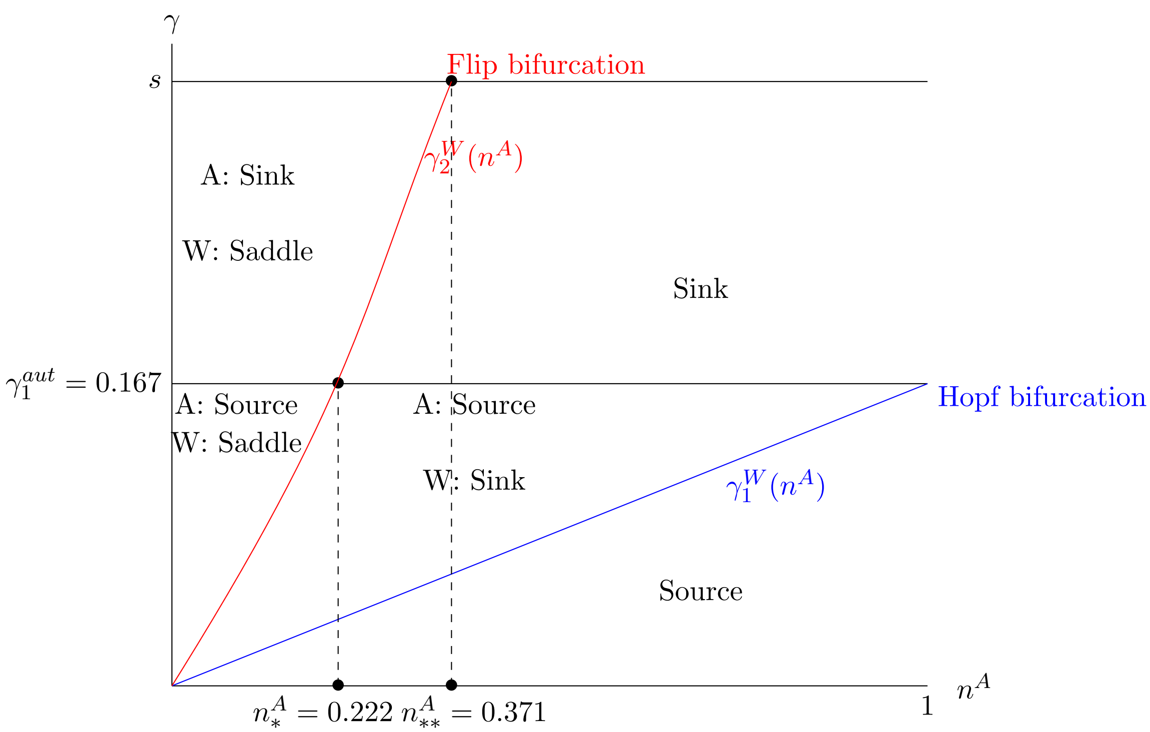

Figure 2: Local dynamics for $\alpha \in (\underline{\alpha}, \tilde{\alpha})$, $\beta = 0.01$, $n^A = 0.1$, $\alpha = 0.6$ and $s = 1/3$.

The first result we highlight is that, after opening the borders, country A may become saddle determinate for empirically plausible values of the parameters. In terms of Figure 2, this will occur in the region to the left of the red line representing γ_2^W. Since γ_2^W decreases as n_A decreases, we conclude that the bigger the size of country B, the more likely is local saddle determinacy

[26]Most of empirical estimates for the degree of increasing reurns to scale point to small values only slightly higher than zero, and values higher that 1/3 are usually considered highly implausible. See Basu and Fernald (1997) and Burnside (1996).

in a globalized world. In this case the local stability properties of country B are exported to country A. In contrast, in the region to the right of the γ_2^W schedule, country B may become locally indeterminate, or even a source. In this case the local stability properties of country A, are exported to country B.

Let us now consider values of $\gamma > \gamma_1^{aut} = 0.167$ so that in autarky the steady state was locally indeterminate in country A. We find that, after opening the borders, the set of values of γ under which indeterminacy emerges shrinks as the steady state in both countries becomes a saddle for $\gamma > \gamma_2^W$. Moreover if n^A is sufficiently small, $n^A < n_*^A = 0.222$, local indeterminacy and therefore sunspots fluctuations are totally eliminated in the world.[27] However, if n^A is sufficiently big, namely $n^A > n_{**}^A = 0.371$, local indeterminacy prevails in the world. In this case there exists a transmission of local indeterminacy from country A to country B, so that expectation driven fluctuations, with origin in country A will be exported to country B, which was stable in autarky.[28]

We consider now lower and more plausible values of $\gamma < \gamma_1^{aut} = 0.167$. After opening the borders, indeterminacy which was not possible in autarky can now emerge. Indeed, the lower bound on γ required for indeterminacy is lower in a globalized world, i.e. $\gamma_1^W < \gamma_1^{aut}$. Therefore, with free-trade and capital movements it is possible to obtain fluctuations driven by self-fulfilling volatile expectations with small values of labor externalities consistent with empirical evidence.[29] To obtain γ_1^W and γ_2^W we will consider two values for n^A, $n^A = 0.1$ and $n^A = 0.3$, respectively below and above n_*^A. For $n^A = 0.1$, we obtain $\gamma_1^W = 0.0136$ and $\gamma_2^W = 0.0658$, while for $n^A = 0.3$ we obtain $\gamma_1^W = 0.043$, and $\gamma_2^W = 0.246$. We confirm therefore that, after opening the borders to *intra-industry* trade and capital mobility, indeterminacy becomes possible for lower values of γ, in accordance with empirical evidence.

Another important result is that, after opening the borders, bifurcations, which in our framework did not occur in autarky, become possible. When γ crosses the critical value γ_1^W a Hopf bifurcation occurs, whatever the relative size of the two countries provided $\underline{\alpha} < \alpha < \tilde{\alpha}$. In all our simulations the Hopf bifurcation is supercritical, so that the invariant closed curve appears when the steady state is a source. With $n^A = 0.1$, the Hopf bifurcation occurs when $\gamma = \gamma_1^W = 0.0136$, and for $\gamma = 0.0132$ we obtain an invariant closed curve surrounding the steady state, which we depict in Figure 3.[30] This means that non-explosive deterministic and stochastic[31] fluctuations become possible in the world economy for small and plausible values of γ. To our knowledge, ours is the first paper highlighting that, by opening the economy, fluctuations due to a Hopf bifurcation emerge.

[27] The same result has been found in Sim and Ho (2007) who consider inter-industry trade and different technologies across countries.

[28] The same result was obtained by Nishimura et al. (2010) who, using a two-country, two-good, two-factor general equilibrium model with sector specific externalities, found that some country's expectation-driven fluctuations can spread throughout the world once inter-industry trade opens, even if the other country has determinacy under autarky.

[29] This result was also emphasized in Aloi and Lloyd-Braga (2010) with perfect capital mobility but no trade.

[30] For $n^A = 0.3$ the supercritical Hopf occurs for $\gamma = \gamma_1^W = 0.043$ and the invariant closed curve appears for $\gamma = 0.0419$.

[31] Around the invariant closed curve, there exist infinitely many equilibria exhibiting bounded stochastic fluctuations. See Grandmont et al. (1998).

Moreover, for $n^A < n^A_{**}$, when γ crosses the critical value γ_2^W, a *flip* bifurcation occurs.[32] In our simulations the *flip* bifurcation is supercritical. With $n^A = 0.1$, the *flip* bifurcation occurs when $\gamma = \gamma_2^W = 0.0658$.[33] In Figure 4 we depict the corresponding bifurcation diagram for values of γ sufficiently close but above γ_2^W, i.e. in the saddle region. We can observe the cascade of doubling periodic cycles, leading eventually to chaos.[34]

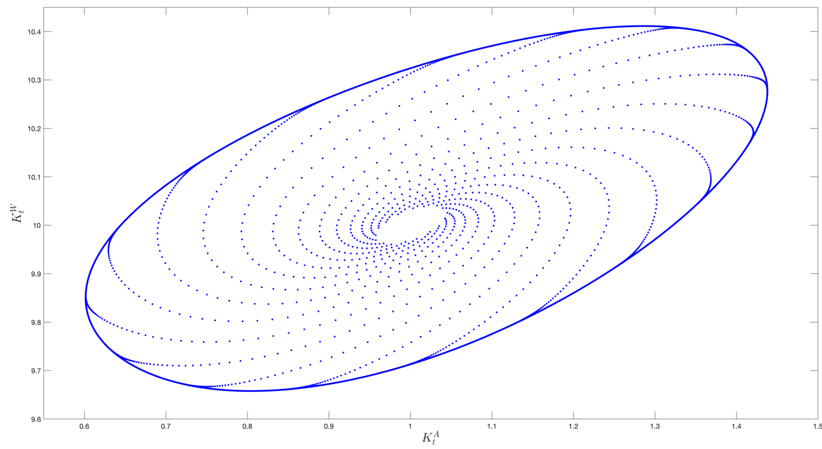

Figure 3: Invariant closed curve surrounding the steady state for $\gamma = 0.0132$

It is also important to discuss the role of *taste for variety*, β, on the impact of globalization on stability, and to compare our results with those obtained in Aloi and Lloyd-Braga (2010), with international capital mobility but no trade ($\beta = 0$). While γ_1^W does not directly depend on β, γ_2^W is a decreasing function of it, so that in Figure 2 we would observe a rightward shift of the γ_2^W schedule when β increases. We can also see that the critical value n^A_{**} increases with β. All this implies, other things being equal, that a higher *taste for variety*, and therefore *intra-industry* trade, shrinks the sink region, reducing the likelihood of local sunspots fluctuations, and enlarges the region where we obtain saddle path stability, exerting therefore a (local) stabilizing effect. However, Hopf and *flip* bifurcations, and their associated deterministic and stochastic cycles, remain possible. As γ_2^W is smaller when β is high, we conclude that obtaining a *flip* bifurcation is more likely in the presence of *intra-industry* trade and that its likelihood increases with *taste for variety.*

Finally, we address the case where $\underset{\sim}{\tilde{\alpha}} < \alpha < 1$, so that Proposition 11 $\lfloor ii \rfloor$ applies. We can see that after opening the borders, country A may become saddle determinate for plausible and sufficiently small values of γ. However, *flip* bifurcations occur. In our simulations these bifurcations were subcritical, so that endogenous fluctuations appear when the steady state is a saddle. Therefore we can not simply conclude that opening the borders exerts a stabilizing influence.

[32] Nishimura et al. (2014), with inter-industry trade and capital movements, also obtain a *flip* bifurcation, in a two-factor, two-sector, two-country model with decreasing returns to scale technologies. However, they do not have local indeterminacy.

[33] With $n^A = 0.3$, the *flip* bifurcation occurs when $\gamma = \gamma_2^W = 0.246$.

[34] Stochastic bounded fluctuations around the periodic cycles also appear. See Grandmont et al. (1998).

Summarizing, although indeterminacy that existed under autarky in country A can be eliminated after opening the borders, the steady state becoming saddle stable, this only happens if country B is sufficiently large. On the other hand, for lower and plausible values of γ, local indeterminacy and sunspots arbitrarily near the steady state can now emerge. Also, bounded deterministic and stochastic fluctuations associated with a supercritical Hopf bifurcation are now possible. Furthermore, for higher values of γ, but still within the plausible range, deterministic/chaotic and stochastic fluctuations when the steady state is a saddle, due to a supercritical *flip* bifurcation also occur. We also notice that, after opening the borders, local indeterminacy and *flip*/Hopf bifurcations also appear in country B, triggering endogenous fluctuations that could not exist under autarky. Moreover, *intra-industry* trade enlarges the parameters' region (in terms of relative size of country A and the degree of labor externalities) with saddle stability, but it also reduces the value of γ for which *flip* bifurcations occur rendering them more plausible.

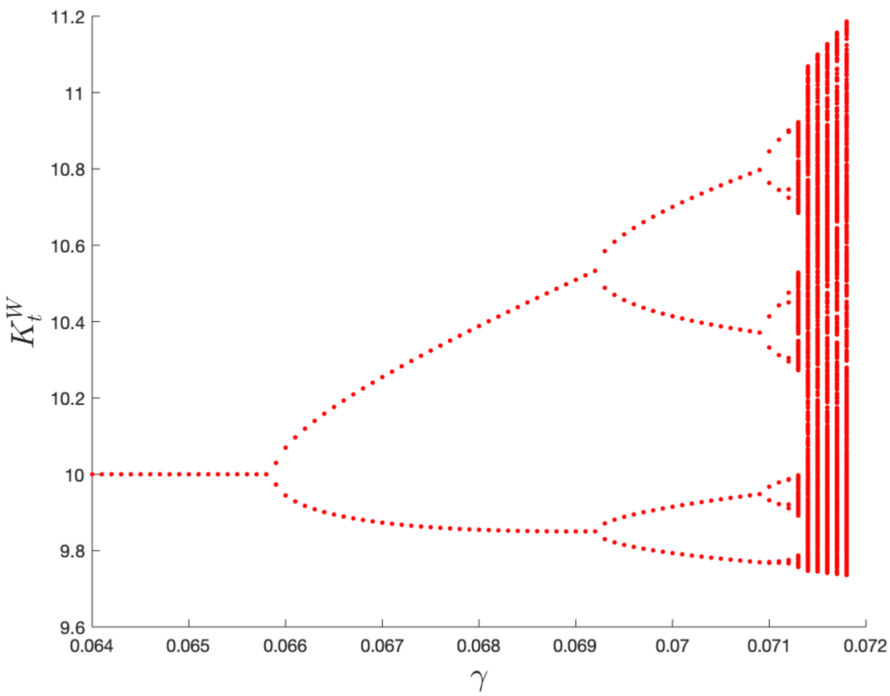

Figure 4: Bifurcation diagram: Supercritical *flip* bifurcation at $\gamma = 0.0658$.

It is also worth emphasizing that, in our framework, net trade may emerge along trajectories that exhibit endogenous fluctuations, driven by changes in expectations. At the steady state, in each country, households save in capital what is needed to use in production. Hence, as we have seen, there are no capital movements across countries at the steady state, and net trade is zero as well. However, changes in expectations in the rigid wage country lead to fluctuations in activity, inducing net trade and changes in the remuneration of factors in both countries. In the following we provide an example of an expectation shock. Departing from the steady state, suppose that, suddenly, expectations of the future real interest rate increase. In country A, the reservation wage decreases (see (9)) and so does the real wage. For a given level of capital stock, K_t^A, and number of varieties produced, N_t^A, and considering that at a symmetric equilibrium the labor demand is downward sloping (which occurs with β and γ small), employment at the firm level in country A, l_t^A, increases. Hence, ceteris paribus, average costs decrease and profits increase at the firm level. This induces the entry of new firms and therefore the production of more varieties in country A,[35] a part of which is exported. Country A becomes therefore a net exporter of goods. At the same time, the observed increase in l_t^A leads, if everything else is equal, to an increase in the marginal productivity of capital in country A, and consequently in r_t^A, the current interest rate in country A. This in turn triggers inflows of capital from country B until interest rates are equalized in the two countries, which leads to fluctuations in wages and activity in country B as well.

6 Concluding remarks

In this paper we consider a two-country, two-factor, overlapping generations model with *taste for variety*, imperfect competition and increasing returns to scale. We also assume that the two countries have different labor market characteristics: in one country, A, there are efficiency wages and unemployment, while in the other country, B, there exists full employment. We first show that in autarky country B is locally stable, while in country A local indeterminacy, and therefore belief driven fluctuations, may emerge, provided the propensity to consume and the degree of increasing returns to scale take intermediate values, although bifurcations are not possible. When trade and capital movements are liberalized, the effects on stability depend on the relative size of the countries and on the existing degree of increasing returns to scale. Considering a parameterization under which indeterminacy existed in country A in autarky, we show that, if country A is sufficiently big, it will export local fluctuations to the full employment country B, globalization inducing local macroeconomic instability in the world. In contrast, provided country B is big enough, local indeterminacy that existed in autarky in country A is eliminated, globalization having in this case a (local) stabilizing effect in the world economy. However, whatever the relative size of the two countries, indeterminacy, and therefore local sunspots fluctuations at the world level, require a degree of externalities smaller than the one needed in autarky. Also bounded deterministic and stochastic fluctuations associated with Hopf and *flip* bifurcations, which did not exist in autarky, become possible in the world economy for sufficiently small values of increasing returns consistent with empirical estimates. Finally, we show that *taste for variety* shrinks the set of parameters under which local indeterminacy occurs, but renders *flip* bifurcations, and therefore two-period cycles, more likely after opening the borders. In terms of steady state welfare, we prove that the full employment country unambiguously gains from opening its borders, while unemployment increases in the country with labor market rigidities, reducing country welfare. Furthermore, we show that *intra-industry* trade alone is responsible for these welfare gains and losses.

Very few papers in the literature have simultaneously addressed the effects of trade on welfare and on stability properties. Two examples are Nishimura et al. (2010) and Le Riche (2020).

[35] Since output per firm is constant, this implies that aggregate output in country A unambiguously increases.

However, they consider inter-industry trade. Moreover, the models used and the mechanisms emphasized are different from ours. They consider perfectly competitive labor markets in both countries and assume that countries have different technologies. Both papers find that opening to inter-industry trade with capital mobility increases the likelihood of local indeterminacy, that one country will gain in terms of stationary welfare while the other country always looses, although Nishimura et al. (2010) also shows that at the world level steady state welfare increases. Considering instead *intra-industry* trade and introducing labor market imperfections in one local market, our findings, while mostly supporting these previous insights, highlight the role of the relative size of the countries and of the degree of increasing returns to scale on shaping the effects of globalization on stability. This suggests that the influence of trade on stability may depend on the type of trade and on its sources. Therefore, a fruitful extension of the model could be to understand how the interaction between comparative advantage (inter-industry trade), increasing returns to scale and *taste for variety* (*intra-industry* trade) affect the stability and the welfare of the trading economies.

7 Appendix

7.1 Proof of Proposition 2

From (20) and (21) we obtain:

$$\tfrac{d\omega}{dK^A} = \tfrac{\beta(1+\gamma)+\gamma}{K^A}\omega,\ \tfrac{d\omega}{da^A} = \tfrac{s-\gamma-\beta(1+\gamma-s)}{a^A}\omega \tag{59}$$

$$\tfrac{d\rho}{dK^A} = \tfrac{\beta(1+\gamma)+\gamma}{K^A}\rho,\ \tfrac{d\rho}{da^A} = -\tfrac{(1+\beta)(1+\gamma-s)}{a^A}\rho. \tag{60}$$

Substituting equations (20) and (21) into the dynamic system (22)-(23), linearizing it and using (59) and (60) we obtain

$$\widehat{K}^A_{t+1} = \underbrace{(1+\beta)(1+\gamma)}_{z_1}\widehat{K}^A_t \ \underbrace{-(1+\beta)(1+\gamma-s)}_{z_2}\widehat{a}^A_t$$

and

$$\underbrace{(1-\alpha)\left[\beta(1+\gamma)+\gamma\right]}_{x_1}\widehat{K}^A_{t+1}\underbrace{-(1-\alpha)(1+\beta)(1+\gamma-s)}_{x_2}\widehat{a}^A_{t+1} = \underbrace{-\left[\beta(1+\gamma)+\gamma\right]}_{z_3}\widehat{K}^A_t + \underbrace{\left[\beta(1+\gamma-s)-(s-\gamma)\right]}_{z_4}\widehat{a}^A_t$$

where $\widehat{K}^A_t$ and $\widehat{a}^A_t$ denote percentage deviations of K^A and a^A from the steady state. We now rewrite the linear system above in matrix form

$$\underbrace{\begin{bmatrix} 1 & 0 \\ x_1 & x_2 \end{bmatrix}}_{J_1} \begin{bmatrix} \widehat{K}^A_{t+1} \\ \widehat{a}^A_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix}}_{J_0} \begin{bmatrix} \widehat{K}^A_t \\ \widehat{a}^A_t \end{bmatrix}.$$

The Jacobian matrix, J, is then

$$J = J_1^{-1} \cdot J_0 = \begin{bmatrix} 1 & 0 \\ -\frac{x_1}{x_2} & \frac{1}{x_2} \end{bmatrix} \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ \frac{z_3 - z_1 x_1}{x_2} & \frac{z_4 - z_2 x_1}{x_2} \end{bmatrix}.$$

The trace, T, and determinant, D, of matrix J, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P(\lambda) \equiv \lambda^2 - \lambda T + D$.

Results follow.

7.2 Proof of Proposition 3

Local indeterminacy emerges when the steady state is a sink, i.e. when $D < 1$, $1 + T + D > 0$ and $1 - T + D > 0$. Local determinacy will arise for any other configuration. In particular, as $D > 0$, the steady state is a saddle when $1 - T + D < 0$ or $1 + T + D < 0$. In any other configuration the steady state will be a source.

From (30), we get that the determinant is lower than one if and only if $\gamma > [\alpha(1 - s) - (1 - 2s)]/(1 - \alpha) \equiv \gamma_1^{aut}$. Such a threshold is positive under Assumption 1. Furthermore, from (30), we can compute $1 - T + D$ and $1 + T + D$:

$$\begin{aligned} 1 - T + D &= \tfrac{\gamma + \beta(1+\gamma)}{(1-\alpha)(1+\gamma-s)(1+\beta)} > 0, \\ 1 + T + D &= \tfrac{1 + (1+\beta)[1+\gamma-2\alpha(1+\gamma-s)]}{(1-\alpha)(1+\gamma-s)(1+\beta)} \gtrless 0. \end{aligned} \tag{61}$$

We have $1 + T + D > 0$ when $\gamma < \gamma_2^{aut} \equiv \frac{2(1-\alpha+\alpha s) - \beta[2\alpha(1-s)-1]}{(1+\beta)(2\alpha-1)} > 0$ under Assumption 1.[36] As $1 - T + D > 0$, when $\gamma < \min\ \{\gamma_1^{aut}, \gamma_2^{aut}\}$ we get $D > 1$ and $1 + T + D > 0$, and thus the steady state is a source. When $\gamma_1^{aut} < \gamma < \gamma_2^{aut}$, we obtain $D < 1$ and $1 + T + D > 0$. It follows that in this configuration the steady state is a sink. When $\gamma > \gamma_2^{aut}$, we get $1 + T + D < 0$ so that the steady state is a saddle. Noting that $\gamma_1^{aut} < \gamma_2^{aut}$ requires $\alpha < \tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$, results follow.

7.3 Proof that $n^A = \chi^A = S^A$

Denote by $n^A \equiv N^A/N^W$ the percentage of varieties produced in country A at the steady state, by $S^A \equiv (1 - \alpha)\omega^{A,W} L^A/K^W$ the steady state share of country A savings in world savings, and by $\chi^A \equiv K^A/K^W$ the share of capital used in country A in total capital used in the world at the

[36] Indeed the numerator of γ_2^{aut} is positive under Assumption 1. Either $2\alpha(1 - s) < 1$, so that the numerator is positive or since Assumption 1 implies $\beta < 1$, we have $2\,[1 - \alpha(1 - s)]\,/\,[2\alpha(1 - s) - 1] > 1$.

steady state. We will start by showing that $n^A = \chi^A$. First, substituting (42) in (44) evaluated at the steady state we obtain

$$N^W = \underbrace{\frac{(a^A)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\varepsilon\phi}}_{N^A}\left(1+\frac{K^B}{K^A}\right). \tag{62}$$

As $\chi^A = \frac{K^A}{K^W}$ we can rewrite (62) as $N^W = N^A\left(1+\frac{1-\chi^A}{\chi^A}\right)$, which, as $n^A = \frac{N^A}{N^W}$, gives $n^A = \chi^A$.

We now show that $n^A = S^A$. Using (20), (21), (31), and (32) at the steady state we have that :

$$\frac{N^A}{N^B} = \frac{\left(a^A\right)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\left(H^B\right)^{1+\gamma-s}\left(K^B\right)^s} = \frac{\omega^{A,W}\frac{K^A}{a^A}}{\omega^{B,W}H^B} = \frac{\omega^{A,W}L^A}{\omega^{B,W}H^B} = \frac{(1-\alpha)\omega^{A,W}L^A}{K^W}\frac{K^W}{(1-\alpha)\omega^{B,W}H^B} = \frac{S^A}{1-S^A} \tag{63}$$

As $\frac{N^A}{N^B} = \frac{n^A}{1-n^A}$ this obviously means that $n^A = S^A$ so that the claim $n^A = \chi^A = S^A$ is true.

7.4 Proof of Proposition 6

Denote by $S^A \equiv (1-\alpha)\omega^{A,W}L^A/K^W$ the steady state share of country A savings in world savings, and by $\chi^A \equiv K^A/K^W$ the share of capital used in country A in total capital used in the world at the steady state. As $S^A \equiv \frac{(1-\alpha)\omega^{A,W}L^A}{K^W} = \frac{(1-\alpha)\omega^{A,W}K^A}{a^A K^W} = \frac{(1-\alpha)\omega^{A,W}}{a^A}\chi^A$ and from Appendix 7.3 we have $S^A = \chi^A$, we conclude that $\frac{\omega^{A,W}}{a^A} = \frac{1}{1-\alpha}$. Also as $1-S^A = \frac{(1-\alpha)\omega^{B,W}H^B}{K^W} = 1-\chi^A = \frac{K^B}{K^W}$ we conclude that $\frac{\omega^{B,W}H^B}{K^B} = \frac{1}{1-\alpha}$ implying that

$$\frac{\omega^{B,W}H^B}{K^B} = \frac{\omega^{A,W}}{a^A} = \frac{1}{1-\alpha}. \tag{64}$$

Combining (45) and (46), and using the last expression we have that at the steady state

$$\rho^W = \frac{s\omega^{A,W}}{(1-s)a^A} = \frac{s}{(1-s)(1-\alpha)} \tag{65}$$

so that $\rho^W = \rho^A = \rho^B$. Now, using also (52), we obtain the steady state value of the capital-labor ratio in country A given by (55). Note that it is identical to the value obtained in autarky.

We now prove uniqueness of the steady state. First, substituting equations (44), (45) and (46) into (48)-(49) we rewrite our dynamic system in terms of K^A and K^W:

$$K^W_{t+1} = \Psi_1(K^W_t)^{(1+\beta)}(K^W_t - K^A_t)^{(s-1)(1+\beta)} \tag{66}$$

$$(K^W_{t+1} - K^A_{t+1}) = \Psi_2\left[(K^W_t)^{\beta}(K^W_t - K^A_t)^{\frac{(s-1)[s-\gamma+\beta(s-1-\gamma)]}{(s-1-\gamma)}}(K^A_t)^{\frac{-\gamma}{(s-1-\gamma)}}(K^W_{t+1})^{\beta(1-\alpha)}\right]^{\frac{1}{(1-\alpha)(1+\beta)(1-s)}}$$

where

$$\Psi_1 \equiv \frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^{\beta}}(H^B)^{(1+\beta)(1+\gamma-s)}$$

$$\Psi_2 \equiv \left\{\frac{\lambda}{\nu}\frac{\alpha^{\alpha}s^{1-\alpha}}{(1-\alpha)(1-s)^{1-\alpha}}\left[\frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^{\beta}}\right]^{(2-\alpha)}(H^B)^{[(1+\gamma-s)[\beta+(1+\beta)(1-\alpha)]+\gamma-s]}\right\}^{\frac{1}{(1-\alpha)(1+\beta)(1-s)}}.$$

At the steady state system (66) becomes:

$$K^W - K^A = \Psi_1^{\frac{1}{(1+\beta)(1-s)}} K^{W\frac{\beta}{(1+\beta)(1-s)}} \tag{67}$$

$$(K^W - K^A)^{\frac{(s-1-\gamma)[(1-\alpha)(1+\beta)+\beta)]+s-\gamma}{(s-1-\gamma)(1-\alpha)(1+\beta)}} = \Psi_2 K^{W\frac{\beta(2-\alpha)}{(1-\alpha)(1+\beta)(1-s)}} K^{A\frac{\gamma}{(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}. \tag{68}$$

Substituting the first equation in the second we obtain

$$K^W = \left(\frac{\Psi_3}{(K^A)^{\gamma(1+\beta)}}\right)^{\frac{1}{\beta}}$$

where

$$\Psi_3 \equiv \Psi_1^{(1+\gamma-s)[(1-\alpha)(1+\beta)+\beta)]-s+\gamma}\Psi_2^{-(1-\alpha)(1+\beta)^2(1+\gamma-s)(1-s)}$$

Substituting now this last expression in (67) we obtain:

$$\left[(K^A)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + [\Psi_1\Psi_3]^{\frac{1}{(1+\beta)(1-s)}}(K^A)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^{\beta} = \Psi_3$$

Substituting Ψ_1 and Ψ_3 in the previous expression, and using (55), we finally obtain equation (54) that we rewrite below:

$$\frac{\varepsilon^{1+\beta}\phi^{\beta}\left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)} = H(K^A) \equiv \left[(K^A)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + \left(H^B a^A\right)^{\frac{1+\gamma-s}{1-s}}(K^A)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^{\beta}.$$

Under Assumption 1 the derivative of the RHS of the previous expression is unambiguously positive. Moreover, the RHS tends to 0 when K^A tends to 0 and it tends to $+\infty$ when K^A tends to $+\infty$, implying that the RHS is an increasing function going from 0 to $+\infty$. Since the LHS is a positive constant, it follows that there exists a unique K^A solution of that equation. As from (67) we have that K^W is uniquely determined by K^A we conclude that the steady state is unique.

7.5 Proof of Proposition 7

Steady state savings in country A are given by $(1-\alpha)\omega^{A,W}L^A$. From (64) we have that $\omega^{A,W} = a^A/(1-\alpha)$. Substituting this in the expression for savings we immediately have that $(1-\alpha)\omega^{A,W}L^A = a^A L^A = K^A$, i.e. capital accumulation in country A equals savings in that country. Of course, as world capital accumulation is equal to world savings, this implies that the same happens in country B. Moreover, from (50), a balanced trade account implies that net capital inflows are zero in both countries at the steady state.

7.6 Proof that K^A and L^A decrease with β

We can rewrite (54) as:

$$\frac{\varepsilon\left(a^A\right)^{(1+\gamma-s)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)}=\left(K^A\right)^{\gamma}\left[\frac{\left(K^A\right)^{1+\gamma}}{\varepsilon\phi\left(a^A\right)^{(1+\gamma-s)}}\left(1+\left(\frac{H^B a^A}{K^A}\right)^{\frac{1+\gamma-s}{1-s}}\right)\right]^{\beta}.$$

We have proved above that the RHS of this expression is an increasing function of K^A. Therefore the steady state level of capital in country A is determined by the intersection of this increasing function with the constant on the LHS. We can show that $\frac{\left(K^A\right)^{1+\gamma}}{\varepsilon\phi\left(a^A\right)^{(1+\gamma-s)}}>1$. Indeed, using (20) this inequality can be rewritten as $N^A>1$, which is always satisfied. We conclude that the term in square brackets in the RHS is higher than one. Therefore, the increasing function of K^A in the RHS of this last expression shifts up when β increases. This implies that K^A decreases with β. As a^A is not affected by β, L^A also decreases with β.

7.7 Proof of Proposition 8

With *intra-industry* trade and free capital movements the number of varieties in the world which are consumed by residents in countries A and B at the steady state, $N^W=N^A+N^B=N^A\left[1+\frac{N^B}{N^A}\right]$, is given by (44) evaluated at steady state, that, using (42), we can rewrite as:

$$N^W=\underbrace{\frac{\left(a^A\right)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\varepsilon\phi}}_{N^A}\left[1+\left(\frac{H^B}{L^A}\right)^{\frac{1+\gamma-s}{1-s}}\right] \tag{69}$$

so that

$$\frac{N^B}{N^A}=\left(\frac{H^B}{L^A}\right)^{\frac{1+\gamma-s}{1-s}}. \tag{70}$$

Let N^B_{aut} denote the number of varieties produced at the steady state in country B in autarky. In country B, as $N^B>N^B_{aut}$, we immediately conclude that $N^W>N^B_{aut}$, i.e. at the steady state residents in country B consume more varieties after opening its borders to free trade and capital movements. The number of varieties in country A under autarky is given by (20). Since a^A is the same in autarky and after opening the borders, and denoting by N^A_{aut} the number of varieties produced at the steady state in country A in autarky and by K^A_{aut} the steady state capital stock of country A in autarky, we have that

$$N_{aut}^A = \left(\frac{K_{aut}^A}{K^A}\right)^{1+\gamma} N^A.$$

Now, using (27) we can rewrite (54) as:

$$\frac{K_{aut}^A}{K^A} = \left[1 + \left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta}{\gamma+(1+\gamma)\beta}}. \tag{71}$$

Note that as the RHS is higher than one it follows that the steady state capital stock of country A decreases with trade. Moreover using the previous expression we obtain

$$N_{aut}^A = N^A \left[1 + \left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta(1+\gamma)}{\gamma+(1+\gamma)\beta}},$$

i.e. the number of varieties produced in A decreases with free *intra-industry* trade. Combining now (69) and the previous expression, we obtain:

$$\frac{N^W}{N_{aut}^A} = \frac{\left[1 + \left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]}{\left[1 + \left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta(1+\gamma)}{\gamma+(1+\gamma)\beta}}} = \left[1 + \left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\gamma}{\gamma+(1+\gamma)\beta}} > 1.$$

We conclude that, although the number of varieties produced in A decreases with free *intra-industry* trade and capital mobility, the residents of country A have access and consume more varieties at the steady state after opening the borders.

Finally, we have that $N^A/N^B = n^A/(1-n^A)$. As N^A decreases with trade and N^B increases with trade, N^A/N^B decreases with trade. Since N^A/N^B is an increasing function of n^A, it follows that n^A decreases and $n^B = 1 - n^A$ increases with trade. As $n^A = S^A = \chi^A$ (see Appendix 7.3) this also means that the steady state share of savings of country A, S^A, and the steady state share of capital of country A, χ^A, decrease, while the steady state share of saving of country B, $1 - S^A$, and the steady state share of capital stock of country B, $1 - \chi^A$, increase.

7.8 Proof of Proposition 10

Linearizing the dynamic system (66) we obtain

$$\widehat{K}_{t+1}^W = \underbrace{\frac{(1+\beta)(s-n^A)}{1-n^A}}_{z_1^W} \widehat{K}_t^W + \underbrace{\frac{(1+\beta)(1-s)n^A}{1-n^A}}_{z_2^W} \widehat{K}_t^A$$

and

$$\underbrace{\frac{1-s-\beta s+\beta n^A}{(1-n^A)(1+\beta)(1-s)}}_{x_1^W}\widehat{K}_{t+1}^W+\underbrace{-\frac{n^A}{1-n^A}}_{x_2^W}\widehat{K}_{t+1}^A=$$

$$\underbrace{\frac{\beta(1+\gamma-s)(s-n^A)+(1-s)(s-\gamma)}{(1-n^A)(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}_{z_3^W}\widehat{K}_t^W+\underbrace{\frac{\gamma-n^A\left\{\gamma+(1-s)[s-\gamma-\beta(1+\gamma-s)]\right\}}{(1-n^A)(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}_{z_4^W}\widehat{K}_t^A$$

where $\widehat{K}_t^W$ and $\widehat{K}_t^A$ denote percentage deviations of K^W and K^A from the steady state.

We now rewrite the linear system above in matrix form

$$\underbrace{\begin{bmatrix}1 & 0\\ x_1^W & x_2^W\end{bmatrix}}_{J_1^W}\begin{bmatrix}\widehat{K}_{t+1}^W\\ \widehat{K}_{t+1}^A\end{bmatrix}=\underbrace{\begin{bmatrix}z_1^W & z_2^W\\ z_3^W & z_4^W\end{bmatrix}}_{J_0^W}\begin{bmatrix}\widehat{K}_t^W\\ \widehat{K}_t^A\end{bmatrix}.$$

The Jacobian matrix, J^W, is then

$$J^W=\left(J_1^W\right)^{-1}\cdot J_0^W=\begin{bmatrix}1 & 0\\ -\frac{x_1^W}{x_2^W} & \frac{1}{x_2^W}\end{bmatrix}\begin{bmatrix}z_1^W & z_2^W\\ z_3^W & z_4^W\end{bmatrix}=\begin{bmatrix}z_1^W & z_2^W\\ \frac{z_3^W-z_1^W x_1^W}{x_2^W} & \frac{z_4^W-z_2^W x_1^W}{x_2^W}\end{bmatrix}.$$

The trace, T^W, and determinant, D^W, of matrix J^W, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P^W(\lambda^W)\equiv\left(\lambda^W\right)^2-\lambda^W T^W+D^W$.

Results follow.

7.9 Proof of Proposition 11

The steady state is a sink when, at a same time, $D^W<1$, $1+T^W+D^W>0$ and $1-T^W+D^W>0$. In that case local indeterminacy emerges. Local determinacy will arise in the remaining cases. In particular, the steady state is a saddle when, simultaneously, $1+T^W+D^W>0$ and $1-T^W+D^W<0$ or $1+T^W+D^W<0$ and $1-T^W+D^W>0$. In any other configurations, the steady state will be a source.

Using (58) we have that D^W is lower than one if and only if $\gamma>\frac{n^A(1-s)[\alpha(1-s)-(1-2s)]}{s-n^A[\alpha(1-s)-(1-2s)]}\equiv\gamma_1^W$, where γ_1^W is the value of γ for which $D^W=1$. Such a threshold is positive under Assumption 1.[37] Furthermore, from (58), we can compute

$$1-T^W+D^W=\frac{n^A\beta(1+\gamma-s)+\gamma[1-s(1+\beta)]}{(1+\gamma-s)(1+\beta)(1-s)(1-\alpha)n^A}>0 \text{ under Assumption 1}$$

[37] The denominator of γ_1^W is positive under Assumption 1, as the term in square brackets is positive and $\frac{s}{\alpha(1-s)-(1-2s)}>1$.

and

$$1 + T^W + D^W = \frac{n^A(1+\gamma-s)\left\{2(1+\beta)\left[(1-\alpha)(1-s)+s\right]-\beta\right\}-\gamma[1+s(1+\beta)]}{(1+\gamma-s)(1+\beta)(1-s)(1-\alpha)n^A}.$$

We have that $1+T^W+D^W>0$ when $\gamma < \frac{n^A(1-s)[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}{1+s(1+\beta)-n^A[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]} \equiv \gamma_2^W > 0$ under Assumption 1.[38] Remark that γ_2^W is the value of γ for which $D^W=-1-T^W$. As we always have $1-T^W+D^W>0$, we conclude that:

(i) the steady state is a source when $D^W>1$ and $1+T^W+D^W>0$, i.e. when $\gamma < \min\left\{\gamma_1^W, \gamma_2^W\right\}$;

(ii) the steady state is a sink when $D^W<1$ and $1+T^W+D^W>0$, i.e. when $\gamma_1^W<\gamma<\gamma_2^W$;

(iii) the steady state is a saddle when $1+T^W+D^W<0$, i.e. when $\gamma>\gamma_2^W$.

Noting that $\gamma_1^W<\gamma_2^W$ requires $\alpha<\frac{1+2s(1+\beta)}{1+3s(1+\beta)}$, results follow.

[38] Note that the numerator of γ_2^W is identical to the numerator of γ_2^{aut} (which is positive under Assumption 1) multiplied by n^A. The denominator of γ_2^W is also positive under Assumption 1. Indeed the term in square brackets is positive and $\frac{1+s(1+\beta)}{2(1-\alpha+s)+\beta[1-2\alpha(1-s)]}>1$.

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