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IZA DP No. 12567

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ABSTRACT

Interest Rate Bands of Inaction and Play-Hysteresis in Domestic Investment: Evidence for the Euro Area*

The interest rate represents an important monetary policy tool to steer investment in order to reach price stability. Therefore, implications of the exact form and magnitude of the interest rate-investment nexus for the European Central Bank's effectiveness in a low interest rate environment gain center stage. We first present a theoretical framework of the hysteretic impact of changes in the interest rate on macroeconomic investment under certainty and under uncertainty to investigate whether uncertainty over future interest rates in the Euro area hampers monetary policy transmission. In this non-linear model, strong reactions in investment activity occur as soon as changes of the interest rate exceed a zone of inaction, that we call 'play' area. Second, we apply an algorithm describing path-dependent play-hysteresis to estimate investment hysteresis using data on domestic investment and interest rates on corporate loans for 5 countries of the Euro area in the period ranging from 2001Q1 to 2018Q1. We find hysteretic effects of interest rate changes on investment in most countries. However, their shape and magnitude differ widely across countries which poses a challenge for a unified monetary policy. By introducing uncertainty into the regressions, the results do not change much which may be due to the interest rate implicitly incorporating uncertainty effects in investment decisions, e.g. by risk premia.

JEL Classification: C32, E44, E49, E52, F21

Keywords: European Central Bank, interest rate, investment, monetary policy, non-ideal relay, path-dependence, play-hysteresis, uncertainty

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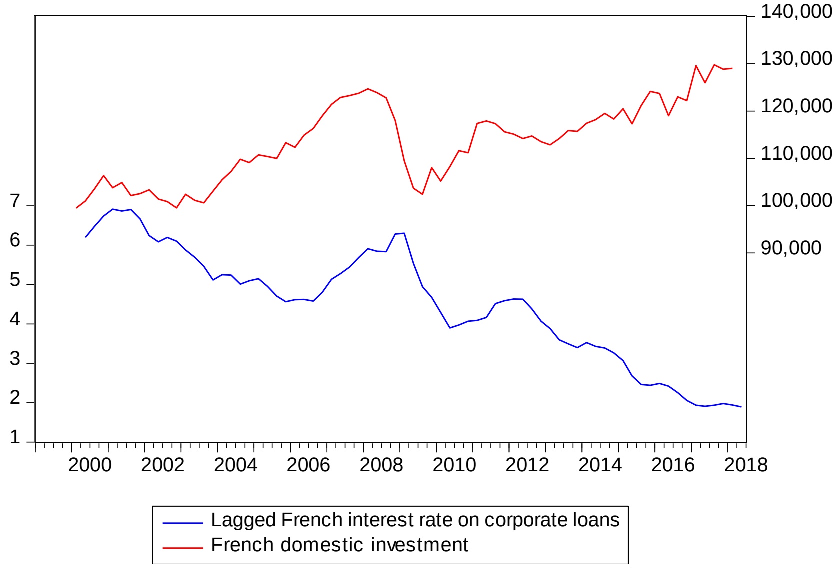
1 Introduction

The European Central Bank (ECB) has been reducing its key interest rate since the financial crisis in 2008, even hitting the zero lower bound steadily in the more recent past. Alongside this development, domestic interest rates on the credit market, for private as well as for corporate lending, have been falling since the key interest rate works as a main driver of the market interest rate. However, this decline happened at different levels across the individual member states of the currency union. The ECB's intention behind this unconventional monetary policy is to stimulate demand in the Euro area, i.e. consumption and investment, and, by this, preserving price stability. Against this background, the negative relation between investment and investment costs leads to the question if and if so, where exactly the interest rate thresholds for additional investment (as well as for disinvestment) are located. In terms of the hysteresis concept, the question is: how wide is the band of inaction or to what extent does the interest rate underlying our study have to decrease until it induces significantly additional investment. The interest rate is of great importance, since interest rates as the main determinant of the cost of capital are of crucial relevance for profitability. In other words, we shed light on an important element of the monetary policy transmission process in the Euro area.

We start with a visual inspection of the development over time of the interest rate relevant for corporate lending and of domestic investment for France in Figure 1 from 2000 onwards. The negative trend of the interest rate is mirrored by a positive trend in investment as theoretically expected. However, the slopes of the trend lines do not correspond to each other with merely opposing signs, hinting at a non-linear relationship. We even find periods of parallel development of the two variables as for example in 2002 or in the quarters leading up to the financial crisis.

In the following, we focus on the hysteretic nature of the relation between investment and the relevant credit market interest rate, following Belke & Göcke (2009,2019). We start with the presentation of a microeconomic model and show the inherently path-dependent nature of investment decisions. In this regard, we derive the difference between the (low) interest rate triggering investment activity and the (high) interest rate triggering a disinvestment of a single firm. Sunk costs may be the reason for this divergence between the two interest triggers, if the investment is firm-specific for example and no sale is possible at the full purchasing price. Additionally, we come up with an extension of the model by introducing uncertainty related to stochastic future revenue and/ or future interest rate changes. Option

Figure 1: French interest rate on corporate lending (lagged by one quarter) and French domestic investment



Notes: Quarterly data from Eurostat and the European Central Bank. Investment on right axis in million €, interest rate on left axis in percent.

value effects then amplify the hysteresis characteristics.[1] Next, we present an approach which incorporates the path-dependent non-linear dynamics on the macroeconomic level. It is called play-hysteresis, since it shows an analogy to mechanical play. The integration of play into a standard regression framework enables the use of macroeconomic data. Hereby, the theory developed can be empirically tested using more readily available macroeconomic data, which is a step into the direction of bringing the concept of hysteresis closer to applicability and increasing its policy relevance. To the best of our knowledge, this study is the first to empirically examine the theoretical concept of interest rate hysteresis in macroeconomic investment in Belke & Göcke (2019).

The remainder of this paper proceeds as follows: we present the theoretical concept of sunk cost hysteresis and non-ideal relays in section 2, capturing the non-linear hysteresis-type dynamics inherent in the relationship between interest rates and investment if sunk adjustment costs matter for investment market entry and exit decisions. In addition, option value effects of waiting due to uncertainty are introduced into the model. Section 3 addresses

[1] Dias & Shackleton (2005) also present a model of investment hysteresis with stochastic interest rate changes. Stochastic differential equations are the foundations of their mathematically complex approach, which allows for numerical solutions. Nevertheless, our model is simpler, hereby enabling algebraic closed form expressions that can be directly interpreted.

the aggregation of microeconomic hysteresis loops and the macroeconomic investment - hysteresis. A description of play-hysteresis is in section 4, while section 5 presents an algorithm capturing linear play, which allows an estimation of the macroeconomic hysteresis loop. In section 6, we introduce the data and estimate the impact of the corporate lending interest rate on domestic investment in five Euro area countries. Section 7 concludes and derives some (monetary) policy conclusions.

2 Sunk cost hysteresis and non-ideal relays

2.1 Sunk cost hysteresis and interest rate changes

In order to illustrate the hysteresis effects on a microeconomic level, we will apply a simple microeconomic model presented by Belke & Göcke (2009,2019). A price-taking firm j decides in period t whether or not to invest into one unit of capital $K_j(=1)$ and to start production. If it invests, the firm has to pay the sunk investment costs $H_j(\geq 0)$. H_j is completely firm specific and cannot be regained if the firm is disinvesting. The specific part H_j decays immediately as soon as the firm does not produce and sell, however, selling the unspecific capital stock K_j at price 1 is possible. Using capital, the firm produces and sells the production immediately resulting in a revenue $e_{j,t}$, which is the (marginal gross) rate of return of the additional unit of capital. Interest costs of the firm are related to both types of capital. Using unspecific capital K_j as an input factor, the interest rate i_t has to be paid as an opportunity cost. If the firm has not produced in the preceding period, it has to pay additionally for the starting costs $H_j(\geq 0)$. However, if it has been active and is just continuing production only the interest costs on K_j are relevant.

The net rate of profit[2] (disregarding the adjustment costs) in period t is:

$$R_{j,t} = e_{j,t} - i_t \cdot K_j = e_{j,t} - i_t \tag{1}$$

As a simple example we assume the firm is expecting for the next period $(t+1)$ a single once and forever change in the interest rate by ρ, which remains constant for the whole infinite future: $i_{t+\tau} = i_t + \rho$ (for all $\tau > 0$). The future gross rate of return is expected constant as well: $e_{j,t+\tau} = e_{j,t} = e_j$. Thus, the future net rate of return is:

$$R_{j,t+\tau} = e_{j,t} - i_{t+\tau} = e_j - (i_t + \rho) \qquad (with\ \tau > 0) \tag{2}$$

[2] We do not explicitly account for depreciation of assets. Nevertheless, depreciation is implicitly included when interpreting the net rate of return.

In the case of an ongoing activity the present value of future revenues as an annuity (with payments at the end of the periods) is:

$$V_{j,t} \equiv \frac{e_j + \frac{e_j}{i_t+\rho}}{1+i_t} - K_j = \frac{(1+i_t+\rho)\cdot e_j}{(1+i_t)\cdot(i_t+\rho)} - 1 \tag{3}$$

Under certainty the present value of revenues has to cover (at least) the value of the capital stock $K_j = 1$ plus the sunk entry costs ($V_{j,t} > K_j + H_j$) to make an entry a profitable investment. If the interest rate is assumed to be the single input variable, while the rate of return is ceteris paribus expected as constant ($e_{j,t} = e_j$), the entry condition can be solved for the interest rate, and an entry trigger interest rate a_j can be calculated.[3] For a first illustration, we assume the simplest case of $\rho = 0$. The entry/investment trigger interest rate solving the entry condition ($V_{j,t} = 1 + H_j$) in this simple case is:

$$for \quad \rho = 0: \quad a_{j,\rho=0} = \frac{e_j}{1+H_j} \qquad\qquad entry\ if\ i_t < a_j \tag{4}$$

A low interest rate results in low capital costs for K_j and H_j and in a high present value of future revenues, which makes an investment profitable.

A firm that was active in the preceding period will leave the market and sell the unspecific capital K_j, if the interest rate is too high compared to the revenue. Thus, an exit of the firm is optimal if ($V_{j,t} < 1$), and the exit/disinvestment trigger interest rate for a scenario with a high interest rate is:

$$for \quad \rho = 0: \quad b_j = e_j \qquad\qquad exit\ if\ i_t > b_j \tag{5}$$

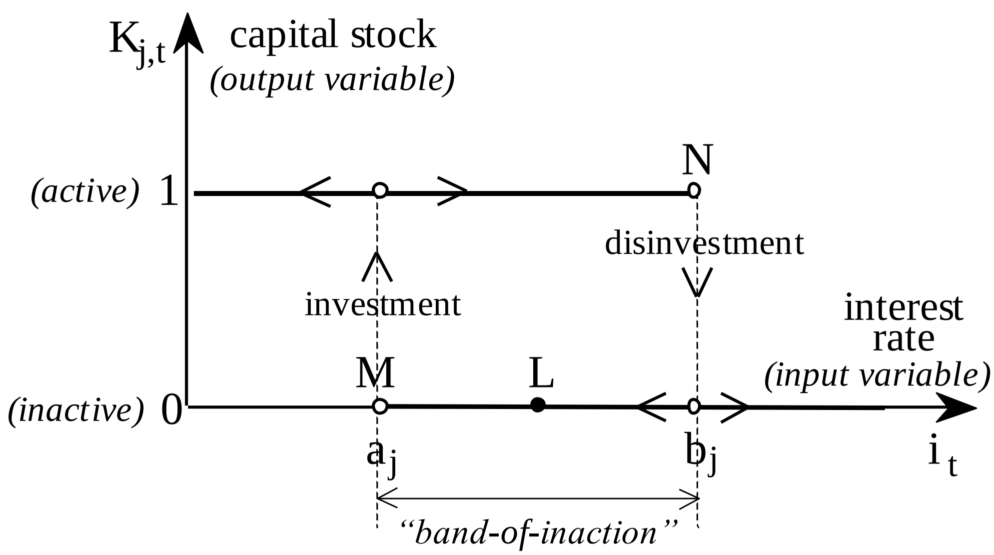
The results show, that an entry requires interest costs on both, unspecific capital (normalized to $K_j = 1$) and sunk investment costs ($H_j \geq 0$), to be covered by the revenue, i.e. by the gross rate of return e_j, to make an investment profitable. This is the case for low interest rates. On the other hand an exit will only occur, if the rate of return e_j no longer covers the interest opportunity costs just on the unspecific part of the capital K_j. Thus, disinvestment takes place if the interest rate exceeds the rate of return.

[3] See Göcke (2019) for a discussion of economic vector-hysteresis with multiple input variables, i.e. a situation with simultaneous revenue and interest rate changes.

For a constant level of the rate of return ($e_{j,t} = e_j$), an (unexpected) change of the interest rate i_t results in a non-ideal relay pattern with respect to the interest rate as the input variable, which is depicted in Figure 2 (see (Göcke 2019, p. 6) for a similar representation):

$$K_{j,t} = \begin{cases} 1 & if \quad (K_{j,t-1} = 0 \wedge i_t < a_j) \\ 1 & if \quad (K_{j,t-1} = 1 \wedge i_t \leq b_j) \\ 0 & if \quad (K_{j,t-1} = 0 \wedge i_t \geq a_j) \\ 0 & if \quad (K_{j,t-1} = 1 \wedge i_t > b_j) \end{cases} \tag{6}$$

Figure 2: Non-ideal relay related to the interest rate as the input variable



A non-ideal relay describes a path-dependent multiple-equilibria characteristic. Starting in an inactivity situation at point L in Figure 2, by reducing interest costs on K_j and H_j, a decreasing interest rate makes an investment profitable, triggering investment at interest rate a_j (entry) a point M. This results in a jump from the (K_j=0)-inactivity-line to the (K_j=1)-activity-line. A later interest rate increase will result in disinvestment if the opportunity costs only on the unspecific K_j are no longer covered (exit at point N at trigger rate b_j). A switch between the two equilibrium-branches takes place when the triggers are passed - otherwise the activity status remains the same. Therefore, the area between both triggers

a_j and b_j can be called a "band of inaction" or hysteresis-band (Baldwin 1989, Baldwin & Lyons 1989). Dependent on the past, two different equilibria are possible: The current level of the input variable (interest rate) does not unambiguously determine the current state of the output/dependent variable (firms activity). A temporary change of the input variable results in a switch between these equilibria and causes a permanent effect on the output variable (called remanence), which is the constituting feature of hysteresis.

2.2 Option value effects in a scenario with stochastic revenue changes

In a situation with uncertainty, a real option approach applies (Belke & Göcke 1999, 2009, 2019, Bentolila & Bertola 1990, Dixit 1989, Dixit & Pindyck 1994, Pindyck 1988, 1991). We again refer to the simple case of no expected change in the interest rate ($\rho = 0$) to illustrate the option effects related to revenue uncertainty. A firm which is currently inactive has to decide whether to invest now or not, having the option to invest later. The option to decide in the future results in risk reduction based on a wait-and-see strategy. A firm can avoid future losses by staying passive, if the stochastic future rate of return will be unfavourable. This wait-and-see-option to enter later is eliminated by an instantaneous investment. Thus, in order to trigger an immediate investment, the respective option value has to be covered in addition to the sunk costs.

The option value effects are demonstrated based on a simple example: assume a single non-recurring stochastic change of the rate of return, which can be either negative ($-\varepsilon_d$, with the index d indicating a downward movement) with probability P, or a positive change ($+\varepsilon_u$, with the index u indicating an upward movement) with probability $(1 - P)$ and $[\varepsilon_{u,d} \geq 0]$. From period $t + 1$ on, the firm will decide under certainty again. In a scenario with uncertainty, the option to wait and to decide on the investment in the next period has to be taken into account. If the future revenue level will turn out to be favourable ($+\varepsilon_u$) the firm still has the option to invest in the next period. On the other hand, potential future losses can be avoided by staying passive in the case of a negative change ($-\varepsilon_d$). The firm receives zero profits in the current period t if it stays passive. In case of a ($+\varepsilon_u$)-realization, the firm will use its option to invest in $t + 1$ and will receive an annuity of $(e_j + \varepsilon_u - i_t)$ by paying discounted sunk investment costs. If a ($-\varepsilon_d$)-realization emerges, the firm will remain passive.

The decision is based on a comparison of the expected present value of the wait-and-see strategy $E(W_{j,t}^{entry})$ with the expected present value $[E(V_{j,t}) - H_j]$ of an immediate entry (without a re-exit):

$$E(W_{j,t}^{entry}) = \frac{1-P}{1+i_t} \cdot \left(\frac{e_j + \varepsilon_u - i_t}{i_t} - H_j \right) \tag{7}$$

$$E(V_{j,t}) - H_j = P \cdot \left(\frac{e_j + \frac{e_j - \varepsilon_d}{i_t}}{1+i_t} \right) + (1-P) \cdot \left(\frac{e_j + \frac{e_j + \varepsilon_u}{i_t}}{1+i_t} \right) - 1 - H_j \tag{8}$$

Indifference between immediate entry and wait-and-see $[E(W_{j,t}^{entry}) = E(V_{j,t}) - H_j]$ results in an entry trigger:

$$e_j = i_t \cdot (1 + H_j) + \frac{P \cdot \varepsilon_d}{i_t + P} \quad \Leftrightarrow \quad i_t = \frac{1}{1+H_j} \cdot \left(e_{j,t} - \frac{P \cdot \varepsilon_d}{i_t + P} \right) \tag{9}$$

A comparison with the entry trigger in a situation with no uncertainty from equation (4) shows that the interest rate resulting in an entry is lower, due to an option value effect of the size $[-(P \cdot \varepsilon_d)/(i_t + P)]$.

A currently active firm, deciding to disinvest now or to stay active with an option to exit later if an unfavourable $(-\varepsilon_d)$ revenue change emerges, has an analogous decision problem. Remaining active for one period results in a current profit of $(e_j - i_t)$. In case of a $(-\varepsilon_d)$-realization, the firm will use its option to exit in $t+1$. Conditional on $(+\varepsilon_u)$ the firm will continue activity with a future annuity of $(e_j + \varepsilon_u - i_t)$. The expected present value of the wait-and-see strategy $E(W_{j,t}^{exit})$ is:

$$E(W_{j,t}^{exit}) = P \cdot \left(\frac{e_j - i_t}{1+i_t} \right) + (1-P) \cdot \left(\frac{e_j + \frac{e_j + \varepsilon_u}{i_t}}{1+i_t} - 1 \right) \tag{10}$$

$E(W_{j,t}^{exit})$ is compared with the expected present value of an immediate exit (without a re-entry), which is zero. Indifference $[E(W_{j,t}^{exit}) = 0]$ between wait-and-see and immediate exit gives the condition for an exit/disinvestment-trigger:

$$i_t = e_{j,t} + \frac{(1-P) \cdot \varepsilon_u}{i_t + (1-P)} \tag{11}$$

For the exit trigger a comparison with the certainty case in equation (5) shows a positive option value effect $[(1-P) \cdot \varepsilon_u]/[i_t + (1-P)]$.

2.3 Option value effects in a scenario with stochastic interest rate changes

Qualitatively similar option value effects will result if other stochastic impacts on the profitability may happen, e.g. if stochastic future changes of the interest rate itself are included. This is demonstrated by an analogous example: A single non-recurring stochastic change of the interest rate, which can be either positive ($+\rho_u$) or negative ($-\rho_d$) (and $\rho_{u,d} \geq 0$). The probability of a positive interest change ($+\rho_u$) is P, and $(1-P)$ for a negative change ($-\rho_d$). From period $t+1$ on, the situation is assumed to be constant again. The option to wait and to decide on the investment later has again to be taken into account. If the future interest change is negative ($-\rho_d$), and thus favourable for investors, the firm can still invest in the next period. However, by staying passive, potential future losses can be avoided if the positive interest rate change ($+\rho_u$) leads to increasing interest costs. The current net profit is ($R_{j,t} = e_j - i_t$. The future interest rate, relevant for future interest costs and discounting, is ($i_{t+\tau} = i_u \equiv i_t + \rho_u$) with probability ($P$) and it is ($i_{t+\tau} = i_d \equiv i_t - \rho_d$) with probability $(1-P)$. From equation (2) the future profit follows if $i_t + \rho$ is substituted by either ($i_t + \rho_u$) or ($i_t - \rho_d$). For both cases, the present values of future revenues can be calculated based on equation (3), applying the same substitution:

$$V_{t,u} \equiv \frac{(1+i_t+\rho_u)\cdot e_{j,t}}{(1+i_t)\cdot(i_t+\rho_u)} - 1 \qquad and \qquad V_{t,d} \equiv \frac{(1+i_t-\rho_d)\cdot e_{j,t}}{(1+i_t)\cdot(i_t-\rho_d)} - 1 \tag{12}$$

A currently inactive firm compares the expected present value of an immediate entry $[E(V_{j,t}) - H_j]$ with the expected present value of the wait-and-see strategy $E(W_{j,t}^{entry})$, which is based on the option to invest in $t+1$. It will invest if the interest costs are decreasing i.e. conditional on a ($-\rho_d$)-realization of the stochastic interest rate, which has a probability of $(1-P)$.

$$E(V_{j,t}) - H_j = P \cdot V_{t,u} + (1-P)\cdot V_{t,d} \tag{13}$$

$$E(W_{j,t}^{entry}) = \frac{1-P}{1+i_t}\cdot\left(\frac{e_j}{i_t-\rho_d} - H_j\right) \tag{14}$$

Under interest rate uncertainty, the entry trigger condition $[E(W_{j,t}^{entry}) = E(V_{j,t}) - H_j]$ leads to:

$$e_j = i_t\cdot(1+H_j) + \frac{\rho_u\cdot P\cdot(1+H_j)}{i_t+\rho_u+P} \quad\Leftrightarrow\quad i_t = \frac{e_j}{1+H_j} - \frac{\rho_u\cdot P}{i_t+\rho_u+P} \tag{15}$$

Comparing this result with equations (4) and (9) shows again an option effect lowering the entry trigger interest rate, now by $[-(\rho_u \cdot P)/(i_t + \rho_u + P)]$. This option value effect is only related to the potential increase of the interest rate. In this case, an immediate investment later will turn out to be wrong due to increasing costs of capital. On the other hand, the chance of a decreasing interest rate is not relevant for an immediate entry, since in this case there is no risk of having conducted the wrong investment.

A currently active firm can disinvest now or stay active. Remaining active results in a current profit of $(e_j - i_t)$. For a $(-\rho_d)$-realization, the firm will continue activity with a future annuity of $(e_{j,t} - i_t + \rho_d)$, resulting in a present value $V_{t,d}$. With probability P the firm will use the disinvestment option in the case of a $(+\rho_u)$-realization in $t+1$. The expected present value of the wait-and-see strategy $E(W_{j,t}^{exit})$ is:

$$E(W_{j,t}^{exit}) = P \cdot \left(\frac{e_j - i_t}{1 + i_t} \right) + (1 - P) \cdot V_{t,d} \tag{16}$$

An immediate disinvestment results in zero profits. Hence, indifference is given by $[E(W_{j,t}^{exit}) = 0]$, determining the exit trigger:

$$i_t = e_j + \frac{\rho_d \cdot (1 - P)}{i_t - \rho_d + 1 - P} \tag{17}$$

Comparing this exit trigger with equations (5) and (11) shows again a positive option value effect $[\rho_d \cdot (1 - P)]/[i_t - \rho_d + 1 - P]$. This option effect is only related to the potential risk of a decreasing interest rate, since an immediate disinvestment would turn out as the wrong decision for this realization.

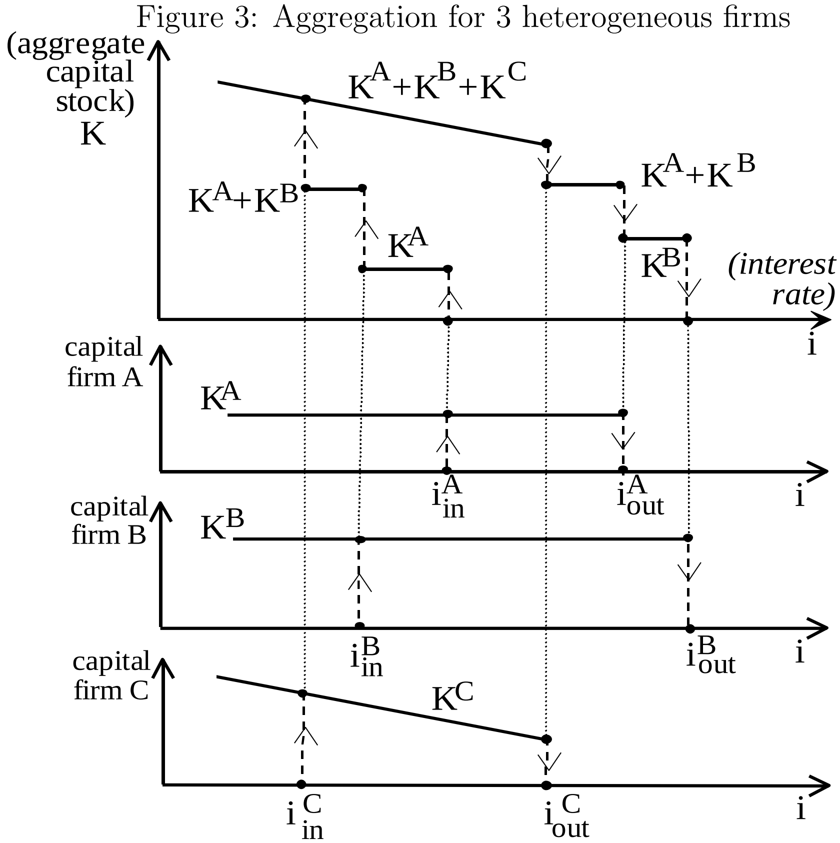
2.4 Non-ideal relay related to interest rates in a stochastic scenario

In the previous two subsections, we showed that the entry/exit triggers are affected by option values. Explicitly solving the trigger conditions for i_t to calculate the interest rate triggers a_j and b_j will lead to a mathematically complex result including some root expressions. Nevertheless, the direction of the option value effects on the level of the interest rate triggers is unambiguous for both different types of stochastic effects. Equations (9) and (11) make clear that there is a negative effect of the option value of a potentially decreasing future rate of return and of a potentially increasing future interest rate on the interest rate that triggers an immediate investment. In Figure 2 this would be depicted as a shift of the investment trigger a_j to the left. For the disinvestment decision, equations (15) and (17) state a positive option value effect on the level of the exit trigger interest rate in both uncertainty scenarios. The risk of a rising future rate of return and the risk of a future interest rate decrease creates an option value of waiting with the disinvestment. Hence, the immediate exit interest rate trigger b_j is shifted to the right in Figure 2 in both scenarios. Summarizing, in all uncertainty scenarios the underlying option value effects lead to a widening of the band of inaction, i.e. the distance between the entry and exit triggers a_j and b_j is increased due to uncertainty. However, the general non-ideal relay characteristic of two different path-dependent equilibria and remanence related to passing specific triggers remains under uncertainty.

3 Aggregation and macroeconomic investment hysteresis

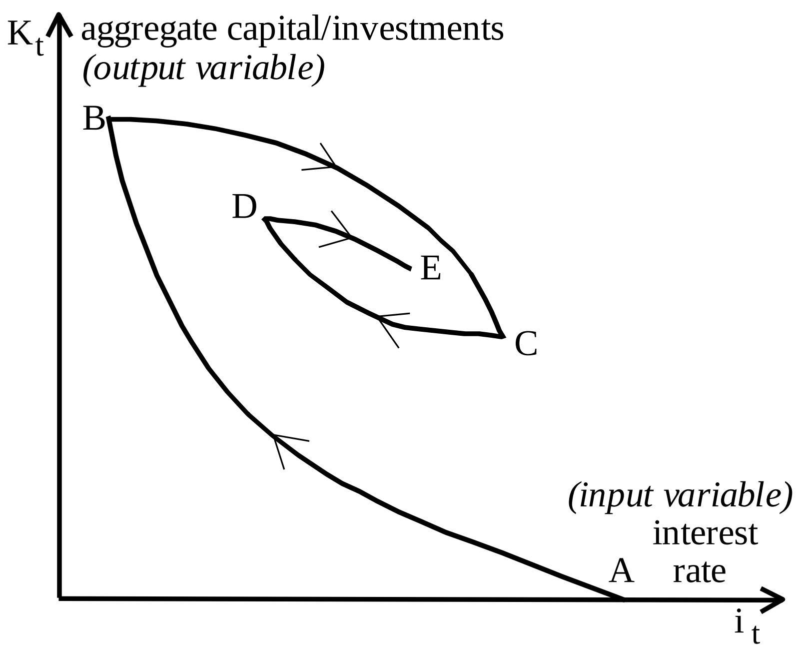
With the microeconomic (investment) behaviour of firms being characterized by non-ideal relays, aggregation is not trivial if there is heterogeneity concerning the entry/exit triggers of different firms. An adequate explicit aggregation procedure, the Preisach (1935) - Mayergoyz (1986) model, is presented by Belke & Göcke (2019) as a modified version for the application of investment hysteresis. This aggregation procedure results in an aggregated macroeconomic hysteresis loop with some differences concerning the quality of path-dependent features.

Figure 3: Aggregation for 3 heterogeneous firms



In this paper, we do not want to explicitly present the aggregation procedure, but only to give an impression of the effects and problems of aggregation in the case of heterogeneous firms. To this purpose, an example with 3 heterogeneous firms with non-ideal relay type reactions is depicted in Figure 3. The individual firm's activity is depicted for 3 firms (A, B and C) in the lower part of the diagram, while the dynamics of the aggregate capital stock resp. production is illustrated in the upper part of Figure 3.[4] If no firm was initially active (i.e. for a very high initial interest rate i), a monotonously decreasing interest rate i will result in an entry of firm A at interest rate i_{in}^{A}, firm B will start activity at $i_{in}^{B} < i_{in}^{A}$, and an interest rate below $i_{in}^{C} < i_{in}^{B}$ will cause an entry of firm C. If subsequently the interest rate increases monotonously, firm C will exit at an interest rate above i_{out}^{C}, and firm A will exit an interest rate above $i_{out}^{A} > i_{out}^{C}$. If the interest rate rises to $i > i_{out}^{B}$, no firm will be active anymore.

Figure 4: The continuous "macroeconomic" hysteresis loop for the aggregate investment activity related to the interest rate for a large number of firms



If the micro behaviour is characterized by non-ideal relays, the aggregate loop for all firms together shows a kind of stairway (i.e. a step function) for decreasing and for increasing input variables with a band-of-inaction between the stairway-up and the stairway-down region. The higher the number of hysteretic firms underlying the aggregation procedure, the

[4] Firm C depicts a more general case of a non-ideal relay, where not only the typical jump at the trigger points but, additionally, a marginal effect of the interest rate on the size of unspecific capital is allowed. Such a generalization is generally possible, but for simplicity reasons was not presented in the micro model in this paper.

smaller is the relative size of the individual firm's steps, converging towards a more and more continuously looking smooth aggregate reaction on both stairways. For a large number of non-ideal relay firms the aggregate loop will converge to a continuous macroeconomic loop as it is depicted in Figure 4 if the different triggers of the heterogeneous firms are distributed continuously. The qualitative characteristics of this continuous macro loop are different: At the micro/firm level, a passing of triggers is necessary to induce a discontinuous jump as a permanent effect. However, for a continuous macro loop, every local extremum in the time-path i.e. every switch in the direction of the development of the input variable will have a persisting/remanence effect. For this reason, the macro hysteresis pattern has been called "strong hysteresis" (e.g. Amable et al. (1991)).

4 Play-hysteresis as an approximation

Applying the explicit Preisach-Mayergoyz procedure, Belke & Göcke (2001,2005) show that even the aggregate behaviour is characterized by areas of weak reactions and that the continuous macro loop can be approximated by a relatively simple dynamic pattern which - corresponding to backlash in mechanics - is called "play".[5] As far as changes occur inside some play area, there are no persistent effects from small changes in the input variable. However, if changes go beyond the play area, sudden strong reactions of the output variable (and remanence effects) result.[6] In contrast to the microeconomic band-of-inaction of Figure 2, the play area is shifted with the history of the forcing input variable: Every change in the direction of the movement starts with traversing an inaction area. After this play area is passed, a stronger reaction (called spurt) results if the input variable continues to move in the same direction.

[5] For a mathematical presentation of play-hysteresis, see (Krasnoselski & Pokrovski 1989, pp. 6 ff.), and (Brokate & Sprekels 1996, pp. 24 f. and pp. 42 ff.).

[6] See (Pindyck 1988, pp. 980 ff.) and (Dixit & Pindyck 1994, pp. 15 f.) for a non-technical description of macroeconomic spurts due to a microeconomic sunk cost mechanism.

Figure 5: Linear (demand side) play-hysteresis and spurt areas

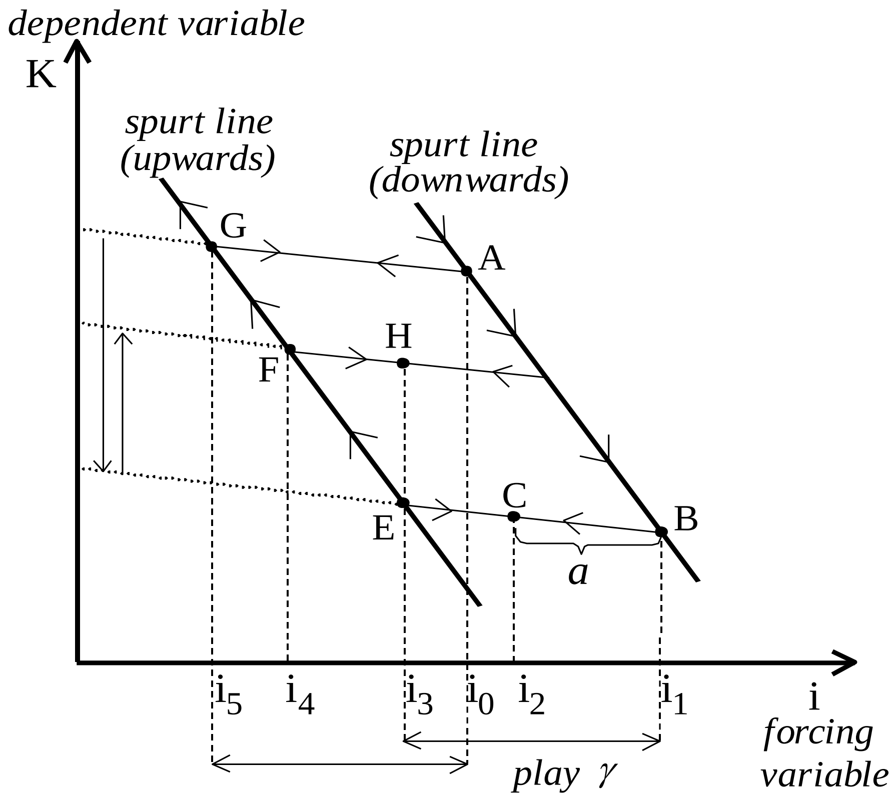


Figure 5 gives an impression of play dynamics for a simple case with linear segments.[7] In our example, the dependent variable is the aggregate capital K and the forcing variable is the interest rate i. A preceding interest rate increase had led to an initial situation in starting point A (i_0) located on the downward leading (right) spurt-line. Changing direction (i.e. now an i-decrease) results in entering the play area. A weak play reaction results until the entire play area of absolute width $\gamma(> 0)$ is passed. The upward leading spurt-line starts in point G at i_5 (with: $\gamma = i_0 - i_5$). In the play area (between points A and G) only a weak reaction of the dependent variable K results from changes in the forcing variable i. A further decrease of i would induce a strong response of K along the (left) upward leading spurt-line.

[7] Actually, the presented play loop is an example of demand-side hysteresis, since the demand of the factor capital of the firms is modelled. On the demand side a price increase is reducing the quantity: i.e. an interest rate increase reduces capital demand. Thus, the play loop shows negative slopes in the case of demand-hysteresis. Contrary, in the supply-side hysteresis case the play loop has positive slopes. For a differentiation of both cases see Göcke & Werner (2015).

Alternatively, think of an interest increase starting from i_0 (A) up to i_1 (point B) and a subsequent decrease to i_2 (C). The corresponding reaction of K first evolves along the right spurt-down line from A $\rightarrow$ B. With this movement, the relevant play area is vertically downward-shifted, from line GA to line EB ($\gamma = i_0 - i_5 = i_1 - i_3$). Now a decrease from i_2 (C) to i_3 (E) takes place in a play area.[8] This play area is partially penetrated in point C by an extent 'a'. A further interest decrease $i_2 \rightarrow i_3 \rightarrow i_4$ (with trajectory points C $\rightarrow$ E $\rightarrow$ F) leads to passing the entire play width γ at point E (i_3), followed by a strong reaction on the upward leading (left) spurt-line until point F. On this spurt-up line, a further input decrease suddenly leads to a strong increase of the dependent variable. However, this (continuous) change in behaviour is not related to a constant trigger level as in the micro loop, but path-dependent, since the play lines are vertically shifted by movements along the spurt-lines. The play area now is shifted in the opposite direction as before, so that for a subsequent increase back to $i_4 \rightarrow i_3$, a weak play reaction (F $\rightarrow$ H) describes the reaction. Summarizing, persistent effects emerge if movements go beyond a play area and lead to reactions on a spurt-line, resulting in a shift of the play area. In contrast, there are no permanent effects from small variations taking place only inside the play-areas.

Actually, interpreted in terms of Figure 3, the spurt-lines are a kind of continuous "stairway-up/-down" reaction due to aggregation over a large number of heterogeneous firms, and the width γ of the play area is related to the distance between both stairways. Of course, using play dynamics with linear segments and a constant play width is a simplified way to capture macro/aggregate dynamics. The slope of the branches of the macro loop depends on the distribution of the firms' triggers and is in general non-linear. However, even a curved aggregate loop (as depicted in Figure 4) can be seen as approximated by the kinked play-loop.[9]

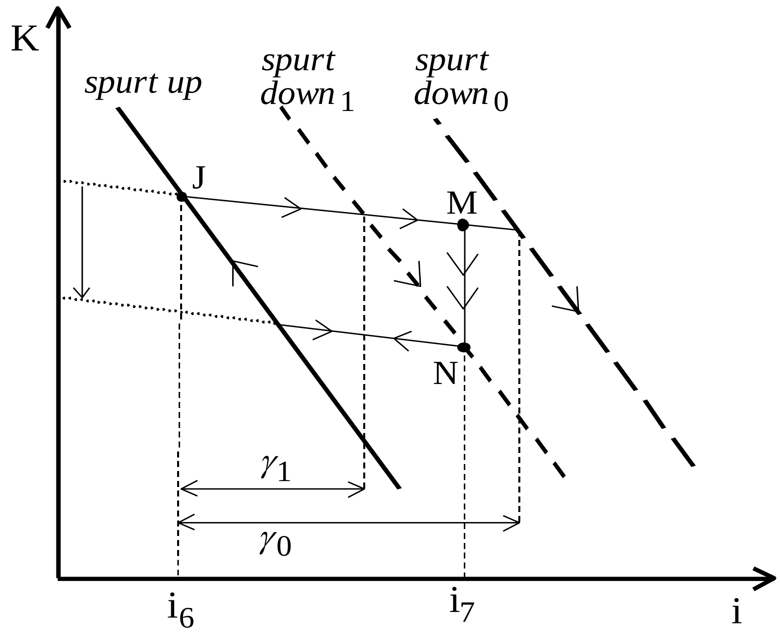
Moreover, since the band of inaction is due to option value effects widened by uncertainty on a microeconomic level, increased uncertainty that is prevalent for all firms on the market will result in a widening of the play area on the aggregate level.[10] Thus, for capturing changing uncertainty effects, changes in the play width must be implemented into the play model, which is illustrated in Figure 6. After some upward movement on the (left) spurt up line we start from point J (i_6). If the width of play is changing, the opposite (right) spurt-line is horizontally shifted, while the location of the left spurt-line is fixed as long as an upwards movement is continued. Generally, the spurt-line on which the most recent movement has

[8] In the original case of mechanical play there would be even no reaction of K inside a (horizontal) play area (Krasnoselski & Pokrovski 1989, p. 8).

[9] For a discussion of the quality of the approximation of the non-linear hysteresis loop by linear segments see Hallett & Piscitelli (2002).

[10] For an integration of these effects into a play loop see Belke & Göcke (2005).

Figure 6: Linear spurt-lines and variable play width



taken place serves as an anchor (here the spurt up line) while the opposite spurt-line (in our example the right spurt down line) is shifted horizontally via play variations. If, e.g. an initially high degree of uncertainty leads to a large play width γ_0, this leads to a right spurt located at (spurt down$_0$). An interest increase from $i_6 \rightarrow i_7$ in the γ_0-situation results in a weak play reaction (points J $\rightarrow$ M). However, if later on the uncertainty is reduced starting from a situation described by point M, this causes a reduction of the play width $\gamma_0 \rightarrow \gamma_1$. Thus, a horizontal shift of the right (spurt down$_0$) to (spurt down$_1$) occurs. Due to the reduction of the play width the system ends up in point N, and at the same time the play area is shifted downwards. Summarizing, movements on the spurt-line resulting from changes in the forcing variable i as well as variations of the width of the play area result in vertical shifts of the relevant play line, i.e. in persistent hysteresis effects.

5 An algorithm capturing linear play

Based on summing up the input variable movements on the spurt-line as well as effects of changes in the play width, an algorithm presented by Belke & Göcke (2001) calculates an artificial shift variable. This spurt variable s_t sums up all spurt movements which had led to shifts of the play area. Actually, the spurt variable s_t is just the series of the original input variable i_t where all small changes inside the play areas are filtered out. Since this filtering is based on the play width, the resulting spurt variable s_t depends on the size of γ_t. After the filtering algorithm is applied on i_t to calculate the spurt variable s_t, a standard linear equation of the following type can be estimated (e.g. by OLS):

$$\text{investment}_t = c + \alpha \cdot i_{t-1} + \beta \cdot s_t(\gamma_t) + \text{function(further variables)} \tag{18}$$

The slope of the play area is stated by the parameter α of the original input variable and the difference in the slope between play areas and spurt-lines is given by the parameter β of the spurt variable s_t. Thus, the resulting slope of the spurt-lines is stated by the parameter sum $(\alpha + \beta)$. In the case of play hysteresis we would expect a low (absolute) level of α, i.e. flat play areas, and a steep spurt-line, with a higher size of $(\alpha + \beta)$. I.e. a typical play hysteresis loop follows, if the following condition holds:

$$|\alpha| < |\alpha + \beta|, \text{ i.e. "flat" play areas and "steep" spurt-lines} \tag{19}$$

Moreover, in the case of demand hysteresis, the slope of the spurt-line is expected to be negative.

For a detailed presentation of the filtering algorithm (and for a translation into an EViews batch program) we refer to Belke & Göcke (2001). Our implementation of the play algorithm allows to control the play width by another variable. In order to capture the positive impacts of uncertainty on the play width, we model the variable play width in a simple linear way as a function of an uncertainty proxy variable u_t.

$$\gamma_t = \mu + \delta \cdot u_t \qquad \text{with: } \mu, \delta \geq 0 \quad \text{and } u_t \geq 0 \Rightarrow \gamma_t \geq 0 \tag{20}$$

Applying an OLS-Procedure, we suppose that the standard regression model assumptions hold: the error term is independently, identically and normally distributed with a constant finite variance over all sections, and the regressors are measured without any error and are not correlated with the error term. However, our model is non-linear in its parameters, since the knots are not known a-priori and since the play width γ has to be estimated in order to determine the spurt variable s. In such a (spline) model with unknown but continuous switches, the OLS-/ML-estimator leads to consistent and asymptotically normally distributed estimates. Unfortunately, the finite sample properties of the play regression model remain problematic: The parameter estimates are not even approximately normally distributed for small samples and local maxima in the likelihood function may occur.[11] Moreover, non-stationary variables might imply non-finite variance, while at the same time play dynamic represents a mixture of short-term and long-term dynamics, which obstructs the

[11] See Hujer (1986, pp. 231 ff.), Poirier (1976, pp. 108 ff., pp. 117 ff. and p. 129), Hudson (1966) and Hinkley (1969).

application of standard cointegration analysis. As a consequence, the small sample distribution and the critical values of the estimators have to be interpreted with caution. However, a solution to these econometric problems is beyond the scope of this paper. In order to minimize the residual sum of squares (i.e. applying OLS), a grid search over the parameters of the play is executed.

6 Empirical application

6.1 Data

In order to check for the empirical relevance of the hysteresis model for domestic investment in several countries of the Euro area, we estimate equation (18) which generalizes hysteretic behavior of investment dependent on movements in the interest rate. In the empirical application, we use data on domestic investment in five countries of the Euro-area - namely Austria, Finland, France, Germany, and Spain - as the dependent variable and the interest rate as the hysteretic input variable. Data availability of the hysteretic input variable led to the final selection of the countries in the empirical analysis. To be as parsimonious as possible, we include real GDP, a linear trend and seasonal dummies as additional non-hysteretic control variables.

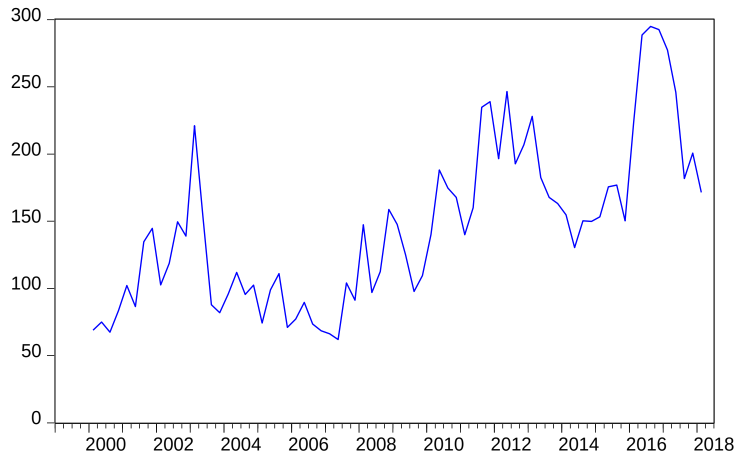
The GDP variable enters with a lag of one quarter into the regression, because lagged GDP data produce the best fit in our regressions and assist in avoiding problems of reverse causation. We choose the same approach for the interest rate, which enters in its lagged form.[12] The spurt variable then is calculated based on the lagged interest rate. This general setting is applied in all estimations in this paper.

The exact specifications of the time series are as follows: investment is covered by quarterly gross fixed capital formation data which comprises fixed assets acquisitions of resident producers, less disposals. It is seasonally and calendar adjusted, as well as deflated since the time series are chain-linked volumes with 2010 as base year. We select this variable to picture investment due to its availability for all countries in the sample in the national accounting database of Eurostat. We consider the bank interest rate on loans to corporations of up to 1 million Euro with a duration of 1 to 5 years as most relevant for the purpose of our analysis, as it is probably the crucial rate for investment of small and medium sized enterprises. The interest rate time series are taken from the ECB database which provides monthly values for the different countries separately. The series are available from January 2000 on, but

[12] Using lagged GDP avoids problems related to endogeneity effects of the dependent variable (investment) and the regressor (GDP). Since investment could theoretically contemporaneously affect the interest rate on bank loans, we have also decided to use the lagged value of the interest rate in the estimations.

they only cover all months up to 2018 for the five countries stated above.[13] We convert the time series to a quarterly format and limit our investigation to the five countries, so that our estimation period ranges from 2001Q1 to 2018Q1. The corresponding real GDP time series are taken from Eurostat. We employ all variables in levels so that the regression results show the absolute change of the variables.

Figure 7: Policy Uncertainty Index



Notes: Data from www.policyuncertainty.com

We apply a political uncertainty index (Baker et al. 2016) to measure economic uncertainty in Europe. Figure 7 displays its development since 2000. In its construction, three components are considered. The first one quantifies newspaper-coverage of policy-related economic uncertainty. The second component counts federal tax code provisions which are about to expire in future years. And the third component accounts for disagreement between economic forecasters to proxy uncertainty. The series has been seasonally adjusted using the Census X-12 procedure in EViews.

Additionally, we employ the variance in the interest rates as another driver of uncertainty to test the robustness of the results applying variable play. Hence, we take the positive log of the variance of the past twelve months for the interest rate on corporate loans and introduce it as another measure of uncertainty.

6.2 Estimation without play (i.e. the standard linear model)

A simple OLS regression allows for the estimation of the linearized model. We start with a regression of French investment on the French interest rate on corporate bank loans, the

[13] The Estonian time series for the interest rate is also complete. However, we chose to include only countries into our regressions that did belong to the Euro area from the beginning.

French real GDP and, additionally, a linear trend dummy plus dummy variables for the first 3 quarters (Q1-Q3) in the upper panels of the following tables containing results. In the two lower panels, we show the results of the regressions for Germany and Austria respectively. At a first stage, we exclude spurt or play effects, i.e. we apply the restriction $\beta = 0$ (or equivalent: $\gamma_t = 0$). Table 1 displays the respective results.

For France and Austria, the estimated coefficients of all regressors are significant except for some seasonal dummy variables. The direction of both effects matches the expectation from theoretical considerations, since we find a negative effect of the interest rate on bank loans on investment, as well as a positive effect of real GDP. However, in the case of Germany, the interest rate appears to not exert any impact on the investment decision, since the corresponding coefficient remains insignificant. The sample period includes 17 years, hereby clearly covering more than one business cycle which is connected to the development of interest rates. Nevertheless, the interest rates of most countries in our sample display a declining trend over the period 2000 until 2018 with the exception of a steep increase from 2006 until 2008 which peaked below the sample maxima in 2000. This is different for Spain, with the maximum value being reached in 2008 at 6.7%, followed by another peak in 2012 at 6.4%, and the interest rate of 6% in 2000 only ranking third in terms of positive extrema. Appendix A.1 shows the development of the several domestic interest rates used in our study.

Table 1: Standard OLS regression without play
(restriction $\beta = 0$ or $\gamma_t = 0$)

(a) Panel A: France

Dependent variable: Domestic investment in France				
Sample: 2001Q2 2018Q1				
Included observations: 68				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-204272.9	16618.98	-12.29154	0.0000
Interest Rate(-1)	**-2560.535**	**492.1037**	**-5.203242**	**0.0000**
GDP_FRANCE(-1)	0.766205	0.041746	18.35413	0.0000
TREND	-855.4151	72.57962	-11.78589	0.0000
D1	433.2822	771.3645	0.561709	0.5764
D2	-557.2394	771.5814	-0.722204	0.4729
D3	74.73736	771.2872	0.096900	0.9231
R-squared	0.932512	Mean dependent var		114310.6
Adjusted R-squared	0.925874	S.D. dependent var		8256.929
Prob(F-statistic)	0.000000			

(b) Panel B: Germany

Dependent variable: Domestic investment in Germany Sample: 2001Q1 2018Q1 Included observations: 69				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-136262.6	24072.32	-5.660549	0.0000
Interest Rate(-1)	**704.9281**	**1216.351**	**0.579544**	**0.5643**
GDP_GERMANY(-1)	0.458217	0.046396	9.876211	0.0000
TREND	-626.8394	129.6259	-4.835758	0.0000
D1	1239.151	1639.012	0.756036	0.4525
D2	1345.661	1664.169	0.808608	0.4218
D3	1063.053	1661.592	0.639780	0.5247
R-squared	0.764493	Mean dependent var		128581.9
Adjusted R-squared	0.741701	S.D. dependent var		9528.944
Prob(F-statistic)	0.000000			

(c) Panel C: Austria

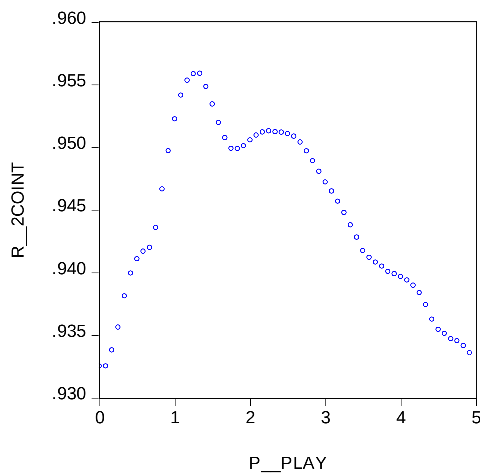
Dependent variable: Domestic investment in Austria Sample: 2001Q2 2018Q1 Included observations: 68				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-10440.41	3483.344	-2.997238	0.0039
Interest Rate(-1)	**-497.8705**	**179.1237**	**-2.779479**	**0.0072**
GDP_AUSTRIA(-1)	0.491722	0.069915	7.033101	0.0000
TREND	-98.76296	23.83483	-4.143640	0.0001
D1	-142.3476	212.4997	-0.669872	0.5055
D2	-59.88853	213.0137	-0.281149	0.7795
D3	-21.02287	212.9566	-0.098719	0.9217
R-squared	0.767901	Mean dependent var		17801.05
Adjusted R-squared	0.745071	S.D. dependent var		1225.858
Prob(F-statistic)	0.000000			

6.3 Estimation with constant play ($\gamma_t = \gamma$)

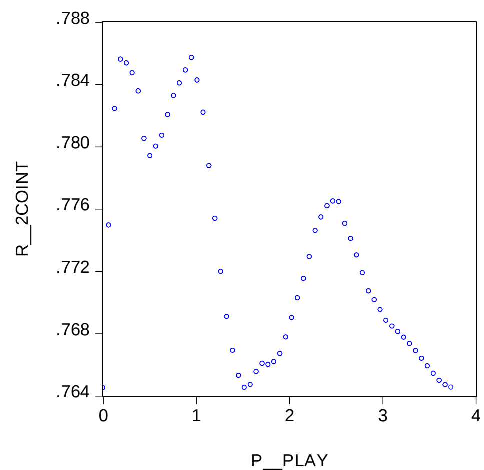
In a second step, we estimate the model for the simple case where the (filtered) spurt variable is calculated based on a constant play width ($\gamma_t = \gamma = \text{const}$). A grid search over different sizes of the play width γ is used to find the play width with the highest R^2.

Figure 8: Different R^2 resulting from a one-dimensional grid search over different values of constant play γ

(a) Panel A: France



(b) Panel B: Germany



(c) Panel C: Austria

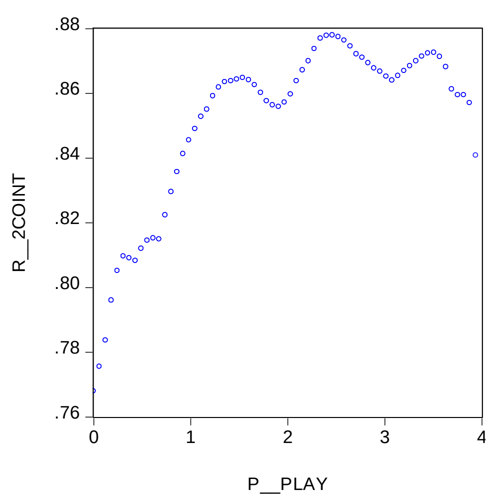


Figure 8 displays the corresponding plot: The R^2 sequence in panel A shows an absolute maximum at $\gamma = 1.333$ (with $R^2 = 0.956$) for France. The R^2 minimum at $\gamma = 0$ (with $R^2 = 0.933$) exactly corresponds to the linear standard model stated in panel A of table 1. For the German case in panel B, we find an absolute maximum at $\gamma = 0.95$ (with $R^2 = 0.786$) and again, the R^2 minimum at $\gamma = 0$ (with $R^2 = 0.765$) equals the linear standard model in panel B of table 1. Panel C has the results for Austria: the absolute maximum is at $\gamma = 2.462$ (with $R^2 = 0.878$) and the R^2 minimum at $\gamma = 0$ (with $R^2 = 0.768$) corresponds to the results in panel C of table 1. Table 2 shows the estimation results of the spurt/play regressions with an artificial spurt-variable (SPURT) based on the constant play-widths $\gamma = 1.333$ for France in panel A, $\gamma = 0.95$ for Germany in panel B, and $\gamma = 2.462$ for Austria in panel C.

Table 2: Standard OLS regression with constant play

(a) Panel A: OLS regression for France with constant play $\gamma = 1.333$

Dependent variable: Domestic investment in France
Sample: 2001Q2 2018Q1
Included observations: 68

Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-174963.1	14510.33	-12.05783	0.0000
Interest Rate(-1)	621.6708	692.3860	0.897867	0.3728
SPURT	**-6595.217**	**1169.605**	**-5.638842**	**0.0000**
GDP_FRANCE(-1)	0.661586	0.038759	17.06914	0.0000
TREND	-874.5072	59.26202	-14.75662	0.0000
D1	449.5134	628.8050	0.714869	0.4775
D2	-546.0547	628.9783	-0.868161	0.3888
D3	107.5536	628.7623	0.171056	0.8648
R-squared	0.955889	Mean dependent var		114310.6
Adjusted R-squared	0.950742	S.D. dependent var		8256.929
Prob(F-statistic)	0.000000			

(b) Panel B: OLS regression for Germany with constant play $\gamma = 0.95$

Dependent variable: Domestic investment in Germany Sample: 2001Q1 2018Q1 Included observations: 69				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-119972.1	24080.71	-4.982082	0.0000
Interest Rate(-1)	6755.456	2726.073	2.478091	0.0160
SPURT	**-10167.49**	**4137.801**	**-2.457220**	**0.0169**
GDP_GERMANY(-1)	0.364661	0.058655	6.217019	0.0000
TREND	-589.1451	125.6003	-4.690633	0.0000
D1	1279.826	1576.310	0.811912	0.4200
D2	1278.784	1600.648	0.798916	0.4274
D3	879.7811	1599.678	0.549974	0.5843
R-squared	0.785704	Mean dependent var		128581.9
Adjusted R-squared	0.761113	S.D. dependent var		9528.944
Prob(F-statistic)	0.000000			

(c) Panel C: OLS regression for Austria with constant play $\gamma = 2.462$

Dependent variable: Domestic investment in Austria Sample: 2001Q2 2018Q1 Included observations: 68				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-14679.57	2611.912	-5.620238	0.0000
Interest Rate(-1)	-195.3476	137.3090	-1.422686	0.1600
SPURT	**-1534.329**	**208.6950**	**-7.352017**	**0.0000**
GDP_AUSTRIA(-1)	0.557385	0.051905	10.73849	0.0000
TREND	-137.9340	18.22720	-7.567483	0.0000
D1	-139.6894	155.4078	-0.898857	0.3723
D2	-51.46682	155.7875	-0.330366	0.7423
D3	-0.393380	155.7667	-0.002525	0.9980
R-squared	0.877898	Mean dependent var		17801.05
Adjusted R-squared	0.863653	S.D. dependent var		1225.858
Prob(F-statistic)	0.000000			

For panels A and C, the variables GDP and SPURT display the theoretically expected sign while being highly significant. With respect to the hypothesis (H1) $\beta \neq 0$, the estimated coefficient of the spurt variable is $\beta = -6595$ with a t-value of -5.639 for France. Note that, as expected, the spurt variable substitutes the effects of the original interest rate on corporate bank loans, which in the linear standard regression in table 1 was $\alpha = -2561$ ($t = -5.203$), and now vanishes to an insignificant (weak play) effect $\alpha = 622$ ($t = -0.898$) in the play regression. Additionally, the absolute effect of spurt in the play regression surpasses the original interest rate effect in the linear regression. However, since the small sample properties of our regression model are unknown, the t-values are most probably not student-t distributed. Nevertheless, this high empirical t realization (which is about three times as high as the 5 per cent critical value in case of a standard student-t distribution) represents at least a strong hint at the relevance of hysteretic play. The empirical evidence of spurt effects in Austria appears to be similar: the highly significant coefficient of the spurt variable is estimated to be $\beta = -1534$ ($t = -7.352$), hereby exceeding the effect of the interest rate in the regression without play at $\alpha = -498$ ($t = -2.780$). In addition, the interest rate variable turns insignificant $\alpha = -195$ ($t = -1.423$) in the regression with constant play. Additionally, the expectation about the relationship between the slope of the spurt-lines and the slope of the play area in the case of play hysteresis is met with equation (19) holding for France and Austria respectively:

$$\text{France: } (|\alpha| = 622) < (|\alpha + \beta| = |622 - 6595|)$$

$$\text{Austria: } (|\alpha| = 195) < (|\alpha + \beta| = |195 - 1534|)$$

The estimation of German investment dependent on the interest rate reveals in panel B, that the estimated coefficient of the spurt variable is $\beta = -10168$ with a t-value of $t = -2.457$. Even though the interest rate emerges as insignificant in the regression without play ($\alpha = 705$, $t = 0.580$), it turns significant in the regression with constant play with a coefficient of $\alpha = 6755$ ($t = 2.478$). The direction of this effect does not correspond to the theoretical expectation of a negative relationship between the cost of an investment and the investment itself. Furthermore, equation (19) does not hold with

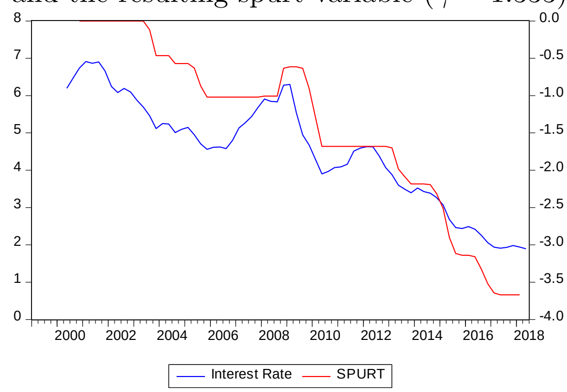
$$\text{Germany: } (|\alpha| = 6755) > (|\alpha + \beta| = |6755 - 10168|)$$

Moreover, the grid-search of the German play-regression shows two local maxima of nearly the same level, which makes an estimation based on the maximum of the R^2 very problematic. Here, the small sample properties of the play regression - potential local maxima in the Likelihood function - actually obstruct an interpretable estimation. Therefore, the evidence does not confirm hysteretic effects to be present in the German investment behavior.

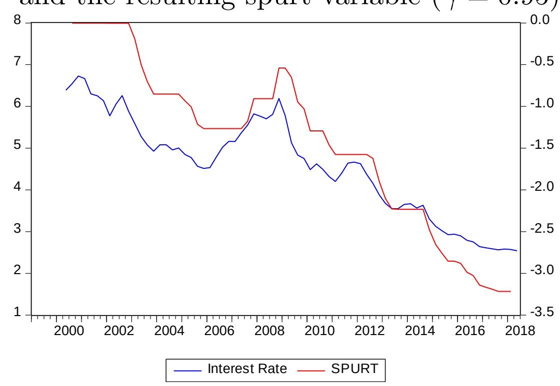
Finally, Figure 9 shows the graphical impression of the time sequence of the original interest rate on corporate bank loans (left scale) and of the respective SPURT (right scale) which captures the strong impact of interest rate changes after passing the play area, i.e. after passing a kind of pain threshold. The time path of the spurt variable shows inevitably similarities to the original interest rate path, since spurt s_t is based on the interest rate i_t less small changes of the latter, occurring inside the play area. However, limited variability of the original interest rate on corporate bank loans series inside the play area (of width $\gamma = 1.333$ for France, $\gamma = 0.95$ for Germany, and $\gamma = 2.462$ for Austria) is filtered away and periods of inaction emerge, exhibiting no variation of the spurt variable due to play/ inaction effects. The artificial spurt series reflects only large/ monotonous changes in the interest rate.

Figure 9: Interest rate on corporate bank loan and spurt variable

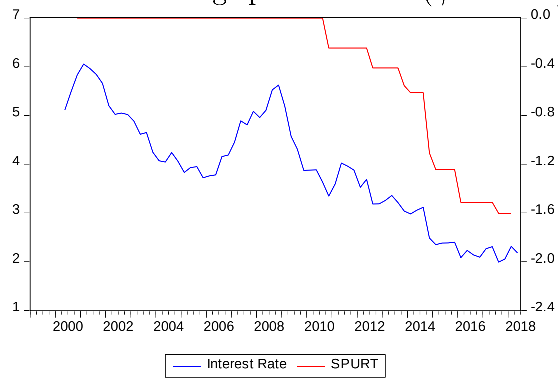
(a) Panel A: French interest rate on corporate bank loans and the resulting spurt variable ($\gamma = 1.333$).



(b) Panel B: German interest rate on corporate bank loans and the resulting spurt variable ($\gamma = 0.95$).



(c) Panel C: Austrian interest rate on corporate bank loans and the resulting spurt variable ($\gamma = 2.462$).



A change in the spurt variable simultaneously shifts the current position and the borders of the play area. The up to now most recent shift of the play position/ borders for domestic investment in France corresponds to the interest rate extremum of the first quarter in 2018 at the end of the estimation period. Thus, the corresponding lower bound, which would result in a strong spurt reaction for French investment, equals this last extremum of the interest rate at an interest rate of 1.89%. Once passed, this interest rate threshold induces a strong spurt reaction of French domestic investment. For Germany and Austria, these lower thresholds correspond to the minimum interest rates of 2.54 and 2.18 respectively in Q1 2018. The upper (exit) triggers, that - once passed - evoke disinvestment, result from an addition of the individual play widths to the lower triggers. Table 3 summarizes the top and bottom thresholds for the three countries.

Table 3: Investment triggers for interest rates with constant play

	France	Germany	Austria
Upper trigger (exit)	3.22	3.53	4.78
Lower trigger (entry)	1.89	2.54	2.18

6.4 Estimation with variable play width γ_t

Next, we turn to an estimation using variable instead of constant play, hereby making the play area of weak investment reaction dependent on the degree of uncertainty. At a first stage, we model uncertainty by introducing political uncertainty into the regression.

Table 4: Standard OLS regression with variable play including political uncertainty

(a) Panel A: OLS regression for France with variable play $\gamma_t = 0.286 + 0.217 \cdot u_t$

Dependent variable: Domestic investment in France
Sample: 2001Q2 2018Q1
Included observations: 68

Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-169041.9	14820.55	-11.40591	0.0000
Interest Rate(-1)	946.8217	731.6595	1.294074	0.2006
SPURT	**-7027.523**	**1228.617**	**-5.719864**	**0.0000**
GDP_FRANCE(-1)	0.642596	0.040169	15.99726	0.0000
TREND	-857.9389	58.87250	-14.57283	0.0000
D1	450.1839	625.6768	0.719515	0.4746
D2	-525.6064	625.8702	-0.839801	0.4044
D3	80.90657	625.6081	0.129325	0.8975
R-squared	0.956327	Mean dependent var		114310.6
Adjusted R-squared	0.951231	S.D. dependent var		8256.929
Prob(F-statistic)	0.000000			

(b) Panel B: OLS regression for Germany with variable play $\gamma_t = 0.543 + 0.09 \cdot u_t$

Dependent variable: Domestic investment in Germany
Sample: 2001Q1 2018Q1
Included observations: 69

Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-113333.2	24885.37	-4.554212	0.0000
Interest Rate(-1)	7096.905	2819.083	2.517452	0.0145
SPURT	**-10781.54**	**4327.491**	**-2.491407**	**0.0155**
GDP_GERMANY(-1)	0.348916	0.062534	5.579610	0.0000
TREND	-555.7183	127.7338	-4.350598	0.0001
D1	1297.044	1574.410	0.823829	0.4132
D2	1286.693	1598.576	0.804899	0.4240
D3	873.9557	1597.729	0.546999	0.5864
R-squared	0.786244	Mean dependent var		128581.9
Adjusted R-squared	0.761714	S.D. dependent var		9528.944
Prob(F-statistic)	0.000000			

(c) Panel C: OLS regression for Austria with variable play $\gamma_t = 2.027 + 0.077 \cdot u_t$

Dependent variable: Domestic investment in Austria Sample: 2001Q2 2018Q1 Included observations: 68				
Variable	Coefficient	Std. Error	t-Statistics	Prob.
C	-14726.78	2613.177	-5.635586	0.0000
Interest Rate(-1)	-195.3284	137.3010	-1.422628	0.1600
SPURT	**-1533.911**	**208.6126**	**-7.352919**	**0.0000**
GDP_AUSTRIA(-1)	0.558436	0.051927	10.75425	0.0000
TREND	-138.5733	18.25164	-7.592373	0.0000
D1	-139.3449	155.3989	-0.896692	0.3735
D2	-51.15982	155.7787	-0.328413	0.7437
D3	0.293704	155.7594	0.001886	0.9985
R-squared	0.877912	Mean dependent var		17801.05
Adjusted R-squared	0.863669	S.D. dependent var		1225.858
Prob(F-statistic)	0.000000			

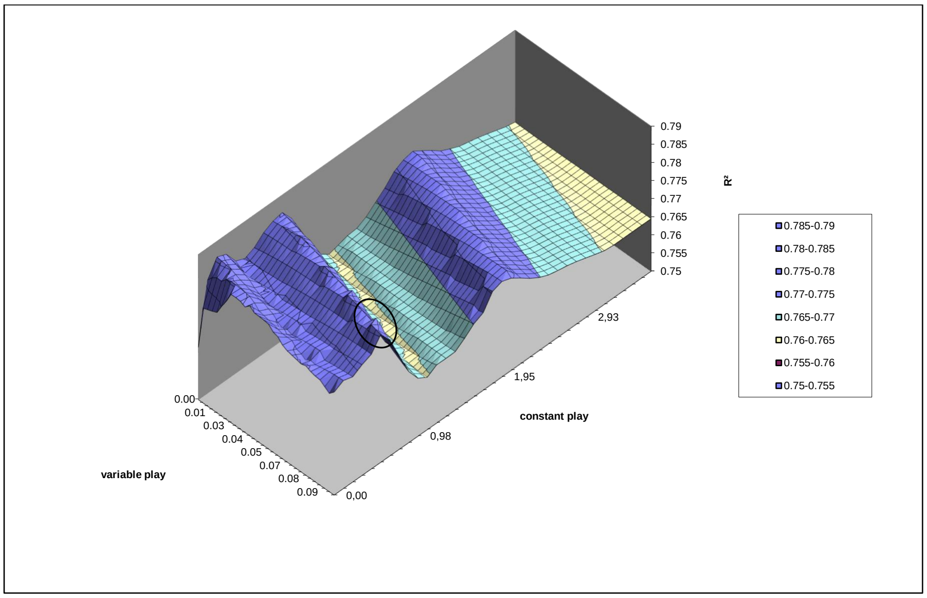
Again, a grid search serves our estimation purposes. However, the grid search now employs a second dimension which depends on the uncertainty variable used (Belke & Göcke 2001). The algorithm allows to control the play width by adding another variable. To capture the positive impact of the uncertainty variable u_t on the play width, we employ the linear function (20). Table 4 shows the estimation output for investment resulting from a two-dimensional grid search with variable play.

The regression equation including the spurt variable in panel A of table 4 again outperforms the original specification without play which can be read from the empirical realization of R^2. It rises from 0.93 in the original specification without play for France (in panel A of table 1) to 0.96 with constant as well as with variable play. Hence, it seems to emerge from the empirical evidence that political uncertainty has no influence on the relationship between interest rates and investment in France indicated by the absence of any further increase in the R^2 from the estimation with constant play to the estimation with variable play. Nevertheless, the spurt variable gains in magnitude in comparison to the regression with constant play, increasing from -6595 to -7,028.

In the German case in panel C, the results show a similar pattern with no distinct increase in the R^2 from the constant play estimation to the variable play estimation, but with a rise in the magnitude of the negative spurt variable. The influence of political uncertainty on the role the interest rate in the determination of Austrian investment seems to be negligible:

neither R^2 nor the magnitude of the spurt variable change noticeably in panel B. Figure 10 shows exemplary the three-dimensional grid search to identify the highest R^2 for the case of Germany. From the perspective of the x-axis, which represents constant play, the picture resembles the corresponding one-dimensional grid search in Figure 8 with the play-area being defined without any variable component. The legend at the right contains the colors indicating the R^2 values. The black ellipse highlights the estimated play-area.

Figure 10: Grid search for German investment (variable play $\gamma_t = 0.543 + 0.09 \cdot u_t$)



To check the results, we apply the variance of the interest rates as another measure of uncertainty. Table 5 summarizes the results for domestic investment in the three countries. Regressions with a theoretically unexpected sign or differing significance are marked by grey shading. In total, this outcome widely confirms the results obtained using political uncertainty.

Table 5: Summary of results employing variance in interest rates as uncertainty measure

Investment in	α	β	γ	R^2
France	847.74	-6826.6***	$1.286 + 0.097 \cdot u_t$	0.96
Germany	18300.77**	-19166.68**	0.217	0.79
Austria	-195.5	-1531.84***	$2.4 + 0.10 \cdot u_t$	0.88

α: estimated coefficient for the interest rate(-1)

β: estimated coefficient for the spurt variable

γ: estimated play width

Several reasons may provide an explanation for the limited effect of the introduction of uncertainty on our empirical results. First, uncertainty is likely to not impact the play width immediately since current uncertainty changes may need some time to spill over to expectations on future cash flows. This is why we do not find statistical evidence in the form of a better fit of the estimation for a widening of the band of inaction. Second, the current interest rate uncertainty, that we employ, does possibly not correspond with the uncertainty about the long-term future, which in turn is of relevance in investment decisions. The search for and implementation of a measure of uncertainty directed at the future and correlating with the lifetime of the investment leaves room for future research. Hence, third, we cannot rule out the possibility that our measures of uncertainty are not completely adequate. What is more, the policy uncertainty index applied relates to uncertainty in Europe, and not specifically to the individual countries in our analysis. The question remains whether uncertainty is a national or international phenomenon. In addition, the algorithm used to estimate hysteretic effects (Belke & Göcke 2001) restricts the possible implementation of uncertainty to one variable. However, uncertainty is likely to be multi-dimensional. Fourth, the interest rate changes themselves may via risk-premia already contain the uncertainty related to investment why no further widening of the bands of inaction and no distinct increase in the goodness of fit measure can be observed by introducing another uncertainty variable. Fifth and lastly, the interest rates show a sustained negative trend over our observation period, why only one side of the play area is actually effective. Hence, changes in the play width due to uncertainty may be difficult to capture.

Apart from the three countries estimated above in detail, we calculate further play regressions for the remaining two countries Finland and Spain, for which we dispose of the time series of the relevant interest rate for the complete sample period. Table 6 summarizes the results which suggest that hysteretic effects are present in both countries. However, for fairly large sizes of the play width, the computed spurt variable reduces to a kind of binary/ dummy variable, only capturing a one-time shift. The results for Spain hint at such an effect, even though the spurt variable contains more than two values. Hereby, a play-hysteretic shift cannot be separated from a one-time structural break. The graphical representation of the interest rate and spurt variable development for Finland and Spain is attached in the appendix A.2. The structural break in Spain coincides with the time of the onset of the financial crisis.

Table 6: Overview of regression results for investment in Finland and Spain

Status of play	Finland	Spain
$\beta = 0$	$\alpha = -394^{***}$ R^2=0.65	$\alpha = -2507^{***}$ R^2=0.89
$\gamma_t = \gamma =$ const. with $\gamma \geq 0$	$\alpha = -300^{**}$ $\gamma = 3.53$ $\beta = -1142^{***}$ R^2=0.70	$\alpha = -557$ $\gamma = 3.93$ $\beta = -13971^{***}$ R^2=0.95
$\|\alpha\| < \|\alpha + \beta\|$	$300 < 1440$	$557 < 14527$
$\gamma_t = \mu + \delta \cdot u_t$ (political uncertainty)	$\alpha = -295^{**}$ $\gamma_t = 1.257 + 0.467 \cdot u_t$ $\beta = -1056^{***}$ R^2=0.71	$\alpha = -674^{**}$ $\gamma_t = 3.84 + 0.087 \cdot u_t$ $\beta = -18439^{***}$ R^2=0.97

α: estimated coefficient for the interest rate(-1)

β: estimated coefficient for the spurt variable

γ: estimated play width

In summary, the investment regressions are in line with the typical play dynamics. We complement the estimations with regressions including variable play, using the log of the political uncertainty index as a proxy for interest rate uncertainty. The R^2 value increases only slightly in comparison to the respective value in the regressions using constant play for the case of Finland (0.702 to 0.707), while it rises more distinctly for the case of Spain (0.95 to 0.97). Hence, the inclusion of political uncertainty to determine the play width (which corresponds to an area of weak investment reaction) increases the goodness of fit of the Spanish domestic investment equation.

7 Conclusions

In this paper, we have empirically examined the relationship between interest rates and investment in five countries of the Euro area. The aim has been the identification of bands of inaction for domestic investment in the individual countries with only weak reactions of investment activity to interest rate reductions. To this end, we have first presented a non-linear path-dependent model in which strong reactions of investment occur when the interest rate passes certain thresholds that border the band of inaction. On an aggregate macro-level, this band becomes the so-called play-area which we capture in a simplified linearized way.

We have then applied an algorithm integrating play-hysteresis into a regression framework. We find hysteretic effects and therefore areas of weak investment reaction to changes in the interest rate for all countries analyzed except for Germany where no effect of interest rate changes on investment can be identified in the first step. It seems as if changes at the very low market interest rate level that has already been reached in Germany do not exert influence on investment. In other words, neither the policy rate nor the corporate market interest rate represents the dominant driver of investment in German (anymore). Much more important are the degree of structural reforms and, thus, expected growth. The play-width identified for Spain is quite large, implying that the spurt variable possibly might capture a one-time structural shift just reflecting the financial crisis. The latter clearly transformed quite a lot of costs into sunk costs due to the burst of Spain's excessive real estate bubble and the emanating banking-debt crisis. What is more, in the context of Finland and, even more so, of Spain our findings indicate political uncertainty enlarges the "band of inaction".

Hence, the existence of "bands of inaction" in domestic investment in several Euro area countries should not be neglected by central banks. This result suggests that not every decrease of the interest rate entails additional investment activity. However, a large decrease of the interest rate implies passing the border of a play (inaction) area, resulting in a strong reaction of investment. In addition, we show that the play area is path-dependent and moves its position with strong interest rate movements. Therefore, no unique exit and entry triggers of the interest rate exist.

To the extent that the ECB is able to affect corporate interest rates, these new findings thus have important implications for the monetary policy transmission process in the Euro area. First, changes in interest rates affect investment demand in a non-linear fashion. The magnitude of a necessary monetary stimulus depends on the historical time-path of the variables. Second, the shape and the effect of a change in interest rates is heterogeneous in the Euro area, i.e. member country-specific. And, third, any stepwise exit from zero interest rate policies by the ECB (interest rate reversal) will not cause great harm for investment demand in the Euro area just because, i.e. the Euro area finds itself within a play area.

Finally, it is important to note that, given the zero lower bound scenario currently prevailing in the Euro area, any further decrease in the nominal policy rates may not be a realistic scenario for the near future. However, the corporate interest rates focused upon in our paper still have some downward leeway, i.e. are still significantly larger than zero due to factors at least partly beyond the realm of the ECB. Any further lowering of the corporate spreads should thus lead to significant positive reactions of investment demand, since the Euro area still finds itself in a "spurt area".

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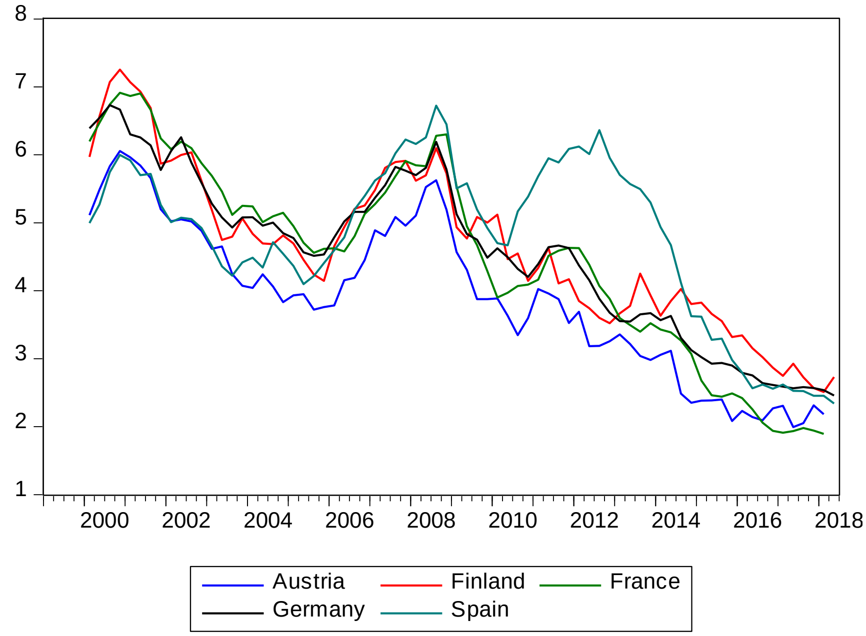
A Appendix

A.1 Development of interest rates for corporate lending since 2000



A.2 Graphical presentation of interest rates and spurt variables for Finland and Spain

(a) Panel A: Finnish interest rate on corporate bank loans and the resulting spurt variable ($\gamma = 3.53$).

(b) Panel B: Spanish interest rate on corporate bank loans and the resulting spurt variable ($\gamma = 3.93$).