Inequality in an OLG Economy with Heterogeneous Cohorts and Pension Systems

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I Z A Institute
of Labor Economics
Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

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ABSTRACT

Inequality in an OLG Economy with Heterogeneous Cohorts and Pension Systems

We analyze the consumption and wealth inequality in an OLG model with mandatory pension systems. Our framework features within cohort heterogeneity of endowments and heterogeneity of preferences. We allow for population aging and gradual decline in TFP growth. We show four main results. First, increasing longevity translates to substantial increases in aggregate consumption inequality and wealth inequality. Second, a pension system reform from a defined benefit to a defined contribution works to reinforce consumption inequality and reduce wealth inequality. Third, minimum pension benefits are able to partially counteract an increase in inequality introduced by the defined contribution system, at a fiscal cost. Fourth the minimum pension benefit guarantee mostly addresses the sources of inequality which stem from differentiated endowments rather than those which stem from heterogeneous preferences.

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1 Introduction

Pension systems are an important force responsible for shaping income, consumption and wealth distributions (Storesletten et al. 2004). Pension systems can affect not only income and consumption inequality but also wealth inequality. Indeed, changes in the pension system and longevity are reflected in changes of wealth and consumption distribution over the life-cycle (Song 2011). For example, longevity should encourage higher wealth accumulation during the working period, whereas a generous pension system reduces the incentives to do so. The net effect of these two opposite mechanisms remains an empirical question.[1] Moreover, the extent to which households accommodate for the changes in the redistributive properties of the pension system depends on within cohort heterogeneity (Hairault and Langot 2008). Consequently, the cross-sectional measures of inequality – prevalent in empirical studies – encompass complex dynamics within the life-cycle combined with composition effects and general equilibrium effects. The detailed analysis of these forces remained outside the scope of analysis so far and our study partially fills this gap.

The focus of this study is on the effects of aging and pension system reforms on inequality. We show that under a defined benefit pension system, where the link between individual labor supply and pension benefit is relatively weak, aging leads to rising consumption and wealth inequality. Making the link between pensions and contributions stronger, reinforces further consumption inequality and curbs wealth inequality. We study the extent to which redistribution within the pension system may counteract those changes. We investigate how minimum pension benefit guarantee and lump sum indexation affect consumption and wealth inequality. Furthermore, we look at the welfare effects of those instruments.

Decreasing fertility and increasing longevity in many advanced and emerging economies put a strain on their pension systems. Many governments responded with pension system reforms which shift the consequences of aging from the pension system to individual pensions. Indeed, these reforms, which typically make the link between the contributions and pension benefits stronger, were subject to a large body of analysis. Initially, studies in this strand of literature frequently relied on the relatively restrictive assumption of within cohort homogeneity (see overview by Fehr 2009). With individuals identical within the cohorts, life-cycle pattern and composition effects are the only sources of potential consumption and wealth inequality. Since the analyzed pension system reforms usually also involved a change in the distribution of pensions within cohort, the studies introduced within cohort inequality *via* idiosyncratic shocks to income, labor supply or lifespan.[2] This way of operationalizing heterogeneity is valuable from many research perspectives,[3] but it is not without limitations (see a discussion by Bourguignon

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[1]For example, Domeij and Klein (2002) show that the generosity of the Swedish pension system effectively crowds out private savings among a large fraction of the population, making wealth inequality measures twice as high as income inequality measures in this country. Clearly, the fact that a certain fraction of population actually accumulates positive wealth also suggests that there are important within cohort differences in ability and/or willingness to save.

[2]See for example Fehr et al. (2008), Hairault and Langot (2008), Fehr and Kindermann (2010), Bucciol (2011), Cremer and Pestieau (2011), Kumru and Thanopoulos (2011), Kaganovich and Zilcha (2012), Fehr and Uhde (2014), St-Amant and Garon (2015), Kindermann and Krueger (2014)

[3]As noted by Nishiyama and Smetters (2007), while privatizing social security can improve labor supply

and Spadaro 2006). Most notably, it neglects other sources of heterogeneity like preferences or differences of abilities.[4] Therefore, it does not offer a full picture of the effects of redistributive instruments within the pension system.

We follow an alternative approach. Our economy is populated by agents *ex ante* heterogeneous within the same birth cohort. This heterogeneity is realized at birth and accounts for both earning abilities and preferences. We introduce complex, multidimensional patterns of heterogeneity, while keeping the computational time within reasonable limits. It allows us to replicate cross-sectional distributions observed in the data. This modeling approach is similar to Hénin and Weitzenblum (2005) and McGrattan and Prescott (2013). In such an economy we study the distributional effects of aging, pension system reform and – finally – the effectiveness of instruments aimed at mitigating old-age poverty.

Our contribution to the literature is threefold. First, in a setup calibrated to the features of the Polish economy, we document that under defined benefit (DB) pension system, increasing longevity leads to a substantial increase in wealth and consumption inequality measured in cross-section, despite stable within cohort heterogeneity. Moreover, we show that reforming the pension system from the DB to defined contribution (DC) further increases consumption inequality, and limits the increase of wealth inequality. These results are appealing for their policy relevance, since gradual increases in longevity are affecting almost every developed and emerging economy and many countries have, in response to these developments, responded by implementing DC systems or are transitioning to one.[5]

Second, we test the redistributive effectiveness and fiscal consequences of policy instruments aimed at reducing old-age inequality: minimum pension benefit guarantee and lump sum indexation. In essence, both these instruments redistribute from high earning individuals to the low earning ones, but the scope of the redistribution is limited to retirees. Minimum pension benefit guarantee may induce high fiscal costs, while the lump sum indexation is fiscally neutral, but also constitutes an intervention of smaller magnitude. These two instruments were analyzed because of their policy relevance. The former exists in most advanced and emerging economies (Gruber and Wise 2004). The latter is widely considered a useful way to reduce inequality in pension benefits, and hence old-age consumption inequality. It was introduced in Russia in 2016 (Sinelnikov-Murylev and Radygin 2017) and Poland in 2012 (Jabłonowski and Müller 2013). In Portugal, there is a higher indexation for lower pensions. Prior to the 2011 reform, Italy had a three-tier system, of 100%, 90% and 75% indexation depending on the pension level (OECD 2014). Our simulations reveal that a minimum pension benefits guarantee is indeed effective in reducing inequality, cutting in half the increase in consumption inequality induced

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incentives, it can also reduce risk sharing. With randomized and non-insurable shocks to individual productivity, the original conclusions from a highly stylized model by Feldstein (1995) do not necessarily hold. Similar conclusions originate from models incorporating time inconsistency into the consumer choice, İmrohoroğlu et al. (2003), Bassi (2008), Fehr et al. (2008), van de Ven and Weale (2010), Fehr and Kindermann (2010), Kumru and Thanopoulos (2011)

[4]As demonstrated by Castañeda et al. (2003), an economy populated with identical agents faced with uninsured idiosyncratic shocks to individual productivity cannot reproduce the actual inequality distributions. See also Benhabib et al. (2015), De Nardi and Yang (2016), Kanbur and Stiglitz (2016).

[5](See e.g. Gruber and Wise 2004), following the advice of international financial institutions, most transition economies in Central and Eastern Europe – including Poland – introduced such reforms.

by the analyzed pension system reform. Meanwhile, the lump sum indexation does not affect inequality measures in any meaningful way. Welfare gains tend to be higher for agents who work less hours and have a lower productivity endowment.

Finally, we provide a novel method to analyze, which *types of inequality* are best addressed by an instrument, which is particularly relevant from a policy perspective (Fleurbaey and Maniquet 2006). Our model explicitly separates preference heterogeneity[6] and endowment heterogeneity. Unequal opportunities, which are operationalized as heterogeneous abilities, constitute a case for socially desirable redistribution (Fleurbaey and Maniquet 2005). Meanwhile, preference heterogeneity reduces the scope of desirable redistribution, due to perverse incentives (Lockwood and Weinzierl 2015). We propose a novel approach to disentangle the channels through which redistribution instruments operate and we quantify the extent to which a given instrument helps to alleviate the consequences of dispersion in endowments and the extent to which it addresses the dispersion stemming from preference heterogeneity. In our setting, inequality of opportunity – that which stems from differentiated endowments – contributes less to overall inequality than heterogeneous preferences. Yet, it is the endowments channel that is effectively addressed *via* minimum pension benefits.

Our paper is structured as follows. We begin by presenting the model in section 2, discussing similarities and differences with reference to previous studies in the field. We then move to describing in detail the model calibration in section 3. Subsequently, in section 4, we analyze the effectiveness of two policy instruments: the minimum pension benefits guarantee and the lump sum taxation. Step by step we show the results for the DB and the DC system, comparing them explicitly. Finally, in section 5 we discuss the results of our decomposition experiment, which isolates the inequality of endowments and inequality of preferences. We conclude the paper by discussing the limitations of our study and policy implications.

2   The model

We develop a general equilibrium overlapping generations model, with exogenous but time varying technological progress, decreasing fertility and increasing longevity. Economy is populated by $\mathcal{K}$ classes of agents with differentiated endowments and preferences (within one family of the utility function), who live for $j = 1, 2, \ldots, J$ periods facing time and age specific stochastic mortality. We denote the unconditional probability of survival until age j in period t for an individual born in period $t - j + 1$ as $\pi_{j,t}$.

2.1   Consumers

Consumers are born at the age of 20, which we denote $j = 1$, at which time they are randomly assigned with individual productivity multiplier ω_κ, the discount factor δ_κ, as well as preference for leisure $1 - \phi_\kappa$. These values do not change until the agent dies. Thus, a subcohort $\kappa =$

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[6]Heterogeneous preferences become increasingly used in macroeconomic research. As early as in 1990s, Krusell and Smith (1997, 1998) introduced differentiated time preference.

$1, 2, \ldots, \mathcal{K}$ of agents of age $j = 1, 2, \ldots, J$ is described uniquely by assigned values of ω, δ and ϕ.

The year of birth determines fully the survival probabilities at each age j. At all points in time, consumers who survive until the age of $j = J$ die with certitude. The share of population surviving until older age is increasing, to reflect changes in longevity. Decreasing fertility is operationalized by a falling number of births.[7]

At each point in time t, an individual of age j and subcohort κ born at time $t - j + 1$, consumes a non-negative quantity of a composite good $c_{j,\kappa,t}$ and allocates $l_{j,\kappa,t}$ time to work (total time endowment is normalized to one). In each period t, agents at the age of $j = \bar{J}$ retire. Agents accumulate assets $a_{j,\kappa,t}$ that earn the interest rate r_t. Consequently, agents' lifetime utility is defined as follows:

$$U_{j,\kappa,t} = u_\kappa(c_{j,\kappa,t}, 1 - l_{j,\kappa,t}) + \sum_{s=1}^{J-j} \delta_\kappa^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u_\kappa \left(c_{j+s,\kappa,t+s}, 1 - l_{j+s,\kappa,t+s} \right) \tag{1}$$

where discounting takes into account the discount factor δ_κ and probability of survival. The instantaneous utility function is given by:

$$u_k \left(c_{j,\kappa,t}, l_{j,\kappa,t} \right) = \phi_\kappa \ln c_{j,\kappa,t} + (1 - \phi_\kappa) \ln \left(1 - l_{j,\kappa,t} \right) \quad \text{with} \quad l_{j,\kappa,t} = 0 \ \ \forall j \geq \bar{J}. \tag{2}$$

Household income consists of earned labor income, capital gains, pension income, bequests and lump-sum taxes/transfers. Labor income tax τ^l and social security contributions τ are deducted from gross earned labor income to yield disposable labor income.[8] Interest earned on assets r_t is taxed with τ^k. In addition, there is a consumption tax τ_t^c as well as a lump sum tax/transfer Υ equal for all subcohorts, which we use to balance the government budget in the initial steady state. Thus, agent of age j in period t maximizes her lifetime utility function $U_{j,\kappa,t}$ subject to the following sequence of budget constraints:

$$
\begin{aligned}
(1 + \tau_t^c)c_{j,\kappa,t} + a_{j,\kappa,t} &= (1 - \tau^l)(1 - \tau)w_t \omega_\kappa l_{j,\kappa,t} && \leftarrow \text{ labor income} && (3) \\
&+ (1 + (1 - \tau^k)r_t)a_{j-1,\kappa,t-1} && \leftarrow \text{ capital income} \\
&+ (1 - \tau^l)b_{j,\kappa,t} && \leftarrow \text{ pension income} \\
&+ beq_{j,\kappa,t} && \leftarrow \text{ bequests} \\
&- \Upsilon_t && \leftarrow \text{ lump-sum tax}
\end{aligned}
$$

When working, the agent is constrained by earned disposable income, bequests and assets accumulated in previous periods with net interest. When retired, the agent is constrained by disposable pension benefit, bequests and assets accumulated in previous periods with net

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[7]The availability of the demographic data and forecast until the age of 100 puts a limit at $J = 80$. The data for mortality and births come from a demographic projection until 2060. As of this year we gradually stationarize model population to reach the final steady state of unchanging population size and structure.

[8]Note that labor is taxed only once social security contribution is paid. This setup follows legislation in many countries and makes pension benefits subject to labor income tax, τ^l, during the retirement.

interest. Agents have no bequest motive, but since survival rates are lower than one, in each period t a certain fraction of subcohort (j, κ) leaves unintended bequests, which are distributed within their subcohort.[9]

In our setting, the agents share a family of utility functions, but the actual trade-off between consumption and leisure (the intra-temporal choice) as well as preference for the future (the inter-temporal choice) are heterogeneous within cohort. The agents are, hence, fully rational in the sense that their decisions follow directly from solving the lifetime utility optimization problem. This approach is similar to multi-agent systems with all the advantages of the general equilibrium setting. An agent is a program/routine capable of optimization, as suggested by standard representative agent first order conditions.[10]

2.2 Production

Perfectly competitive producers supply a composite final good with the Cobb-Douglas production function $Y_t = K_t^\alpha (z_t L_t)^{1-\alpha}$, with K denoting capital and L denoting labor. This production function features labor augmenting exogenous technological progress denoted as $\gamma_t = z_{t+1}/z_t$. The standard maximization problem of the firm yields the real wage $w_t = (1-\alpha)K_t^\alpha z_t^{1-\alpha} L_t^{-\alpha}$ and the return on capital $r_t = \alpha K_t^{\alpha-1}(z_t L_t)^{1-\alpha} - d$, where d denotes the depreciation rate of capital.

2.3 Pension system

We consider a pay-as-you-go defined benefit system (DB), with an exogenous contribution rate τ and an exogenous replacement rate ρ, thus

$$
b_{j,\kappa,t} = \begin{cases} \rho \cdot (w_{t-1} \cdot \omega_\kappa \cdot \bar{l}_{\kappa,t} \cdot (1-\xi) + w_{t-1} \cdot \xi) & \text{if } j = \bar{J} \\ (1 + 0.25 r_t^I) b_{j-1,\kappa,t-1} & \forall j > \bar{J}, \end{cases} \tag{4}
$$

where w_{t-1} denotes previous period average wage, $\bar{l}_{\kappa,t}$ denotes labor supply averaged over lifetime[11] and with $r_t^I = \frac{w_t L_t}{w_{t-1} L_{t-1}} - 1$. We denote by ξ the redistributive share of the DB pension system, i.e. the share of the pension that is determined by an average wage in the economy rather than by the individual optimization problem. The indexation with 25% of payroll growth as well as the size of the redistributive part were both stipulated by the law.

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[9]This assumption limits the scope for within-cohort redistribution back-doors. Moreover, it appears plausible: households are typically formed by individuals of similar income and social status.

[10]Such a program can be run on a system populated by 'agents' with differentiated preferences and endowments, see Russell and Norvig (1995), Wooldridge and Jennings (1995), Kirman (1997). Whereas a standard OLG model aggregates over cohorts to obtain general equilibrium conditions, an OLG model with MAS aggregates over classes of economic agents within a cohort and only then over cohorts, see Ferber (1999), Tesfatsion (2002), Wooldridge (2009). Thus, the solution in the equilibrium relies on the same premises as in a standard OLG model with a representative agent, as long as agents share the family function for preferences.

[11]$\bar{l}_{\kappa,t} = \sum_{j=1}^{\bar{J}} l_{j,\kappa,t-\bar{J}+j}/(\bar{J})$

The system collects contributions from the working and pays benefits to the retired:

$$\sum_{j=\bar{J}}^{J}\sum_{\kappa=1}^{\mathcal{K}} N_{j,\kappa,t}b_{j,\kappa,t} = \tau w_t L_t + subsidy_t \tag{5}$$

where $subsidy_t$ is a subsidy/transfer from the government to balance the pension system.

We also consider a pay-as-you-go defined contribution system (DC). In parallel to the DB system, the DC system collects contributions and uses them to cover contemporaneous benefits. However, unlike the DB system, in the DC system each agent has an account $f_{j,\kappa,t}$ which collects information about all individual contributions. The contributions are indexed annually with the payroll growth: $f_{j,\kappa,t} = (1 + r_t^I)f_{j-1,\kappa,t-1} + \tau\omega_\kappa w_t l_{j,\kappa,t}$. The DC system then pays out the pension benefits based on the accumulated contributions and conditional life expectancy at retirement, $LE_{\bar{J},t} = \sum_{s=\bar{J}}^{J} \frac{\pi_{s,t-\bar{J}+s}}{\pi_{\bar{J},t}}$

$$b_{j,\kappa,t} = \begin{cases} f_{j,\kappa,t}/LE_{j,t} & \text{if } j = \bar{J} \\ (1 + 0.25r_t^I)b_{j-1,\kappa,t-1} & \forall j > \bar{J}. \end{cases} \tag{6}$$

Our economy starts with the DB system and gradually shifts to the DC system, with an unchanged contribution rate τ. The transition is gradual in a sense that the cohorts with $j \geq \bar{J}$ at the time of the reform receive pensions in an unchanged manner. The same applies to the oldest 20 cohorts of workers, who remain in the DB system. Hence, only workers with $j < \bar{J} - 20$ in the first year of the transition path receive pensions from the formula given by equation (6) whereas older cohorts receive pension benefits according to equation (4). For all the years of work under the DB system, we compute implied contributions to the DC system *as if* these workers were covered by the DC system from the first contribution.

2.4 The government

The government collects taxes (τ^k on capital, τ^l on labor and τ^c on consumption, as well as a lump-sum tax/transfer Υ) and spends a fixed share of GDP on (unproductive) consumption $G = g \cdot Y$. Government balances the pension system. Given that the government is indebted, it naturally also services the outstanding debt D_t.

$$T_t = \sum_{j=1}^{J}\sum_{\kappa=1}^{\mathcal{K}} N_{j,\kappa,t}\left[\tau^l((1-\tau)w_t\omega_\kappa)l_{j,\kappa,t} + b_{j,\kappa,t}) + \tau_t^c c_{j,\kappa,t} + \tau^k r_t a_{j-1,\kappa,t-1} + \Upsilon_t\right] \tag{7}$$

$$T_t + (D_t - D_{t-1}) = G_t + subsidy_t + r_t D_{t-1} \tag{8}$$

In the initial steady state, we close the government budget with lump sum tax Υ_t to match the initial deficit and debt to GDP ratios. On the transition path, we hold the long run debt/GDP ratio and the values of Υ and G (per effective unit of labor) fixed, at the level from the initial steady state. In order to balance the government budget, on the transition path and in the final steady state (8), we adjust consumption taxes.

2.5 Market clearing and equilibrium conditions

In a competitive equilibrium, the goods market clearing condition is satisfied:

$$\sum_{j=1}^{J}\sum_{\kappa=1}^{\mathcal{K}} N_{j,\kappa,t}c_{j,\kappa,t} + G_t + K_{t+1} = Y_t + (1-d)K_t, \tag{9}$$

and the labor market clears when

$$L_t = \sum_{j=1}^{\bar{J}-1}\sum_{\kappa=1}^{\mathcal{K}} N_{j,\kappa,t}\omega_\kappa l_{j,\kappa,t}. \tag{10}$$

Additionally, an asset market clearing condition states:

$$K_{t+1} + D_t = \sum_{j=1}^{J}\sum_{\kappa=1}^{\mathcal{K}} N_{j,\kappa,t}a_{j,\kappa,t}, \tag{11}$$

where $a_{j,\kappa,t}$ denotes private assets.

A competitive equilibrium is an allocation $\{(c_{1,\kappa,t}, ..., c_{J,\kappa,t}), (a_{1,\kappa,t}, ..., a_{J,\kappa,t}), (l_{1,\kappa,t}, ..., l_{J,\kappa,t}),$ $K_t, Y_t, L_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that:

- for all $t \geq 1$, for all $j \in [1, J]$ for all $\kappa \in [1, \mathcal{K}]$ $((c_{j,\kappa,t}, ..., c_{J,\kappa,t+J-j}), (a_{j,\kappa,t}, ..., a_{J,\kappa,t+J-j})$ and $(l_{1,\kappa,t}, ..., l_{J,\kappa,t+J-j})$ solve the problem of an agent at the age of j from subcohort κ in period t, given prices;

- prices are given by: $r_t = \alpha K_t^{\alpha-1}(z_t L_t)^{1-\alpha} - d$ and $w_t = (1-\alpha)K_t^{\alpha}z_t^{1-\alpha}L_t^{-\alpha}$

- government sector is balanced, i.e. (5), (7) and (8) are satisfied;

- markets clear.

2.6 Policy instruments

We consider two instruments: minimum pension guarantee and lump sum indexation. With the **minimum pension benefit**, if an agent's pension is lower than the minimum pension threshold, the agent receives minimum pension b_t^{min}, i.e. equation (6) becomes:

$$b_{j,\kappa,t} = \begin{cases} \max\{b_t^{min}, f_{j,\kappa,t}/LE_{j,t}\} & j = \bar{J} \\ \max\{b_t^{min}, (1+0.25r_t^I)b_{j-1,\kappa,t-1}\}, & \forall j > \bar{J}, \end{cases} \tag{12}$$

where $b_t^{min} = \underline{b} \cdot w_t$.

With the **lump sum indexation**, the total funds used for indexation are divided by the number of pensioners and each pension is increased by a universal increment Δb_t for all agents from cohorts retired in t. Hence, the aggregate total pension expenditure grows at the same rate as in a pure DC system, but agents with lower pensions receive larger relative indexation and agents with higher pensions receive smaller relative indexation. Hence, with lump sum

indexation, the pension benefit computed at $j = \bar{J} + 1$ is the same as in equation (6), but the indexation formula $\forall j > \bar{J} + 1$ is changed to:

$$b_{j,\kappa,t} = b_{j-1,\kappa,t-1} + \Delta b_t, \;\; \text{where} \;\; \Delta b_t = 0.25 r_t^I \cdot \frac{\sum_{j=\bar{J}+1}^{J} N_{j,\kappa,t} b_{j-1,\kappa,t-1}}{\sum_{j=\bar{J}+1}^{J} N_{j,\kappa,t}}. \tag{13}$$

2.7 Model solving

Subcohorts represent parts of a cohort that share the same parametrization, that is productivity ω, preference for leisure $1 - \phi$ and the discount factor δ. In each year of the simulation, the agents within a subcohort either divide their time between labor and leisure given equations (1)-(2), and receive salary from the private sector agent ($\forall j < \bar{J}+1$) or receive benefits from the government ($\forall j \geq \bar{J}+1$). In both cases, the agents pay taxes to the government and spend some of their resources on consumption. Every agent in a subcohort decides about intra-temporal division of time to work and leisure and inter-temporal choice of consumption and assets in order to maximize lifetime utility, following the budget constraint (3).

Government represents pension system and tax-collecting. Its responsibilities in the system consist of collecting taxes from all the subcohorts and paying benefits to retired subcohorts. The government can run a DB or a DC pension system. If the government runs a DC pension system, the system can be modified to account for a minimum pension benefit guarantee or lump sum indexation. The government agent is given a fiscal closure rule, which translates government expenditures and tax base to tax rates, described by equations (7)-(8).

In each simulation scenario we calculate the transition path of the economy, given the assumptions on the pension system. To achieve this objective, we calculate two steady states, representing the initial and final year of modeled period, and then compute the transition path between them. General algorithms for computing the steady state and the transition path are similar. It is an iterative process using the Gauss-Seidel method. In each iteration, agents' choices are updated. The process stops when the difference between the capital from the new iteration is indiscernible from the previous iteration, i.e. smaller than a given parameter ϵ. On a transition path the optimization criterion relies on a sum of ϵ from each period. The parameter ϵ has been set to 10^{-14} in the steady states and a sum over all T set to 10^{-4} for the transition path.

Every iteration consists of the same major steps. Basing on capital calculated in the previous iteration (or initial capital in first iteration) and parametrization of the model, we compute tax rates. Given these rates and the structure of the pension system, we compute pension benefits for the retired cohorts. Given the amount of capital and labor, we obtain interest rates and wages. Given the tax rates, interest rates, and wages (as well as received bequests), we solve each individual problem and find labor supply, consumption and assets for each period and each subcohort. Next, we aggregate assets to obtain capital, to be compared with capital from the previous iteration. If the two values satisfy the norm condition, the process finishes. Otherwise, a new iteration starts.

3 Calibration

The model is calibrated to replicate features of the Polish economy and pension system. An economy begins with the DB system, which functioned in Poland until 1999. Pensions were granted to all individuals above age 65 (60 for women, with the average effective retirement age at 61), with at least 25 years of working experience (20 for women). Post-secondary education accrued to the working experience. Part-time employment counted the same way as full-time employment. The nominal replacement comprised a formula which had a social component and a private component. The social component was related to an average wage in the economy. Meanwhile, the rules for the private component have changed many times since 1990. Initially, the rule was to utilize as a base the monthly wage from the last month prior to claiming the pension. With subsequent legislative changes, that base was changed to an average over the last year, then over the last three years, and eventually, the best 10 out of the last 20 years of a working career, including career breaks such as unemployment or temporary disability leaves.

As of 1999, Poland adopted a defined contribution system for all cohorts born in 1958 or later.[12] All prior pension obligations were continued, whereas individuals close to retirement at the moment of the reform were to participate in a mixed system, which partly uses DB rules and partly uses DC rules, with proportions of the DC rules growing for younger cohorts. The mixed system was to become effective as of 2009 for women and 2014 for men.[13]

Our calibration of the initial steady state replicates the state of the economy just prior to the 1999 pension system reform. Macroeconomic flow aggregates match the averages obtained for the available time series, to average out the cyclical fluctuations (i.e., the time series cover 1995-2005). The capital market interest rate is computed using the data for the pension funds and thus it covers 1999-2009. Finally, the public debt was calibrated to match the 1999 ratio in GDP, i.e. 45%.

Below we discuss in detail the macroeconomic assumptions as well as the underlying sources of microeconomic heterogeneity. First, we describe the structural parameters of the model, i.e. those parameters that do not change between the baseline and reform scenarios. All structural macroeconomic parameters are presented in Table A1. Second, we discuss the calibration of the within cohort heterogeneity.

Demographics. The Eurostat demographic projection for Poland serves as a source for the size of new cohort arriving each year in the economy, i.e. $j = 1$ at each point in time t. The same source provides size of each birth cohort at consecutive ages, which serves as the basis for computing the survival probabilities $\pi_{j,t}$. The projection is available until 2060. We gradually stationarize after that period, so our population stabilizes after 2140.

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[12]Cohorts born in 1968 or later were also to start accruing pensions savings in a capital pillar, cohorts born between 1948 and 1968 could opt in to accruing pension savings in a capital pillar. Since the capital pillar was effectively abandoned in 2013, with the majority of accumulated savings nationalized and added to the pay-as-you-go pillar, we abstract from this feature of the pension system in our paper.

[13]In 2009 the legislation regulating the retirement age was changed (with subsequent changes to the whole pension system in 2011, 2013 and 2017). Accordingly, the pensions for the cohorts in the mixed system will typically be determined by DC rules.

Technological progress. The model accounts for labor augmenting technological progress. We input the projected path on future technological growth rates based on the forecast by the Aging Work Group of the European Commission, which provides this time series for all EU Member States. The overall assumption behind the forecast is that countries with lower *per capita* income will be catching up in terms of labor productivity, but eventually there is a steady convergence towards the long term value of 1.5% exogenous rate of labor productivity growth *per annum*, common for all EU countries.

Government. There are no tax redemptions on capital income tax, so *de iure* and *de facto* tax rates were set equal, which implies $\tau_k = 19\%$. Labor income tax (τ_l) was set at effective 19%, which matches the ratio of labor income tax revenues to GDP in 1999 (4.9%). Consumption tax τ_c was set at 11%, which matches the ratio of revenues from this tax to GDP in 1999 (7.4%). The consumption tax is allowed to increase in the baseline and reform scenarios to keep the government budget constraint given in equation (8) in balance. We keep the debt to GDP ratio at 45%, which corresponds to the level in Poland prior to the 1999 pension system reform.

Pension system. The original replacement rate ρ in the DB PAYG system was set to match the ratio of the pension benefits in GDP to 1999, i.e. 5%. Subsequently, the social security contributions τ were set to reflect the size of the deficit in the pension system (denoted in our model as $subsidy_t$) in 1999, which amounted to 0.8% of GDP. In the DB simulation, ρ and τ are kept constant. In the DC simulations, only τ is relevant and it is fixed.

We calibrate the minimum pension benefit threshold (denoted $\underline{b}$) in order to match the take up rate in the initial steady state: 4% of all new pensions are minimum pensions, according to the Social Insurance Fund reports. The pension benefit threshold is the 4th percentile of the initial steady state pensions distribution. In the initial steady state this value was equivalent to 6.5% of the earnings. We keep this ratio constant throughout the simulation.

For cohorts already working under the defined benefits system, there is no data on accrued savings that could be used to compute pension benefits under defined contribution. We simulate the amount of the contributions using the formula implicit in equation (6) for all the cohorts active prior to the reform but participating in the reformed pension system.[14]

3.1 Calibration of the productivity endowment and the preferences

We have three dimensions of individual within cohort heterogeneity: two for preferences (leisure and time preference) and one for endowments (productivity). Following the previous insights from Hénin and Weitzenblum (2005), McGrattan and Prescott (2013) as well as Kindermann and Krueger (2014), we calibrate them using micro datasets. Unlike McGrattan and Prescott (2013), we rely on individual rather than household data for two main reasons. First, we cannot

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[14]Instead of implied savings, one could consider using data on actual accrued savings. However, inferring from the released sample of 1% of the records of the Social Insurance Fund, for many future retirees the records are empty. Although the legislation set a date until which citizens are obliged to claim their pensionable work experience, in practice they can do so at any point in time, including just prior to the retirement.

obtain reliable indicators of individual productivity from household budget surveys (individual income earned is not recoverable for many types of households). Second, both household budget survey data and the labor force survey data are self-reported, thus featuring all the well known problems such as rounding the reported values of earnings and hours. For these two reasons we rely on linked employer-employee data, collected biennially by central statistical offices of the UE, the Structure of Earnings Survey. It covers the enterprise sector and comprises a sample approximately 20 times larger than labor force or household budget surveys. The values of hours worked as well as earnings are reported in actual terms by the employers, which results in a substantially smoother distribution of the two variables. Finally, this way we also avoid confusion of wage income and capital income (see McGrattan and Prescott 2013).

Productivity endowment (ω_κ). Since, in our model, the productivity endowment is allocated once for the entire lifetime, we use the early years in the career to obtain the distribution of endowments. We estimate a standard Mincerian wage regression with education levels, occupation, industry and region controls, as well as the form of contract (fixed term or indefinite duration), form of employment (part-time, full-time, weekends, etc.). We use total hourly wage, including overtime and bonuses. The Mincerian wage regression was estimated for all individuals in the sample, so we had controls for age and experience (both linear and squared). Subsequently, we use fitted value of log earned hourly wage against the mean of this prediction for the individuals up to five years after labor market entry. This yields the final distribution of productivity multipliers of ω as depicted in Figure A3a, i.e.

$$\omega_\kappa \in \{0.70\omega, 0.76\omega, 0.84\omega, 0.93\omega, 0.98\omega, 1.03\omega, 1.08\omega, 1.14\omega, 1.20\omega, 1.26\omega\}^{[15]}.$$

These individual productivity endowment multipliers do not change during the lifetime, i.e. age-productivity profile is flat (see e.g. Deaton 1997, Börsch-Supan and Weiss 2016).

Leisure preference ($1 - \phi_\kappa$). Agents' preference for leisure/consumption is directly responsible for the labor supply decisions, so we calibrate it to replicate the employment ratio of 56.8% in 1999. The final value of aggregate ϕ amounts to 0.500, which seems plausible: average hours worked in the Polish economy amount to approximately 2050[16], i.e. 51.5% of the total workable time.

However, individual preference for leisure is likely to be heterogeneous, with a fraction of population working part-time or not at all. Since preference for leisure is set once for the whole lifetime, we cannot directly replicate the distribution of working/non-working population (i.e. pick from data a share of individuals who do not *currently* participate in the labor market to proxy for a share of individuals who in the model *never* participate in the labor market), because we would automatically translate the initial structure of the inequality to the future

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[15]We run a similar analysis if median fitted value was to be the metric of endowments, the distribution is similar. The results are available upon request. The raw ω multipliers obtained from a regression on logarithms of wages were subsequently recalculated via the exponential function and normalized.

[16]Conference Board, averaged for 1999-2012 (*ahwpol* from The Conference Board Total Economy Database).

via non-participation. Thus, we rely on reported hours actually worked in the Structure of Earnings Survey which range from 31% to 206% of the regular working time. We thus obtain the individual multipliers of ϕ, i.e. $\phi_\kappa \in \{0.5\phi, 1.0\phi, 1.5\phi, 2.0\phi\}$. The distribution scaled by the mean hours worked is depicted in Figure A3b.[17]

The discount factor (δ_k). The aggregate value of δ was set at 0.981 to match the interest rate of 7.0% on the asset portfolio, as observed in the data.[18] Depreciation rate d is calibrated to match the investment rate in the economy given δ. The observed investment rate in the period between 1995 (first reliable post-transition data) and 2010 fluctuated between 19% and 23%, yielding an average of approximately 21%. We take this as a target value for calibrating the depreciation rate.

There are no empirical counterparts for the individual δ_κ for Poland. We rely on two sources of data about wealth inequality, to match the calibrated heterogeneity of δ parameter to replicate wealth inequality in the initial steady state. Davies et al. (2011) estimate the Gini coefficient for wealth inequality in Poland in 2000 at 65.7. This estimation overlaps with the timing of the initial steady state. The data underlying the estimation, however, comes from a relatively small sample and a simplified survey of household wealth. A more thorough way to recording household wealth is pursued by the Household Finance and Consumption Network in the European System of Central Banks. The survey conducted by the National Bank of Poland in 2015 yields an estimate of wealth Gini coefficient of roughly 57.9. We calibrate the multipliers for δ to match the mid-range of these two values. We arbitrarily assume three classes for δ_κ to encompass 40% of the cohort in the mid class, 30% of the cohort to be more impatient than average and the remaining 30% of the cohort to be more patient than average. This assumption combined with the target value yields $\delta_\kappa \in \{0.988\delta, 1.0\delta, 1.012\delta\}$.

3.2 The implied within-cohort heterogeneity

The adopted parametrization of the utility function generates substantial variation in income. In fact, the Gini coefficient for consumption in the initial steady state reaches approximately 21.5, which is close enough to approximately 23.5 observed in the data, see Brzeziński (2012). In terms of individual level differences, they are best observed in life cycle wealth profiles. More patient subcohorts with higher endowments and lower preference for leisure have substantially higher savings path than less patient subcohorts with lower endowments and higher preference for leisure.

In addition to a standard subcohort, which has no multipliers on preference for time δ, for leisure ϕ and endowments ω, Figures 1a to 2b depict life-time savings and labor supply

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[17]This calibration abstracts from within household specialization between market and non-market work, see Stigler and Becker (1977).

[18]While this value may seem high, please note that we are calibrating to the case of a catching-up economy. For example, Nishiyama and Smetters (2007) calibrate interest rate to 6.25% for the US economy. Also, the average real annual rate of return at the level of 7.4%, net of all the fees, with a balanced portfolio strategy was achieved on average by the open pension funds in the period 1999-2009. Thus, this value is not excessive, when compared to data or to the literature.

Figure 1: Life cycle path of wealth accumulation (assets, $a_{j,k,1}$) – initial steady state

(a) different endowments, identical preferences   (b) identical endowments, different preferences
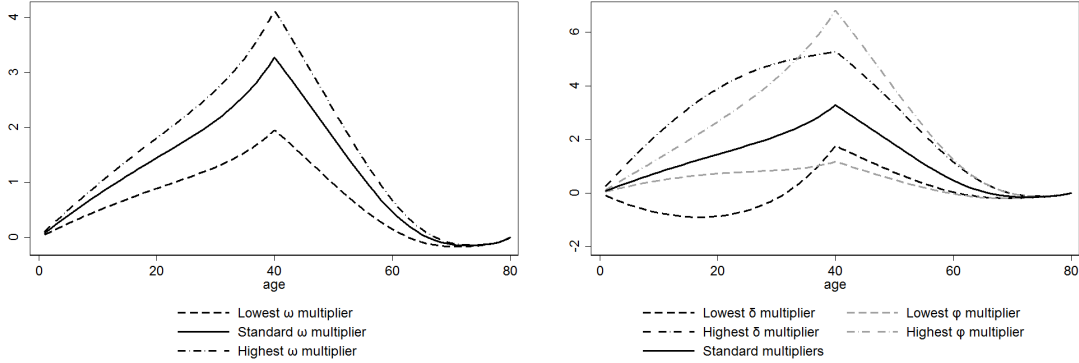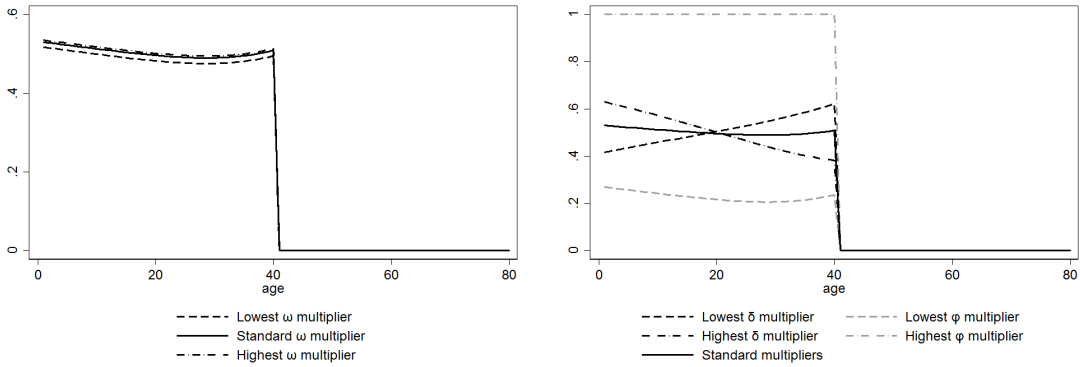

Figure 2: Life cycle path of labor supply ($l_{j,k,1}$) – initial steady state

(a) different endowments, identical preferences   (b) identical endowments, different preferences

patterns for several types of subcohorts. First, in Figures 1a and 2a we show life cycle paths for subcohorts with all multipliers over preferences set to 1, but with differing multipliers on endowments.[19] Analogously, in Figures 1b and 2b we keep endowment multipliers at 1 and display subcohorts with differentiated preferences. Heterogeneity in preferences translates to substantial differentiation in the propensity to save. In fact, at the retirement age, patient agents have wealth (i.e. accumulated lifetime savings) 2.3 times larger than 'median' agents, whose wealth is 4 times larger than for the impatient agents (computed for cohorts with multiplier of 1 for ω and ϕ).

3.3 Simulation scenarios

The baseline scenario of no policy change involves the demographic and productivity change, see Section 3, but the pension system remains DB. In the reform scenario, the economy is in transition from the DB to the DC system. In the baseline and in the reform scenario we obtain wealth and consumption distributions for each point in time. In the reform scenarios, we introduce the two mechanisms designed to curb inequality: minimum pension benefits and

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[19]The budget constraint requires no debt at death, but permits negative savings throughout the life-cycle.

lump-sum indexation.

The **minimum pension** stipulates that an individual receives a minimum pension irrespective of individual pre-retirement earnings. Note that this instrument automatically redistributes towards low productivity and high leisure preference subcohorts. Since, in principle, pure DC systems are balanced, this redistribution necessitates a taxation surge. Namely, paying out pensions in excess of accumulated savings to some subcohorts will generate a deficit in the pension system (as denoted by $subsidy_t$). Following the assumed government budget constraint, this will imply an increase in consumption taxes in our setup, relative to the scenario of no minimum pensions. Notably, this increase in taxes may be lower than in the baseline scenario of DB pension system, which in general is not balanced.

The second instrument, **lump sum indexation**, stipulates that the total funding for indexation in a given year is spread equally among all pension benefit recipients. Following the formula for pension benefits given in equation (6), the total growth in pensions between year t and year $t + 1$ follows from two sources: (a) some of the retirees die and a new cohort of retirees arrives; and (b) pensions of the surviving retirees are indexed. With lump sum indexation, the relative growth in pension benefits is higher for low-pension benefit recipients than in the proportional indexation case. By the same token, high income earners receive less than proportional pension benefit increase. This instrument is also fiscally neutral in a sense that the total pension expenditure remains intact (there may naturally be some general equilibrium effects).

4 Results

The general feature of perfect foresight models is that agents adjust savings to the expected path of future incomes. This implies that longevity translates to increased savings and thus assets regardless of the pension system, although the reasons differ. Under the defined benefit system, agents expect a stark increase in taxation, which lowers their future net income. In the defined contribution scheme agents expecting to live longer need higher savings to supplement the relatively low pension benefits. Consequently, there are likely to be substantial effects of longevity on wealth inequality.[20] We also expect the instruments to affect the inter-temporal choices of agents. By contrast, the effects for consumption inequality should be smaller, because they are indirect and occur *via* consumption smoothing and *via* labor supply adjustments.

4.1 Aggregate inequality

In the DB system, wealth inequality grows substantially larger during the demographic transition, to level off once the new population structure stabilizes, see Figure 3a. The transition to a defined contribution system necessitates adjustments in individual voluntary savings, leading to a starker increase in consumption inequality, but less wealth inequality. This is a result of the fact that a lower fraction of all subcohorts is in debt at any point in time.

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[20]Wealth inequality is measured as inequality of assets. For brevity, henceforth we use the term wealth inequality rather than inequality of assets.

The minimum pension benefit reduces consumption inequality permanently, by roughly 50% of the original increase due to the pension system reform from the DB to the DC. By analogy, the decline in wealth inequality is weaker. This suggest that the incentives from the minimum pension are rather strong, lowering the individual voluntary savings, and thus slowing down the rate of capital accumulation permanently. However, wealth inequality is lower with the DC system than in the case of the DB system, which suggests that impatient subcohorts increase savings more in response to demographic changes under the DC system than in the case of the DB system. This effect stems from lower expected pension benefits under the DC system. The deterrence from private savings is confirmed in Figure 3b, with inequality measures at retirement (for subsequent cohorts reaching that age). The majority of the effect on consumption inequality comes from the demographic transition (changes in life expectancy), whereas the majority of the effect on wealth inequality comes from the pension system reform and may be largely influenced by minimum pension benefits

By contrast, the lump sum indexation has virtually no effect on inequality of either consumption or wealth. Although this instrument redistributes within cohort after retirement, the scope of inequality accumulated until retirement cannot be visibly influenced by a fiscally neutral instrument. While many countries consider it as a viable way to reduce old age poverty, the effect of lump sum indexation on poverty is negligible even if one analyzes only poverty among the retirees, as portrayed by Figure 3c. Clearly, minimum pension benefits reduce poverty and old age poverty, relative to the transition to the DC system with no redistribution. This result is a flip side of lower incentives for private voluntary savings among the individuals with lower life-time income profiles.

The results for the Gini coefficient are reflected in alternative inequality measures, see Figure 4. For brevity, we present the synthetic comparison of the distributional and dispersion measures for selected years of the transition (left axis denotes measures of inequality for consumption, full bars and right axis denotes measures of inequality for wealth, empty bars).[21] While the introduction of the DC system leaves older cohorts with less room to adjust to the new rules, gradually the difference in the consumption Gini coefficient decreases to stabilize with the new demographic structure. However, wealth inequality diverges between DB and DC systems. The instrument with the strongest power to reduce consumption inequality and at the same time increase wealth inequality is the minimum pension. In fact, given the wide coverage of the minimum pension, wealth inequality is substantially higher with the minimum pension benefit than with no instrument. These effects for wealth inequality appear to stem mostly from the lower part of the wealth distribution, whereas for consumption inequality the effects are the largest at the top part of the distribution.

4.2 Macroeconomic effects

Although wealth inequality grows, the aggregate effect on capital is negligible. The fiscal consequences are considerable, though. Overall, the fiscal cost of an unchanged pension system
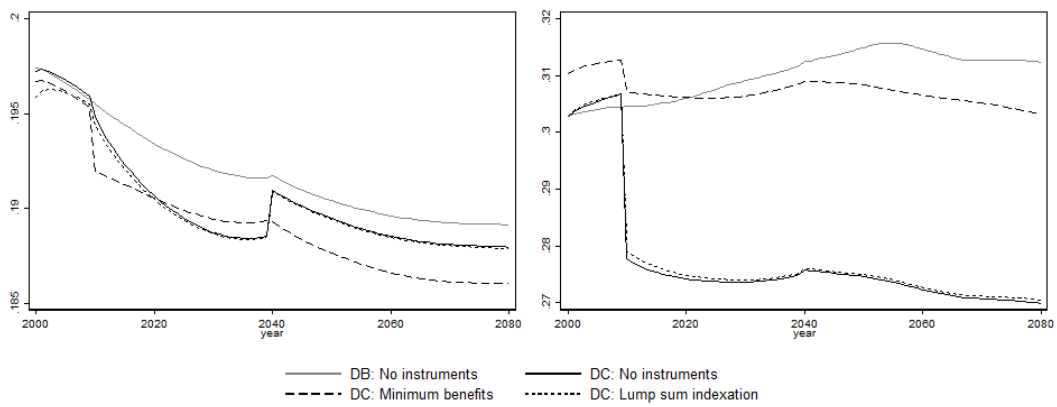
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[21]The complete set of measures for all periods and across all methods is available at `http://grape.org.pl/data/aging-and-inequality`.

Figure 3: Evolution of Gini coefficients and poverty
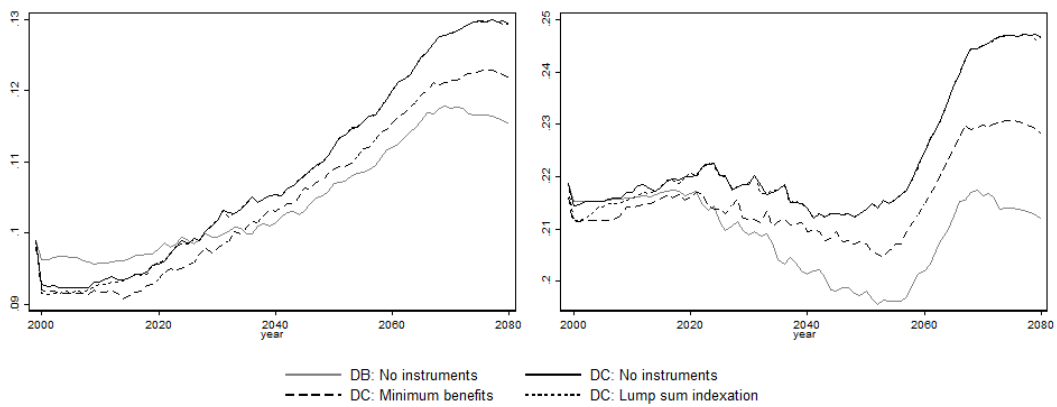
(a) Gini coefficient, all subcohorts, left: consumption, right: wealth
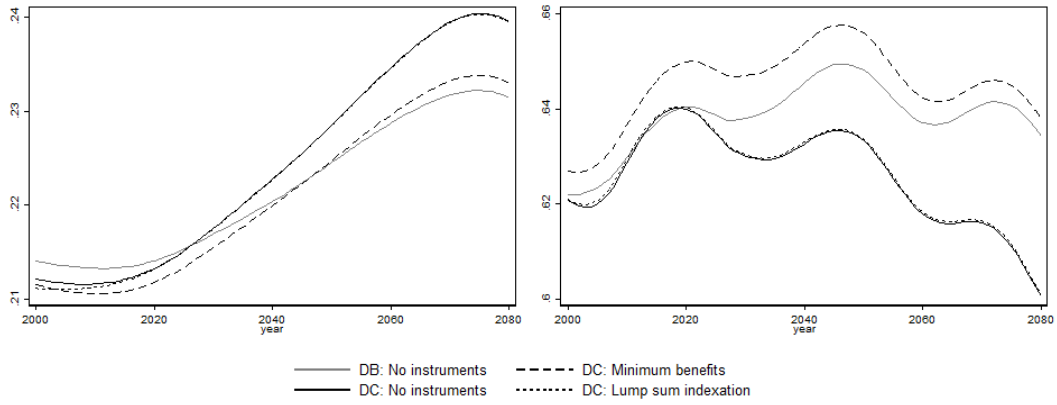
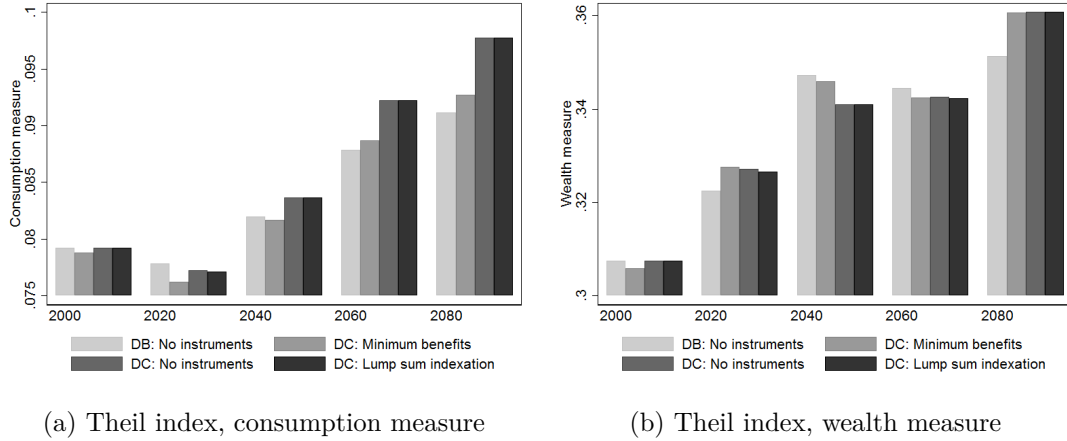(b) Gini coefficient, at retirement, left: consumption, right: wealth

(c) population below poverty line, left: overall, right: retirees

Note: Poverty line set at consumption below 50% of median consumption in a given year.

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Figure 4: Evolution of inequality measures over time, Theil index

(a) Theil index, consumption measure

(b) Theil index, wealth measure

Note: results for 50-20 and 80-20 ratios reported in Figures A6 and A7.

(DB) under demographic transition is substantial, requiring a large increase in consumption taxes by year 2080. In principle, a pure DC system is balanced, so by the year 2080 there is no deficit in the pension system. Also, tax rates can be somewhat lowered once DC is implemented. Clearly, minimum pensions in the DC system are quite costly from a fiscal perspective, however the fiscal cost is much smaller when compared to the potential imbalance in the DB system. Lump sum indexation, due to fiscal neutrality and very modest scope of redistribution has almost no macroeconomic effects compared to the pure DC system.

Table 1: Macroeconomic effects (until $t = 2080$)

	No instrument		DC with minimum	DC with lump sum
	DB	DC	benefits	indexation
K/AL	184%	198%	191%	198%
K/A	106%	115%	111%	115%
Labor	57.3%	58.4%	57.9%	58.4%
Tax rates				
initial steady state	18.5%	18.5%	18.5%	18.5%
$t = 2080$	27.5%	19.6%	22.2%	19.6%
Pension system deficit				
initial steady state	0.54%	0.54%	0.81%	0.54%
$t = 2080$	6.35%	0.44%	2.68%	0.43%

Notes: Capital reported in relation to the initial steady state (ratio, in %). Pension system deficit as a share of GDP, negative numbers indicate surplus. DC denotes transition from DB to DC. The values for the final steady state are reported in Table A2 in the Appendix. Labor supply effects for agents from each subcohort for agents retiring in 2080 reported in Figure A10.

Figure 5: Aggregate welfare effects
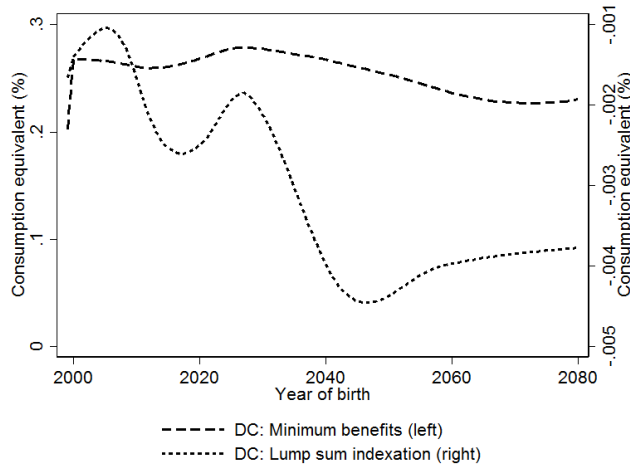
Note: consumption equivalent measured in percent of lifetime consumption.

4.3 Welfare effects

We display welfare effects as a comparison of individual utilities in the world with a redistributive instrument to those from the world where there are no redistribution instruments. Hence, the 'baseline' for welfare comparisons is the transition from DB to DC with no instruments, whereas 'reform' for welfare comparisons is transition with minimum pension benefits or transition with lump sum indexation. We measure change in utility as compensating variation and express it in terms of percent of lifetime consumption discounted to $j = 1$. To obtain the aggregate compensating variation we sum (with appropriate weights) over the lump sum transfers needed to maintain lifetime utility of agents within each subcohort at unchanged level and express it relative to the aggregate lifetime consumption of the entire cohort in a year when this cohort comes to the model. The aggregate welfare effects are displayed in Figure 5. The value of 0.2%-0.25%, means that the aggregate welfare with minimum pensions is equal to aggregate welfare with lifetime consumption increased by 0.2%-0.25% for all subcohorts. Figure 6 shows the welfare effects by subcohorts entering the model in year 2040. We pick this cohort, since it arrives in the model after the transition from DB to DC is effectively over, hence the welfare effect stems from the presence of the redistributive instrument in the pension system.

As displayed in Figure 5, welfare effects of lump sum indexation are negative but negligible. Meanwhile, the minimum pension benefits actually yield positive welfare effects. Notice that (as revealed by Figure A4) a large fraction of pensioners obtains minimum pensions. Figure 6 reveals that the positive effects arise predominantly from individuals with low productivity endowment. Apparently, the gains from higher pension benefits in old age outweigh the fiscal burden associated with this instrument. The group which loses on the introduction of the minimum pension benefits are agents characterized by low preference for leisure and high impatience. For welfare, minimum pensions transfer resources from the individuals with low marginal utility to the ones with high marginal utility, at the expense of taxation distortion.

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Figure 6: Disaggregated welfare effects, by subcohort, cohort born in $t = 2040$

(a) minimum benefits

(b) lump sum indexation
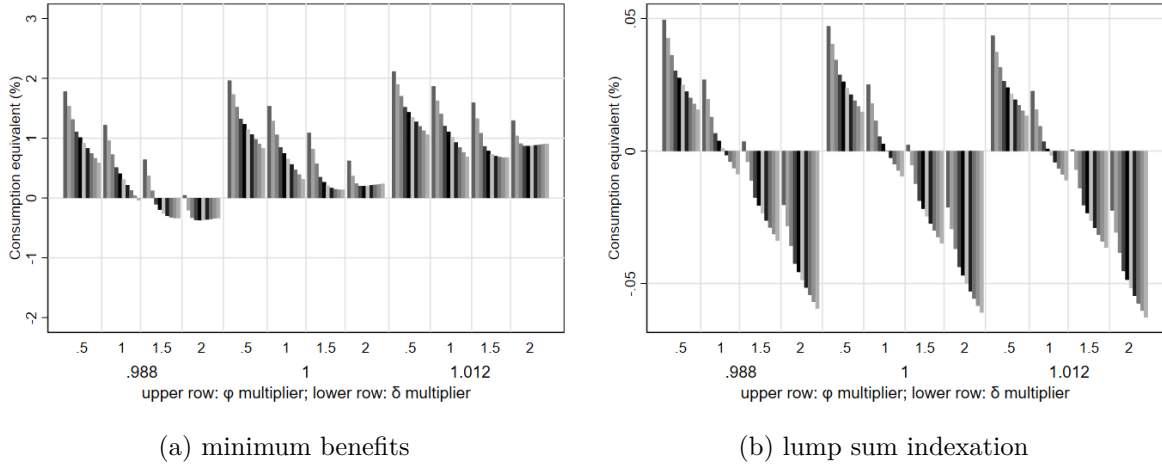
Note: Figure depicts welfare effects, expressed as percentage points of lifetime consumption for a cohort born in $t = 2040$. This cohort was selected because it is the first with the entire life-time after the transition from DB to DC is completed in a sense that all pensions from the old pension system are already terminated. For each given level of ϕ and δ, the bars are ordered in terms of the individual productivity (ω), with the subcohorts with the lowest productivity most to the left in each panel, subsequent panels of subcohorts separated by grid lines. Full set of results for each birth cohort available at `http://grape.org.pl/data/aging-and-inequality`.

Our numerical simulations indicate that, overall, such a transfer results in welfare gain. In the case of lump sum indexation, agents with low preference for leisure observe welfare loss, due to relatively higher implicit taxation of labor in the pension system design. Especially the patient agents with low leisure preference, who assign high weight to future pension benefits, but also contribute to the pension system a lot, observe welfare losses. However, welfare effects of lump sum indexation are one order of magnitude smaller than those of minimum pension benefits, effectively negligible.

Summarizing, there are considerable wealth and consumption consequences of introducing minimum pension benefits. If consumption equality was the policy objective, this instrument is helpful in in the defined contribution system, but at the expense of fostering wealth inequality. Additionally, there are high fiscal costs, in the form of pension system deficit and thus imposing higher taxes on all cohorts. Overall, welfare effects of minimum pensions are positive. Lump sum indexation has a negligible effect on inequality, old-age poverty and thus also welfare.

These overall results help build an intuition on the effects introduced by two instruments: minimum pension benefit and lump sum indexation. However, we are unable to judge if these effects – especially in the case of minimum pension – stem from large response to the instrument or rather from the heterogeneity of agents combined with the demographic transition. To put it differently: are minimum pension benefits overcoming the dispersion in endowments or is the extent of redistribution providing sufficiently strong perverse incentives that the preference channel dominates. In the next section we separate these two effects. Since effects for lump sum indexation are negligible, in the interest of brevity, we focus on the minimum pensions.

5 Identifying the channels for changes in inequality

To identify whether redistribution addresses the inequality which stems from dispersion of endowments or rather inequality which stems from differentiated preferences, we propose the following experiment. In a partial equilibrium setup (i.e. keeping the prices intact) we eliminate one of the channels of heterogeneity and recalculate the equilibrium for two paths: the transition to the DC system with no instruments and the transition to the DC system with minimum pension benefits. We can subsequently compute the inequality measures for those implied new paths (keeping prices fixed at levels from a respective path with all channels of heterogeneity on partial equilibrium). We depict the results for Gini coefficient and share measures in Figure 7, whereas other measures of inequality are deferred to the Appendix (see Figure A9). Solid and dashed thick lines plot the level of inequality measure in the scenario of transition to DC with no instruments and the scenario of transition to DC with minimum pension benefits, respectively. Thin dashed lines depict the results of the partial equilibrium with only one dimension of heterogeneity: either preference heterogeneity (dashed black line) or endowment heterogeneity (dashed gray line).

When we eliminate any dispersion in endowments, the measures of inequality are fairly similar across scenarios – dashed thin black line hovers in the vicinity of zero. However, eliminating the heterogeneity of preferences implies a substantial decrease in consumption inequality. This conclusion is robust to the way inequality is measured. For wealth inequality, both preferences and endowments channels appear to have effects of similar magnitude on the reduction of the aggregate inequality due to minimum pension benefits.[22] However, the preference channel is much stronger at the bottom of the wealth distribution.

Although the majority of changes in inequality stems from the demographic transition, the instruments tend to reduce inequality more in the case of a simulation without preference heterogeneity than for a simulation with no endowments heterogeneity. Consequently, the effects are larger for what could be called "equality of opportunity". Notably, inequality which stems from heterogeneity of preferences is a large part of overall inequality, whereas inequality which stems from heterogeneity of endowments is a small part of overall inequality. Since minimum pension benefits operate mostly along the channel which contributes relatively less to overall inequality, then clearly the scope for reducing inequality is also constrained.

6 Conclusions

We study the effects of aging and pension system reforms on inequality. Declining fertility and increasing longevity forced many governments to reform pension systems. Usually, such reforms feature linking pension benefits to contributions. The reform we study involves switching from the PAYG DB system to the PAYG DC. We supplement the standard overlapping generations model with within cohort heterogeneity. While some of the previous studies allowed for dif-
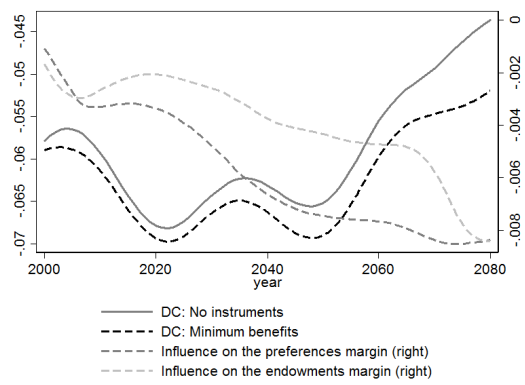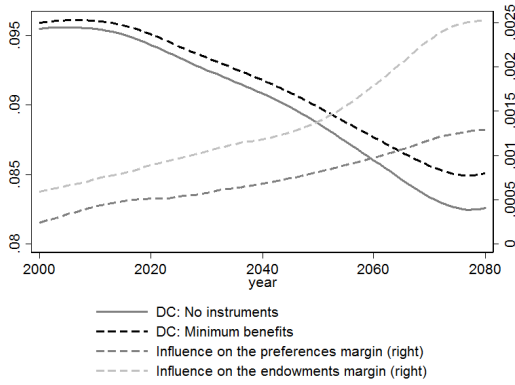
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[22]Wealth inequality increases under minimum benefits due to strong general equilibrium effects, which dominate the opposite direction of change in the partial equilibrium analysis.

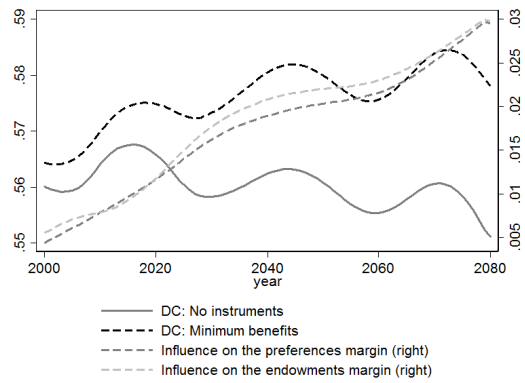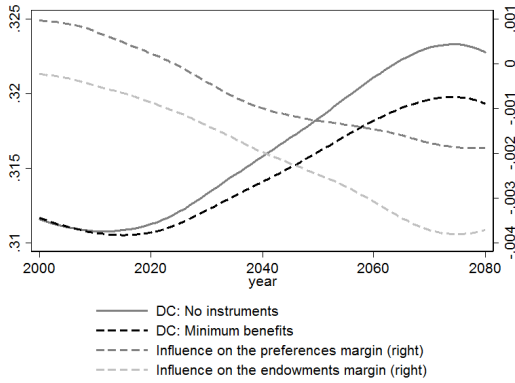Figure 7: Changes in inequality due to the minimum pension benefits
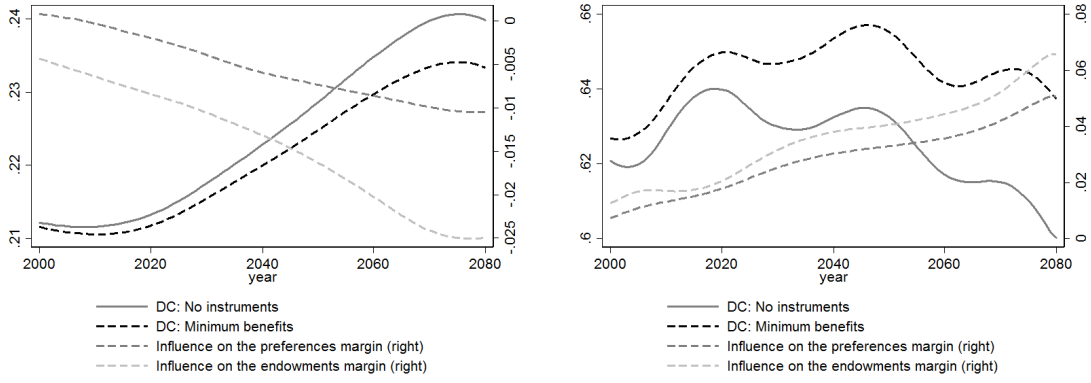
(a) consumpion Gini coefficient

(b) wealth Gini coefficient

(c) share of bottom 20% on consumption

(d) share of bottom 20% on wealth

(e) share of top 20% on consumption

(f) share of top 20% on wealth

Note: Solid and dashed thick lines plot the level of inequality measure in the scenario of transition to DC with no instruments and the scenario of transition to DC with minimum pension benefits, respectively. Dashed gray and light gray lines depict the relative (percentage) difference between these two scenarios and a reform to DC scenario under partial equilibrium with only one dimension of heterogeneity: either preference heterogeneity (dashed black line) or endowment heterogeneity (dashed gray line).

ferences in endowments, our setting allows agents to differ also in terms of time preference as

well as leisure preference. Our study uses an overlapping generations framework which permits comparing two types of pension systems: defined contribution and defined benefit. We account for changing demographics as well as a gradually decreasing rate of the technological progress. We nest two instruments which are popular policy solutions into our model: minimum pension guarantee and lump sum indexation.

We find that increasing longevity and declining fertility is *the* main force behind changes in wealth and consumption inequality. There are two main drivers of this change. The first driver relies on behavioral adjustments: expecting longevity, households adjust their savings profiles, raising the maximum accumulated wealth prior to retirement. The second driver relies on a change of the population structure: over time more middle aged people hold a fair amount of assets.

The pension system reform brings about two major changes from the point of view of an individual. First, it lowers pension benefits and consumption taxes. Second, it links future pensions benefits to labor supply, which means that the implicit tax on labor income falls. These incentives result in higher labor supply and higher voluntary private savings. As such, they lead to higher consumption inequality and lower wealth inequality.

We also experiment with two instruments that are typically considered as means of reducing inequality, especially old-age inequality. These are minimum pension benefits and lump sum indexation. The effects of lump sum indexation are negligible, as this instrument is too weak to affect choices of agents and distribution of consumption among retirees. Minimum pension benefits are very effective in reducing old-age and overall consumption inequality. They also contribute to higher wealth inequality, as they eliminate part of the precautionary motive. For a policymaker concerned with consumption equality rather than wealth equality these effects are noticeable. The reduction of inequality comes with positive overall welfare effects, despite the fiscal cost.

Analyzing the channels through which these changes occur, we find that this instrument could be particularly effective towards overcoming the 'inequality of opportunity'. In partial equilibrium we run simulations in which inequalities are driven by only (1) endowments; and (2) preferences. We find that even though the larger share of consumption inequality is driven by preferences rather than by endowment, surprisingly, the minimum pensions are particularly effective in reducing inequality stemming from endowments, i.e. 'inequality of opportunity'. Our results suggest that if inequality of old-age consumption is a policy concern, then minimum pension benefits may constitute a suitable policy option, so long as the eligibility criteria prevent early retirement.

We can summarize our results as follows. **First**, under increasing longevity a reform from the DB to the DC system results in higher consumption inequality. But, the even greater force changing aggregate inequality is longevity and changing population structure. **Second**, the increase in inequality due to the pension system reform can be substantially attenuated by minimum pension benefit guarantee. Minimum pension guarantee results in a decline in labor supply and brings about a considerable welfare cost, but in terms of welfare is welfare improving. **Third**, a way to reduce the fiscal costs of post-retirement income redistribution is the lump

sum indexation. We show that the effect of such a instrument is negligible – inequality remains unaffected by this fiscally neutral solution. **Fourth** and final, minimum pension benefits mostly address the inequality which stems from differentiated endowments and not that which stems from heterogeneous preferences.

Our results could be related to recent findings by Buyse et al. (2017) who examine the impact of pension system schemes on macroeconomic and welfare effects in an economy with endogenous human capital accumulation and individuals differing ex ante with respect to their human capital accumulation ability. In their setup, labor supply of lowest ability individuals drops sharply with minimum pensions, whereas we argue that the labor supply disincentives are small relative to other effects observed in the economy after the introduction of minimum pensions. The main reason for this decline in labor is the possibility of early retirement in their setup, which is unavailable in our framework.[23]

Our findings provide intuitions which could be an interesting starting point for further research. For example, in our setup there are no social assistance benefits during working life. This implies that agents – even low ability and high preference for leisure agents – have to work to finance instantaneous consumption. However, modern states provide a variety of income support instruments during the working period (e.g. means-tested and unconditional transfers), which may additionally influence labor supply decisions. Second, many countries introduce age and work experience eligibility criteria for minimum pension benefits. Policy experiments concerning the access to minimum pension benefits can reveal the extent to which these two dimensions – value of minimum pension benefits and when it can be accessed – can alter the consumption and wealth distribution within societies. Third, the long-run link between capital and labor may change, as has been observed over the past decades with declining labor share and changes in capital-labor complementarity. It was established in the literature that these processes stand behind changes in income and wealth inequality. Modeling such mechanisms in an OLG setup may provide additional insights on the links between longevity, inequality and pension system design.

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[23]In a review, Jiménez-Martín (2014) argues that the labor supply disincentives of the minimum pension benefits display in early labor market exits to retirement.

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A Calibration

Table A1: Calibrated parameters for the initial steady state

	Macroeconomic parameters	Calibration	Target	Value	Source
ϕ_k	preference for leisure	0.500	employment ratio	56.8%	NA
α	capital share	0.310	conventional value		
δ	discounting rate	0.981	interest rate	7%	(*)
d	depreciation rate	0.043	investment % in GDP	21%	NA
τ_l	labor tax	0.190	revenue as % of GDP	4.9%	OECD
τ_c	consumption tax	0.110	revenue as % of GDP	7.4%	OECD
τ_k	capital tax	0.190	*de iure*		
ρ	replacement rate	0.280	benefits as % of GDP	5%	NA
τ	social security contr.	0.061	deficit of pension sys. as % of GDP	0.8%	NA
ξ	DB pension social share	0.24	*de iure*		
$\underline{b}$	min. pensions mult.	0.065	minimum pen. as % of all pensions	4%	SIF

(*) this is the real effective net rate of return recorded by the private pension funds between 1999 and 2009. Pension funds were obliged by the law to hold a balanced portfolio. NA stands for National Accounts. SIF denotes Social Insurance Fund.

Figure A1: Number of 20-year-old and their survival probabilities to the age of 65 over time

(a) Number of 20 year old

(b) Survival probability to 65 at 20
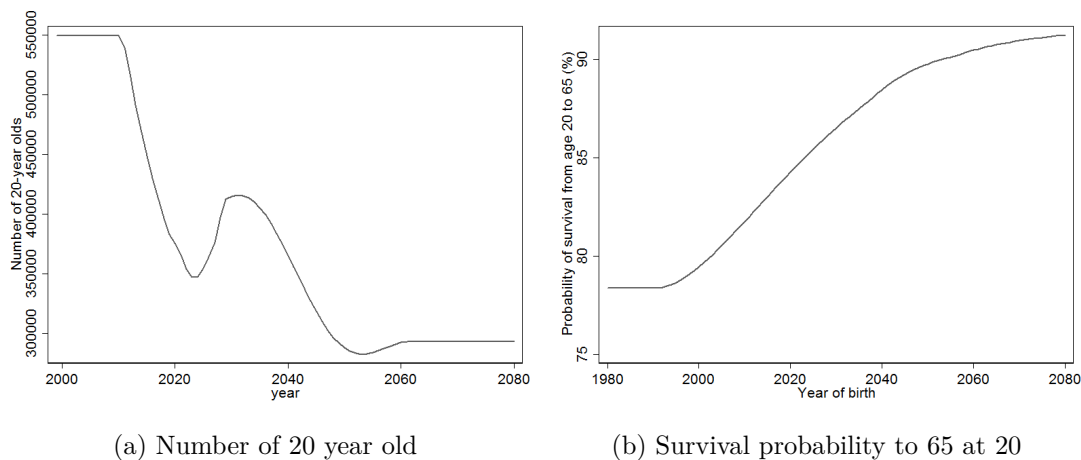
Figure A2: Labor augmenting TFP growth
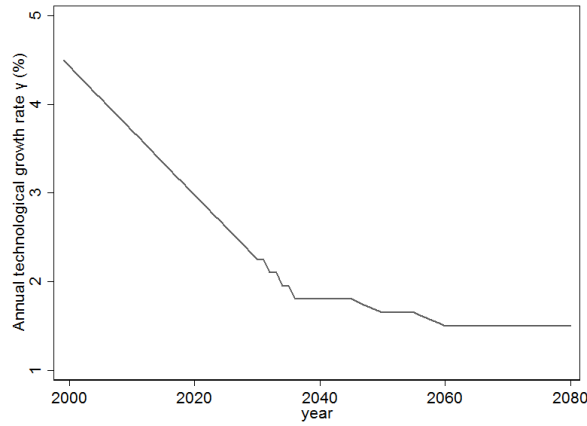
Figure A3: Calibrations based on Structure of Earnings Survey, 1998
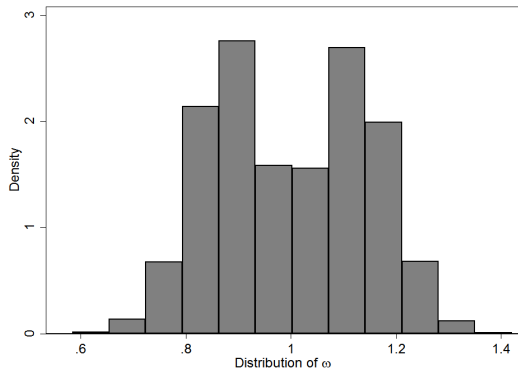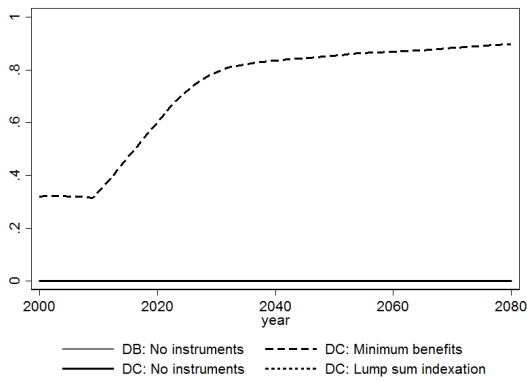

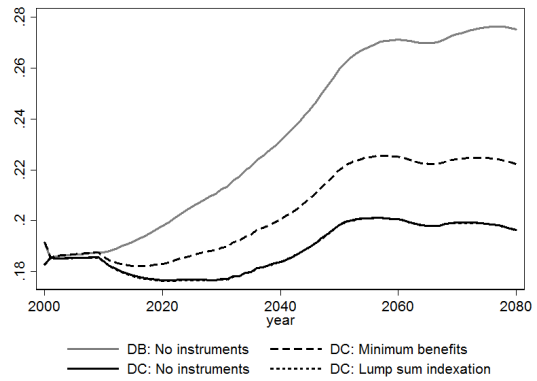
(a) calibration of productivity
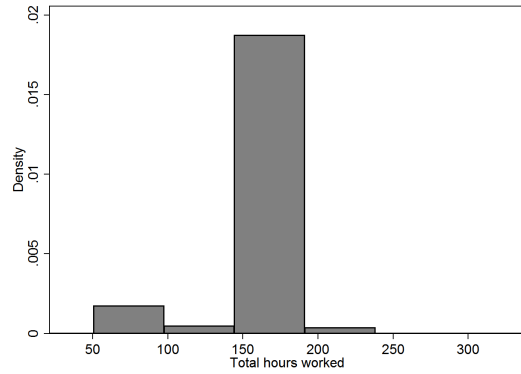
(b) calibration of preference for leisure

Figure A4: Coverage of the minimum pensions and fiscal adjustment

(a) minimum pension benefit coverage

(b) consumption tax

B   Appendix

Table A2: Macroeconomic effects (the final steady state)

	No instrument		DC with minimum	DC with lump sum
	DB	DC	benefits	indexation
K/AL	177%	188%	183%	188%
K/A	99%	107%	104%	107%
Labor	56.1%	57.1%	56.6%	57.1%
Tax rates				
initial steady state	18.5%	18.5%	18.5%	18.5%
final steady state	23.4%	17.0%	18.9%	17.0%
Pension system deficit				
initial steady state	0.54%	0.54%	0.81%	0.54%
final steady state	3.95%	-0.76%	0.92%	-0.77%

Notes: Capital reported in relation to the initial steady state (ratio, in %), expressed per effective unit of labor. Pension system deficit as a share of GDP, negative numbers indicate surplus. DC denotes transition from DB to DC.

Figure A5: Macroeconomic effects of instruments
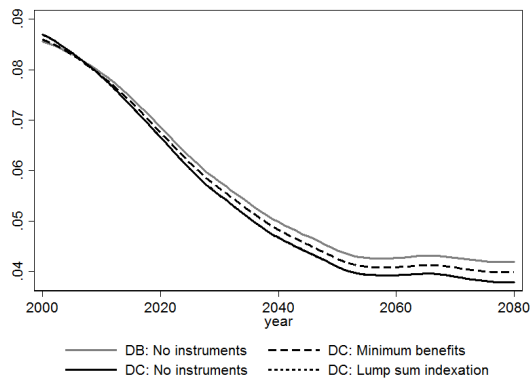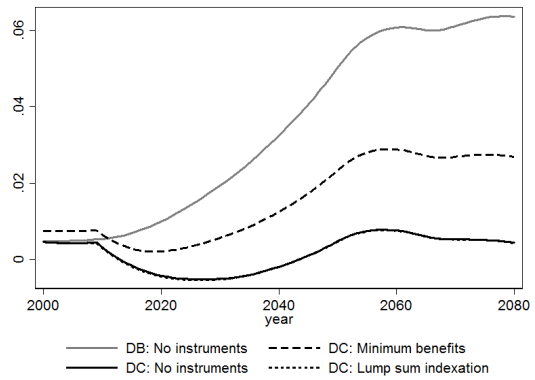

(a) interest rate
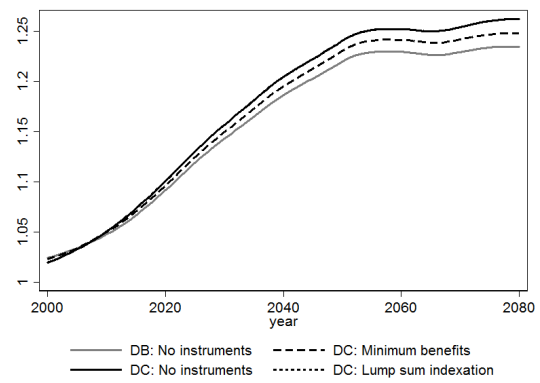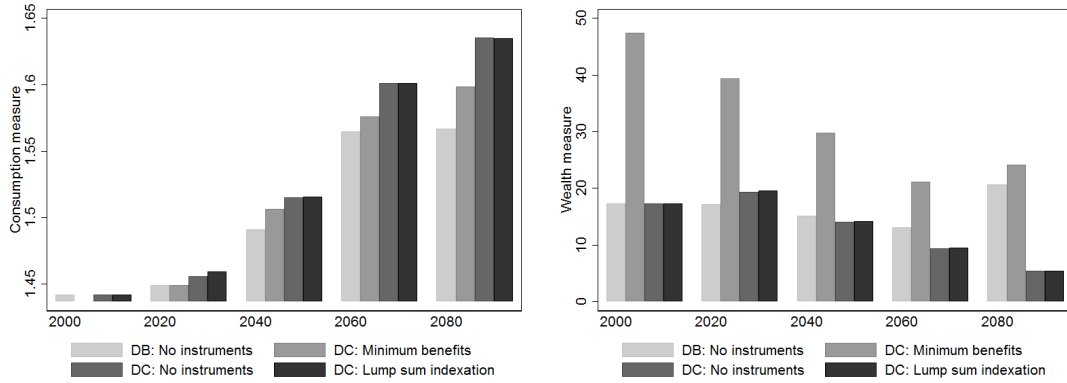

(b) wages
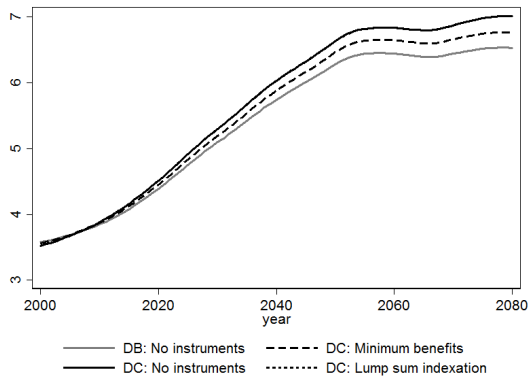
(c) capital
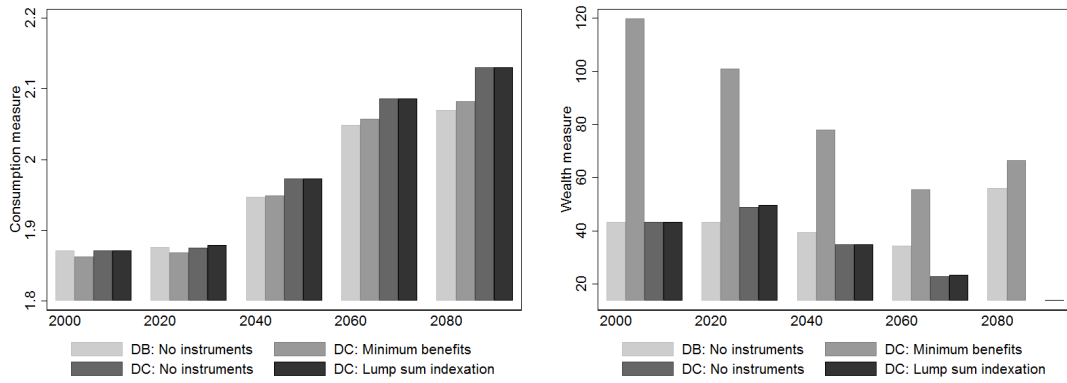

(d) pension system deficit (as % of GDP)

Figure A6: Evolution of inequality measures over time, 50-20 ratio

(a) 50-20 ratio, consumption measure
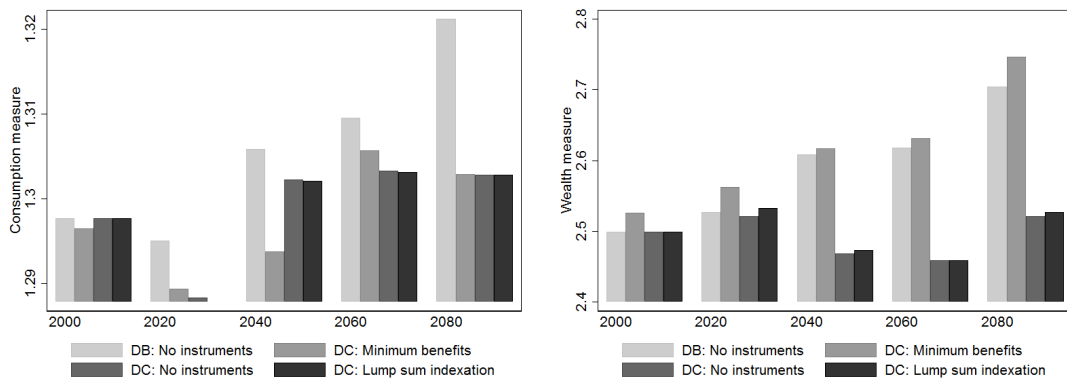
(b) 50-20 ratio, wealth measure

Figure A7: Evolution of inequality measures over time, 80-20 ratio

(a) 80-20 ratio, consumption measure
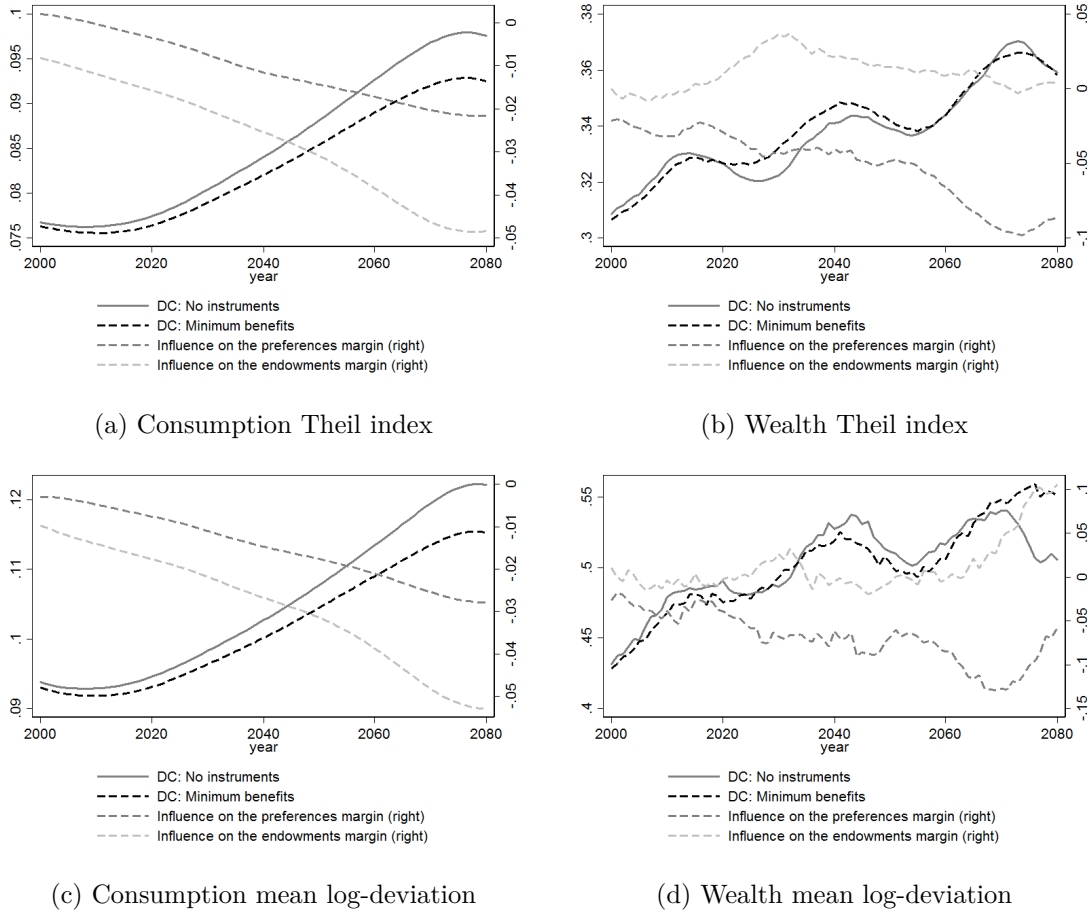
(b) 80-20 ratio, wealth measure

Figure A8: Evolution of inequality measures over time, 80-50 ratio


(a) 80-40 ratio, consumption measure

(b) 80-40 ratio, wealth measure

Figure A9: Sources of changes in inequality due to the minimum pension benefits

(a) Consumption Theil index

(b) Wealth Theil index

(c) Consumption mean log-deviation

(d) Wealth mean log-deviation

Note: Solid and dashed thick lines plot the level of inequality measure in the scenario of transition to DC with no instruments and the scenario of transition to DC with minimum pension benefit, respectively. Dashed gray and light Grey lines depict the relative (percentage) difference between the reform and baseline scenarios under partial equilibrium with only one dimension of heterogeneity: either preference heterogeneity (dashed black line) or endowment heterogeneity (dashed gray line).

Figure A10: Labor supply effects of instruments by subcohorts born in 2040
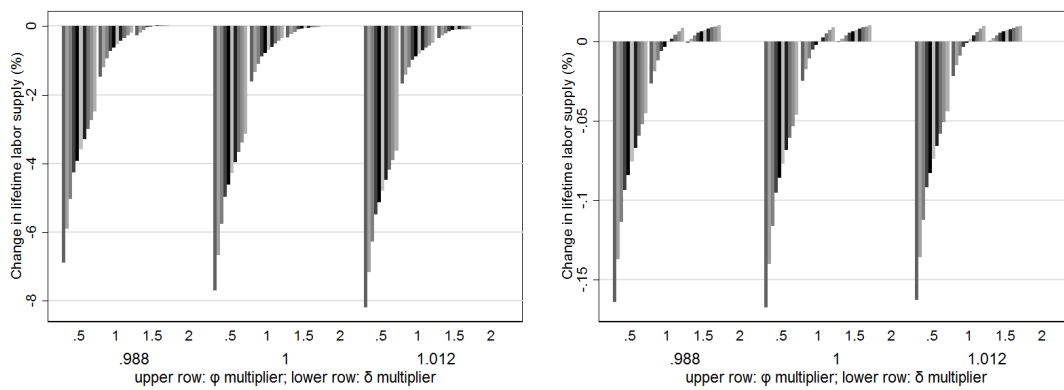
(a) Minimum pensions

(b) Lump sum indexation