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Ugo Colombino
University of Turin, LISER and IZA

Nizamul Islam
LISER

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IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Optimal Tax-Transfer Rules under Equilibrium and New Labour Demand Scenarios

We present an extension of the numerical approach to empirical optimal taxation allowed by a peculiar structure of a microeconometric model of labour supply that includes a representation of the demand side. This makes it possible to identify optimal tax-transfer rules while accounting for equilibrium constraints and to evaluate the effects of exogenous labour demand shocks. We provide illustrative examples using the 2015 EU-SILC data set for Italy.

JEL Classification: H21, C18

Keywords: empirical optimal taxation, microsimulation, microeconometrics, evaluation of tax-transfer rules, equilibrium, labour demand shocks

Corresponding author:
Ugo Colombino
Dipartimento di Economia e Statistica "Cognetti De Martiis"
University of Turin
Lungo Dora Siena 100
10153 Torino
Italy
E-mail: ugo.colombino@unito.it

1. Introduction

In Islam and Colombino (2018) we have used a numerical approach to the identification of optimal tax-transfer rules (TTRs) within the class of negative income tax – flat tax rules in a sample of European countries. The method consists of a combination of microeconometrics, microsimulation and numerical optimization. In Colombino and Islam (2020) we presented a generalization of the previous exercise by considering flexible TTRs represented by a 4th degree polynomial. In this paper we extend Colombino and Islam (2020) according to the simulation methodology suggested by Colombino (2013). The microeconometric model belongs to the RURO type (Aaberge and Colombino 2014). It can be interpreted as a matching model since, besides modelling labour supply decisions, it contains a representations of the demand side (number and types of available jobs). Colombino (2013) argues that in such a model, assuming we start from an equilibrium initial condition, we should impose an equilibrium constraint when simulating the effects of a reform in order to consistently compare the new allocation with the initial one, according to the principle of comparative statics. Using Italian data (EU-SILC 2015), we apply the equilibrium methodology to the identification of optimal tax-transfer rules. We also illustrate the possibility of simulating the effects of exogenous alternative labour demand scenarios.

2. The empirical model of household labour supply

We model the households as agents who can choose within an opportunity set Ω containing jobs or activities characterized by hours of work *h*, wage rate *w* and sector of market job *s* (wage employment or self-employment) and other characteristics (observed by the household but not by us). We define $\mathbf{h}$ and $\mathbf{w}$ as vectors with one element for the singles and two elements for the couples, $\mathbf{h}=\begin{pmatrix} h_F \\ h_M \end{pmatrix}, \mathbf{w}=\begin{pmatrix} w_F \\ w_M \end{pmatrix}$, where the subscripts F and M refer to the female and the male partner respectively. Analogously, in the case of couples, $\mathbf{s}=\begin{pmatrix} s_F \\ s_M \end{pmatrix}$. The above notation assumes that each household member can work only in one sector. We write the utility function of the i-th household at a (*h, s*) job as follows (Coda Moscarola et al. 2014):

$$U_i(\mathbf{h},\mathbf{s},\varepsilon;\boldsymbol{\tau}) = \mathbf{Y}_i(\mathbf{w_i},\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \varepsilon \qquad (1)$$

where:

$\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ are parameters to be estimated;

$\mathbf{Y}_i(\mathbf{w_i},\mathbf{h},\mathbf{s};\boldsymbol{\tau})$ is a vector including

- $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ = household disposable income on a (**h,s**) job given the TTR represented by the vector of parameters $\boldsymbol{\tau}$;
- the square of the household disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ defined above;
- the product of disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ and household size *N* (interaction term);

$\mathbf{L}_i(h)$ is a row vector including

- the leisure time (defined as the total number of available weekly hours (80) minus the hours of work *h*) of the two partners (for a couple) or of the individual (for a single): $L_{ig}=80-h_{ig}$, where $g=F,M$.
- the square of leisure time(s), $(L_{ig})^2$;

- the interaction(s) of leisure time(s) with household disposable income ($L_{ig} \times C_i$), with age of the couple's partners of the single, age square and three dummy variables indicating presence of children of different age range (any age, 0-6, 7-10);

ε is a random variable that measures the effect of unobserved (by the analyst) characteristics of the job-household match.

The opportunity set a single can choose among is

$\Omega = \{(0,0),(h_1, s=1),(h_2, s=1),(h_3, s=1),(h_1, s=2),(h_2, s=2),(h_3, s=2)\}$, where (0,0) denotes a non-market "job" or activity (non-participation), h_1,h_2,h_3 are values drawn from the observed distribution of hours in each hour interval 1-26 (part time), 27-52 (full time), 52-80 (extra time) and sector indicator *s* is equal to 1 (wage employment) or 2 (self-employment). In the case of couples, each partner can choose within $\Omega_g = \{(0,0)_g,(h_1, s=1)_g,(h_2, s=1)_g,(h_3, s=1)_g,(h_1, s=2)_g,(h_2, s=2)_g,(h_3, s=2)_g\}$, with $g = F(\text{wife}), M(\text{husband})$. Therefore the opportunity set of couples can be defined as the cartesian product of Ω_F and Ω_M .

A (**h,s**) job is "available" to household *i* with p.d.f. $p_i(\mathbf{h,s})$, which we call "opportunity density".

We estimate the labour supply models of couples and singles separately. In the case of singles, we have 7 alternatives, while in the case of couples, who make joint labour-supply decision, we combine the choice alternatives of two partners, thus getting 49 alternatives.

When computing the earnings of any particular job (**h, s**) we face the problem that the wage rates of sector *s* are observed only for those who work in sector *s*. Moreover, for individuals who are not working we do not observe any wage rate. To deal with this issue, we follow a two-stage procedure presented in Dagsvik and Strøm (2006) and also adopted in Coda-Moscarola et al. (2014). The procedure is analogous to the well-known Heckman correction for selectivity but is specifically appropriate for the distribution assumed for ε .

By assuming the ε is i.i.d. Type I extreme value we obtain the following expression for the probability that household *i* chooses a $(\mathbf{h},\mathbf{s})$ job (e.g. Aaberge and Colombino 2013)

$$P_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau}) = \frac{\exp\{\mathbf{Y}_i(\mathbf{w}_i,\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \ln p_i(\mathbf{h},\mathbf{s})\}}{\sum_{(\mathbf{H},\mathbf{S})\in\Omega} \exp\{\mathbf{Y}_i(\mathbf{w}_i,\mathbf{H},\mathbf{S};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{H})'\boldsymbol{\lambda} + \ln p_i(\mathbf{H},\mathbf{S})\}} \qquad (2)$$

By choosing a convenient ("uniform with peaks") specification for the opportunity density *p(.,.)* it turns out that expression (2) can be rewritten as follows (e.g. Aaberge and Colombino 2013, Colombino 2013),

$$P_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau}) = \frac{\exp\left\{\mathbf{Y}_i(\mathbf{w}_i,\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h},\mathbf{s})'\boldsymbol{\delta}\right\}}{\sum_{(\mathbf{H},\mathbf{S})\in\Omega} \exp\left\{\mathbf{Y}_i(\mathbf{w}_i,\mathbf{H},\mathbf{S};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{H})'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{H},\mathbf{S})'\boldsymbol{\delta}\right\}} \tag{3}$$

where, for a single household, $\mathbf{D}_i$ is the vector (with **1**[.] denoting the indicator function)

$$\begin{aligned}
D_{1,0} &= 1[s=1, h>0], \\
D_{1,1} &= 1[s=1, 1 \le h \le 26], \\
D_{1,2} &= 1[s=1, 27 \le h \le 52], \\
D_{2,0} &= 1[s=2, h>0], \\
D_{2,1} &= 1[s=2, 1 \le h \le 26], \\
D_{2,2} &= 1[s=2, 27 \le h \le 52].
\end{aligned} \tag{4}$$

and $\boldsymbol{\delta}$ is vector of parameters to be estimated. The hour ranges $1 \le h \le 26$ and $27 \le h \le 52$ correspond to part-time and full-time respectively. For couples, $\mathbf{D}_i$ contains two analogous sets of variables, one for each partner (*F* = wife, *M* = husband):

$$
\begin{aligned}
&D_{F,1,0} = 1[s_F = 1, h_F > 0],\\
&D_{F,1,1} = 1[s_F = 1, 1 \le h_F \le 26],\\
&D_{F,1,2} = 1[s_F = 1, 27 \le h_F \le 52],\\
&D_{F,2,0} = 1[s_F = 2, h_F > 0],\\
&D_{F,2,1} = 1[s_F = 2, 1 \le h_F \le 26],\\
&D_{F,2,2} = 1[s_F = 2, 27 \le h_F \le 52]\\
&D_{M,1,0} = 1[s_F = 1, h_F > 0],\\
&D_{M,1,1} = 1[s_F = 1, 1 \le h_F \le 26],\\
&D_{M,1,2} = 1[s_F = 1, 27 \le h_F \le 52],\\
&D_{M,2,0} = 1[s_F = 2, h_F > 0],\\
&D_{M,2,1} = 1[s_F = 2, 1 \le h_F \le 26],\\
&D_{M,2,2} = 1[s_F = 2, 27 \le h_F \le 52].
\end{aligned}
\tag{5}
$$

The model is a simplified version of the so-called RURO model (Aaberge and Colombino 2014). The main simplification with respect to the RURO model concerns the wage rates. In the most general versions of the RURO model (e.g. Aaberge et al. 1995, 1999, 2013) the wage rates densities are estimated simultaneously with the preference parameters and the hours' opportunity density. In this paper we use instead pre-estimated wage densities.

The datasets used in the analysis are the EUROMOD input data based on the European Union Statistics on Income and Living Conditions (EU-SILC) for the year 2015 in Italy. The input data provide all required information on demographic characteristics and human capital, employment and wages of household members, as well as information about various sources of non-labour income. We select individuals in the age range 18-55 who are not retired or disabled. EUROMOD[1] is used for two different operations. First, for every household in the sample, it computes the net available income under the current TTR at each of the 49 (7) alternatives available to the couples (singles). The net available incomes are used in the estimation of the labour supply model. Second, for each household, it computes the gross income at each alternative. Gross incomes are used in the simulation and optimization steps, where

[1] EUROMOD is a large-scale pan-European tax-benefit static micro-simulation engine (e.g. Sutherland and Figari, 2013). It covers the tax-benefit schemes of the majority of European countries and allows computation of predicted household disposable income, on the basis of gross earnings, employment and other household characteristics.

EUROMOD is not used anymore and new values of net available incomes are generated by applying the new TTRs to the gross incomes.

The estimates for couples (32 parameters), singles females (17 parameters) and single males (17 parameters) in Italy are reported in Tables A.1 and A.2 of the Appendix.

3. Polynomial tax-transfer rule

We look for optimal TTRs within the class of rules defined as a polynomial functions of total taxable income $y_i = \mathbf{w}_i^{'}\mathbf{h}_i + I_i - S_i$, where S_i denotes social security contributions:

$$C_i = \tau_0 \sqrt{N_i} + \tau_1 y_i + \tau_2 y_i^{\,2} + \tau_3 y_i^{\,3} + \tau_4 y_i^{\,4}$$

where yi = household total taxable income and N_i = household size. The parameter τ_0 is constrained to be greater than or equal to zero (lump-sum taxes are ruled-out). A pure flat tax rule is the special case $C_i = \tau_1 y_i$. A negative income tax matched with a flat tax corresponds to $C_i = \tau_0 \sqrt{N_i} + \tau_1 y_i$. This special cases have been analysed in Islam and Colombino (2018). In general, the rule can be interpreted or implemented as a negative income tax ((where $\tau_0 \sqrt{N_i}$ is the guaranteed minimum income) or a universal basic income (where $\tau_0 \sqrt{N_i}$ is the non means-tested basic income transfer). The term $\sqrt{N_i}$ rescales the guaranteed minimum income or the basic income according to the household size (square root rule). The rule is sufficiently flexible to represent many alternative versions of non-linear tax rules. The tax, the marginal tax rate and the average tax rate are, respectively:

$$T(y_i;\boldsymbol{\tau}) = y_i - \tau_0 \sqrt{N} - \tau_1 y_i - \tau_2 y_i^{\,2} - \tau_3 y_i^{\,3} - \tau_4 y_i^{\,4}$$

$$MT(y_i;\boldsymbol{\tau}) = \frac{\partial T(y_i;\boldsymbol{\tau})}{\partial y_i} = 1 - \tau_1 - 2\tau_2 y_i - 3\tau_3 y_i^{\,2} - 4\tau_4 y_i^{\,3}$$

$$AT(y_i;\boldsymbol{\tau}) = \frac{T(y_i;\boldsymbol{\tau})}{y_i} = 1 - \frac{\tau_0 \sqrt{N}}{y_i} - \tau_1 - \tau_2 y_i - \tau_3 y_i^{\,2} - \tau_4 y_i^{\,3}$$

4. Comparable Money-metric Utility

Based on the estimated model described in Section 2, hereafter we define the Comparable Money-metric Utility (CMU). This index transforms the household utility level into an inter-household comparable monetary measure that will enter as argument of the Social Welfare function (to be described in Section 5). First, we calculate the expected maximum utility attained by household *i* under TTR $\boldsymbol{\tau}_i$ (e.g. McFadden 1978):

$$E(\max U_i \setminus \boldsymbol{\tau}) = \ln\left(\sum_{\mathbf{s}} \sum_{\mathbf{h}\in\Omega} \exp\left\{ \mathbf{Y}_i(\mathbf{w}_i, \mathbf{h}, \mathbf{s}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} \right\} \right) \tag{6}$$

Analogously, we define

$$E(\max U_R \setminus \boldsymbol{\tau}_R) = \ln\left(\sum_{\mathbf{s}} \sum_{\mathbf{h}\in\Omega} \exp\left\{ \mathbf{Y}_R(\mathbf{w}_i, \mathbf{h}, \mathbf{s}; \boldsymbol{\tau}_R)'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} \right\} \right) \tag{7}$$

as the expected maximum utility attained by the "reference" household R under the "reference" TTR $\boldsymbol{\tau}_R$. The reference household is the couple household at the median value of the distribution of $E(\max U \setminus \boldsymbol{\tau}_R)$. The CMU of household i under TTR $\boldsymbol{\tau}$, $\mu_i(\boldsymbol{\tau})$, is defined as the gross income that a reference household under a reference TTR $\boldsymbol{\tau}_R$ would need in order to attain an expected maximum utility equal to $E(\max U_i \setminus \boldsymbol{\tau})$. The CMU is analogous to the "equivalent income" defined by King (1983).[2] Although the choice of the reference household is essentially arbitrary, some choices make more sense than others. Our choice of the median household as reference household can be justified in terms of representativeness or centrality of its preferences. Aaberge and Colombino (2006, 2013) adopt a related, although not identical, procedure that consists of using a common utility function as argument of the social welfare function (Deaton and Muelbauer, 1980). A significant portion of the empirical policy evaluation literature is silent upon the issue of interpersonal preference comparability. Theoretical models or general equilibrium models typically assume identical preferences or a representative individual, so that the problem is absent by construction. In the empirical literature based on microdata and micro-modelling, frequently either income is interpreted as an index of welfare or the utility levels are directly used, maybe under the assumption that the solution of the comparability problem is somehow implicitly

[2] The basic idea is using the preferences of the "reference household" in the same way as reference prices are used in computing equivalent or compensating variations for comparing utility levels attained under different budget sets.

accounted for by the social welfare function. We follow here the tradition of Deaton and Muellbauer (1980) and King (1983), which, in our view, is both practical and theoretically sound.

5. Social Welfare function

We choose Kolm (1976) Social Welfare index, which can be defined as:

$$W = \bar{\mu} - \frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\left\{-k\left(\mu_{\mathrm{i}} - \bar{\mu}\right)\right\}}{\mathrm{N}}\right] \tag{8}$$

where

$\bar{\mu} = \frac{1}{N}\sum_{i}\mu_i$ is an index of Efficiency ,

$\frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\left\{-k\left(\mu_{\mathrm{i}} - \bar{\mu}\right)\right\}}{\mathrm{N}}\right] =$ Kolm Inequality Index,

$k =$ Inequality Aversion parameter,

μ_i = comparable money-metric utility of household i (defined in Section 4.1).

W has limit $\bar{\mu}$ as $k \to 0$ and $\min\left\{\mu_1, ..., \mu_N\right\}$ as $k \to \infty$.

The meaning of k might be clarified by the following example. Let us take two households with $\mu_2 - \mu_1 = 1$. Given the social marginal evaluation of μ_i, $\frac{\partial W}{\partial \mu_i} = \frac{e^{-k\mu_i}}{e^{-k\mu_1} + e^{-k\mu_2}}$, we get the social marginal rate of substitution: $SMRS_{1,2} = e^{k(\mu_2 - \mu_1)} = e^k$. Now let us consider a (small) transfer $\tau < 1$ from household 2 to household 1 in order to reduce the inequality. Note that the social planner would be willing to take $\exp\left\{k\right\}\tau$ from household in order to give τ to household1. Since $\exp\left\{k\right\} \geq 1$, $\exp\left\{k\right\} - 1$ measures (approximately) the “excess willingness to pay” for a “inequality reducing” transfer from household 2 to household 1:

k	0.05	0.10	0.25	0.50
$\exp\{k\}-1$	0.051	0.105	0.284	0.649

Kolm Inequality Index is an absolute index, meaning that it is invariant with respect to translations (i.e. to adding a constant to every μ_i). Absolute indexes are less popular than relative indexes (e.g. Gini's or Atkinson's), although there is no strict logical or economic motivation for preferring one to the other. Blundell and Shephard (2012) adopt a social welfare index which turns out to be very close to Kolm's index. Their main motivation for their index seems to be the computational convenience, since it handles negative numbers (random utility levels, in their case). Our motivation in choosing Kolm's index is analogous. In our case, μ_i is a monetary measure, yet it can turn out to be negative when the utility level of household *i* is sufficiently lower than the utility level of the reference household. Kolm's index handles negative arguments. Moreover, it is also possible to shift the μ_i-s by adding a constant (which would not be allowed with a relative index).

6. Equilibrium simulation

Let us consider again the expressions (3) and (4) of Section 2. Colombino (2013) shows that the dummies' coefficients δ of expression (3) have the following interpretation:

$$\delta_{s,0} = \ln\left(A_{s,0} J_s\right), s = 1, 2 \tag{9}$$

$$\delta_{s,l} = \ln\left(A_{s,l} \frac{J_{s,l}}{J_s}\right), s = 1, 2; l = 1, 2 \tag{10}$$

where

$\delta_{s,0}$ = coefficient of $D_{s,0}$
$\delta_{s,l}$ = coefficient of $D_{s,l}$
$l = 1$ denotes part time $(1 \leq h \leq 26)$
$l = 2$ denotes full time $(27 \leq h \leq 52)$
J_s = number of jobs in sector s
$J_{s,l}$ = number of jobs in sector s and hour range l

and

$A_{s,0}, A_{s,l}$ are constants that can be retrieved from the data.

Analogously, for couples we have:

$$\delta_{g,s,0} = \ln\left(A_{g,s,0} J_s\right), \delta_{g,s,l} = \ln\left(A_{g,s,l} \frac{J_{g,s,l}}{J_{g,s}}\right), g = F, M, s = 1, 2; l = 1, 2 \tag{11}$$

This interpretation of the parameters δ permits to simulate the effects of policies or of exogenous events that imply changes in the number of available jobs. A first case is represented by tax-transfer reforms that change the households' willingness to be match to the various jobs. If we require that in equilibrium the number of available jobs of a given type is equal to the number of individuals willing to choose that type of jobs, it follows that in general we will have to appropriately adjust the number of available jobs and the corresponding value of the parameters δ. The most common assumption is that the number of jobs depends on the wage rates. Using the analogy with the traditional demand-supply cross, the tax-transfer reform shifts the labour supply curve. Then the new equilibrium requires a new number of job and a new value of the wage rate (a movement along the labour demand curve). This Section 6 and the next Session 7 explain how this case can be modelled. Section 8 illustrates instead an example where we want to account for an exogenous change in the number of jobs.

To start with, let us consider singles and the simple case with only one dummy D_0 (i.e. employment of any type). Let δ represent its coefficient. According to expression (9) above we have

$$\delta = \ln J + \alpha$$

where $\alpha = \ln(A)$ is a constant and J = number of available jobs.

The policy in general will induce a change in the number of people willing to work. Equilibrium requires that the number of available jobs J is equal to the number of people willing to work. J depends on the wage rates and δ depends on J. Therefore, the policy will determine a change in the values of J, of δ, of the wage rate and of the number of employed.

Let e^v be the proportional change in J, where v denotes a parameter that will have to be determined in order attain equilibrium. The changed value of J is

$$J(v) = Je^v \tag{12}$$

The corresponding changed value of δ is:

$$\delta(v) = \ln(Je^v) + \alpha = \ln J + \alpha + v = \delta + v \tag{13}$$

By assuming the J depends on the wage rate w according the labour demand function $J = Kw^{-\eta}$ or $w = K^{1/\mu} J^{-1/\eta}$, we get the wage rate corresponding to Je^v:

$$w(v) = K^{1/\eta}(Je^v)^{-1/\eta} = K^{1/\eta} J^{-1/\eta} e^{-v/\eta} = we^{-v/\eta} \tag{14}$$

The new values of $\delta(v)$ and $w(v)$ determine new choice probabilities.

Let $M(\boldsymbol{\tau}, v)$ be the number of people willing to work given the policy $\boldsymbol{\tau}$ and the parameter v.

(which determines the new values of $\delta(v)$ and $w(v)$ according to expressions (11) and (12)). The equilibrium value v^* is such that

$$M(\boldsymbol{\tau}, v^*) = Je^{v^*} \tag{15}$$

As a matter of fact, we have different types of households and different types of jobs. For each couple we have $J_{F,s}$, $J_{F,s,l}$, $J_{M,s}$, $J_{M,s,l}$ with the corresponding dummies' coefficients $\delta_{F,s,0}, \delta_{F,s,l}$, $\delta_{M,s,0}$, $\delta_{M,s,l}$ and the wage rates $w_{F,s}$, $w_{M,s}$ with s = 1, 2 and l = 1, 2. Analogously, for the singles we have

$J_{g,s,0}, J_{g,s,l}$, $\delta_{g,s,0}, \delta_{g,s,l}, w_{g,s}$ with $g = f, m$.

Ideally, we might want to account for some degree of substitution/complementarity between all different types of jobs, e.g.: $J_{g,s} = K_{g,s} \prod_{\substack{k=F,M,f,m \\ z=1,2}} w_{k,z}^{-\eta_{k,z}(g,s)}$

Moreover, in principle we might allow for different equilibrium adjustment $v_{g,s}$ for the specific job types. The illustrative exercise that follows adopts three drastic simplifications. First, we impose that the number of any type of jobs only depends on its own wage rate, with a common elasticity: $J_{g,s} = K_{g,s} w_{g,s}^{-\eta}$. Second, we impose a common equilibrium parameter *v* for all job types. Third, we only account for aggregate equilibrium, i.e. total number of jobs equal to total (expected) number of people willing to work.

If we denote with $P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}, v)$ the probability that the *i*-th household chooses alternative $(\mathbf{h}_i, \mathbf{s}_i)$ given the policy $\boldsymbol{\tau}$ and the parameter *v*, expression (3) can be rewritten as follows:

$$P_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau}, v) = \frac{\exp\left\{\mathbf{Y}_i(\mathbf{w}_i(v), \mathbf{h}, \mathbf{s}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h}, \mathbf{s})'\boldsymbol{\delta}(v)\right\}}{\sum_{(\mathbf{H},\mathbf{S})\in\Omega} \exp\left\{\mathbf{Y}_i(\mathbf{w}_i(v), \mathbf{H}, \mathbf{S}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{H})'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{H}, \mathbf{S})'\boldsymbol{\delta}(v)\right\}} \qquad (16)$$

where $\boldsymbol{\delta}(v)$ and $\mathbf{w}_i(v)$ are defined according to expressions (13) and (14). Note that since the parameter *v* is common to all type of jobs, for singles, $\delta_{s,0}$ (s = 1,2) are the only components of vector $\boldsymbol{\delta}(v)$ affected by *v* (see expressions (9) and (10)). Analogously, for couples, only the components $\delta_{F,s,0}$ and $\delta_{M,s,0}$ are affected. The expected number of individuals willing to work is

$$\sum_i \sum_{(\mathbf{h},\mathbf{s})\, with\ h_F>0} P_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau}, v) + \sum_i \sum_{(\mathbf{h},\mathbf{s})\, with\ h_M>0} P_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau}, v) \equiv M(\boldsymbol{\tau}, v) \qquad (17)$$

The two terms on the left-hand side are respectively the expected number of wives who are willing to work and the expected number of husbands who are willing to work. In the case of singles, only one of the two terms on the left-hand side is present. The equilibrium value *v** is defined by:

$$M(\boldsymbol{\tau}, v^*) = \sum_{g=F,M} \sum_{s=1,2} J_{g,s,0}(v^*) \qquad (18)$$

where the term on the right-hand side is the total number of jobs.

7. Identifying the optimal policies in equilibrium

In order to identify the optimal TTR in equilibrium we proceed as follows.

1. Start with initial guesses $\boldsymbol{\tau}^0$ and v^0

2. Compute by microsimulation the comparable money metric measures $\mu_1(\boldsymbol{\tau}^0, v^0), ..., \mu_H(\boldsymbol{\tau}^0, v^0)$ and the social welfare function $W\left(\mu_1(\boldsymbol{\tau}^0, v^0), ..., \mu_H(\boldsymbol{\tau}^0, v^0)\right)$

3. Compute the total expected net tax revenue $\sum_{i=1}^{N} \sum_{\mathbf{h},\mathbf{s}\in\Omega} P_i\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^0, v^0\right) T\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^0, v^0\right)$, where $T\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^0, v^0\right)$ denotes the net tax paid by the i-th household at job ($\mathbf{h},\mathbf{s}$).

4. Compute the total number of jobs $\sum_{g=F,M} \sum_{s=1,2} J_{g,s,0}(v^0)$ and the total number of individuals willing to work $M(\boldsymbol{\tau}, v^0)$

5. Iterate (1) – (4) updating $(\boldsymbol{\tau}^0, v^0), (\boldsymbol{\tau}^1, v^1)....$ until $W\left(\mu_1(\boldsymbol{\tau}^*, v^*), ..., \mu_H(\boldsymbol{\tau}^*, v^*)\right)$ is maximized and $\sum_{i=1}^{N} \sum_{\mathbf{h},\mathbf{s}\in\Omega} P_i\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^*, v^*\right) T\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^*, v^*\right) = R$ and $M(\boldsymbol{\tau}, v^*) = \sum_{g=F,M} \sum_{s=1,2} J_{g,s,0}(v^*)$ are both satisfied, where R is the net tax revenue required by public budget constraint.[3]

We performed the exercise with two different values of the labour demand elasticity: -0.5 and -∞. In Table 1 we show (for Kolm's k = 0.075) the results without equilibrium and with equilibrium corresponding to the two values of the elasticity.

Different assumptions upon the value of labour demand elasticity are most frequently interpreted as reflecting the length of the period considered (e.g. short-run vs long-run). However, the so-called replication argument leading to a scenario with constant returns and perfectly elastic demand in principle is valid even in the short-run. If it does not apply, it must be due to limitations that might act whatever the period length. The case of inelastic demand can be interpreted as a situation where the firms that would like to increase production and employment face rising costs due to the organizational and institutional environment. In

[3] For the numerical maximization we used the application CO (Constrained Optimization) provided by the package GAUSS.

order to compensate for those costs the firms would need lower wages (hence a steep demand curve). However, lower wages might make equilibrium impossible. At the opposite, an highly elastic demand represents a situation where the firms can change the level of activity without significant changes in unit production costs.

Table 1 reports the results with no equilibrium and with equilibrium (η = 0.5, ∞). The first column also reports the parameters of the polynomial approximation to the current TTR and the actual current economic and fiscal results. When we assume η = 0.5, equilibrium requires only a very small adjustment and as a consequence the results are almost identical to those obtained when not accounting for equilibrium. There are minor changes in the tax-transfer parameters (beyond the third decimal position), not shown in Table 1, that cause the modest effects in economic and behavioural effects. For higher (absolute) values of the elasticity, equilibrium requires a much higher adjustment. Here we report the limit case of a perfectly elastic demand, i.e. $\eta = \infty$. As a matter of fact, we observed that this limit is approximately reached as soon as η is larger than 1. The results produced by an elastic demand are strikingly different. It is important to remember that we simultaneously reach an equilibrium and an optimal TTR. An inelastic demand constrains the solution to remain close to the current system: the optimized TTR is an improvement though a modest one. An elastic demand opens the space for a much larger improvement by the optimized TTR. Employment increases by 26%. Disposable income increases by 32%, more than the increase in employment since the net tax is lower. Poverty is reduced by more than 10 percentage points.[4]

[4] Given our interpretation of the different values of the demand elasticity, the results appear to be particularly interesting for a country like Italy, where the costs limiting the entrepreneurial activity are well documented (Doing Business 2020).

8. Simulating the effects of alternative exogenous labour demand scenarios

This section illustrates a different exercise made possible by the peculiar structure of the microeconometric model. Rather than looking for an equilibrium allocation together with an optimal TTR, we introduce an exogenous demand shock (a 10% reduction of available jobs) and compare the performance of the current rule and of the optimized rule under Kolm's $k = 0.075$. From the policy perspective, the exercise is relevant for investigating whether and to what extent the optimized rule (compared to the current one) contributes to moderating the economic damages caused by an exogenous labour demand shock.[5] Rather than a strict equilibrium, we impose the feasibility of the allocation, i.e. more available jobs than individuals willing to work. We leave the wage rates unchanged. We do not impose fiscal neutrality. Table 2 present the results. After the demand shock, the optimized TTR is welfare superior to the current rule. However, the optimized rule induces a larger public budget deficit. Although in principle we could account for the larger cost of public funds entailed by the current rule, we are not able to evaluate the lost benefits from the lower public expenditure caused by the optimized rule. In order to make the results comparable, we rescale the optimized rule, i.e. we multiply the parameters $\boldsymbol{\tau}$ (i.e. the coefficients of the polynomial that represents the disposal income) by a constant such that the net tax revenue is identical to the one collected through the current rule. The required constant turns out to be 0.8625, which means that for a given level of taxable income net taxes would be increased by about 14%. The rescaled optimized rule is still welfare superior to the current one, although by a small amount, and it performs slightly better also under other dimensions (poverty gap, disposable income, winners). A more sophisticated comparison would consist of re-optimizing (instead of simply rescaling) the optimized rule under the constraint of a public budget deficit equal to the one entailed by the current rule. This is left for future work.

[5] There are many current economic processes that envisage scenarios with less jobs at least in some countries (automation, globalization etc.). And of course, currently, the Covid-19 pandemic.

Table 1. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects with no equilibrium and with equilibrium. Italy. Kolm's *k* = 0.075.

	Approximated Current	No equilibrium	Equilibrium with η =0.5	Equilibrium with η =∞
τ_0	217.24	236.69	236.69	297.89
τ_1	0.745	0.655	0.655	0.742
τ_2	-1.98×10^{-6}	0.002×10^{-6}	0.002×10^{-6}	0.010×10^{-6}
τ_3	0.69×10^{-11}	0.004×10^{-11}	0.004×10^{-11}	0.014×10^{-11}
τ_4	-0.07×10^{-16}	-0.000×10^{-16}	-0.000×10^{-16}	0.003×10^{-16}
Gross income	3215.89	3213.70	3220.23	3806.58
Taxable income	2279.76	2278.90	2283.309	2687.42
Disposable income	1851.96	1849.77	1852.62	2442.84
Weekly hours	28.68	28.59	28.71	35.29
Employment %	79.58	79.27	79.65	100.00
Poverty gap %	19.00	15.06	14.77	4.22
Welfare	9001.23	9059.93	9058.57	9220.06
Efficiency	9451.87	9518.37	9518.37	9709.47
Inequality	450.64	459.80	459.80	489.40
v*	---	---	0.001	0.229
Willing to work	10715	10673	10723	13470
Available jobs	10715	10715	10723	13470

Table 2. Effects of an exogenous reductions of available jobs (-10%) under alternative TTRs. Italy. Kolm's *k* = 0.075

	Approximated Current	Optimal TTR ($\eta=\infty$)	Rescaled optimal TTR ($\eta=\infty$)
τ_0	217.24	297.89	257.93
τ_1	0.745	0.742	0.640
τ_2	-1.98×10^{-6}	0.010×10^{-6}	0.009×10^{-6}
τ_3	0.69×10^{-11}	0.014×10^{-11}	0.012×10^{-11}
τ_4	-0.07×10^{-16}	0.003×10^{-16}	0.002×10^{-16}
Gross income	2739.77	2781.54	2745.31
Taxable income	1949.67	1980.09	1954.06
Disposable income	1635.15	1921.48	1640.46
Weekly hours	23.53	23.73	23.52
Employment %	64.29	64.73	64.23
Poverty gap %	21.46	17.12	20.56
Welfare	9038.51	9220.00	9053.94
Efficiency	9492.63	9709.38	9512.78
Inequality	454.12	489.39	458.84
Net tax revenue	1105.00	860	1105.00
Willing to work	8906	8964	8898
Available jobs	9696	9696	9696

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Appendix

A.1 – Maximum likelihood estimates – couples (Italy)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	-2.227042	0.3359151
	Self-employed_Man	-1.793772	0.3327547
	Employee_Woman	-4.205803	0.3711781
	Self-employed_Woman	-3.159583	0.3091701
	Part-time_Employee_Man	1.810835	0.2235256
	Full-time_Employee_Man	3.457804	0.1466732
	Part-time_Self-employed_Man	-1.142189	0.2861769
	Full-time_Self-employed_Man	1.827801	0.1352579
	Part-time_Employee_Woman	3.522802	0.3488772
	Full-time_Employee_Woman	4.233018	0.3257372
	Part-time_Self-employed_Woman	0.2200945	0.3028192
	Full-time_Self-employed_Woman	1.989132	0.2580389
Y vector		γ	
	Household_Disposable_income	0.0005129	0.0001534
	Hosuhold_Disposable_income squared	1.36E-08	7.25E-09
	Household_size X Household_disposable_income	-0.0001608	0.0000251
L vector		λ	
	Leisure_Male	0.0030689	0.05153
	Leisure_Man squared	-0.0000926	0.0001607
	Leisure_Woman	0.2598116	0.0365898
	Leisure_Woman squared	-0.000653	0.0001763
	Leisure_Man X Household_disp_income	4.38E-06	1.43E-06
	Leisure_Woman X Household_disp_income	-5.81E-07	1.01E-06
	Leisure_Man X Age_Man	-0.0015349	0.0025113
	Leisure_Woman X Age_Woman	-0.0097254	0.0016741
	Leisure_Man X Age_Man squared	0.0000135	0.0000318
	Leisure_Woman X Age_Woman squared	0.0001141	0.0000223
	Leisure_Man X No. Children	-0.0081218	0.0022336
	Leisure_Woman X No. Children	0.0078869	0.0017578
	Leisure_Man X No. Children0-6	0.0076125	0.0026554
	Leisure_Man X No. Children7-10	0.0002707	0.0028172
	Leisure_Woman X No. Children0-6	-0.0054445	0.0020634
	Leisure_Woman X No. Children7-10	-0.0009139	0.0020886
	Leisure_Woman X Leisure_Man	0.0003854	0.0000964
Other			
	N. observations (N. couples*49 alternatives)	188405	
	N. couples	3845	
	LR chi2(32)	10209.91	
	Prob > chi2	0	
	Pseudo R2	0.3411	
	Log likelihood	-9859.09	

Table A.2– Maximum likelihood estimates – singles (Italy)

Model component	Variable	Male Coef.	Male Std. Err.	Female Coef.	Female Std. Err.
Opportunity density		δ		δ	
	Employee	-1.22117	0.331639	-3.43019	0.3787
	Self_employed	-0.47643	0.315555	-2.81075	0.350903
	Part-time_Employee	1.263827	0.268794	3.554593	0.34008
	Full-time_Employee	3.310487	0.207522	4.654217	0.303264
	Part-time_Self-employed	-2.2652	0.32631	0.618142	0.341357
	Full-time_Self-employed	1.473456	0.180946	2.786139	0.266647
Y vector		γ		γ	
	Disposable income	0.000114	0.000145	0.0003	0.000255
	Disposable income squared	5.12E-09	1.08E-08	6.55E-09	3.11E-08
	Household size X Disp_income	-5.5E-05	4.01E-05	-0.00011	4.77E-05
L vector		λ		λ	
	Leisure	0.280595	0.024332	0.312801	0.030346
	Leisure2	0.000164	0.000173	0.000428	0.000198
	Leisure X Disposable income	1.36E-06	1.55E-06	-1.91E-07	2.59E-06
	Leisure X Age	-0.01438	0.001037	-0.01841	0.001297
	Leisure X Age squared	0.000176	1.51E-05	0.000225	1.84E-05
	Leisure X No. Children	-0.0191	0.01175	0.005966	0.003381
	Leisure X No. Children 0-6	0.007813	0.020605	0.00305	0.005703
	Leisure X No. Children 7-10	0.011513	0.022161	-0.00433	0.005772
Other					
	N. observations (N. single*7 alternatives)	22190		18270	
	N. single	3170		2610	
	LR chi2(17)	4055.02		3501.41	
	Prob > chi2	0		0	
	Pseudo R2	0.3287		0.3447	
	Log likelihood	-4141.03		-3328.12	