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IZA DP No. 12770

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Hans Gersbach
ETH Zurich, CEPR and IZA

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ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Skills, Tasks, and Complexity*

We introduce a new task-based framework to describe production. It focuses on the fact that certain tasks are too complex for many workers to perform. In such an environment, the relationship between wages of workers may significantly differ from their relative productivities. The labor market partitions itself into submarkets, in which wages and productivity are aligned, and high-skilled workers may obtain wage premia above their productivity level. The framework offers a variety of applications. For instance, the automation of low-complexity tasks tends to reinforce labor market segmentation. Moreover, comparatively small changes in a task's complexity, via new products, processes or automation, may have either a negligible or a large impact on wage inequality.

JEL Classification: O31, O38

Keywords: skill, technological change, tasks, complexity

Corresponding author:
Hans Gersbach
EHT Zürich
Zürichbergstrasse 18
8092 Zürich
Switzerland

E-mail: hgersbach@ethz.ch

* We would like to thank David Hémous, Jan-Egbert Sturm, Martin Tischhauser and seminar participants at ETH Zurich for helpful comments.

1 Introduction

For almost any worker, there exist jobs involving tasks that are too complex for the worker to perform.[1] We argue that such inability to perform complex tasks can have profound consequences for wages, segmentation of labor markets, and the incentives for and consequences of new products and processes.

Our framework involves task-based production in which tasks are characterized by their complexity level. Tasks of high complexity require higher skills than tasks of low complexity. Thus, our framework is built on the assumption that an individual endowed with a particular skill level cannot perform a task that is above a certain complexity level in a production process. We use this link between the skill level and the complexity of a task—which we call "*task-complexity*"—to capture skill requirements in production.[2]

Our model is based on three observations. First, empirical evidence (Autor et al., 2003; Frey and Osborne, 2017), and daily experience suggest that a task's complexity determines whether a worker with a given skill level can execute it successfully or not.[3] Second, workers with higher skills are not only more productive than lower-skilled workers, they can also execute a larger set of tasks which comprises higher complexities.[4] Third, with technological innovations, workers are subject to changes in labor demand, through job creation and destruction, and automation (Katz and Murphy, 1992; Davis et al., 1998). Different skill levels are impacted differently by such changes (Bauer and Bender, 2004).

Our main insights are as follows: The limitations of workers to perform complex tasks trigger various labor market patterns, such as labor market segregation with wage premia paid to the workers who are able to perform higher task-complexities. In such circumstances, the wage scheme is a monotonically increasing step-function along the skill dimension, featuring upward jumps whenever the labor market is segregated. This

[1] Of course, with training, the space of jobs which can be performed becomes larger, but this space typically is comparatively small in the short-term.

[2] The task-complexity model developed in this paper is a generalization of the model presented in Schmassmann (2018).

[3] Intuitively, not every individual in the economy can execute high complexity tasks, such as working as a lawyer, an engineer, or a mathematician. Of course, in practice there are many dimensions along which tasks can be ordered according to complexity. Typically, individuals specialize on one or more dimensions. In our paper high-skilled individuals can perform high complexity tasks only on some dimensions. For simplicity, we only look at one dimension in our baseline model.

[4] The different sets of tasks that can be performed by workers of different skill levels can lead to segregated labor markets, as we will see. Skill segregation was also documented by Kremer and Maskin (1996).

happens if there is a high demand for high-skilled workers.

Our framework can be used to study the consequences of innovations in products and processes. In particular, the introduction of high task-complexities through innovation directs the labor market towards (more) separation and increasing scale effects. The introduction of low task-complexities has the opposite effect. Increases in the lowest task-complexities, or the automation of the corresponding tasks, can generate a workforce that is unemployable.

Our framework is complementary to the task-based model developed by Acemoglu and Autor (2011). Instead of a continuum of tasks and the assumption that workers can perform any task—albeit with different productivities—, we assume that the specific complexity of a task determines which workers can perform this task and which cannot. This has different consequences for the way small skill differences translate into labor market outcomes and how technological progress affects wages. This complementarity of the frameworks will be discussed in detail in Appendix A.3.

In Section 2, we will introduce the basic model. In Section 3, we provide numerical examples. In Section 4, we show applications of the model. We conclude in Section 5.

2 The Model

2.1 Macroeconomic Environment

Labor and Skills

There is a continuum of households, each endowed with L^r units of labor, where the index r reflects the household's productivity when employed as worker in production. We will use the terms "household" r, "worker" r and "labor of skill level" r interchangeably. We assume that the skill level is distributed according to some density function $f(r)$—and cumulative distribution function $F(r)$—with support $\mathcal{R} = [\underline{r}, \bar{r}]$.

Product Space and Firms

The product space comprises two dimensions, an industry dimension denoted by i and a variety dimension denoted by j. Varieties are differentiated products within an industry. In each industry, there is an exogenously-given amount of firms n_i. Each firm produces one variety. A firm of an industry i producing a particular variety is simply called "firm" i, and a representative product variety of each industry can also be identified by i. A firm is atomistic and takes the wage rate as given. Thus, i equally indicates a representative product, a representative firm, and an industry. An industry i and variety j combination is denoted by (i, j) and simply called a product.

Production and Industries

In each industry i firms use a production process that consists of tasks of a particular complexity. We call the complexity level of a task *"task-complexity"*. It indicates the degree of difficulty to successfully complete a task, i.e., the higher the task-complexity, the more difficult the production process. Higher task-complexities require, ceteris paribus, higher skill levels from workers. For simplicity, we assume that there is a one-to-one mapping from task-complexities to industries.[5] Hence, the defining characteristic of an industry is its task-complexity. Thus, we can denote the task-complexity and its corresponding task also by i (and use it as an industry-identifier) and the set of task-complexity levels by $\mathcal{I} = \left\{\underline{i}, ..., \bar{i}\right\}$, where $\underline{i}$ and $\bar{i}$ denote the smallest and largest elements in the set, respectively. Hence, industry $\bar{i}$ ($\underline{i}$) offers the product that requires the highest (lowest) complexity of tasks to be produced.

We will assume that workers of different skill levels are substitutable for a certain task, as long as they have a skill level high enough to master the corresponding task-complexity level. In other words, either a worker is able to perform the task, or the worker is unable to perform the task, in which case he is not hired in the first place. A formal definition will be given below.[6]

Households and Consumption

Households derive utility from the consumption of products. Each product (i, j) is a variety j that belongs to industry i. The utility of household r is described by a nested CES-function

$$U^r\left(\left\{c^r_{i,j}\right\}_{(i,j)\in\mathcal{I}\times[0,n_i]}\right) = C^r, \tag{1}$$

where

$$C^r := \left[\sum_{i\in\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_0^{n_i} {c^r_{i,j}}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}.$$

[5] This assumptions is not necessary but eases the notation significantly, however, in general different industries could display the same task-complexity in production.

[6] This assumption is in stark contrast to the often-used Constant Elasticity of Substitution (CES) production in the skill-biased technological change literature. However, Graetz and Feng (2015) also assume that at task level, all factors of production are perfect substitutes. In that sense, if a worker is endowed with a skill level that is too low for a certain task-complexity, the worker can not be considered as a production factor in a production process with the corresponding task.

C^r equally represents the utility and the consumption basket of a household r. In the consumption basket, $c^r_{i,j}$ is the amount of product (i,j) consumed by household r. The parameter ψ_i is an industry-specific demand shifter, σ_I describes the elasticity of substitution *between* industries, and σ_v describes the elasticity of substitution between varieties *within* an industry. We assume that $\sigma_I < \sigma_v$, which means that products *within* an industry are closer substitutes than products *between* industries.

The wage household r receives is denoted by w^r and the profits he obtains from ownership are denoted by Π^r. We do not make any assumption about the distribution of firm ownership across households since equilibria can be determined independently of the distribution of ownership, and thus of the profit income distribution. The budget constraint of household r is therefore

$$\sum_{i\in\mathcal{I}} \int_0^{n_i} p_{i,j} c^r_{i,j} dj \leq L^r w^r + \Pi^r \ , \tag{2}$$

where $p_{i,j}$ denotes the price of product (i,j). The demand of household r for a product (i,j) is

$$c^r_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C^r \ ,$$

where $P_i := \left[\int_{n_i} p_{i,j}^{1-\sigma_v} dj\right]^{\frac{1}{1-\sigma_v}}$ and $P := \left[\sum_{i\in\mathcal{I}} \psi_i P_i^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}$ are the price indices for industry i and the aggregate price index, respectively. The derivation of household r's demand is presented in Appendix A.1. Total demand for the product (i,j) is

$$c_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \tag{3}$$

where $C := \int_{\mathcal{R}} C^r f(r) dr$ is the total consumption. Aggregation of budget constraints yields

$$PC := \int_{\mathcal{R}} L^r w^r f(r) dr + \int_{\mathcal{R}} \Pi^r f(r) dr \ ,$$

where PC denotes the total nominal consumption expenditures.

Production Technology

We assume that every firm holds a patent for its product. A representative firm i e.g. holds a patent for a product (i,j).[7] Specifically, if it hires an amount $l_i(r)$ labor of skill level r its output—denoted by x_i—is given by

$$x_i = \int_{r\in\mathcal{R}_i} \kappa_1(r)\kappa_2(i) l_i(r) dr \ , \tag{4}$$

[7] We discard the subscript j whenever convenient.

where $\kappa_1(r)\kappa_2(i)$ are the skill-dependent and complexity-dependent productivities of the employed labor, respectively. The functions $\kappa_1(r) : [\underline{r}, \bar{r}] \rightarrow \mathbb{R}_+$ and $\kappa_2(i) : \mathcal{I} \rightarrow \mathbb{R}_+$ have the following properties: $\kappa_1'(.) > 0$ (the higher the skill, the more productive is labor) and $\kappa_2'(.) < 0$ (the higher the task-complexity, the more difficult is production).

Several remarks are in order. First, in Appendix A.2 we provide an alternative functional form for the production function, based on the O-ring theory, that differs from $\kappa_1(r)\kappa_2(i)$ insofar that the two parts of the production function are (initially) not multiplicatively separable. Second, our results do not depend on the assumption that $\kappa_2(i)$ differs across industries, but typically industries differ with regard to the complexity of tasks which is accounted for by $\kappa_2(i)$. Essential is the following assumption:

Assumption 1 (Appropriate Skill Condition)
Labor of skill level r can only perform at task-complexities i if $r \geq \tilde{r}(i)$, where $\tilde{r}(i)$ is the threshold ability for the task-complexity (and industry) i.[8]

The function $\tilde{r}(i) : \mathcal{I} \rightarrow \mathbb{R}_+$ has the following property: $\tilde{r}'(.) > 0$. We assume that $\tilde{r}(\bar{i}) < \bar{r}$ always holds, i.e., there are always skills in the economy that can perform the task with the highest task-complexity. We denote by $\tilde{i}(r)$ the highest task-complexity a worker of skill r can perform, i.e., $\tilde{i}(r) = \max\{i \mid \tilde{r}(i) \leq r\}$.

Firms

The profit maximization problem of a representative firm i is

$$\begin{aligned} \max_{\mathcal{R}_i, p_i, \{x_i(r)\}_{r\in\mathcal{R}_i}, \{l_i(r)\}_{r\in\mathcal{R}_i}} \quad & \int_{r\in\mathcal{R}_i} \left[p_i x_i(r) - l_i(r) w^r\right] dr \ , \\ s.t. \quad & x_i(r) = \kappa_1(r)\kappa_2(i) l_i(r) \ , \\ & x_i = \int_{r\in\mathcal{R}_i} x_i(r) dr = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \\ & r \geq \tilde{r}(i) \quad \forall r \in \mathcal{R}_i \ , \\ & \mathcal{R}_i \subseteq \mathcal{R} \ . \end{aligned} \tag{5}$$

Firm i chooses a set of skill levels in production, denoted by $\mathcal{R}_i$, an amount of labor input, $l_i(r)$, for each skill level in the set $\mathcal{R}_i$, and a price p_i. Labor $l_i(r)$ produces quantity $x_i(r)$. Total output of firm i then is $x_i := \int_{r\in\mathcal{R}_i} \kappa_1(r)\kappa_2(i) l_i(r) dr$.

Firm i's maximization problem is solved by dividing it into two sub-problems:

(i) *Cost Minimization:* The firm chooses skill levels $\mathcal{R}_i$ suitable for production that minimize the cost per unit of output.

[8] In Appendix A.2 we show a micro-foundation of this assumption.

(ii) *Profit Maximization:* Given the minimal cost per unit of output, the firm chooses a price p_i to maximize its profits.

The price, in turn, determines the output and the amount of labor input. Note that the cost per unit of output might be minimized for different skill levels. Then a firm is indifferent between these skill levels since they are perfect substitutes. We next study each of these two sub-problems of firm i in detail.

(i) *Cost Minimization*

Firm i minimizes the cost per unit of output by choosing a subset of skills, $\mathcal{R}_i \subseteq \mathcal{R}$, that fulfills the minimization problem

$$\min_r \quad \frac{w^r}{\kappa_1(r)\kappa_2(i)} \qquad s.t. \quad r \geq \tilde{r}(i) \ .$$

Note that the firm takes Assumption 1 into account.

(ii) *Profit Maximization*

Given the cost-minimizing set of skill levels in production, $\mathcal{R}_i$, firm i chooses a price to solve its profit maximization problem given in (5). Without loss of generality, we can assume that all of firm i's production is performed by a single skill level, i.e., $\mathcal{R}_i = \{r\}$. Firm i's profit maximization problem then is

$$\begin{aligned} \max_{p_i} \qquad & p_i x_i - x_i \frac{w^r}{\kappa_1(r)\kappa_2(i)} \ , \\ s.t. \qquad & x_i = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ . \end{aligned}$$

This yields

$$p_i = \frac{\sigma_v}{\sigma_v - 1} \frac{w^r}{\kappa_1(r)\kappa_2(i)} \ . \tag{6}$$

The price equals the constant mark-up, $\frac{\sigma_v}{\sigma_v - 1}$, times the marginal cost, $\frac{w^r}{\kappa_1(r)\kappa_2(i)}$. Note that only the elasticity of substitution within a given industry is relevant for the price setting of a firm.[9] We next establish the equilibrium.

[9]The firm's price decision also determines the quantity produced and the labor employed in equilibrium.

2.2 Equilibrium

We start with the definition of an equilibrium.

Definition 1 (Equilibrium) *An equilibrium is*

(i) a set of skill levels $\mathcal{R}_i \subseteq \mathcal{R}$ for each representative firm i, with $i \in \mathcal{I}$, that this firm is willing to employ,

(ii) output levels, $\{x_i\}_{i\in\mathcal{I}}$, and labor, $\{l_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$, for representative firm i,

(iii) a set of consumption levels, $\left\{c^r_{i,j}\right\}_{(i,j,r)\in\mathcal{I}\times[0,n_i]\times\mathcal{R}}$, for each household r's consumption of each product (i,j),

(iv) a set of goods prices, $\{p_{i,j}\}_{i\in\mathcal{I}\times[0,n_i]}$,

(v) a set of wages, $\{w^r\}_{r\in\mathcal{R}}$,

such that

(A) x_i, $\{l_i(r)\}_{r\in\mathcal{R}_i}$, and p_i solve the representative firm i's profit maximization problem (5), $\forall i \in \mathcal{I}$,

(B) $\left\{c^r_{i,j}\right\}_{i\in\mathcal{I}\times[0,n_i]}$ maximizes the utility of the household r in (1), subject to the household's budget constraint (2) $\forall r \in \mathcal{R}$,

(C) goods markets clear for all products,

(D) labor markets clear, and

(E) $\mathcal{R}_i$ fulfills Assumption 1 for all $i \in \mathcal{I}$.

Before we derive the equilibrium, we relate skill level r to the productivity of the highest skill level in the economy,

$$\tilde{l}(r) = l(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})} \ . \tag{7}$$

We note that $\tilde{l}(r)$ expresses labor input, normalized by productivity across skill levels. We call $\tilde{l}(r)$ *"effective"* labor. Furthermore, we denote effective labor demand of a representative firm i by

$$\tilde{l}_i = \int_{\mathcal{R}_i} l_i(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})}dr \ .$$

Depending on the distribution of skills and the demand for these skills, different types of equilibria with the following properties may occur:

(i) Unemployment, if $\underline{r} < \tilde{r}(\underline{i})$.

(ii) Always integrated labor markets, if $\tilde{r}(\bar{i}) < \underline{r}$.

(iii) Integrated or disintegrated labor markets, if $\underline{r} < \tilde{r}(\bar{i})$.

If there is unemployment, the first type (i), then essentially a fraction of households $F(\tilde{r}(\underline{i}))$ cannot be used in production. The skills of these households are too low even for the task with the lowest skill requirement. Brynjolfsson and McAfee (2014) state:

> *"If neither the worker nor any enrepreneur can think of a profitable task that requires that worker's skills and capabilities, then that worker will go unemployed indefinitely [...]. In other words, just as technology can create inequality, it can also create unemployment. And in theory, this can affect a large number of people, even a majority of the population, and even if the overall economic pie is growing"* (p. 179).
>
> 10

Thus, if technological progress leads to increases in the complexity levels of the lowest task-complexities, such an unemployable class may emerge.[11] If labor markets are always integrated, the second type (ii), all labor can be used in the production process of all industries. Finally, if labor markets can be either integrated or disintegrated, the third type (iii), only labor with sufficiently high skill can be used in industries with high complexity levels.[12] Next, we take a closer look at the labor market clearing condition—henceforth (LMCC) represented by inequality (8). For this purpose we assume that all firms in industry i producing n_i different varieties employ the same labor distribution across skills.[13]

[10] Similarly Harari (2016) writes:

> *"In the nineteenth century the Industrial Revolution created a huge new class of urban proletariats [...]. In the twenty-first century we might witness the creation of a massive new unworking class: people devoid of any economic, political or even artistic value, who contribute nothing to the prosperity, power and glory of society. This useless class will not be merely unemployed—it will be unemployable"* (p. 379).

Furthermore Keynes (1931) famously states:

> *"We are being afflicted with a new disease of which some readers may not yet have heard the name, but of which they will hear a great deal in the years to come—namely, technological unemployment. This means unemployment due to our discovery of means of economising the use of labour outrunning the pace at which we can find new uses for labour"* (p. 325).

[11] We will analyze this in Section 4.

[12] Note that in this equilibrium some skill levels can also remain unemployed if (i) holds too.

[13] This is merely to simplify the formal description.

LMCC (Labor Market Clearing Condition)

$$\int_{\tilde{r}(\hat{i})}^{\bar{r}} \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L^r f(r) dr \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} n_i \tilde{l}_i (\mathcal{W}) \quad \forall \, \hat{i} \in \mathcal{I} \quad \text{(LMCC)} \; , \tag{8}$$

where $\mathcal{W} = \{w^r\}_{r \in \mathcal{R}}$ denotes the "wage scheme" that describes the wages workers of different skill levels will receive in an equilibrium. The left-hand side is the supply of effective labor that is able to at least perform the task-complexity of industry $\hat{i}$. The right-hand side is the demand of all firms of industries $i \geq \hat{i}$ for effective labor, given wage scheme $\mathcal{W}$. In an equilibirum, condition (8) must hold for every industry $\hat{i} \in \mathcal{I}$ and with equality for $\underline{i}$ (the least complex industry).

From (6) we know that a firm i is indifferent between producing a product with skill levels r' or r—under the condition that both are greater or equal to $\tilde{r}(i)$—if and only if the wages reflect their relative productivity differences, i.e., if

$$w^r = \frac{\kappa_1(r)}{\kappa_1(r')} w^{r'} \; . \tag{9}$$

Whenever this relation holds for all $r \in \mathcal{R}$, we say labor markets are integrated; thus we speak of an *Integrated Labor Market* equilibrium, henceforth ILM. However, if the demand for high-skilled workers exceeds their supply, given the wage scheme of an integrated labor market, then (8) must be violated. In such a case, we speak of a *Disintegrated Labor Market* equilibrium, henceforth DLM.

For both cases we assume the following wage scheme of *potential* wages:

$$\mathbf{W} = \left\{ \omega_i \frac{\kappa_1(r)}{\kappa_1(\bar{r})} \right\}_{(i,r) \in \mathcal{I} \times \mathcal{R}} \; , \tag{10}$$

where ω_i denotes the "scaling factor" in industry i and $\frac{\kappa_1(r)}{\kappa_1(\bar{r})}$ is the "productivity factor" of skill r. The wage scheme $\mathbf{W}$ is a menu of wages and describes the wage of a worker of skill r when he is working in industry i. The *realized* wages in an equilibrium is a subset of the *potential* wage scheme, i.e., $\mathcal{W} \subseteq \mathbf{W}$ and describes the wages of workers in the industries they are actually working. Note that the wages always depict nominal wages.

We denote the households' demand for representative product i by c_i and we use firms' optimal price choice (6) and wage scheme (10) to obtain

$$\begin{aligned}
p_i =& \frac{\sigma_v}{\sigma_v - 1} \frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} \quad \forall\, i \in \mathcal{I} \;, \\
P_i =& \frac{\sigma_v}{\sigma_v - 1} \frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I} \;, \\
P =& \frac{\sigma_v}{\sigma_v - 1} \left[\sum_{i \in \mathcal{I}} \psi_i \left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \right]^{1-\sigma_I} \right]^{\frac{1}{1-\sigma_I}} \;.
\end{aligned}$$

Using (3) and the price indices from above, the households' demand for representative product i is

$$c_i = \psi_i \left[\frac{\kappa_1(\bar{r})\kappa_2(i)}{\omega_i} \right]^{\sigma_I} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} n_{\hat{i}}^{\frac{1}{1-\sigma_v}} \right]^{1-\sigma_I} \right]^{\frac{\sigma_I}{1-\sigma_I}} C \;.$$

Goods market clearing implies that $c_i = x_i$. Then, effective labor demand of firm i is

$$\tilde{l}_i = \psi_i \omega_i^{-\sigma_I} \left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I - 1} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} n_{\hat{i}}^{\frac{1}{1-\sigma_v}} \right]^{1-\sigma_I} \right]^{\frac{\sigma_I}{1-\sigma_I}} C \quad \forall\, i \in \mathcal{I} \;. \tag{11}$$

Labor market clearing implies that total wages paid by firms must equal total wages earned by households, i.e.,

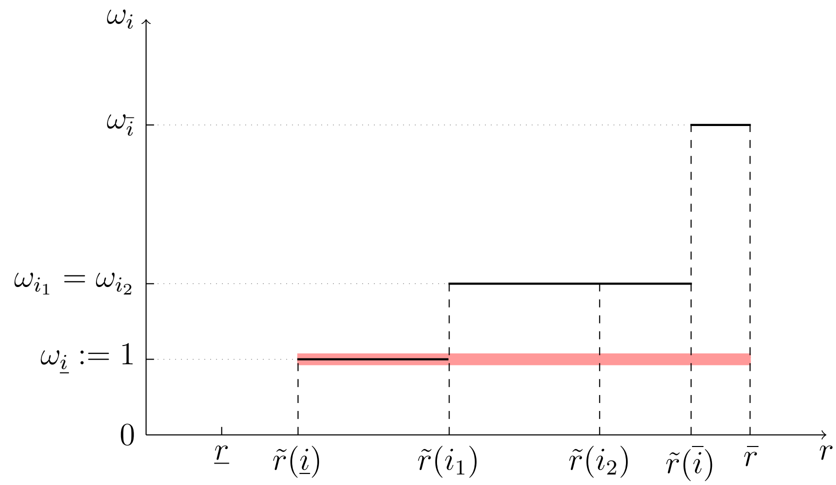
$$\sum_{i \in \mathcal{I}} n_i \omega_i \tilde{l}_i = \sum_{i \in \mathcal{I}} \int_{\tilde{r}(i)}^{\tilde{r}(i_+)} \omega_i \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L^r f(r) dr \;,$$

where i_+ denotes the next greater task-complexity than i in the set $\mathcal{I}$. Henceforth, we denote total wages by TW and the effective labor supply that is employable in industry i but unemployable in industry i_+ by $\tilde{L}_i^{\Delta} := \int_{\tilde{r}(i)}^{\tilde{r}(i_+)} \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L^r f(r) dr$. Hence, $TW = \sum_{i \in \mathcal{I}} \omega_i \tilde{L}_i^{\Delta}$. In equilibrium, goods markets must clear for every $(i, j) \in \mathcal{I} \times n_i$ and labor markets must clear, i.e., Equation (11) and (8) must be fulfilled, respectively. We next multiply (11) with the scaling factor ω_i and the number of firms n_i, and sum over all industries, yielding total consumption

$$C = \left[\sum_{i \in \mathcal{I}} \psi_i \left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \right]^{1-\sigma_I} \right]^{\frac{1}{\sigma_I - 1}} TW \;.$$

We normalize the scaling factor of the industry with the lowest complexity to one, i.e.,

Figure 1: Two wage schemes with different scaling factors



$\omega_{\underline{i}} := 1$ and we set $\omega_i := 0 \ \forall \ r < \tilde{r}(\underline{i})$. Thus, the lowest-skilled worker that can be employed always earns his productivity factor, i.e., $w^{\underline{r}} = \frac{\kappa_1(\underline{r})}{\kappa_1(\bar{r})}$. The scaling factors ω_i have to be non-decreasing in i. Otherwise, wages as a function of skill levels would not be increasing everywhere. Consequently, the *highest* skilled worker always earns a wage equal to the *highest* scaling factor, i.e., $w^{r} = \omega_{\bar{i}}$, as the productivity factor of this worker equals one. In Figure 1, we show two possible wage schemes in an economy with $\mathcal{I} = \{\underline{i}, i_1, i_2, \bar{i}\}$, where $\underline{i} < i_1 < i_2 < \bar{i}$. The red line shows a case where wages are only determined through the productivity factor, i.e., $w^r = \frac{\kappa_1(r)}{\kappa_1(\bar{r})} \ \forall \ r \in [\tilde{r}(\underline{i}), \bar{r}]$, thus the red line displays an ILM where $\omega_i = 1 \ \forall i \in \mathcal{I}$. The black step function shows a case where the scaling factor jumps twice (in the range of employed workers), at $\tilde{r}(i_1)$ and at $\tilde{r}(\bar{i})$, thereby displaying a DLM. Note that in this case $\omega_{\underline{i}} = 1 < \omega_{i_1} = \omega_{i_2} < \omega_{\bar{i}}$. In the scenarios shown, a measure of $F(\tilde{r}(\underline{i}))$ of workers is unemployed. The two cases in Figure (1) must represent different skill distributions.

We can call each set of task-complexities with a common scaling factor a *"group"*. A group is denoted by g. Thus, a group can encompass different task-complexities. We order groups according to skill levels and g_+ denotes the next higher group to g. We call a particular set of groups a group structure denoted by $\mathcal{G}$. Then, $\mid \mathcal{G} \mid$ equals the number of labor market separations plus 1 in such a group structure, i.e., the number of jumps in the scaling factor plus 1.[14] The scaling factor within a group is denoted by w_g and by construction,

$$\omega_{g_+} > \omega_g \quad \forall \ g \in \mathcal{G} \ , \tag{12}$$

[14] Thus, whenever $\mid \mathcal{G} \mid= 1$, there is no labor market separation and the economy is in an ILM equilibrium, and whenever $\mid \mathcal{G} \mid> 1$, the economy is in a DLM equilibrium.

i.e., the higher a group's index, the higher its scaling factor. The following proposition establishes the existence and uniqueness of the equilibrium.

Proposition 1 (Existence and uniqueness)
There exists a unique equilibrium with group structure $\mathcal{G}^\star$.[15]

The proof is given in Appendix B.1. In the proof we also establish the following properties regarding the equilibrium wages:

Corollary 1 (Wage Scheme)

1. *The uniquely determined wage scheme $\mathcal{W}^\star$ can be charactereized by*

   $\mathcal{W}^\star := \left\{\omega^\star(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})}\right\}_{r\in\mathcal{R}}$, *where* $\omega^\star(r) = \omega^\star_i$ *if* $r \geq \tilde{r}(i)$ *and* $r < \tilde{r}(i_+)$.

2. *If two workers of skill levels r and r' are in the same group, then $\omega^\star(r) = \omega^\star(r')$,*

We note that if two workers of skill levels r and r' are in the same group, their relative wages reflect their relative productivities. From the previous considerations we obtain the characterization of the entire equlbrium:

Proposition 2 (Equilibrium)
The equilibrium is characterized by a group structure $\mathcal{G}^\star$ and by

(i) $\mathcal{W}^\star = \left\{\omega^\star(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})}\right\}_{r\in\mathcal{R}}$,

(ii) $\mathcal{R}^\star_i \subseteq \{\{r\in\mathcal{R} \mid r \geq \tilde{r}(i)\} \cap \{r\in\mathcal{R} \mid \omega^\star(r) = \omega^\star_i\}\} \ \ \forall\, i\in\mathcal{I}$,

(iii) $p^\star_i = \frac{\sigma_v}{\sigma_v-1}\frac{\omega^\star_i}{\kappa_1(\bar{r})\kappa_2(i)} \quad \forall\, i\in\mathcal{I}$,
$P^\star_i = \frac{\sigma_v}{\sigma_v-1}\frac{\omega^\star_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i\in\mathcal{I}$,
$P^\star = \frac{\sigma_v}{\sigma_v-1}\left[\sum_{i\in\mathcal{I}}\psi_i\left[\frac{\omega^\star_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}$,

(iv) $TW^\star = \sum_{i\in\mathcal{I}}\omega^\star_i \tilde{L}^\Delta_i$,

(v) $\tilde{l}^\star_i = \frac{\psi_i \omega_i^{\star\,-\sigma_I}[\kappa_1(\bar{r})\kappa_2(i)]^{\sigma_I-1} n_i^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}}{\sum_{\hat{i}\in\mathcal{I}}\psi_{\hat{i}}\left[\frac{\omega^\star_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} n_{\hat{i}}^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}} TW^\star$,

(vi) $x^\star_i = \kappa_1(\bar{r})\kappa_2(i)\tilde{l}^\star_i$,

(vii) $\pi^\star_i = \frac{\tilde{l}^\star_i}{\sigma_v-1}$,

[15] An equilibrium is unique up to the exact allocation of skill levels within $\mathcal{R}_i$ to firms.

(viii) $C^\star = \left[\sum_{i\in\mathcal{I}} \psi_i \left[\frac{\omega_i^\star}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}\right]^{\frac{1}{\sigma_I - 1}} TW^\star$,

and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} TW^\star$,

where $\pi_i^\star$ *denotes the equilibrium profit of representative firm* i*.*

Given $i \in \mathcal{I}$, varieties, $\{n_i\}_{i\in\mathcal{I}}$, and demand shifters $\{\psi_i\}_{i\in\mathcal{I}}$, different skill distributions can have different labor market outcomes. Then, ceteris paribus, an economy endowed with a large share of high-skilled labor is more likely to be in an ILM equilibrium, or an equilibrium with only few and moderate scaling factor increases, whereas an economy with a small share of high-skilled workers tends to turn into a DLM equilibrium with high wage premia captured by high scaling factors.

We note that the equilibrium characterized by Proposition 2 is Pareto efficient. There are no externalities and all conditions of the first welfare theorem are fulfilled. Of course, an equilibrium may display large wage differences reflected in large scaling factors, but this is no contradiction to Pareto efficiency.

Note that within each group, the higher skills within the group naturally tend to work on higher complexities, whereas the lower skills within a group tend to work on the lower complexities.

3 Numerical Examples

In this section, we provide numerical examples to illustrate how the model works. First, we specify the functional forms and parameters of the model. In all examples, there are four industries and skills are distributed according to a Beta-distribution. Further, the skill-dependent productivity is linear in the skill level and the complexity-dependent productivity is inversely related to the task-complexity.[16] The industry set is $\mathcal{I} = \left\{\underline{i}, i_1, i_2, \bar{i}\right\} = \{1, 1.5, 2, 2.25\}$ and the number of firms in each industry is $N = \left\{n_{\underline{i}}, n_{i_1}, n_{i_2}, n_{\bar{i}}\right\} = \{1, 5, 5, 3.5\}$.

The set of skills employed in an industry i that cannot be employed in industry i_+ is called a *"category"*—or, equivalently, a skill category. There are four different task-complexities which transform the skill distribution into five categories which we will call: "Unemployed", "Low", "Low-Mid", "Mid-High", and "High". Table 1 presents the skill categories and their respective relative labor masses.

[16] A detailed description of our parameter assumptions for the numerical examples is given in Appendix C.

Table 1: Skill Categories

Unemployed	: $F(\tilde{r}(\underline{i}))$
Low	: $F(\tilde{r}(i_1)) - F(\tilde{r}(\underline{i}))$
Low-Mid	: $F(\tilde{r}(i_2)) - F(\tilde{r}(i_1))$
Mid-High	: $F(\tilde{r}(\bar{i})) - F(\tilde{r}(i_2))$
High	: $1 - F(\tilde{r}(\bar{i}))$

In all examples, the threshold skill levels are chosen such that there are no unemployed workers. Thus, we focus on the remaining four categories.

In Figure 2, we display the consequences of an increase in the task-complexity in industry i_2 from 2 to 2.1.[17] We show the resulting re-categorization of labor, the changes for the distribution of effective labor across categories, the scaling factors, and the wage scheme. The sky-blue graphs and bars describe State 1, where $i_2 = 2$, whereas the red graphs and bars describe State 2, where $i_2 = 2.1$. The vertical red lines mark $\tilde{r}(2.1)$. Figure 2a presents the skill distribution and the skill categories.

We observe that the increase in the task-complexity i_2 re-categorizes some of the *lowest-skilled* Mid-High workers as the *highest-skilled* Low-Mid workers. Figure 2b displays the amount of effective labor in each category. The increase of task-complexity i_2 increases the effective labor in the Low-Mid category and decreases correspondingly the effective labor in the Mid-High category, while the other two categories remain unchanged. Figure 2c displays the scaling factor of each category.

We next observe that in State 1 labor markets are separated twice. Low workers and Low-Mid workers form a group, i.e., Low-Mid workers are employed in industries $\underline{i}$ and i_1, whereas all other skill categories are only employed in one industry (Low in $\underline{i}$, Mid-High in i_2, and High in $\bar{i}$). In State 2, by contrast, labor markets are separated only once. Low and Low-Mid workers form a group, and Mid-High and High workers form another group. Thus, High workers are employed in industries i_2 and $\bar{i}$.

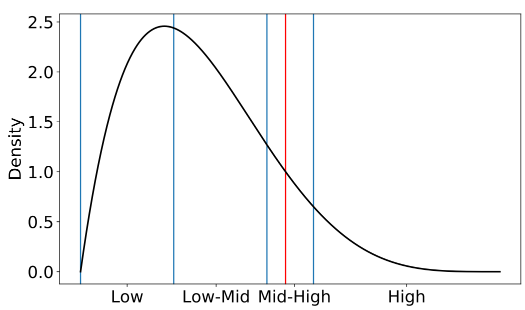
Figure 2d shows the wage scheme in both states. Note that the scaling factor scales wages in comparison to productivity relationships of the workers. Thus, the higher the scaling factor, the steeper the slope of the wage scheme. In State 1, there are two jumps in the wage scheme, whereas in State 2 there is only one jump. However, the jump in State 2 is significantly larger. Intuitively, the supply of effective labor in the

[17]Note that the exercise is symmetrical to a decrease in the task-complexity in industry i_2 from 2.1 to 2.

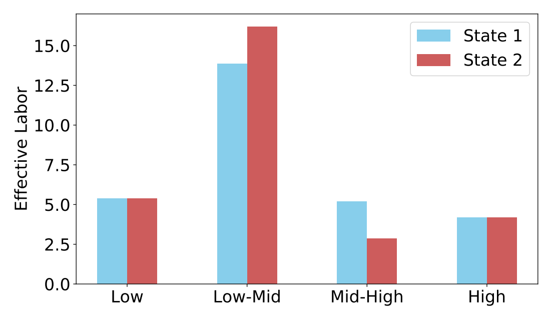
Mid-High category decreases significantly due to the increase in i_2. In contrast, the demand from industry i_2 for labor able to perform on i_2 remains high. The decrease of effective labor in the Mid-High category is sufficiently strong to integrate the Mid-High and High workers into one group. Thus, firms in industry i_2 become indifferent between employing Mid-High and High workers.

Figure 2: Equilibrium DLM - Increase in i_2 from 2 to 2.1

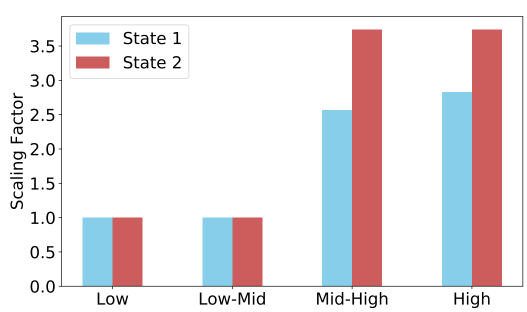
(a) Skill Distribution



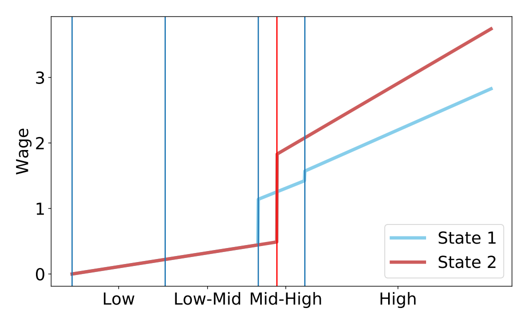
(b) Effective Labor by Category



(c) Scaling Factor by Category



(d) Wage Scheme



Notes: i_2 is the upper (lower) bound of the Low-Mid (Mid-High) skill group.

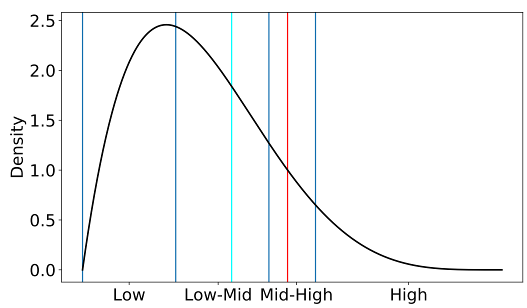
In Figure 3, we add to Figure 2 the impact of a decrease in i_2 from 2 to 1.8, which we call State 3 (displayed in dark blue). The vertical lines in cyan mark $\tilde{r}(1.8)$. The *highest-skilled* Low-Mid workers are re-categorized as the *lowest-skilled* Mid-High workers. Now, the increase in effective labor supply in category Mid-High drives down wages in this category. This effect is sufficiently strong such that the workers in categories Low-Mid and Mid-High form one group. This can be seen in Figure 3c, where the scaling factors of the two skill categories are the same. Equivalently, the wage scheme in Figure 3d displays no jump at $\tilde{r}(1.8)$ (the vertical line in cyan).[18]

[18]Note that if we further decreased i_2 (e.g., to 1.6) the economy would remain in the exact same

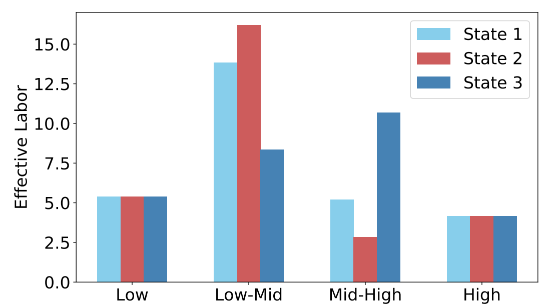
To sum up, in State 1, there are three groups, the categories Low and Low-Mid form one group. In State 2, there are two groups, the two lower and the two upper skill categories form each a group. In State 3, there are again three groups, where the middle skill categories form a group. Computing the gini coefficients, we can show that wages in State 2 are most unequally distributed, whereas wage inequality in States 1 and 3 is roughly the same (wage inequality being slightly higher in State 1).[19]

Figure 3: Equilibrium DLM - Decrease in i_2 from 2 to 1.8

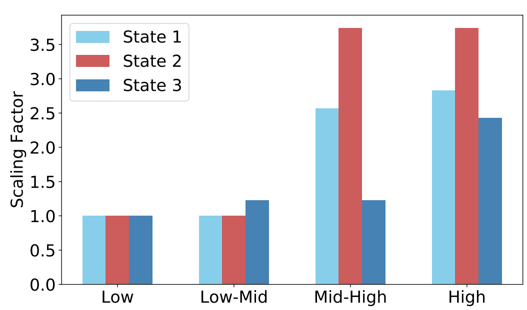
(a) Skill Distribution



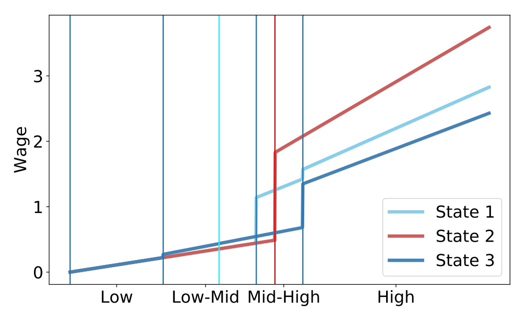
(b) Effective Labor by Category



(c) Scaling Factor by Category



(d) Wage Scheme



Notes: i_2 is the upper (lower) bound of the Low-Mid (Mid-High) skill group.

In Figure 4, we display the effects of an increase in the number of firms in industry i_2, n_{i_2}, from 5 to 9. Thus, more varieties are offered in industry i_2 which impacts the number of groups and the wage jumps. The sky-blue graphs and bars display State 1 from the previous figures, while State 4 (red graphs and bars) depicts the situation

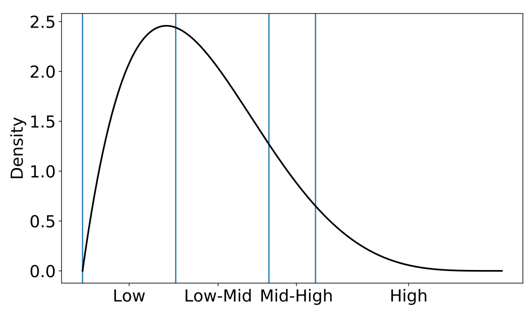
state. This is because a *decrease* in an interior task-complexity, i.e., a task-complexity that does not act as a bound of a group, only affects categories within one group. In this case, groups and scaling factors remain unaffected. Note that this must not be true for an *increase* in an interior task-complexity of a group.

[19]We obtain a gini coefficient of 0.44696951 for State 1, of 0.4782155 for State 2, and of 0.44650342 for State 3.

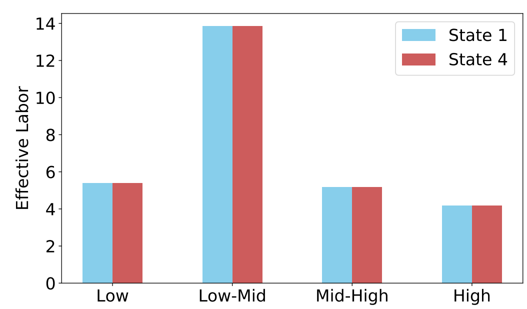
in which the firm number is increased. As a consequence of this increase, the demand for labor of category Mid-High increases, thereby pushing the scaling factor upwards. Another consequence of the increased demand for the Mid-High skill category is that the Mid-High category and the High category now form one group. The lower skill categories form the other group. Thus, in State 4, the Mid-High skill category earns higher wages, but also the scaling factor of the High skill category slightly increases, and there are only two groups instead of the three groups in State 1.

Figure 4: Equilibrium DLM - Increase in n_{i_2} from 5 to 9

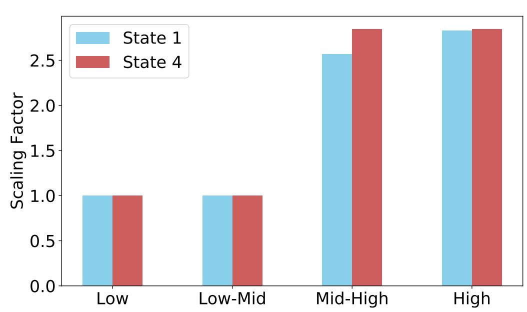
(a) Skill Distribution



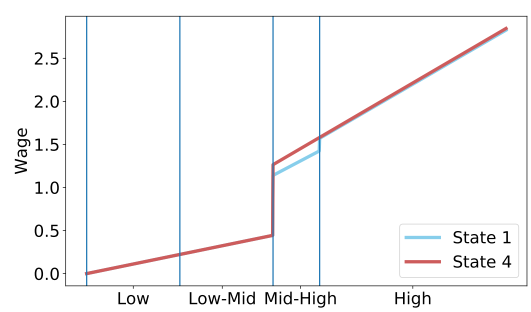
(b) Effective Labor by Category



(c) Scaling Factor by Category



(d) Wage Scheme



Notes: An increase in n_{i_2} increases the demand for the Mid-High skill group.

4 Applications

The presented task-complexity framework offers a variety of extensions and applications. We briefly provide an account of three possible applications: Automation, innovation to reduce task-complexity, and jobs.

Automation

Automation can be understood in the context of our model as the ability of machines to execute tasks so far performed by human beings. We examine what happens if automation takes place.[20] We distinguish several cases. First, automation could address the high-complexity tasks—when for instance software tools are developed by software machines or trading algorithms replace traders. Second, automation could also eliminate low-complexity tasks—when for instance driving is automated by self-driving cars. In both cases we encounter subtle general equilibrium effects which go in very different directions for both cases. While in the first case, scaling factors and wage inequalities will tend to decline, the opposite tends to occur when low-complexity tasks are automated. However, the precise consequences of automation depend on the interplay between the entire skill distribution and the distribution of task-complexities. Moreover, the incentives for innovation may be quite different in both cases. An R&D firm that anticipates that its blueprint will help to automate high-complexity tasks and reduce wage scaling factors will expect large gains from such automation blueprints. However, technological difficulties and costs for such innovations may also be high. Both, the gains and the costs are considerably lower for automating low-complexity tasks for a similar market size, but the gains may also be large if the market is large. To sum up, the task-complexity model may shed an interesting light on incentives and consequences of automation of tasks.

Task-facilitating Innovations and the Task Life-cycle

Another interesting aspect of technological progress are innovations that facilitate the performance of tasks and thus help workers to perform them (henceforth "task-facilitating innovations"). Together with automation of low-complexity tasks and the emergence of new production methods with high task-complexities, we could examine

[20]This happens if machines can perform these tasks at lower costs than human beings.

a task life-cycle. Such a task life-cycle thus describes the emergence of a task with relatively high task-complexity, the following descend of this task-complexity due to task-facilitating innovations, and the final automation of the task. For instance, suppose a new coating material has been invented. Initially, it requires special knowledge to mix the raw materials and the physical conditions for successful coating. Over time, the task is facilitated by machines controlling and adjusting the physical conditions which reduces the task-complexity of coating. Finally, the entire process is automated.

Jobs and Wages

Typically, a set of tasks can be divided among a set of workers which facilitates to match task-complexity to worker skills. However, this division has technological limits and subsets of tasks cannot be further divided. Such limits give rise to the concept of jobs. A job encompasses a set of tasks which cannot be split further into subsets and thus has to be performed by one person. For instance, a person that creates a homepage must be able to use the underlying program tools, must be able to structure the activities for the homepage in a collectively exhaustive and mutually exclusive way and must be able to write the text in the languages required. With jobs, the skill requirement is determined by the highest task-complexity of the tasks involved. Technological progress typically changes the boundaries of jobs and creates new jobs. This, in turn, may facilitate or make it more difficult to match skills and jobs. Whether for instance automation that may make it easier to redefine jobs will lower or increase the scaling factors of wages across skill groups is of course a central issue in the debate about the impact of automation on wages. Note that with jobs, the incentives to automate a task depend on the frequency this task features across jobs, the wage scheme, and the market size of the corresponding jobs.

5 Conclusion

We have developed a new task-based model that can shed light on labor market outcomes and how they are affected by changes in the complexity of tasks—or automation. The framework offers a variety of extensions and applications. First, the stark predictions about the segmentation of labor markets could be relaxed by allowing smooth employability possibilities of workers below the minimum skill-requirement. Second, a detailed analysis of possibilities and incentives to automate tasks of specific complexities is of crucial importance to understand how further automation (and robotization) will impact wage inequalities. Third, extending the setting to more than one dimension of task-complexities along which worker skills can be ordered would be another useful extension. With multiple types of task-complexity, besides minimal skill requirements for performing complex tasks, the match of skills to task-complexity types becomes essential for the working of the labor market. This is left for further research.

Appendix

A Derivation and Comments

A.1 Micro-foundation of Households' Demand

Lagrangian Derivation

To derive the demand of a household r, we solve the household's optimization problem by setting up the Lagrangian

$$\mathcal{L} = \left[\sum_{i\in\mathcal{I}} \left[\psi_i^{\frac{1}{\sigma_I-1}} \left[\int_{n_i} c_{i,j}^{r}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}$$
$$- \lambda \left[\sum_{i\in\mathcal{I}} \int_{n_i} p_{i,j} c_{i,j}^r dj - L^r w^r - \Pi^r\right] .$$

We take the derivative with respect to consumption of a single good (i, j)

$$\frac{\partial \mathcal{L}}{\partial c_{i,j}^r} = \frac{\sigma_I}{\sigma_I - 1} \left[\sum_{\rangle\in\mathcal{I}} \left[\psi_i^{\frac{1}{\sigma_I-1}} \left[\int_{n_i} c_{i,j}^{r}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{1}{\sigma_I-1}}$$
$$\frac{\sigma_I - 1}{\sigma_I} \left[\psi_i^{\frac{1}{\sigma_I-1}} \left[\int_{n_i} c_{i,j}^{r}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{-\frac{1}{\sigma_I}}$$
$$\frac{\sigma_v}{\sigma_v - 1} \psi_i^{\frac{1}{\sigma_I-1}} \left[\int_{n_i} c_{i,j}^{r}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{1}{\sigma_v-1}}$$
$$\frac{\sigma_v - 1}{\sigma_v} c_{i,j}^{r}{}^{-\frac{1}{\sigma_v}} - \lambda p_{i,j} = 0 .$$

The ratio of the marginal utility of consumption of good $c_{i,j}^r$ and of good $c_{i',j'}^r$ must be proportional to their respective prices

$$\frac{p_{i,j}}{p_{i',j'}} = \left[\frac{c_i^r}{c_{i'}^r}\right]^{\frac{\sigma_I-\sigma_v}{\sigma_I\sigma_v}} \left[\frac{c_{i,j}^r}{c_{i',j'}^r}\right]^{-\frac{1}{\sigma_v}} \left[\frac{\psi_i}{\psi_{i'}}\right]^{\frac{1}{\sigma_I}} , \tag{A.1}$$

where $c_i^r := \left[\int_{n_i} c_{i,j}^{r}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}$. Using equation (A.1) and the budget we can derive the following condition

$$c^r_{i',j'} = p_{i',j'}^{-\sigma_v} c_{i'}^{r\frac{\sigma_I-\sigma_v}{\sigma_I}} \psi_{i'}^{\frac{\sigma_v}{\sigma_I}} \left[\sum_{i\in\mathcal{I}} \psi_i^{\frac{\sigma_v}{\sigma_I}} \int_{n_i} p_{i,j}^{1-\sigma_v} c_{i,j}^{r\frac{\sigma_I-\sigma_v}{\sigma_I}} dj \right]^{-1} [L^r w^r + \Pi^r] \ , \tag{A.2}$$

and the consumption choice of the single good $c^r_{i',j'}$ depends on its price $p_{i',j'}$, and on the choice of consumption of the industry basket $c^r_{i'}$. We multiply optimality condition (A.2) with $p_{i',j'}$ and integrate over the variety domain. Using the industry price index $P_{i'} := \left[\int_{n_{i'}} p_{i',j}^{1-\sigma_v} dj\right]^{\frac{1}{1-\sigma_v}}$, and industry-specific expenditures $P_{i'}c^r_{i'} := \int_{n_{i'}} p_{i',j} c^r_{i',j} dj$, we rearrange and obtain

$$c^r_{i'} = P_{i'}^{-\sigma_I} \psi_{i'} \left[\sum_{i\in\mathcal{I}} \psi_i^{\frac{\sigma_v}{\sigma_I}} \int_{n_i} p_{i,j}^{1-\sigma_v} c_{i,j}^{r\frac{\sigma_I-\sigma_v}{\sigma_I}} dj \right]^{-\frac{\sigma_I}{\sigma_v}} [L^r w^r + \Pi^r]^{\frac{\sigma_I}{\sigma_v}} \ . \tag{A.3}$$

Next we multiply optimality condition (A.3) with the price index of industry i', $P_{i'}$ and sum over the industry set. The overall price index is $P := \left[\sum_{i'\in\mathcal{I}} \psi_{i'} P_{i'}^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}$ and the budget constraint must be binding, i.e., $\sum_{i'\in\mathcal{I}} P_{i'}c^r_{i'} = L^r w^r + \Pi^r$. Thus, rearranging yields

$$[L^r w^r + \Pi^r]^{1-\frac{\sigma_I}{\sigma_v}} P^{\sigma_I - 1} = \left[\sum_{i\in\mathcal{I}} \psi_i^{\frac{\sigma_v}{\sigma_I}} \int_{n_i} p_{i,j}^{1-\sigma_v} c_{i,j}^{r\frac{\sigma_I-\sigma_v}{\sigma_I}} dj \right]^{-\frac{\sigma_I}{\sigma_v}} \ .$$

We plug this expression into (A.3) and obtain

$$c^r_{i'} = \psi_{i'} \left[\frac{p_{i'}}{P}\right]^{-\sigma_I} \frac{L^r w^r + \Pi^r}{P} \ . \tag{A.4}$$

Using conditions (A.2), (A.3) and (A.4), the following allocation choice for any $(i', j) \in \mathcal{I} \times n_{i'}$ is derived:

$$c^r_{i',j} = \psi_{i'} \left[\frac{p_{i',j}}{P_{i'}}\right]^{-\sigma_v} \left[\frac{P_{i'}}{P}\right]^{-\sigma_I} \frac{L^r w^r + \Pi^r}{P} \ .$$

The latter condition can then be substituted into the definition of C^r to obtain

$$\sum_{i'\in\mathcal{I}} \int_{n_{i'}} p_{i',j} c^r_{i',j} dj = PC^r = L^r w^r + \Pi^r \ ,$$

and the optimal consumption allocation is subsumed in the condition stated in the main text

$$c^r_{i',j} = \psi_{i'} \left[\frac{p_{i',j}}{P_{i'}}\right]^{-\sigma_v} \left[\frac{P_{i'}}{P}\right]^{-\sigma_I} C^r \ .$$

Intuitive Derivation

A more intuitive, though less rigorously derived approach to obtain the optimal consumption allocation function is shortly outlined. The utility function (1) can be rewritten and rearranged into two separate CES-utilities, which are

$$U^r := \left[\sum_{\mathcal{I}} \psi_i^{\frac{1}{\sigma_I}} c_i^{r\,\frac{\sigma_I - 1}{\sigma_I}} \right]^{\frac{\sigma_I}{\sigma_I - 1}}$$

with

$$c_i^r = \left[\int_{n_i} c_{i,j}^{r\,\frac{\sigma_v - 1}{\sigma_v}} dj \right]^{\frac{\sigma_v}{\sigma_v - 1}} \quad .$$

We now proceed in two steps. First we derive household r's demand for industry i's products, c_i^r, given some demand level C^r. Second we derive the demand for a single product (i, j), $c_{i,j}^r$, given some demand level c_i^r. For households, prices $p_{i,j}$, P_i and P are taken as given. Performing these steps for the nested CES-utility function yields the consumption decision of household r across industries,

$$c_i^r = \psi_i \left[\frac{P_i}{P} \right]^{-\sigma_I} C^r \ ,$$

and the consumption decision within an industry for a variety,

$$c_{i,j}^r = \left[\frac{p_{i,j}}{P_i} \right]^{-\sigma_v} c_i^r \ .$$

Hence, household r's demand for the single product (i, j) is

$$c_{i,j}^r = \psi_i \left[\frac{p_{i,j}}{P_i} \right]^{-\sigma_v} \left[\frac{P_i}{P} \right]^{-\sigma_I} C^r \ .$$

A.2 Micro-foundation of Firms' Production

In this section we provide a micro-foundation for Assumption 1, that is based on a minimum-quality requirement in production.

O-ring Production Technology

We choose a specific functional form for our production function. This functional form is based on Kremer (1993) who developed the O-ring theory of economic development and has been derived in Schetter (2016). The O-ring theory assumes that the production process fails, if one task of the production process is *not* executed successfully.[21] We deviate from the O-ring theory in one aspect as we assume, that the production process—as an entity—is reflected by a single task, characterized by its task-complexity. Furthermore, we will assume that $\mathcal{R} = [\underline{r}, \bar{r}]$, where $0 < \underline{r} < \bar{r} < 1$.

In contrast to the main text, a firm also chooses a quality level of its production process, denoted by q ($q \in \mathbb{R}_+$). The quality level the firm chooses must be greater or equal to unity, i.e., $q \geq 1$. This is a functional *minimum-quality* constraint that is normalized to 1 across task-complexities (industries). The impact of quality in the production process on output is twofold: Quality linearly scales output, but also increases the complexity of the production process.

Specifically, firm i's expected output denoted by $\mathbb{E}[x_i]$ is given by

$$\mathbb{E}[x_i] = \int_{r \in \mathcal{R}} q[r]^{iq^\lambda} l_i(r) \ . \tag{A.5}$$

λ ($\lambda > 0$) is a parameter we explain below. The rationale for this formula is as follows: The production of a product with task-complexity i and quality q using skill level r is successful with probability $[r]^{iq^\lambda} \in (0, 1)$. Higher quality in the production process leads to greater output, conditional on its successful completion. Intuitively, on the one hand, higher quality increases the complexity of the production process and thus lowers the probability of its successful completion. On the other hand, higher quality increases the output of the production process, if successfully completed. The success probability of the production process is higher if the worker's skill level is higher. We define $\zeta_i(q) := iq^\lambda$, and $\zeta_i(q)$ is called *"total complexity"*. Total complexity is determined by task-complexity i, the chosen quality q, and the parameter λ.

We observe that the production function (A.5) is increasing in r and decreasing in i in accordance with our assumptions in Section 2. The higher the chosen quality of the production process the higher the total complexity of production, $\zeta_i(q)$. The parameter λ measures how much complexity rises when quality is increased. Note that $\zeta_i(q)$ can be convex or concave, depending on the value of λ.

We adapt Assumption 1 to the O-ring environment:

[21] Based on this theory Kremer (1993) explains assortative matching.

Assumption 2 (Appropriate Skill Condition: O-ring)
Labor of skill level r can only successfully perform total complexity if

$$\zeta_i(q) \leq -\frac{1}{\lambda \log(r)} \ . \tag{A.6}$$

Assumption 2 states that for total complexity $\zeta_i(q)$ a minimum skill level r is required. Intuitively, a firm chooses the labor it wants to employ, taking 2 into account, and accordingly chooses a certain quality for the production process, which must satisfy the minimum-quality constraint.[22] Observe that Assumption 2 imposes a constraint on the total complexity of the production process, $\zeta_i(q)$, and *not* directly on the task-complexity i itself.[23] Total complexity $\zeta_i(q)$ is minimized for $q = 1$, i.e., when the quality choice hits the minimum-quality constraint. Thus, the limiting total complexity level for a worker of skill level $\tilde{r}$ is $\zeta_i(1) = -\frac{1}{\lambda \log(\tilde{r})}$. This skill level constraints the set of skill levels that can be used by firm i, namely the skill levels $r \geq \tilde{r} = \exp(-\frac{1}{\lambda i})$.[24] Since a continuum of workers are employed by firm i, we can apply an appropriate version of the law of large numbers to (A.5) and thus dispense with the expectation operator in the rest of the paper.

Firms

A firm i selects a set of skills for production, denoted $\mathcal{R}_i$, and chooses an amount of labor input, $l_i(r)$, for each skill level in the set. This labor input produces quantity $x_{i,q_i(r)}$ at chosen quality $q_i(r)$. The firm also chooses the price p_i. Considering the skill set in production that the firm chooses, $\mathcal{R}_i$, total output of firm i then is $x_i := \int_{r \in \mathcal{R}_i} q_i(r) \, [r]^{i q_i(r)^\lambda} \, l_i(r)$. The profit maximization problem of a representative firm of

[22]One specific interpretation of Assumption 2 is that it summarizes institutionalized knife-edge conditions in the education and labor market system, such as licenses needed for acting as a doctor, a lawyer, an engineer, a teacher or a translator.

[23]We will discuss later that for the qualitative results Assumption 2 is not needed.

[24]This derivation is shown below in more detail.

industry i is then

$$\begin{aligned}
\max_{\mathcal{R}_i, p_i, q, x_i, q, l_i} \quad & \int_{r\in\mathcal{R}_i} [p_i x_i - l_i(r) w^r] \ , && \text{(A.7)} \\
s.t. \quad & x_i = \int_{r\in\mathcal{R}_i} x_i(r) = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \\
& x_i(r) = q\,[r]^{iq^\lambda}\, l_i(r) \ , \\
& q \geq 1 \ , \\
& r \geq \exp\left(-\frac{1}{\lambda i}\right) \quad \text{Assumption (2)} \ , \\
& \mathcal{R}_i \subseteq \mathcal{R} \ .
\end{aligned}$$

Firm i's decision problem is solved by dividing it into the following three sub-decisions:

(i) *Quality Choice:* The optimal quality in the production process $q_i(r)$ is chosen for every skill level r.

(ii) *Cost Minimization:* Given the optimal choice of quality, the firm chooses a set of skill levels $\mathcal{R}_i$ suitable for production that minimize the cost per unit of output.

(iii) *Profit Maximization:* Given the minimal cost per unit of output, the firm chooses a price, p_i.

The price, in turn, determines the output, $\{x_i(r)\}_{\mathcal{R}_i}$, as well as the labor input, $\{l_i(r)\}_{\mathcal{R}_i}$, of firm i. The cost per unit of output might be minimized for multiple skill inputs. Then, a firm is indifferent regarding hiring different skill levels to produce its output and hence, the skill levels are perfect substitutes. We next study each of the three sub-decision problems of firm i in detail.

(i) *Quality Choice*

Given a skill level r, the firm chooses a quality of the production process which best complements the skill level, by maximizing $q\,[r]^{iq^\lambda}$. The first-order-condition with respect to the quality choice yields

$$[r]^{iq^\lambda} = -\lambda q^\lambda i \log(r)[r]^{iq^\lambda} \ . \tag{A.8}$$

The optimality condition trades off higher production chances against a higher probability of failure in the production process.[25] From (A.8) and from the exogenously-given minimum-quality constraint ($q \geq 1$) we obtain a uniquely-determined cost-minimizing

[25] Quality could also be understood as product quality which the consumer values.

quality choice of the production process for each skill level, defined by

$$q_i(r) = \max\left\{1, \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}}\right\} \qquad \forall (i,r) \in \mathcal{I} \times \mathcal{R}_i \ . \tag{A.9}$$

Assumption 2 and (A.9) determine boundary values on the maximum task-complexity that can be produced by a certain skill level and, equivalently, on the minimum skill level that can be employed by firms in a particular industry. Specifically, these boundary values are $\tilde{i}(r) = -\frac{1}{\lambda \log(r)}$ and $\tilde{r}(i) = e^{-\frac{1}{\lambda i}}$, where $\tilde{i}(r)$ denotes the highest task-complexity that skill level r can master without violating Assumption 2. In turn, $\tilde{r}(i)$ denotes the minimal skill level needed to execute a task with complexity i.

(ii) *Cost Minimization*

Firm i minimizes the cost per unit of output $\frac{w^r}{q_i(r)[r]^{iq_i(r)^\lambda}}$, where w^r denotes the wage of a worker with skill level r. Using the optimal quality choice (A.9), the cost per unit of output when labor of skill level r is employed is $w^r\left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}}$. Firm i chooses a subset of skills, $\mathcal{R}_i \subseteq \mathcal{R}$, that fulfills the following minimization problem:

$$\min_r \quad w^r\left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}} \qquad s.t. \quad r \geq e^{-\frac{1}{\lambda i}} \ .$$

(iii) *Profit Maximization*

Firm i chooses a price to solve its profit maximization problem in (A.7), given (i) the optimal quality choice and (ii) the optimal set of skill levels in production, $\mathcal{R}_i$. We next assume that all of firm i's production is performed by a single skill level $r \in \mathcal{R}_i$ for which the cost per unit of expected output is minimal. Firm i's optimization problem then is

$$\begin{aligned} \max_{p_i} \qquad & p_i x_i - x_i w^r\left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}} \ , \\ s.t. \qquad & x_i = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \end{aligned}$$

with the solution

$$p_i = \frac{\sigma_v}{\sigma_v - 1} w^r[-e\lambda i \log(r)]^{\frac{1}{\lambda}} \ . \tag{A.10}$$

The price equals the constant mark-up, $\frac{\sigma_v}{\sigma_v - 1}$, times the marginal cost, $w^r[-e\lambda i \log(r)]^{\frac{1}{\lambda}}$. Note that only the elasticity of substitution within a given industry is relevant for the price setting of a firm.

Equilibrium

To obtain an equilibrium for the O-ring production, we can use the same wage scheme as in Definition 1, i.e.,

$$\mathcal{W} = \left\{ \omega(r) \left[\frac{\log(\bar{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \right\}_{r \in \mathcal{R}} . \tag{A.11}$$

Then, in accordance to (7) we transform labor demand into effective labor demand, i.e.,

$$\tilde{l}(r) = l(r) \left[\frac{\log(r)}{\log(\bar{r})} \right]^{\frac{1}{\lambda}} . \tag{A.12}$$

Using (A.10), (A.11), and (A.12) we can derive the price indices (see main text). Then, plugging these price indices into (3), the households' demand for a product i is derived,

$$c_i = \psi_i \omega_i^{-\sigma_I} i^{-\frac{\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{\sigma_I}{1-\sigma_I}} C \; .$$

Goods market clearing implies that $c_i = x_i$. We use (A.5), (A.9), and (A.12) to obtain the effective labor demand of a firm i,

$$\tilde{l}_i = \psi_i \left[-e\lambda \log(\bar{r}) \right]^{\frac{1}{\lambda}} \omega_i^{-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{\sigma_I}{1-\sigma_I}} C \; . \tag{A.13}$$

Replacing $[\kappa_1(r)\kappa_2(i)]^{-1}$ with $[-e\lambda i \log(r)]^{\frac{1}{\lambda}}$ in the main text, we can formulate a Proposition closely related to Proposition 2.

Two Tasks

Assume that there are only two tasks, a *routine* task with corresponding task-complexity i_R and a *non-routine* task with corresponding task-complexity i_N, where $i_N > i_R$. We call workers that can perform both i_R and i_N *"high-skilled workers"* and workers that can only perform i_R *"low-skilled workers"*. The skill sets of these two groups are $\mathcal{R}_H = \{r \in \mathcal{R} \mid r \geq \tilde{r}(i_N)\}$ and $\mathcal{R}_L = \{r \in \mathcal{R} \mid \tilde{r}(i_N) > r \geq \tilde{r}(i_R)\}$, respectively. Then, the scaling factor in (A.11) can only take on value $\omega(r) = \omega$ for all $r \in \mathcal{R}_H$ and we have $\omega(r) = 1$ for all $r \in \mathcal{R}_L$. Observe that the task-complexity i_N affects the supply of skills in both sets, $\mathcal{R}_L$ and $\mathcal{R}_H$. The skill sets depend on task-complexities i_N and i_R. The greater i_N the smaller the set of labor $\mathcal{R}_H$ able to manage this task-complexity and the larger the set of labor $\mathcal{R}_L$ able to manage lower task-complexities, for a given skill distribution. We define the effective labor supply by

$$\tilde{\phi}_L = \int_{\tilde{r}(i_R)}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} L^r f(r) dr \quad \text{and} \quad \tilde{\phi}_H = \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} L^r f(r) dr \ .$$

We multiply (A.13) by the number of firms n_{i_N} and n_{i_R} in each industry. Implying labor market clearing, i.e., $n_{i_R}\tilde{l}_{i_R} \geq \tilde{\phi}_L$ and $n_{i_H}\tilde{l}_{i_N} \leq \tilde{\phi}_H$ and $n_{i_R}\tilde{l}_{i_R} + n_{i_N}\tilde{l}_{i_N} = \tilde{\phi}_L + \phi_H$, we obtain

$$\begin{aligned}
\tilde{\phi}_L \leq & \psi_{i_R} \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} i_R^{\frac{1-\sigma_I}{\lambda}} n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{\sigma_I}{1-\sigma_I}} C \\
\tilde{\phi}_H \geq & \psi_{i_N} \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \omega^{-\sigma_I} i_N^{\frac{1-\sigma_I}{\lambda}} n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{\sigma_I}{1-\sigma_I}} C \ .
\end{aligned}$$

In such an environment the scaling factor is

$$\omega = \max\left\{1, \left[\frac{\tilde{\phi}_L}{\tilde{\phi}_H}\frac{\psi_{i_N}}{\psi_{i_R}}\left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}}\left[\frac{n_{i_N}}{n_{i_R}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I}}\right\} \ . \tag{A.14}$$

We observe that shifts of either task-complexity, i_R or i_N, *directly* impact wage inequality via the scaling factor, ω, and also, *indirectly*, the amount of labor suitable for a task, and hence, the labor supply. Thus, the labor supply is partially determined by the *technology*. Hence, the technology impacts both labor demand and labor supply.

A.3 Relation to the Task-based Model

The model presented in the main text is related to the task-based models pioneered by Acemoglu and Zilibotti (2001), Autor et al. (2003), and principally Acemoglu and Autor (2011). Such task-based models differ from earlier models developed to understand wage inequality by analyzing demand and supply of skills, as they incorporate the assignments from skills to tasks within the model. Following Acemoglu and Autor (2011), we speak of the *canonical model* whenever referring to the first models which formally analyzed the demand and supply of skills and their effects on wage dynamics (Tinbergen, 1974, 1975; Welch, 1973; Katz and Murphy, 1992; Card and Lemieux, 2001a,b).

Acemoglu and Autor (2011) present a task-based model in which particular skill groups have a comparative advantage for certain tasks. We will call their model the *"A/A-model"* and our model the *"task-complexity model"*.

There are three main differences between the A/A-model and the task-complexity model.

First, the A/A-model assumes that any skill level is able to perform any task, however, skills have comparative advantages in performing certain tasks. In the task-complexity framework the micro-level task-complexity is decisive as to whether a worker can perform a certain task-complexity. Hence, workers with different skills are perfect substitutes as long as they are able to perform a certain task-complexity, and as long as they are part of the same labor market group g. As a consequence small skill differences may be associated with large wage differences—well beyond the productivity differences for the same tasks—when the higher skilled worker's wage is scaled up. In contrast to the A/A-model, the task-complexity model assumes that the assignment of skills to task-complexities is technologically given, i.e., whether or not a skill is sufficiently high to perform a certain task-complexity cannot change through dynamics in supply and demand for skills.

Second, there is only one distribution of skills in the task-complexity model while the A/A-model involves different skill groups, each with a factor-augmenting technology. Such factor-augmenting technologies are used to explain skill-biased technological change. In principle, factor-augmenting technologies could be added to our model. However, skill-biased technological change is directly implied by the task-complexity model under a very weak condition. For instance, assume that technological progress is biased towards high task-complexities. Since the execution of high task-complexities is only possible for high-skilled households, demand for the high-skilled increases, when such new task-complexities emerge due to new technologies. The difference to factor-augmenting technology is subtle but important. According to our model there is only one factor that can be augmented, which is labor, and e.g., high-skilled and low-skilled households are mere labels for two skill groups when the labor market is separated once. Thus, even without factor-augmenting technologies, we can have skill-biased technological change through the emergence of new task-complexities of high complexity.

Third, the A/A-model and the task-complexity model have also different implications when technological progress takes place. For instance, the implications of automation for certain tasks-complexities or tasks are different in the two models. In the A/A-model, workers who lose their job because of automation can always switch to tasks previously performed by other skill groups, if they accept lower wages. The A/A-model is very useful to understand wage dynamics that can explain the empirical patterns of wage inequality and the recently observed wage polarization (Acemoglu and Autor, 2011). In the task-complexity model, such shifts may not be possible if low-skilled workers are unable to produce higher task-complexities. Such a barrier to the reallocation of low-skilled workers, together with the occurring automation of low task-complexities, can greatly aggravate inequality or can lead to unemployment, for instance to the predicted "useless" and "unemployable class" described by Harari (2016).[26,27]

We believe that the two frameworks complement each other and should both be considered when analyzing the implications of a given policy.

[26] Cf. the quote of Harari (2016) at the beginning of the chapter.

[27] Not only automation on low task-complexities may aggravate inequality, but also automation in the middle range of task-complexities—and even on some high task-complexities. The industry-composition of the model—the workers' task-complexity-composition, i.e., the task-complexities that remain being executed by workers—is decisive for the evolution of inequality in combination with the skill distribution.

B Proofs

B.1 Proof of Proposition 1

We proof the uniqueness of the equilibrium. First note, that there are $2^{|\mathcal{I}|-1}$ possible arrangements of the labor market.

- For $|\mathcal{I}| = 2$ there are 2 potential divisions, 1× ILM and 1× DLM.
- For $|\mathcal{I}| = 3$ there are 4 potential divisions, 1× ILM, 2× a single separation, and 1× a double separation.
- For $|\mathcal{I}| = 4$ there are 8 potential divisions, 1× ILM, 3× a single separation, and 3× a double separation, and 1× three separations.

Next we use the group notation.[28] We can rewrite the problem as a system of non-linear equations. We denote the demand for a group g's effective labor by $\tilde{L}_g^d$. Using (11) we can derive $\tilde{L}_g^d = \sum_{i\in g} n_i \tilde{l}_i$, which is

$$\begin{aligned}\tilde{L}_g^d =&\omega_g^{-\sigma_I}\left(\sum_{i\in g}\psi_i\left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I-1}n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right)\left[\sum_{\hat{g}\in\mathcal{G}}\sum_{i\in\hat{g}}\psi_i\left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)}n_i^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}\right]^{-1}\sum_{\hat{g}\in\mathcal{G}}\sum_{i\in\hat{g}}\omega_i\tilde{L}_i^{\Delta}\\ =&\omega_g^{-\sigma_I}\left(\sum_{i\in g}\psi_i\left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I-1}n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right)\left[\sum_{\hat{g}\in\mathcal{G}}\omega_{\hat{g}}^{1-\sigma_I}\sum_{i\in\hat{g}}\psi_i\left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I-1}n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{-1}\sum_{\hat{g}\in\mathcal{G}}\omega_{\hat{g}}\sum_{i\in\hat{g}}\tilde{L}_i^{\Delta}\ ,\end{aligned}$$

where $\tilde{L}_i^{\Delta} := \int_{\tilde{r}(i)}^{\tilde{r}(i_+)}\frac{\kappa_1(r)}{\kappa_1(\bar{r})}L^r f(r)dr$ is the effective labor supply of skills in the interval $[\tilde{r}(i), \tilde{r}(i_+))$.

We define two constants: $\gamma_g = \sum_{i\in g}\psi_i\left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I-1}n_i^{\frac{1-\sigma_I}{1-\sigma_v}}$ and $\tilde{L}_g^{\Delta} = \sum_{i\in g}\tilde{L}_i^{\Delta}$ (this is the effective labor supply in group g). Using these constants, we rewrite the demand

[28] E.g. when $|\mathcal{I}| = 4$, there might be a solution with a double separation, i.e., with three groups. Then, the condition above states that this is only a solution to our model if $\omega_1 < \omega_2 < \omega_3$. Note that in this example

(a) $\{i_1, i_2\} \in g_1$, $\{i_3\} \in g_2$, and $\{i_4\} \in g_3$, or

(b) $\{i_1\} \in g_1$, $\{i_2, i_3\} \in g_2$, and $\{i_4\} \in g_3$, or

(c) $\{i_1\} \in g_1$, $\{i_2\} \in g_2$, and $\{i_3, i_4\} \in g_3$,

i.e., there are three group structures, each with three groups. We can denote the three different group structures (a), (b) and (c) by $\mathcal{G}^a$, $\mathcal{G}^b$ and $\mathcal{G}^c$.

for the group $\hat{g}$'s effective labor,

$$\tilde{L}_g^d = \omega_g^{-\sigma_I}\gamma_g \left[\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I}\gamma_{\hat{g}}\right]^{-1} \sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}\tilde{L}_{\hat{g}}^{\Delta} \ . \tag{B.1}$$

In equilibrium, for each group the labor market must clear, i.e., $\tilde{L}_g^d = \tilde{L}_g^{\Delta}$, and thus

$$1 = \frac{\omega_g^{1-\sigma_I}\gamma_g}{\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I}\gamma_{\hat{g}}} \frac{\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}\tilde{L}_{\hat{g}}^{\Delta}}{\omega_g \tilde{L}_g^{\Delta}} \ . \tag{B.2}$$

This system of equations can be written as

$$\omega_g^{\sigma_I} \frac{\tilde{L}_g^{\Delta}}{\gamma_g} = \frac{\sum_{\hat{g}\in\mathcal{G}\setminus g} \omega_{\hat{g}}\tilde{L}_{\hat{g}}^{\Delta}}{\sum_{\hat{g}\in\mathcal{G}\setminus g} \omega_{\hat{g}}^{1-\sigma_I}\gamma_{\hat{g}}} \quad \forall\, g \in \mathcal{G} \ , \tag{B.3}$$

where the right-hand side is independent of ω_g. Thus, we need to find a solution to system (B.2) or (B.3) with a number of $\mid \mathcal{G} \mid$ unknowns under the constraint given in (12). Without loss of generality we can normalize $\omega_{g_1} \equiv 1$. Furthermore, for each group g it can be shown that the partials for the demand for a group's effective labor, denoted by $\tilde{L}_g^d$, fulfill the following properties in the auxiliary Lemma 1.

Lemma 1
The partial derivatives with respect to scaling factors for a group's effective labor demand are

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_g} < 0 \text{ for } \mid \mathcal{G} \mid \geq 2 \text{ and } \frac{\partial \tilde{L}_g^d}{\partial \omega_g} = 0 \text{ for } \mid \mathcal{G} \mid = 1, \tag{B.4}$$

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_{\hat{g}}} = \frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} \frac{\tilde{L}_{\hat{g}}^{\Delta}}{\tilde{L}_{g'}^{\Delta}} > 0 \ \forall \hat{g}, g' \in \mathcal{G} \setminus g, \tag{B.5}$$

A proof is given in Section B.2. Lemma 1 implies that groups are gross substitutes. Furthermore, (B.2) and (B.3) are homogeneous of degree 0 in wages and Walras' Law holds. Thus, the system is equivalent to a demand system with fixed supply.

The proof of Proposition 1 now proceeds in three steps. First, we show that for each group structure $\mathcal{G}$ the system described in (B.2) has a unique solution. Since there are $2^{|\mathcal{I}|-1}$ groups structures, there are $2^{|\mathcal{I}|-1}$ potential solutions. Second, we show using (12) that all potential solutions are mutually exclusive, i.e., only one can qualify as an equilibrium solution for the wage scheme. Third, we show that there exists a equilibrium solution which, given Step 2, must be unique.

Step 1: Unique solution for given $\mathcal{G}'$

EXISTENCE: An economy with a group structure $\mathcal{G}'$ can be interpreted as a pure exchange economy with $\mid \mathcal{G}' \mid$ goods and $\mid \mathcal{G}' \mid$ agents, where there is a demand for labor and a given labor supply. Then the aggregate excess demand can be denoted by

$$z(\vec{\omega}) = \sum_{g \in \mathcal{G}'} \left[\tilde{L}^d_g(\vec{\omega}, \omega_g \tilde{L}^\Delta_g) - \tilde{L}^\Delta_g \right] \equiv 0 \ ,$$

where $\vec{\omega}$ denotes the vector of scaling factors. Since preferences are described by CES utiliy functions, demand functions are continuous and we can define continuous functions $s_g(\vec{\omega})$,

$$s_g(\vec{\omega}) = \frac{\omega_g + \max\{0, z_g(\vec{\omega})\}}{1 + \sum_{\hat{g} \in \mathcal{G}'} \max\{0, z_{\hat{g}}(\vec{\omega})\}} \ \forall \ g \in \mathcal{G}' \ ,$$

where $z_g(\vec{\omega}) = \tilde{L}^d_g(\vec{\omega}, \omega_g \tilde{L}^\Delta_g) - \tilde{L}^\Delta_g$. The Brouwer fix-point theorem guarantees that there exitst $\vec{\omega}^\star$ with $\vec{\omega}^\star \equiv s(\vec{\omega}^\star)$. This implies

$$z_g(\vec{\omega}^\star) = \tilde{L}^d_g(\vec{\omega}^\star, \omega^\star_g \tilde{L}^\Delta_g) - \tilde{L}^\Delta_g = 0 \ \forall \ g \in \mathcal{G}' \ ,$$

and thus $\vec{\omega}^\star$ is a price vector solution for the economy with a group structure $\mathcal{G}'$ that satisfies the equilibrium condition (B.2).[29]

UNIQUENESS: Partial derivatives expressed in (B.5) and (B.4) show that labor demand of groups are gross substitutes. Thus the solution of (B.2) for a given group structure $\mathcal{G}$ is unique.

NUMBER OF SOLUTIONS: There are $2^{|\mathcal{I}|-1}$ ways to build a group structure and for every particular group structure $\mathcal{G}'$ there is a unique solution. Thus there must be $2^{|\mathcal{I}|-1}$ unique solutions.

[29]If $\mid \mathcal{G}' \mid = 3$, this implies

$$\begin{aligned}
\tilde{L}^d_{g_1,g_1}(\vec{\omega}^\star, \omega^\star_{g_1}\tilde{L}^\Delta_{g_1}) + \tilde{L}^d_{g_1,g_2}(\vec{\omega}^\star, \omega^\star_{g_2}\tilde{L}^\Delta_{g_2}) + \tilde{L}^d_{g_1,g_3}(\vec{\omega}^\star, \omega^\star_{g_3}\tilde{L}^\Delta_{g_3}) - \tilde{L}^\Delta_{g_1} &= 0 \ , \\
\tilde{L}^d_{g_2,g_1}(\vec{\omega}^\star, \omega^\star_{g_1}\tilde{L}^\Delta_{g_1}) + \tilde{L}^d_{g_2,g_2}(\vec{\omega}^\star, \omega^\star_{g_2}\tilde{L}^\Delta_{g_2}) + \tilde{L}^d_{g_2,g_3}(\vec{\omega}^\star, \omega^\star_{g_3}\tilde{L}^\Delta_{g_3}) - \tilde{L}^\Delta_{g_2} &= 0 \ , \\
\tilde{L}^d_{g_3,g_1}(\vec{\omega}^\star, \omega^\star_{g_1}\tilde{L}^\Delta_{g_1}) + \tilde{L}^d_{g_3,g_2}(\vec{\omega}^\star, \omega^\star_{g_2}\tilde{L}^\Delta_{g_2}) + \tilde{L}^d_{g_3,g_3}(\vec{\omega}^\star, \omega^\star_{g_3}\tilde{L}^\Delta_{g_3}) - \tilde{L}^\Delta_{g_3} &= 0 \ ,
\end{aligned}$$

where $\tilde{L}^d_{g_2,g_1}(\vec{\omega}^\star, \omega^\star_{g_1}\tilde{L}^\Delta_{g_1}) := {\omega^\star_{g_2}}^{-\sigma_I} c_{g_2} \left[\sum_{g \in \mathcal{G}} {\omega^\star_g}^{1-\sigma_I} \gamma_g\right]^{-1} \omega^\star_{g_1} \tilde{L}^\Delta_{g_1}$ denotes the demand for services of group g_2 from group g_1.

Step 2: Mutually exclusive solution

Given a set of task-complexities $\mathcal{I}$ and corresponding parameters, ψ_i and n_i, we show that Inequality (12) implies that potential solutions are mutually exclusive.

We first note that since production is linear in labor, the total amount of effective labor employed is constant, no matter how it is allocated across industries and groups. We define two group structures:

(a) $\mathcal{G}^a = \{g_1^a, g_2^a, ...\}$, where $g_1^a = \{\underline{i}\}$ and $g_2^a = \{i_1, ...\}$,

(b) $\mathcal{G}^b = \left\{g_1^b, g_2^b, ...\right\}$, where $g_1^b = \{\underline{i}, i_1\}$ and $g_2^b = \{i_2, ...\}$.

We need to analyze two cases:

- *CROSSING:* Assume that the two group structures $\mathcal{G}^a$ and $\mathcal{G}^b$ correspond to Figure 5, i.e., their wage schemes cross at i_2. Then, $\mathcal{G}^a$ implies that $\tilde{L}_{g_1}^{d,a} = \tilde{L}_{g_1}^{\Delta,a}$ and $\mathcal{G}^b$ implies that $\tilde{L}_{\underline{i}}^{d,b} > \tilde{L}_{\underline{i}}^{\Delta,b}$, i.e., the demand for effective labor from industry $\underline{i}$ is greater than the supply of low-skilled effective labor, thus, also higher skill levels (with $r \geq \tilde{r}(i_1)$ and $r < \tilde{r}(i_2)$) work in this industry. Thus, with $\mathcal{G}^b$ the labor market is locally integrated and $\underline{i}$ and i_1 form the group $g_1^b \in \mathcal{G}^b$. Because of homothetic preferences, linear production and equal effective labor supply ($\tilde{L}_{g_1}^{\Delta,a} = \tilde{L}_{i_1}^{\Delta,b}$), the high demand for effective labor in industry $\underline{i}$ with $\mathcal{G}^b$ can only originate from $TW^a < TW^b$.

  Now, $\mathcal{G}^a$ also implies that $\tilde{L}_{i_1}^{d,a} \geq \tilde{L}_{i_1}^{\Delta,a}$ and $\mathcal{G}^b$ implies that $\tilde{L}_{i_1}^{d,b} \leq \tilde{L}_{i_1}^{\Delta,b}$. Equal labor supply ($\tilde{L}_{i_1}^{\Delta,a} = \tilde{L}_{i_1}^{\Delta,b}$) implies that $\tilde{L}_{i_1}^{d,a} \geq \tilde{L}_{i_1}^{d,b}$. We now know that labor $\tilde{L}_{i_1}^{\Delta,a}$ earns the scaling factor $\omega_{g_2}^a > 1$. Thus, knowing from before that $TW^a < TW^b$, there is a contradiction with the higher demand for effective labor of industry i_1, despite higher wages.

- *DOMINANCE:* Assume now that the two group structures $\mathcal{G}^a$ and $\mathcal{G}^b$ correspond to Figure 6. Again, $\mathcal{G}^a$ implies that $\tilde{L}_{g_1}^{d,a} = \tilde{L}_{g_1}^{\Delta,a}$ and $\mathcal{G}^b$ implies that $\tilde{L}_{\underline{i}}^{d,b} \geq \tilde{L}_{\underline{i}}^{\Delta,b}$. We conclude again that $TW^a \leq TW^b$. But the area of the scaling factor step function of $\mathcal{G}^b$ is always below the scaling factor step function of $\mathcal{G}^a$ and equal effective labor supply total wages under $\mathcal{G}^a$ must be greater than under $\mathcal{G}^b$, i.e., $TW^a > TW^b$. This contradicts our conclusion before.

Figure 5: Crossing

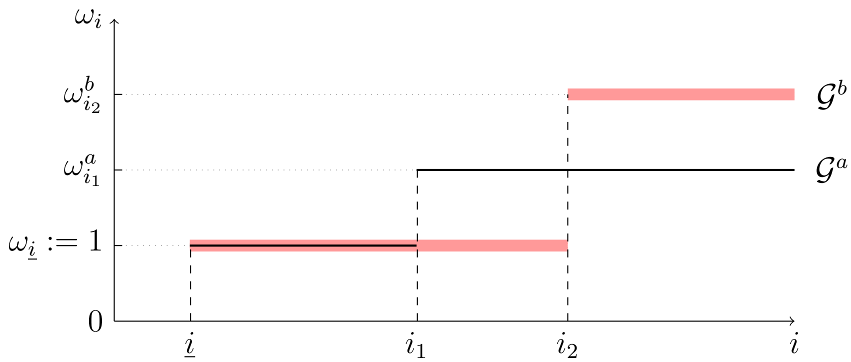
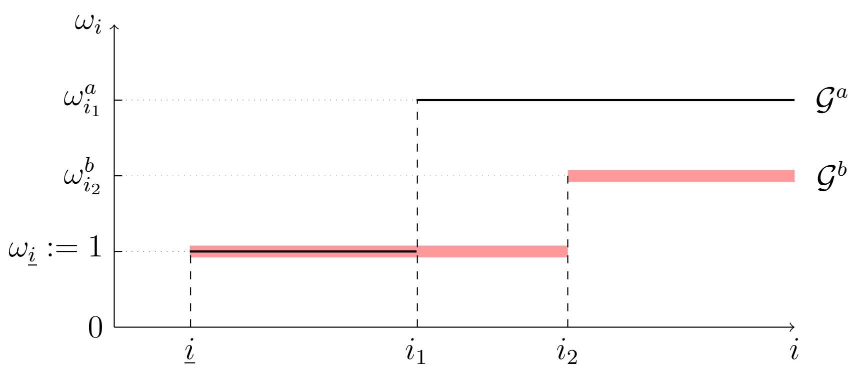


Figure 6: Dominance



Thus, we have proven that under Condition (12) potential solutions are mutually exclusive, i.e., there is at most one solution fulfilling Condition (12).

Step 3: At least one solution

We now show that there must be at least one solution that fulfills Condition (12). We start with the integrated labor market, i.e., $\mid \mathcal{G} \mid= 1$. If this is not an equilibrium, we know that there must be excess demand for higher skills. Thus, we can try all separations with $\mid \mathcal{G} \mid= 2$ under Condition (12) (which yields $\mid \mathcal{I} \mid -1$ possibilities). If there is no solution, there must be excess demand for higher skills and we continue with $\mid \mathcal{G} \mid= 3$ under Condition (12) and so on. At the latest with $\mid \mathcal{G} \mid=\mid \mathcal{I} \mid$ we must have found at least one solution.

Thus, we have proven that there is one unique solution $\mathcal{W}^\star$ for any set of task-complexities $\mathcal{I}$, demand shifters $\{\psi_i\}_{i\in\mathcal{I}}$, amount of firms $\{n_i\}_{i\in\mathcal{I}}$, and skill distribution $F(r)$, given that $\sigma_I > 1$.

□

B.2 Proof of Lemma 1

We use (B.1). We first proof (B.4), then we proof (B.5).

For $\mid \mathcal{G} \mid\geq 2$, the derivative of (B.1) with respect to its own scaling factor ω_g is given by

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_g} =\omega_g^{-\sigma_I}\gamma_g \left[\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I}\gamma_{\hat{g}}\right]^{-1} \left[-\sigma_I\omega_g^{-1}TW + [\sigma_I - 1]\,\gamma_g\omega_g^{-\sigma_I} \left[\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I}\gamma_{\hat{g}}\right]^{-1} TW + \tilde{L}_g^\Delta\right] ,$$

where $TW = \sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}\tilde{L}_{\hat{g}}^\Delta$. We now show that the sum of the terms in brackets is negative. We multiply with the scaling factor $\omega_{\hat{g}}$ and obtain

$$\begin{aligned} & -\sigma_I + [\sigma_I - 1]\,\frac{\gamma_g\omega_g^{1-\sigma_I}}{\sum_{\hat{g}\in\mathcal{G}}\gamma_{\hat{g}}\omega_{\hat{g}}^{1-\sigma_I}} + \frac{\omega_g\tilde{L}_g^\Delta}{\sum_{\hat{g}\in\mathcal{G}}\omega_{\hat{g}}\tilde{L}_{\hat{g}}^\Delta} \\ = \quad & \sigma_I\left[\frac{\gamma_g\omega_g^{1-\sigma_I}}{\sum_{\hat{g}\in\mathcal{G}}\gamma_{\hat{g}}\omega_{\hat{g}}^{1-\sigma_I}} - 1\right] - \frac{\gamma_g\omega_g^{1-\sigma_I}}{\sum_{\hat{g}\in\mathcal{G}}\gamma_{\hat{g}}\omega_{\hat{g}}^{1-\sigma_I}} + \frac{\omega_g\tilde{L}_g^\Delta}{\sum_{\hat{g}\in\mathcal{G}}\omega_{\hat{g}}\tilde{L}_{\hat{g}}^\Delta} \end{aligned}$$

From (B.2) we know that $-\frac{\gamma_g \omega_g^{1-\sigma_I}}{\sum_{\hat{g}\in\mathcal{G}} \gamma_{\hat{g}} \omega_{\hat{g}}^{1-\sigma_I}} + \frac{\omega_g \tilde{L}_g^{\Delta}}{\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^{\Delta}} = 0$. Observe that $\sigma_I > 0$ and that $\frac{\gamma_g \omega_g^{1-\sigma_I}}{\sum_{\hat{g}\in\mathcal{G}} \gamma_{\hat{g}} \omega_{\hat{g}}^{1-\sigma_I}} - 1 < 0$. Thus, we have proven that $\frac{\partial \tilde{L}_g^d}{\partial \omega_g} < 0$.

The derivative of (B.1) with respect to some scaling factor $\omega_{g'}$ $(g' \neq g)$ is given by

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} = \omega_g^{-\sigma_I} \gamma_g \left[\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I} \gamma_{\hat{g}} \right]^{-1} \left[[\sigma_I - 1] \gamma_{g'} \omega_{g'}^{-\sigma_I} \left[\sum_{\hat{g}\in\mathcal{G}} \omega_{\hat{g}}^{1-\sigma_I} \gamma_{\hat{g}} \right]^{-1} TW + \tilde{L}_{g'}^{\Delta} \right] > 0 \ .$$

Thus we have proven that $\frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} > 0$.

□

C Parameters for the Numerical Examples

The following list of parameters describes the assumptions made for the numerical analysis in Section 3:

- $F(r) = Beta(2, 5)$, i.e., Beta-distribution with $a = 2$ and $b = 5$
- $\mathcal{R} = [0, 1]$
- $\mathcal{I} = \left\{ \underline{i}, i_1, i_2, \bar{i} \right\} = \{1, 1.5, 2, 2.25\}$
- $N = \left\{ n_{\underline{i}}, n_{i_1}, n_{i_2}, n_{\bar{i}} \right\} = \{1, 5, 5, 3.5\}$
- $\Psi = \left\{ \psi_{\underline{i}}, \psi_{i_1}, \psi_{i_2}, \psi_{\bar{i}} \right\} = \{1, 1.5, 2, 2\}$
- $\kappa_1(r) = \frac{r}{2}$ and $\kappa_2(i) = i^{-1}$
- $\tilde{r}(i) = [i - \underline{i}]\, \bar{i}^{-1}$

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