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ABSTRACT

School Choice Design, Risk Aversion, and Cardinal Segregation*

We embed the problem of public school choice design in a model of local provision of education. We define cardinal (student) segregation as that emerging when families with identical ordinal preferences submit different rankings of schools in a centralised school choice procedure. With the Boston Mechanism (BM), when higher types are less risk-averse, and there is sufficient vertical differentiation of schools, any equilibrium presents cardinal segregation. Transportation costs facilitate the emergence of cardinal segregation as does competition from private schools. Furthermore, the latter renders the best public schools more elitist. The Deferred Acceptance mechanism is resilient to cardinal segregation.

JEL Classification: I21, H4, D78

Keywords: school choice mechanisms, cardinal segregation, segregation, peer effects, local public goods

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Short Title: School Choice, Risk and Segregation.

1 Introduction

Many OECD countries use public school choice systems that assign children to schools in a centralized manner (Musset, 2012). However, the impact of these systems on students, schools, and neighbourhoods more broadly remains little understood. This paper seeks to fill this gap by connecting two important areas of the economic literature that study the impact of school choice on the educational landscape.

The literature on local public goods dates back to Tiebout (1956), though it wasn't until de Bartolomé (1990) and Epple and Romano (2003), among others, that the latter was explicitly applied to education. This literature endogenizes school quality through school finance or the peer group effect, but it simplifies the assignment problem by either matching children to their local school or allowing frictionless choice (i.e., with zero transportation costs and no capacity constraints).[1] Under realistic conditions, socioeconomic segregation arises between neighbourhoods and their public schools, though it can be avoided with frictionless school choice.[2] Epple and Romano (2003) suggest that this would also apply to a model in which public schools had limited capacity and overdemand was resolved through lotteries. However, their school choice mechanism lacks important details, such as what happens to children excluded from their first choice.[3]

In contrast, the literature on market design takes school quality and residence as given and focuses on the specific features of the algorithms that determine families' behaviour and final school

[1] Early contributions of the literature on local public goods explain how decentralized school finance can lead to income segregation across the school districts of a metropolitan area (e.g., Epple et al., 1984). More recent studies explain that the peer group effect and other neighbourhood externalities may trigger segregation across schools and their catchment areas within a single district, or between private and public schools, examining the equity, efficiency and policy implications of such phenomena (e.g. Bénabou, 1996; Epple and Romano, 1998; De Fraja and Martinez-Mora, 2014).

[2] Epple and Romano (2003) also study the impact of transportation costs on the outcome of open school choice and find them to be sufficient to generate residential and school segregation by income.

[3] Other important contributions to this literature include Bénabou (1993), who illustrates how socioeconomic segregation may create poverty traps and ghettos, and Nechyba (2000), who shows how the existence of private schools and private school vouchers may reduce socioeconomic segregation by severing the link between a household's location and the school the child attends.

assignments. Applied for the first time to school choice by Abdulkadiroglu and Sönmez (2003), the literature reveals the importance of the rules employed to resolve overdemand when limited school capacities make it impossible for every parent's first choice to be immediately satisfied. It formally analyzes the game generated by a centralized system in which families submit a ranking of schools and a set of rules then determines who is assigned to a school with overdemand and what options remain for rejected applicants.

We embed the mechanism design problem in a city model of centralized public school choice. In our model, there is a continuum of households characterized by their socioeconomic (or ability) type and endowed with preferences for student achievement and money. Schools combine student (home), peer and school inputs to produce education. Peer effects are determined endogenously by the school's student composition, while the quality of school inputs and the distribution of student types are exogenously given.[4] Observable exogenous quality differences across schools vertically differentiate them. We later extend the model to introduce additional realistic features of school markets: private school competition and preference for nearby schools, as well as school priorities for local students.

Our results centre on a specific source of school segregation that we term cardinal segregation. This arises when households with identical ordinal preferences react to the choice mechanism with diverse strategies and so end up with different school assignments. We show that, under the well-known Boston Mechanism (BM), school choice can be characterized as a choice between a risky lottery over schools and a safe school. Variance in the degree of risk aversion towards schools may, therefore, result in differences in the optimal applications to submit. When that variance takes the form of *decreasing absolute risk aversion over types (DARAT)*, such differences in optimal strategies may lead to an equilibrium where ex-ante identical schools are segregated by type. Intuitively, this condition requires that advantaged families be willing to take greater risks.

The existence of private schools exacerbates these effects and further hampers equality of

[4] In line with Epple and Romano (2011), we define the peer group effect as any influence that a student has on the learning of her class or schoolmates. A large and growing body of literature studies the empirical relevance of peer effects and the mechanisms through which they affect the educational process. There is a clear consensus that these effects are important, and that a "better" peer group enhances performance (Epple and Romano, 2011; Sacerdote, 2011).

educational opportunity by lowering the peer quality of the weakest public schools. Higher types, who can afford or value *private* schools, meanwhile have increased chances of getting their child into the best *public* schools. Particularly disturbing is the fact that the assignment to public, tuition-free schools is affected by differences in households' willingness to pay for a private school.

When households have a preference for nearby schools – due to transportation costs, say – this expands the parameters for which an equilibrium with cardinal segregation may occur. Here, a safer school offers parents an opportunity to avoid paying the cost of the child attending a distant school. Moreover, the peer quality of the safe school unambiguously worsens with transportation costs.

Cardinal segregation does not emerge in equilibrium when the Deferred Acceptance mechanism is used. There is a wide literature on the appealing properties of this mechanism: strategy-proofness, stability-constrained efficiency under strict priorities, and protection of nonstrategic families (see, among others, Gale and Shapley, 1962; Roth, 1985; Erdil and Sönmez, 2006; and Pathak and Sönmez, 2010).[5] Still, empirical papers including those by He (2017), Calsamiglia, Fu, and Güell (2018), or Agarwal and Somaini (2018) show that the Boston Mechanism performs better if parental welfare is considered.

Our paper isolates a novel and important dimension that differs across school choice mechanisms and that may be relevant to policymakers when comparing their performance. To this regard, there is an empirical literature that studies the impact of catchment areas on levels of segregation across neighbourhoods and schools.[6] Various aspects of school choice can affect segregation in schools. For example, several empirical studies show that lower-income families apply for lower-quality schools.[7] Such research demonstrates that lower-income families lack information and have lower

[5] As shown in Pathak and Sönmez (2013), in most cities around the world where choice mechanisms have been remodelled, the mechanisms have become less manipulable.

[6] A series of papers reviewed in Black and Machin (2010) provide evidence that the implementation of school choice reduces price differences across neighbourhoods with different levels of school quality. Oosterbeek et al (2018) empirically decompose segregation as arising from different sources such as school choice design or parental preferences, among others.

[7] For instance, Bobba and Frisancho (2016), Hastings et al. (2010), and Ajayi and Freedman (2017) show how families from disadvantaged backgrounds do not apply to high-performing schools, even in strategy-proof environments. However, they also show that a lack of information about their actual chances of being accepted plays a major role for many such families. Pallais and Turner (2007), Hoxby and Avery (2013), Smith et al. (2013) and Bowen et al. (2013) show similar results for colleges, where the system is not centralized.

perceptions of their ability and, as a result, their chances of being accepted. They also see the admissions process as a large bureaucratic burden that they are not willing to navigate. In this paper, we isolate an additional factor behind disadvantaged families'tendency to apply to lower-quality schools. More specifically, we show that even when types are equally rational and informed, the BM causes additional segregation due to a difference in the shape of the utility function, especially if there are complementarities between types and school productivity or preferences.

In many cities, giving priority to families living in a school's catchment area has been used to resolve situations of overdemand for a given school. In such circumstances, we show that the relevance of the student assignment mechanisms drastically declines: both the Boston and Deferred Acceptance mechanisms lead to intense socioeconomic segregation across the city's neighbourhoods and schools. We demonstrate that this can be the case even if schools do not reserve all their seats for local students.

Calsamiglia and Güell (2018) provide empirical evidence showing that priorities play a fundamental role in the final allocation of students to schools where residential priorities are enforced.[8] They also show that, while only 4% of the schools in Barcelona are private, 14% of families that adopt risky school choice strategies opt for a private school if they are not assigned to their preferred choice, suggesting that the risk-taking population is predominantly comprised of families with an outside option. Although the authors do not consider peer effects or residential choices explicitly, the segregation effects that we identify in the BM with outside options seem empirically plausible in light of their results.

Segregation is of concern to governments and social actors. The principal worry is that it inhibits social cohesion and equality of opportunity by increasing achievement gaps (Card and Rothstein, 2007; Billings et al., 2014), raising inequality, and reducing integration; see Wouters (2017) for a review. Durlauf (1996), for instance, explains how socioeconomic segregation can perpetuate income inequality across generations. In terms of efficiency, the literature on peer effects and tracking is still inconclusive as to the optimal mix of students (see Carrell, Sacerdote, and West, 2013; and Burgess and Platt, 2018). One of the main proposed instruments to control the mix of

[8] This the case in most OECD countries (OECD, 2012).

students in schools has been to introduce reserves or quotas for different groups, as advocated by Abdulkadiroglu and Sönmez (2003) and Echenique and Yenmez (2015). Wouters (2017) analyzes the impact of implementing such reserves in Flanders at the kindergarten level. Oosterbeek, Sovago, and van der Klaauw (2018) identify the different channels through which school choice has affected segregation in Amsterdam. Basteck and Mantovani (2018) show experimentally that individuals with lower cognitive ability get assigned to worse schools more often under the BM than DA, resulting in some degree of segregation of cognitive ability. In this paper, we show how cardinal segregation may arise under the Boston Mechanism, resulting in segregated schools even when these are ex-ante identical, and even under circumstances when there are no priorities, reserves, or quotas, and assuming the same cognitive abilities across types.

Two recent pieces of related theoretical work are especially relevant to our study. Ongoing research by Cantillon (2014) suggests that group admission quotas can avoid the emergence of segregation when preferences are endogenously determined by peer quality. Avery and Pathak (2015) compare the heterogeneity of the schools of a city when the school assignment is neighbourhood-based to that emerging with flexible choice, in a setting with a residential choice between the city and an adjacent one. They find that choice narrows the quality gap between the best and the worst schools.

More generally, the expansion of school choice responds to other socioeconomic concerns. Advocates who defend expanding school choice argue for its potential to be "a rising tide that lifts all boats," allowing equal access to higher-quality schooling for all. A central argument supporting this view is that choice infuses competition into the market, pushing schools to be more productive.[9] Another contention is that affluent families always had choice since they could afford private schooling or housing in an expensive area. Thus, school choice could improve equity by expanding choice to disadvantaged households (see, e.g., Friedman, 1955; Hoxby, 2003).[10] In sharp

[9] While several theoretical contributions explain why school competition may *harm* school productivity in the presence of reputation effects or asymmetric information (De Fraja and Landeras, 2004; MacMillan, 2004, MacLeod and Urquiola, 2015, but see also Hoxby, 1999), recent empirical evidence supports the existence of positive productivity effects of school competition (see Hoxby, 2000, 2003, 2007; Rothstein, 2007; Gibbons et al., 2010, OECD, 2014).

[10] Indeed, it has been shown that, under certain stylized conditions, specific forms of school choice could be the solution to school and neighbourhood segregation (Epple and Romano, 2003, 2008).

contrast, critics argue that expanding choice could exacerbate educational inequality and harm vulnerable students, *leaving them behind* in lower-quality schools and increasing segregation across institutions. Arguments on this side of the debate highlight that schools usually prefer wealthier households, better-off parents exercise choice more often and make more informed choices, and the choice sets of low-income households are more restricted since they may not be able to afford transportation and other indirect costs (e.g., Smith and Meier, 1995; Hastings, Kane and Staiger, 2010; Musset, 2012; OECD, 2012; Burgess, Greaves, Vignoles and Wilson, 2015).

The rest of the paper is organized as follows. Section 2 presents the baseline model, the main assumptions, and the assignment mechanisms. Section 3 discusses the relationship between risk and strategic manipulation in the Boston Mechanism and presents our main result. Section 4 introduces competition from private schools. Section 5 considers preferences for geographical proximity of the school. Section 6 briefly assesses the presence of residential priorities, taste shocks, and non-strategic players. Section 7 explores Deferred Acceptance, an alternative mechanism that avoids cardinal segregation. The concluding section provides a brief summary and discussion of our results. The appendix contains one proof and an illustrative example with a common CES achievement production function. An Online Appendix includes long proofs, a model with more than three schools, an extension of Section 6, and other numerical examples.

2 Baseline Model and Concepts

A population of households (or agents, families, and students) with mass normalized to 1 lives in a city. Every household consists of a parent, who makes decisions, and a school-aged child. Households are *uniformly* distributed along a single-dimension type $t \in [0, 1]$ that represents the socioeconomic type, with greater t corresponding to wealthier households.[11]

The city is divided into three equal-sized neighbourhoods, indexed with $j = 1, 2, 3$, each hosting

[11] Our interpretation of this model of the household population closely follows Bénabou (1996): t measures parental human wealth, which determines parental income, $y(t)$, with $y'(t) > 0$, as well as the household's ability to benefit from education – determined by the availability of parental and home inputs and the child's school readiness. See the numerical examples in the appendix and online appendix.

a public school s_j with capacity mass $1/3$.[12]

Following the empirical literature (e.g., Coleman et al., 1966; Rivkin et al., 2005; Epple and Romano, 2011; Sacerdote, 2011, Hanushek, 2020) we assume that a student's educational achievement depends on a combination of school inputs (denoted by Δ), peer inputs (q) and home or personal inputs (t). Schools may, therefore, differ in up to three dimensions: the quality of the student body, the quality of school inputs, and the cost of attendance. Let $c_j(t)$ denote the latter for a child of type t attending school j.[13]

Peer effects are central to our analysis and endogenous: let Φ_j be the (nonatomic) distribution of student types being assigned to school j. We define **peer quality** as a function $q_j \equiv q(\Phi_j)$ that is continuous and *monotonic in the first-order stochastic dominance sense*: Φ_j FOSD Φ_i implies $q_j > q_i$.[14] This flexible functional form captures many characterizations of peer effects, from a standard setup in which quality equals the average type in the distribution of students to much richer ones where quality depends on the proportion of high types (Summers and Wolfe, 1977) and heterogeneity (Bénabou, 1996).

Other relevant school inputs, such as the quality of teachers or school leadership, are aggregated into an (exogenous) index Δ. We order schools (and their neighbourhoods) according to school input quality, $\Delta_1 \geq \Delta_2 \geq \Delta_3$, such that school s_1 represents top schools, school s_2 represents intermediate schools, and school s_3 represents bottom-ranked schools.

Every school has access to the same technology to produce education. Technology is captured by the educational production function $a(\Delta, q, t)$, which yields the school achievement of a child of socioeconomic type t who attends a school of quality Δ along with a group of peers of quality q. We assume that a is continuous, increasing in all its arguments, and twice-differentiable.

Households have identical preferences over combinations of a private composite good x (with price normalized to one) and the child's school achievement a. A utility function, $u(x, a)$, also continuous, increasing in both arguments and twice-differentiable, represents household prefer-

[12] The reader could conceptualize this model as a simplification of a more realistic environment in which schools are stratified *a priori* into three quality types: top, intermediate, and bottom.

[13] While public schools are tuition-free, attending a school may require incurring some cost in the form of transportation expenditure, higher housing prices, or private tutoring.

[14] The function is continuous according to distance $d(\Phi, \Phi') = \int_0^1 |\Phi(t) - \Phi'(t)|dt$. Also, Φ_j FOSD Φ_i if for all $t \in [0, 1]$ we have $\Phi_i(t) \geq \Phi_j(t)$, and inequality is strict for some t.

ences. The *induced utility function*, denoted with some abuse of notation by $v_j(t)$, is defined as $v_j(t) \equiv u(y(t) - c_j(t), a(\Delta_j, q_j, t))$. When no cost is charged or incurred from attending a school, we also use the shortened notation $u(\Delta_j, q_j, t)$. If a household of type t faces probabilities $\pi = (\pi_j)_{j=1,2,3}$ of having their child assigned to each school, the *expected utility* they obtain is given by $V(\pi, t) \equiv \sum_{j=1,2,3} \pi_j v_j(t)$.

We show that, under the BM, the choice between alternative strategies is ultimately one between lotteries. Thus, how families react to risk is key to our analysis. In particular, our results depend on preferences exhibiting a property we call **decreasing absolute risk aversion over types** (**DARAT**): Suppose all types prefer school i over school j, and the latter over school k. Consider two lotteries π and π' such that $\pi_j = 0$, $\pi_i \in (0, 1)$, $\pi_k \in (0, 1)$ and $\pi'_j = 1$. DARAT is satisfied whenever $V(\pi, t) - V(\pi', t)$ is increasing in t. DARAT implies that the willingness to take risks relating to schools is larger for higher types. [15] There are several empirically relevant specifications of preferences and technology that satisfy DARAT. These are described and discussed at the end of Section 3.

We also assume that the worst school is "sufficiently bad" based on exogenous inputs. We call this condition **sensitivity to failing school**, which is formalized as: $\frac{v_1(0)+v_3(0)}{2} < v_2(0)$ when $q_1 = q_2 = q_3$. This condition implies that school s_2 is preferred (by the lowest type) to a lottery that gives 1/2 probability to the best school and 1/2 to the worst one when peer qualities are identical. An extension of this assumption to a model with more than three schools can be found in the online appendix.

Let $s_i \succ s_j \succ s_k$ denote a (pure) ranking of schools where school s_i is ranked first, school s_j second and school s_k last. $R(t)$ denotes the (possibly mixed) ranking strategy of household t. R, the mapping from the type space to the probability space over pure rankings, is a *ranking strategy profile*. It is a cutoff strategy profile with cutoff t if all types below the cutoff play a (pure) ranking strategy while all types above the cutoff play a different (pure) ranking strategy.

We analyze two kinds of assignment mechanisms $M = BM, DA$, the Boston Mechanism and

[15] It is a common assumption that "wealthier people are willing to bear more risk than poorer people" (Mas-Colell, Whinston, and Green, 1995, page 192). This assumption has long been supported by empirical evidence, as in Friend and Blume (1975).

Deferred Acceptance, described at the end of this section. Throughout the paper, we mostly assume that these mechanisms do not use any neighbourhood priority criteria. Instead, a unique lottery number determines individual student priorities for all schools. We discuss the role of neighbourhood priorities in Section 8.

Given a mechanism M, a *(Nash) equilibrium* is a strategy profile R^* that is a best-response profile to itself: no $t-$type household can increase utility by submitting a different ranking of schools other than $R^*(t)$. Schmeidler (1973) guarantees the existence of an equilibrium among pure strategies in this game. Each equilibrium is further required to satisfy a *trembling-hand* stability criterion, from Selten (1975).[16] A **cutoff equilibrium** is a Nash equilibrium in which R^* is a cutoff strategy profile.

The equilibrium allocation of children to schools determines the peer groups and the endogenous quality component of schools, q_1, q_2, q_3. Since our results emerge most clearly when agents have identical ordinal preferences over schools, that is, when schools are vertically differentiated, we focus on parameterizations leading to equilibria where all agents prefer school s_1 to school s_2 and school s_2 to school s_3.[17]

We say that there is **segregation** between schools s_i and s_j if $q_i > q_j$, while there is *no segregation* between schools i and j if $q_i = q_j$. Qualitatively, we distinguish between two types of segregation. We have **cardinal segregation between schools i and j** if there is segregation between schools i and j and all types assigned to either i or j prefer school i to school j. This type of segregation is fully motivated by lottery preferences among agents that share the same ordinal preferences between schools. Strategic choices separate types that otherwise prefer the same schools.

We also compare assignments that generate different peer qualities in the same school j, say q_j

[16] R^* is a trembling-hand equilibrium profile if for any converging sequence of strategy profiles $R_n \to R$ there is a best response (to a given R_n) profile sequence BR_n that also converges to R^*. Convergence is defined over the Euclidean distance $\int_0^1 \|R^*(t) - R^n(t)\| \, dt$, where mixed strategies are expressed as (possibly degenerate) probability vectors with dimensions equal to the number of possible rankings over schools.

[17] The latter requires the difference in the quality of school inputs between schools s_2 and s_3 be sufficiently large to compensate for the difference in equilibrium peer qualities. Example A2 in the online appendix shows that cardinal segregation may still emerge with the BM when some households prefer school s_3 to school s_2.

and q'_j, with $j = 1, 2, 3$. Supposing that school s_1 has the highest peer quality under both assignments, we say that the former assignment is **more elitist** than the latter if $q_1 > q'_1$. Supposing that $q_i > q_j$, we say that the former assignment is **more segregative between schools i and j** than the latter if $q_i > q'_i$ and $q_j < q'_j$.

Boston (Immediate Acceptance) Mechanism vs. Deferred Acceptance.

Under the Boston Mechanism (BM), parents are required to submit a complete ranking of the available schools to the school authority. The following multi-round algorithm is then applied: in the first round, each student is considered for the school ranked by her parents in the top spot. If the number of students considered for a school exceeds its capacity (i.e., the school is *over-demanded*), applicants are then ranked according to a list of pre-established priorities and accepted following that order until the school's capacity is exhausted. The assignment is final for those accepted. In the second round (or any round $k > 1$), any student remaining unassigned is considered for her second- (or k-th) ranked school, if the school still has free slots after the previous round, and following the same logic: if a school is over-demanded, the students considered in a given round are admitted according to the priority order until the school's seats are filled. If the school does not have any free slots, the student remains unassigned until at least the following round. The algorithm only finishes when no student remains unassigned. In our model with three schools, any student rejected from her second option is automatically assigned to the school that has not rejected her yet.

The Deferred Acceptance mechanism proceeds almost identically to the Boston Mechanism. The sole, but crucial, difference is that *acceptance at each round of the DA algorithm is only tentative*, as opposed to definitive. This simply means that a student accepted to one school in a given round gains the right to be considered at the same school in the following round. She may well be rejected in further rounds since the school selects from a different pool of students at each step.

3 Risk and the Boston Mechanism

Manipulation of the Boston Mechanism is a response to the risk of an adverse school assignment. Why would a rational agent lie by ranking a school higher than the one she actually prefers? Such a strategy unambiguously entails a smaller probability of admission to the preferred school. However, it may still provide the agent with greater expected utility by decreasing the risk of an even worse assignment. The appeal of such safer choice, therefore, depends on the agent's risk aversion.

Consider the following example:

Example (SAFE strategy). *Suppose (i)* $\frac{\partial a}{\partial q} = 0$ *(i.e., no peer effects); (ii)* $\Delta_1 > \Delta_2 > \Delta_3$; *and (iii) (almost) every agent reports preferences over schools* ***truthfully****:* $s_1 \succ s_2 \succ s_3$*. The following trade-off emerges: an agent choosing to be truthful faces a one-third probability of being assigned to the worst school, a risk she can avoid by misreporting her preferences over schools* s_1 *and* s_2 *to secure a seat in the latter.*

In a **SAFE** strategy, an agent ranks one of the schools above another that she prefers but where her probability of admission is lower, in order to reduce the risk of being assigned a seat in the third, least-preferred alternative. An expected-utility maximizer may only wish to manipulate the BM by playing a SAFE strategy. In our model, a SAFE strategy implies ranking school s_2 in the first position, despite school s_1 actually being the favourite school.

The Choice between TRUE and SAFE. Our formal analysis confirms the intuition that attitudes towards school assignment risk play a pivotal role. The next lemma is proven in the Appendix:

Lemma 1 *The choice between TRUE and SAFE is equivalent to a choice between the safe school* (s_2) *and a lottery between the best* (s_1) *and worst* (s_3) *alternatives.*

We refer to such a lottery as the **Equivalent Lottery.** This equivalence results in the following lemma, which leads us directly to our main result:

Lemma 2 *If the preference profile satisfies DARAT, then any equilibrium strategy profile will be a cutoff strategy profile with higher types choosing to play TRUE and lower types choosing to play SAFE.*

Proof. It is implied by the definition of DARAT. ■

Theorem 1 *Under the conditions of the base model, i.e., DARAT and sensitivity to failing school, there exists a trembling-hand equilibrium with peer qualities* $q_1 \geq q_3 > q_2$ *and cardinal segregation between schools* s_1 *and* s_2*.*

Proof. DARAT establishes a cutoff strategy profile, where the cutoff type is indifferent between SAFE and TRUE. Sensitivity to failing school precludes the possibility of such cutoff type being 0 (in which case all peer qualities would be equal). Monotonicity of peer quality in the FOSD sense obtains the desired ranking of peer qualities. ■

Remark 1 *The theorem shows that the emergence of cardinal segregation requires not only DARAT but also that the worst school be bad enough relative to its alternatives. Otherwise, no agent would respond with SAFE to a strategy profile in which (almost) every other agent chooses TRUE, and the equilibrium would display no cardinal segregation.*

Remark 2 *The equilibrium is trembling-hand since it is best-response even after small changes in the strategy profile. Indeed, DARAT is a sufficient condition for the trembling-hand refinement to hold, since any cutoff equilibrium is trembling-hand.*

Remark 3 *A trembling-hand equilibrium without segregation cannot exist under the conditions of the previous Theorem.*

The DARAT property is central to our analysis. The following proposition identifies the empirically relevant specifications of the model that yield DARAT preferences. Notice that DARAT is not equivalent to the standard notion of decreasing risk aversion with one-dimensional preferences.

In our model, households have preferences over combinations of money and school achievement, while the latter depends on peer, school and home quality. Therefore, our definition of risk aversion requires specifying the source of risk.[18] It is consequently more cumbersome to detect DARAT in our setting than with one-dimensional preferences (in which case r decreasing characterizes DARAT). Indeed, as intuition suggests, DARAT results from decreasing risk aversion for educational achievement (hence the conditions in the first part of the proposition). But it also emerges when the benefits of peer quality relative to school quality are more substantial for higher types (hence the conditions in its second part). The reason is that the risky lottery in the Boston Mechanism provides assignments with better peer groups (since higher types are those taking the risk). Furthermore, even if none of the above conditions holds (e.g. with linear utility and CES production), DARAT will still characterize preferences if the household type and peer and school inputs are gross complements (and so the third part of the proposition). Let the analogue of Arrow-Pratt's coefficient of absolute risk aversion: $r_f(s;\cdot) \equiv -\dfrac{\partial^2 f/\partial s^2}{\partial f/\partial s}$ measure risk aversion of function f over attribute s. Denote the marginal rate of technical substitution between school inputs with $MRTS(\Delta_j, q_j) \equiv \dfrac{\partial a/\partial q}{\partial a/\partial \Delta}$:

Proposition 1 *If preferences are described by* $v_j(t) \equiv u(y(t) - c_j, a(\Delta_j, q_j, t))$*, the following cases guarantee that decreasing absolute risk aversion over types (DARAT) is satisfied.*

I. Preferences over money and achievement. *Assume* $a(q, \Delta, t) = f(q) + g(\Delta) + h(t)$*. Then, risk-aversion over achievement,* $r_u(a) \equiv -\dfrac{\partial^2 u(x,a)/\partial a^2}{\partial u(x,a)/\partial a} > 0$*, nonincreasing in* a *or* x *and strictly decreasing in one of them is a sufficient condition for DARAT preferences.*

II. Technology of Education Production. *Assume risk-neutrality of* u *over achievement,* $r_u(a) = 0$ *, and* $q_1 \geq q_3 > q_2$*. The following are then sets of sufficient conditions on the achievement function,* $a(\Delta, q, t)$ *for DARAT preferences:*

1. *Strict supermodularity over* (q, t) *and additive separability over* (Δ, t)

[18] When more than one attribute defines the outcome of a lottery, "attitudes towards multidimensional risk depend both on the shape of the indifference map under certainty and on the degree of concavity of the utility function representing preferences under risk. [...] It is well-known in the risk literature (see Kihlstrom and Mirman, 1974) that when lotteries are defined on many attributes, the properties of the Von Neumann-Morgenstern (VNM) utility function can be confused with changes in the degree of substitutability between goods, which is an ordinal property of individual preferences." (Eeckhout et. al., 2017, page 2.)

2. Supermodularity over (q, t), strict submodularity over (Δ, t) and $r_a(\Delta)$ non-increasing in t.

3. Supermodularity over (q, t) and (Δ, t) and all of $-r_a(\Delta)$, $-r_a(q)$ and $MRTS(q, \Delta)$ increasing in t (and at least one expression strictly increasing.)

III. Technology of Education Production. CES achievement function case. *Assume risk-neutrality of u over achievement, $r_u(a) = 0$, and that the achievement function displays constant elasticity of substitution: $a(\Delta, q, t) = A\left[\alpha\Delta^\rho + \beta q^\rho + \gamma t^\rho\right]^{K/\rho}$, with $A, K, \alpha, \beta, \gamma > 0$. In that case, preferences satisfy DARAT* if and only if *either $\rho < 0$ or $\rho > K$.*

Proofs can be found in the online appendix.

Proposition 1 isolates sets of sufficient conditions on preferences and technology for DARAT to hold: preferences satisfy DARAT if (at least) one of the following holds: (I) Household preferences display decreasing risk aversion for educational achievement, or risk aversion for educational achievement falls with current consumption. (II) Either better-off parents care more about peer quality relative to school input quality than worse-off parents or, if every parent has the same relative valuation of peer and school inputs, (III) when the type is a gross complement of peer and school inputs.

Concerning the first part of the proposition, while there is no available direct evidence on risk aversion for school achievement, it is a common assumption that "wealthier people are willing to bear more risk than poorer people" (Mas-Colell et al. 1995, p. 192). This assumption has long been supported by empirical evidence, as in Friend and Blume (1975).

Conditions in parts II and III of the proposition are harder to verify empirically since that entails the identification of the specific drivers of achievement. Identifying the determinants of school achievement and parental preferences over them poses significant empirical challenges, especially due to the unobservability of relevant school inputs. The literature's preferred option for identifying the impact of a school on individual achievement is to examine systematic changes in children's achievement, sometimes referred to the value-added of a school.[19] However, evidence on the specific

[19] See See Rivkin et al. (2005). Imberman and Lovenheim (2016) analyse whether parents are willing to pay more for additional school value-added.

mechanisms through which a school impacts student performance is limited. Efforts have focused on identifying teacher value-added and peer effects on performance and preferences.[20]

Particularly valuable for our results is the evidence in Rothstein (2006). He finds that sorting into schools occurs according to peer effects more than to differences in school effectiveness. Such result is consistent with the idea that higher types – better-off households with the ability to choose schools in housing market equilibrium – have a greater valuation of peer effects relative to school effectiveness.[21] Also of relevance to our model are recent studies that analyze how school preferences change across children of different backgrounds. They find evidence that more advantaged families care more about schools with greater value-added, even when their children are not more productive in them (Hastings et al., 2010; Burgess et al., 2015; Abdulkadiroglu et al., 2017; Walters, 2018).

4 Private school competition

Does private school competition alter segregation patterns across public schools?[22] In this section, we extend the baseline model to accommodate private school competition and compare the properties of the equilibrium with and without it.

Extended Model. The extended game of this section has an additional stage that takes place after the Boston Mechanism (BM) game. At that stage, profit-maximizing private schools may enter the market to attract some public school students. To model this problem, we adapt Epple and Romano's (1998) canonical model of public vs private school competition to an economy (i) with public schools of differing quality, and (ii) where students vary in a single dimension. Our

[20] Chetty et al. (2014, 2017) and Jackson (2012), for example, investigate the impact that having better teachers has on life outcomes. Jacob and Lefgren (2007) study parents' teacher choices to understand what they value in an instructor.

[21] Note that, even if the achievement production function of disadvantaged students might be more sensitive to peer quality gains, the induced utility function of advantaged students could still be more sensitive to peer quality gains. Parts II and III of proposition 1 require complementarity either in the production function or in preferences, both of which lead to cardinal segregation.

[22] We do not assess the initial choice between taking part in the mechanism or opting out of the public school system altogether; an issue that has already been extensively studied (Epple and Romano, 1998, 2008; Martinez-Mora, 2006; Epple et al., 2004). Only agents who participate in the public school assignment game are thus part of our model, and we can rule out the possibility that private schools attract students assigned to their top choice.

equilibrium concept includes the following three *additional* conditions: 1) A profile of decisions on whether to join the private school or not, contingent on the assigned public school $P^*(t) \in \{public, private\}^3$, $t \in [0,1]$ and which maximizes the utility of each household type, accompanies the ranking decision profile; 2) private school admission and tuition policies maximize profits; 3) no other private school can enter the market and make a positive profit.[23] Moreover, to follow Epple and Romano's formulation, we assume that peer quality, including that of the private school, is determined by the average student type at the school $q_j = \mathbb{E}_{\Phi_j}(t)$. The extended model explicitly accounts for the fixed (F) and variable (W) costs of running a school, assumed to be identical across schools. For a school serving a student body of size k, costs are given by:

$$C(k) = F + W(k),$$

with $F > 0$, $W' > 0$, $W'' > 0$.[24] Finally, for simplicity, we also assume that Δ_p, private school input quality, is exogenously given.[25]

The private school maximizes profits as a utility-taker and may condition tuition and admission policies on type.[26] That means it observes student types and preferences and chooses tuition and admission policies with the belief that it can attract any student by offering admission for tuition no greater than her reservation price. In contrast, public schools are passive, simply providing education to their assigned students who do not opt for the private alternative.

Following Epple and Romano (1998), we define the private school's **effective marginal cost** (EMC) of admitting a type t student as the sum of the marginal custodial cost and the peer-externality cost of admitting the student:

[23] For simplicity's sake, and given that our results do not depend on the number of private schools entering the market, we assume the cost structure is such that only one private school enters in equilibrium.

[24] Still, public finance is not modelled explicitly: public schools are financed with an exogenous city-wide proportional income or property tax so that $y(t)$ represents after-tax household income.

[25] We could follow Epple and Romano (2008) and model it as educational spending per student (spending above custodial costs) but that extension would complicate the analysis without adding to our insights.

[26] However, it cannot price discriminate between otherwise identical students who have been assigned to different public schools. In other words, the private school may practice third-degree price-discrimination (as in Epple et al., 2016) but not first-degree price discrimination (as in Epple and Romano, 1998, 2008). We consider this the most reasonable assumption in our setting: conditioning prices on public school assignment seems like a practice that would be precluded by consumer protection regulations.

$$EMC(t) = W'(k_p) + \eta(q_p - t).$$

$EMC(t)$ captures the effect the admission of a type t student has on the school's profits via its impact on the school's custodial costs, its peer quality, and what parents are willing to pay. $\eta(> 0)$ – the Lagrange multiplier associated to the school's peer quality constraint in the solution of the profit maximization problem – is equal to the average (per-student) increase in parental willingness to pay for the school due to a quality rise. Thus, ηk_p equals the growth in the school's revenues due to an increase in quality, while the change in quality due to admitting a student of type t is $(t - q_p)/k_p$. Consequently, the negative of their product, $\eta(q_p - t)$, is the peer-externality cost of admitting a student of type t. Notice that the peer-externality cost of a student with ability above the school average is negative. The reason is that the admission of better students rises the school's peer quality, allowing it to charge a higher price to everyone.

Before proceeding with the analysis, we introduce some additional notation: we denote the first-stage (BM) first-round cutoff of the extended game with $\hat{t}_p$. Additionally, we denote the reservation price of a student of type t who has been assigned to school s_j with r_j, $j = 1, 2, 3$. This is the maximum price her household is willing to pay to attend the private school. It is implicitly defined by:

$$u(y - r_j(t), a(\Delta_p, q_p, t)) \equiv u(y, a(\Delta_j, q_j, t)),\ \ j = 1, 2, 3.$$

Analysis. In our problem, demand for the private school stems from parents who are disappointed with their child's public school assignment. Therefore, the candidates most likely to opt-out of public education after the BM game are students assigned to the worst public school. Note the private school may still want to attract students from an additional group: those who ranked s_1 first but were assigned to s_2 instead.

Attracting students from the latter group is, however, less appealing to the private school for several reasons: first, their reservation prices are lower. Second, since the private school cannot price discriminate between identical students from different public schools, the school would need to charge tuition to students from the worst school at rates below their reservation price. Third, this group of students is smaller (except when every agent plays TRUE in the BM game) and vanishes when school s_2 fills its seats in the first round.

The following lemma establishes a sufficient condition whereby the private school would not attract any students from the safe school. The ensuing corollary characterizes profit-maximizing tuition and admission policies under such conditions.

Lemma 3 *(i) There exists a cutoff type, denoted with $t^* < 1/3$, such that, for any first-stage cutoff $\hat{t}_p \in [t^*, 1/3]$ (in which types $t > \hat{t}_p$ play TRUE and types $t < \hat{t}_p$ play SAFE) and the resulting matching of students to public schools, the solution to the second-stage profit-maximization problem entails the admission of students from school s_3 only. (ii) The threshold t^* falls (respectively, goes up) when Δ_3 goes down or Δ_2 goes up (respectively, when Δ_3 goes up or Δ_2 goes down).*

Assuming an interior solution to the profit-maximization problem[27], we can further state:

Corollary 1 *Consider any first-stage cutoff $\hat{t}_p \geq t^*$ (such that types $t > \hat{t}_p$ play TRUE and types $t < \hat{t}_p$ play SAFE). Then:*

(i) The solution to the second-stage profit-maximization problem is characterized by a (unique) cutoff type $l \in (0,1)$ satisfying $r_3(l) = EMC(l)$, such that the optimal admissions policy has

$$\alpha_2^*(t) = 0 \;\forall t \text{ and } \alpha_3^*(t) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{for } t \begin{Bmatrix} \geq \\ < \end{Bmatrix} l \iff \text{for } t \mid r_3(t) \begin{Bmatrix} \geq \\ < \end{Bmatrix} EMC(t)$$

(ii) The optimal private school's pricing policy is $p^*(t) = r_3(t)\ \forall t \mid \alpha_j^*(t) = 1$ (hence admitted students are indifferent between school s_3 and the private school.)

We are now ready to present our result.

Theorem 2 *Suppose that agents share their ordinal ranking of schools $s_1 \succ s_2 \succ s_3$ and that equilibrium of the baseline model satisfies $\hat{t} \geq t^*$. It follows that an equilibrium of the extended model with a private school exists with the first-stage cutoff $\hat{t}_p > \hat{t}$.*[28]

[27]The assumption is consistent with the evidence (e.g. Epple, Figlio and Romano, 2004), and means the private school prefers to be selective rather than admit all students assigned to s_3.

[28]Note that the impact of entry by a selective private school on the first-stage cutoff is qualitatively equivalent to the effect of a drop in Δ_3 in the baseline model.

Remark 4 *Fewer parents play the TRUE strategy in an equilibrium of the extended model than in the corresponding equilibrium of the baseline model. In other words, with private school competition, more parents give up the chance of entering the best school, following instead the SAFE strategy. Thus, the best public school becomes more elitist, and risk-takers enjoy a higher probability of obtaining a seat in the best school.*

Competition from private schools unambiguously widens the quality gap between the top and bottom public schools. On the one hand, competition from the private alternative weakens the quality of the worst public school, as explained. On the other hand, since a larger proportion of households strategically misrepresent their preferences, the best school becomes more elitist and of greater peer quality. In contrast, the quality of the safe school may fall or rise. It will improve or deteriorate depending on whether the school is oversubscribed or undersubscribed in the first round of the game without the private school.

Remark 5 *Entry by a private school may generate socioeconomic segregation across public schools where before there was none.*[29]

Remark 6 *Suppose private schools were not allowed to price discriminate between students. Would private school competition have the same qualitative impact? With uniform pricing, lower types extract less surplus from private schooling and are the last to opt for the private option. They also impose a greater peer-externality cost so that the private school still prefers higher types. Moreover, the school has an additional incentive to be selective, since admitting students with a smaller willingness to pay requires reducing the price charged to everyone else. Hence the logic of the result does not change: if the school does not want to attract students assigned to school s_2, the equilibrium cutoff type in the baseline model will still prefer to play SAFE in the corresponding equilibrium with the private school.*

[29] Example A1 in the online appendix illustrates this point. In this example, our sensitivity to failing school assumption fails to hold and so no cardinal segregation emerges in the baseline model. With private school competition, however, the quality of school s_3 deteriorates, which induces about 6% of parents to play the SAFE strategy, resulting in cardinal segregation.

5 Preference for nearby schools and endogenous location

In this section, we discuss an extension of our baseline model that includes transportation costs, local housing markets, and residential location choices.

A strategy in this extended model consists of two elements: a specification of the neighbourhood residential choice and a ranking of schools.[30] An equilibrium of this extended model not only comprises the best response of every agent according to the strategy profile but also requires residential market clearing. Therefore, it includes a set of equilibrium rents, one per neighbourhood, such that each family's strategy choice is optimal and exactly 1/3 of the population resides in each neighbourhood. By Schmeidler (1973) again, we can safely claim that there is an equilibrium in pure strategies.

To facilitate the analysis, we assume preferences display **quasilinearity** in consumption: $u(x, a) = x + z(a)$. Consumption is reduced by a transportation cost c whenever the child attends a school that is outside of her neighbourhood of residence, as well as by the residential rent in the chosen neighbourhood. Finally, we assume **attribute-type complementarity**, that is, complementarity between the attributes of a school and individual types in the (utility from) academic achievement function: $\frac{\partial^2 z}{\partial q \partial t} > 0, \frac{\partial^2 z}{\partial \Delta \partial t} \geq 0$.

We consider three ranges of transportation costs. The first encompasses small transportation costs that do not overcome the input advantage that school s_2 has over school s_3. The second case involves moderate costs, which may overcome the input advantage of school s_2 over school s_3 for some types, but does not negate the advantage that school s_1 has over school s_3. The last case considers high transportation costs that are sufficient to offset any potential quality differences among schools. Not surprisingly, such an extreme case leads to what is known as *positive assortative matching (PAM)*: the lowest third of types live in neighbourhood 3 and attend school s_3, the upper third live in neighbourhood 1 and attend school s_1, and intermediate types live in neighbourhood 2 and attend school s_2.

Theorem 3 *Assuming DARAT, sensitivity to failing school, quasilinearity, and complementarity*

[30] A natural timing in this model would require households to make location decisions first and then play the school choice game. We make this double decision simultaneous for the sake of simplicity. We do not lose generality because, for every equilibrium of our simplified model, there is an equilibrium path in the extended game.

attributes-type:

1) There is a value of c, $c^* > 0$*, such that for every* $c \in (0, c^*]$*, a (trembling-hand) cutoff equilibrium with cutoff* $\hat{t}$ *exists under the Boston Mechanism. Compared to the case with* $c = 0$*, this equilibrium yields a lower peer quality for school* s_2*.*

2) Moreover, there is another value of c, $0 < c^{**} < c^*$*, such that all cases with* $c \in [c^{**}, c^*]$ *yield the lowest possible peer quality for school* s_2 *(i.e.,* $\hat{t} = 1/3$*).*

3) The existence of an equilibrium displaying positive assortative matching requires $c > c^*$*.*

While all the proofs can be found in the online appendix, here we highlight several intuitions. It is paramount to understand which neighbourhood residential market becomes the cheapest in equilibrium. The answer depends on the cutoff $\hat{t}$ under consideration. When the cutoff is below 1/3, neighbourhood 2 becomes the cheapest.[31] Types playing TRUE would constitute a mass above 2/3 and would have to spread across all three neighbourhoods (and be indifferent among all of them.) Since school s_2 is the least likely allocation for them, neighbourhood 2's rent should be lowest. When the cutoff is above 1/3, neighbourhood 3 becomes the cheapest. If it were more expensive than neighbourhood 1, no types playing TRUE would prefer neighbourhood 1 to neighbourhood 3 for residence purposes, leading to excess supply in neighbourhood 3. Analogously, neighbourhood 3 housing cannot be more expensive than in neighbourhood 2. The implication is a tendency of $\hat{t}$ towards 1/3. Cutoffs below 1/3 give a premium to the SAFE strategy, which in turn raises the cutoff. Cutoffs above 1/3 deliver a premium to playing TRUE,[32] which lowers the cutoff.

In summary, small to intermediate transportation costs damage the safe school (school s_2) by reducing its peer quality. This is because more students tend to rank school s_2 in the first position when it is under-demanded, while fewer rank it first when it is over-demanded due to housing market effects. The existence of positive assortative matching equilibria is incompatible with sufficiently low transportation costs. A preference for neighbourhood schooling thus does nothing to deter segregation under the Boston Mechanism.

[31] Transportation costs facilitate the emergence of cardinal segregation in BM. The SAFE strategy provides full insurance, not only against the child being assigned to the worst school but also against having to incur transportation costs.

[32] The equilibrium cutoff type is indifferent between SAFE and TRUE and across all residential choices. We can thus assume that she chooses neighbourhood 3. School s_3 is a more likely allocation for those who play TRUE, hence the premium for such strategy.

A series of papers have studied how the introduction of intra- and inter-district school choice affects housing prices across districts. Reback (2005), Ferreyra (2007), Black and Machin (2010), Brunner, Cho, and Reback (2012) and Gibbons, Machin, and Silva (2013) all find that reducing the link between housing and school assignment decreases income segregation across districts worldwide. This suggests that although distance to school does matter, reducing the link imposed by school assignment does, in fact, lead to children moving across districts to attend school if quality differentials are large enough. From an economic perspective, a family that is willing to pay a premium for a house near a high-quality school with a neighbourhood assignment rule would also be willing to pay some amount of transportation costs to cross district boundaries for that same school. Our transportation cost model does represent such a reality. This paper is the first to investigate how the interaction between the school choice mechanism and transportation costs may foster segregation across schools.

6 Other extensions: neighbourhood priorities and non-strategic players

One realistic extension to the baseline model is to introduce neighbourhood priorities. It is clear that prioritizing all seats in favour of residents yields positive assortative matching at the neighbourhood and school levels.[33]

In many real cases, factors such as how many seats are prioritized and the order in which seats are allocated affect residential priorities. Instead of considering every possible model, let us consider a small deviation from full neighbourhood priority (that is, where living in a neighbourhood "almost guarantees" applicants being allocated to that neighbourhood's school.[34] The assumptions of Sections 3 and 4 hold for the following proposition.

Proposition 2 *(Assuming DARAT, sensitivity to failing school, quasilinearity, and complementarity attributes-type.) For close to full neighbourhood priorities, the Boston Mechanism displays*

[33] Positive assortative matching at the neighbourhood and school levels also result from prohibitively high transportation costs. In that case, the residential market allocates full priority rights.

[34] A precise depiction of "almost full priority" is provided in the online appendix.

positive assortative matching in the housing market, but not in the resulting school assignment.

The online appendix contains a more technical depiction of the above result and its proof. This proposition means that segregation is greater in the residential market than between schools when priority structures are close (but not identical) to full neighbourhood priority.

It is, furthermore, worth considering the implications of the literature on strategic sophistication in school choice, for which Pathak and Sönmez (2008) provide a seminal contribution. Our model, however, delivers unclear conclusions with respect to a potential lack of strategic sophistication among lower types. While such a lack might be expected to foster segregation under the Boston Mechanism, segregation in the BM results from relatively low types playing SAFE strategically. Somewhat surprisingly, segregation is thus reduced between schools s_1 and s_2 if they play TRUE non-strategically at higher rates.

However, a model with more than three schools would introduce another source of segregation in the form of overrepresentation of low types in the worst school, as in the experimental set up designed by Basteck and Mantovani (2018). More than three schools would also make it possible for a non-strategic student to mistakenly apply for an over-demanded school in the second round.

7 Deferred Acceptance

Deferred Acceptance has been suggested as an appealing alternative in the school choice debate, most prominently by Abdulkadiroglu and Sönmez (2003). Here, we demonstrate that Deferred Acceptance is immune to cardinal segregation and also more resilient to other sources of inter-school segregation than the Boston Mechanism.

Theorem 4 *Using the Deferred Acceptance algorithm to assign children to schools guarantees that* ***no cardinal segregation*** *arises.*

Moreover:

(a) There is no segregation of students between any pair of schools in equilibrium of the baseline model.

(b) There is no segregation of students between schools s_1 and s_2 in equilibrium of the extended model with private schools.

Furthermore, under quasilinearity and attributes-type complementarity:

(c) In the extended model with a preference for nearby schools: for low enough transportation costs (i.e. $c \in [0, \bar{c}]$), DA generates no segregation, though BM does. For somewhat higher transportation costs (i.e. $c \in [0, \breve{c}]$, where $\breve{c} > \bar{c}$), the student assignment is less segregative with respect to schools s_1 and s_2 under DA, and school s_1 is less elitist than it is under BM. When c is large enough, DA leads to PAM whenever it happens under BM.

(d) In the extended model with a preference for local students and almost full priority, neither DA nor BM results in equilibrium assignments that are unambiguously less elitist than the other.

All proofs are found in the online appendix.

Deferred Acceptance is more resilient to segregation as a result of its strategy-proofness. Suppose parents expect that two *a priori* identical schools will have different peer qualities; since the mechanism is strategy-proof, everyone would rank these two schools according to those expectations. But that leads to a contradiction: there is no difference between the distribution of those students assigned to the higher-quality school and those who are rejected from it and so then apply to the lower-quality one. Thus, peer qualities will not be different.

It is worth noting, however, that although DA does not exacerbate segregation, families from a larger range of types choose private schools under DA, since the peer quality of the worst school before the last stage is lower under DA. Moreover, given that the best public school is better under BM than under DA, the former may be a preferred option if the goal is to retain high types in the public system in the first place.

Part (c) of the proposition indicates that the existence of some preference for nearby schools gives rise to segregation provided lower types care less about peer quality than about geographical proximity. Calsamiglia and Güell (2018) find that 21% of the families do not choose any of the 6 closest schools in the system when they are given the option to do so. Thus, while distance may be important, it is not the sole determinant of choice. However, several studies have shown that poorer families care more about distance than wealthier ones (Hastings, Kane and, Staiger, 2010; He, 2017; Agarwal and Somaini, 2018; Calsamiglia, Fu, and Güell, 2019). Even so, part (c) of the proposition notes that segregation decreases as c falls to zero, something that does not happen under BM. By continuity, then, for low transportation costs, we show that equilibrium cutoffs satisfy $t^{DA} < t^{BM}$, meaning that DA generates less segregation between schools s_1 and s_2 than the BM.

The comparison between the two mechanisms becomes ambiguous for higher transportation costs, in the sense that the two equilibrium cutoffs can no longer be ordered (i.e., $t^{DA} \leq t^{BM}$ is not guaranteed). This is intuitively explained by two forces working in opposite directions: on the one hand, agents under the BM consider the risk of being assigned to school s_3 when comparing the TRUE and SAFE strategies, while under DA they ignore such risk because of its strategy-proofness. This provides incentives for segregation under the BM (pushing t^{BM} above 0) but not under DA (keeping $t^{DA} = 0$). On the other hand, given the same cutoff, the peer quality of school s_2 is higher under DA than under the BM. This is due to the fact that acceptance to school s_2 is only tentative and may be revoked by the arrival of higher types that have been rejected from school s_1 in a previous round. This might trigger some intended segregation since low types might honestly prefer school s_2 (and neighbourhood 2) to school s_1 in equilibrium, given the transportation costs.

Part (d) states that, when priorities for local students are sufficiently close to "full priority for all seats," it is not clear whether DA is less elitist than the BM. In such cases, the details of the assignment mechanism do not matter, since they are dominated by the effects of the priority structure. A similar idea underlies the equality of outcomes under both mechanisms when transportation costs are prohibitive: they simply dominate the details of the assignment mechanism (see Epple and Romano, 2003).

Deferred Acceptance under idiosyncratic shocks.[35] Deferred Acceptance is not immune to segregation when we introduce additional elements in our baseline model. For example, consider a simple variation in which each household's preferences are independently affected by a preference shock. We illustrate a very simple case in which this preference shock affects school s_2's input by

[35] We thank a referee for urging us to consider this variation of our model as a source of segregation in DA.

a positive amount δ, with probability λ. Otherwise, school s_2's inputs remain at Δ_2.

Such a model can result in segregation in Deferred Acceptance. Suppose that, in equilibrium, agents' true ranking of schools is $1 \succ 2 \succ 3$ for all households unaffected by the shock, as well as those belonging to a type above some cutoff $t_\delta$who are affected by the shock. Households affected by the shock belonging to a type below t_δ have the ranking $2 \succ 1 \succ 3$. Unless $t_\delta \in \{0, 1\}$, the resulting DA allocation would yield $q_1 > q_2$ since Φ_2 would put more weight on types below t_δ than Φ_1 would. However, segregation along these lines is distinct from the notion of cardinal segregation in this study.

We do suggest, however, that for sufficiently low values of δ or λ, the BM would generate more segregation between schools s_1 and s_2. This follows a continuity argument, since the BM was already generating segregation when either δ or λ is zero.

We do not present an extensive analysis of such cases. The reader may notice that this is not particularly different, in a formal sense, from the analysis on transportation costs, which may indeed generate segregation in DA. Similarly, sufficiently low transportation costs result in more segregation (across schools s_1 and s_2) under the Boston Mechanism.

8 Summary and concluding remarks

In what follows, we briefly summarize our results and provide some final remarks.

Baseline model. In our baseline model with neither priorities nor other factors such as outside options (private schooling) or preference for nearby schools, the Boston Mechanism generates cardinal segregation *between* the best schools in the system, under the following two conditions: absolute risk aversion over achievement decreasing in household type, and sufficient vertical differentiation (embedded in our sensitivity to failing school assumption).

Competition from private schools. If we add private school competition, student assignment under the Boston Mechanism becomes more elitist compared to the base model. In particular, the peer quality of the top-quality school improves and the chances of top types being assigned to the best public school increase. Also, the peer quality of bottom-quality schools decreases.

Preference for proximity to the school. If we add a preference for nearby schools (e.g., due to transportation costs) to the base model, peer segregation between non-bottom schools under the Boston Mechanism becomes more likely, and the peer quality of intermediate-quality schools worsens (as compared to the base model). Moreover, segregation arises in the residential market. A sufficiently intense preference for nearby schools (i.e., dominating the effect of exogenous quality differences) generates positive assortative matching (PAM) both at the neighbourhood and school assignment levels.

Residential Priorities. If we add generic residential priorities to the base model, we find that for priority criteria sufficiently close to full priority for all seats, positive assortative matching (PAM) arises in the residential market, and the Boston Mechanism generates segregation close to PAM.

Deferred Acceptance. Deferred Acceptance is immune to cardinal segregation and so prevents cardinal segregation in the baseline model. With private schooling, DA is immune to segregation between public schools that do not lose students to the private alternative. In the model with a weak preference for nearby schools, Deferred Acceptance generates less segregation and less elitism than the Boston Mechanism. With a sufficiently intense preference for nearby schools or with residential priorities sufficiently close to full priority for all seats, Deferred Acceptance produces qualitatively similar results to the Boston Mechanism, which are arbitrarily close to PAM at both the residential and school assignment levels.

This paper introduces a theory of segregation in public schools with centralized school choice. In doing so, it endogenizes preferences and school quality and incorporates the role of private options, transportation costs, and the housing market. We show that the choice of assignment mechanism plays a crucial role in the resulting distribution of children across public schools and the degree of equality of opportunity offered by the education system. We do so in a parsimonious context, with no informational asymmetries or naïve players. We thus provide a solid theoretical underpinning for a novel equity concern that to date has yet to be explicitly addressed by policy-makers and dedicated empirical research. The paper remains silent about the welfare and fairness implications of the implied segregation, questions already explored by the literature (e.g. Arnott and Rowse, 1977; Bénabou, 1996).

9 Appendix

Proof. Lemma 1

Let R be the TRUE ranking and R' be the SAFE one. $b(t)$ is the best school for a student of type t, $s(t)$ denotes her safe (second-best) school, and $w(t)$ is her worst school. Type t agents prefer to play TRUE if and only if

$$\pi^R_{b(t)} v_{b(t)}(t) + \pi^R_{s(t)} v_{s(t)}(t) + \pi^R_{w(t)} v_{w(t)}(t) \geq \pi^{R'}_{b(t)} v_{b(t)}(t) + \pi^{R'}_{s(t)} v_{s(t)}(t) + \pi^{R'}_{w(t)} v_{w(t)}(t), \tag{1}$$

and play SAFE otherwise. We can rewrite this condition as follows:

$$\left(\pi^R_{b(t)} - \pi^{R'}_{b(t)}\right) v_{b(t)}(t) + \left(\pi^R_{w(t)} - \pi^{R'}_{w(t)}\right) v_{w(t)}(t) \geq \left(\pi^{R'}_{s(t)} - \pi^R_{s(t)}\right) v_{s(t)}(t), \tag{2}$$

or:

$$\frac{\pi^R_{b(t)} - \pi^{R'}_{b(t)}}{\pi^{R'}_{s(t)} - \pi^R_{s(t)}} v_{b(t)}(t) + \frac{\pi^R_{w(t)} - \pi^{R'}_{w(t)}}{\pi^{R'}_{s(t)} - \pi^R_{s(t)}} v_{w(t)}(t) \geq v_{s(t)}(t). \tag{3}$$

The two ratios on the LHS add up to 1 because, by definition, $\pi^R_{b(t)} + \pi^R_{s(t)} + \pi^R_{w(t)} = \pi^{R'}_{b(t)} + \pi^{R'}_{s(t)} + \pi^{R'}_{w(t)} (= 1)$, so that $\pi^R_{b(t)} - \pi^{R'}_{b(t)} + \pi^R_{w(t)} - \pi^{R'}_{w(t)} = \pi^{R'}_{s(t)} - \pi^R_{s(t)}$. Moreover, $\frac{\pi^R_{b(t)} - \pi^{R'}_{b(t)}}{\pi^{R'}_{s(t)} - \pi^R_{s(t)}} > 0$, since $\pi^R_{b(t)} > \pi^{R'}_{b(t)}$ and $\pi^{R'}_{s(t)} > \pi^R_{s(t)}$: given the nature of the Boston Mechanism, the chance of being accepted to a particular school is greater if it is ranked higher. To conclude the proof, note that

$$\frac{\pi^R_{b(t)} - \pi^{R'}_{b(t)}}{\pi^{R'}_{s(t)} - \pi^R_{s(t)}} > 1 \Leftrightarrow \pi^R_{b(t)} + \pi^R_{s(t)} > \pi^{R'}_{s(t)} + \pi^{R'}_{b(t)} \Leftrightarrow \pi^R_{w(t)} < \pi^{R'}_{w(t)} \tag{4}$$

implies TRUE is preferred ($\pi^R FOSD \pi^{R'}$). The reason is that the SAFE strategy in that case implies, along the usual smaller probability of being assigned to the best school, a larger probability of being assigned to the worst one. We can then express (2) as:

$$\pi_R v_{b(t)}(t) + (1 - \pi_R)\, v_{w(t)}(t) \geq v_{s(t)}(t),$$

with $\pi_R \equiv \min\{1, \frac{\pi^R_{b(t)} - \pi^{R'}_{b(t)}}{\pi^{R'}_{s(t)} - \pi^R_{s(t)}}\}$, since $\pi_R = 1$ entails that TRUE is preferred (since then the LHS simplifies to $v_{b(t)}(t)$ which is greater than $v_{s(t)}(t)$ by definition.) $\blacksquare$

Table 1: A complete example including baseline model, private school, preference for nearby schools and full residential priorities.

[INSERT TABLE 1 ABOUT HERE]

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