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ABSTRACT

Invisible Geniuses: Could the Knowledge Frontier Advance Faster?*

The advancement of the knowledge frontier is crucial for technological innovation and human progress. Using novel data from the setting of mathematics, this paper establishes two results. First, we document that individuals who demonstrate exceptional talent in their teenage years have an irreplaceable ability to create new ideas over their lifetime, suggesting that talent is a central ingredient for the production of knowledge. Second, such talented individuals born in low- or middle-income countries are systematically less likely to become knowledge producers. Our findings suggest that policies to encourage exceptionally-talented youth to pursue scientific careers - especially those from lower income countries - could accelerate the advancement of the knowledge frontier.

JEL Classification: O31, J24, I25

Keywords: talent, knowledge frontier, innovation, IMO, mathematics

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1 Introduction

A group of the world's leading mathematicians gathered at the Collège de France in Paris in June of 2000, where the Clay Mathematics Institute (CMI) announced seven Millennium Prize Problems. The prizes – along with a million dollars for the solution of each problem – were conceived "to spread the news that in mathematics hard, significant problems still abound – the frontiers of knowledge are still wide open." Andrew Wiles, famous for proving Fermat's Last Theorem and a member of the CMI Scientific Advisory Board, said that the Millennium Prize Problems "will excite and inspire future generations of mathematicians and non-mathematicians alike...[w]e are convinced that the resolution of these prize problems will open up a new world of mathematics which as yet we cannot even imagine."

In scientific fields there is an important open question about how to motivate future generations to engage in knowledge production, and to possibly advance the knowledge frontier in the process. Philanthropists and governments have tried to promote innovation through rewarding research outputs ('pull' incentives) or providing up-front support for research inputs ('push' programs) (Kremer & Williams 2010).[1] A common pull incentive used to promote discoveries in science is the use of prizes, which can be designed as targeted prizes that request solutions to prespecified problems (e.g. the Millennium Prize), or blue-sky prizes that are granted for innovations not specified in advance (e.g. the Nobel Prize) (Maurer & Scotchmer 2004). On the other hand, there is also widespread use of push programs to stimulate discoveries in science – such as grants that cover research costs in advance (Stephan 2012, Williams 2012). Fellowships for gifted students to pursue scientific careers is another type of push program. For instance, one of CMI's primary objectives is "to encourage gifted students to pursue mathematical careers," and it runs various programs towards this aim.

In practice, various combinations of pull and push incentives have been used simultaneously, but we are only starting to learn about the effectiveness of each tool in advancing the knowledge frontier. One notable instance of the knowledge frontier moving is the proof of the Poincaré conjecture (one of the seven Millennium Problems) provided by the Russian mathematician Grigori Perelman in 2002. This was striking because the problem had eluded the best minds for over a century, and many mathematicians speculated that another century could pass before a solution was found (Overbye 2006).[2] For his solution Perelman was

[1]Pull and push incentives have often been discussed in the context of pharmaceutical innovation, but they are also relevant for innovation in other fields and knowledge production more generally.

[2]Bruce Kleiner, a Yale mathematician, said "It's really a great moment in mathematics...[it] could have happened 100 years from now, or never."

awarded the Fields Medal in 2006, the Millennium Prize in 2010, and several tenured positions at the world's top mathematics departments – all of which he declined. It appears that at least in this particular case, pull incentives were not explicitly at work (Overbye 2006). Instead, biographers suggested that it was his "mind of unrivaled computational power" honed from an early age in special Russian schools that enabled Perelman to solve the problem (Gessen 2009, Szpiro 2008). Indeed, Perelman had been recognized as an exceptional talent in his youth, and had achieved a perfect score at the 1982 International Mathematics Olympiad (IMO). Perelman's success raises the question of whether exceptional youth talent could also be important for advancing the knowledge frontier more generally? And, if so, are there many other exceptional talents like Perelman out there that have been lost or are at-risk of being lost to the world of science?

Motivated by this anecdotal evidence, we ask two research questions about the relationship between talent and knowledge production.[3] First, how does knowledge produced over a lifetime depend on talent displayed in teen years? Second, how does the country in which a talented youth is born influence the knowledge she can produce over her lifetime? We focus on these questions because they are central to understanding both the process of knowledge creation and whether the world is utilizing the pool of talented youth optimally to advance the knowledge frontier.[4]

Answering these questions raises three major empirical challenges: (1) how to measure talent; (2) how to make this measurement of talent comparable across multiple countries and over time; and (3) how to construct a broad sample of talented youth without selecting on eventual lifetime success in knowledge production. To address these challenges, we focus on knowledge production in mathematics and use a unique institutional feature of this discipline: the International Mathematics Olympiads (IMO), a prominent worldwide competition for high-school students. This setting allows us to measure talent in teenage years (as proxied by IMO scores) as well as to conduct direct comparisons of talent in teenage years across countries. Thus, in the paper we use the word talent to refer to an individual's problem-solving capacity in their teenage years. This could be a product of innate ability, practice, or both. By connecting multiple sources, we are able to build an original database covering

[3]Knowledge may be produced routinely by the systematic application of known methods. In other cases, the production of knowledge may involve or require a novel step. In this paper, we are specifically interested in the latter type of knowledge production.

[4]There has been little systematic study of these questions with some notable exceptions. Aghion et al. (2018) find a significant but relatively weak correlation between visiospatial IQ (from an army entrance exam) and the propensity to become an inventor in Finland. Bell et al. (forthcoming) also report a correlation between 3rd grade math scores and the propensity to become a patent inventor in the U.S. There is also a psychology literature investigating the nexus between intelligence, creativity and scientific achievement. For instance, Cox (1926) estimates IQ scores for 300 'geniuses' who made outstanding contributions to science.

the education history and publications of the population of IMO participants participating across 20 years of the competition (1981-2000; n=4,710).

We first document a salient positive correlation between the points scored at the IMO and subsequent mathematics knowledge production. Even in this group of teenagers in the extreme right tail of the talent distribution, small differences in talent are associated with sizeable differences in long-term achievements. Each additional point scored on the IMO (out of a total possible score of 42) is associated with a 2.6 percent increase in mathematics publications and a 4.5 percent increase in mathematics citations. These correlations reflect both the extensive and intensive margins: strong IMO performers are more likely to become professional mathematicians (as proxied by getting a PhD in mathematics); and conditional on becoming professional mathematicians, they are more productive than lesser IMO performers, and are significantly more likely to produce frontier research in mathematics. The conditional probability that an IMO gold medalist will become a Fields medalist is *fifty* times larger than the corresponding probability for a PhD graduate from a top 10 mathematics program.

We then investigate whether success in the IMO might have a causal effect on subsequent achievements in mathematics. To do so, we exploit the fact that IMO medals are an important summary of IMO performance, and are allocated solely based on the number of points scored at the IMO. We implement a regression discontinuity research design comparing those who nearly made a medal threshold versus those who nearly missed them. We find no evidence of a causal effect of crossing medal thresholds on subsequent performance. Additionally, we find a positive gradient between points scored and long-term performance within medal bins. We thus interpret the medal-math achievement relationship as reflecting underlying differences in talent instead of an effect of initial success.

Next, we investigate the role of country of origin on career outcomes and knowledge production of IMO participants controlling for IMO score. We find that there is a developing country penalty throughout the talent distribution in our sample. That is, compared to their counterparts from high-income countries who obtained the same score in the IMOs, participants born in low- or middle-income countries produce considerably less knowledge over their lifetime. A participant from a low-income country produces 35% fewer mathematics publications and receives over 50% fewer mathematics citations than an equally talented participant from a high-income country. The cross-country income group differences in mathematics knowledge produced in large part reflect differences in the propensity to get a PhD in mathematics. IMO participants from low- and middle-income are considerably less likely to do a PhD in their home country (and not more likely to do a PhD abroad). The lower propensity to earn a PhD at home is in turn correlated with the home country having weak mathematics research and training capacity.

Finally, we present three pieces of evidence to assess the broader implications of mathematics losing a few talented individuals. First, we perform a back of the envelope calculation asking how much more mathematics knowledge could be produced if IMO participants from developing countries produced knowledge at the same rate as those from developed countries. We conclude that the knowledge production (from IMO participants) could be 10% higher in terms of publications and 17% higher in terms of cites. Second, we show that strong performers at the IMO have a disproportionate ability to produce frontier mathematical knowledge compared to PhD graduates and even PhD graduates from elite schools. Third, we use manually collected data on the current occupations of IMO medalists to see what types of careers these individuals are pursing when they do not become professional mathematicians. We find that, for developing country medalists, the lower incidence of employment in mathematics academia is not offset by a greater propensity to be employed in non-mathematics academia, or in prominent industry positions. Instead, medalists from developing countries are more likely to become 'invisible' or at least not have prominent careers.

This work builds upon the macroeconomic literature on talent allocation and the microeconomic literature on the origin of knowledge producers..[5] Baumol (1990) and Murphy, Shleifer & Vishny (1991) emphasize the allocation of talent across different sectors of the economy as being key for economic growth. More recently, Hsieh et al. (2013) attribute part of aggregate wage growth in the U.S. to the integration of talented women and blacks into the U.S. labor market. Most relevant for us, recent empirical literature investigates how children's socioeconomic and geographic backgrounds influence their likelihood of becoming a patent inventor in the U.S. (Bell et al. forthcoming, Akcigit, Grigsby & Nicholas 2017, Celik 2017) and in Finland (Aghion et al. 2018). A consistent finding is that children of low-income parents are much less likely to become inventors than their higher-income counterparts. Bell et al. (forthcoming) also report considerable differences between states of birth in the likelihood of becoming an inventor - for instance, they find that children born in Massachusetts are five times more likely than children born in Alabama to become inventors. Our results echo these differences at the international level. Besides the cross-country dimension, a distinguishing feature of our data is that we have a sample of individuals in the very right tail of the ability distribution; and we document that, even there, different backgrounds result in substantial differences in knowledge produced. The evidence we present

[5]We also build on the literature on the role of place in knowledge production (Kahn & MacGarvie 2016), and on the determinants of high math achievement (Andreescu et al. 2008, Ellison & Swanson 2010, Ellison & Swanson 2016)

on the importance of talent in the production of frontier knowledge also sheds further light on the costs of having talented individuals who do not participate in knowledge producing careers.

More generally, our results also relate to the study of the determinants of the rate of knowledge production. The endogeneous growth literature has studied the size of the knowledge production sector (see e.g. Jones 2002, Freeman & Van Reenen 2009, Bloom et al. 2017) but has given less attention to its composition. Similarly, the literature on the economics of science has typically focused on how institutions and incentives affect the productivity of existing researchers rather than on who becomes a scientist or knowledge producers in the first place.[6] Our study suggests that the selection of talented individuals into knowledge production may be important for the rate of scientific progress. Lastly, our paper also builds on the pull vs. push incentives literature (Kremer & Williams 2010, Maurer 2006, Fu, Lu & Lu 2012). While pull incentives can be potentially important for stimulating scientific discoveries, our study suggests that push programs targeted at talented youth could be an effective complementary tool for the advancement of the knowledge frontier.

The paper proceeds as follows. Section 2 describes the International Mathematics Olympiad. Section 3 presents the data. The results on the link between IMO success and long-term achievements appear in section 4 and those on cross-country comparisons in section 5. Section 6 provides additional evidence on the importance of losing a few talented individuals and section 7 concludes.

2 The International Mathematics Olympiads

Since the International Mathematics Olympiad (IMO) plays an important role in our research design, we describe it in some detail in this section. The IMO is a competition held annually since 1959. Participants travel to the location of the IMO (a different city every year) together as part of a national team. Initially, only Eastern European countries sent participants, but over time participation expanded to include over 100 countries.[7] The competition is aimed at high school students, and requires that participants be younger than 20 years of age and

[6] For instance, Borjas & Doran (2012) study the productivity of U.S. mathematicians following a large influx of Russian mathematicians into the U.S. Other studies on the determinants of scientific productivity among established scientists include Azoulay, Graff-Zivin & Manso (2011), Azoulay, Graff Zivin & Wang (2010), Waldinger (2010), Waldinger (2011), Jacob & Lefgren (2011), Ganguli (2017), Iara, Schwarz & Waldinger (forthcoming). For a more general survey on the economics of science, see Stephan (2012)

[7] The United Kingdom and France joined in 1967, the U.S. joined in 1974 and China in 1985. The only countries with population above 20 million that have never participated are Ethiopia, Sudan and the Democratic Republic of Congo.

not enrolled at a tertiary education institution.

The IMO participants are selected by their national federation (up to six per country), often on the basis of regional and national competitions. Some participants compete several years in several successive years but the majority of participants only compete once. They solve a total of six problems drawn from geometry, number theory, algebra and combinatorics. Each problem is worth seven points and participants can score up to 42 points. Appendix A provides additional information on IMO problems, including two examples. Medals are awarded based solely on the sum of points collected across problems. Slightly fewer than half of the participants receive medals, which are gold, silver or bronze. An "honourable mention" recognizing a perfect solution to one problem has been awarded out to non-medalists since 1987.

Several IMO participants are known to have had outstanding careers as professional mathematicians. Maryam Mirzakhani, an IMO gold medalist with a perfect score, was the first woman to win the Fields medal - the most prestigious award in mathematics. Terence Tao received a gold medal at the 29th IMO, went on to win the Fields medal, and is one of the most productive mathematician in the world. As noted in the introduction, another IMO gold medalist, Gregori Perelman, solved the Poincaré conjecture and famously declined the Fields medal as well as a Millenium Prize and its associated one million dollar award. Of the 26 Fields medals awarded between 1994 and 2018, 14 went to former IMO medalists (see Table 1).

(insert Table 1 about here)

3 Data

Multiple sources of data were combined to create the original data for this paper. We started with the official IMO website: http://www.imo-official.org. For each participant, the website lists name, the country (s)he represented and the year of participation, the number of points obtained on each problem and the type of medal awarded, if any. We extracted data on all IMO participants from the website and then selected those who participated between 1981 and 2000, inclusive.[8] Some participants compete in multiple years, in which case we kept only the last participating year. We ended up with a list of 4,710 individuals.

[8] We did not include later cohorts of participants as we wanted enough time to have elapsed to construct meaningful long-term outcomes. In principle, we could include earlier cohorts but those are small as relatively few countries were participating in the 1970s.

We then constructed long-term performance outcomes in mathematics for these individuals using PhD and bibliometric data. For PhD theses, we relied on the Mathematics Genealogy Project, a volunteer effort aimed 'to compile information on all mathematicians in the world'.[9] It has achieved broad coverage, with information on more than 200,000 mathematicians. For each graduating student, it lists university, advisor name, graduation year and dissertation topic. For bibliometric data, we used MathSciNet data which is produced by Mathematical Reviews under the auspices of the American Mathematical Society. While the underlying publication data is richer, our outcomes are based on total publications and cites by author as computed by MathSciNet (and reflecting the manual author disambiguation by the publishers of Mathematical Reviews). Both of these databases have been used in prior research (Borjas & Doran 2012, 2015a, 2015b, Agrawal, Goldfarb & Teodoridis 2016). We complemented the publication data by collecting a list of speakers at the International Mathematics Congresses (IMC) and tagging IMO participants who were speakers at the IMC congress. Being invited to speak at the IMC congress is a mark of honor for mathematicians and we use it a measure of community recognition independent of bibliometrics. Similarly, we tag IMO participants who have received the Fields medal.

While most of this paper focuses on those bibliometric and PhD data, we also manually searched the names of IMO participants online to construct current employment measures. Given that this part of the data collection is particularly time-intensive, this information was only collected for IMO medalists (2,272 people out of the 4,710 participants). We coded whether the person is employed in (1) mathematics academia, (2) non-mathematics academia, (3) the IT industry, (4) the finance industry, (5) in another industry. Academics leave a sizeable digital footprint, so we are confident that we are measuring academic employment accurately. However, the industry employment measures are capturing only a part of total industry employment.

Our final database covers the population of IMO participants who participated between 1981 and 2000 (4,710 people). Besides information on IMO participation (year, country, points scored, type of medal), we know whether the person holds a PhD in mathematics, and, if so, what year and from what school it was earned, their mathematics publications and cites counts until 2015 and whether that person has been a speaker at the IMC Congress or a Fields Medalists. For IMO participants who hold a PhD in mathematics, we are interested in whether they have graduated from an elite school; we proxied this by graduating from

[9] One might worry that the coverage of the Mathematics Genealogy Project might be worse for developing countries, which would be problematic for cross-country comparisons. However, we have encountered only a handful of individuals with math publications (or with a faculty appointment in mathematics) that were not listed in the genealogy project, and they were mostly from developed countries.

one of the top ten schools in the Shanghai 2010 mathematics rankings (cf Table A8 for the list).

Table 2 presents descriptive statistics on our sample. Around 8% of IMO participants earn a gold medal, while 16% have a silver medal and 24% have a bronze medal; a further 10% have an honourable mention. Around 22% of IMO participants hold a PhD in mathematics; of those around a third a PhD in mathematics from a top 10 school. One percent of IMO participants became IMC speakers, and 0.2% became Fields medalists. Collectively, the IMO participants in our sample produced more than 15,000 publications and received more than 160,000 cites.

(insert Table 2 about here)

We proxied country of origin by the country individuals represented at the IMO. Around half of the participants were from high-income countries (as per the 2000 World Bank classification), 23% from upper middle income countries, 16% from lower middle-income countries, and 11% from low-income countries. Historically, the most successful countries at the IMO have been China (receiving 54 gold medals in our sample), followed by USSR/Russia (43), the U.S.A. (27) and Romania (26). Germany, Bulgaria, Iran, Vietnam, the U.K., Hungary and France also have more than 10 gold medalists in our sample.

(insert Figure 1 and 2 about here)

Finally, we produced an ancillary dataset that has all the PhD graduates (irrespective of IMO participation) listed in the Math Genealogy Project who graduated between 1990 and 2010 (n=89,086). For those, we know the school and year they graduated, how many math publications and cites they produced (from MathSciNet) and whether they have been IMC speakers or Fields Medalists.

4 How much does teenage talent affect long-term performance?

In this section, we investigate whether teenage talent – as proxied by IMO scores – is correlated with long term performance in the field of mathematics. On one hand, it is natural to expect performance at the IMO to be positively correlated with becoming a professional mathematician, mathematical knowledge production, and outstanding achievements in mathematics. After all, both IMO problems and research in mathematics are two activities that involve problem solving in the field of mathematics. Moreover, there is ample anecdotal evidence of distinguished mathematicians having won IMO medals.

However, the link between IMO score and long-term performance is less obvious than it seems. Mathematical research encompasses activities other than problem solving - such as conceptualization and hypothesis generation. Moreover, the IMOs problems are quite different from research problems, in that they are known to have a solution and they have to be solved in a very short time frame, without access to literature or the help of other mathematicians.[10] Even if talent is indeed important for the production of knowledge, IMO scores may not be a good measure of talent. If some measure of luck or extraeneous factors affect IMO scores, this will introduce classical measurement error when considering IMO scores as a measure of talent. We also have to allow for the possibility that IMO scores may be correlated with performance in mathematics research even if IMO scores are uninformative about talent. That scenario could occur if IMO performance had a causal effect on performance, a possibility we will investigate in subsection 4.2.

4.1 Link between IMO score and long-term performance

We begin with some graphical evidence: Figure 3 plots the mean achievements of Olympians by the number of points they scored at the IMO, with a linear fit superimposed.[11] Six achievements are considered: obtaining a PhD degree in mathematics, obtaining a PhD in mathematics from a top 10 school, the number of mathematics publications (in logs), the number of mathematics cites (in logs), being a speaker at the International Congress of Mathematicians, and earning a Fields medal.

(insert Figure 3 about here)

The graphs in the first two rows of Figure 3 all display a clear positive gradient: IMO participants with higher IMO scores are more likely to have a PhD in mathematics or a PhD in mathematics from a top school; and they produce more mathematical knowledge measured in terms of publications and cites. The two graphs in the bottom rows measure exceptional achievements in mathematics. By definition, these are rare outcomes and the graphs are less smooth, but the broad pattern is similar.

[10] Similarities and differences between IMO problems and research problems are often discussed among professional mathematicians. See e.g. Smirnov (2011)

[11] This figure, as others in the paper, benefits from the 'plotplain' Stata scheme developed by Daniel Bishof (see Bischof 2017).

We also investigate the relationship between points scored at the IMO and subsequent achievements in a regression format at the individual level. We regress each achievement on points scored at the IMO, cohort fixed effects, and country of origin fixed effects. Specifically we run specifications of this type:

$$Y_{it} = \beta IMOscore_{it} + \delta X_{it} + \epsilon_{it} \quad (1)$$

where i indexes IMO participants and t indexes Olympiad years. Y_{it} is one of the six outcomes variables as previously defined, $IMOscore_{it}$ is the number of points scored at the IMO; X_{it} includes cohort (Olympiad year) fixed effects and country of origin fixed effects.

(insert Table 4 about here)

The results suggest (see Table 4) that each additional point scored at the IMO (out of a total possible score of 42) is associated with a 1 percent point increase in likelihood of obtaining a Ph.D., a 2.6 percent increase in publications, a 4.5 percent increase in citations, a 0.1 percent point increase in the likelihood of becoming an IMC speaker, and a 0.03 percent point increase in the likelihood of becoming a Fields medalist.[12, 13] Overall, the results of this subsection suggest that there is a close link point scored at the IMO and future achievements.

4.2 Link between IMO score and long-term performance conditional on a PhD

We proceed by analyzing whether long term performance is correlated once we condition on getting a PhD in mathematics. We have documented in the last subsection that higher scorers are more likely to undertake a PhD in mathematics. Is the link between IMO points and knowledge produced entirely driven by this extensive margin or is there an intensive margin story as well?

To investigate this intensive margin, we restrict the sample to IMO participants who have a PhD in mathematics ($n = 1,023$). In Table 5 panel A, we regress publications (in logs), cites (in logs), becoming an IMC speaker, and receiving the Fields medal on IMO

[12] IMO scores alone explain around 8% of the variation in math PhD, publications and citations (not shown on the table).

[13] In appendix Table A2, we break down the IMO score between the number of points scored on less and more difficult problems (problems 1, 2 , 4, and 5 are normally less difficult and problems 3 and 6 more difficult). Interestingly, the coefficients for the score on more difficult problems is consistently larger than for the score on less difficult problems, though the latter also tends to be significant.

scores, cohort fixed effects and Olympiad fixed effects. The point estimates are positive and significant: conditional on a PhD, an additional IMO point is associated with a 2% increase in publications, 4% increase in cites, a 0.2 percentage point increase in the propensity to become an IMC speaker and a 0.07 percentage point in the propensity to become a Fields Medalist. Interestingly, these estimates are hardly lower than those of Table 4 panel A.

(insert Table 5 about here)

In Table 5 panel B, we run the same regressions but add graduate school by Olympiad year fixed effects. That is, we compare IMO participants who competed in the same Olympiad and completed a mathematics PhD in the same school. Even in this demanding specification, we find a positive correlation between IMO scores and long-term performance. The point estimates are somewhat lower than in panel A for publications and cites, but similar for becoming an IMC speaker, and receiving the Fields medal.

4.3 Is there a causal effect of medals on long-term performance?

We have so far documented a link between IMO scores and long-term performance. Could this be because earning a high score at the IMO boosts one's self-confidence in mathematics or facilitates access to better schools? Could IMO scores – by themselves – have a causal effect on long-term performance? If scoring well at the IMO generates a success-begets-success dynamic, IMO scores could affect long term performance even if talent is not relevant to the production of knowledge.

While we cannot directly test for a causal effect of scores, we can investigate whether IMO *medals* may have a causal effect on performance.[14] At the IMO, medals are awarded based on explicit cutoffs in the IMO score. For example, in the year 2000, all participants who scored 30 points and above received a gold medal, those scoring between 21 and 29 received a silver medal, and those scoring between 11 and 20 received a bronze medal. To the extent that medals play a useful role in summarizing and communicating IMO performance to outsiders - as one might expect given that IMO medals are frequently mentioned on CVs and LinkedIn profiles, whereas raw IMO scores are not - the causal effect of IMO medals may be informative about the causal effect of IMO performance more generally.

The IMO medal allocation mechanism is a natural setting for a regression discontinuity (RD) design comparing those who just made the medal threshold (or threshold for a better

[14] Our outcomes are getting a PhD in mathematics and/or mathematical knowledge produced since the data is too sparse to consider exceptional achievement such as being awarded the Fields Medal

medal) versus those who nearly missed it. The assignment variable here is the number of points scored which solely determines medal awards. Importantly, the number of points scored cannot be precisely manipulated by participants, and in any case the medal thresholds are not known when the participants solve the problems.[15] Moreover, since the thresholds are different each year, these results are likely to be robust to any sharp non-linearities in the function linking IMO score and performance.

We implement a simple regression discontinuity design by estimating linear regressions of the following type:

$$\begin{aligned} y_{it} = \alpha + \beta AboveThreshold_{it} + \delta_1(IMOScore_{it} - Threshold_t) \\ +\delta_2 AboveThreshold_{it} * (IMOScore_{it} - Threshold_t) + \lambda X_{it} + \epsilon_{it} \end{aligned} \quad (2)$$

Where i indexes IMO participants and t indexes Olympiads. y_{it} is either obtaining a PhD in mathematics, obtaining a PhD in mathematics from a top 10 school, mathematics publications in logs or mathematics cites in logs. $AboveThreshold_{it}$ is an indicator variable for being at or above the medal threshold and our variable of interest. We control for linear distance to the cutoff $(IMOScore_{it} - Threshold_t)$, allowing it to have a different coefficient on each side of the threshold. Additional controls in X_{it} include cohort fixed effects and country of origin fixed effects.

We have three different time-varying thresholds corresponding to gold, silver and bronze medals. To maximize power, we pool observations across the three thresholds and analyze data at the individual IMO participant-threshold level. Specifically, we generate three copies of the data corresponding to each of the three medals thresholds, and express the IMO score as a distance to the respective threshold. The effect of being above the threshold is thus a weighted average of the effect of being above the gold threshold, being above the silver threshold and being above the bronze threshold. For each outcome variable, we select a set of individuals narrowly above and below the cutoff using the optimal bandwidth selector of Calonico, Cattaneo & Titiunik (2014).[16]

(insert Table 6, Figure 4 about here)

[15] Figure A1 plots the distribution of IMO scores expressed in terms of distance to medal threshold. The Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete does not reject the null of no manipulation (p-value=0.974 for k=0.02).

[16] The validity of the regression discontinuity design rests upon the assumption that there is no precise manipulation and that the density of cases is smooth around the cutoff. Figure A1 displays the distribution of scores by distance to the cutoff. We also use the Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete. The test does not reject the null of no manipulation (p-value=0.974, k=0.02).

The results are presented in Table 6, while a graphical version is displayed in Figure 4. The point estimates for the effect of a (better) medal are imprecisely estimated but generally close to zero for all four outcome variables. That is, controlling for score, being awarded a better medal appears to have no additional impact on becoming a professional mathematician or future knowledge production.

We present two complementary pieces of evidence to conclude this subsection. First, receiving an honorable mention – an award for solving one of the six IMO problems perfectly – does not appear to have a causal effect on long-term performance (see appendix for details). Second, there appears to be a positive gradient between points scored and long-term performance even within medal bins. For instance, in the sample of gold medalists, the number of IMO points scored is positively correlated with each of the four outcomes (Table A4 panel A). The same holds for bronze medalists (Table A4 panel A), while for silver medalists, the number of points scored is significantly correlated with two of the four outcomes. Taken together, these results suggest that the link between IMO scores and long-term performance reflect differences in the underlying talent of medalists rather than a causal effect of IMO success.

5 Does the link between talent and performance depend on country of origin?

5.1 Link between IMO score and long-term performance by country income group

The previous section establishes that performance at the IMOs is strongly correlated with getting a PhD in mathematics and mathematics knowledge produced. We now proceed to study how this link varies according to the country of origin of IMO participants. Because we have relatively few participants for any country, we group countries in terms of income group levels (according to the 2000 World Bank classification) as a broad proxy of differences in opportunities and environment across countries. We consider specifically how IMO participants from low- and middle-income countries – about half of our sample – perform in the long-run compared to observationally equivalent participants from high-income countries. While our regressionsexplicitly control for IMO scores, it is worth noting that participants from developing countries do not score lower at the IMO than participants from developed countries.[17]

(insert Figure 6 about here)

We begin by exploring graphically the link between points scored at the IMO and the propensity to a PhD in mathematics for participants from different groups of countries. In Figure 6, we plot the share of IMO participants obtaining a PhD in math by points scored at the IMO (five-points bands) across groups of countries. The general pattern we observe is that, for a given number of points, the share of participants getting a PhD in math is typically highest for high-income countries, followed by upper middle-income, then by lower middle income, with low-income countries having the lowest share.

We investigate cross-country differences more formally using the following specification:

$$Y_{it} = \beta_1 IMOscore_{it} + \beta_2 CountryIncomeGroup_i + \eta_t + \epsilon_{it} \tag{3}$$

where i indexes medalists and t indexes Olympiad years. Y_{it} is an indicator variable for getting a PhD in mathematics, getting a PhD in mathematics from a top school, publications in logs and cites in logs. Our variable of interest is the income group of the country that a participant represented at the Olympiad. We include indicator variables for low-income, lower middle-income and upper middle-income with high-income the omitted category. Crucially, we control for the number of points scored at the IMO, our proxy for talent.[18] We also control for Olympiad year fixed effects (η_t).

(insert Table 7 about here)

Results (see Table 7) suggest that across all long-term productivity outcome variables IMO participants from low- and middle-income countries significantly underperform compared to their high-income counterparts. For instance, IMO participants from low-income countries are 16 percentage points less likely to do a PhD and 3.4 percentage points less likely to do a PhD in a top school; they produce 35% fewer publications and 57% fewer cites. To put things in perspective, the low-income penalty for getting a math PhD is equivalent to that of scoring 15 fewer points at the IMO. A similar, though less pronounced, pattern can be observed for participants from middle-income countries. We find similar results if

[17] Cf appendix Table A5 for details.

[18] We find similar results throughout if we control for points fixed effects instead of the linear number of points scored.

we replace country income groups by linear income per capita (cf Table A6) or indicator variables for deciles in the income per capita distribution (cf Table A7). If we interact the number of points score with the country income group (cf Table 9), we find that the low income penalty is larger for individuals who score more points.

(insert Table 9 about here)

We also check whether there is a difference in the mathematics knowledge produced by developing country and developed country IMO participants considering only those who have a PhD in mathematics (Table 10).[19] The point estimates for low-income, lower middle income and upper middle income, albeit negative, are not significant (or only marginally significant in the case of low-income). When we compare participants who competed in the same year and went to the same graduate school (IMO participation year by graduate school fixed effects, panel B), the point estimates are positive though not significant (with the exception of upper middle-income in the cites regression). Taken together, this evidence suggests that the extensive margin (getting a math PhD) may be more important than the intensive margin (productivity conditional on having a math PhD) in explaining the relationship between country of origin and mathematics knowledge produced.

(insert Table 10 about here)

5.2 Cross-country differences in IMO training

A possible explanation for the fact that developing country participants are less likely to get a PhD in mathematics and produce less mathematics is that there may be systematic differences in the intensity and quality of training for the IMO across countries.[20] We do control for the number of points scored in all regressions. However, if it was the case that IMO participants from developing countries train more extensively for the IMO than those from developed countries, they may in fact have lower mathematical ability conditional on achieving the same IMO scores.

While we do not directly observe differences in training across countries, we are nevertheless able to shed some light on training differences from individuals who participate at the

[19] Although we control for the IMO score, this comparison is complicated by the fact that the selection into doing a PhD in mathematics is different for developing country and developed country participants.

[20] IMO scores partly reflect training that participants undertake for the purpose of competing at the IMO. For instance, national teams typically run training camps where participants practice solving past IMO problems.

Olympiad multiple times. The majority of participants in our sample compete only once, but a sizeable minority compete twice or more. We may think of participants who compete multiple times as having received more training, as they would have been through training camp multiple times, and they may have learned from their own past IMO participation(s).

(insert Table 8 about here)

We find that participants from low and middle-income countries are less likely to have participated at the IMO multiple times than those from developed countries (Table 8 column 1). On this important observable dimension of the quantity of training received, IMO participants from low- and middle-income countries receive generally less training, not more, than those from high-income countries.

5.3 Migration and home country environment

We now examine the role of migration and home country environment in explaining why IMO participants from low/middle income countries are less likely to become professional mathematicians. To do so, we distinguish between doing a PhD abroad versus in the home country (cf Table 11 panel A). We find that the difference in the propensity to do a math PhD between high and low/middle income countries is driven entirely by the propensity to do a PhD at home. That is, IMO participants from developing countries are significantly less likely to do a PhD in their home country, with no corresponding increase in the propensity to do a PhD abroad. Next, we control for the capacity of the home country in two ways: (1) whether the home country has at least one university ranked among the 100 best in the world in mathematics (2009 Shanghai mathematics rankings) and (2) the production of mathematics articles in the home country in log (panel C). Either indicator of mathematics capacity is strongly correlated with doing a PhD at home, and including them reduces the coefficient on the coefficients for low- and middle-income countries. Taken together these two findings suggest that relatively weak mathematics research and training capacity in their home countries could play a role in explaining why developing country medalists are less likely to become knowledge producers.

(insert Table 11 about here)

5.4 Is the importance of country of origin decreasing over time?

We now explore whether the low- and middle-income country penalty has changed over time. We repeat the last specifications, but now include an interaction term between the country income group and an indicator variable for 'late' cohorts (IMO participants who competed between 1991 and 2000, with those who competed between 1981 and 1990 being the omitted category). Results are displayed in Table 12.

(insert Table 7 about here)

The results are somewhat mixed. On the one hand, the interaction between late and low-income is positive (and significant for three out of the four outcomes), indicating that the penalty associated with coming from a low-income country has decreased over time. However the interaction term between late and upper middle-income is negative; suggesting an increasing gap over time between upper middle-income and high-income countries.

(insert Figure 7 about here)

Finally, we present some graphical evidence on how the low-income penalty has evolved over time for different parts of the talent distribution. In Figure 7, we plot the difference in the share of medalists getting a PhD in math between high- and low-income countries. We plot this difference separately by five-years bands (of IMO participation) and type of medal. Consistent with the regression results, the difference is always positive. For bronze and silver medalists, the difference has considerably diminished over time. However, for gold medalists we observe no such decrease.

Overall, the evidence suggests that coming from a low-income country is less detrimental than it used to be to becoming a professional mathematician and producing mathematical knowledge. However, the gap between high- and low-income countries at the top of the talent distribution has not narrowed.

6 Does losing a few talented individuals matter?

6.1 Quantifying the size of the lost knowledge production

The previous section documented that IMO participants from low- and middle-income countries produce consistently less knowledge than equally talented participants from high-income countries. We now proceed to quantify the size of the knowledge production lost. Specifically, we ask how much knowledge production (from IMO participants) there could be if IMO participants from low-income countries were producing knowledge at the same rate as those from high-income countries. The size of the loss depends on the share of participants in each income group and the penalty for that group. Around half of the IMO participants and medalists are from low- and middle-income countries (cf descriptive statistics Table 2). We multiply the coefficients on the country income groups in our main specifications by the share of IMO participants in each group, and then aggregate across the country income groups (cf Table 13). We conclude that the knowledge production (of IMO participants) could be 10% higher in terms of publications and 17% in terms of cites if IMO participants from low-income countries were producing knowledge at the same rate as IMO participants from high-income countries.

(insert Table 13 about here)

While this calculation suggests that the benefits of enabling individuals from developing country to produce knowledge at the same rate as those from developed countries may be sizeable, we are unable to quantify the costs of doing so, and such costs may be substantial.[21] Moreover, enabling developing country individuals to produce more knowledge could reduce the knowledge produced by individuals from developed countries (for instance if the number of important mathematical problems to be solved is fixed, or if native mathematicians in developed countries are crowded out from research positions (Borjas & Doran 2012)). We also need to contend with the possibility that inducing individuals to engage in mathematical careers may reduce distinctive contributions that they may otherwise make outside mathematics. The next two subsections seek to partially address these last two points.

6.2 Comparing IMO participants with other mathematicians

We have shown that a set of particularly talented individuals from developing countries (the IMO participants and, in particular, the IMO medalists) produce less mathematical knowledge than a similar set of individuals from developed countries. However, perhaps that knowledge can be replaced by other individuals. This could occur if factors such as effort, luck and/or training could substitute for talent in the production of knowledge. It

[21] On the one hand, a number of targeted fellowships for particularly talented individuals, or spots in highly ranked mathematical programs would not be particularly expensive. On the other hand, improving mathematical training and research in developing countries could involve larger costs.

could also be that there are enough very talented individuals overall that losing a fraction of them is inconsequential for the overall rate of knowledge production.

A first insight into these issues might be gained from the fact that more than half of the Fields medalists were IMO medalists, as mentioned previously (cf Table 1), and all but two of these were gold medalists. We take this as suggestive evidence that talent is important for the production of the most groundbreaking mathematical discoveries, as more than half of the Fields medalists had displayed elite problem solving ability (as measured by having an IMO gold medal) when they were teenagers. Moreover, this is likely to be an underestimate, as other Fields medalists might have had high talent in a way we do not measure. We also take it as suggestive evidence that there are not many individuals who are as talented as the IMO gold medalists.

Extreme achievements are by nature very rare, so evidence can only be anecdotal. As we discussed in the introduction, the Poincaré conjecture was solved by Grigori Perelman, a Russian mathematician who had won a gold medal with a perfect score at the 1982 IMO. It turns out that, as of 2018, this is the only 'Millenium Prize Problem' to have been solved (cf Table A9 for a list of the problems).

In order to compare IMO medalists with other mathematicians more systematically, we constructed a sample with all PhD students obtaining a PhD in mathematics between 1990 and 2010 (n=89,068). We also constructed the subsample of PhD students graduating from top 10 schools (n=9,049). We then constructed the sample of IMOs bronze and silver medalists with a mathematics PhD (n=520) and that of gold medalists with a mathematics PhD (n=145). We plot (see Figure 8) the average number of publications, the average number of citations, the share becoming IMC speakers and the share becoming Fields medalists across the four groups.

(insert Figure 8 about here)

For each outcome, we observe the same pattern: the medalists (especially the gold medalists) outperform both other PhD graduates and the PhD graduates from top schools. While the IMO medalists produce more papers and receive more cites than other graduates, we observe a much larger difference for exceptional achievements such as being invited to the IMC Congress and receiving the Fields medal. This evidence suggests that talent may be more important for exceptional research achievements than more routine knowledge production.

We proceed by comparing IMO medalists to other PhD graduates using regressions. Using the sample of 89,068 mathematics PhD graduates, we regress each of the four outcomes on an indicator variable taking value one for IMO bronze or silver medalists; and an indicator variable taking value one for IMO gold medalists. We run these regressions without other controls in Panel A and include PhD graduation year fixed effects and PhD graduate school fixed effects in column B.

(insert Table 14 about here)

We find positive and significant coefficients for both bronze/silver medalists and for gold medalists. The magnitude of the effect is sizeable in the regressions with papers and citations as outcomes. But it is considerably larger still for the regressions with exceptional achievements: for the propensity to become an IMC speaker, the coefficient for IMO gold medalist is an order of magnitude larger than the mean propensity to become IMC speaker; for the propensity to become a Fields medalist, it is two orders of magnitude larger than the mean propensity to become a Fields medalist. The conditional probability that an IMO gold medalist will become a Fields medalist is fifty times larger than the corresponding probability for a PhD graduate from a top 10 mathematics program. Interestingly, the coefficients are roughly similar when we control for PhD graduate school fixed effects. Overall, we find that IMO medalists who get a PhD tend to outperform both other mathematics PhD graduates and their classmates from the same school.

(insert Figure 9 about here)

Finally, we take a brief look at how IMO medalists sort themselves across graduate schools. To do this, we compute the number of IMO medalists graduating with a PhD degree from each school, and plot that against the rank of the school as proxied by the Shanghai university ranking for mathematics (see Figure 9). The medalists tend to cluster in the very best schools, with 36% of the IMO medalists who get a PhD in math graduating from the top 10 schools. We see this as further evidence that highly talented individuals (as proxied by IMO medals) are scarce.

6.3 Careers of IMO participants outside mathematics

The last two subsections suggested that the quantity of lost knowledge production arising from cross-country differences in the productivity of IMO participants is sizeable, and that this lost knowledge production is not easily replaceable by that of other mathematicians. These observations do not necessarily imply that the resulting allocation of talent is inefficient. The efficient allocation of talent would clearly depend (among other considerations) on how valuable mathematical knowledge is to society compared to other goods and services, as well as the comparative advantage of IMO participants in the production of mathematical knowledge.

In their influential paper on talent allocation and growth, Murphy, Shleifer & Vishny (1991) make a distinction between people who have a strong natural comparative advantage in one activity versus those can excel in many occupations. We think of IMO participants as being more in the first category given the high specificity of mathematics as a discipline and occupation. Still, our views on whether it is efficient that individuals from developing countries with a talent for mathematics do not participate in mathematics knowledge production may be influenced by what else they are doing. Measuring the non-mathematical activities of IMO participants is intrinsically challenging, but for the subsample of IMO medalists, we have manually collected data on current occupations using information gathered from online web profiles. We classified occupations in five broad categories - mathematics academia, academia outside mathematics, occupations in the finance industry, occupations in the IT industry, other occupations (mostly other industries), and 'no online profile'. We then regress indicator variables corresponding to these categories on the country income group indicator variables, points scored and cohort dummies (cf Table 15 for the results).

(insert Table 15 about here)

Unsurprisingly, given the earlier results, low- and middle-income country medalists are significantly less likely to be employed in mathematics academia. There are some differences across country income groups in the propensity to be employed in non-math academia or in finance, but no systematic pattern.[22] Low-income and lower-middle income medalists are more likely to have no online presence. The coefficients are quantitatively large (around 9 percentage points) and highly significant. Such medalists do not produce either mathematical or non-mathematical knowledge (at least in academia) nor are they otherwise visible online.

7 Conclusion

This paper studies two questions about the role of talent in the advancement of the knowledge frontier. First, how does knowledge produced over a lifetime depend on talent displayed in teen years? Second, conditional on a given level of talent in teenage years, what is the

[22] Medalists from low-income countries are more likely to be employed in finance than those of high income countries; but those from upper middle-income countries are less likely. Participants from upper-middle income countries are more likely to be employed in non-mathematics academia.

impact of country of birth on knowledge produced? We focus on knowledge production in mathematics, as this allows us to use a unique institutional feature of the discipline – the IMO – to overcome empirical challenges to answering these two questions. By following IMO participants over their lifetime, we are able to measure talent in a comparable way across multiple countries and also to construct a sample without selecting on eventual lifetime success in knowledge production.

We document a strong and consistent link between IMO scores and a number of achievements in mathematics, including getting a PhD, mathematics publications and cites, and being awarded a Fields medal. That is, even in this group of teens, who fall in the extreme right tail of the talent distribution, small differences in talent translates into sizeable differences in long-term achievements. We provide evidence that this relationship reflects the underlying talent distribution and not a signaling effect of medals. We then show that IMO participants from low- and middle-income countries produce consistently less mathematical knowledge than equally talented participants from high-income countries. Our results suggest that the quantity of lost knowledge production arising from cross-country differences in the productivity of IMO participants is sizeable, and that this lost knowledge production is not easily replaceable by that of other mathematicians.

While we have devoted considerable attention to the link between IMO scores and future performance in mathematics, we are in no way implying that high ability in problem solving in late teenage years – much less IMO participation or performance – is a necessary condition to become a successful mathematician. Some individuals may excel at mathematics knowledge production without scoring well on IMO-style tests or having a taste for that type of competition. Others may become interested in mathematics after their teenage years. We simply use the IMO medals as a tool to observe part of the extreme right tail of the ability distribution. What we are suggesting is the mathematics knowledge frontier could advance faster if individuals in the extreme right tail of the ability distribution – some of whom are IMO participants and some of whom are not – do not drop out of mathematics.

One might question whether less mathematical knowledge produced is in any way bad for welfare. An in-depth analysis or even discussion of the contributions of mathematics to the economy is beyond the scope of this paper. However, we note that there is plenty of anecdotal evidence of mathematical discoveries having direct or indirect practical applications, such as in weather simulations, cryptography and telecommunications.[23,24]

[23]The following quote from Laurent Schwartz presents the indirect benefits of mathematics thus: "What is mathematics helpful for? Mathematics is helpful for physics. Physics helps us make fridges. Fridges are made to contain spiny lobsters...."

[24]A Deloitte report quantified the benefits of mathematical research to the UK economy as above 200

Even if mathematical knowledge production did contribute to welfare, the lost knowledge production arising from the under-utilization of developing-country talent is more palatable (or perhaps even desirable) if talent from developing countries is used to produce other types of knowledge. We cannot rule out the possibility that developing country talent ends up in valuable occupations (outside mathematical and non-mathematical knowledge production) where they might make distinctive contributions. However, if we think of IMO participants as having a strong natural comparative advantage in one very particular activity (mathematics) – as we do – then this makes it more likely that the current allocation is inefficient.

Having more developing country talent engaged into mathematics knowledge production might have side effects on the production of developed country mathematicians. On the one hand, there might be learning and spillovers (Azoulay et al. 2010). On the other hand, competition from developing country talent might induce displacement and crowding out of developed country mathematicians if there is a limited number of spots in graduate schools or faculty ranks. Borjas & Doran (2012) document such effects among American mathematicians following the influx of talented Russian mathematicians after the collapse of the Soviet Union. These effects may be important for the distribution of welfare among knowledge producers (and potential knowledge producers). From the perspective of advancing the knowledge frontier, however, it is highly desirable to have the most talented people engaged in knowledge production: as we show in this paper, they have a disproportionate ability to make ground-breaking contributions.

This paper falls short of identifying precisely why developing country participants are less likely to become professional mathematicians and produce less mathematical knowledge. Research and training capacity in the home country seems to play a role. However, other factors may also be at play. For instance, developing country participants may have different preferences or private incentives to enter different types of careers, in particular if careers outside mathematics pay more. Future research may further elucidate the role of different factors in cross-country differences in the utilization of talent. For now, we briefly mention several types of push programs (or supply-side policies) that could be useful in light of the findings of the paper. First, fellowships for high-end talent to study mathematics at undergraduate and/or graduate levels may alleviate resources constraints and make mathematics careers more attractive. Second, top schools could encourage applications from developing countries; and recruiting elite talent to their student programs is probably in their interest. Third, strengthening mathematics research and training capacity in developing countries could not only improve the training of those who prefer to stay in their home

billion pounds (Deloitte, 2012).

country, but would also make mathematics research careers more attractive to them. While this paper has focused on mathematics, there are other disciplines – such as biomedicine and computer science – where knowledge production is perhaps more obviously important for welfare. We suspect that developing country talent might also be under-utilized in those fields, though at this stage it is less clear whether talent is as important in those fields as it appears to be in mathematics.

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Tables

Table 1: IMO Medalists and Fields medalists

Year	Fields medalists by award year (Former IMO medalists in **bold**			
1994	Jean Bourgain	**Pierre-Louis Lions**	**J-C Yoccoz**	Efim Zelmanov
1998	**Richard Borcherds**	**Timothy Gowers**	Maxim Kontsevich	Curtis McMullen
2002	**Laurent Lafforgue**	Vladimir Voevodsky		
2006	Andrei Okounkov	**Grigori Perelman**	**Terence Tao**	Wendelin Werner
2010	**Elon Lindenstrauss**	**Ngo Bao Chau**	**Stanislav Smirnov**	Cedric Vilani
2014	**Artur Avila**	Manjul Bhargava	Martin Hairer	**M Mirzakhani**
2018	**Peter Scholze**	Alessio Figalli	**A. Venkatesh**	Caucher Birkar

Notes: The Fields medal is a highly prestigious prize awarded every four years to up to four mathematicians under the age of 40.

Table 2: Summary statistics on IMO participants (1981-2000)

Variable	Obs	Mean	Std. Dev.	Min	Max
IMO Score	4,710	16.0	11.3	0	42
Gold Medal	4,710	0.08	0.27	0	1
Silver Medal	4,710	0.16	0.16	0	1
Bronze Medal	4,710	0.24	0.24	0	1
Honourable Mention	4,710	0.10	0.30	0	1
Olympiad Year	4,710	1992.4	5.5	1981	2000
Math PhD	4,710	0.22	0.41	0	1
Math PhD (top 10)	4,710	0.07	0.25	0	1
Pubs	4,710	3.3	11.5	0	264
Cites	4,710	34.6	221.1	0	11,062
IMC speaker	4,710	0.01	0.09	0	1
Fields medalist	4,710	0.002	0.04	0	1
High-income country	4,710	0.50	0.50	0	1
Upper middle-income country	4,710	0.23	0.42	0	1
Lower middle-income country	4,710	0.16	0.37	0	1
Low-income country	4,710	0.11	0.31	0	1

Notes: The table displays descriptive statistics on the sample of all individuals who participated in any IMO from 1981 to 2000. IMO medals are based on the number of points scored (IMO score). Multiple gold, silver and bronze medals are awarded at every IMO. Math PhD is based on the Mathematics Genealogy Project. Math PhD (top 10) is based on the list of the 10 top schools listed in appendix table A8. Publication and cites are from MathSciNet. IMC speaker stands for speaker at the International Mathematics Congress. Country income groups are based on the 2000 World Bank classification.

Table 3: Summary statistics on IMO medalists (1981-2000)

Variable	Obs	Mean	Std. Dev.	Min	Max
IMO Score	2,272	25.4	8.3	11	42
Math PhD	2,272	0.32	0.47	0	1
Math PhD (top 10)	2,272	0.11	0.32	0	1
Pubs	2,272	5.3	14.8	0	264
Cites	2,272	59.3	305.7	0	11,062
IMC speaker	2,272	0.02	0.12	0	1
Fields medalist	2,272	0.004	0.06	0	1
High-income country	2,272	0.39	0.48	0	1
Upper middle-income country	2,272	0.18	0.38	0	1
Lower middle-income country	2,272	0.23	0.42	0	1
Low-income country	2,272	0.13	0.33	0	1
Manually constructed current employment					
Academia (math)	2,272	0.24	0.43	0	1
Academia (not math)	2,272	0.13	0.33	0	1
Finance industry	2,272	0.05	0.21	0	1
IT industry	2,272	0.09	0.29	0	1
Other industry	2,272	0.04	0.50	0	1
No online profile	2,272	0.46	0.50	0	1

Notes: The table displays descriptive statistics on the sample of all individuals who won a medal at any IMO from 1981 to 2000. IMO medals are based on the number of points scored (IMO score). Multiple gold, silver and bronze medals are awarded at every IMO. Math PhD is based on the Mathematics Genealogy Project. Math PhD (top 10) is based on the list of the 10 top schools listed in appendix table A8. Publications and cites are from MathSciNet. IMC speaker stands for speaker at the International Mathematics Congress. Country income groups are based on the 2000 World Bank classification. The last six rows on current employment of IMO medalists are based on manually collected data from LinkedIn and other online sources.

Table 4: IMO scores and subsequent achievements

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)	(5) IMC speaker	(6) Field medalist
IMO Score	0.0101***	0.0054***	0.0261***	0.0434***	0.0012***	0.0003***
	(0.0008)	(0.0006)	(0.0021)	(0.0035)	(0.0003)	(0.0001)
Olympiad Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,710	4,710	4,710	4,710	4,710	4,710
Adjusted R2	0.1463	0.1094	0.1400	0.1465	0.0202	0.0012
Mean of D.V.	0.219	0.068	0.431	0.713	0.009	0.002

Notes: These regressions are run on the sample of all IMO participants who competed at any point between 1981 and 2000. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log plus one of mathematics publications (column 3), the log plus one of mathematics cites (column 4), becoming an IMC speaker at the IMC Congress (column 5), becoming a Fields medalist (column 6). The variable of interest is the number of points scored controlling for cohort (olympiad year) fixed effects and country fixed effects. All regressions are estimated by OLS. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Performance conditional on PhD

Panel A	(1) Pubs (log)	(2) Cites (log)	(3) IMC speaker	(4) Fields medalist
IMO Score	0.0239***	0.0404***	0.0026***	0.0007**
	(0.0054)	(0.0087)	(0.0009)	(0.0003)
Country FE	Yes	Yes	Yes	Yes
Olympiad Year FE	Yes	Yes	Yes	Yes
Observations	1,032	1,032	1,032	1,032

Panel B	(1) Pubs (log)	(2) Cites (log)	(3) IMC speaker	(4) Fields medalist
IMO Score	0.0173**	0.0282**	0.0025**	0.0008
	(0.0087)	(0.0135)	(0.0012)	(0.0005)
Country FE	Yes	Yes	Yes	Yes
Graduate School by Olympiad Year FE	Yes	Yes	Yes	Yes
Observations	1,023	1,023	1,023	1,023

Notes: These regressions are run on the subset of IMO participants who have a PhD in mathematics (n=1,023). The dependent variables are: the log plus one of mathematics publications (column 1), the log plus one of mathematics cites (column 2), becoming an IMC speaker at the IMC Congress (column 3), becoming a Fields medalist (column 4). The variable of interest is the number of points scored controlling for cohort (olympiad year) fixed effects and country fixed effects. Panel B also includes graduate school by olympiad year fixed effects - comparing participants who participated in the same year and went to the same school for their PhD. All regressions are estimated by OLS. Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 6: Regression discontinuity estimates of the effect of (better) medals

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Above (better) medal threshold	0.0138	0.0147	0.0117	0.0114
	(0.0250)	(0.0142)	(0.0719)	(0.1043)
Distance from threshold	0.0098***	0.0038**	0.0202*	0.0385***
	(0.0032)	(0.0015)	(0.0104)	(0.0133)
Distance from threshold X above threshold	-0.0031	0.0011	0.0085	0.0039
	(0.0035)	(0.0018)	(0.0129)	(0.0146)
Country FE	Yes	Yes	Yes	Yes
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Threshold FE	Yes	Yes	Yes	Yes
Bandwidth	[-9;9]	[-10;10]	[-8;8]	[-9;9]
Observations	5,208	5,775	4,564	5,208
Mean of D.V.	0.2757	0.0868	0.5680	0.9169

Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO, with the medal thresholds varying from year to year. To maximize power, we pool observations across the three thresholds and analyze data at the individual IMO participant-threshold level. Specifically, we generate three copies of the data corresponding to each of the three medals thresholds, and express the IMO score as a distance to the respective threshold. The effect of being above the threshold is thus a weighted average of the effect of being above the gold threshold, being above the silver threshold and being above the bronze threshold. For each outcome variable, we select a set of individuals narrowly above and below the cutoff using the optimal bandwidth selector of Calonico, Cattaneo & Titiunik (2014). Because the optimal bandwidth depends on the dependent variable, the number of observations varies across specifications. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log plus one of mathematics publications (column 3), the log plus one of mathematics cites (column 4). All regressions are estimated by OLS and include country fixed effect, cohort (olympiad year) fixed effects and threshold fixed effects. Standard errors clustered by olympiad participant in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Link between IMO score and long-term performance by country income group

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Income group of country of origin:				
Low-income	-0.152***	-0.031***	-0.337***	-0.560***
	(0.017)	(0.012)	(0.041)	(0.067)
Lower middle-income	-0.101***	-0.022**	-0.194***	-0.321***
	(0.014)	(0.009)	(0.034)	(0.057)
Upper middle-income	-0.040**	-0.025**	-0.083**	-0.171***
	(0.017)	(0.010)	(0.041)	(0.066)
High-income: omitted category				
IMO Score	0.011***	0.005***	0.027***	0.045***
	(0.001)	(0.000)	(0.001)	(0.002)
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	4,710	4,710	4,710	4,710
Mean of D.V.	0.219	0.068	0.431	0.713

Notes: The variables of interest are the income group levels of a participant's country of origin. The income level groups based on the World Bank 2000 classification and the omitted country income level category is high income countries. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log plus one of mathematics publications (column 3), the log plus one of mathematics cites (column 4). All regressions are estimated by OLS, control for the number of points scored at the IMO and include cohort (olympiad year) fixed effects. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Evidence on differences in training across countries

	(1) Participated multiple times
Low-income	-0.160***
	(0.019)
Lower middle-income	-0.037**
	(0.015)
Upper middle-income	-0.020
	(0.017)
IMO score (last participation)	0.012*** (0.001)
N	4,710
Mean of D.V.	0.254

Notes: The dependent variable is an indicator variable taking value one if the individual has participated in any prior IMO. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 9: Country income groups interacted with IMO score

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Income group of origin country:				
Low-income	-0.094***	-0.031**	-0.086*	-0.117
	(0.022)	(0.014)	(0.048)	(0.081)
Lower middle-income	-0.086***	-0.016	-0.114**	-0.172**
	(0.020)	(0.012)	(0.049)	(0.081)
Upper middle-income	-0.048**	-0.001	-0.066	-0.099
	(0.024)	(0.014)	(0.056)	(0.090)
IMO Score	0.012***	0.006***	0.031***	0.052***
	(0.001)	(0.000)	(0.004)	(0.006)
Low-income X IMO Score	-0.003**	-0.000	-0.014***	-0.025***
	(0.001)	(0.001)	(0.004)	(0.006)
Lower middle-income X IMO Score	-0.001	-0.000	-0.005	-0.010*
	(0.001)	(0.001)	(0.004)	(0.006)
Upper middle-income X IMO Score	0.001	-0.002	-0.001	-0.005
	(0.001)	(0.001)	(0.004)	(0.007)
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	4,710	4,710	4,710	4,710
Mean of D.V.	0.219	0.068	0.431	0.713

Notes: These regressions repeat those of table 7 but include interactions between the country income group and the number of points scored at the IMO. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Cross-country comparisons conditional on math PhD

Panel A	(1) Pubs (log)	(2) Cites (log)
Income group of origin country:		
Low-income	-0.285*	-0.468*
	(0.167)	(0.268)
Lower middle-income	-0.054	-0.077
	(0.109)	(0.181)
Upper middle-income	-0.051	-0.235
	(0.112)	(0.180)
High-income: omitted category		
IMO Score	0.022***	0.037***
	(0.004)	(0.006)
Olympiad Year FE	Yes	Yes
Observations	1,032	1,032

Panel B	(1) Pubs (log)	(2) Cites (log)
Income group of origin country:		
Low-income	0.134	0.057
	(0.469)	(0.734)
Lower middle-income	0.226	0.369
	(0.326)	(0.536)
Upper middle-income	-0.331	-0.722
	(0.350)	(0.588)
High-income: omitted category		
IMO Score	0.020**	0.033**
	(0.013)	(0.021)
Graduate School by Olympiad Year FE	Yes	Yes
Observations	594	594

Notes: These regressions are run on the subset of IMO participants who have a PhD in mathematics. The dependent variables are: the log plus one of mathematics publications (column 1), the log plus one of mathematics cites (column 2). The variables of interest are the country income group dummies (high-income omitted) and we control the number of points scored and cohort fixed effects. Panel B also include graduate school by olympiad year fixed effects - comparing participants who participated in the same year and went to the same school for their PhD; the number of observations is lower as some IMO years/graduate school cells only have one observations and are dropped. All regressions are estimated by OLS. Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 11: Doing a PhD at home and abroad

Panel A	(1) PhD	(2) PhD abroad	(3) PhD home
Low-income	-0.152***	-0.004	-0.148***
	(0.017)	(0.015)	(0.010)
Lower middle-income	-0.101***	0.038***	-0.139***
	(0.014)	(0.012)	(0.008)
Upper middle-income	-0.040**	0.025*	-0.065***
	(0.017)	(0.013)	(0.012)
IMO Score	0.011***	0.006***	0.005***
	(0.001)	(0.000)	(0.000)
Panel B	PhD	PhD abroad	PhD home
Low-income	-0.152***	-0.022	-0.130***
	(0.018)	(0.017)	(0.011)
Lower middle-income	-0.101***	0.015	-0.116***
	(0.016)	(0.014)	(0.010)
Upper middle-income	-0.040**	0.001	-0.040***
	(0.018)	(0.015)	(0.013)
IMO Score	0.011***	0.007***	0.005***
	(0.001)	(0.000)	(0.000)
Any top university	0.001	-0.051***	0.051***
in the home country	(0.018)	(0.014)	(0.014)
Panel C	PhD	PhD abroad	PhD home
Low-income	-0.112***	-0.017	-0.095***
	(0.021)	(0.019)	(0.012)
Lower middle-income	-0.059***	0.024	-0.082***
	(0.017)	(0.016)	(0.010)
Upper middle-income	-0.022	0.021	-0.043***
	(0.017)	(0.014)	(0.013)
IMO Score	0.010***	0.007***	0.003***
	(0.001)	(0.001)	(0.000)
Home country math	0.017***	-0.007**	0.024***
publications (log)	(0.004)	(0.003)	(0.003)
Obs. (all 3 panels)	4,710	4,710	4,710
Mean of D.V. (all 3 panels)	0.219	0.122	0.097

Notes: The dependent variables are getting a PhD in mathematics (column 1), getting a PhD in mathematics from a university outside the home country (2), getting a PhD from a university located in the home country (3). Panel B includes as control an indicator variable for whether the home country has at least one university ranked among the 100 best in the world in mathematics (2009 ARWU/Shanghai ranking for mathematics). Panel C includes as control the log of the number of mathematics publications produced by the home country. The omitted country income category is high income. All regressions include cohort (Olympiad year) fixed effects. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 12: Does the importance of the country of origin diminish over time?

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Income group of origin country:				
Low-income	-0.237***	-0.061**	-0.624***	-1.024***
	(0.030)	(0.021)	(0.069)	(0.119)
Lower middle-income	-0.081***	-0.008	-0.207***	-0.345***
	(0.024)	(0.016)	(0.067)	(0.112)
Upper middle-income	0.023	0.017	0.036	0.016
	(0.030)	(0.020)	(0.086)	(0.140)
Low-income X late cohort	0.106**	0.035	0.373***	0.604***
	(0.036)	(0.025)	(0.084)	(0.141)
Lower middle-income X late cohort	-0.032	-0.022	0.019	0.035
	(0.030)	(0.019)	(0.078)	(0.130)
Upper middle-income X late cohort	-0.090**	-0.061***	-0.165*	-0.256
	(0.036)	(0.023)	(0.097)	(0.158)
IMO Score	0.012***	0.006***	0.028***	0.046***
	(0.001)	(0.000)	(0.001)	(0.002)
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	4,710	4,710	4,710	4,710
Mean of D.V.	0.219	0.068	0.431	0.713

Notes: These regressions repeat those of table 7 but include interactions between the country income group and an indicator variable for 'late cohort'. Late cohort takes value one for individuals who participated in the IMO between 1991 and 2000; the omitted category is those who participated between 1980 and 1990. The main effect of late cohort is absorbed in the Olympiad year fixed effects. Standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 13: Back-of-the-envelope calculation on the size of lost knowledge production

	Share of IMO	Coeff (pubs)	Coeff (cites)	Loss (pubs)	Loss (cites)
Low-income	0.11	-0.337	-0.561	-0.037	-0.062
Lower middle-income	0.23	-0.194	-0.321	-0.045	-0.075
Upper middle-income	0.23	-0.083	-0.171	-0.019	-0.039
Total				-0.101	-0.176

Notes: This back-of-the-envelope calculation seeks to estimate how much mathematics knowledge production is lost due to developing country participants producing at a lower rate than those of developed countries. To do this we take a weighted sum of coefficients from the main cross-country comparison regressions (7) where the weights are the share of IMO participants in low-income, lower middle-income and upper middle-income countries respectively. This calculation ignores spillovers to other researchers and other general equilibrium effects.

Table 14: Comparison with non medalists

	(1) Pubs (log)	(2) Cites (log)	(3) IMC speaker	(4) Fields medalist
Gold medalist	1.2597***	2.1425***	0.0799***	0.0333**
	(0.1087)	(0.1764)	(0.0231)	(0.0147)
Silver or Bronze Medalist	1.1248***	1.8256***	0.0365***	0.0036
	(0.0577)	(0.0932)	(0.0088)	(0.0027)
Graduate from top 10 schools	0.1128***	0.2645***	0.0098***	0.0005***
	(0.0138)	(0.0231)	(0.0012)	(0.0003)
PhD Graduation Year FE	Yes	Yes	Yes	Yes
Observations	89,068	89,068	89,068	89,068
Mean of D.V.	0.9754	1.6089	0.0040	0.0002
	(1) Pubs (log)	(2) Cites (log)	(3) IMC speaker	(4) Fields medalist
Gold medalist	1.1780***	1.9934***	0.0747***	0.0269*
	(0.1332)	(0.2171)	(0.0271)	(0.0152)
Silver or Bronze Medalist	1.0029***	1.6561***	0.0430***	0.0057
	(0.0752)	(0.1253)	(0.0130)	(0.0042)
Graduate School by graduation year FE	Yes	Yes	Yes	Yes
Observations	37,501	37,501	37,501	37,501
Mean of D.V.	0.9892	1.6793	0.0070	0.0003

Notes: These regressions are based on an ancillary sample including all math PhD graduates listed in the Math Genealogy Project graduating between 1990 and 2010. The dependent variables are the log plus one of mathematics publications (column 1), the log plus one of mathematics cites (column 2), becoming a speaker at the International Mathematics Congress (column 3) and being awarded the Fields medal (column 4). The top 10 schools are defined according to the Shanghai Math Ranking and are listed in appendix table A8. All regressions estimated by ordinary least squares. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 15: Current occupations of IMO medalists

	(1) Academia (math)	(2) Academia (not math)	(3) Finance industry	(4) IT industry	(5) Other industry	(6) No online profile
Low-income	-0.111***	-0.011	0.031*	0.024	-0.016	0.087***
	(0.026)	(0.021)	(0.017)	(0.021)	(0.013)	(0.033)
Lower middle-income	-0.049**	-0.004	-0.002	-0.018	-0.024**	0.095***
	(0.023)	(0.017)	(0.011)	(0.016)	(0.010)	(0.027)
Upper middle-income	0.002	0.082***	-0.030**	-0.005	-0.029***	-0.020
	(0.026)	(0.022)	(0.010)	(0.017)	(0.011)	(0.029)
High-income: omitted						
IMO Score	0.007***	0.002***	0.001*	-0.001*	-0.001	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)
N	2,272	2,272	2,272	2,272	2,272	2,272
Mean of D.V.	0.241	0.125	0.045	0.093	0.040	0.457

Notes: These regressions are based on the subsample of IMO participants for whom we have manually collected information on their current occupations, i.e. all IMO medalists. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figures

Figure 1: IMO medalists by country, 1981-2000

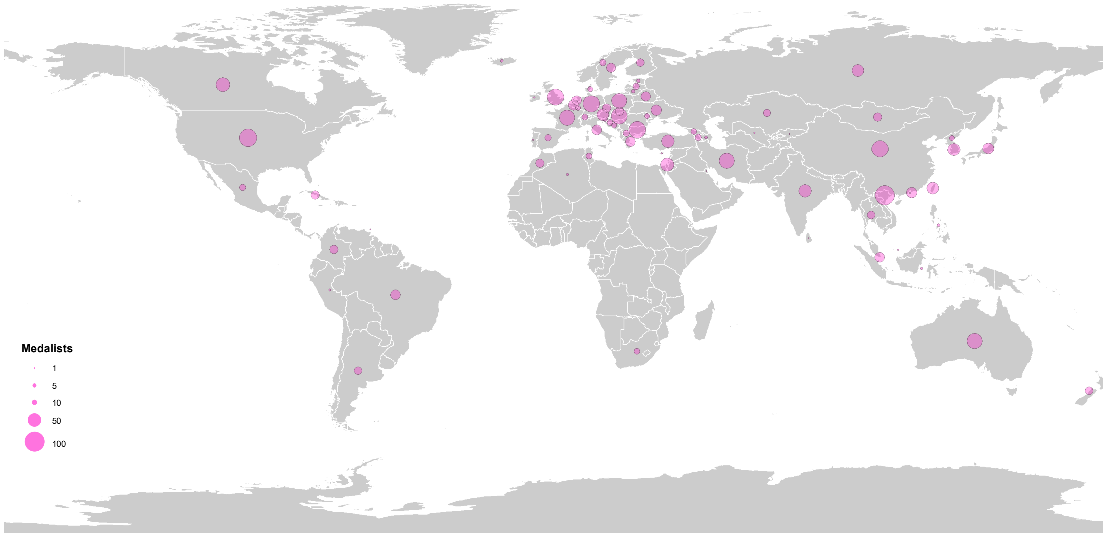


Figure 2: IMO gold medalists by country, 1981-2000

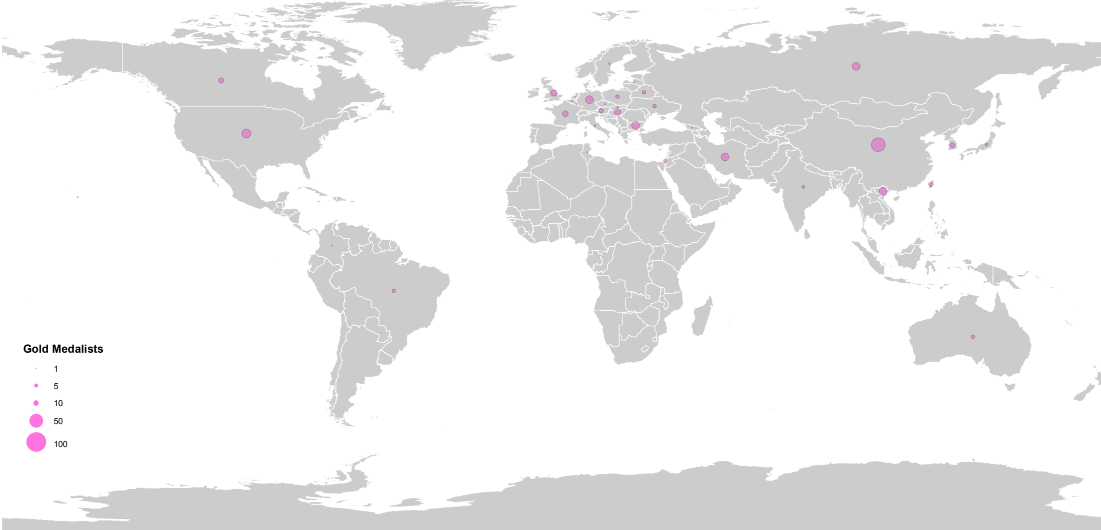
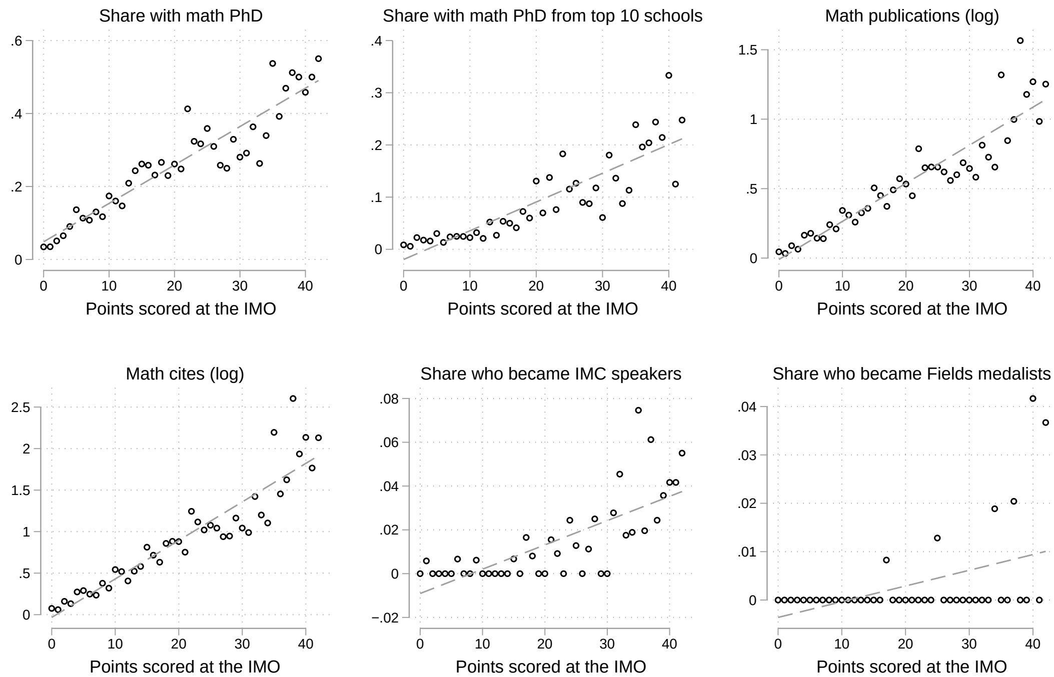
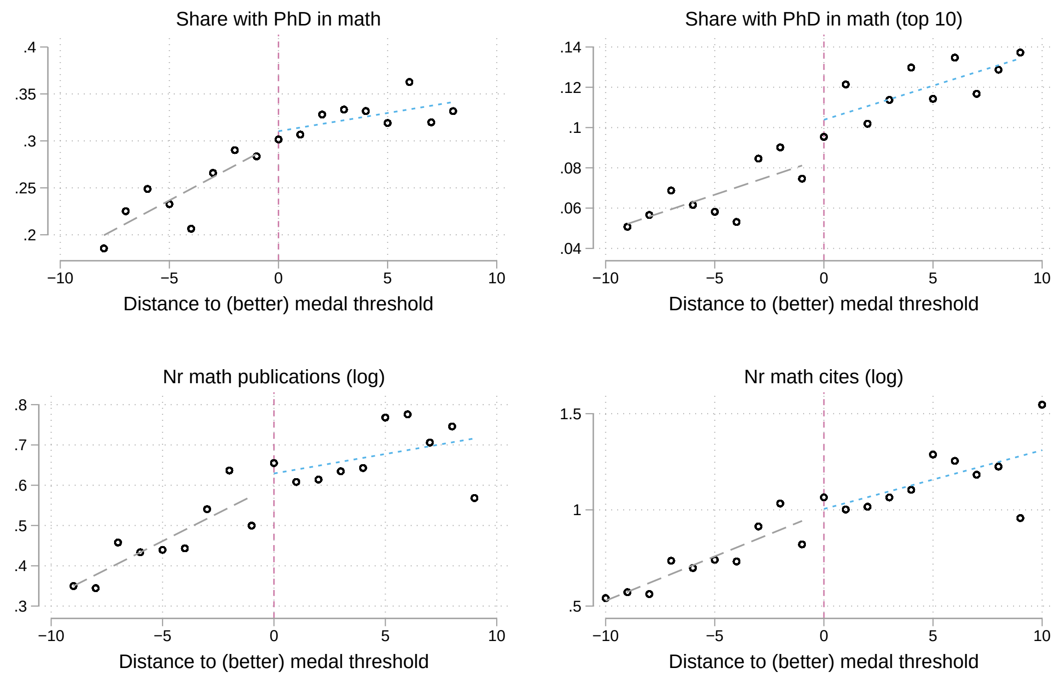


Figure 3: Relationship between points scored at the IMO and subsequent achievement



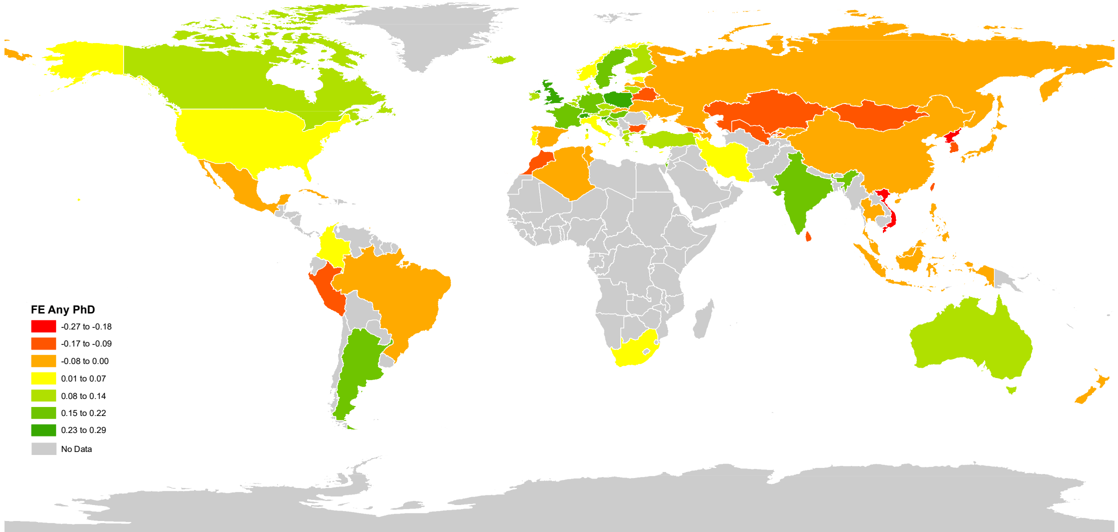
Notes: We compute the sample means of each of the six outcomes variables by the number of points scored at the IMO (for publications and cites, we add one before taking the logs, and then compute the sample means). We then plot the resulting number against the number of points scored. A linear fit is superimposed.

Figure 4: Distance to medal threshold and long-term performance



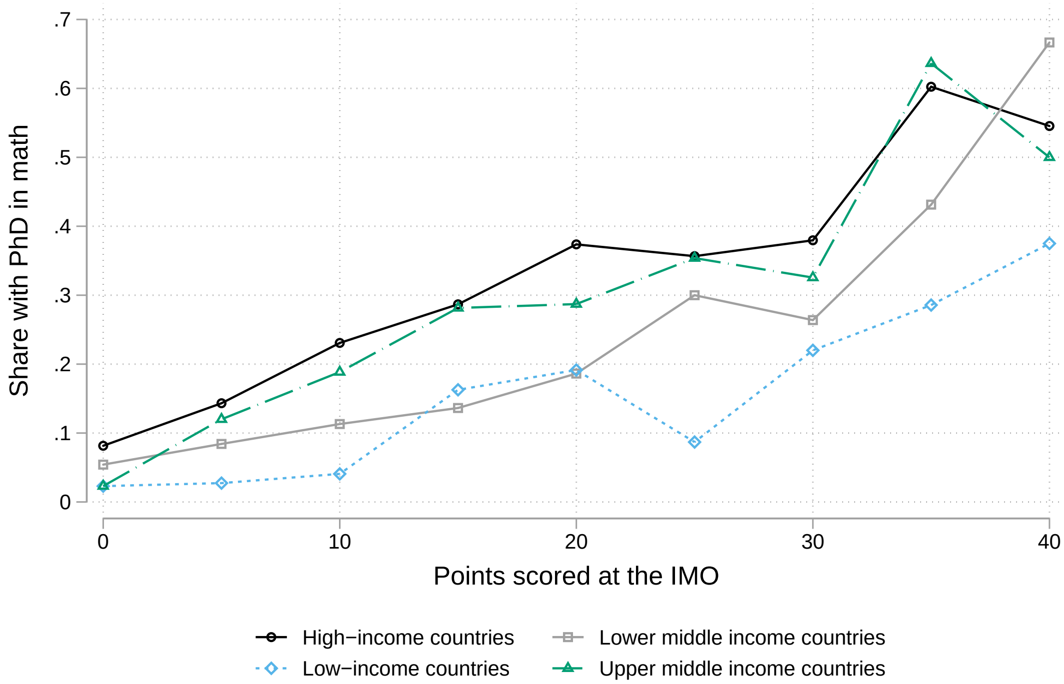
Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO. The medal thresholds for a gold, silver, or bronze medal vary from year to year. For each medal threshold, we construct the sample of participants no more than 5 points from the threshold. We then stack these three samples and construct a unique distance (number of points) to the threshold for a (better) medal. The graph displays samples means by distance to the threshold for a (better) medal, with linear fits superimposed.

Figure 5: Talent conversion by country



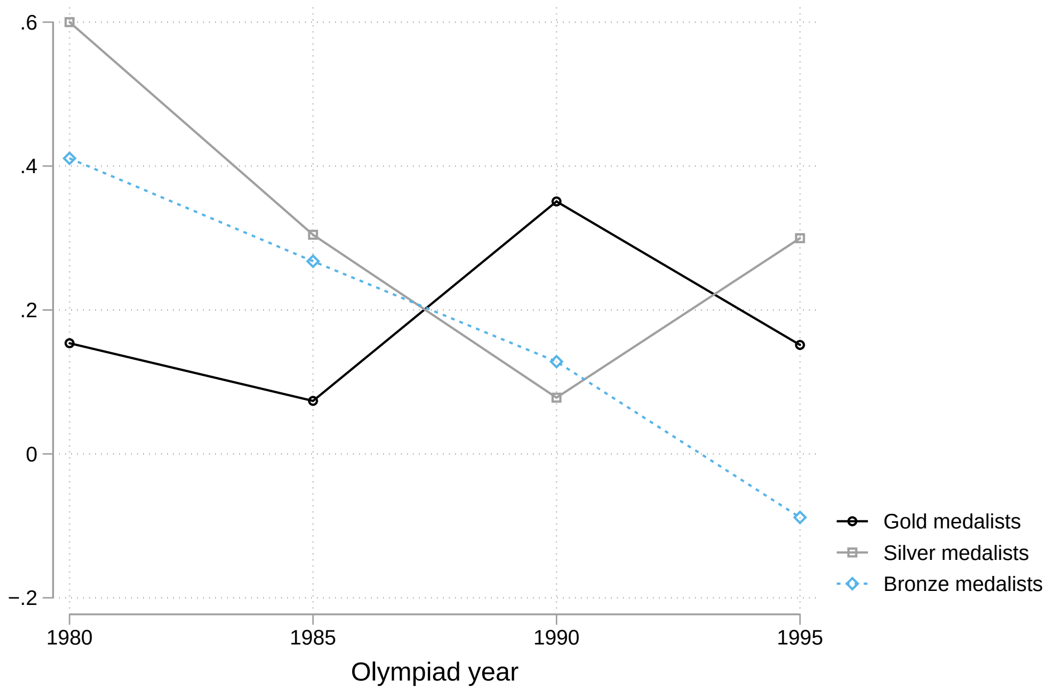
Notes: The figure is based on the country fixed effects arising from a regression of the propensity of doing a mathematics PhD on country fixed effects, cohorts fixed effects and points fixed effects. Countries whose IMO participants have a low propensity of doing a PhD mathematics are shown in red, and countries whose IMO participants have a high propensity of doing a PhD a mathematics are shown in green. The omitted country in these regressions is the U.S.

Figure 6: Share getting a PhD in mathematics across country income groups



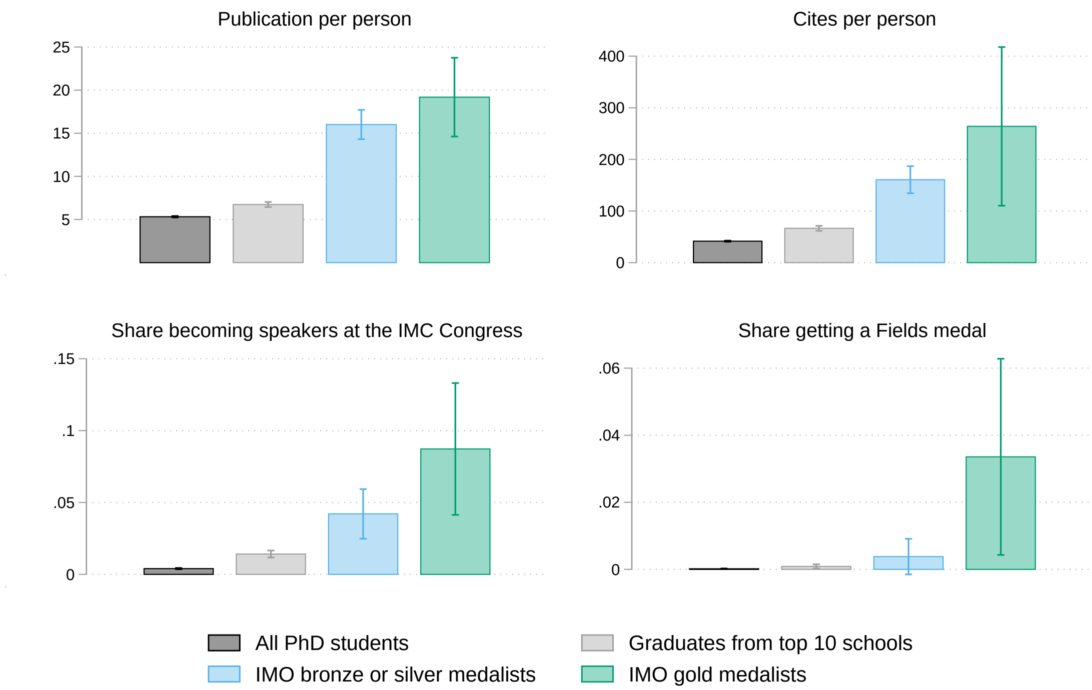
Notes: We compute the share of IMO participants getting a PhD in mathematics by the number of IMO points scored (5-year bands) and plot the resulting share against the number of points scored.

Figure 7: Difference in share getting a PhD between high and low income countries for different cohorts of medalists



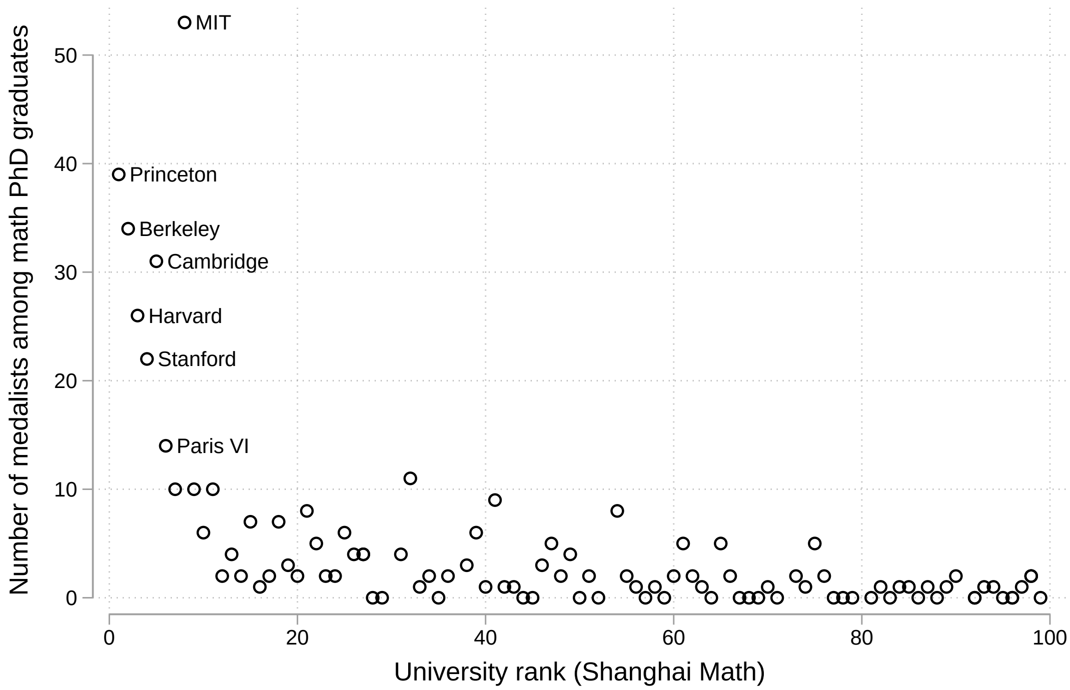
Notes: We first compute the share of IMO medalists getting a PhD in mathematics by Olympiad year (5-year band), medal type and country income group. We then plot the difference between high- and low-income countries in the share of IMO medalists getting a PhD.

Figure 8: Comparing IMO medalists with other professional mathematicians



Notes: These figures are based on an ancillary sample including all mathematics PhD graduates listed in the Mathematics Genealogy Project. The graphs display sample means across four outcomes (publications, cites, becoming a speaker at the IMC congress, and becoming a Fields medalist) for four groups of PhD graduates (all PhD graduates, PhD graduates from top ten schools, IMO bronze or silver medalists, IMO gold medalists).

Figure 9: Number of IMO medalists graduating from math PhD programs, by graduating school



A Appendix: IMO problems

IMO problems are meant to be solveable without knowledge of higher level mathematics such as calculus and analysis taught at tertiary level. They are typically drawn from geometry, number theory or algebra. Potentials problems are submitted by national mathematical federations and then selected by a problem selection committee from the host country.

Compendia of past problems are available and often used as training by future participants. The topic of the most difficult problems is often discussed on online forums such as Quora.[25]

An example of an 'easy' problem. IMO 1964 Problem 1: (a) Find all natural numbers n such that the number $2^n - 1$ is divisible by 7. (b) Prove that for all natural numbers n the number $2^n + 1$ is not divisible by 7.

The following solution is suggested on the https://artofproblemsolving.com:

"We see that 2^n is equivalent to $2, 4$ and $1 \pmod 7$ for n congruent to $1, 2$ and $0 \pmod 3$, respectively. From the statement above, only n divisible by 3 work. Again from the statement above, 2^n can never be congruent to $-1 \pmod 7$ so there are no solutions for n."

An example of a difficult problem. IMO 1988 Problem 6. Let a and b be positive integers such that $(1 + ab)|(a^2 + b^2)$. Show that $(a^2 + b^2)/(1 + ab)$ must be a perfect square.

Engel (1998:138) recounts the following story regarding this problem.

> "[The] problem was submitted in 1988 by the FRG [Federal Republic of Germany]. Nobody of the six members of the Australian problem committee could solve it. Two of the members were Georges Szekeres and his wife, both famous problem solvers and problem creators. Since it was a number theoretic problem it was sent to the four most renowned Australian number theorists. They were asked to work on it for six hours. None of them could solve it in this time. The problem committee submitted it to the jury of the XXIX IMO marked with a double asterisk, which meant a superhard problem, possibly too hard to pose. After a long discussion, the jury finally had the courage to choose it as the last problem of the competition. Eleven students gave perfect solutions."

[25]See for instance https://www.quora.com/What-according-to-you-is-the-easiest-problem-ever-asked-in-an-IMO or https://www.quora.com/What-is-the-toughest-problem-ever-asked-in-an-IMO

B Appendix: estimating the causal effect of honourable mentions

IMO participants who do not win a medal but solve one problem perfectly (7 points out of 7) receive an honourable mention. To estimate the causal effect of receiving an honourable mention, we rely on the fact that the honourable mention award was introduced at the 1988 IMO (and given out in subsequent years) but did not exist previously. This enables us to identify a set of 'counterfactual honourable mention awardees' that would have received the award if it had existed in the year in which they competed, but did not actually receive the award. While comparing the actual awardees to the counterfactual awardees is conceptually appealing, we must reckon with the fact that the actual and counterfactual awardees necessarily belong to different cohorts. We thus implement a difference-in-differences approach comparing those who solved one problem perfectly before and after the honourable mentions were introduced, using the rest of non-medalists participants as a control group to infer a counterfactual time trend. Specifically, we run regressions of the following type:

$$y_{it} = \alpha + \beta * Scored7_{it} + \delta * Honourable_{it} + \eta_t + \epsilon_{it} \tag{4}$$

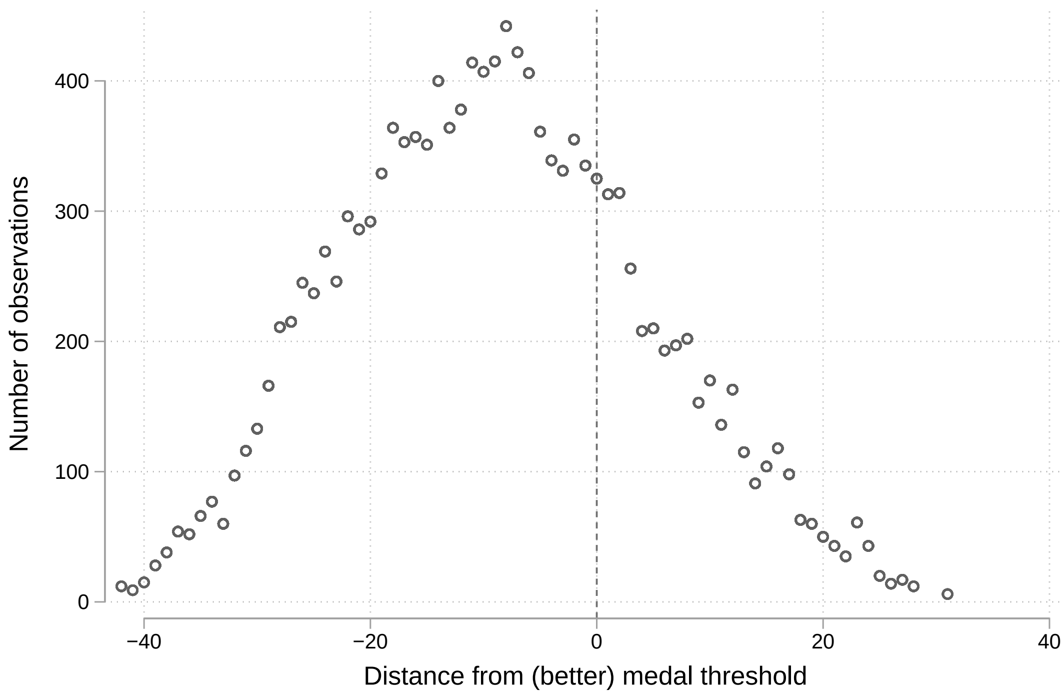
where y_{it} is one of obtaining a PhD in mathematics, obtaining a PhD in mathematics from a top 10 school, mathematics publications in logs or mathematics cites in logs. $Score7_{it}$ is an indicator variable for having solved one problem perfectly. $Honourable_{it}$ is an indicator effect for having received an honourable mention. $Honourable_{it}$ also corresponds to the interaction between $Score7_{it}$ and an indicator variable for competing after 1988. Finally η_t is a full set of Olympiad year fixed effects. The sample for these regressions is the set of non-medalists among all IMO participants.

(insert table A1 about here)

The results are displayed in table A1. We find no significant effect of getting an honourable mention. While the estimates are noisy, the point estimate for the effect of getting an honourable mention is actually negative for all four outcomes. We conclude that honourable mentions do not appear to have a causal effect on getting a PhD in mathematics and other mathematics-related career achievements.

C Appendix figures

Figure A1: Distribution of IMO scores expressed in terms of distance to the threshold for a (better) medal



Notes: For the regressions evaluating the causal effects of medals to be valid, there should be no discontinuity in the density of cases across the threshold for a (better) medal. To test this formally, we use the Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete. The test does not reject the null of no manipulation (p-value=0.974, k=0.02). As in the main analysis, we pool observations across the three medal thresholds (gold, silver, bronze) with each individual appearing three times.

D Appendix tables

Table A1: Effect of obtaining a honourable mention at the IMO

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Honourable mention	-0.0550	-0.0238	-0.0443	-0.0769
	(0.0359)	(0.0204)	(0.0882)	(0.1423)
Perfect score	0.0613*	0.0320*	0.0678	0.1124
on one problem	(0.0313)	(0.0188)	(0.0814)	(0.1309)
Observations	2,438	2,438	2,438	2,438
Adjusted R2	0.0021	0.0029	0.0059	0.0060

Notes: Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A2: IMO scores on less and more difficulty problems and subsequent achievements

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)	(5) IMC speaker	(6) Field medalist
Score on less difficult problems	0.0087***	0.0044***	0.0346***	0.0208***	0.0010***	0.0002
	(0.0011)	(0.0007)	(0.0044)	(0.0027)	(0.0003)	(0.0001)
Score on more difficult problems	0.0129***	0.0077***	0.0616***	0.0375***	0.0019***	0.0007**
	(0.0022)	(0.0015)	(0.0095)	(0.0059)	(0.0006)	(0.0003)
Olympiad Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,491	4,491	4,491	4,491	4,491	4,491
Adjusted R2	0.1456	0.1112	0.1495	0.1428	0.0250	0.0041

Notes: These regressions are similar to those of table 4 but distinguish between the number of points scored on the less difficult problems (1, 2, 4 and 5) and those scored on the more difficult problems (3 and 6); we do not have the score breakdown for the 1981 and 1983, hence the number of observations is slightly lower than in table 4. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log plus one of mathematics publications (column 3), the log plus one of mathematics cites (column 4), becoming an IMC speaker at the IMC Congress (column 5), becoming a Fields medalist (column 6). All regression are estimated by OLS. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A3: Regression discontinuity estimates of the effect of (better) medals - robustness to not including country and cohort fixed effects

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Above (better) medal threshold	0.0079	0.0154	0.0128	-0.0054
	(0.0257)	(0.0147)	(0.0729)	(0.1065)
Distance from threshold	0.0130***	0.0041***	0.0240**	0.0504***
	(0.0032)	(0.0016)	(0.0104)	(0.0132)
Distance from threshold X above threshold	-0.0065*	0.0013	0.0029	-0.0069
	(0.0036)	(0.0018)	(0.0132)	(0.0148)
Country FE	No	No	No	No
Cohort (Olympiad Year) FE	No	No	No	No
Threshold FE	Yes	Yes	Yes	Yes
Bandwidth	[-9;9]	[-10;10]	[-8;8]	[-9;9]
Observations	5,208	5,775	4,564	5,208
Mean of D.V.	0.2757	0.0868	0.5680	0.9169

Notes: These regressions are identical to those of table 6 except that country fixed effects and cohort fixed effects are not included. Robust standard errors clustered by olympiad participant in parentheses. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A4: IMO score gradient within medal bins

Panel A

	Sample: IMO Gold Medalists			
	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
IMO Score	0.0355***	0.0207**	0.0933***	0.1446***
	(0.0105)	(0.0082)	(0.0284)	(0.0469)
Country FE	Yes	Yes	Yes	Yes
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	371	371	371	371
Adjusted R2	0.0633	0.1398	0.1329	0.1266

Panel B

	Sample: IMO Silver Medalists			
	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
IMO Score	0.0017	-0.0012	0.0323*	0.0537*
	(0.0067)	(0.0049)	(0.0185)	(0.0300)
Country FE	Yes	Yes	Yes	Yes
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	775	775	775	775
Adjusted R2	0.1070	0.1300	0.1216	0.1422

Panel C

	Sample: IMO Bronze Medalists			
	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
IMO Score	0.0104**	0.0052*	0.0287**	0.0495**
	(0.0049)	(0.0027)	(0.0132)	(0.0211)
Country FE	Yes	Yes	Yes	Yes
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	1,126	1,126	1,126	1,126
Adjusted R2	0.0547	0.0641	0.0535	0.0569

Notes: These regressions repeat those of table 4 separately on the subsample of gold medalists (column A), silver medalists (panel B) and bronze medalists (panel C). The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log plus one of mathematics publications (column 3), the log plus one of mathematics cites (column 4). All regressions are estimated by OLS and include country fixed effect and cohort (olympiad year) fixed effects. Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A5: IMO score by country income group

	(1) IMO Score
Income group of origin country:	
Low-income	3.206***
	(0.583)
Lower middle-income	0.623
	(0.403)
Upper middle-income	0.030
	(0.430)
High-income: omitted category	
Cohort (Olympiad Year) FE	Yes
Observations	4710
Mean of D.V.	16.007

Notes: Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A6: IMO score, long-term performance and GDP per capita of the origin country

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
GDP per capita	0.003***	0.001***	0.006***	0.011***
(1000 USD)	(0.000)	(0.000)	(0.001)	(0.001)
IMO score	0.011***	0.006***	0.028***	0.046***
	(0.001)	(0.000)	(0.002)	(0.003)
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	4,378	4,378	4,378	4,378
Mean of D.V.	0.213	0.069	0.417	0.692

Notes: Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. GDP per capita is as of 2000 and expressed in thousands of U.S. dollars.

Table A7: Link between IMO score and long-term performance by country income deciles

	(1) Math PhD	(2) Math PhD (top 10)	(3) Pubs (log)	(4) Cites (log)
Decile 1	-0.1240***	-0.0419***	-0.2271***	-0.3873***
(poorest)	(0.0204)	(0.0133)	(0.0482)	(0.0778)
Decile 2	-0.1283***	-0.0347**	-0.2552***	-0.4164***
	(0.0212)	(0.0146)	(0.0479)	(0.0795)
Decile 3	-0.0768***	-0.0245*	-0.1824***	-0.2969***
	(0.0230)	(0.0154)	(0.0526)	(0.0872)
Decile 4	0.0071	-0.0068	0.0368	0.0510
	(0.0240)	(0.0160)	(0.0606)	(0.0992)
Decile 5	0.0406*	-0.0316**	0.1439**	0.1927*
	(0.0259)	(0.0152)	(0.0668)	(0.1073)
Decile 6	0.0003	-0.0142	0.0410	0.0512
	(0.0235)	(0.0141)	(0.0575)	(0.0929)
Decile 7	-0.0644***	-0.0079	-0.1301**	-0.2170***
	(0.0216)	(0.0148)	(0.0505)	(0.0833)
Decile 8	0.0532**	-0.0157	0.1495***	0.2709**
	(0.0253)	(0.0157)	(0.0660)	(0.1097)
Decile 9	0.0966***	0.0091	0.2536***	0.4306***
	(0.0263)	(0.0168)	(0.0705)	(0.1153)
Richest decile omitted				
IMO score	0.0108***	0.0054***	0.0262***	0.0436***
	(0.0005)	(0.0004)	(0.0015)	(0.0025)
Cohort (Olympiad Year) FE	Yes	Yes	Yes	Yes
Observations	4,710	4,710	4,710	4,710
Mean of D.V.	0.219	0.068	0.431	0.713

Notes: Robust standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The income deciles are computed based on 2000 GDP per capita.

Table A8: Top ten schools in mathematics according to the 2010 Shanghai (ARWU) subject ranking

University	**Rank**
Princeton University	1
University of California, Berkeley	2
Harvard University	3
Stanford University	4
University of Cambridge	5
Pierre and Marie Curie University (Paris 6)	6
University of Oxford	7
Massachusetts Institute of Technology	8
University of Paris Sud (Paris 11)	9
University of California, Los Angeles	10

Table A9: The Millenium Prizes Problems in Mathematics

Millenium Prize Problem	Status	Solver
Poincaré conjecture	Solved	Grigori Perelman, formerly IMO gold medalist with a perfect score
P versus NP	Unsolved	
Hodge conjecture	Unsolved	
Riemann hypothesis	Unsolved	
Yang-Mills existence and mass gap	Unsolved	
Navier-Stokes existence and smoothness	Unsolved	
Birch and Swinnerton-Dyer conjecture	Unsolved	

Notes: The Clay Mathematics Institute has identified seven of the most important difficult problems with which mathematicians were grappling at the turn of the second millennium. A one million dollar prize was allocated to the solution of each problem. The discovery of a solution for each problem would be 'an achievement in mathematics of historical magnitude'. As of 2018, only one of the problems had been solved. The solver was Grigori Perelman, a Russian mathematician, who has solved the Poincaré conjecture in 2003. Mr Perelman had won a gold medal with a perfect score at the 1982 IMO.