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The Optimal Graduated Minimum Wage and Social Welfare

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ABSTRACT

The Optimal Graduated Minimum Wage and Social Welfare

This paper analyzes the effects of introducing a graduated minimum wage in a model with optimal income taxation in which a government seeks to maximize social welfare. It shows that the optimal graduated minimum wage increases social welfare by increasing the low-productivity workers' consumption and bringing it closer to the first-best. The paper also describes how the graduated minimum wage in a social welfare optimum depends on important economy characteristics such as the government's revenue needs, the social-welfare weight of low-productivity workers, and the numbers and productivities of the different types of workers.

JEL Classification: D60, H21, J30

Keywords: graduated minimum wage, optimal income taxation, social welfare

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1 Introduction

It has long been recognized that in a competitive economy with optimal nonlinear income taxation and variable working hours, a *constant* minimum wage cannot improve social welfare (Allen, 1987; Guesnerie and Roberts, 1987). Recently, however, Danziger and Danziger (2015) have shown that a *graduated* minimum wage (that ties the minimum wage a firm must pay to the firm's size) can provide a strict Pareto improvement even in the presence of an optimal nonlinear income tax. Essentially, the graduated minimum wage forces firms to pay low-productivity workers above their productivity and thereby slackens the high-productivity workers' incentive-compatibility constraint. Since that paper focussed on showing only that a Pareto improvement is feasible, it was ill-equipped to address questions about the optimal graduated minimum wage when the goal is to maximize social welfare.[1]

Our purpose in the present paper, therefore, is to explore the properties of the optimal graduated minimum wage. Indeed, it can be shown that when the goal is to maximize social welfare, the optimal graduated minimum wage need not provide a Pareto improvement over the allocation with only an optimal nonlinear income tax. Nevertheless, the fact that a graduated minimum wage *can* provide a strict Pareto improvement guarantees that even when it does not, it still must strictly increase social welfare compared to the allocation with only an optimal nonlinear income tax.

[1] Boadway and Cuff (2001) show that a minimum wage may be desirable if the welfare system can deny unemployment benefits to workers who refuse job offers. For analyses of a constant minimum wage with variable working hours if the environment is not competitive or taxation not optimal, see Rebitzer and Taylor (1995), De Fraja (1999), Bhashar et al. (2002), Blumkin and Sadka (2005), Strobl and Walsh (2007, 2011), Kaas and Madden (2008, 2010), Hungerbühler and Lehmann (2009), Danziger (2009a), and Basu et al. (2010). If working hours are fixed and employment is rationed such that the involuntary unemployment induced by the minimum wage falls entirely on the low-productivity workers with the smallest surplus from working, then a constant minimum wage is optimal (Lee and Saez, 2012). See also Cahuc and Laroque (2014). The importance of cultural and institutional factors in the determination of a minimum wage is emphasized in Sobel (1999), Cahuc et al. (2001), Belot et al. (2007), Boeri and Burda (2009), Aghion et al. (2011), and Boeri (2012). Taking into account that the minimum wage may affect the human-capital formation and hence the skill distribution, Gerritsen and Jacobs (2016) show that the benefits of a minimum wage may outweigh its costs.

In order to explore the properties of the optimal graduated minimum wage we employ a modified Stiglitz (1982) framework with low- and high-productivity workers and optimal nonlinear income taxation. This framework allows us to address several interesting questions. In particular, if the goal is to maximize social welfare, what effect does the introduction of a graduated minimum wage have on the low-productivity workers' consumption? And how does the graduated minimum wage in a social optimum depend on central characteristics of the economy such as the government's revenue needs, the social-welfare weight of low-productivity workers, and the numbers and productivities of the different types of workers?

In our model, a graduated minimum wage ties the minimum wage a firm must pay to the employment of its eligible workers. Accordingly, it is a nonlinear function of the working hours of a firm's low-productivity workers that forces firms to choose between different combinations of minimum wage and corresponding working hours. The graduated minimum wage is thus analogous to a nonlinear income tax that forces workers to choose between different combinations of consumption and corresponding income. We follow the optimal tax literature in assuming that income taxes can only depend on a worker's income and not be directly conditioned on wages.[2] As in Lee and Saez (2012) we also assume that a graduated minimum wage can be enforced by a system of self-reporting and potential whistle blowing by disgruntled workers leading to inspections and heavy penalties for noncompliance.

Admittedly, a graduated minimum wage may be more difficult to enforce than a constant minimum wage due to the need to keep track of the low-productivity workers' hours. However, in this respect the graduated minimum wage would not be different from other government programs – such as the Work Opportunity Tax Credit program and the Welfare-to-Work Tax Credit program – that also require keeping track of hourly wages and hours worked. Importantly, the Fair Labor Standards Act facilitates compliance by requires em-

[2] Even though it would be preferable that taxes (including transfers) depend on wages and thus be a function of worker productivities, there seems to be a strong taboo against such type-based taxation.

ployers to keep accurate records of each worker's hours worked every day, total number of hours worked each workweek, rates paid, and total daily and weekly earnings.[3]

A major finding is that a government that strives to maximize social welfare will always want to use a graduated minimum wage to increase the low-productivity workers' consumption. This is not trivial in view of the fact that an optimal nonlinear income tax does not necessarily do so. We also obtain the intuitively appealing results that the optimal graduated minimum wage decreases with the government's revenue needs and the number of low-productivity workers, but increases with the social-welfare weight of the low-productivity workers, the number of high-productivity workers and the productivities of both types of workers.

2 The Model

The economy contains a continuum $n_1 > 0$ of low-productivity workers. Their utilities are given by $u(c_1) - h_1$, $u' > 0$ and $u'' < 0$, where c_1 and h_1 denote a low-productivity worker's consumption and working hours. The economy also contains a continuum $n_2 > 0$ of high-productivity workers whose utilities are given by $u(c_2) - h_2$, where c_2 and h_2 denote a high-productivity worker's consumption and working hours.[4]

In addition, there is a unit continuum of identical firms that produce a single consumption good whose price is normalized to unity. A firm's output is given by $af(\ell_1) + b\ell_2$, where ℓ_1 and ℓ_2 are the total hours of low- and high-productivity workers, respectively, hired by a firm; $a > 0$ and $b > 0$ their productivity levels; $f(0) = 0$; and $f' > 0$ and $f'' < 0$. The

[3] The analysis of the effects of noncompliance with a graduated minimum wage is beyond the scope of the current paper. However, just as in the case of noncompliance with a constant minimum wage when labor is adjusted on the intensive-margin (see Danziger, 2009b), the possibility of detection and consequent backpay including awards introduces an element of income uncertainty which, given workers' risk aversion, will affect welfare both directly and through the choice of working hours.

[4] The assumption that preferences are quasi-linear in working hours greatly facilitates the analysis of problems involving optimal taxation; see Boadway et al. (2000).

production function therefore exhibits decreasing returns to scale, with the low-productivity workers having a decreasing marginal product. Consistent with the characterization of workers as having either high or low productivity, it is assumed that $af'(\ell_1) < b$ in the relevant allocations.[5] Since there is a unit continuum of firms, in equilibrium labor-market clearing implies $\ell_1 = n_1 h_1$ and $\ell_2 = n_2 h_2$.

The government has a weighted utilitarian social-welfare function

$$\gamma n_1 \left[u(c_1) - h_1 \right] + n_2 \left[u(c_2) - h_2 \right], \tag{1}$$

where $\gamma \geq 1$ is the social-welfare weight of a low-productivity worker relative to that of a high-productivity worker. The resource constraint of the economy is

$$af(\ell_1) + b\ell_2 - n_1 c_1 - n_2 c_2 = R, \tag{2}$$

where $R \geq 0$ is the government's exogenous revenue needs to finance public expenditures. The government determines a nonlinear income tax and possibly also a graduated minimum wage, and its goal is to maximize social welfare. Following the income-tax literature, we assume that the government can condition the income tax a worker pays only on her total income and not on her hourly wage.

2.1 The Benchmark Case: Social Welfare without a Graduated Minimum Wage

Suppose that wages are competitively determined so that the low-productivity workers' wage is $w_1 = af'(\ell_1)$ and the high-productivity workers' wage is $w_2 = b$. Also, in the benchmark case, suppose that the government enacts a nonlinear income tax but not a graduated minimum wage. Since the government can only distinguish workers based on

[5] Low-productivity workers must have a decreasing marginal product in order to create a rent that makes a graduated minimum wage desirable. Our results would not change if high-productivity workers also have a decreasing marginal product or if low- and high-productivity workers are employed in different firms.

their income, the income-tax scheme consists of the consumption-income bundles they are offered. The government can then differentiate between the two types of workers by designing the income tax so that each type of worker prefers the consumption-income bundle meant for her own type rather than for the other type (and making any other available bundle less attractive). This leads to the following incentive-compatibility constraints of the low- and high-productivity workers

$$u(\hat{c}_1) - \hat{h}_1 \geq u(\hat{c}_2) - \frac{b\hat{h}_2}{\hat{w}_1}, \tag{3}$$

$$u(\hat{c}_2) - \hat{h}_2 \geq u(\hat{c}_1) - \frac{\hat{w}_1\hat{h}_1}{b}, \tag{4}$$

where a circumflex is used to denote the optimal value of a variable. The last term on the right-hand side of the constraints are the hours that one type of worker must work in order to earn the same income as the other type of worker.

The government seeks to maximize social welfare (1) by devising an income-tax function that determines the consumption-income bundles for the low- and high-productivity workers subject to the resource constraint (2) and the incentive-compatibility constraints (3) and (4).[6] Assuming an internal solution, it is straightforward to show that in a social-welfare optimum with only a nonlinear income tax, constraints (2) and (4) but not (3) are binding. Standard derivations establish that the low- and high-productivity workers' consumption satisfy

$$\begin{aligned} u'(\hat{c}_1) &= \frac{\gamma n_1 b + n_2 a f'(\hat{\ell}_1) + n_2 \hat{\ell}_1 a f''(\hat{\ell}_1)}{b\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\hat{\ell}_1) - \gamma n_2 b + \gamma n_2 \hat{\ell}_1 a f''(\hat{\ell}_1)\}}, \\ u'(\hat{c}_2) &= \frac{1}{b}. \end{aligned} \tag{5}$$

Put in words, for each type of worker the marginal utility of consumption is equal to the marginal social cost of producing that consumption. While the social cost of additional

[6] Firms' profits are taxed away. Alternatively, profits could be taxed by less than 100% and the remainder distributed among the firm owners with the income tax taking the distributed profits into account. If all high-productivity workers own the firms in the same proportion and the same is true for the low-productivity workers, then our results remain unchanged. In reality the profit tax may, among other things, affect a firm's location decision and foreign direct investments.

high-productivity worker consumption is exactly equal to their disutility from the additional work required to produce this additional consumption, the situation is different for low-productivity workers. Indeed, because additional low-productivity worker consumption tightens the incentive-compatibility constraint of high-productivity workers, the marginal social cost of additional low-productivity worker consumption exceeds the utility cost from the additional work required to produce this consumption.[7] Hence, the well-known result that the optimal tax rate is zero for high-productivity workers and positive for low-productivity workers.

2.2 Social Welfare with a Graduated Minimum Wage

Suppose that in addition to a nonlinear income tax, the government can enact a graduated minimum wage $m(\ell_1)$ that sets the minimum wage as a function of the total working hours of a firm's low-productivity workers. As we shall see, due to the optimal income tax, the government then effectively sets both the realized minimum wage and the actual working hours for low-productivity workers. Thus, in addition to using the income tax to offer workers a choice between different consumption-income bundles, the government can now also use a graduated minimum wage to offer firms a choice between different minimum wage-hours bundles. Accordingly, the wage for low-productivity workers is no longer competitively determined. However, the wage for high-productivity workers is still competitively determined and equal to b.

[7] Formally,

$$
\begin{aligned}
& & u'(\hat{c}_1) &> \frac{1}{af'(\hat{\ell}_1)} \\
&\Leftrightarrow & \frac{\gamma n_1 b + n_2 af'(\hat{\ell}_1) + n_2 \hat{\ell}_1 af''(\hat{\ell}_1)}{b\{[\gamma n_1 + (1+\gamma)n_2]\, af'(\hat{\ell}_1) - \gamma n_2 b + \gamma n_2 \hat{\ell}_1 af''(\hat{\ell}_1)\}} &> \frac{1}{af'(\hat{\ell}_1)} \\
&\Leftrightarrow & \left[\gamma b - af'(\hat{\ell}_1)\right]\left[b - af'(\hat{\ell}_1) - \hat{\ell}_1 af''(\hat{\ell}_1)\right] &> 0,
\end{aligned}
$$

since $b - af'(\hat{\ell}_1) > 0$, $\gamma \geq 1$, and $f''(\hat{\ell}_1) < 0$.

The incentive-compatibility constraints of the low- and high-productivity workers are modified to

$$u(c_1^*) - h_1^* \geq u(c_2^*) - \frac{bh_2^*}{m(\ell_1^*)}, \tag{6}$$

$$u(c_2^*) - h_2^* \geq u(c_1^*) - \frac{m(\ell_1^*)h_1^*}{b}, \tag{7}$$

where an asterisk is used to denote the optimal value of a variable.

The government, however, is constrained in its choice of the realized minimum wage, that is, the one actually paid by firms, $m(\ell_1^*)$. The reason is that if $m(\ell_1^*)$ is set too high, then firms may prefer some other point on the graduated minimum-wage schedule. Also, since, by definition, the minimum wage is the lowest wage in the economy, it must be the case that $m(\ell_1) \leq b$ for all values of ℓ_1. Therefore, the most attractive that the government can make ℓ_1^* relative to any alternative ℓ_1 is to set the graduated minimum wage at all points other than ℓ_1^* of ℓ_1 to be as prohibitive as possible, i.e., $m(\ell_1) = b$ for $\ell_1 \neq \ell_1^*$. Such a graduated minimum-wage schedule leaves a firm with two alternatives: The firm can either hire ℓ_1^* hours of low-productivity labor at wage $m(\ell_1^*)$, or it can choose to pay its low-productivity workers b and be free to set their working hours as it desires. For the graduated minimum wage to be relevant, therefore, the realized minimum wage, $m(\ell_1^*)$, must be less than b and satisfy *the minimum-wage constraint;* that is, a firm must prefer to hire ℓ_1^* hours of low-productivity labor at a wage of $m(\ell_1^*)$, rather than choose a different ℓ_1 and pay the higher b to earn the alternative profit $K \equiv \max_{\ell_1} [af(\ell_1) - b\ell_1]$,[8]

$$af(\ell_1^*) - m(\ell_1^*)\ell_1^* \geq K. \tag{8}$$

The government's optimization problem is then to maximize social welfare (1) by designing an income-tax function that determines the consumption-income bundles for the low- and high-productivity workers, and a graduated minimum-wage schedule, $m(\ell_1)$. The allocation

[8] If $af'(0) \leq b$, then $K = 0$; if $af'(0) > b$, then $K > 0$.

must satisfy the resource constraint (2), the modified incentive-compatibility constraints (6) and (7), and the minimum-wage constraint (8). Similarly to the case of only a nonlinear income tax, it can be shown that in any internal solution for a social-welfare optimum with both a nonlinear income tax and a graduated minimum wage only constraints (2), (7), and (8) are binding. The solution to the government's problem of maximizing welfare subject to these constraints implies that low- and high-productivity workers' consumptions are such that

$$
\begin{aligned}
u'(c_1^*) &= \frac{\gamma n_1 b + n_2 a f'(\ell_1^*)}{b\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b\}}, \\
u'(c_2^*) &= \frac{1}{b}.
\end{aligned}
\tag{9}
$$

As in the benchmark case with only an optimal nonlinear income tax, the marginal social cost of high-productivity worker consumption equals the utility loss from producing the additional consumption, while the marginal social cost of low-productivity-worker consumption exceeds the utility loss from producing the additional consumption due to the tightening of the incentive-compatibility constraint on high-productivity workers. Hence, it is still the case that the optimal tax rate is zero for high-productivity workers and positive for low-productivity workers. However, as we will show in the next section, the optimal graduated minimum wage reduces the marginal social cost of low-productivity-worker consumption relative to the scenario with only an optimal nonlinear income tax, thereby facilitating an improvement in social welfare. As is clear from a comparison of (5) and (9), the benefit from a graduated minimum wage is due to $f'' < 0$, i.e., the decreasing returns to scale, which generates the rent that the graduate minimum wage can utilize.

3 Optimal Consumption with a Graduated Minimum Wage

Since the high-productivity workers' consumption is at the first-best level in a social-welfare optimum with only a nonlinear income tax (due to the quasi-linearity of utility in working hours) and the social cost of their consumption ($= 1/b$) is not affected by the graduated minimum wage, when a graduated minimum wage is introduced their consumption in a social optimum remains unchanged at the first-best level. However, as we now show, the low-productivity workers' consumption is positively affected by the introduction of the graduated minimum wage:

Proposition 1 *Low-productivity workers' consumption is higher with both an optimal graduated minimum wage and an optimal nonlinear income tax than with only an optimal nonlinear income tax; that is,* $c_1^* > \hat{c}_1$.

Proof. First, if $h_1^* = \hat{h}_1$, then (5) and (9) imply that

$$\begin{aligned} & u'(\hat{c}_1) - u'(c_1^*) \\ = \; & \frac{\gamma n_1 b + n_2 a f'(\ell_1^*) + n_2 \ell_1^* a f''(\ell_1^*)}{b\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b + \gamma n_2 \ell_1^* a f''(\ell_1^*)\}} - \frac{\gamma n_1 b + n_2 a f'(\ell_1^*)}{b\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b\}} \\ = \; & \frac{n_2 \ell_1^* a f''(\ell_1^*)\, (\gamma n_1 + n_2)\, [a f'(\ell_1^*) - \gamma b]}{b\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b + \gamma n_2 \ell_1^* a f''(\ell_1^*)\}\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b\}} \\ > \; & 0. \end{aligned}$$

Accordingly, if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Next, if $h_1^* < \hat{h}_1$, the derivative of the marginal social cost of c_1^* (the right-hand side of (9)) with respect to h_1^* is

$$-\frac{\gamma n_1 \left(\gamma n_1^2 + n_2^2 + (1+\gamma) n_1 n_2\right) a f''(\ell_1^*)}{\{[\gamma n_1 + (1+\gamma) n_2]\, a f'(\ell_1^*) - \gamma n_2 b\}^2}.$$

Hence, the marginal social cost of c_1^* increases with h_1^*. As the marginal utility of c_1^* decreases with c_1^* and since we have just shown that if $h_1^* = \hat{h}_1$, then $c_1^* > \hat{c}_1$, it follows that if $h_1^* < \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Finally, if $h_1^* > \hat{h}_1$, we use that the total differential of the social welfare (1) given the resource constraint (2) and the optimal choice of c_2 is

$$n_1 \left\{ \left[\gamma u'(c_1) - \frac{1}{b} \right] dc_1 + \left[\frac{af'(\ell_1)}{b} - \gamma \right] dh_1 \right\}.$$

Since $\gamma u'(c_1) - 1/b > 0$ and $af'(\ell)/b < \gamma$, the graduated minimum wage would decrease social welfare if $c_1^* < \hat{c}_1$ in addition to $h_1^* > \hat{h}_1$. Therefore, it must be the case that if $h_1^* > \hat{h}_1$, then $c_1^* > \hat{c}_1$.

Consequently, it can be concluded that $c_1^* > \hat{c}_1$. ■

The introduction of a graduated minimum wage increases the pre-tax income of low-productivity workers, thereby making it harder for high-productivity workers to mimic low-productivity workers' income. This loosens the incentive-compatibility constraint of the high-productivity workers, which allows the government to increase the after-tax income, or consumption, of the low-productivity workers compared to the optimum with only an optimal nonlinear income tax.

The proposition guarantees that a graduated minimum wage increases social welfare. After all, the government could impose a trivial graduated minimum wage that leaves the equilibrium allocation unchanged compared to the allocation with only optimal taxes. Indeed, this is precisely the scenario with a constant minimum wage, which is always optimally nonbinding in the presence of optimal income taxation. The fact that the introduction of a graduated minimum wage leads to a different allocation means that it is both nontrivial and binding, implying that it must increase social welfare.

In essence, by loosening the incentive-compatibility constraint of the high-productivity workers, the introduction of a graduated minimum wage reduces the marginal social cost of the low-productivity workers' consumption, c_1^*. As a consequence, a graduated minimum wage allows the government to raise c_1^*. However, since the high-productivity workers' modified incentive-compatibility constraint still binds, the graduated minimum wage cannot raise c_1^* so much that it reaches the high-productivity workers' consumption and becomes first-best. Thus, the graduated minimum wage mitigates, but does not completely eliminate, the social-welfare loss that stems from the inability of the income-tax system to raise low-productivity workers' consumption to its first-best level.

The introduction of the optimal graduated minimum wage may either increase or decrease low-productivity workers' hours. On the one hand, by increasing the low-productivity workers' income for unchanged working hours, the graduated minimum wage makes it less attractive for the high-productivity workers to mimic their income. This reduces the government's need to resort to increasing the working hours of low-productivity workers as a means of loosening the high-productivity workers' incentive-compatibility constraint. On the other hand, the graduated minimum wage boosts the gain of the low-productivity workers' pre-tax income associated with an increase in their working hours. This increases the scope for further loosening of the high-productivity workers incentive-compatibility constraint through an increase in low-productivity workers' working hours. As a result of these opposing forces, the graduated minimum wage has an ambiguous effect on the low-productivity workers' hours.

That the working hours of low-productivity workers may decrease implies that the optimal graduated minimum wage, while strictly increasing social welfare, does not necessarily provide a Pareto improvement. Since the graduated minimum wage does not change the high-productivity workers' consumption and increases the low-productivity workers' consumption, total consumption increases. Therefore, at least in those cases where low-productivity workers' hours decrease, high-productivity workers' hours must increase and their utilities necessarily decrease. That is, social welfare increases even though high-productivity workers are made worse off.

4 The Optimal Graduated Minimum Wage

In Propositions 2-4 which follow, we determine how the characteristics of the economy affect the optimal graduated minimum wage.[9] The fact that the minimum-wage constraint (8) binds implies that $m(\ell_1^*) = \{af(\ell_1^*) - \max_{\ell_1}[af(\ell_1) - b\ell_1]\}/\ell_1^*$. Therefore, due to the low-productivity workers' diminishing marginal product, the optimal realized minimum wage is inversely related to their optimal working hours. This fact plays a key role in the logic underlying the propositions in this section. We begin with Proposition 2, which in the first part relates the revenue needs of the government and in the second part the low-productivity workers' social-welfare weight to the optimal realized minimum wage.

Proposition 2 *The optimal realized minimum wage:*

1. *decreases with the government's revenue needs; that is, $dm(\ell_1^*)/dR < 0$;*
2. *increases with the social-welfare weight of a low-productivity worker; that is, $dm(\ell_1^*)/d\gamma > 0$.*

The higher the government's revenue needs, the less output is available for the low-productivity workers' consumption for any given working hours. As the marginal utility of low-productivity workers' consumption decreases with an increase in their consumption, the social gain from having them work more will be higher.[10] Accordingly, the low-productivity workers' hours increase with the government's revenue needs. Since the optimal realized minimum wage is reversely related to the working hours of low-productivity workers, an increase in the government's revenue needs lowers the optimal realized minimum wage.

The higher the social-welfare weight that the government attaches to the low-productivity workers, the more utility they will attain in the optimum. Since the low-productivity workers'

[9] The proofs of Propositions 2-4 are in the Appendix.

[10] Of course, the low-productivity workers may benefit from higher public expenditures if used for public goods that are of value to them.

optimal tax rate is positive (so that an increase in their working hours implies that they will receive less of a transfer from the high-productivity workers) and the high-productivity workers' consumption is independent of γ, an increase in the low-productivity workers' hours would cause the high-productivity workers' hours to go down. Hence, both the low- and the high-productivity workers' utility would go up, inconsistent with the fact that the economy was in a social optimum. It must therefore be the case that the low-productivity workers' hours decrease.[11] Due to the inverse relationship between the optimal realized minimum wage and the working hours of low-productivity workers, this will increase the optimal realized minimum wage.

We now examine the impact of the number of workers of each type on the optimal realized minimum wage.

Proposition 3 *The optimal realized minimum wage:*

1. *decreases with the number of low-productivity workers; that is,* $dm(\ell_1^*)/dn_1 < 0$*;*
2. *increases with the number of high-productivity workers; that is,* $dm(\ell_1^*)/dn_2 > 0$*.*

The first part of the proposition shows that the optimal realized minimum wage is inversely related to the number of low-productivity workers. The more low-productivity workers there are, the lower is their per-capita consumption for a fixed total number of low-productivity worker hours. Since workers have decreasing marginal utility from consumption, the social gain from additional working hours for low-productivity workers increases. Similarly to the logic underlying the first part of Proposition 2, this will increase the socially optimal number of total working hours for low-productivity workers (although each individual worker's hours may be lower). In the social optimum, therefore, the total hours of

[11] Their hours cannot remain unchanged since the minimum wage would then also be unchanged. However, an unchanged h_1 would, together with an increase in c_1 (in order to increase the low-productivity workers' utility) and an increase in h_2 to produce the additional c_1, violate the high-productivity workers' incentive-compatibility constraint (7).

low-productivity workers within each firm is greater when the number of low-productivity workers is greater. Consequently, the optimal realized minimum wage decreases with the number of low-productivity workers.

The second part of the proposition shows that the number of high-productivity workers affects the optimal realized minimum wage positively. Since high-productivity workers produce more than they consume, an increase in their numbers facilitates the transfer of consumption to low-productivity workers. Again, since workers have decreasing marginal utility from consumption, this transfer reduces the marginal social gain from additional work of the low-productivity workers. Therefore, low-productivity workers will work less, which raises the optimal realized minimum wage.

In our final proposition, we establish the relationship between each type of worker's productivity and the optimal realized minimum wage.[12]

Proposition 4 *The optimal realized minimum wage:*

1. *increases with the low-productivity workers' productivity; that is,* $dm(\ell_1^*)/da > 0$*;*
2. *increases with the high-productivity workers' productivity; that is,* $dm(\ell_1^*)/db > 0$*.*

When wages are competitively determined, then it is, of course, unsurprising that an increase in workers' productivity leads to an increase in their wages. However, because here the wage paid to low-productivity workers is not competitively determined, it is far less obvious that the optimal realized minimum wage should increase with the productivity of low-productivity workers. Nevertheless, the first part of the proposition confirms that this is indeed the case.

The intuition can be summarized as follows: An increase in the productivity of low-productivity workers increases their consumption because it reduces the social cost of their

[12] For simplicity, the proof requires that $af'(0) \leq b$, and, for the first part of the proposition, that the elasticity of f is nonincreasing, i.e., $d\left[f'(\ell_1)\ell_1/f(\ell_1)\right]/d\ell_1 \leq 0$.

consumption, both by lowering the disutility associated with the production of additional output and by loosening the high-productivity workers' incentive-compatibility constraint. At the same time, as low-productivity workers' consumption increases (and their marginal utility from consumption decreases), a greater marginal product of low-productivity labor is required to make additional working hours socially beneficial. Therefore, since the increase in productivity leads to an increase in consumption for low-productivity workers, it must also lead to an increase in their marginal product in the social optimum. This then implies that they will work less and, consequently, that the optimal realized minimum wage will be greater.

The logic underlying the second part of the proposition, that the optimal realized minimum wage is positively related to the productivity of high-productivity workers, is as follows: The higher the productivity of the high-productivity workers, the less attractive are additional low-productivity working hours relative to high productivity working hours from a social-welfare point of view. This will tend to decrease the working hours of the low-productivity workers, and, as a result, the optimal realized minimum wage.

5 Conclusion

This paper is the first to study the properties of a graduated minimum wage introduced to increase social welfare in a competitive environment. Our starting point is the well-established theoretical insight that a constant minimum wage cannot increase welfare beyond what can be obtained by an optimal income tax alone. The government's dilemma is that although it would like to redistribute resources from high-productivity workers to low-productivity workers, it is limited in its ability to do so by an incentive-compatibility constraint. Specifically, the government's redistribution scheme cannot be so generous that high-productivity workers find it preferable to mimic low-productivity workers by earning the same income and working fewer hours. However, unlike a constant minimum wage, a graduated minimum wage, by increasing the pre-tax income of low-productivity workers, makes it more difficult for high-productivity workers to mimic low-productivity workers since the former would need to work more hours to earn the income of the latter. This allows the government to further redistribute to the low-productivity workers by increasing their after-tax income by more than it could have otherwise.

An important result is, therefore, that when the government's toolbox of available policies is expanded to include a nonconstant minimum wage, welfare can be improved beyond what is achievable with only income taxes and a constant minimum wage. And why should the government not have such a policy tool available to it? After all, if the government can impose nonlinear income taxes in the interest of social welfare, why not a nonconstant minimum wage as well? It seems rather arbitrary to allow one but prohibit the other.

What are the properties of an optimal size-dependent, or as we refer to it, graduated, minimum wage? In this paper we have shown that a welfare-maximizing graduated minimum wage increases the low-productivity workers' consumption above its level with an optimal income tax alone, bringing it closer to the first-best. We have also shown that the realized minimum wage in a social-welfare optimum depends on important economy characteristics. On the one hand, the optimal realized minimum wage decreases with government's revenue needs and with the number of low-productivity workers. On the other hand, it increases with the number of high-productivity workers, the social-welfare weight of the low-productivity workers, and the productivities of the different types of workers.

We have abstracted from the possibility that firms differ in their productivities. Such firm heterogeneity would require the graduated minimum wage to also satisfy the constraint that firms prefer the minimum wage-hours bundle intended for them rather than a bundle intended for other firms. The government can potentially circumvent this additional constraint by targeting the graduated minimum wage to particular sectors where firms are relatively homogeneous. As an example, a graduated minimum wage that applies specifically to fastfood restaurants at the establishment level may be desirable as these restaurants tend to be rather homogeneous and employ a large number of minimum-wage workers. This scenario would be close to the one analyzed in this paper, and, indeed, many countries, including e.g. Indonesia, Philippines, and South Africa, have enacted sectoral (albeit constant) minimum wages.

Introducing additional firm and worker heterogeneity into our framework may also introduce social costs due to potential search and queuing of workers that vie for the high minimum-wage jobs and due to additional constraints on the income-tax schedule. Fully incorporating these considerations is a worthwhile direction for future research for which the model developed in this paper can serve as a basis. After all, even in these more complex situations a graduated minimum wage may still entice some firms to pay low-productivity workers more than their marginal product, thereby increasing their income and loosening the high-productivity workers' incentive-compatibility constraint. Hence, the basic mechanism that makes a graduated minimum wage a policy tool for improving social welfare would remain intact.

Appendix

We prove Propositions 2-4 by differentiating Condition (9) and the binding constraints (2), (7), and (8). Let

$$\begin{aligned}
A &\equiv \frac{\left[af'(\ell_1^*) - m(\ell_1^*)\right] B}{a\ell_1^* \left\{(n_2 + n_1)\, u''(c_1^*) f'(\ell_1^*) B + \gamma n_1 f''(\ell_1^*) C D\right\}}, \\
B &\equiv \left\{\left[\gamma n_1 + (1+\gamma) n_2\right] af'(\ell_1^*) - \gamma n_2 b\right\}^2, \\
C &\equiv n_1 + b n_2 u'(c_1^*), \\
D &\equiv \gamma n_1^2 + n_2^2 + (1+\gamma) n_1 n_2,
\end{aligned}$$

where $A > 0$ (since $af'(\ell_1^*) - m(\ell_1^*) < 0$, $u''(c_1^*) < 0$, and $f''(\ell_1^*) < 0$); $B > 0$; $C > 0$; and $D > 0$.

Differentiating with respect to R yields

$$\frac{dm(\ell_1^*)}{dR} = n_1 u''(c_1^*) A.$$

It follows that $dm(\ell_1^*)/dR < 0$, which proves the first part of Proposition 2.

Differentiating with respect to γ yields

$$\frac{dm(\ell_1^*)}{d\gamma} = \frac{n_1 n_2 \left(n_1 + n_2\right) \left[b - af'(\ell_1^*)\right] af'(\ell_1^*) AC}{bB}.$$

Since $b - af'(\ell_1^*) > 0$ if follows that $dm(\ell_1^*)/d\gamma > 0$, which proves the second part of Proposition 2.

Differentiating with respect to n_1 yields

$$\begin{aligned}
\frac{dm(\ell_1^*)}{dn_1} = &\left\{-\gamma n_1 n_2 \left[b - af'(\ell_1^*)\right]\left[\gamma b - af'(\ell_1^*)\right] C\right. \\
&\left. + \left[n_1 c_1^* + h_1^* n_2 m(\ell_1^*)\right] b u''(c_1^*) B + \gamma n_1 n_2 \ell_1^* a^2 f'(\ell_1^*) f''(\ell_1^*) C\right\} \frac{A}{bB}.
\end{aligned}$$

It follows that $dm(\ell_1^*)/dn_1 < 0$, which proves the first part of Proposition 3.

Differentiating with respect to n_2 yields

$$\frac{dm(\ell_1^*)}{dn_2} = \{C\gamma n_1 [b - af'(\ell_1^*)] [yb - af'(\ell_1^*)] - (bh_2^* - c_2^*) bu''(c_1^*)B\} \frac{n_1 A}{bB}.$$

Since there is redistribution away from the high-productivity workers in a social-welfare optimum, $bh_2^* > c_2^*$. Hence, $dm(\ell_1^*)/dn_2 > 0$, which proves the second part of Proposition 3.

Differentiating with respect to a yields

$$\begin{aligned}\frac{dm(\ell_1^*)}{da} \quad = \quad & \left((n_1 + n_2) f(\ell_1^*)m(\ell_1^*)u''(c_1^*) + \{af(\ell_1^*)f''(\ell_1^*) - f'(\ell_1^*) [af'(\ell_1^*) - m(\ell_1^*)]\} \frac{\gamma n_1 CD}{B} \right) \\ & \frac{A}{af'(\ell_1^*) - m(\ell_1^*)},\end{aligned}$$

where we have used that $af'(0) \leq b \Rightarrow K = 0$. Since $K = 0 \Rightarrow m(\ell_1^*) = af(\ell_1^*)/\ell_1^*$, this equals

$$\left\{ (n_1 + n_2) f(\ell_1^*)m(\ell_1^*)u''(c_1^*) + \frac{d [f'(\ell_1^*)\ell_1^*/f(\ell_1^*)]}{d\ell_1^*} \frac{\gamma af^2(\ell_1^*)CD}{h_1^* B} \right\} \frac{A}{af'(\ell_1^*) - m(\ell_1^*)}.$$

Using that $d [f'(\ell_1^*)\ell_1^*/f(\ell_1^*)] / d\ell_1^* \leq 0$, it follows that $dm(\ell_1^*)/da > 0$, which proves the first part of Proposition 4.

Differentiating with respect to b yields

$$\begin{aligned}\frac{dm(\ell_1^*)}{db} \quad = \quad & \left[\left(\gamma n_1 \left\{ \gamma b^2 - [af'(\ell_1^*)]^2 \right\} + [2\gamma b - (1+\gamma)af'(\ell_1^*)] \, n_2 af'(\ell_1^*) \right) \frac{n_2 C}{bB} \right. \\ & \left. - [bh_2^* - m(\ell_1^*)h_1^*] \, n_2 u''(c_1^*) \right] \frac{n_1 A}{b},\end{aligned}$$

where we have used that $K = 0$. The term in braces is positive since $b - af'(\ell_1^*) > 0$ and $\gamma \geq 1$ imply that $\gamma b^2 - [af'(\ell_1^*)]^2 > 0$ and $2\gamma b - (1+\gamma)af'(\ell_1^*) > 0$, and the high-productivity workers' modified incentive-compatibility constraint implies that $bh_2^* - m(\ell_1^*)h_1^* > 0$. Consequently, $dm(\ell_1^*)/db > 0$, which proves the second part of Proposition 4. ■

References

Aghion, P., Algan, Y., Cahuc, P., 2011. Civil society and the state: The interplay between cooperation and minimum wage regulation. Journal of the European Economic Association 9, 3-42.

Allen, S.P., 1987. Taxes, redistribution, and the minimum wage: A theoretical analysis. Quarterly Journal of Economics 102, 477-489.

Basu, A.K., Chau, N.H., Kanbur, R., 2010. Turning a blind eye: Costly enforcement, credible commitment and minimum wage laws. Economic Journal 120, 244-269.

Belot, M., Boone, J., van Ours, J., 2007. Welfare-improving employment protection. Economica 74, 381-396.

Bhaskar, V., Manning, A., To, T., 2002. Oligopsony and monopsonistic competition in labor markets. Journal of Economic Perspectives 16, 155-174.

Blumkin, T., Sadka, E., 2005. Income taxation and wage policy: An application to minimum wage. International Tax and Public Finance 12, 713-722.

Boadway, R., Cuff, K., 2001. A minimum wage can be welfare-improving and employment-enhancing. European Economic Review 45, 553-576.

Boadway R., Cuff, K., Marchand, M., 2000. Optimal income taxation with quasi-linear preferences revisited. Journal of Public Economic Theory 2, 435-60.

Boeri, T., 2012. Setting the minimum wage. Labour Economics 19, 281-290.

Boeri, T., Burda, M.C., 2009. Preferences for collective versus individualised wage setting. Economic Journal 119, 1440-1463.

Cahuc, P., Laroque, G., 2014. Optimal taxation and monopsonistic labor market: Does monopsony justify the minimum wage? Journal of Public Economic Theory 16, 259-273.

Cahuc, P., Zylberberg, A., Saint-Martin, A., 2001. The consequences of the minimum wage when other wages are bargained over. European Economic Review 45, 337-352.

Danziger, L., 2009a. The elasticity of labor demand and the minimum wage. Journal of

Population Economics 22, 757-772.

Danziger, L., 2009b. Noncompliance and the effects of the minimum wage on hours and welfare in competitive labor markets. Labour Economics 16, 625-630.

Danziger, E., Danziger, L., 2015. A Pareto-improving minimum wage. Economica 82, 236-253.

De Fraja, G., 1999. Minimum wage legislation, productivity and employment. Economica 66, 473-488.

Gerritsen, A., Jacobs, B., 2016. Is a minimum wage an appropriate instrument for redistribution? Working Paper.

Guesnerie, R., Roberts, K., 1987. Minimum wage legislation as a second best policy. European Economic Review 31, 490-498.

Hungerbühler, M., Lehmann, E., 2009. On the optimality of a minimum wage: New insights from optimal tax theory. Journal of Public Economics 93, 464-481.

Kaas, L., Madden, P., 2008. Holdup in oligopsonistic labour markets - a new role for the minimum wage. Labour Economics 15, 334-349.

Kaas, L., Madden, P., 2010. Minimum wages and welfare in a Hotelling duopsony. Economic Theory 43, 167-188.

Lee, D., Saez, E., 2012. Optimal minimum wage policy in competitive labor markets. Journal of Public Economics 96, 739-749.

Rebitzer, J.B., Taylor, L.J., 1995. The consequences of minimum wage laws: Some new theoretical ideas. Journal of Public Economics 56, 245-255.

Sobel, R.S., 1999. Theory and evidence on the political economy of the minimum wage. Journal of Political Economy 107, 761-785.

Stiglitz, J.E., 1982. Self-selection and Pareto efficient taxation. Journal of Public Economics 17, 213-240.

Strobl, E., Walsh, F., 2007. Dealing with monopsony power: Employment subsidies vs.

minimum wages. Economics Letters 94, 83-89.

Strobl, E., Walsh, F., 2011. The ambiguous effect of minimum wages on hours. Labour Economics 18, 218-228.