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ABSTRACT

From Classes to Copulas: Wages, Capital, and Top Incomes[1]

Public debates about the rise in top income shares often focus on the growing dispersion in earnings and the soaring pay for top executives and financial-sector employees. But can the change in the marginal distribution of earnings on its own explain the rise in top income shares? Are top executives replacing capital owners in the group of top-income earners, or are we rather witnessing a fusion of top capital and top earnings? This paper proposes an extension of the copula framework and uses it for exploring the changing composition of top incomes. It illustrates that changes in top income shares can easily be decomposed into respective changes in the marginal distributions of labour and capital income and the changing association between the two types of income. An application using tax record data from Norway shows that the association between top labour and capital incomes grew stronger between 1995 and 2005 in the top half of the wage and capital income distribution, though it declined for the top 1 per cent of capital income receivers. A gender decomposition demonstrates that the association of wage and capital incomes at the top is particularly striking for men, while women are largely under-represented in the top halves of the two marginal distributions.

JEL Classification: C14, D31, D33
Keywords: income distribution, top incomes, income composition, copulas

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[1] This paper has been accepted for publication in the *The Journal of Economic Inequality*. Tony Atkinson sadly passed away in January 2017 before this paper could be finalized. Tony initiated this paper, and it would not exist without his passion and commitment. Subsequent revisions were carried out by the other two authors. The authors are grateful to Christoph Lakner, who contributed to the drafting of an earlier version of this paper and to Tom Wennemo for his help with preparing the empirical results. We are very grateful to Cecilia García-Peñalosa, Facundo Alvaredo and two anonymous referees for helpful comments. The opinions expressed in the paper are those of the authors and do not necessarily represent the views of Statistics Norway, the OECD or its member countries.

1. Introduction

For classical economists, there was a straightforward relationship between the factor distribution of income and the distribution of income among persons. There were workers, capitalists and landlords as separate classes, receiving wages, profits and rent, respectively. Workers were assumed to be at the bottom of the ladder, and a rise in the wage share reduced inequality in the personal distribution. A rise in the share of investment income – combining profits and rent – increased inequality in the personal distribution.

While this classical model gives a stylised view of the factor composition of incomes, strong elements of the class system remained at the beginning of the twentieth century in that top incomes were made up predominantly of investment income. Piketty and Saez (2007, Table 5A.7) show that in the United States in 1916 the capital income share for the top 0.5 per cent was over 50 per cent, and that the share of earned income for the top 0.1 per cent was only 10 per cent. As they observe, "top corporate executives at the beginning of the century were only a tiny minority within the top taxpayers" (2007, page 152). In the United Kingdom in 1911 investment income made up 72.3 per cent of the income of those assessed to super-tax (Atkinson, 2007, page 109).[2]

This has now changed. Over the twentieth century there was in the US a "dramatic evolution of the composition of top incomes" (Piketty and Saez, 2007, page 152). There has been a surge in top wage earnings, and the "working rich" are now to be found in the top income ranges, along with the top capital owners ("rentiers") who populated the top 1 per cent in earlier times. According to Wolff and Zacharias (2009, page 108), "the two groups now appear to cohabitate the top end of the income distribution". In France, Piketty and Saez (2003) found that the top capital incomes had not been able to recover from a succession of adverse shocks over the period 1914 to 1945; post-war progressive income and inheritance taxation prevented the re-establishment of large fortunes.

Recent studies suggest a different evolution in the Nordic countries. Jäntti et al. (2010) concluded that "the main factor that has driven up the top 1 per cent income share in Finland after the mid-1990s is an unprecedented increase in the fraction of capital income". In Sweden, Roine and Waldenström (2008) report that "between 1945 and 1978 the wage share at all levels

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[2] The super-tax was an additional income tax levied on top incomes introduced by the UK Government in 1909. A positive side-effect of its introduction from today's perspective is that it provided information on total incomes which had previously not been available on a regular basis.

of top incomes became more important . . . But in 2004 the pattern is back to that of 1945 in terms of the importance of capital, in particular when we include realized capital gains". In a study of top income mobility in Norway, Aaberge, Atkinson and Modalsli (2013) show that between 60 and 80 per cent of the richest 0.1 per cent had their main income from capital during the period 1993 - 2005, while an additional 20 per cent had their main income from self-employment. The tax reform of 2005, however, changed the composition of top income earners - in 2005, 81 per cent of the top 0.1 per cent derived their largest income component from capital, down to 50 per cent in 2006 and a maximum for the post-reform period of 55 per cent in 2008. Self-employment income was largely unaffected, whereas wage shares rose.

This paper further explores the changing composition of top incomes, by examining in greater depth the roles of earned (labour) and investment (capital) income in Norway. Not only will this two-way decomposition help link the changes in top shares to macro-economic developments (the changes in the wage share), but it will also aid our understanding of the wider social implications of distributional change. The substantial rise in top income shares that has taken place in many (but not all) advanced countries means that it is important to understand the underlying mechanisms. Can the change in the marginal distribution of earnings on its own explain the rise in top income shares? Or in terms of our earlier stylised example: Are top executives elbowing capital owners out of the top income group? If that is the case, then it is the distribution of earnings that should have first claim on our attention. But if those at the top are increasingly receiving income from both sources, then we have to pay greater attention to the ownership and transmission of wealth.

To address these questions, we explore the changes in the wage-capital composition of top incomes employing copulas, a known method which however has not yet been applied to the problem at hand.[3] Copulas are functions of ranks in marginal distributions that connect the bivariate distribution functions to the associated marginal distribution functions. This means that a two-way table showing the proportions with wage income below the u-quantile of the wage income distribution and capital income below the v-quantile of the capital income distribution for different values of u and v displays a copula. The two-way table reveals whether there is an over- or underrepresentation of people in the different cells of the table; i.e. whether there is positive association, negative association or independence between wage and capital income.

[3] Atkinson and Lakner (2017), which has been developed in parallel, use copula to study the association between capital and labour incomes in the United States.

In our application, we do not seek to estimate parametric copula functions, but rather to compare the degree of association, proposing a straightforward procedure for the implementation of non-parametric copula dominance criteria. First-degree copula dominance applies for non-intersecting copulas. When copulas intersect, the paper proposes to use second-degree copula dominance, which is defined by the cumulative integrated copula function. However, even second-degree copula dominance might fail to provide rankings of copulas in empirical applications. Thus, summary measures might be helpful both for ranking purposes and for quantifying and comparing the degree of association between variables. The Spearman coefficient emerges as an attractive summary measure of association because it is found to be consistent with second-degree copula dominance.

The paper is structured as follows: Section 2 describes the data used for the empirical analysis, which are drawn from Norwegian tax returns in 1995 and 2004/2005, and discusses their advantages and limitations. In Section 3, we set out the analytical framework. Section 4 presents evidence on the changing association of top labour and capital incomes in Norway between 1995 and 2004/5, and provides a breakdown of top income shares by gender and age group. Section 5 concludes.

2. Data

In our empirical analysis, we use tax return micro data from the administrative registers of Statistics Norway for the years between 1995 and 2005. The tax unit in Norway is the individual, and both labour and capital incomes are recorded in the data at the personal level.

We define labour income as earnings, to which we add two-thirds of self-employment income. Capital income is defined as personal investment income derived from profits and rents as well as from interest received on government and other debt. The allocation of self-employment income to labour and capital income in shares two-thirds and one-third respectively is arbitrary but does not seem unreasonable (e.g. see Johnson, 1954, and Gollin, 2002). Appendix A.1 provides further results on the joint distribution of capital and self-employment income and on the effects of dropping self-employment income from the analysis and looking at earnings and capital income only.

In the first part of the paper, we aggregate labour and capital income across all household members and carry out the empirical analysis for household income. We include both single and multi-person households in the analysis without adjusting for household size. Our main results continue to hold, however, if we limit the sample to two-adult households. Observations with negative income from any of the three sources (labour, capital or self-employment) are excluded.

Again, however, our main results are robust to retaining observations with negative income from capital or self-employment in the sample.[4] Later parts of the paper, where we present a decomposition of the results by gender and age group (in Appendix A.2), are based directly on individuals' tax records. For this analysis, we trim the sample by excluding below-25 year-olds.

The data we use have several advantages: First, there is no attrition from the original sample due to refusal by participants to consent to data sharing. Second, our income data pertain to all individuals, and not only to jobs covered by social security or individuals who respond to income surveys. And third, most income components are third-party reported, with little measurement error and without any top or bottom coding.

We are fully aware also of the limitations of using tax data, however: Income tax data generally do not adequately capture the full return to capital.[5] Moreover, the extent of coverage has fallen over time as there has been erosion of capital income from the progressive income tax base.[6] To address this issue, we rely on the existing estimates of Hicksian measures of capital income (Aaberge and Atkinson, 2010). As Norway introduced dividend taxation in 2006, capital incomes in 2005 are higher and distributed slightly differently than in previous years due to the fact that a large part of the top income earners adjusted their income through legal means such as for owner-managers of closely held firms to increase dividends in 2005 (the tax on dividends was to be increased in 2006 from 0 to 28 per cent). We therefore average results for 2004 and 2005 instead of using the year 2005 in our analysis. Again, all results are robust, however, to using either 2004 or 2005 only.

3. Analytical framework

We are concerned with the decomposition of total personal income into two components: earned (labour) income and investment (capital) income. With this two-way division, the personal distribution of total income depends on three factors:

a) The marginal distribution of earned incomes;

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4 2.9% of all observations have negative income from capital or self-employment.

5 For the United States, Piketty, Saez and Zucman (2018) attempt to provide a more complete estimate of capital incomes which includes undistributed corporate profits.

6 At the outset, a number of income tax systems (such as those of France or the UK) included imputed rents of homeowners in the tax base, but today imputed rents are typically excluded (Spain being one exception). Where the tax base has been extended, this has in some cases taken the form of separate taxation (as with capital gains in the UK), so that the income is not covered in the income tax data. As a result of these developments, the share of capital income that is reportable on income tax returns, and hence included in the series presented, has significantly decreased over time.

b) The marginal distribution of capital incomes;

c) The association between earned and capital incomes.

The developments in top shares described earlier have highlighted these elements. In the US context, for instance, it would be natural to suppose that a substantial fraction of the rise in top incomes was due to a surge in top wage incomes (factor (a)). The decline of the rentier also reflects reduced concentration of wealth (factor (b)). The simultaneous distribution of earnings and capital income has moved in ways that has increased the association between earnings and capital income among top income earners.

In contrast to the elements (a) and (b), the third factor – the pattern of association – has received very little attention.[7] Yet it is potentially important. The observed change in the composition of the top income group may also result from changes in the association between earnings and capital income. There may no longer be a sharp distinction between workers and rentiers (capitalists). In the pure class model, the association (correlation) between labour and capital income is minus 1. The association is now obviously greater than this, and may even be positive.

It is therefore tempting to measure the third element in terms of the association. The Pearson correlation coefficient, defined by the ratio of the covariance between the two variables and the product of their standard deviations, is not however well-suited for this purpose, since it is not independent of changes in the marginal distributions. Suppose, for example, in the class model, workers are divided into two sub-classes, with the same mean wage, but with one class earning β times as much as the other. An increase in β means that the correlation coefficient between wage and capital income moves away from minus 1, but there is no change in rankings or in the composition of the top income group. This objection does not apply to the rank-dependent association measures. In this paper, we therefore consider the cross-association in terms of the *copula function*, which provides a non-parametric approach of isolating changes in association independent of changes in the marginal distribution.

Proceeding more formally, let us denote labour income by x and capital income by y. The marginal distributions of, respectively, earned income and capital income are denoted by F and G. Since we are interested in the top incomes, we consider the survival distributions $\tilde{F}(x) = 1 - F(x)$, the proportion of the population with earned income of x or higher, and $\tilde{G}(y) = 1 - G(y)$, the proportion with capital income of y or higher. The joint distribution of x

[7] See Aaberge et al. (2000), García-Peñalosa and Orgiazzi (2013) and Milanovic (2017) for a discussion of similar issues, based on a factor income decomposition of either the Gini coefficient or the coefficient of variation.

and y is denoted by $H(x,y)$, and the joint survival function by $\tilde{H}(x,y)$. Note that $\tilde{H}(x,y) = 1 - F(x) - G(y) + H(x,y)$.

The copula is the function that binds together the two marginal distributions and is defined by $C(u,v) = H(F^{-1}(u), G^{-1}(v))$, where $F^{-1}(u)$ and $G^{-1}(v)$ are the left inverses of *F* and *G*. Or rather, since our purpose is to study the top shares, it seems better to use the *survival copula* $\tilde{C}(u,v)$ associated with $\tilde{H}$ (see Nelsen, 2006, pages 32-33). This shows the proportion of the population whose rank is *u* or higher in terms of labour income and *v* or higher in terms of capital income. Note that the survival copula is defined by

$$\tilde{C}(u,v) = \tilde{H}(\tilde{F}^{-1}(u), \tilde{G}^{-1}(v)), \quad (1)$$

where $\tilde{F}^{-1}(u)$ and $\tilde{G}^{-1}(v)$ are the left inverses of $\tilde{F}$ and $\tilde{G}$. In other words, we can obtain the survival copula from the survival function by substituting $\tilde{F}^{-1}(u)$ and $\tilde{G}^{-1}(v)$. It should be noted that the copula is invariant with respect to strictly increasing transformations of these functions, which makes copula-based measures of association insensitive to the tail properties of the marginal distributions. Note that we have the following convenient relationship between the *copula* and the *survival copula*,

$$\tilde{C}(u,v) = u + v - 1 + C(1-u, 1-v), \quad (2)$$

and thus that $\tilde{C}(0.5, 0.5) = C(0.5, 0.5)$ .The properties of the copula function are described clearly by Dardanoni and Lambert (2001) in their analysis of the measurement of horizontal equity.

The attraction of the copula is that it allows us to separate cleanly the changes in the relative rankings of individuals from changes in the marginal distributions. Suppose that there is a shift away from capital income towards labour income, in such a way that all capital income components are reduced proportionately, and all labour incomes are increased proportionately. This leaves the ranks in each dimension unchanged. The copula function is therefore unchanged.

3.1. Partial orderings

First-degree copula dominance

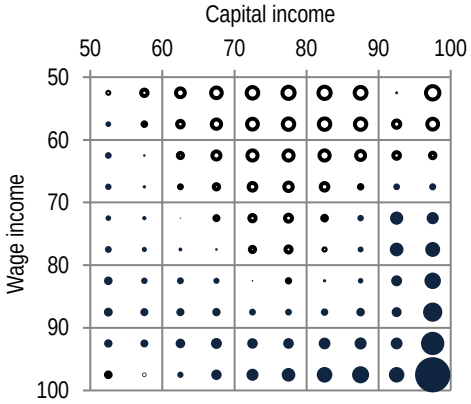
The implementation of the copula may proceed parametrically or non-parametrically. However, since parametric specifications impose considerable structure on the joint distribution, it appears more attractive to proceed non-parametrically when empirical analyses rely on large number of

observations.[8] One advantage is that such an approach is closer to that adopted in studies of social mobility, where a distinction is drawn between structural mobility and exchange mobility (holding the marginal distributions constant). Exchange mobility is studied directly in terms of transition matrices $[p_{ij}]$ where the cells are defined in terms of percentile position. The counterpart of such a matrix in the present application – the association matrix – is illustrated in Table 1 for Norway in 2004/5. The table shows for example that 0.07 per cent of households were in the top 0.5 per cent of both wages and capital incomes. In other words, 14 per cent of households in the top 0.5 per cent of wages were also in the top 0.5 per cent of capital incomes. Of the households in the top 0.5 per cent of wages, 44 per cent were in the top 5 per cent of the capital distribution; whereas only 32 per cent of the top 0.5 per cent of capital incomes was in the top 5 per cent of wages. This underlines the need to allow for asymmetry.

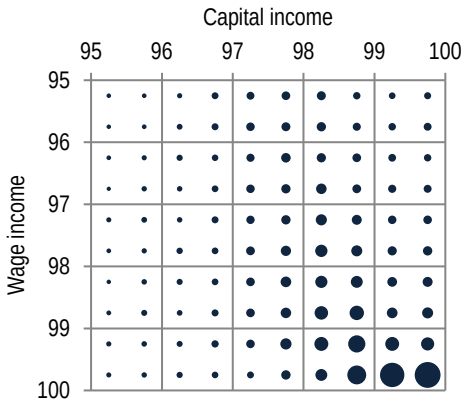
This result can be illustrated more plastically in graphical form. Figure 1 shows the simultaneous density of capital and wage income for Norway in 2004/5, once for the top halves of the two distributions and once for the top 5 per cent only. The area of each circle is proportional to the frequency of observations in that specific cell. Shaded circles indicate cells in which households are overrepresented relative to the case of a uniform distribution of households across cells; blank circles give those in which households are underrepresented.[9]

Figure 1. Normalised simultaneous density of capital and wage income, 2004/5

Panel A: The top halves of both distributions



Panel B: The top 5 per cent of both distributions



[8] See Bonhomme and Robin (2006, 2009) and Jäntti, Sierminska and Van Kerm (2015) for applications of the Placket parametric family of copulas in cases where samples of observations form the informational basis. Arellano and Bonhomme (2017) use a more flexible parametric copula as a basis for correcting quantile regression estimation of wage distributions for sample selection.

[9] The circles' areas have been calculated as (percentage of observations in the given cell) - (1 / the number of cells). Cells with negative values, in which households are underrepresented relative to the case of a uniform distribution of households across cells, are shown as blank.

Note: The area of each circle is proportional to the frequency of observations in the specific cell. Shaded circles indicate cells in which households are overrepresented; blank circles give those in which households are underrepresented.

There is an overrepresentation of households among the top 15 per cent of wage income earners (across the top half of capital income receivers) and a strong concentration in the top 5 per cent of both marginals (Panel A). Within the top 5 per cent, the pattern is particularly pronounced in top 2 to 3 per cent of capital and wage income earners (Panel B), and somewhat stronger among top capital than among top wage income earners.

We are interested in how far societies have moved from having a negative diagonal pattern to the transition matrix, as with the class society, to a situation where the two sources of income are independent or positively associated. In order to assess such a movement, it is helpful to move from the frequencies to the cumulative distribution, as with the survival copula $\tilde{C}(u,v)$. The survival function is shown in Table 2. This shows, for example, that nearly all of the top 0.5 per cent of the wage distribution (92%) were in the top half of the distribution by capital income, whereas over 20 per cent of the top 0.5 per cent of the capital income distribution were in the bottom half of the distribution by wage income. Such a cumulative distribution can be used to compare the degree of diagonality. If we define a diagonalizing switch as one that adds and subtracts δ from adjacent cells in the frequency matrix [p_{ij}]:

(3)
$$\begin{matrix} p_{i,j} + \delta & p_{i,j+1} - \delta \\ p_{i+1,j} - \delta & p_{i+1,j+1} + \delta \end{matrix}$$

(where *i* denotes the *i*-th percentile group, where *i* is counted from the top). The effect of the diagonalizing switch is to raise the survival copula by δ at $(i+1,j+1)$ and to leave it elsewhere unchanged (in particular, the marginal distributions are unchanged) - see Atkinson (1981) and Atkinson and Bourguignon (1982). On this basis, one distribution is closer to a positive diagonal (further from a class distribution) if its survival copula is everywhere higher or no lower. If, as we do below, we compare Table 2 for 2004/5 with the same table for earlier years, then this provides a simple dominance test as to the effect of the third factor: the changing degree of association between incomes from different sources.

More formally speaking, the standard ranking criterion of non-intersecting copulas, called *first-degree copula dominance*, is based on the following definition:

Definition 3.1. *A copula C_1 is said to first-degree dominate a copula C_2 if*

$$C_1(u,v) \geq C_2(u,v) \quad \textit{for all} \quad u,v \in [0,1]$$

and the inequality holds strictly for some $u,v \in \langle 0,1 \rangle$.

Note that first-degree copula and survival copula dominance orderings are equivalent, which follows straightforward from equation (2).[10] However, as demonstrated by Decancq (2014), this is not the case for multivariate copulas of dimension higher than two. And similarly as the Pigou-Dalton principle of transfers can be used to justify the criterion of first-degree Lorenz dominance, the principle of correlation-increasing transfers introduced by Boland and Proschan (1988) can be used to justify the criterion of first-degree copula dominance.[11] Note that the correlation-increasing rearrangement relies on the condition of fixed marginal distributions.

Second-degree copula dominance

The test just described is one for first-degree dominance, and there may be situations in which this does not allow matrices to be ranked. When we compare the survival copulas for two years, we may find that there are both positive and negative differences. As with one dimensional inequality measurement, the dominance criteria can be extended to second and higher degrees. In the inequality measurement case, the second-degree condition is obtained by integrating the cumulative distribution or the inverse cumulative distribution, which leads to a readily implementable test in terms of comparing Lorenz curves. In the present case, the second-degree dominance condition can be obtained by integrating the copula function, and this leads to a readily implementable test in terms of comparing rank correlations.

To deal with cases where there are both positive and negative differences between two *copulas*, we hence employ a criterion weaker than first-degree dominance comparing the upwards-cumulative integrated copulas. This is parallel to the concept, in one dimension, of second-degree upwards Lorenz dominance in Aaberge (2009).

Definition 3.2. *A copula C_1 is said to second-degree dominate a copula C_2 if*

$$\int_0^v \int_0^u C_1(s,t)\,ds\,dt \geq \int_0^v \int_0^u C_2(s,t)\,ds\,dt \quad \text{for all } u,v \in [0,1] \tag{4}$$

and the inequality holds strictly for some $u,v \in (0,1]$.

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[10] First-order copula dominance is also referred to as "concordance ordering" of copulas in the literature (see Nelson (2006), Definition 2.8.1).

[11] See also Tchen's (1980) discussion of positive association (or concordance) for bivariate probability measures and Decancq (2012, 2014) for a generalization of these principles and an analysis of their links to stochastic dominance.

It follows from equation (2) that

$$\int_{1-v}^{1}\int_{1-u}^{1}\tilde{C}(s,t)dsdt = \frac{uv}{2}(2-u-v)+\int_{0}^{v}\int_{0}^{u}C(s,t)dsdt\,, \tag{5}$$

which means that second-degree copula and survival copula dominance are equivalent ranking criteria, and can be considered as a parallel to first-degree upwards and downwards Lorenz dominance. Note however that second-degree upwards and downwards Lorenz dominance are different ranking criteria (see Aaberge, 2009).

First and second-degree partial copula dominance

Although analyses of copulas will normally be concerned with the entire bivariate distribution of two variables it might be of particular interest to focus on specific parts of the copula; e.g. comparing copulas for a specific value of *u* or *v*. To deal with such cases (see Figure 2 in Section 4) we introduce the following definitions of partial copula dominance:

Definition 3.3. *Let* $\underline{u},\overline{u},\underline{v},\overline{v}\in[0,1]$ *where* $\underline{u}<\overline{u}$ *and* $\underline{v}<\overline{v}$*. A copula* C_1 *is said to first-degree partial dominate a copula* C_2 *if*

$$C_1(u,v)\geq C_2(u,v) \quad \textit{for all} \quad u\in[\underline{u},\overline{u}]$$

and the inequality holds strictly for some $u\in[\underline{u},\overline{u}]$,

or

$$C_1(u,v)\geq C_2(u,v) \quad \textit{for all} \quad v\in[\underline{v},\overline{v}]$$

and the inequality holds strictly for some $v\in[\underline{v},\overline{v}]$.

Definition 3.4. *Let* $\underline{u},\overline{u},\underline{v},\overline{v}\in[0,1]$ *where* $\underline{u}<\overline{u}$ *and* $\underline{v}<\overline{v}$*. A copula* C_1 *is said to second-degree partial dominate a copula* C_2 *if*

$$\int_{0}^{v}\int_{0}^{u}C_1(s,t)dsdt \geq \int_{0}^{v}\int_{0}^{u}C_2(s,t)dsdt \quad \textit{for all} \quad u\in[\underline{u},\overline{u}]$$

and the inequality holds strictly for some $u\in[\underline{u},\overline{u}]$,

or

$$\int_{0}^{v}\int_{0}^{u}C_1(s,t)dsdt \geq \int_{0}^{v}\int_{0}^{u}C_2(s,t)dsdt \quad \textit{for all} \quad v\in[\underline{v},\overline{v}]$$

and the inequality hold strictly for some $v \in [\underline{v}, \overline{v}]$.

3.2. A complete ordering

To achieve rankings of intersecting (survival) copulas (or cumulative distribution functions) there are two possible strategies. As just discussed, one is to use weaker partial orderings than first-degree copula dominance, where second-degree dominance emerges as an appropriate alternative. The other alternative is to apply summary measures of association, which provide complete orderings and offer a method for quantifying the degree of association between random variables. As is noted by Nelsen (2006, page 170), we may integrate $H(x,y)$ over (x,y), which leads to the Pearson correlation coefficient (see Schweizer and Wolff, 1981, page 879).[12] This measure has however already been rejected on the grounds that it is not independent of changes in the marginal distributions and only is valid for distributions with finite second-order moments. By contrast, these attractive features are captured by Spearman's association measure[13], which is defined by

$$\rho_C = 3\left\{\Pr\left[(X_1 - X_2)(Y_1 - Y_3) > 0\right] - \Pr\left[(X_1 - X_2)(Y_1 - Y_3) < 0\right]\right\} = \tag{6}$$

$$12\iint C(u,v)\,du\,dv - 3,$$

where (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) are independent random vectors with distribution H. The standard definition of the Spearman coefficient is given by the former expression of ρ_C, while the latter expression provides the copula version. From expressions (5) and (6) we get that $\rho_{\tilde{C}} = \rho_C$, which means that the Spearman coefficient either can be defined as a functional of the copula or the survival copula. Note that $\rho_C \in [-1,1]$ and $\rho_C = 0$ when the bivariate variables are independent. Expression (6) shows that Spearmans's ρ_C has a similarly relationship to the copula as the Gini coefficient has to the Lorenz curve; ρ_C is a linear transformation of the volume between the copula and the unit square. As demonstrated by Proposition 3.1 Spearman's association measure is consistent with second-degree copula dominance.[14]

Proposition 3.1. *Let C_1 and C_2 be copulas. Then*

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12 Use of the incomplete covariance is discussed in Atkinson and Bourguignon (1982, Section 4).

13 See Decancq (2014) for a generalization of Spearman's bivariate association measure to multivariate distributions.

14 Note that Kendal's measure of association is not consistent with second-degree copula dominance.

(i) C_1 *second-degree upward dominates* C_2

implies

(ii) $\rho_{C_1} > \rho_{C_2}$.

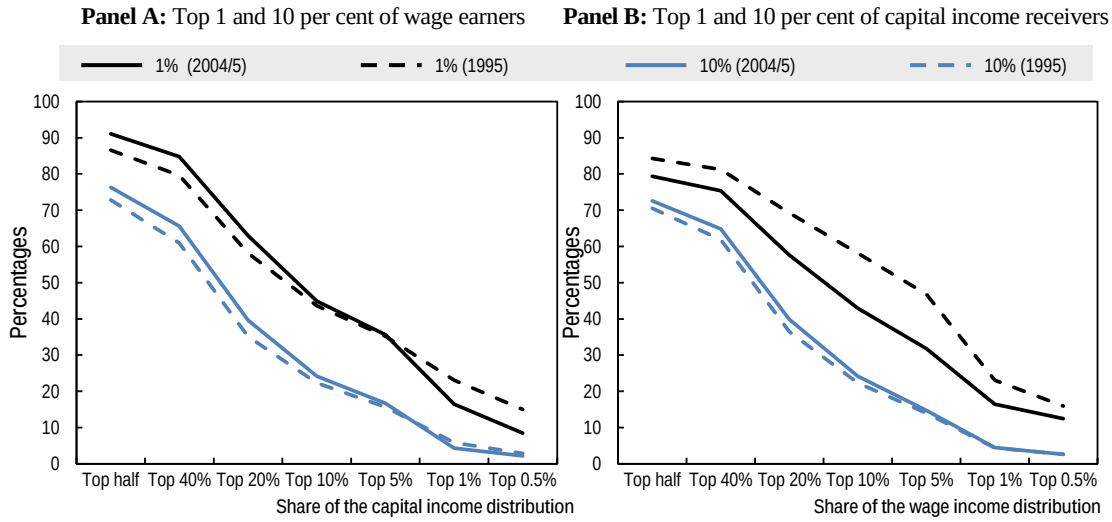
By applying Definition 3.2, Proposition 3.1 follows directly from the latter expression of ρ_C defined by (6).

4. Empirical results

4.1. The association of top labour and capital incomes in 2004/5

We have already shown the association matrix for Norway in the year 2004/5 in Table 1 and Figure 1. The degree of association appears strong. If we sum the entries for the top two groups, to look at the top 1 per cent, then nearly half (43 per cent) of the top 1 per cent of capital income recipients find themselves in the top tenth of earners – see also the graphical representation in Figure 2. 16 per cent of these taxpayers are in the top 1 per cent for both. Also, as noted earlier, the matrix is slightly asymmetric: The share of top 1 per cent wage income recipients who find themselves in the top parts of capital income distribution is always a little larger than vice versa – the sold black line in Panel A of Figure 2 lies slightly above that in Panel B. Put the other way round, only 9 per cent of those in the top 1 per cent of wage earners find themselves in the bottom half of the capital income distribution. Being well paid seems to almost guarantee being well placed in terms of capital income. In contrast, 21 per cent of those in the top 1 per cent of capital income are located in the bottom half of the wage income distribution. One in five is close to being old-style capitalists. There is a positive association, but Norway in 2004/5 is far from complete alignment of wage and capital incomes.

Figure 2. Changes in the conditional survival function of top earned income for various groups of top capital income receivers (and *vice versa*), 2004/5 vs. 1995



Note: Panel A: Of those in the top 1 per cent of wage income, 63 per cent were in the top 20 per cent of earned income in 2004/5 (compared to 58 per cent in 1995). Of those in the top 10 per cent, the respective shares were 40 per cent and 35 per cent. Panel B: Of those in the top 1 per cent of capital income, 58 per cent were in the top 20 per cent of earned income in 2004/5 (compared to 69 per cent in 1995). Of those in the top 10 per cent, the respective shares were 40 per cent and 37 per cent.

4.2. Changing association patterns: 1995 to 2004/5

How has the extent of association changed over time? There is no unambiguous trend. A comparison of the survival copulas for top wage earners and capital income receivers shows, overall, a slightly rising association in the top halves of the two distributions. The opposite pattern can however be observed in the very top, notably among the top 1 per cent of capital income receivers.

Indeed, Figure 2, which also gives the relationship for 1995, shows that for the top 1 per cent of capital income earners (Panel B), the dashed black line for 1995 lies always well above the solid black one for 2004/5. In other words, the top-half segment of the 1995 copula for the top 1 per cent capital income receivers first-degree dominates the corresponding segment of the 2004/5 copula. The degree of association in this group hence declined substantially between 1995 and 2004/5: for instance, the proportion of the top 1 per cent of capital income receivers who were in the top 10 per cent of wage earners fell from 58 per cent to 43 per cent. For the top 10 per cent capital income receivers, the top-half segment of the 2004/5 copula first-degree dominates the corresponding 1995 copula segment – the solid blue line lies above the dashed blue line – though by a much narrower margin.

No similar trend can be observed for the top wage earners: the dashed and the solid black lines in Panel A cross, as the line for 2004/5 is steeper than that for 1995, and blue lines for the top 10 per cent follow the same pattern. The association between top wage and capital incomes hence rose except for the very top of capital income receivers. The criterion of first-degree partial

copula dominance does neither apply for the top 1 per cent nor for the top 10 per cent wage income earners. We do however find that the top-half segment of the 2004/5 copula for both the top 1 per cent and the top 10 per cent wage earners second-degree dominates the corresponding segments for the 1995 copula.

This pattern of a strengthening association between labour and capital incomes can also be observed for the remainder of the upper half of the two distributions. Table 3 shows the difference between the survival matrix in 2004/5 and that in 1995, where a positive entry implies that the (inverse) cumulative distribution is greater in 2004/5 than in 1995. We observe that outside the top 1 per cent of capital income receivers, all entries except one (they shaded cell) are positive. The shaded entries however imply that we do not have first-degree dominance, but we find that 2004/5 copula second-degree dominates the 1995 copula[15]. This is also reflected by the Spearman coefficient, which increases by 40 per cent from 0.169 in 1995 to 0.236 in 2004/5. A decomposition of the Spearman coefficient into four segments, where the median wage income and median capital income define the dividing lines, is displayed in Table 4. Note that each segment consists of 25 per cent of the population when wage and capital income are independent variables. Table 4 shows that the upper half – upper half segment provides the largest contribution to the positive overall Spearman coefficient, whereas the lower half – upper half and the upper half – lower half segments display negative association between wage and capital income. The positive contribution from both the lower half – lower half segment and upper half – upper half segment has increased significantly from 1995 to 2004/5.

4.3. Does household size matter? Results for two-adult households

We have so far considered the joint distribution of capital and labour incomes for the entire household population without accounting for household size and type. Household composition may be an important determinant of this joint distribution, however: larger households tend to have higher incomes from both labour and capital, and couples make different labour supply and investment decisions than singles.

In spite of this, the findings just presented continue to hold if we focus on a population that is homogenous in terms of household composition. When breaking down our analysis by household type, we observe that two-adult households find themselves more often in the upper half of

[15] Atkinson and Lakner (2017) show that the trend towards greater association is significantly stronger in the United States and more pronounced over the longer run.

the wage and capital income distributions than persons living in other types of households (*not shown*).[16] This is what we would expect given that resources in a two-adult household are pooled. Our main results remain largely unaffected, however: The survival function for capital and labour income in the income distribution of two-adult households looks very similar to that for all households in the overall income distribution. For instance, 26.1 per cent of taxpayers were in the top 50 per cent of both wages and capital incomes in the income distribution of two-adult households in 2004/5 (Table 5) compared to 27.6 per cent of taxpayers in the overall income distribution (Table 2). The shares in the top 10 per cent of wage and capital income are 2.2 per cent in the income distribution of two-adult households and 2.4 per cent in the overall distribution.

Also the time trend in the association of top capital and wage income is similar for the income distribution of two-adult households as for the overall income distribution. The association between top wage and capital incomes has grown over the observation period, as illustrated by the positive differences between the 2004/5 and the 1995 survival functions for two-adult households (Table 6). As in the overall population, it however weakened in the top 1 to 1.5 per cent of the capital income receivers, as indicated by the shaded cells. Differences in household size and changes in the household composition during the observation period hence do not heavily affect our results. As suggested by Table 6 we find that the 2004/5 copula second-degree dominates the 1995 copula, which is also confirmed by a 15 per cent increase in the Spearman coefficient; from 0.136 in 1995 to 0.156 in 2004/5. A similar decomposition as for all households is provided by Table 7. The results of Table 7 are roughly similar to those displayed in Table 4, where the lower half – lower half and upper half – upper half segments provide positive contributions and the lower half – upper half and upper half – lower half segments provide negative contributions.

4.4. The decomposition by gender

An obvious question to consider is whether there is a significant gender differential in the joint distribution of top labour and capital incomes. Men and women may differ in the frequency with which they find themselves in the top parts of the labour and capital income distributions because of earnings differentials and differences in investment decisions[17]. Also the strength of the association between the two income sources may vary between men and women. To be able to provide evidence on this issue, we abandon our earlier approach of aggregating incomes at the

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[16] 29 per cent of two-person households were in the upper halves of the wage and capital income distributions for the overall population in 2004/5 compared to only 20 per cent of other households.

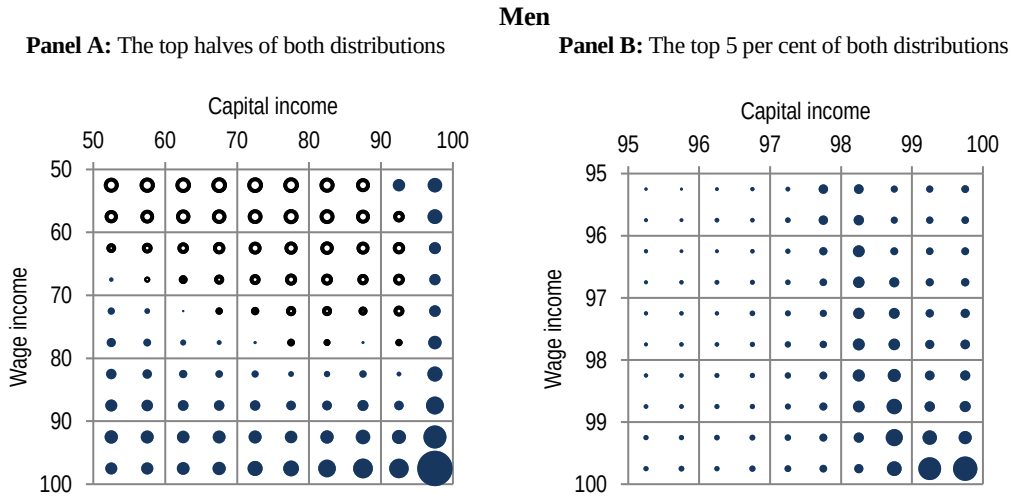
[17] See Atkinson et al (2018) who demonstrate that women are strongly under-represented in the distribution of total income.

household level in favour of studying the distribution of earnings and capital incomes of individual persons. Any gender differences in the distribution of earnings and capital income that we find in our empirical analysis hence result from gender differences across single-person households as well as within-household gender differences in the distribution of earnings and capital incomes for multi-person households.

Figure 3 again shows the simultaneous density of capital and wage income for the top halves and the top 5 per cent of the distributions. This time, we have however broken down our population by gender depicting the position of men and women in the overall distribution separately. Each circle's area is again proportional to the frequency of observations in that cell, with shaded (blank) circles indicating the cells in which persons are overrepresented (underrepresented).[18]

We find the joint distribution of top labour and capital incomes to differ substantially between men and women. Men are systematically overrepresented among the top 20 per cent of wage earners (across top capital income receivers) and the top 5 per cent of capital income receivers (across top earners), see Panel A. By contrast, women tend to be underrepresented in the top halves of the wage and capital income distribution, except between the 50th and 70th percentiles of both labour and capital jointly (the top-left cells in Panel C) and the top percentile of the two distributions (the bottom-right cell in Panel D).

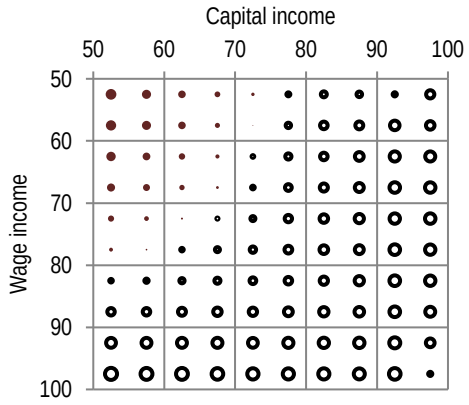
Figure 3. Normalised simultaneous density of capital and wage income, 2004/5, by gender



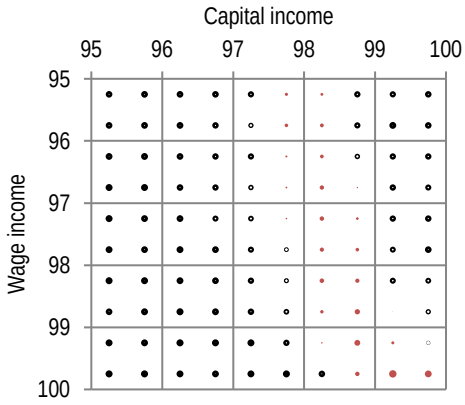
[18] The circles' areas in Panels A and B have been calculated as (percentage of observations in the given cell) - (1 / the number of cells)*(population share of males), and accordingly for women in Panels C and D.

Women

Panel C: The top halves of both distributions



Panel D: The top 5 per cent of both distributions



Note: The area of each circle is proportional to the frequency of observations in the specific cell. Shaded circles indicate cells in which persons are overrepresented; blank circles give those in which persons are underrepresented.

The resulting survival functions, which we obtain again by aggregating from the top the simultaneous densities of wage and capital income separately for men and women, imply that women are underrepresented relative to men in the entire top half of the overall wage and capital income distribution. The share of taxpayers who are in the top half of both distributions is more than 13 percentage points lower for women than of men (Table 8), at 19.3 per cent vs. 32.8 per cent, respectively. The size of this gap has significantly declined since 1995, however, by 2.6 percentage points (*not shown*). The underrepresentation of women relative to men tends to be systematically stronger for wage than for capital incomes: the below-diagonal values in Table 8 are virtually always larger than their above-diagonal counterparts. In other words, the gender earnings gap at given levels of top capital incomes is larger than vice versa.

Also the association between top labour and capital incomes is stronger for men than it is for women (Table 9). Among male taxpayers who were in the top 10 per cent of wage earners and in the top half of capital income receivers of the overall distribution in 2004/5, over one-third were also in the top 10 per cent of capital income receivers (4.2 / 12.1 = 35 per cent); for women, the corresponding share was only about one-quarter (0.6 / 2.4 = 26 per cent). Similarly, of the top 10 per cent of capital income earners who were also in the top half of wage earners in the overall distribution, 41 per cent of male taxpayers were in the top 10 per cent of wage earners compared to only 22 per cent of female taxpayers. Note that we can no longer produce a figure with meaningful conditional survival functions, as done in Figure 2 for the overall population. The reason is that the shares of men (and women) in the top 1 per cent of either the overall wage income distri-

bution or the overall capital income distributions do not longer sum up to 1 per cent. They are moreover different for the top wage and capital income distributions.[19]

5. Conclusion

Public debates about rising top income shares often focus on the growing dispersion of wage incomes. Another potential driver is the growing dispersion of capital incomes. In this paper, we focused on a third mechanism, namely the changing *association* between wage and capital incomes. We have proposed extensions of the copula framework as tools for exploring the association between wage and capital income among top income earners, and illustrated its usefulness in examining the changing composition of top incomes in Norway between 1995 and 2004/5.

Top wage and capital incomes in Norway are clearly positively associated. Top wage income earners – such as executives or financial-sector employees – occupy virtually always also a high position in the capital income distribution: for example, of those in the top 1 per cent of wage income distribution, 91% were among the top half of capital income receivers, and over one-third (36 per cent) even among the top 5 per cent. The association between top wage and capital income is slightly asymmetric, however: the share of the 1 per cent of capital income earners who were also in the top half of the wage income distribution was somewhat lower, at 79 per cent, and asymmetry that has become more pronounced over time. This is also consistent with the importance of capital incomes at the top of the income distribution in a number of Scandinavian countries, including Norway, as suggested in earlier research.

The association between wage and capital incomes in the top half of the respective marginal distributions has generally grown stronger, hence contributing to increasing top income shares. An exception to this trend have been the top 1 per cent of capital income earners, who were much less likely to find themselves among top wage earners in 2004/5 than in 1995. One way of interpreting these results would be that of increasing class differentiation at the very top of the capital income distribution, with a growing class of "old-school capitalists" with little to no income from labour. Our gender decomposition of individual-level – which of course disregards any within-household resource sharing – demonstrates that the association of wage and capital

[19] Specifically, 1.7 per cent of men are in the top 1 per cent of capital incomes while 1.9 per cent are in the top 1 per cent of wage incomes. With 0.3 per cent of men being in the top 1 per cent of both distributions, the conditional probability of being in the top 1 per cent of labour income (capital income) for those in the top 1 per cent of capital income (labour income) is 0.3 / 1.7 = 20.4 per cent (0.3 / 1.9 = 18.6 per cent).

incomes at the top is particularly striking for men, while women are largely under-represented in the top halves of the two marginal distributions.

While a detailed policy analysis lies beyond the scope of this paper, the growing association between top wage and capital incomes is consistent with the development of top marginal tax rates. Today's capital income is the return to a stock of wealth accumulated over earlier periods. Top marginal tax rates in Norway were gradually reduced in the late 1980s and early 1990s, like elsewhere, thus allowing for increased savings and more wealth accumulation for high earners, and raising future capital incomes (Aaberge, Atkinson and Modalsli, 2013; Alvaredo et al., 2013).

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Table 1. Simultaneous density of capital and wage income 2004/5 (frequencies in per cent)

	Capital income								
Wage income	**Up to P50**	**P50-P59**	**P60-P79**	**P80-P89**	**P90-P94**	**P95-P99**	**P99-P99.5**	**P99.5-P100**	**TOTAL**
Up to P50	27.63	4.62	10.16	4.85	1.90	0.64	0.10	0.11	50
P50-P59	6.16	0.98	1.44	0.64	0.47	0.27	0.02	0.02	10
P60-P79	9.89	2.15	3.60	1.87	1.24	1.08	0.09	0.08	20
P80-P89	3.95	1.17	2.20	1.11	0.64	0.77	0.08	0.07	10
P90-P94	1.49	0.59	1.29	0.69	0.34	0.49	0.06	0.05	5
P95-P99	0.78	0.42	1.10	0.67	0.32	0.56	0.08	0.08	4
P99-P99.5	0.05	0.04	0.13	0.09	0.05	0.10	0.02	0.02	0.5
P99.5-P100	0.04	0.02	0.09	0.09	0.04	0.09	0.06	0.07	0.5
TOTAL	50	10	20	10	5	4	0.5	0.5	100

Note: The numbers may not add up exactly on account of rounding.
Reading: 27.63 per cent of households were in the bottom half of the wage and capital income distributions. In other words, of those households who were in the bottom half of wages, 55.26 per cent were in the bottom half of capital incomes.

Table 2. Survival function for capital and wage income 2004/5 (frequencies in per cent)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	27.63	22.26	12.41	7.26	4.15	0.79	0.39
Top 40 per cent	23.79	19.40	11.00	6.48	3.85	0.75	0.37
Top 20 per cent	13.68	11.44	6.64	3.99	2.59	0.58	0.29
Top 10 per cent	7.63	6.56	3.96	2.42	1.67	0.43	0.22
Top 5 per cent	4.13	3.64	2.33	1.47	1.07	0.32	0.16
Top 1 per cent	0.91	0.85	0.63	0.45	0.36	0.16	0.08
Top 0.5 per cent	0.46	0.44	0.35	0.26	0.22	0.12	0.07

Reading: 0.26 per cent of households were in the top 0.5 per cent of wages and the top 10 per cent of capital incomes. That is, 52 (=100*0.0026/0.005) per cent of those households who were in the top 0.5 per cent of the wage distribution were also in the top 10 per cent of the capital income distribution.

Table 3. Difference in survival function for capital and wage income 2004/5 compared with 1995 (in percentage points)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	1.19	1.07	0.77	0.21	-0.09	-0.05	-0.02
Top 40 per cent	1.06	1.10	0.87	0.30	0.04	-0.06	-0.03
Top 20 per cent	0.59	0.78	0.74	0.33	0.13	-0.12	-0.05
Top 10 per cent	0.35	0.47	0.44	0.19	0.10	-0.15	-0.07
Top 5 per cent	0.20	0.25	0.23	0.08	0.04	-0.15	-0.08
Top 1 per cent	0.05	0.05	0.05	0.01	0.00	-0.07	-0.07
Top 0.5 per cent	0.02	0.03	0.02	0.00	0.00	-0.03	-0.05

Note: The shading shows the cells whose size declined between 1995 and 2004/5.
Reading: In 2004/5, 0.03 percentage points *fewer* households were in the top 1 per cent of capital incomes and the top 0.5 per cent of wages, compared to 1995.

Table 4. Decomposition of the Spearman coefficient of wage-capital for households in 1995 and 2004/5

			Capital	
			Lower half	**Upper half**
Wage	**Lower half**	1995	.206	-.164
		2004/5	.230	-.154
	Upper half	1995	-.117	.243
		2004/5	-.103	.260

Table 5. Survival function for capital and wage income for *two-person households*, 2004/5 (frequencies in per cent)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	26.13	20.56	10.69	6.05	3.95	0.74	0.37
Top 40 per cent	21.99	17.45	9.18	5.16	3.49	0.70	0.35
Top 20 per cent	12.73	10.48	5.91	3.43	2.37	0.56	0.28
Top 10 per cent	7.16	6.08	3.68	2.25	1.61	0.43	0.22
Top 5 per cent	3.91	3.42	2.20	1.40	1.03	0.32	0.17
Top 1 per cent	0.88	0.81	0.60	0.43	0.34	0.17	0.09
Top 0.5 per cent	0.45	0.42	0.33	0.25	0.21	0.13	0.07

Reading: 0.25 per cent of two-person households were in the top 0.5 per cent of wages and the top 10 per cent of capital incomes. That is, 50 (=100*0.0025/0.005) per cent of those households who were in the top 0.5 per cent of the wage distribution were also in the top 10 per cent of the capital income distribution.

Table 6. Difference in survival function for capital and wage income for *two-person households*, 2004/5 compared with 1995 (in percentage points)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	0.48	0.55	0.59	0.11	-0.21	-0.08	-0.04
Top 40 per cent	0.66	0.83	0.99	0.52	0.23	-0.09	-0.04
Top 20 per cent	0.61	0.73	0.79	0.43	0.26	-0.14	-0.07
Top 10 per cent	0.41	0.45	0.44	0.20	0.14	-0.19	-0.09
Top 5 per cent	0.24	0.25	0.22	0.08	0.05	-0.20	-0.09
Top 1 per cent	0.05	0.04	0.03	-0.01	-0.02	-0.09	-0.08
Top 0.5 per cent	0.03	0.02	0.02	0.00	-0.01	-0.04	-0.06

Note: The shading shows the cells whose size declined between 1995 and 2004/5.
Reading: In 2004/5, 0.04 percentage points *fewer* two-person households were in the top 1 per cent of capital incomes and the top 0.5 per cent of wages, compared to 1995.

Table 7. Decomposition of the Spearman coefficient of wage-capital for two-person households in 1995 and 2004/5

			Capital	
			Lower half	**Upper half**
	Lower half	1995	.227	-.172
		2004/5	.224	-.171
Wage	**Upper half**	1995	-.135	.215
		2004/5	-.130	.232

Table 8. Difference in survival function for capital and wage income Norway men compared to women, 2004/5 (in percentage points)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	13.42	12.48	9.93	7.25	5.26	1.14	0.58
Top 40 per cent	15.20	13.26	9.29	6.29	4.55	1.10	0.56
Top 20 per cent	13.91	11.76	7.47	4.75	3.30	0.96	0.49
Top 10 per cent	9.66	8.28	5.33	3.40	2.36	0.77	0.40
Top 5 per cent	5.80	5.09	3.42	2.24	1.60	0.59	0.31
Top 1 per cent	1.46	1.35	1.03	0.74	0.57	0.30	0.16
Top 0.5 per cent	0.77	0.72	0.58	0.43	0.35	0.22	0.12

Note: Results are for the persons aged 25 years and over.
Reading: In 2004/5, 0.57 percentage points *fewer* female than male taxpayers were in the top 5 per cent of capital incomes and the top 1 per cent of wages.

Table 9. Survival function of top earned income for various groups of top capital income receivers (and vice versa): men (and women in brackets), 2004/5 (frequencies in per cent)

	Capital income			
Wage income	**Top 10 per cent**	**Top 20 per cent**	**Top 40 per cent**	**Top half**
Top 10 per cent	4.2 (0.6)	6.6 (1.1)	10.4 (2.0)	12.1 (2.4)
Top 20 per cent	6.1 (1.1)			
Top 40 per cent	8.6 (2.1)			
Top half	10.2 (2.8)			

Reading: 10.2 per cent of men and 2.8 per cent of women were in the top half of the overall wage income distribution and the top 10 per cent of overall capital income.

Appendix A.1 – The treatment of self-employment income

In our standard analysis, we allocate self-employment income to labour and capital income in shares two-thirds and one-third. While this choice is largely uncontroversial when looking at aggregate income shares, it seems admittedly more *ad hoc* when studying the distribution of households' or individuals' incomes. In this appendix, we therefore provide some further details on the joint distribution of capital and self-employment incomes, and illustrate the impact of our results of simply ignoring income from self-employment.

Top capital and self-employment income shares are somewhat less strongly associated than those for capital and wage income. This can be seen in Table A.1.1, where we compare the survival probabilities for the top income shares of capital and *self-employment* and capital and *wage* income. More specifically, the table gives the ratio of the joint survival probabilities – in these calculations, we no longer attribute income from self-employment to capital and wage income. Most of the values in Table A.1.1 are lower than unity – this means that the cell sizes are smaller for the top capital and *self-employment* income shares than for the top capital and *wage* income shares. Especially the top 1 per cent of capital income earners are more likely to be also among the top wage earners than among the top self-employment income receivers.

Since having high income from self-employment is positively associated with having high capital income, our decision to attribute self-employment income to capital and wage income rather than to ignore it in our analysis led to a boost in the association of top capital and wage incomes. Table A.1.2 illustrates that the share of households in the top 1 per cent of both capital and wage income that we have calculated is more than twice as high as it would be had we simply ignored income from self-employment. In other words, the association in top wage and capital income shares that we found was reinforced by the positive relationship between being in top part of these income distributions and having high income from self-employment.

Our time trend of a strengthening association of top capital and wage incomes is however not heavily affected by our decision to allocate income from self-employment to capital and wages. The observed difference in survival functions for top capital and wage incomes between 1995 and 2004/5 looks very similar if we ignore income from self-employment (Table A.1.3). As in our standard analysis (Table 3), we find a strengthening in the association of top capital and wage incomes over time except for the top 1 per cent of capital income earners – in fact, the strengthening becomes more visible once self-employment income is ignored.

Table A.1.1 Ratio of the survival probabilities for capital and self-employment over capital and wage income 2004/5

	Capital income						
Self-employment / wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	1.00	1.05	1.12	1.05	0.95	0.86	0.85
Top 40 per cent	0.96	1.00	1.06	0.99	0.90	0.79	0.79
Top 20 per cent	0.87	0.88	0.89	0.84	0.77	0.64	0.62
Top 10 per cent	0.84	0.84	0.86	0.85	0.81	0.65	0.61
Top 5 per cent	0.80	0.79	0.78	0.77	0.75	0.59	0.56
Top 1 per cent	0.90	0.89	0.89	0.90	0.93	0.84	0.77
Top 0.5 per cent	0.97	0.97	1.00	1.04	1.10	1.06	0.96

Reading: Among households who were in the top 1 per cent of capital income receivers, the share who were also in the top 10 per cent of self-employment income receivers is 35 per cent lower than that of those who were also in the top 10 per cent of wage earners.

Table A.1.2 Ratio of the survival probabilities for capital and wage income, with and without including income from self-employment, 2004/5

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	1.01	1.03	1.13	1.21	1.37	1.13	1.11
Top 40 per cent	1.00	1.02	1.08	1.11	1.27	1.15	1.14
Top 20 per cent	1.01	1.02	1.06	1.05	1.18	1.21	1.17
Top 10 per cent	1.02	1.03	1.07	1.07	1.22	1.34	1.26
Top 5 per cent	1.02	1.03	1.07	1.08	1.25	1.51	1.39
Top 1 per cent	1.03	1.05	1.11	1.16	1.39	2.40	1.96
Top 0.5 per cent	1.03	1.06	1.12	1.20	1.46	2.92	2.37

Reading: The share of households who were both in the top 1 per cent of capital income receivers and wage earners is 2.4 times higher when income from self-employment is accounted for in the analysis then when it is not.

Table A.1.3 Survival function for capital and wage income without including income from self-employment, 2004/5 minus 1995 (frequencies in per cent)

	Capital income						
Wage income	**Top half**	**Top 40 per cent**	**Top 20 per cent**	**Top 10 per cent**	**Top 5 per cent**	**Top 1 per cent**	**Top 0.5 per cent**
Top half	1.67	1.42	1.23	1.03	0.67	0.05	0.00
Top 40 per cent	1.52	1.39	1.28	1.02	0.65	0.05	0.00
Top 20 per cent	0.96	1.00	1.05	0.82	0.48	0.02	-0.01
Top 10 per cent	0.59	0.62	0.66	0.49	0.26	-0.02	-0.03
Top 5 per cent	0.34	0.36	0.39	0.28	0.13	-0.03	-0.03
Top 1 per cent	0.07	0.08	0.08	0.06	0.01	-0.03	-0.02
Top 0.5 per cent	0.03	0.04	0.04	0.02	0.00	-0.02	-0.02

Note: The shading shows the cells whose size declined between 1995 and 2004/5.
Reading: In 2004/5, 0.02 percentage points *fewer* households were in the top 1 per cent of capital incomes and the top 0.5 per cent of wages, compared to 1995.

Appendix A.2 – A decomposition by age

Another way of decomposing the joint distribution of labour and capital incomes is to look at the contributions of the different age groups. Figure A.2.1 shows the survival copulas for the top 10 per cent of wage earners and capital income receivers, "sliced" for the top half of the capital and wage income distribution, respectively. Taxpayers in their late 40s to 50s are most strongly represented in the top 10 per cent capital and wage income earners; persons above 70 are essentially not represented, since they typically do not make it into the upper half of the wage income distribution.

Figure A.2.1. Survival functions for capital and wage income, by age, 2004/5

Frequencies in per cent