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ABSTRACT

Monetary Policy and Inequality under Labor Market Frictions and Capital-Skill Complementarity*

In order to improve our understanding of the channels through which monetary policy has distributional consequences, we build a New Keynesian model with incomplete asset markets, asymmetric search and matching (SAM) frictions across skilled and unskilled workers and, foremost, capital-skill complementarity (CSC) in the production function. Our main finding is that an unexpected monetary easing increases labor income inequality between high and low-skilled workers, and that the interaction between CSC and SAM asymmetry is crucial in delivering this result. The increase in labor demand driven by such a monetary shock leads to larger wage increases for high-skilled workers than for low-skilled workers, due to the smaller matching frictions of the former (SAM-asymmetry channel). Moreover, the increase in capital demand amplifies this wage divergence due to skilled workers being more complementary to capital than substitutable unskilled workers are (CSC channel). Strict inflation targeting is often the most successful rule in stabilizing measures of earnings inequality even in the presence of shocks which introduce a trade-off between stabilizing inflation and aggregate demand.

JEL Classification: E24, E25, E52, J64

Keywords: monetary policy, search and matching, capital-skill complementarity, inequality

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1 Introduction

During the last two decades growing inequality has become one of the most discussed topics in the public debate, mainly pointing to long-term trends driven by technological change and globalization. However, following the financial crisis and the extreme measures central banks took to fight it, questions have also arisen about how monetary policy might affect inequality over the business cycle. There are contrasting views on this issue. On the one hand, there are concerns that the highly accommodative monetary policy stance in advanced economies, as with unconventional quantitative easing, favors richer households disproportionately, thereby contributing to a more unequal income and wealth distribution. On the other hand, there are opinions supporting the opposite view, namely, that expansionary monetary policy reduces inequality because borrowers become better off than savers.

Of course, central banks consider the economy *as a whole* when setting monetary policy, their objectives being price stability and the stabilization of aggregate real economic activity over the business cycle. As Bernanke (2015) points out, distributional issues should not be the concern of unelected monetary policymakers but rather be addressed by other policy tools, such as fiscal policy. Monetary policy is a blunt tool which is ill-equipped to target various measures of inequality even if it does have an impact on them. Given that inequality has been growing over a much wider horizon than the one over which monetary policy typically operates, it is thought that its influence on inequality should be transitory, canceling out over the course of the business cycle. In addition, Bivens (2015) and Bell et al. (2012) have argued that any distributional consequence of monetary policy actions should be assessed in light of a counterfactual scenario in which central banks abstain from stabilizing the economy. For instance, even if some population groups have benefited more than others from the exceptional monetary easing following the financial crisis, it is generally acknowledged that society as a whole is possibly better off as a result of such active policies. A corollary to this way of thinking is therefore that the most monetary policy can contribute to social welfare is by promoting aggregate economic stability which is also likely to be beneficial from an inequality perspective.

Notwithstanding the above, it is increasingly acknowledged that the short-run effects of monetary policy on inequality could be relevant for its optimal design. Taking these effects into consideration might influence the welfare implications of various systematic monetary strategies; likewise, inequality might also interact with the different channels of the monetary transmission mechanism. As a result, there has been a recent body of literature dealing with how these issues are related. Our paper aims to contribute to this literature by introducing some new features in a New Keynesian model with incomplete markets; in particular, we focus on the role played by capital-skill complementarity (CSC, hereafter) and labor market heterogeneity in the form of asymmetric search-and-matching (SAM) frictions.

The channels through which monetary policy affects inequality are complex. As highlighted in Bell et al. (2012), monetary policy directly affects interest rates received by savers on deposits and paid by borrowers on loans (*income effect*), as well as assets such as bonds, equities and real estate (*wealth effect*). A change in real interest rates influences the intertemporal consumption-saving choice of households (*substitution effect*), which is the main channel of monetary transmission in mainstream representative-agent New Keynesian models, through changes in aggregate demand followed by changes in employment, wages and profits. The main channels through which monetary policy can affect inequality are the following (see also, Amaral (2017) and Coibion et al. (2012)): (a) The **savings redistribution channel** since the income effect of a monetary expansion affects differently borrowers and savers; (b) The **interest-rate exposure channel** concerns the maturity structure of assets and liabilities as their duration determines how much their value increases in response to a fall in the discount rate; (c) The **inflation tax channel** concerns the effects of expected inflation on the cash holdings of different households (see Erosa and Ventura (2002)); (d) The **income composition channel** relates to the fact that households differ in the share of their income from business and financial dividends, and insofar as a monetary expansion leads to a steeper rise in profits than in wages, it can contribute to a rise in income inequality; (e) The **"intertemporal substitution heterogeneity channel"** resulting form the fact that households differ either with respect to their preferences (patient vs impatient), or as regards their access to financial markets (Ricardian vs hand-to-mouth consumers, or credit constraints); and (f) The **earnings heterogeneity channel** which involves several factors here, like unemployment risk, asymmetric wage rigidities and labor market institutions, or different complementarity with capital across the skill distribution, the latter being one of the main mechanisms we focus on here. For instance, Heathcote et al. (2010) show that labor earnings at the bottom of the distribution are the ones most exposed to business cycle fluctuations, leading to larger fluctuations in working hours and employment than at the top of the distribution (i.e. unemployment falls disproportionately on the poor). Hence, a monetary expansion which decreases unemployment (more so than it increases wages) will reduce income inequality.

The balance of all these forces is ambiguous and thus it can only be determined by quantitative methods. Yet, while "empirically it is hard to control for all the sources of endogeneity, theoretically it is cumbersome to include all the relevant heterogeneity" (Amaral, 2017, p.5). Therefore, our main goal in the present paper is to focus on the earnings heterogeneity channel, leaving aside other sources of heterogeneity (like the wealth distribution). More specifically, we wish to investigate how the interaction of CSC with monetary policy affects inequality between high and low-skilled workers. Skill-biased technological change has been traditionally considered as one of the main driving factors behind the growing trend of income inequality, as reflected by an increasing skill wage premium. However, to the best of our knowledge, no theoretical model has yet analyzed the business-cycle effects of this feature of the production process with the aim of studying its implications for monetary policy.

If high-skill labor is a complementary input to capital while low-skill labor can be substituted by a combination of high-skill labor and capital, then cyclical changes induced by monetary policy can have markedly different effects on the relative income shares of these labor inputs than when CSC is ignored. This is especially the case when these two types of workers are also subject to different labor-market frictions which damage low-skilled workers' prospects of finding or keeping jobs. In particular, to the extent that a monetary expansion leads to a fall in the relative cost of capital through lower nominal interest rates, which translate into lower real interest rates given price stickiness, it might stimulate demand for complementary skilled labor relatively more than for substitutable unskilled labor. This could show up in a combination of larger wage (*skill*) premium and larger employment gains for high-skilled workers, and could even push the income shares of skilled and unskilled workers in opposite directions (Koczan et al., 2017). Given that high-skilled workers are already richer, an expansionary monetary policy shock is then *increasing* inequality through this specific channel, which contrasts with how most researchers interpret the effects of the earnings heterogeneity channel.

To illustrate our previous reasoning, we build a New Keynesian model which combines CSC and asymmetric SAM frictions as its two key ingredients. We include these frictions to account for other potential sources of earnings heterogeneity which could interact with CSC. As regards their asymmetric nature, one rationalization could be that, while low-skilled workers are only able to undertake simple tasks, high-skilled workers can undertake both complex and simple tasks, so that they experience lower job-separation rates. In addition, to the extent that high-skilled workers may look for better jobs when they land in simple jobs, these jobs become more unstable and more costly for firms to open them. Furthermore, to the extent that high-skilled workers have more stable jobs than less-skilled workers, they are likely to have larger networks helping them to find jobs when unemployed, therefore leading to more efficient search intensity. Finally, it is also plausible that high-skilled workers have larger bargaining power than less-skilled ones since they are more valuable for the firm. Notice that we do not explicitly model these mechanisms here (see Dolado et al., 2009, for a detailed discussion) but rather use them as informal motivation devices for heterogenous SAM frictions by worker-type. With regard to CSC, it is assumed that the elasticity of substitution between skilled labor and capital is below unity (making them complements), while it is above unity between unskilled labor and capital or skilled labor (making them substitutes). Under incomplete markets the idiosyncratic effect of unemployment or earnings risk on the consumption of these heterogenous workers cannot be fully insured. First, we look at the effect of *unexpected* expansionary monetary policy shocks on the relative income shares of high and less-skilled labor, which we use as a measure of inequality. We then compare this setting against the benchmark cases without CSC and with symmetric SAM frictions in order to identify the role of different sources of heterogeneity. Finally, we also look at the effect of various other expansionary shocks under different rules for *systematic* monetary stabilization to examine the implications of different monetary policy strategies for inequality.

Our main finding is that an expansionary monetary policy shock *increases* earnings inequality by inceasing the labor share of income for high-skilled workers decreasing it for lower-skilled workers. This is mainly driven by an increase in the wage premium for the high skilled (i.e., the skill premium), who also fare better in terms of employment rates. Relative to the benchmark cases without CSC or with symmetric SAM frictions, we find that introducing CSC on its own is not enough to generate these results: it is it's *interaction* with asymmetric SAM frictions which leads to a larger rise in inequality compared to a standard Cobb-Douglas (CD) production function. It is important to stress at this stage that these results are not specific to monetary policy shocks but are qualitatively similar for any positive aggregate demand shock leading to higher investment. Nonetheless, since monetary shocks have a stronger impact on the relative price of capital than, eg., government expenditure shocks, due to the investment crowding-out effects of the latter, we show that their effect on income inequality via capital accumulation is much larger.

Looking at the performance of alternative monetary policy rules in the face of various other shocks, we find that strict inflation targeting is more successful in stabilizing the economy than a more flexible Taylor rule. It does so by managing expectations in a way which improves potential trade-offs and limits variations in relative income shares of the two types of workers. However, increasing the Taylor-reaction parameter to unemployment/ output (i.e. making inflation targeting "less strict") has the same effect (although at the cost of higher inflation), suggesting that unemployment fluctuations are also important for inequality and it might be worth limiting them. In contrast, *absent* CSC and asymmetric SAM, the optimal monetary policy rule might put less weight on unemployment, even in the face of cost-push shocks, confirming the results of Coibion et al. (2012) or Gornemann et al. (2012)).

Finally, we analyze the welfare effects of monetary policy shocks for the different groups of individuals in our economy and in the aggregate. In the absence of CSC and asymmetric SAM, a monetary expansion is beneficial for both types of agents. However, with SAM and CSC a monetary expansion reduces aggregate welfare by increasing inequality. The high skilled gain 2.06 percent while the low skilled lose 0.17 percent of their respective steady-state consumption levels.

The rest of the paper is organized as follows. Section 2 revises the extant literature and stresses our contribution in relation to the available studies. We use Section 3 to motivate our further analysis; we do so by estimating the dynamic effects of monetary policy shocks on the wage premium and the relative employment of skilled and unskilled workers in structural VAR (SVAR) models. Section 4 lays out the theoretical model, while Section 5 explains our calibration strategy. Results are presented and discussed in Section 6. Finally, Section 6 concludes.

2 Related Literature

In carrying out the above exercises, we relate to several strands in the literature. First, our model belongs to the family of **New Keynesian models with SAM frictions** á la Mortensen-Pissarides in the labor market. We have endogenous participation choice as in Ravn (2006), Campolmi and Gnocchi (2016) and Christiano et al. (2016). Labor market heterogeneity is captured by asymmetric SAM frictions across various groups of workers as in Brückner and Pappa (2012), and Pappa et al. (2015). The implications of SAM frictions for optimal monetary policy are derived by Ravenna and Walsh (2011). Second, we rely on the literature which looks at the relationship between **capital-skill complementarity** and inequality. Krusell et al. (2000) were the first to model the effects of skill-biased technological change on the U.S. skill premium in the medium and long run by introducing CSC via a two-level CES production function, a formulation that we borrow for our analysis. Lindquist (2004) has shown that CSC is crucial in explaining the behavior of the skill premium and wage inequality at business cycle frequencies. Angelopoulos et al. (2017) use the same production function and SAM frictions to investigate the effects of policies supporting on-the-job-learning. However, none of these studies consider nominal rigidities which are crucial for monetary policy analysis. Third, this paper is also part of the recently growing literature on **monetary policy and inequality**. Most of these studies combine an incomplete market Aiyagari-type heterogenous agent framework with New Keynesian nominal rigidities, resulting in what is now referred to as HANK models (**????**). This allows for a rich analysis of distributional issues, as the model yields wealth and income distributions over a wide range of values. Gornemann et al. (2012) use this framework, augmented by SAM frictions, to make unemployment risk endogenous to monetary policy. They show that contractionary monetary shocks increase income inequality –via a rise in precautionary savings by poorer households which leads to a higher value of the assets held by the wealthy rich – and therefore have larger welfare costs than thought before. Gornemann et al. (2016) show that a systematic monetary policy rule which puts larger weight on stabilizing unemployment (shifting from "hawks" to "doves"), is relatively more beneficial for poorer than for rich households, as it provides partial insurance against unemployment risk. However, they do not account for CSC and their SAM frictions are symmetric across skills (even if idiosyncratic risk generates labor market heterogeneity ex post). The HANK framework is crucial to analyze the above mentioned savings redistribution and income composition channels through which monetary policy influences inequality, since asset and income distribution are needed to address these issues. However, since our focus here lies only in the earnings heterogeneity channel, it suffices to use a less rich setting of heterogeneity, already present in the form of CSC and asymmetric SAM frictions.

The main contribution of our paper is to *combine* these three strands of literature, which proves to be crucial to derive our results. Introducing CSC to the relationship between monetary policy and inequality, and analyzing its interaction with heterogeneity in the labor market is something that, to the best of our knowledge, is novel in this area of research.

On the empirical side, the evidence on the effects of monetary policy on inequality is also mixed. Coibion et al. (2012) find that contractionary monetary policy shocks (identified as in Romer and Romer (1998)) systematically increase inequality as rising unemployment falls disproportionately on low-income workers. Furceri et al. (2016) also argue that unexpected monetary tightening increases inequality, but the effect depends on the labor share of income and the degree of redistributive policies. In addition, they show that increases in interest rates that *systematically respond to* an economic upturn actually reduce inequality. O'Farrell et al. (2016) conduct a simple theoretical exercise (in a partial equilibrium framework) based on empirical income and wealth distributions and find that the effects are small, and that a monetary expansion reduces inequality in Canada and the U.S., but increases it in most European countries. Saiki and Frost (2014) found that aggressive monetary easing in Japan has contributed to higher income inequality. Bivens (2015) finds that the overall effect on inequality of the Fed's policies after the financial crisis was very modest, with most of the different channels offsetting each other. He instead emphasizes that without the exceptional monetary stimulus all groups would have been worse-off. Bell et al. (2012) achieve a similar result for the UK.

3 Monetary Policy and Wage Inequality: SVAR evidence

To motivate the research question addressed in this paper, we start by identifying the impact of a monetary policy shock on the relative wages (*skill premium*) and the employment rates of skilled and unskilled workers (*employment rates ratio*) in a conventional SVAR model. Our SVAR consists of six variables: log industrial production, unemployment rate, the log of the skill premium, the relative employment rate ratio, the consumer price index inflation, and the federal funds rate (FFR). Data for both the unemployment rate and consumer price index for all urban consumers are produced by the Bureau of Labor Statistics; data for industrial production index and the effective FFR are produced by the Board of Governors of the Federal Reserve System. For the relative wage and employment data we construct the series using the NBER extracts of the Current Population Survey (CPS) Merged Outgoing Rotation Groups, including in the sample only individuals in working age 15-64 and excluding part-time workers, self-employed and military employees. CPS provides monthly information from 1979:1 until 2016:6 on the participants' employment status, level of education, weekly earnings, and weekly hours of work.

We classify workers as high-skilled and low-skilled according to whether they have a college degree or not. Low-skilled workers are defined as all those employed with a lower educational attainment.[1] Employment is defined as number of monthly hours of work per employee times

[1] Defining high-skilled workers as those with some college education using the NBER harmonization of education in CPS over time leads to a break in the series from 1991 to 1992. To avoid this break we have opted to split workers depending on whether or not they have completed a college degree.

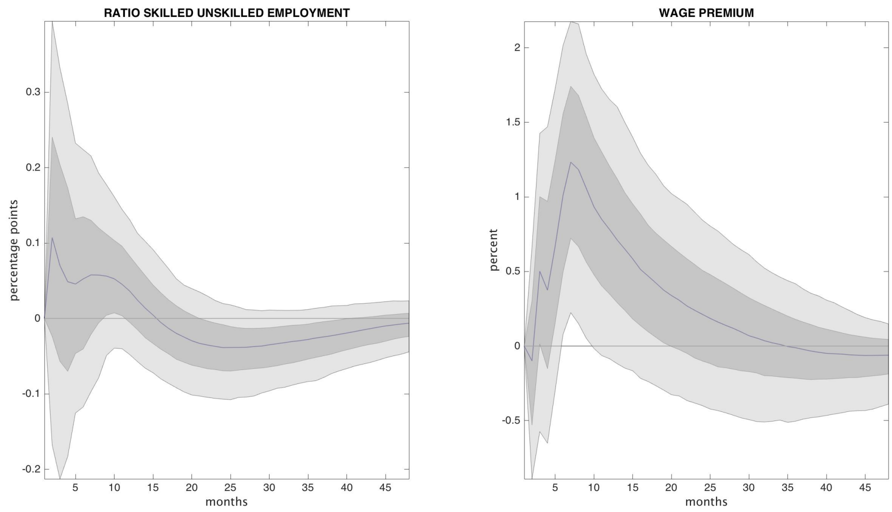

Figure 1: IRFs of employment rates ratio and skill premium to a one percentage point unexpected reduction if the FF interest rate

the number of salaried workers in each skill category.[2] We obtain hourly wages for both types of workers using weekly wage and dividing by the corresponding weekly hours worked for each group. These derived wages are used to compute the skill premium, which is defined as the ratio between the weighted average of hourly wages of the high-skilled and low-skilled workers. All variables except for the FFR are seasonally adjusted and detrended by a fourth order polynomial trend. Using information criteria, we include four lags of each variable in the VAR.

Given the high frequency of the data, the identifying assumption is that the FFR is allowed to respond contemporaneously to shocks in all remaining variables, while real variables and prices do not react to FFR shocks within a month. Hence, we consider a lower triangular Cholesky decomposition using the aforementioned ordering of the variables.

The left-hand side of Figure 1 displays the skilled-unskilled employment rate ratio impulse response function (IRF) to a one percentage point (100 b.p) unexpected cut in the FF rate, while the right-hand side shows the corresponding IRF for the skill- premium. Confidence intervals at 68% (darker bands) and 95% levels (lighter bands) are depicted. As can be observed, both variables increase during the first 12-15 months as a result of the expansionary monetary policy shock and their IRFs exhibit a hump-shaped pattern, especially in the case of the skill premium. Although the IRF of the employment ratio is only marginally significant at 68% level, both IRFs seem to suggest that inequality between high and low-skilled workers (in terms of pay and employment rates) is positively related to an unexpected reduction in interest rates.[3] At the

[2] The weighted average of the weekly earnings and hours worked last week for each skill group are calculated using the proper weights ERNWGT. These weights are computed each month such that, when applied, the resulting counts are representative of the national counts. Thus, this application of weights enables the results to be representative of the US population as a whole, instead of just the participants in the survey.

[3] In the Online Appendix we present the complete set of responses of the SVAR as well as robustness checks. Results are insensitive if we stop the sample in 2007, if we use 2 instead of 4 lags, or if we use a shorter set of

Figure 2: IRFs of employment rates ratio and skill premium to a one percentage point unexpected reduction if the FF interest rate

peaks of the IRFs, the employment rate ratio increases by about 0.1 percentage points while the skill premium goes up more substantially, by 1.3 percent.

In Figure 2, we plot similar IRFs as those above, this time using an external instrument to identify the monetary policy shock. In particular, we use the proxy-SVAR approach advocated by Mertens and Ravn (2013), where the extended time series of the Romer and Romer's (2004) narrative/ Greenbook shocks constructed by Tenreyro and Thwaiter 2015) is used as a noisy measure of the true shocks[4]. By assuming that the narrative monetary policy shocks are contemporaneously correlated with the structural policy shock and that it is contemporaneously uncorrelated with the remaining structural shocks, we recover the shocks to monetary policy.[5] As can be inspected, the IRFs of the skill premium and the employment rate ratio to an an expansionary monetary shock are larger than those in Figure 1, though their hump shapes are fairly similar. Thus, we take this evidence asindicating that these measures of labor market inequality, if anything, grow together when monetary policy becomes unexpectedly laxer.

4 Model

Our model belongs to the family of New Keynesian models with SAM frictions in the labor market. The New Keynesian feature of nominal rigidities ensures that monetary policy has real effects in the economy, while SAM frictions allow us to model unemployment. Heterogeneity

variables in the VAR

[4]The Romer and Romer series pass the weak instrument test at 10 percent significance level.

[5]In the Online Appendix we also present the complete set of responses of the SVAR as well as robustness checks. The optimal lag criteria suggest higher lags for this specification in the baseline model. As before, results are insensitive if we stop the sample in 2007, if we use 4 or 12 instead of 6 lags, or if we use a shorter set of variables in the VAR.

in the population manifests itself along two dimensions, with three different households (high and low-skilled workers, and entrepreneurs) and three different labor-market status for workers. Skill types differ in their labor market frictions (*"asymmetric" SAM*) and also in their role in production, with high-skilled workers having a lower elasticity of substitution with capital than low-skilled workers do (*CSC*). Different households can trade with each other in an single risk-free bond market but the incomplete market setup pevents full insurance against the idiosyncratic effects of shocks, leading to fluctuating consumption inequality.

Perfectly competitive intermediate-good firms produce a homogeneous output by renting capital and the two types of labor from the households. Hiring and firing are subject to SAM frictions and wages are set by Nash bargaining. Intermediate output is then differentiated by monopolistically competitive retail firms who face Calvo-type nominal rigidities in the price of the final good. Final output is used for consumption, investment and (wasteful) government spending. Fiscal policy finances exogenous expenditures, unemployment benefits and production subsidies by lump-sum taxes. Monetary policy sets the short term nominal interest rate.

4.1 Labor market search and matching

There are three different types of households: high and low-skilled workers and entrepreneurs, all of whom have constant masses φ^k, $k \in \{H, L, E\}$. It is assumed that no transition is possible across those types. The population size is normalized to one, i.e. $\sum_k \varphi^k = 1$. For each worker belonging to household $k \in \{H, L\}$ there are three possible labor market status at any period of time: employment n_t^k, unemployment u_t^k, and inactivity (enjoying leisure) l_t^k. Entrepreneurs use all of their time endowment enjoying leisure.

$$1 = n_t^k + u_t^k + l_t^k \qquad k \in H, L \tag{1}$$

Intermediate good firms post vacancies v_t^k requiring different skills, which are then matched by unemployed job-searchers U_t^k according to the following matching technology:

$$m_t^k \left(v_t^k, U_t^k \right) = \rho_m^k \left(v_t^k \right)^{\alpha^k} \left(U_t^k \right)^{1-\alpha^k} \qquad k \in \{H, L\} \tag{2}$$

where ρ_m^k is the matching efficiency parameter for a k skilled unemployed. Aggregate measures of employment and unemployment in each household are $N_t^k = \varphi^k n_t^k$ and $U_t^k = \varphi^k u_t^k$.

Labor market tightness θ_t^k, vacancy filling probabilities $\gamma_{f,t}^k$ and hiring probabilities $\gamma_{h,t}^k$ are defined as follows:

$$\theta_t^k = \frac{v_t^k}{U_t^k} \qquad k \in H, L \tag{3}$$

$$\gamma_{f,t}^k = \frac{m_t^k}{v_t^k} \qquad k \in H, L \tag{4}$$

$$\gamma_{h,t}^k = \frac{m_t^k}{U_t^k} = \rho_m^k \left(\frac{v_t^k}{U_t^k} \right)^{\alpha^k} \qquad k \in H, L \tag{5}$$

An exogenous separation rate σ^k signals the fraction of employed workers losing their job each period, who then become unemployed. Unemployed agents either find a job, stay unemployed or exit the labor force. As a result, the transition dynamics between different labor market status can be expressed as:

$$N_{t+1}^k = (1-\sigma^k)N_t^k + \underbrace{\gamma_{h,t}^k U_t^k}_{m_t^k} \qquad k \in H, L \tag{6}$$

Participation in the labor force is chosen by a given-skilled household (from (1), we have: $1-l_t^k = u_t^k + n_t^k$). However, while the household can only decide to start *searching* for a job (transiting from inactive to unemployed), *getting* a job is constrained by search and matching frictions. Therefore, n_t^k are pre-determined (state) variables at time t, so the participation margin can only be adjusted through choosing u_t^k.[6] Then, the choice of u_t^k can affect future employment through the hiring probabilities in (5). Similarly, the intermediate firm cannot decide directly how many workers to employ in a given period, but it can only affect future employment levels through its current posted vacancies v_t^k (as it also affects hiring probabilities $\gamma_{h,t}^k$ through (5)). Once these choices are made by households and firms, and given the pre-determined levels of n_t^k, future flows into employment are governed by the laws of motions (6), which will act as constraints on the household's and firm's problems.

This also shows that labor market status endogenously interact in this setup with the rest of the economy through two channels. One is the participation choice of the household through u_t^k and the other is the vacancy posting decision of intermediate firms v_t^k, both taking into account future desired levels of employment n_{t+1}^k, which in turn are subject to constraints imposed by SAM frictions.

The potential asymmetry in SAM frictions across skills $k \in \{H, L\}$ are captured by k-specific parameters $\sigma^k, \alpha^k, \rho_m^k$ (see below for further discussion on this issue).

4.2 Households

The three different household types (i.e. high and low-skilled workers and entrepreneurs, $k \in \{H, L, E\}$) have some common features. They all maximize lifetime utility which is a separably additive function of consumption c_t^k and leisure l_t^k. The three households can trade with each other through incomplete financial markets, i.e., in a single risk-free nominal bond B_t^k, which pays a gross nominal return of R_t. All households exhibit external habits in consumption, i.e. their utility depends on past levels $\hat{c}_{t-1}^k$ with a parameter of h, but this is taken as given when choosing consumption. The intertemporal elasticity of substitution is $\frac{1}{\eta}$ for everyone. Lump-sum taxes t^k are collected from the households by the government. The aggregate price level of final consumption goods is p_t.

[6] Notice that, through the time endowment constraint and once employment and unemployment are accounted for, leisure becomes just a residual. As a result, the endogenous participation choice is equivalent to choosing u_t^k.

Households discount future utils from any period t to $t=0$ by using a time varying discount factor $\widetilde{\beta}_t \equiv \beta^t \prod_{s=0}^{t-1} \psi_s$, which is subject to exogenous shocks. From this we can define the discount factor between any two consecutive periods as follows

$$\beta_t \equiv \frac{\widetilde{\beta}_{t+1}}{\widetilde{\beta}_t} = \beta \psi_t \tag{7}$$

$$\ln \psi_t = \rho_\psi \ln \psi_{t-1} + \varepsilon_t^\psi \tag{8}$$

4.2.1 Entrepreneurs

Entrepreneurs do not participate in the labor market, and therefore enjoy leisure equal to 1. In addition to trading in a non-state contingent nominal bond B_t^E, as the workers, they save by investing in physical capital k_t, too. Investment i_t also has to cover depreciation and capital adjustment costs, the latter being governed by parameter, ω. Entrepreneurs then rent capital out to intermediate firms at a rental rate r_t. It is possible to adjust the intensity of capital utilization z_t which boosts the return on capital but also raises depreciation costs according to $\delta(z_t) = \delta\, z_t^\phi, \quad \phi > 1,\ \delta > 0$. Entrepreneurs own the firms in the economy, so they receive all profits π_t^f as dividends. The main reason why they appear in our model is to avoid further asymmetries beyond those in the labor market between high and low-skilled workers that could originate from firm ownership and dividends, as in income composition channel. Thus, entrepreneurs maximimize utility subject to their budget constraint and the capital law of motion.

$$\max_{\{c_t^E, i_t, k_{t+1}, B_t^E, z_t\}} \quad E_0 \sum_{t=0}^{\infty} \widetilde{\beta}_t \left[\frac{(c_t^E - h\, \hat{c}_{t-1}^E)^{1-\eta}}{1-\eta} + \Phi^E \frac{1}{1-\zeta} \right]$$

$$c_t^E + \Delta_t i_t + t_t^E + \frac{B_t^E}{p_t} \leq r_t(z_t k_t) + \frac{R_{t-1} B_{t-1}^E}{p_t} + \pi_t^f$$

$$i_t = k_{t+1} - (1 - \delta(z_t))k_t + \frac{\omega}{2} \left(\frac{k_{t+1}}{k_t} - 1 \right)^2 k_t \tag{9}$$

The relative price of investment Δ_t is assumed to follow an exogenous AR(1) process, where the innovation to this process represents an investment-specific shock.

$$\ln \Delta_t = \rho_\Delta \ln \Delta_{t-1} + \varepsilon_t^\Delta \tag{10}$$

Substituting investment out (by plugging the capital law of motion (9) into the budget constraint), and defining gross inflation as $\Pi_t = p_t / p_{t-1}$ we can take first-order conditions:

$$c_t^E: \qquad (c_t^E - h\, c_{t-1}^E)^{-\eta} = \lambda_t^{c,E} \tag{11}$$

$$k_{t+1}: \quad \lambda_t^{c,E} \Delta_t \left[1 + \omega \left(\frac{k_{t+1}}{k_t} - 1 \right) \right] = \beta_t E_t \lambda_{t+1}^{c,E} \left\{ r_{t+1} z_{t+1} + \Delta_{t+1} \left(1 - \delta(z_{t+1}) + \frac{\omega}{2} \left[\left(\frac{k_{t+2}}{k_{t+1}} \right)^2 - 1 \right] \right) \right\} \tag{12}$$

$$B_t^E: \qquad \lambda_t^{c,E} = \beta_t E_t \lambda_{t+1}^{c,E} \frac{R_t}{\Pi_{t+1}} \tag{13}$$

$$z_t: \qquad r_t = \Delta_t \delta'(z_t) = \Delta_t \delta \phi\, z_t^{\phi - 1} \tag{14}$$

4.2.2 Workers

We have two separate worker households, $k \in \{H, L\}$. In addition to the intertemporal consumption-saving choice, workers also need to decide on intratemporal labor-market participation. The inverse Frisch-elasticity of labor supply is ζ, while Φ^k governs the weight of the leisure of each skill type in their utilities. On top of being able to save in risk-free nominal bonds B_t^k, they can get additional insurance through inflation-indexed unemployment benefits b^k. Labor earnings depend on the real wage w_t^k and employment levels n_t^k for each skill type k.

The utility maximization problem is subject to the budget constraint and constraints on employment flows imposed by SAM frictions (6).[7]

$$\max_{\{c_t^k, B_t^k, u_t^k, n_{t+1}^k\}} \quad E_0 \sum_{t=0}^{\infty} \widetilde{\beta}_t \left[\frac{(c_t^k - h\, \hat{c}_{t-1}^k)^{1-\eta}}{1-\eta} + \Phi^k \frac{\left(l_t^k\right)^{1-\zeta}}{1-\zeta} \right] \qquad k \in H, L$$

$$c_t^k + t_t^k + \frac{B_t^k}{p_t} + \underbrace{\frac{\xi}{2}\left(\frac{B_t^k}{p_t} - \bar{b}^k\right)^2}_{f_t^k} \leq w_t^k n_t^k + \frac{R_{t-1} B_{t-1}^k}{p_t} + b^k u_t^k + f_t^k \qquad k \in H, L$$

$$n_{t+1}^k = (1 - \sigma^H) n_t^k + \gamma_{h,t}^k u_t^k \qquad k \in H, L$$

As discussed before, current employment n_t^k is a state variable, pre-determined by previous labor market conditions, and each household can only choose their next period value taking into account the constraints imposed by SAM frictions. The participation margin today can only be adjusted through unemployment u_t^k. Using the laws of motion for employment (6) as defined above means that the household does not take the number of matches as given, but instead takes into account the effect of its unemployment decisions on matches, at least partially.[8]

A well-known issue with incomplete market models is that the non-stochastic steady state suffers from indeterminacy, while equilibrium dynamics around it are non-stationary (Schmitt-Grohé and Uribe, 2003) because consumption growth does not depend on asset holdings. In order to get around this problem, we introduce a small portfolio adjustment cost (governed by ξ), which penalizes workers in case their real bond holdings deviate from some benchmark level $\bar{b}^k$. To rule out any wealth effects, all of these costs are rebated to the households as a lump- sum rebate f_t^k, but the latter is not taken into account when making the saving/borrowing decision.

[7] The law of motion for employment (6) is expressed here in *per capita* terms, i.e.,divided by the mass of workers φ^k.

[8] It ignores, however, the full effect of its decisions on matches. In particular, its unemployment decisions also affect hiring probabilities $\gamma_{h,t}^{j,k}$ through (5), which the household takes as given in the above formulation; see Brückner and Pappa (2012). The full effect would be taken into account only if we substituted (2) into (6), instead of using the formulation with hiring probabilities.

Substituting leisure out using (1) we can take first-order conditions for $k \in \{H, L\}$:

$$c_t^k: \qquad (c_t^k - h\, c_{t-1}^k)^{-\eta} = \lambda_t^{c,k} \tag{15}$$

$$B_t^k: \qquad \lambda_t^{c,k}\left[1 + \xi\left(\frac{B_t^k}{p_t} - \bar{b}^k\right)\right] = \beta_t E_t \lambda_{t+1}^{c,k}\frac{R_t}{\Pi_{t+1}} \tag{16}$$

$$u_t^k: \qquad \lambda_t^{n,k} = \frac{\Phi^k\left(l_t^k\right)^{-\zeta} - b^k \lambda_t^{c,k}}{\gamma_{h,t}^k} \tag{17}$$

$$n_{t+1}^k: \qquad \lambda_t^{n,k} = \beta_t E_t\left[\lambda_{t+1}^{c,k} w_{t+1}^k + (1-\sigma^k)\lambda_{t+1}^{n,k} - \Phi^k\left(l_{t+1}^k\right)^{-\zeta}\right] \tag{18}$$

where $\lambda_t^{c,k}, \lambda_t^{n,k}$ are Lagrange-multipliers on the budget constraint and the employment laws of motion, respectively. Then, cancelling the portfolio adjustment costs with the lump- sum rebates f_t^k, and defining *real bonds* as $b_t^k \equiv \frac{B_t^k}{p_t}$, we use the budget constraints as:

$$c_t^k + t_t^k + b_t^k + \leq w_t^k n_t^k + \frac{R_{t-1}}{\Pi_t} b_{t-1}^k + b^k u_t^k \qquad k \in \{H, L\} \tag{19}$$

4.2.3 Financial markets

Trading in non-state contigent bonds means that financial markets are incomplete, implying that different households cannot fully insure against idiosyncratic risk or against that part of aggregate shocks which affect them in an asymmetric way.[9] In other words, there is no perfect risk sharing among them, which can give rise to *fluctuating* consumption inequality. Imperfect risk-sharing can be seen through the fact that the ratio of different households' marginal utilities is not constant *ex post*, and stays fixed only in *expectation*. Hence, idiosyncratic shocks will lead to fluctuations in this ratio. In other words, *expected* consumption growth is equalized across agents, but *actual* consumption growth is not.

This is in contrast to the complete market case which is practically equivalent to using only a representative household. In that case, perfect risk-sharing would create a much tighter link between the consumption of different households and keep their ratio of marginal utilities constant at all periods due to the ability to insure against all idiosyncratic shocks by means of a complete set of state-contingent securities. [10]

Our model framework nests different financial market setups in the following way:

a) **financial autarky** can be approximated by $\xi \to \infty$. Another way is by replacing the Euler equations of the workers (16) with $b_t^k = 0$ for $k \in \{H, L\}$. In essence we have to

[9] Even aggregate shocks can have idiosyncratic effects if they influence different households in different ways. For instance, an aggregate demand shock will lead to changes in the skill premium.

[10] Note however, that even under complete markets there would be consumption inequality since, even in the steady state, different households would enjoy different consumption *levels* (through different wages, rent, benefits, initial wealth). Likewise, even complete markets cannot insure against *aggregate* risk, but only against the *idiosyncratic* component, so the *level* of consumption would change. It is just the *ratio* of marginal utilities which would stay constant. Under incomplete markets, this ratio is constant only in expectation terms, and this allows idiosyncratic shocks to give rise to fluctuating relative consumption and fluctuating inequality.

impose that workers are hand-to-mouth, and they cannot save. Then the Euler equation of the entrepreneurs (13) will be used to price the bond (which is not traded in equilibrium). [11]

b) intermediate case of **incomplete markets** is any finite value of $\xi > 0$. Note that, $\xi = 0$ is still the incomplete market case, but then we have non-stationarity and indeterminacy.

c) **complete markets** can be obtained by replacing the Euler equations[12] of the workers (16) with the perfect risk-sharing condition across agents, which keep the ratio of their marginal utilities constant at its steady- state value (labeled by bars) :

$$\frac{\lambda_t^k}{\lambda_t^E} = \frac{\overline{\lambda}^k}{\overline{\lambda}^E} \qquad k \in \{H, L\}$$

4.3 Intermediate goods firms

A continuum of perfectly competitive firms produces a homogeneous intermediate good y_t, using high and low-skill labor N_t^k and *effective* aggregate capital K_t. Just like in the case of the household, N_t^k are state variables, given by matches and employment levels in the previous period. It is only next period's employment levels N_{t+1}^k which can be influenced by choosing how many vacancies v_t^k to post. This influence is subject to the same SAM frictions as in the case of the household, however, (6) is now reformulated by plugging in (4) to reflect how vacancies are affecting the number of matches.[13] Therefore, the firm's problem becomes dynamic.

$$V^F\left(N_t^H, N_t^L, \boldsymbol{s_t}\right) = \max_{K_t, N_{t+1}^H, N_{t+1}^L, v_t^H, v_t^L} x_t F\left(K_t, N_t^H, N_t^L\right) - r_t K_t - w_t^H N_t^H - w_t^L N_t^L - \kappa^H v_t^H - \kappa^L v_t^L +$$
$$+ E_t \Lambda_{t+1} V^F\left(N_{t+1}^H, N_{t+1}^L, \boldsymbol{s_{t+1}}\right)$$
$$\Lambda_{t+1} = \beta_t \frac{\lambda_{t+1}^{c,E}}{\lambda_t^{c,E}}$$
$$N_{t+1}^H = (1 - \sigma^H) N_t^H + \gamma_{f,t}^H v_t^H$$
$$N_{t+1}^L = (1 - \sigma^L) N_t^L + \gamma_{f,t}^L v_t^L$$

where Λ_{t+1} is the stochastic discount factor of the entrepreneurs, reflecting the ownership of the firm. The real price of intermediate goods x_t is taken as given by the firm − this constitutes real marginal costs for retails firms. Posting vacancies has a unit cost of κ^k. Substituting in the constraints and taking derivatives, the first-order conditions yield the capital and labor demand

[11] With financial autarky, the bond price is meaningless anyway since there are no financial markets, and the entrepreneurs will also end up being de facto hand-to-mouth. But this is the bond price which makes it an optimal decision for them.

[12] Under complete markets the budget constraints will have to be adjusted as well since now we have a full set of state-contingent securities, and not a single bond b_t. In this case, however, the budget constraint is only used to recover the value of this portfolio as a residual, and so it can be omitted.

[13] Also, as with households, the firm does not take into account the full effect of its vacancy choices on the number of matches. In particular, it disregards the effect on vacancy filling probabilities through (4).

equations:

$$K_t: \qquad r_t = x_t F_{k,t} \tag{20}$$

$$N_{t+1}^H: \qquad \frac{\kappa^H}{\gamma_{f,t}^H} = \beta_t E_t \frac{\lambda_{t+1}^{c,E}}{\lambda_t^{c,E}} \left[x_{t+1} F_{n,t+1}^H - w_{t+1}^H + (1-\sigma^H) \frac{\kappa^H}{\gamma_{f,t+1}^H} \right] \tag{21}$$

$$N_{t+1}^L: \qquad \frac{\kappa^L}{\gamma_{f,t}^L} = \beta_t E_t \frac{\lambda_{t+1}^{c,E}}{\lambda_t^{c,E}} \left[x_{t+1} F_{n,t+1}^L - w_{t+1}^L + (1-\sigma^L) \frac{\kappa^L}{\gamma_{f,t+1}^L} \right] \tag{22}$$

where $F_{k,t}$ and $F_{n,t}^k$ are the marginal products of respective inputs. Aggregate capital K_t and the production function are defined as

$$K_t = \varphi^E z_t k_t \tag{23}$$

$$y_t = F\left(K_t, N_t^H, N_t^L\right) \tag{24}$$

Our baseline production function is a nested CES composite of production factors, where we can separately control the elasticity of substitution between capital and high-skill labor, ϱ_{k,n^H} on the one hand, and between capital and low-skill labor ϱ_{k,n^L} on the other.[14] This follows Krusell et al. (2000) and Lindquist (2004).

$$F\left(K_t, N_t^H, N_t^L\right) = A_t \left[a^k \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-a^k)(N_t^L)^\alpha \right]^{\frac{1}{\alpha}}$$

The capital intensity of the "skilled input bundle" is controlled by λ, while a^k represents the "skill intensity" of total production. The elasticities of substitution are governed by γ and α, and can be defined as $\varrho_{k,n^H} = \frac{1}{1-\gamma}$, and $\varrho_{k,n^L} = \varrho_{n^H,n^L} = \frac{1}{1-\alpha}$. We restrict these elasticities to be positive in order to preserve strict quasi-concavity of the production function. This means that $\alpha, \gamma \leq 1$.

CSC is captured in the following way:

- $0 < \varrho_{k,n^H} < 1$ represents CSC ($\gamma < 0$, with larger absolute values corresponding to a higher degree of complementarity)
- $1 < \varrho_{k,n^L}$ shows the substitutability of low-skill labor with the skilled inputs ($0 < \alpha \leq 1$ with larger values corresponding to a higher degree of substitutability and $\alpha = 1$ meaning perfect substitutes)
- $\varrho_{k,n^L} = \varrho_{k,n^H} = 1$ correspond to the CD case with a unit elasticity of substitution ($\gamma, \alpha = 0$)

This follows the definition of Koczan et al. (2017), who define production factors as complements when their elasticity of substitution is below unity. In that case a fall in the relative price of one factor should increase the income share of the other factor, and vice versa for substitutes.[15]

[14] The structure of the nesting implies that the elasticity of substitution between high and low-skill labor must be the same as between capital and low-skill labor $\varrho_{n^H,n^L} = \varrho_{k,n^L}$.

[15] However, in these authors' model there is only one type of labor and no SAM frictions (which put a wedge between the wage and the marginal product of labor), so this might not carry straight over to our model.

Lindquist (2004) uses a less strict definition: based on the following formula, he shows that as long as $1 \geq \alpha > \gamma$, we get that a rise in the stock of capital will *ceteris paribus* raise the relative marginal product of skilled labor,[16] and this is what we call *CSC effect*. It can also be seen that a rise in low-skill employment relative to high- skill employment raises the marginal product of high- skilled workers, which is called the *relative supply effect*.

$$\frac{F^H_{n,t}}{F^L_{n,t}} = (1-\lambda)\frac{a^k}{1-a^k}\left[\lambda\left(\frac{K_t}{N^H_t}\right)^\gamma + (1-\lambda)\right]^{\frac{\alpha-\gamma}{\gamma}}\left(\frac{N^L_t}{N^H_t}\right)^{1-\alpha}$$

In order to gauge the effects of CSC relative to the case of no CSC, we can change parameters γ and α, but we also need a different benchmark production function, the structure of which allows for controlling the elasticity of substitutions separately between capital and any labor, and between the two different types of labor.

$$\widetilde{F}\left(K_t, N^H_t, N^L_t\right) = A_t K^\iota_t\left[\mu(N^H_t)^\nu + (1-\mu)(N^L_t)^\nu\right]^{\frac{1-\iota}{\nu}}$$

where, as it is well known, the CD structure between capital and labor is a special case of CES, with the elasticity of substitution between them being unity – i.e. capital and labor are neither complements, nor substitutes, and the income shares are constant, with ι being the share of capital. The two different types of labor are perfect substitutes when $\nu = 1$, i.e., their elasticity of substitution is $\frac{1}{1-\nu} = \infty$. With equal intensity $\mu = 0.5$ labor is basically homogeneous.

Aggregate TFP follows an exogenous AR(1) process:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon^a_t \tag{25}$$

4.4 Wage bargaining

Workers and intermediate firms split the surplus of a match by Nash-bargaining. Wages are negotiated separately in the high and low-skill labor markets.

$$\max_{w^k_t} (1-\vartheta^k)\ln\left(V^{E,k}_t\right) + \vartheta^k \ln\left(V^{F,k}_t\right) \qquad k \in H, L$$

$$V^{E,k}_t = \frac{\partial \mathcal{L}}{\partial n^k_t} = \lambda^{c,k}_t w^k_t - \Phi^k\left(l^k_t\right)^{-\zeta} + (1-\sigma^k)\lambda^{n,k}_t$$

$$V^{F,k}_t = \frac{\partial V^F(N^k_t)}{\partial N^k_t} = x_t F^k_{n,t} - w^k_t + (1-\sigma^k)\frac{\kappa^k}{\gamma^k_{f,t}}$$

where $V^{E,k}_t$ is the marginal value for the household of being employed, and $V^{F,k}_t$ is the expected value for the firm of filling a vacancy. $\mathcal{L}$ is the Lagrangian of the household, while V^F is the value function of the firm. During Nash-bargaining they maximize the weighted sum of log surpluses,

[16] In our model SAM frictions put a wedge between the wage and the marginal product of labor, but they are still closely related. Therefore the above argument can be applied to the skill premium as well.

with the weights ϑ^k representing the bargaining power of the firm in each labor market. The solution to this problem yields the *desired* wage w_t^{k*}:

$$w_t^{k*} = (1-\vartheta^k)\left[x_t F_{n,t}^k + (1-\sigma^k)\frac{\kappa^k}{\gamma_{f,t}^k}\right] + \frac{\vartheta^k}{\lambda_t^{c,k}}\left[\Phi^k\left(l_t^k\right)^{-\zeta} - (1-\sigma^k)\lambda_t^{n,k}\right] \tag{26}$$

Actual wages w_t, however, will be subject to an exogenous constraint which is meant to be a shortcut for wage rigidities, controlled by parameter ρ_w^k. Under this formulation, and unlike the case of Calvo-type frictions, the presence of rigidities is not taken into account in a forward-looking manner either by firms or by households during their bargaining process. However, this allows for a much simpler setup.

$$w_t^k = \rho_w^k w_{t-1}^k + (1-\rho_w^k) w_t^{k*} \qquad\qquad k \in H, L \tag{27}$$

4.5 Retail firms

We have a continuum $i \in [0,1]$ of monopolistically competitive retail firms, each of which buys an amount $y_t(i)$ of the homogenous intermediate good y_t, and produces a differenciated product $y_t^r(i)$ with a linear technology, i.e. $y_t^r(i) = y_t(i)$. These differentiated products are then assembled to become final goods y_t^r according to a CES aggregator:

$$y_t^r = \left[\int_0^1 y_t^r(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}} = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}} = y_t$$

where the last equality is just a scaling assumption, so that we can drop the r superscript, i.e., in the end we have as many final goods as intermediate goods. ϵ is the elasticity of substitution between different products.

Retail firms take the relative price x_t of the intermediate good as given, which is basically their real marginal cost. This depends neither on i (since intermediate goods are homogenous, so retail firms are competitive *buyers*) nor on the amount of goods used (since all the retail firms have an infinitesimally small size). Due to differentiation, retailers have pricing power in setting the price of their own product $p_t(i)$, but take the aggregate price level p_t as given. The latter is defined as $p_t = \left[\int_0^1 p_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}$.

In setting their price, retailers are constrained by Calvo-type nominal rigidities, meaning that in every given period a fraction χ of them cannot adjust prices. The $(1-\chi)$ fraction of firms, who are able to adjust prices in a given period, will choose them ($p_t^*(i)$) so as to maximize the real present value of expected future profits, taking into account both nominal rigidities and the price-elastic demand of households.

$$p_t^*(i) \equiv \underset{p_t(i)}{\arg\max}\; E_t \sum_{s=0}^{\infty} \chi^s\, \Lambda_{t+s}\left[\frac{p_t(i)}{p_{t+s}} - (1-\tau)x_{t+s}\right] y_{t+s}(i)$$

$$y_{t+s}(i) = \left(\frac{p_t(i)}{p_{t+s}}\right)^{-\epsilon} y_{t+s}$$

where τ is a production subsidy used by the government to eliminate the static distortion coming from monopolistic competition. Due to symmetry across retailers, all of them will choose the same price $p_t^* \equiv p_t^*(i)$. The solution to this problem yields:

$$p_t^* = \underbrace{\frac{(1-\tau)\epsilon}{\epsilon-1}}_{(1-\tau)\mathcal{M}} E_t \frac{\sum_{s=0}^{\infty} \chi^s \Lambda_{t+s}\, y_{t+s}(i)\, \overbrace{p_{t+s}x_{t+s}}^{MC_t}}{\sum_{s=0}^{\infty} \chi^s \Lambda_{t+s}\, y_{t+s}(i)}$$

Calvo-rigidities imply that evolution of the aggregate price level will follow

$$p_t = \left[(1-\chi)(p_t^*)^{1-\epsilon} + \chi\, p_{t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Manipulating the previous expression and the FOC of the retailers, we can derive a non-linear substitute for the New Keynesian Phillips Curve,[17] expressed by the following three equations:

$$\frac{K_t}{F_t} = \left[\frac{1-\chi\Pi_t^{\epsilon-1}}{1-\chi}\right]^{\frac{1}{1-\epsilon}} \tag{28}$$

$$K_t \equiv y_t\, (c_t^E - hc_{t-1}^E)^{-\eta}\, x_t\, \frac{(1-\tau)\epsilon}{\epsilon-1} \;+\; \chi\beta_t E_t\left[\Pi_{t+1}^{\epsilon} K_{t+1}\right] \tag{29}$$

$$F_t \equiv y_t\, (c_t^E - hc_{t-1}^E)^{-\eta} \;+\; \chi\beta_t E_t\left[\Pi_{t+1}^{\epsilon-1} F_{t+1}\right] \tag{30}$$

where $\Pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation rate.

4.6 Monetary and fiscal policies

Monetary policy sets short term nominal interest rates following a standard Taylor rule with interest rate smoothing and potential reaction to the deviations of output and unemployment from their steady-state values

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\zeta^R} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\zeta^\pi} \left(\frac{y_t}{\overline{y}}\right)^{\zeta^y} \left(\frac{u_t}{\overline{u}}\right)^{\zeta^u} v_t^R\right]^{1-\zeta^R} \tag{31}$$

$$\ln v_t^R = \rho_R \ln v_{t-1}^R + \varepsilon_t^R \tag{32}$$

where v_t^R captures a (persistent) monetary policy shock which follows an AR(1) process.

Fiscal policy is described by an exogenous and wasteful government consumption G_t, production subsidy τ to retailers, and unemployment benefits b^k, all of which are financed by lump-sum taxes T_t, implying that the government runs a balanced budget in every period.

$$T_t = b^H U_t^H + b^L u_t^L + G_t + \tau x_t y_t \tag{33}$$

$$\ln G_t = (1-\rho_g)\ln(\overline{y}\Gamma) + \rho_g \ln G_{t-1} + \varepsilon_t^g \tag{34}$$

where Γ is the steady-state share of government consumption in output. The distribution of lump-sum taxes is assumed to be equal, i.e. $t_t^k = T_t$ for $k \in \{H, L, E\}$, so that we have $T_t = \sum_k \varphi^k t_t^k = T_t \sum_k \varphi^k$.

[17] For detailed derivations, refer to the online appendix.

4.7 Market clearing

Since households can only trade bonds with each other, and not with the government or foreign agents, the bond market clears as follows:

$$\varphi^E b_t^E + \varphi^H b_t^H + \varphi^L b_t^L = 0 \tag{35}$$

Combining the budget constraints of the households and the government (and using the bonds market clearing condition) we get the goods market-clearing condition. Final output is used for consumption, investment, government expenditures and posting vacancies.

$$y_t = C_t + \Delta_t I_t + G_t + \kappa^H v_t^H + \kappa^L v_t^L \tag{36}$$

where $C_t = \sum_{k\in(H,L,E)} \varphi^k c_t^k$ and $I_t = \varphi^E i_t$.

Equilibrium is described by equations (1) through (36).

This completes the description of the model.

5 Calibration

In parameterizing the model we use the following strategy. First, we consider the model period to be one quarter. Second, we set targets to the steady-state values of participation and unemployment rates – separately for high and low-skill labor markets. In doing this, we set values that track the pre-crisis averages for the U.S. As mentioned earlier, high-skilled workers are regarded to have college degrees or higher[18]. In terms of the model variables, these correspond to

$$partic_t^k \equiv \frac{N_t^k + U_t^k}{\varphi^k}$$
$$unemp_t^k \equiv \frac{U_t^k}{N_t^k + U_t^k}$$

Parameters Φ^k, ϑ^k are calibrated so as to match the above targets.[19] The exact values can be seen in Table 1, with blue color for targeted steady states and with red color for calibrated parameters.

The asymmetry in SAM frictions is captured by skill-specific parameters. We set separation rates $\sigma^H < \sigma^L$ and matching efficiencies $\rho_m^L < \rho_m^H$, which all result in larger frictions for the low-skilled workers, making their steady-state vacancy filling and hiring probabilities lower than that of the high skilled: $\gamma_h^L < \gamma_h^H$, $\gamma_f^L < \gamma_f^H$. The non-equal share of different skill types in the population φ^k and our skill-specific steady-state targets for employment variables result in further asymmetries

[18]According to this classification, 18 percent of our households are high-skilled workers, while 72 percent are low-skilled workers and the share of entrepreneurs in the economy is set to 10 percent.

[19]The details of on how the steady state is calculated can be found in the online appendix.

for calibrated parameters. In particular, the weight of leisure (inactivity) in the households' utility function will be higher for low-skilled workers $\Phi^H < \Phi^L$, while the bargaining power of the firm (worker) will be higher (lower) in the low-skill labor market $\vartheta^H < \vartheta^L$. The latter feature mitigates the relative costliness of low-skilled workers as the firm is able to capture a larger share of the surplus created by filling a low-skill vacancy. So does the slightly higher unit cost of posting a high-skill vacancy $\kappa^L < \kappa^H$. In addition, we also have asymmetry in the wage replacement rate of unemployment benefits b^k/w^k.

Parameters							
σ^H	0.0299	β	0.9900	κ^H	0.1400	ϵ	6.0000
σ^L	0.0505	η	2.0000	κ^L	0.1300	χ	0.8500
ρ_m^H	0.8630	ζ	4.0000	a^k	0.4300	τ	0.1667
ρ_m^L	0.3835	Φ^H	0.8529	λ	0.3500	Γ	0.2000
α^H	0.5000	Φ^L	21.1374	α	0.4000	ζ^R	0
α^L	0.5000	ξ	0.0010	γ	-0.4902	ζ^π	1.5000
ρ_w^H	0.0000	ω	4.0000			ζ^y	0
ρ_w^L	0.0000	δ	0.0100	ϑ^H	0.2515	ρ_ψ	0.7000
φ^H	0.1800	ϕ	2.0101	ϑ^L	0.6983	ρ_a	0.8500
φ^L	0.7200	h	0.8000	b^H	0.2818	ρ_g	0.7000
φ^E	0.1000	$\bar{b}^k$	0.0000	b^L	0.2432	ρ_R	0.7000
Steady state values							
w^H/w^L	1.6242	$partic^H$	0.7100	b^H/w^H	0.4000	K/y	3.0845
$(w^H n^H)/(w^L n^L)$	0.5242	$partic^L$	0.5700	b^L/w^L	0.5500	I/y	0.0308
$(w^H n^H)/(y-\kappa v)$	0.3215	$unemp^H$	0.0340	κ^H/w^H	0.1901	$(\kappa^H v^H + \kappa^L v^L)/y$	0.0417
$(w^L n^L)/(y-\kappa v)$	0.6132	$unemp^L$	0.0680	κ^L/w^L	0.2867	$partic$	0.6382
$(rK)/(y-\kappa v)$	0.0653			z	1.0000	$unemp$	0.0505
$skilled/(y-\kappa v)$	0.3868			x	1.0000		

Table 1: Parameters and selected steady-state values. 8 blue steady states are targeted by 8 red parameters.

Heterogeneity across high and low-skilled workers does not only come from the labor market but, as captured by CSC, it also has to do with their different roles in production. We set elasticities of substitution based on the estimation of our production function by Krusell et al. (2000). This means $\frac{1}{1-\gamma} = 0.67$ and $\frac{1}{1-\alpha} = 1.67$, which makes high-skill labor complementary to capital, while low-skill labor becomes substitute.

Other parameters are set to standard values as it is common in the literature. In particular, the production subsidy is set to eliminate the static distortion coming from monopolistic competition $\tau = 1/\epsilon$, which makes the steady-state real marginal cost (markup) x equal to one. ϕ is calibrated so as to have a steady-tate capital utilization rate z of unity.

As a result, under our baseline parameterization we get a steady-state wage premium of 62%, which matches the average wage premium in our sample. Finally, the low- skill labor market is much tighter $\theta^H < \theta^L$, as a result of its lower matching efficiency.

6 Simulation results

We approximate the model by log-linearizing it around the deterministic steady state, and look at impulse responses to various shocks under different scenarios and parameterizations.

6.1 The effect of expansionary monetary policy shocks

An expansionary monetary-policy shock (100 basis point cut in the nominal interest rate) stimulates aggregate demand, which leads to expanding output and inflationary pressures. Reacting to a higher relative price of the intermediate good x_t, firms increase their demand for capital and labor, which leads to rising investment and higher employment, together with higher wages and larger rent on capital. What happens in our labor market with SAM frictions is that firms start posting more vacancies while households raise their labor market participation in response to better job finding prospects. However, employment cannot suddenly react much (since it is not a jump variable) being subject to SAM frictions according to (6), which is why most of the adjustment takes place through higher wages.[20]

Heterogeneity in our labor market (asymmetric SAM) and different roles in production due to CSC imply that high and low-skilled workers will not experience the same increase in wages and employment. Under our baseline scenario, an expansionary monetary policy shock leads to a rise in the skill premium $\frac{w_t^H}{w_t^L}$ of about 30%. As shown in the left panel of Figure 3, this also implies an increase in the income share $\frac{w_t^H n_t^H}{y_t - \kappa v_t}$ for the high skilled at the expense of a *decreasing* income share for low-skilled labor. This means that the benefits of a monetary easing are not evenly distributed, with high-skilled workers getting *relatively* more of the increase in real income than do low-skilled workers – even though both types are better off in absolute terms. To the extent that the low skilled are poorer to begin with (which is confirmed by a steady-state skill premium of 62%), a monetary expansion raises inequality. The right panel of Figure 3 shows that the increase in the relative income share of high-skill labor $\frac{w_t^H n_t^H}{w_t^L n_t^L}$ is driven mainly by different increases in wages, while changes in employment are negligible.

At this stage a brief note of warning to the reader is due on the much larger responses of the skill premium in the calibrated model than in the estimated SVAR. The insight is that, since employment (i.e. quantities) is a state variable whose current value is restricted due to SAM frictions, most of the adjustment to a shock takes place through wages (i.e. prices), whose changes can be quite large. This is the so-called "Shimer puzzle" and a popular attempt to overcome it is to assume ad-hoc wage rigidities which constrain these excessive wage movements. We do this in the sensitivity analysis and the responses of the skill premium to an unexpected cut in the interest rate becomes much more muted. Thus, we do not really try to match the

[20]This is a standard result in SAM models: as demand pressures run up against SAM frictions, the surplus coming from a match increases a lot. In other words, firms are willing to agree to a much higher wage during the Nash bargaining, since they are compensated by higher revenues.

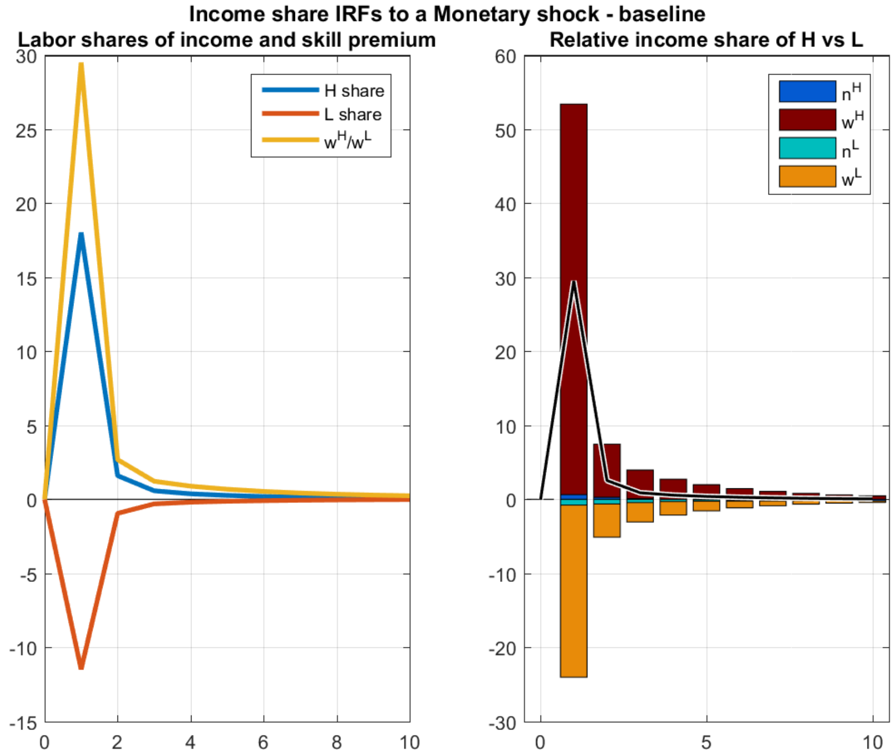

Figure 3: Income shares of labor types

empirical SVAR results to the theoretical predictions, but only use them as a motivation device towards building a model which qualitatively points in the same direction.

It is not clear, however, where lies the source of the rise in inequality. In order to separately identify the effects of asymmetric SAM frictions on the one hand, and CSC on the other, we construct a benchmark case with *symmetric* SAM frictions and a standard CD production function where high and low- skill labor inputs are perfect substitutes (as defined by $\widetilde{F}\left(K_t, N_t^H, N_t^L\right)$ above). Then we add either SAM asymmetry only or CSC only in order to compare the effect of each of these features against our benchmark. Finally, we add both sources of heterogeneity together to get back our original (baseline) scenario. The results of this exercise are shown in Figures 4 and 5.

Blue lines represent our symmetric benchmark (symmetric SAM frictions and a CD production function). The results after introducing CSC only are displayed in red lines. It can be inspected that, by changing the characteristics of the production function, an expansionary monetary policy shock affects aggregate and relative variables by a lesser extent that under the benchmark specification. In particular, output now expands by much less, and there is a smaller reduction in the non-employment rate $\frac{u_t+l_t}{n_t+u_t+l_t}$.[21] Moreover, looking at relative measures between high and low-skilled workers in Figure 5 we see almost negligible changes. The relative income

[21]The steady state share of investment is smaller in the CSC scenario which is why a similar proportional increase in investment leads to a smaller increase in output.

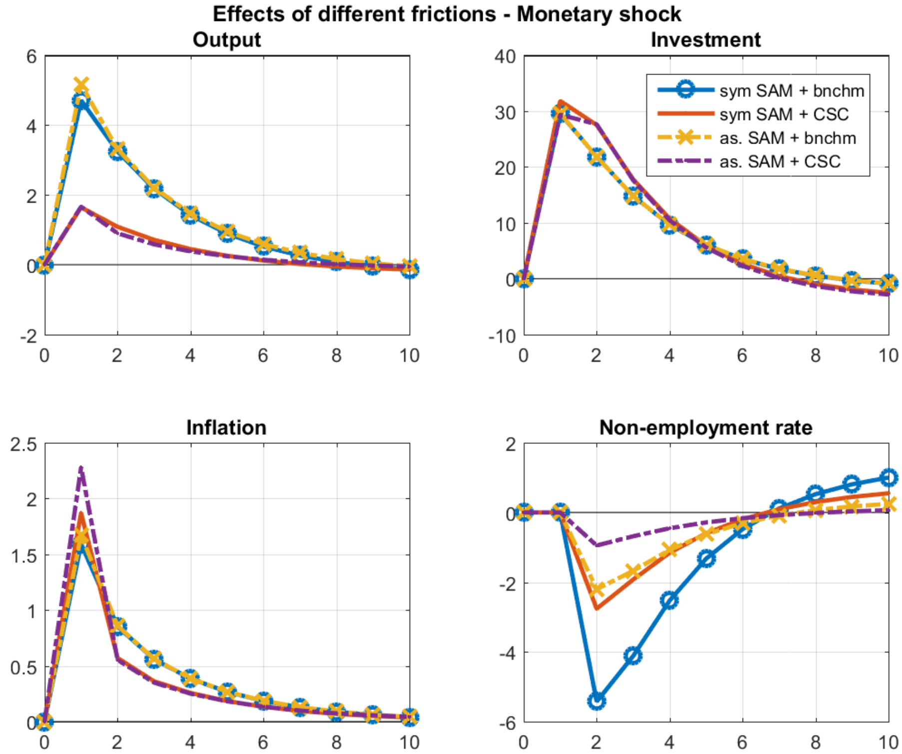

Figure 4: Effects of SAM asymmetry and CSC – aggregate variables

share of high-skilled workers hardly changes. This would be contrary to our expectation that the increased demand for capital induced by an expansionary monetary-policy shock would benefit complementary high-skill labor relatively more than substitutable low-skill labor. In fact, however, the relative marginal product of high-skill labor $\frac{F^H_{n,t}}{F^L_{n,t}}$ does increase (though little) in line with the CSC effect discussed by Lindquist (2004) – we still have some asymmetry in ϑ^k and Φ^k which puts a wedge between marginal products and wages.[22] At any rate, the overall conclusion of this exercise is that introducing CSC *only* only does not seem to have large effects on inequality.

The effects of introducing only SAM asymmetries (and keeping the benchmark CD production function) are considered next in Figures 4 and 5 (plotted in yellow lines) In terms of *aggregate* variables the IRFs to an expansionary monetary shocks are essentially identical to the benchmark case, which suggests that labor market heterogeneity does not have significant effects either at the macro level, provided we have the same production technology. In contrast, high versus low-skill *relative* variables are affected significantly. This is because larger SAM frictions in the low-skill sector make it relatively more costly for firms to open low-skill vacancies and for

[22]This is an artifact of the way we engineered SAM symmetry. Recall from our calibration strategy that ϑ^k and Φ^k are not free parameters but residuals of our steady state targets. Even with symmetric steady-state employment targets, CSC introduces a non-zero steady-state skill premium, which then leads to different values for the above parameters. Fixing these parameters symmetrically instead of our employment targets would force us to abandon our steady-state calculation strategy.

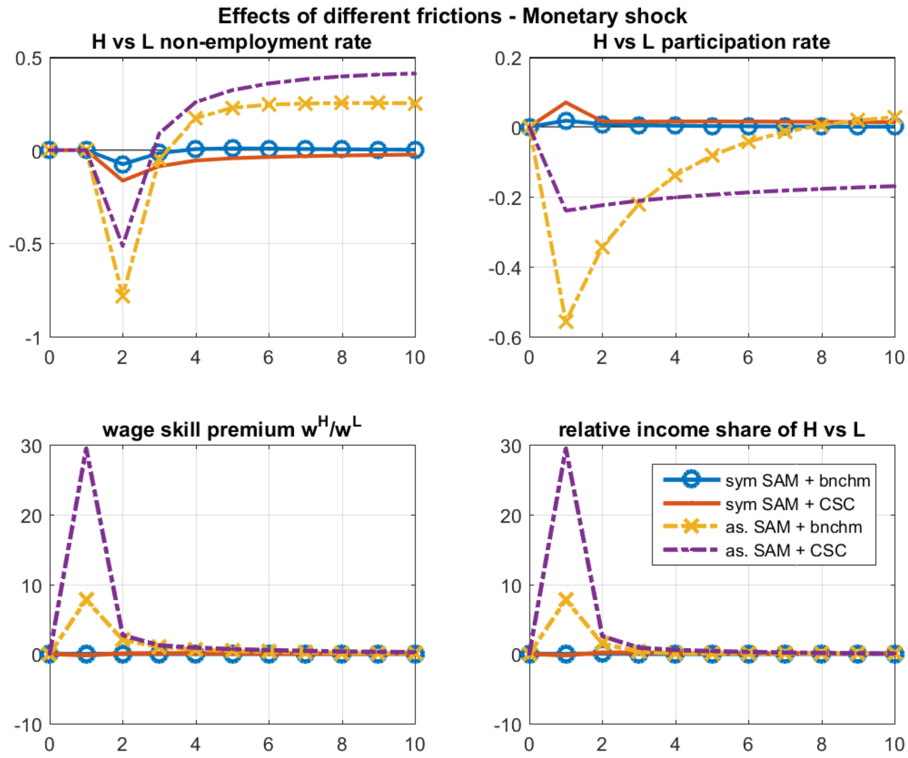

Figure 5: Effects of SAM asymmetry and CSC – relative variables

households to enter the low-skill labor market, in line with the results in Eeckhaut and Philipp (2018). Likewise, the value of an unskilled match is relatively lower since the resulting job is more likely to be terminated, and a subsequent match is less likely to take place. This results in a nearly 8% increase in the skill premium. The picture for employment is more mixed: the non-employment rate of skilled workers falls more initially, but later on becomes slightly higher than the non-employment rate of unskilled workers. The participation *rate* also rises less for high-skilled workers (i.e., their leisure consumption drops by less). In any case though, the magnitudes of these employment changes are small, and most of the increase in the relative income share of high-skilled workers is driven by the skill premium. This leads to the conclusion that a monetary expansion increases inequality under asymmetric SAM frictions but by a modest magnitude.

Finally, we introduce CSC on top of SAM asymmetry, where this scenario is plotted with purple lines. In contrast to what happened when CSC is introduced in a symmetric labor market environment (blue *vs.* red lines), this time one can observe sizeable changes in the responses of relative variables between the yellow and purple lines (while aggregate variables change by a similar amount as before). Both the responses of non-employment and participation rates change in favor of high-skilled workers, while their skill premium rises even more, driving the 30% increase in the relative income share for high-skill labor. Therefore, under a heterogenous labor market, CSC makes the effect of a monetary expansion more beneficial for high-skilled workers, contributing substantially to the rise in inequality.

The *interaction* of CSC with asymmetric SAM frictions, however, is crucial for this result. As argued above, introducing CSC on its own has little effect on the relative income share, while SAM asymmetry alone leads to a modest rise. In fact, CSC more than triples the effect of SAM asymmetry, amplifying the rise in inequality. This indicates that SAM asymmetries are required for CSC to exert its effect in full. The insight for this result is that the increase in labor demand induced by an unexpected monetary expansion leads to larger wage increases for high-skilled workers who have smaller matching frictions (SAM-asymmetry channel). In addition, the increase in capital demand amplifies this wage divergence due to high-skilled workers being more complementary to capital than substitutable unskilled ones are (CSC channel)

6.2 Different monetary policy strategies

In addition to analyzing the effects of an *unexpected* monetary policy shock on inequality, it is also important to know how different types of *systematic* monetary policy strategies perform in the response to other shocks that could also be a source of cyclical fluctuations. To this end, we analyze the impulse responses of the economy to shocks in: (i) TFP, (ii) government spending, (iii) cost-push , and (iv) investment specific and (v) discount factor shocks. Our baseline monetary policy strategy is described by a standard Taylor rule, as in (31), where $\zeta^{\pi} = 1.5$ and the rest of the reaction parameters are set equal to zero. For the sake of brevity, we do not report these results (available upon request) but just briefly describe our main findings in what follows. These are that, while aggregate variables might react differently to these shocks, relative labor market variables tell basically the same story, conditional on whether the shock strengthens or weakens aggregate demand. Another noticeable feature is that a positive to government expenditure increases the skill premium but by much less (around 3 percent) than an expansionary monetary shock does, since it crowds out investment. The rise in inequality, however, is larger under shocks lowering the discount rate and the price of investment which lead to larger investment. In other words, the effects of CSC on inequality are qualitatively similar under a monetary easing than under any other shock which leads to an expansion in demand: inequality rises more with CSC than in its absence.

Given these effects, a monetary policy rule which can stabilize demand could also prevent the distributional consequences of these shocks. It has been shown that, in a basic New Keynesian model, some of the above mentioned shocks exhibit the so called *divine coincidence*, whereby the central bank does not face any trade-off between stabilizing inflation and the welfare-relevant output gap (Blanchard and Galí, 2007). In such cases, strict inflation targeting (IT) is the optimal policy, which also stabilizes aggregate demand. [23] Based on the above considerations, in

[23] Blanchard and Galí (2010) and Ravenna and Walsh (2011) show that, in the presence of labor market frictions, the divine coincidence vanishes, but delivering price stability remains very close to the optimal policy. This is also the case in our model with SAM frictions where the divine coincidence does not hold either but strict

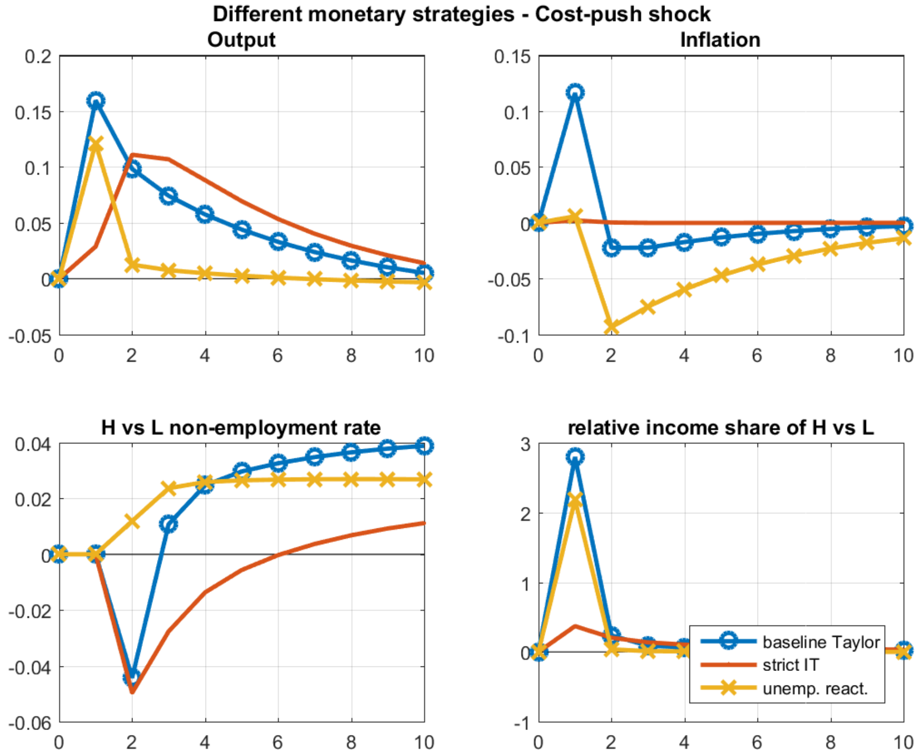

Figure 6: IRFs to a favourable cost push shock under different monetary policy rules – case of capital-skill complementarity

our setup this also means that the relative income share of high-skilled workersr would be stabilized under a strict IT regime in the face of divine coincidence shocks. Indeed, approximating strict IT by setting $\zeta^{\pi} = 100$ in our Taylor rule, monetary policy manages to almost completely stabilize output (hence demand) and the income shares of both types of labor in the face of positive TFP, government expenditure or other expansionary demand shocks.

Some shocks, however, are not subject to the divine coincidence, meaning that a monetary policy rule focused solely on price stability might lead to large output fluctuations, and thereby deviations in the relative income share of high-skilled labor. To examine this, we look at the IRFs of a favourable *cost-push shock*, which introduces a trade-off between inflation and output stabilization.[24] In Figure 6 we compare these responses under various monetary policy regimes. The baseline policy rule features $\zeta^{\pi} = 1.5$, while with strict IT it becomes $\zeta^{\pi} = 100$. We also introduce another regime under which the central bank explicitly reacts to the deviation of unemployment from its steady state, i.e. in terms of (31) we set $\zeta^{u} = 0.5$ together with the baseline $\zeta^{\pi} = 1.5$ – we can call this strategy *flexible* inflation targeting.

As Figure 6 shows, following a favorable cost-push shock, strict IT stabilizes inflation almost fully, but it cannot prevent output from rising above its steady-state value. However, even with a much more agressive stance against inflation than in our baseline Taylor rule, strict IT leads

IT still does a good job in stabilizing the economy in response to these shocks.

[24]For simplicity, cost-push shocks are included as an ad-hoc additive term in the pricing equation

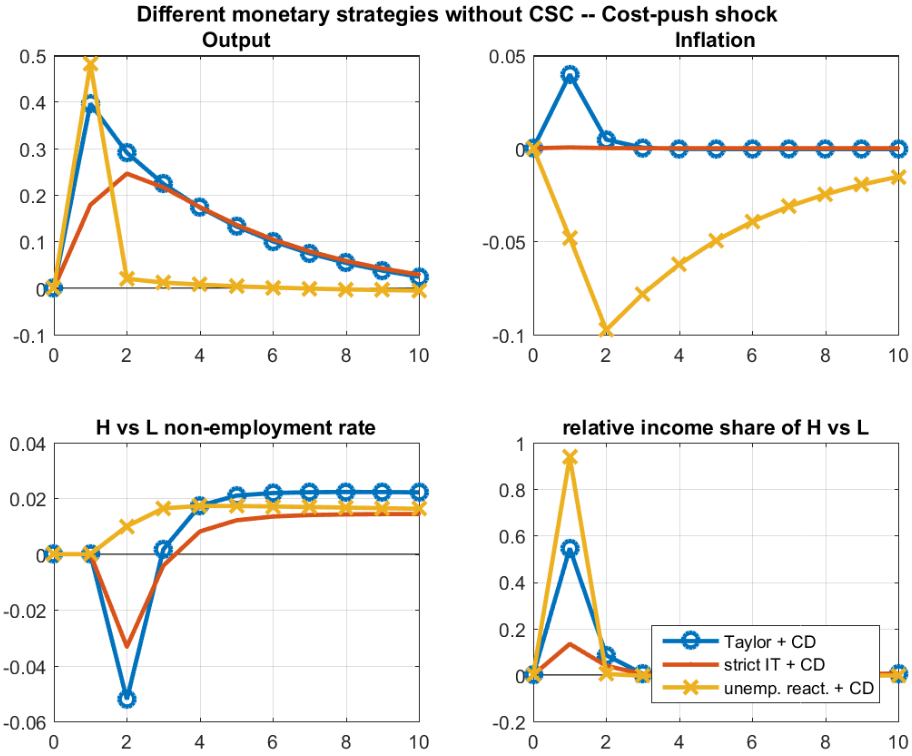

Figure 7: IRFs to a favourable cost push shock under different monetary policy rules – *without* capital-skill complementarity (i.e. standard Cobb-Douglas production functions)

to a smaller increase in output.[25] This is because the strict commitment to price stability helps the central bank manage inflation expectations more efficiently, and this improves the trade-off along the Phillips Curve. This is why a given change in the inflation rate requires a smaller sacrifice in terms of output deviation. In contrast, when monetary policy becomes relatively more lenient on inflation, by explicitly reacting to unemployment as well, as in the flexible rule, the trade-off becomes worse. As can be seen from Figure 6, although output rises less than in the baseline, this comes at the cost of inflation being lower than target. Compared to strict IT, output even rises *more* on impact (despite the explicit reaction), although later on the baseline rule manages to almost completely bring the expansion to an end

These different demand stabilization properties of our alternative policy rules also carry over to their ability to offset the distributional consequences of the cost-push shock. In this respect, our baseline Taylor rule performs the worst. The regime with unemployment reaction delivers slightly more stable paths for the relative income share, while strict IT prevents most of the deviation on impact even though later on it fares a bit worse than flexible IT.

The above ranking of different monetary policy rules in terms of their efficacy in muting the

[25]Under our baseline monetary policy regime, inflation *rises* on impact in response to a favourable cost-push shock, which is unusual. The reason for this move is that the expansionary effects of the shock set off such a large increase in demand that it outweighs the initial deflationary pressure. The latter can dominate, however, if monetary policy becomes relatively less focused on inflation, as is the case with $\zeta^u = 0.5$.

distributional effects of cost-push shocks is derived under CSC. Hence, it is interesting to check how this ranking would change were CSC absent. To do so, we plot impulse responses under our alternative monetary policy regimes using the benchmark CD production function instead of CSC. The results are presented in Figure 7.

The main difference between Figure 6 and Figure 7 is that in the latter the flexible rule (in yellow), now leads to the largest increase in output as well as the largest deviation in the relative income share of high-skill labor. This is in contrast to the case with CSC in Figure 6, where an explicit reaction to unemployment deviation slightly *improved* the performance of monetary policy in stabilizing inequality compared to the baseline Taylor rule. Therefore, from an inequality point of view, the presence of CSC seemingly favors a monetary policy rule which explicitly reacts to unemployment as well. As illustrated earlier, this is so because CSC exacerbates the increase in inequality resulting from expansionary shocks to the economy. In that case, monetary policy can counteract those effects. In the absence of CSC those effects are much smaller making it unworthy for monetary policy to correct them at the cost of higher aggregate instability.

6.3 Sensitivity analysis

6.3.1 Elasticities of substitution

CSC is captured in our model through the elasticity of substitution between capital and high-skill labor, $\frac{1}{1-\gamma}$. This elasticity is below unity in our baseline parameterization signaling the complementary relationship between them. Given the above effects of CSC, increasing complementarity through this parameter should lead to similar changes as the switch from the benchmark CD production function to the one in our baseline scenario with CSC (and vice versa when decreasing it).

As Figure 8 shows, the results of this exercise confirm our previous findings. A larger degree of complementarity (i.e. lower elasticity of substitution, with yellow lines) favors high-skilled workers even more after an expansionary monetary-policy shock. As demand for capital increases, firms need even more skilled labor relatively to unskilled workers. This is reflected in a larger skill premium which in turn drives the relative income share higher.[26] Conversely, decreasing complementarity (red lines) results in the opposite changes. Notice, that in the latter scenario we still maintain the relation $\frac{1}{1-\alpha} = 1.67 > \frac{1}{1-\gamma} = 1.33$, which captures CSC in the sense defined by Lindquist (2004), even though capital and high-skilled workers are now substitutes (but less so than capital and unskilled workers are). If we went up to the point where $\frac{1}{1-\gamma} > \frac{1}{1-\alpha}$, then the CSC channel would switch sign and it would actually *reduce* the increase in the relative income share compared to the case with only asymmetric SAM frictions and a CD production

[26] Non-employment *rate* differences, however, do not change in favor of high skilled workers. This might only be some base effect, as differences in high and low- skilled employment *levels* change in favor of the skilled workers.

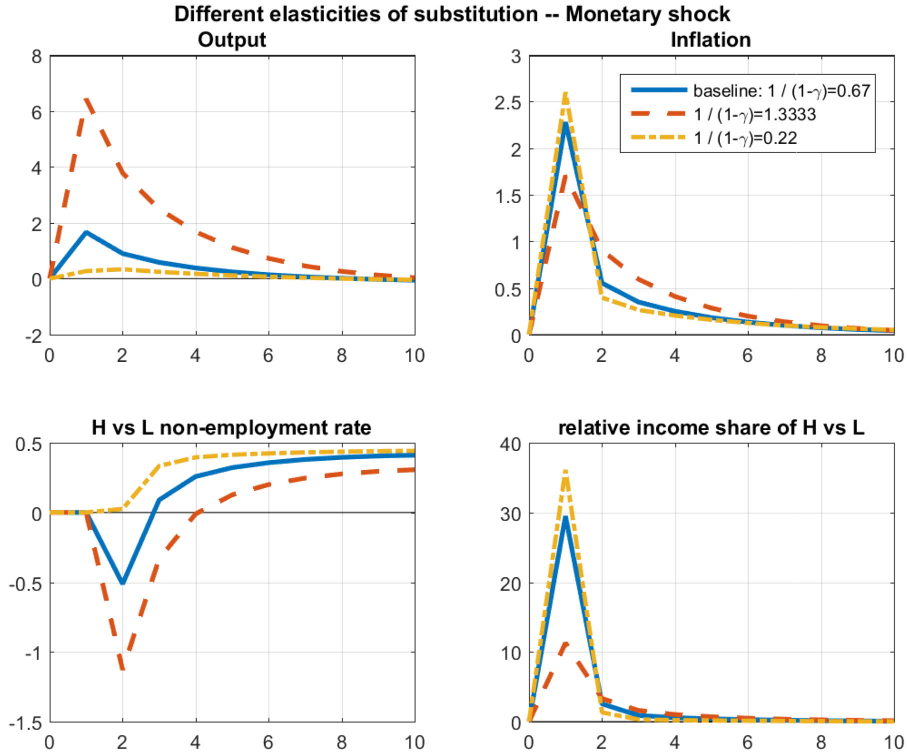

Figure 8: The effects of different elasticities of substitution $\frac{1}{1-\gamma}$ between capital and skilled labor – expansionary monetary policy shock, asymmetric SAM frictions, capital-skill complementarity

function, instead of amplifying it.

Notice also the similar change in the responses of output in Figure 8 compared to Figure 4. Increasing CSC (either through γ or through switching from the benchmark CD to our baseline production function) mutes the expansion of aggregate demand after a monetary easing. This is becase the production function is becoming "more restrictive" in the sense that the same inputs can be combined in fewer ways, or equivalently, most combinations will yield lower output (just think about the isoquants). In contrast, increasing the degree of substitutability in the production function will allow output to react more flexibly – in this case even more than with the benchmark CD production function.

6.3.2 Wage rigidities

All the above results about inequality are driven mainly by large (and unrealistic) changes in wages, as is often the case in SAM models. Therefore it is important to check whether they still hold in the presence of wage rigidities. We do this by considering positive values for ρ_w^k in equation (27) capturing (in an admittedly ad-hoc way) higher wage persistence. After introducing symmetric wage rigidities by setting $\rho_w^H = \rho_w^L = 0.8$, the signs of the effects of CSC and asymmetric SAM frictions in response to an unexpected monetary easing (as identified by the bottom right panel of Figure 5) do not change, although their magnitude is mitigated.

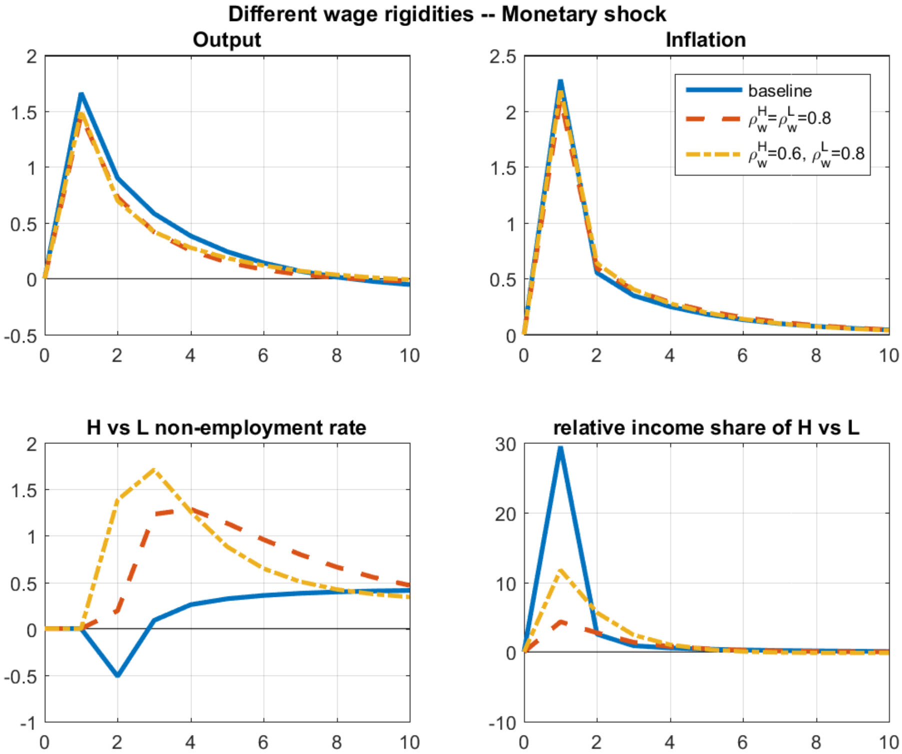

Figure 9: The effects of wage rigidities under an expansionary monetary policy shock – asymmetric SAM frictions, capital-skill complementarity

The same holds in the case of asymmetric wage stickiness where $\rho_w^L = 0.8 > \rho_w^H = 0.6$. In the online appendix we show, however, that wage rigidities do overturn our previous result regarding monetary policy strategies, namely, that from an inequality perspective the presence of CSC makes it slightly more beneficial to have a flexible policy rule which explicitly reacts to unemployment after a cost-push shock – compared to the baseline Taylor rule without such reaction. Since wage rigidities reduce the asymmetric effects of cost-push shocks, a monetary policy that focuses on price stability results more stabilizing.

If we only deviate from the baseline scenario in allowing for wage rigidity, then the effects of an unexpected interest rate cut change according to what is shown in Figure 9. Symmetric wage stickiness (with red lines) mitigates the deviation in the relative income share of high-skilled workers. However, if we increase the stickiness of low- skill wages further (asymmetric case, with yellow lines), then we almost retrieve the original effect. This is not surprising, since it is now relatively easier to increase the wages of high- skilled workers, and a rise in the skill premium would be exactly what economic forces would push for in the absence of wage rigidities. Meanwhile, aggregate variables are not affected much.

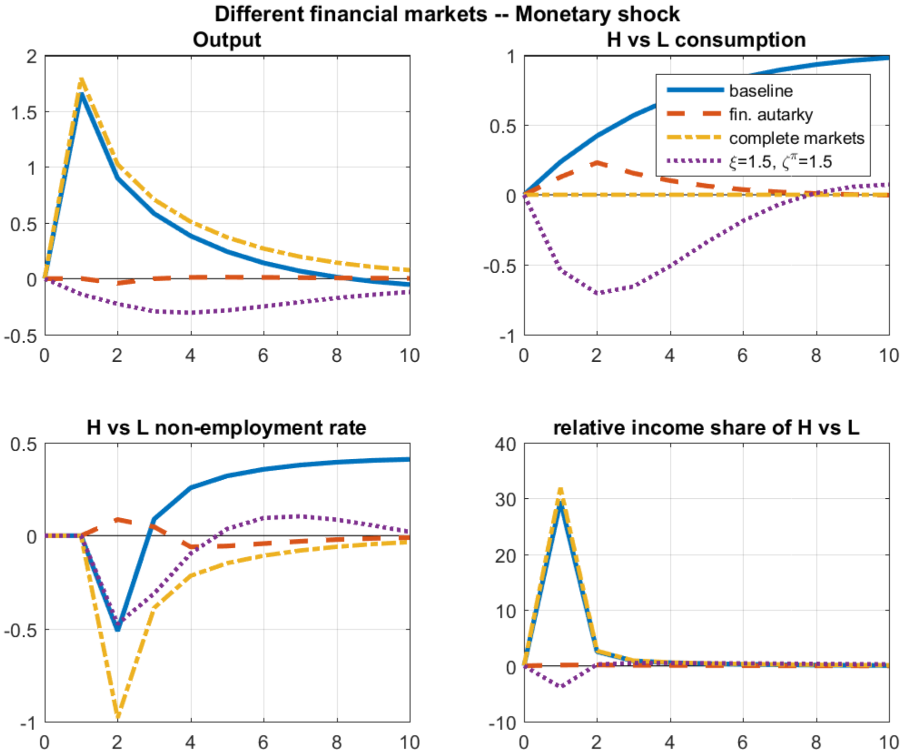

Figure 10: The effects of different financial market structures under an expansionary monetary policy shock – asymmetric SAM frictions, capital-skill complementarity. Baseline scenario is incomplete markets with $\xi = 0.001$ and $\zeta^{\pi} = 1.5$, while the financial autarky case features $\zeta^{\pi} = 27$ in order to impose a unique stable solution.

6.3.3 Effect of financial market structure

As already discussed, the incomplete financial market structure of our baseline scenario prevents perfect risk sharing in consumption among different households. In particular, the idiosyncratic effects of aggregate shocks cannot be fully insured against, leading to changes in the ratio of marginal utilities $\frac{\lambda_t^{c,i}}{\lambda_t^{c,j}}, j \neq i$ and fluctuating consumption inequality. In contrast, complete markets would prevent any such fluctuations, and keep consumption ratios constant across households.[27]

The differences between incomplete and complete financial markets can be inspected in Figure 10. Even though the skill premium and relative income shares change in a similar way under our baseline incomplete (blue lines) and complete markets setup (yellow lines), in the former case the rise in the skill premium leads to a higher consumption path for high-skilled workers relative to their unskilled peers. By contrast, under complete markets there is no such fluctuation in consumption inequality despite leading to a similar rise in the skill premium. This points to the fact that, while an expansionary monetary policy shock can have the same effects on wage inequality, its effect on consumption inequality can differ starkly, depending on the financial market structure. This is why the latter is important from a welfare point of view.

[27] For further details we refer back to Section 4.2.3.

Notice that aggregate variables and prices move very similarly under complete and incomplete markets. This is to be expected, as the main difference is the lack of insurance against the *idiosyncratic* aspect of shocks under incomplete markets, while even complete markets cannot insure against *aggregate* shocks. As a result, individual variables differ noticeably across the two cases, as can be seen from the consumption and non-employment rate responses of different workers. Recall, that complete markets provide insurance for *consumption*, but not for *leisure*, which is why even under this setup we see fluctuating differences in the non-employment rate of skilled versus unskilled workers (bottom left panel). High- skilled workers do not increase their participation as much as low- skilled workers (i.e. they consume relatively more leisure), as they feel wealthier. Notwithstanding, their non-employment rate still drops more because of their better employment prospects. This is the case throughout the complete markets responses while, under incomplete markets, the participation effect will dominate in the long run due to persistence in leisure demand.

Another relevant result to stress is how persistent the IRFs of individual variables are under incomplete markets. By introducing portfolio-adjustment costs ξ only slightly higher than zero, we ensured stationarity of the model, but only barely so. The ratio of marginal utilities still behaves close to a random walk process, so the effect of any idiosyncratic shock will be highly persistent. High-skilled workers can save the unexpected temporary increase in their wages which makes them persistently richer allowing them to consume more goods and leisure for a long time. The complete-markets model does not feature this kind of non-stationarity or persistence due to the perfect risk-sharing property, whereby a full set of securities provide insurance against any type of unexpected idiosyncratic contingency. [28]

Finally, let us consider the case of financial autarky (depicted with red lines) which can be approximated by $\xi \to \infty$. This scenario is not directly comparable to the ones above because in order to ensure a stable unique equilibrium, the Taylor-coefficient of the monetary authority had to be increased to 27. In fact, increasing ξ beyond 1.5 requires to increase the Taylor-coefficient as well.footnote The Taylor coefficient needs to be increased by gradually less, converging to 27 under current parameterization – see the online appendix for details This is an important result, as it shows that, in the absence of financial markets, monetary policy might not be able to pin down the path of prices unless it reacts very aggressively to inflation deviations from its target. The reason is that, with less access to financial instruments, agents become less responsive to interest rate changes when making their consumption-saving decisions, namely, they start behaving in a similar way to hand-to-mouth households. This unresponsiveness to

[28] The incomplete markets model would not exhibit non-stationarity either if we solved it globally in a non-linear way – instead of by local approximation around the non-stochastic steady state. In that case consumption growth would depend on asset holdings even without imposing portfolio adjustment costs. This is due to the combination of idiosyncratic uncertainty and the presence of a borrowing constraint, where the risk of hitting the latter would constrain individual borrowing (precautionary saving motive). Therefore, assets would become an important state variable in the agent's decision problem, providing a feedback into consumption and inducing stationarity.

changes in interest rates compromises the stabilizing effect of the Taylor principle which is needed to prevent sunspot equilibria.

6.4 Welfare analysis

6.4.1 Monetary expansion

The above discussion has shown how monetary policy might affect labor income inequality in the presence of CSC and asymmetric SAM frictions. We have also seen how this inequality might translate into consumption and leisure inequalities under different financial market structures. In this section we proceed to explore the welfare consequences of these fluctuations in inequality.

We consider the effects of an expansionary monetary policy shock for $T = 20$ periods (after which the shock fades away) and calculate the following consumption equivalent welfare measures.

$$\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u(\mu\ c^k, l^k) = \underbrace{\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u\left(c_t^k, l_t^k\right)}_{W_{0,T}}$$

$$\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u(\mu^{\text{level}}\ c^k, l^k) = \underbrace{\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u\left(c^k \frac{C_t}{C}, l^k \frac{L_t}{L}\right)}_{\widetilde{W}_{0,T}}$$

$$\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u(\mu^{\text{ineq}}\ c^k \frac{C_t}{C}, l^k \frac{L_t}{L}) = \underbrace{\sum_{t=0}^{T} \beta^t \sum_k \varphi^k u(c_t^k, l_t^k)}_{W_{0,T}}$$

$$\sum_{t=0}^{T} \beta^t u(\mu_k\ c^k, l^k) = \underbrace{\sum_{t=0}^{T} \beta^t u\left(c_t^k, l_t^k\right)}_{W_{0,T}^k} \quad \text{for } k \in H, L, E$$

where μ is a measure of aggregate welfare comparing the $T = 20$ periods under the monetary shock (an unexpected cut of the interest rate by 100 bp.) with the case of no shock (i.e., the steady state). This can be decomposed into contributions of a change in the average level of consumption and leisure on the one hand (μ^{level}), and of a change in the dispersion of consumption and leisure across different households (μ^{ineq}) on the other hand. For this decomposition we have constructed a synthetic case where the distribution of individual variables mimics that of the steady state but everything is scaled up to reflect average levels in the actual shock scenario. Finally, μ_k shows how individual households' welfare is affected as a result of the shock.

As Table 2 shows, in our baseline scenario a monetary expansion actually *reduces* aggregate welfare by 0.07 % of steady state consumption. The change in average consumption (which increases) and leisure (which falls due to higher participation rates) are such that they exactly offset each other in terms of welfare. Therefore, the whole reduction in welfare is driven by

100∗	$\mu - 1$	$\mu^{\text{lev}} - 1$	$\mu^{\text{ineq}} - 1$	$\mu_H - 1$	$\mu_L - 1$
baseline	−0.07	0.00	−0.07	2.06	−0.17
$\zeta = 100$	0.33	0.07	0.26	1.26	0.35
complete markets	−0.82	−0.91	0.08	−0.57	−0.88
sym SAM + CD	3.15	1.72	1.38	3.15	3.18
asym SAM + CD	1.20	0.46	0.74	2.02	1.12
$\gamma = 0.25$	0.34	−0.12	0.46	1.79	0.23
$\gamma = -3.5$	−0.10	0.08	−0.17	2.27	−0.19

Table 2: Consumption equivalent welfare measures shown in percentages, i.e. in the form $(\mu - 1) * 100$. These compare the effects of an unexpected monetary expansion in the following $T = 20$ periods relative to the no shock scenario.

higher inequality across households. The last two columns show that high-skilled workers are better off by 2.06 % of their steady state consumption, while low- skilled workers are worse off by 0.17 %.[29] This is driven by the aforementioned rise in consumption inequality, driven in turn by an increase in the relative income share. Yet, these measures also include different fluctuations in leisure, which initially falls less, and then increases persistently higher for high-skilled than for low-skilled workers.

To filter our the effects coming from leisure (and focus only on consumption), we next examine a scenario with nearly inelastic labor supply ($\zeta = 100$), so that leisure and participation changes become negligible. This shows that now both types of workers are better off as a result of a rise in their consumption, but this effect is larger for high-skilled workers since they can afford even more consumption due to the rise in the skill premium.[30] In contrast, complete markets prevent any fluctuations in consumption inequality (even with an increasing relative income share of the skilled workers), while both types of workers have to work more which makes them both worse off. This negative effect is again larger for unskilled workers, but they started from a higher leisure level to begin with. Thus, in terms of "leisure-inequality" this actually improves on aggregate welfare (hence the positive μ^{ineq}).

In line with our earlier results, an expansionary monetary policy shock leads to negative welfare effects through its adverse effects on inequality under the baseline scenario of CSC and SAM asymmetry. Similarly to the ranking in the responses of relative income shares of high *vs.* low-skilled workers (which translate into similar differences in consumption inequality), it can be seen that the contribution of inequality to welfare (μ^{ineq}) follows a similar pattern. In the case with symmetric SAM frictions and no CSC, the skill premium remains constant, consumption and leisure inequality do not change, and both types of workers become better off by approximately the same amount (due to increasing consumption levels and persistently falling participation).[31]

[29] Entrepreneurs are also worse off due to falling profits, and therefore falling consumption. The countercyclicality of profits is a well-known feature of New Keynesian models.

[30] This rise in consumption inequality across workers is offset by a fall in entrepreneurs' consumption, which is why μ^{ineq} is positive, i.e., overall consumption inequality falls.

[31] The fact that $\mu^{\text{ineq}} > 0$ is again due to the fall in entrepreneurs' consumption, which improves consumption inequality.

As we introduce the SAM asymmetry only, we see that the rise in the skill premium leads to larger inequality also in consumption which is why μ^{ineq} drops relative to the benchmark model. Finally, when we add CSC on top of asymmetric SAM frictions (our baseline scenario), μ^{ineq} drops even further, just like when we further increase the degree of CSC by making high-skilled workers even more complementary to capital ($\gamma = -3.5$). This illustrates the effect of CSC in making a monetary expansion contribute to rising inequality and decreasing welfare.

6.4.2 Favourable cost-push shock

In Section 6.2 we have already seen how the effects of cost-push shocks on the macroeconomy and inequality can be different under various monetary policy strategies. Table 3 shows how these policy rules perform in terms of welfare and how they interact with CSC.

100*	$\mu - 1$	$\mu^{\text{lev}} - 1$	$\mu^{\text{ineq}} - 1$	$\mu_H - 1$	$\mu_L - 1$
baseline (Taylor)	−0.02	−0.03	0.01	0.21	−0.03
unemp. react	−0.05	−0.08	0.03	0.07	−0.05
strict IT	0.01	0.02	−0.01	0.14	0.00
CD (Taylor)	0.10	0.01	0.09	0.19	0.09
CD + unemp.react.	0.06	−0.02	0.08	0.12	0.05
CD + strict IT	0.10	0.08	0.02	0.14	0.10

Table 3: Consumption equivalent welfare measures shown in percentages, i.e. in the form $(\mu - 1) * 100$. These compare the effects of a favourable cost-push shock in the following $T = 20$ periods relative to the no shock scenario.

Under CSC, an explicit reaction to unemployment by the central bank leads to more favourable welfare outcomes with respect to inequality (μ^{ineq} being higher). In contrast, the relation is reversed in the absence of CSC (with a standard CD production function). This is in line with what Figures 6 and 7 suggest, namely, that flexible inflation targeting stabilizes the skill premium (and therefore consumption inequality) more than the baseline Taylor rule only under CSC, but otherwise it does not. Moreover, both with and without CSC, a flexible Taylor rule delivers lower aggregate welfare since lower average levels of consumption and/or leisure (μ^{lev} being lower) dominate differences in μ^{ineq}. Therefore, in spite of its slightly more favourable inequality features, it is not the preferred policy rule even in the presence of CSC.

Strict IT performs the best in terms of aggregate welfare both under CSC and CD production technology (highest μ). However, this is mainly due to its ability to deliver higher average levels of consumption and/or leisure but not because it fares well in terms of inequality – which becomes higher (lower μ^{ineq}). This is somewhat in contrast to what we would expect based on Figures 6 and 7 where, in the face of a favourable cost-push shock strict IT was the most successful rule stabilizing the skill premium, and hence consumption inequality. The reason is that even though consumption inequality is better under strict IT, "leisure inequality" worsens considerably. As output expands, high-skilled workers increase their labor force participation more than under other policy rules (without actually earning that much more), which affects inequality adversely since they consume less leisure than their low-skilled peers to begin with.

7 Conclusions

In order to improve our understanding of the channels through which monetary policy might affect inequality, we have built a New Keynesian model with capital-skill complementarity (CSC) in the production function and asymmetric search and matching (SAM) frictions between high and low-skilled workers. Our main result is that an unexpected monetary easing increases earnings inequality by raising the relative income share of high- skilled workers. Disentangling the effects of different sources of heterogeneity we find that the *interaction* of CSC and SAM asymmetry is crucial in delivering this result. In effect, while the increase in labor demand induced by an unexpected cut in interest rates leads to larger wage increases for high-skilled workers with smaller matching frictions (SAM-asymmetry channel), the increase in capital demand amplifies this wage divergence due to these workers being more complementary to capital than substitutable unskilled ones are (CSC channel).

Our second main result is that, in the face of shocks which introduce a trade-off between stabilizing inflation and aggregate demand, strict inflation targeting is the most successful rule in terms of stabilizing measures of labor income inequality and maximizing welfare– while, for shocks with the *divine coincidence* property, strict IT manages to completely stabilize the whole economy and maximize welfare. A more unusual finding is that the presence of CSC alters the ranking of policy rules as regards their ability to stabilize the relative income share of high-skill labor: unlike the case of no CSC, reacting explicitly to unemployment slightly improves upon the performance of a basic Taylor rule which only reacts to inflation. The latter result, however, is not robust to different parameterizations, such as higher capital intensity or wage rigidities and does not always deliver higher aggregate welfare.

Our findings are not to be taken as suggestions for what optimal monetary policy should be, nor as proposals that central banks should consider reacting to measures of inequality. Issues of inequality might be best dealt with by other policy areas led by elected officials. Nonetheless, it is worth being aware of the potential distributional consequences of monetary policy actions even if it is not among the objectives in the mandate of central banks. We aimed to contribute to a deeper understanding of these forces.

That said, our main result that monetary easing increases earnings inequality should also be interpreted with caution. We have focused in this paper only on one particular channel, namely the joint effect of CSC and asymmetric SAM frictions, while in reality the channels through which monetary policy affects inequality are more complex than what our model is capable of capturing. For example, as pointed out in the Introduction, an unexpected cut in interest rates should increase inflation which harms lenders and benefits borrowers, and conversely with an unexpected rise in interest rates. As a result, this alternative channel, which is not accounted for in this paper, would operate the other way around: inequality goes down (up) when the interest rate is subject to a negative (positive) shock, whereas our mechanism implies that it goes up (down). However, the alternative mechanism impinges on capital/ debt income whereas ours operates trough wages and employment, which constitute the two key variables we have focused on here to highlight our proposed channel which, to our knowledge, is novel. For a more complete picture and a more comprehensive welfare study, further analysis and different models are needed. Lastly, the implication that inequality would go down in the face of contractionary monetary shock could not hold if high-skilled workers were allowed to search for low-skill jobs, crowding out the latter workers. In our model with a single intermediate good, we do not allow for these spillovers. Yet, this remains high in our current research agenda since this feature could lead to asymmetries in the effects of monetary policies on labor income inequality.

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