DISCUSSION PAPER SERIES

IZA DP No. 10273

Comparative Advantage and Agglomeration of Economic Activity

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October 2016

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

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Discussion Paper No. 10273
October 2016

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ABSTRACT

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The division of labor between and within countries is driven by two fundamental forces, comparative advantage and increasing returns. We set up a simple Ricardian model with a Marshallian input sharing mechanism to study their interplay. The key insight that emerges is that the interaction between agglomeration economies and comparative advantage involves a fundamental tension which is intricately affected by trade costs. A reduction of trade costs fosters the dispersive impact of comparative advantage in sectors governed by this force whilst the impact of agglomeration economies is enhanced by trade cost reductions in the increasing returns sector. The key implication for international trade is that the wage ratio between large and small economies is not only shaped by the primitives that determine agglomeration economies and comparative advantage but also, and differentially, by the sectoral levels of trade costs. The fundamental implication for an economic geography context where labor is mobile across locations is that partial agglomeration emerges when agglomeration economies are strong relative to comparative advantage, and this is more likely the lower are trade costs in increasing returns sectors and the higher are trade costs in sectors governed by comparative advantage. The model may serve as a foundation for an urban system where the endogenously emerging larger city exhibits more diversity in production.

JEL Classification: F12, F22, R11, R12, R13

Keywords: comparative advantage, increasing returns, labor mobility, agglomeration, offshoring, urban systems

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* We thank Johannes Bröcker, Hans Fehr, Masahisa Fujita, Philipp Harms, Carsten Hefeker, Xenia Matschke, Yasusada Murata, Volker Nitsch, Toshihiro Okubo, Shin-Kun Peng, Jens Südekum, Jacques-François Thisse and seminar participants in Augsburg, Darmstadt, Dresden, Hamburg (VfS), St. Petersburg (UEA/ERSA), WU Vienna and Tohoku University for helpful comments and discussion. We gratefully acknowledge financial support from Deutsche Forschungsgemeinschaft (DFG) through PF 360/7-1 and the JSPS Grants-in-Aid for Scientific Research.

1 Introduction

The division of labor between and within countries is driven by two fundamental forces, comparative advantage and increasing returns. For much of the recent past, research was tilted in favor of the latter and so for good reasons. The important role of trade of similar products between similar countries (e.g. France and Germany; the United States and Canada) was hard to explain with the received theories of comparative advantage. Increasing returns were also key for the development of the new economic geography, for the development of micro-foundations for urban agglomerations and for the 'new new trade theory' with heterogeneous firms.[1] Yet, comparative advantage is back. Possibly surprisingly, no one has made this point more forcefully than Paul Krugman in his Nobel Price Lecture, reflecting on the increasing returns revolution in trade and geography (Krugman 2009). The validity of his judgement is strongly manifested by the explosion of trade between advanced economies and much poorer low-wage economies, in particular the emergence of China as a giant in international trade in the last 15 years.[2] The reincarnation of comparative advantage has also led to the development of a seminal Ricardian model of technology, geography and trade (Eaton and Kortum 2002) which inspired an outburst of research on quantitative trade modelling (e.g. Costinot and Rodriguez-Claré 2014).

The aim of this paper is to study the implications of the interplay between comparative advantage and increasing returns for trade and the location of economic activity. We establish a simple unified framework which makes use of the mentioned innovation in Ricardian trade modelling and a Marshallian input sharing mechanism. *The key insight of our analysis is that the interaction between agglomeration economies and comparative advantage involves a fundamental tension which is crucially and differentially affected by trade costs.* A reduction of trade costs fosters the impact of comparative advantage in sectors governed by this force whilst the impact of agglomeration economies is enhanced by trade cost reductions in the increasing returns sector.

The key implication for international trade is that the wage ratio between large and small economies is not only shaped by the primitives that determine agglomeration economies and comparative advantage but also by the levels of trade costs across sectors. We also apply our model to an economic geography context where labor is assumed to be mobile across locations. The fundamental finding for that environment is that partial agglomeration emerges when agglomeration economies are strong relative to comparative advantage and that this is more likely

[1] See e.g. Fujita et al. (1999), Ottaviano and Thisse (2004), Duranton and Puga (2004) and Melitz and Redding (2014).

[2] This development is picked up in an exploding literature focused on the labor market effects of the rise of China and other low-wage economies (e.g. Autor et al. 2012 and Autor et al. 2014).

the lower are trade costs in increasing returns sectors and the higher are trade costs in sectors governed by comparative advantage. The model may also serve as a foundation for an urban system where the endogenously emerging larger city exhibits more diversity in production. We develop and explain these fundamental findings in great detail in sections 3 and 4.

In order to work out the tension between comparative advantage and agglomeration economies and the differential effects of trade costs with utmost transparency we integrate the theory of Eaton and Kortum (2002) with the input sharing mechanism of Ethier (1982). This yields a tractable model with a small set of parameters. Specifically, we use a parameterized version of the Dornbusch et al. (1977) model. With two locations the probabilistic representation of technologies due to Eaton and Kortum (2002) implies a handy specification of comparative advantage in the production of final goods and services. The production of these final outputs is assumed to use both labor and a CES-bundle of intermediate goods and services. The latter are produced under increasing returns by monopolistically competitive firms as in Ethier (1982). This introduces agglomeration economies based on linkages between intermediate suppliers and final output producers as in urban and regional economics (Duranton and Puga 2014). Our two choices grant modelling ease but they are also quite natural. First, comparative advantage and perfect competition are basically 'twins' in classical trade theory. Second, even though increasing returns could be captured by other mechanism than input sharing, focusing on input-output linkages has the merit that it allows us to keep track of the increasing tradability of intermediates (more on this below).[3] Ethier (1982) also pointed out that the sharing of intermediate inputs may be more important than the love of variety on the part of consumers.[4]

We should like to point out that even though we study the interplay between comparative advantage and increasing returns within a specific framework where final outputs are completely governed by the former whereas intermediates are governed by the latter, *the mechanisms that we uncover are fundamental and will apply generally in models that exhibit both comparative advantage and increasing returns*.

Related Literature. We are not the first to jointly consider comparative advantage and increasing returns. We now explain in what ways our analysis differs from previous works.

[3] Ethier (1982) provides a formalization Marshall's (1890) agglomeration economies based on linkages. Marshall's two other agglomeration mechanisms, knowledge spillovers and labor market pooling, have (in the positive, not the normative sense) many isomorphic aggregate implications as input sharing. Duranton and Puga (2004) work out the relationships in terms of sharing, learning and matching mechanisms.

[4] Feenstra et al. (1992) and Kasahara and Rodrigue (2008) provide evidence in favor of a strong productivity-enhancing role of intermediates.

Matsuyama (2013) has recently set up a model with similar ingredients. Consumers are assumed to have preferences for a continuum of final outputs as in Dornbusch et al. (1977) and these inputs are produced with a primary factor (call it labor) and a CES-basket of intermediates produced under increasing returns and monopolistic competition as in Ethier (1982). All further assumptions are different, however, and so is Matsuyama's research line. His key contribution is to endogenously explain comparative advantage. In order to achieve this, he not only assumes that final output is traded at no cost, but, crucially, that intermediates are non-tradable. This differs from our assumptions since we pursue an altogether different research line. We take comparative advantage as exogenously given for locations, reflecting technological capabilities, natural advantages or institutional advantages which favor certain outputs (Nunn and Trefler 2014). Moreover, trade costs both for final outputs and for intermediates are key for our story. In contrast to our analysis, Matsuyama (2013) is also purely concerned with trade and not with geography,

Forslid and Wooton (2003) introduce comparative advantage into a Krugman-type new economic geography model by assuming that the fixed production costs differ across products and can be ordered by size. This heterogeneity in fixed costs is meant to reflect differences in the region's relative endowments of other factors of production which are not explicitly modeled. A key implication of their model is that a lowering of trade costs first leads to increased concentration and then to dispersion of production much as in Helpman (1998). The insight that comparative advantage is a dispersion force also holds true in our analysis. Our model allows for a much more elaborate, yet simple, production structure. Fundamentally, our analysis delivers the novel implication that the location equilibrium depends on the interaction of two types of trade costs and that it is the trade liberalization in final outputs which fosters dispersion by strengthening comparative advantage whereas trade liberalization in intermediates fosters agglomeration. Finally, the parameterization of comparative advantage as in Eaton and Kortum (2002) allows us to pinpoint the effect of comparative advantage becoming more volatile (more on this below).

Our analysis is also related to further works from the new trade literature and the new economic geography where input-output linkages have also been incorporated. Examples for the first literature are the classic piece by Ethier (1982) as well as Helpman and Krugman (1985) and Van Marrewijk et al. (1997). These works typically consider only the borderline cases of full tradability or non-tradability of intermediates. Examples of the latter are the new economic geography models with vertical linkages which are surveyed in Baldwin et al (2003). Apart from Forslid and Wooton (2003) there are also works in the new economic geography tradition which consider comparative advantage in addition to increasing returns. The key examples which focus on comparative advantage due to technology differences are Ricci (1999) and Venables (1999) and the works where comparative advantage arises due to factor proportions are represented by Amiti (2005) and Epifani (2005). Neither of those works shares the focus on the differential effects of various trade costs nor the parameterization of comparative advantage highlighted in our study. Finally, our analysis is also related to the research on the micro-foundations of urban agglomeration economies and the city systems literature in the wake of Henderson (1974) as we explain in detail in section 4.

The structure of the rest of the paper is as follows. Section 2 sets up our model. Section 3 assumes that labor is immobile across locations to study the implications of the interaction of comparative advantage and increasing returns for international trade. Section 4 turns to the implications of our framework for locations which are connected by mobile labor to study its implications for economic geography. Section 5 offers conclusions.

2 The Model

General set-up. Our analysis builds on the Ricardian model of Dornbusch et al. (1977) with two locations (home and foreign), one factor of production, labor, and a continuum of *consumer goods and services.* These *final outputs* are produced with constant returns to scale under perfect competition and they can be traded subject to iceberg trade costs.

We amend this model in three ways. First, we assume that the production of final outputs makes use of labor *and a symmetric CES*-composite of *producer goods and services.* These *intermediates* are produced with labor under increasing returns and monopolistic competition. To work out the implications of this amendment, our baseline case starts with the assumption that intermediates are purely localized as in Marshall (1890). We then turn to the general case where intermediates can be traded subject to an iceberg trade cost. Second, in order to render the model analytically tractable, we parameterize the technologies for final outputs as in Eaton and Kortum (2002). One parameter governs the dispersion of productivity across outputs, a measure of comparative advantage which is assumed to be identical across locations. A second parameter captures locations' total factor productivities. This measure of absolute advantage may differ across locations. Third, we put the model to use for trade and for geography. In the geography analysis we allow for labor mobility across locations and we study the spatial equilibrium.

We now describe home's preferences and technologies. Variables and parameters pertaining to foreign are denoted by an asterisk (*).

Preferences. Preferences are defined over the consumption $c(z)$ of final goods and services $z\epsilon[0,1]$ and are assumed to take the Cobb-Douglas form:

$$U\{c(z)\} = \exp\left[\int_0^1 \ln c(z)\,dz\right]$$

The associated price index is

$$P = \exp\left[\int_0^1 \ln p(z)\,dz\right] \qquad (1)$$

where $p(z)$ is the consumer price which comprises trade costs if z is imported.

Technologies and prices. Each final output z is produced under constant-returns with unit costs

$$\kappa(z) = a(z)w^{1-\beta}P_s{}^{\beta}, \qquad P_s = \left[\int_0^n p_s^{1-\sigma}ds + \int_0^{n^*}(\tau_s p_s^*)^{1-\sigma}ds\right]^{\frac{1}{1-\sigma}} \qquad (2)$$

where w is home's wage and P_s is the CES-price index for intermediates. The mill price for an intermediate s produced in home (foreign) is denoted by p_s (p_s^*), $\tau_s \geq 1$ are the iceberg trade costs for imported intermediates, $\sigma > 1$ is the constant elasticity of substitution between any two intermediates and n (n^*) is the endogenous mass of domestic (foreign) intermediates. The cost share of intermediates is represented by the parameter $0 \leq \beta \leq 1$ across all final outputs, $a(z)$ is an exogenous technology parameter which varies across final outputs and $a^*(z)$ is the foreign analog.[5] We introduce comparative advantage as in Dornbusch et al. (1977) by assuming that final outputs z can be ranked in descending order of $A(z) \equiv a^*(z)/a(z)$. To gain tractability, we build on Eaton and Kortum (2002) and impose the specification $A(z) = [T(1-z)/T^*z]^{\frac{1}{\theta}}$, where $T, T^* > 0$ measure home's and foreign's (absolute) levels of technology, respectively, and where $\theta > 1$ is an inverse measure of the variability of productivities which indicates that comparative advantage is the stronger, the smaller is the value of θ.[6]

Producer goods and services are horizontally differentiated and symmetric, by assumption. We follow Ethier (1982) and assume that each intermediate s is produced by a single firm under increasing returns and monopolistic competition. The labor needed to produce quantity q_s is $l_s = f + mq_s$, with f and m denoting the fixed and variable inputs, respectively, so that total costs are $w(f + mq_s)$. Market clearing commands that $q_s = q_d + \tau_s q_d^*$ where domestic demand for domestic intermediates is given by $q_d = p_s^{-\sigma}P_s^{\sigma-1}\beta wL$ and foreign demand for domestic

[5] With $\beta = 0$ these parameters are labor coefficients as in Dornbusch et al. (1977).

[6] Eaton and Kortum (2002:1746-1747) assume that technologies are realizations of a random variable drawn independently for each output from a region-specific probability distribution of the Fréchet type. The specification stated in the text is implied for the case with two locations.

intermediates is $q_d^* = (\tau_s p_s)^{-\sigma} P_s^{*\sigma-1} \beta w^* L^*$, where L and L^* denote the labor force in home and foreign, respectively.[7] The Dixit-Stiglitz specification in eq. (2) implies that σ is the price elasticity of demand for intermediates and that monopolistically competitive producers charge profit-maximizing mill prices:

$$p_s = \frac{\sigma}{\sigma-1} wm \tag{3}$$

Profits of each intermediate producer are $\pi_s = (p_s - wm)q_s - wf$ which, on substituting p_s from eq. (3), can be rewritten as $\pi_s = w[mq_s/(\sigma - 1) - f]$. Free entry establishes zero profits. The level of output at which an intermediate producer breaks even is given by

$$q_s = \frac{f(\sigma-1)}{m} \tag{4}$$

We can make use of eq. (3) to rewrite unit costs to produce final output z as

$$\kappa(z) = \left(\frac{\sigma}{\sigma-1} m\right)^{\beta} a(z) w^{1-\beta} [n w^{1-\sigma} + n^* (\tau_s w^*)^{1-\sigma}]^{-\gamma} \tag{5}$$

where $\gamma \equiv \beta/(\sigma - 1)$ measures agglomeration economies in the sector of intermediates. These agglomeration economies are strong if intermediates are important in the production of final outputs (high value of β) and if the parameter σ, an inverse indicator of the substitutability of intermediate goods and services, takes on a small value. Unit costs fall as the *overall* number of intermediate goods and services increases and when an intermediate is produced locally rather than imported as in Ethier's (1982) formalization of Marshall's productivity gains associated with specialized suppliers. Eq. (5) also shows that unit costs rise with wages, the inverse productivity coefficient $a(z)$, and the variable labor input m.

3 Trade

3.1 Trade equilibrium

We now characterize the equilibrium with costly trade in final outputs and intermediates. Consumers buy final goods and services from the minimum cost source and they take into account that imports are subject to iceberg trade costs, $\tau_f \geq 1$. Producer (mill) prices for final outputs reflect unit costs. Given the technology ordering $A(z)$ there are two conditions which determine the cutoff thresholds $\bar{z}$ and $\overline{z^*}$ for the specialization across locations as in Dornbusch et al. (1977), $\kappa(\bar{z}) = \tau_f\ \kappa^*(\bar{z})$ and $\kappa^*(\overline{z^*}) = \tau_f\ \kappa(\overline{z^*})$. The first of these formalizes that domestic production is

[7] This follows from applying Shephard's Lemma to eq. (2) and to unit costs in foreign, respectively.

the minimum cost source for home consumers for the range $z \in [0, \bar{z}]$ of final outputs and that home consumers import the range $z \in [\bar{z}, 1]$ from foreign. The second condition states that foreign production is the minimum cost source for foreign consumers for $z \in [\overline{z^*}, 1]$ and that the range of final goods and services imported by foreign is accordingly given by $z \in [0, \overline{z^*}]$. Substituting unit costs (5) and rearranging, these two conditions imply the cutoffs:

$$\bar{z} = \frac{1}{1+\phi_f^{\theta}\Omega} \tag{6}$$

$$\overline{z^*} = \frac{1}{1+\phi_f^{-\theta}\Omega} \tag{7}$$

$$\Omega \equiv \left(\frac{T^*}{T}\right) \omega^{(1-\beta)\theta} \left(\frac{n\omega^{1-\sigma}+\phi_s n^*}{n^*+\phi_s n\omega^{1-\sigma}}\right)^{-\gamma\theta}$$

where $\omega \equiv w/w^*$ is the wage ratio between locations, $\phi_s \equiv \tau_s^{1-\sigma} \in [0,1]$ is a measure of trade freeness for intermediate goods and services and $\phi_f \equiv \tau_f^{-1} \in [0,1]$ is a measure of trade freeness for final outputs. Comparing (6) with (7), we immediately get $\bar{z} > \overline{z^*}$ for all $\phi_f \in [0,1)$. This implies that final goods z are produced in both locations in the interval of $(\overline{z^*}, \bar{z})$, i.e. a range of final outputs is non-traded, unless trade of final outputs is costless ($\phi_f = 1$).

Trade is balanced in the general equilibrium of this static model. Hence, the value of domestic imports of final goods and services, $(1-\bar{z})wL$, and the value of domestic imports of intermediates, $n^*\tau_s p_s^* q_{d^*}$, equal the value of final goods and services exported, $\overline{z^*}w^*L^*$, in addition to the value of exported intermediates, $n\tau_s p_s q_d^*$. Using (3) and (4) the trade balance can be written as:

$$(1-\bar{z})\omega L + \frac{\beta\phi_s n^*(\bar{z}\omega L+\overline{z^*}L^*)}{n\omega^{1-\sigma}+\phi_s n^*} = \overline{z^*}L^* + \frac{\beta\phi_s n\omega^{1-\sigma}\,[(1-\bar{z})\omega L+(1-\overline{z^*})L^*]}{n^*+\phi_s n\omega^{1-\sigma}} \tag{8}$$

The technology for final outputs eq. (2) implies that home's labor costs in final outputs are a constant share $(1-\beta)$ of revenues. Hence, $wL_f = (1-\beta)(\bar{z}wL + \overline{z^*}w^*L^*)$, where L_f denotes home's labor used for the production of final outputs. The conditions of labor market equilibrium and of zero profits for intermediates imply $wL_f = wL - wnl_s = wL - np_s q_s$. Using (3) and (4) this equation can be solved for the mass of domestic intermediates and by analogy also for the mass of foreign intermediates:

$$n = \frac{L}{f\sigma} - \frac{1-\beta}{f\sigma}\left(\bar{z}L + \frac{1}{\omega}\overline{z^*}L^*\right) \tag{9}$$

$$n^* = \frac{L^*}{f\sigma} - \frac{1-\beta}{f\sigma}[(1-\bar{z})\omega L + (1-\overline{z^*})L^*] \tag{10}$$

The general equilibrium is represented by the equation system (6) – (10) which determines the wage ratio ω, the specialization thresholds $\bar{z}$ and $\overline{z^*}$, and the mass of intermediates in home and foreign, n and n^*, respectively. Eqs. (6) and (7) imply the relationship $\phi_f^{2\theta} = \left(\frac{1}{\bar{z}} - 1\right) / \left(\frac{1}{\overline{z^*}} - 1\right)$ between the two cutoffs. Using this result to substitute $\overline{z^*}$ together with n and n^* from (9) and (10) in the domestic cutoff condition (6) and the trade balance (8), the general equilibrium system is reduced to a two-equation system in $\bar{z}$ and ω. These two equations have no closed-form solution except in special cases. In general the domestic cutoff and the wage ratio are implicitly determined and all other endogenous variables are then immediately derived.

It is instructive to explore the properties of the trade equilibrium in two steps. First we assume that intermediates are completely non-tradable, our baseline case. Then we allow for the tradability of intermediates. In both cases we highlight the role of trade costs for final outputs.

3.2 The baseline case: non-tradable intermediates ($\phi_s = 0$).

Alfred Marshall (1890) described a world with purely localized intermediates.[8] The case where $\phi_s = 0$ simplifies the analysis considerably. The trade balance then involves final outputs only, $(1-\bar{z})\omega\, L/L^* = \overline{z^*}$. This in turn implies $n = \beta L/\sigma f$ from (9) and $n^* = \beta L^*/\sigma f$ from (10). Hence, the relative mass of intermediates is proportional to the relative availability of labor in the two locations, $n/n^* = L/L^* = \lambda/(1-\lambda)$, where we denote home's share of labor in the overall economy by λ. We also fix the total amount of labor in the economy at unity from now on.

3.2.1 A simple solvable case. The model simplifies further when trade in final outputs is costless, $\phi_f = 1$. This case allows us to characterize the interplay between comparative advantage and increasing returns most transparently. Intuitively, comparative advantage then plays out most strongly since it is not hindered by trade costs. The two cutoffs then coincide, $\bar{z} = \overline{z^*} \equiv \tilde{z}$, which implies that all final outputs are traded. Using the trade balance we obtain closed-form solutions for the wage ratio and cutoff:

$$\omega = \left(\frac{T}{T^*}\right)^{\frac{1}{\theta+1}} \left(\frac{\lambda}{1-\lambda}\right)^{\frac{\gamma\theta-1}{\theta+1}}, \qquad \tilde{z} = 1 \Bigg/ \left[1 + \left(\frac{T^*}{T}\right)^{\frac{1}{\theta+1}} \left(\frac{1-\lambda}{\lambda}\right)^{\frac{\gamma\theta+\theta}{\theta+1}}\right] \tag{11}$$

[8] "Subsidiary trades grow up in the neighbourhood, supplying it with implements and materials, organizing its traffic, and in many ways conducing to the economy of its material (...)" (Marshall 1890).

In Dornbusch et al. (1977) the larger economy always has the lower wage (all else equal). This is a consequence of the strict focus on comparative advantage. Things are different in our model which introduces increasing returns in the production of intermediates. In fact, under the condition $\gamma > 1/\theta$ the larger economy has the higher wage. Intuitively, this condition is fulfilled if comparative advantage is weak (i.e. θ is big) and if agglomerative economies in the intermediates sector are strong (i.e. $\gamma = \beta/(\sigma - 1)$ is small). It is also immediate from (11) that the more populous economy produces a larger range of final goods and services for all values of γ and θ.

The technologically advanced location always has the higher wage (e.g. $\omega|_{\lambda=1/2} > 1$ if $T > T^*$) and produces a larger range of final outputs ($\tilde{z}|_{\lambda=1/2} > 1/2$ if $T > T^*$). Changes in the primitives representing comparative advantage and agglomeration economies affect the wage ratio and the cutoff only if technology levels or labor endowments differ.[9] A flattening of comparative advantage (an increase in θ) has a positive effect both on the wage ratio and the cutoff if home's labor force is bigger ($\lambda > 1/2$) and if home's initial absolute technology advantage ($T > T^*$) is not too strong. Moreover, when home has the larger labor force, the effect of an increase in the agglomeration parameter γ on both the wage ratio and on the cutoff $\tilde{z}$ is positive.

3.2.2 The general case. Unless $\phi_f = 1$ there are no closed-form solutions for the wage ratio and the two cutoffs for final outputs. However, it is easily shown that the equilibrium relative wage is uniquely determined by the trade balance equation and the two cutoffs are then also uniquely determined (see appendix A). This allows us to explore the effect of parameter changes on the wage ratio and the cutoffs using standard techniques of comparative static analysis. Note that the results derived below hold true for any initial position (wage ratio and cutoffs) of the economy.

Proposition 1. *Assume non-tradable intermediates ($\phi_s = 0$) whilst final goods and services are traded at an iceberg cost ($0 < \phi_f < 1$). Then we have the following comparative statics:*

	$\boldsymbol{\lambda}$	$\boldsymbol{T/T^*}$	$\boldsymbol{\gamma}$	$\boldsymbol{\theta}$	$\boldsymbol{\phi_f}$
$\boldsymbol{\omega}$	$\text{sgn}\,[\gamma - 1/\theta Z]$	$+$	$\text{sgn}(\lambda - 1/2)$	$+/-$	$+/-$
$\bar{\mathbf{z}}$	$+$	$+$	$\text{sgn}(\lambda - 1/2)$	$+/-$	$-$
$\bar{\mathbf{z}}^*$	$+$	$+$	$\text{sgn}(\lambda - 1/2)$	$+/-$	$+$

Legend: The signs indicate how ω, $\bar{z}$ and $\bar{z}^*$ are affected by changes in the respective parameter.

Proof: See appendix B.

[9] From eq. (11) we obtain $\text{sgn}\,(d\omega/d\,\theta) = \text{sgn}\,(d\tilde{z}/d\,\theta) = \text{sgn}\,\{\,(\gamma + 1)\ln[\lambda/(1-\lambda)] - \ln[T/T^*]\,\}$ and also $\text{sgn}\,(d\omega/d\,\gamma) = \text{sgn}\,(d\tilde{z}/d\,\gamma) = \text{sgn}\,[\lambda/(1-\lambda)]$.

We find that an increase in home's share of labor λ leads to an unambiguous increase in its range of final outputs and exports, whilst these ranges shrink in foreign, just as in the simple solvable case. The wage ratio ω is positively affected iff $\gamma > 1/\theta Z$ where $Z > 1$ (see appendix B). Hence, compared to the case considered in section 3.2.1 where final outputs are traded at no cost, the condition for a positive effect on the wage ratio is relaxed. Intuitively, the impact of comparative advantage gets smaller the higher the costs of trading final outputs, and hence, the negative effect of labor force size on the wage ratio is smaller.

An across the board relative technological improvement in final outputs in home raises the wage ratio ω and raises (lowers) the range of goods produced in home (foreign).[10] The effect of an increase in the 'agglomeration economies parameter' γ on the wage ratio and the two cutoffs is positive iff home is the larger economy. A flattening of comparative advantage (increasing θ) has an ambiguous effect on the wage ratio and the cutoff levels. It depends on the technology levels and on the initial cutoff levels around which the increase in θ takes place (appendix B). Finally, increasing trade freeness in final outputs leads to a fall in $\bar{z}$ and an increase in $\bar{z}^*$ so that both economies import a greater range of final outputs. The effect on the wage ratio is ambiguous, it depends on the initial position of the economy (appendix B).

3.2.3 Summary. Summarizing the results for the case where intermediates are non-tradable we note that the effects of changes of the distribution of labor, technologies and the parameters representing comparative advantage and agglomeration economies that we have identified for the simple solvable case carry over with only minor amendments to the general case where trade of final outputs is costly. Most importantly, the higher these trade costs, the smaller the force associated with comparative advantage. Hence, the tension between comparative advantage and increasing returns in determining the wage ratio shifts in favor of the latter and, hence, in favor of the location with the higher labor force.

3.3 Tradable intermediates

Intermediate goods and services have increasingly become tradable due to path-breaking improvements in transport, information and communication technologies. We account for this by now turning from Marshall's world to the case where intermediates are tradable. We start by looking at the opposite case where intermediates are assumed to be tradable at no cost ($\phi_s = 1$)

[10] Note that $T > T^*$ implies that home is not only technologically superior in final outputs generally, but also that home has a comparative advantage in production of final outputs compared to intermediates.

since this case turns out to be analytically very tractable. We then turn to the general case where intermediates are traded at an iceberg cost, $0 < \phi_s < 1$.

3.3.1 A simple solvable case. With $\phi_s = 1$, producers of final outputs in home and foreign are faced with an identical price index for intermediates, $P_s = P_s^*$. Let's now stipulate that the wage ratio is unity, $\omega = 1$. It then follows from (6) and (7) that both $\bar{z}$ and $\bar{z}^*$ are fully determined by the parameters of the model and that by (9) and (10) the number of intermediates in home and foreign are also determined. Their total number is $n + n^* = \beta / f\sigma$. Using these results to calculate home's net imports of final outputs and home's net exports of intermediates yields terms that are identical, so that by (8) trade is balanced at the stipulated wage ratio $\omega = 1$. Hence, we have a trade equilibrium $\omega = 1$ fulfilling (6) – (10) as long as n and n^* are positive which holds true when

$$\max\{\lambda_1, 1 - \lambda_2\} < \lambda < \min\{1 - \lambda_1, \lambda_2\} \tag{12}$$

where $\lambda_1 < \lambda_2$,

$$(\lambda_1, \lambda_2) = \left(\frac{(1-\beta)k\phi_f^\theta\left(k+\phi_f^\theta\right)}{\beta k+(1+k^2)\phi_f^\theta+k(2-\beta)\phi_f^{2\theta}}, \frac{\left(\beta+k\phi_f^\theta\right)\left(k+\phi_f^\theta\right)}{\beta k+(1+k^2)\phi_f^\theta+k(2-\beta)\phi_f^{2\theta}} \right)$$

and $k \equiv T/T^*$. Under the mentioned assumptions the model is solvable and it exhibits "factor price insensitivity" similarly to the Rybczynski-Theorem of the Heckscher-Ohlin model. Our model features only one production factor, labor, however. That a change in λ within this interval goes along with an unchanged wage ratio of unity is essentially due to an adjustment in the size of the intermediate sector in the two locations. The larger region attracts a larger number of intermediates.

If home's share of labor falls outside of the interval (12), we expect it to affect the wage ratio. It is indeed straightforwardly proved that the wage ratio is greater than unity ($\omega > 1$) and the number of intermediates is nil in home ($n^* > n = 0$) if $\lambda < \max\{\lambda_1, 1 - \lambda_2\}$. Similarly, the wage ratio is smaller than unity and the number of intermediates is nil in foreign ($n > n^* = 0$) if $\lambda > \min\{1 - \lambda_1, \lambda_2\}$. Hence, we have:

Proposition 2. *Assume $\phi_s = 1$. If the share of labor does not differ much, then the wages are equal and the larger region attracts more intermediates. Otherwise, the larger and technologically lagging economy has the lower wage and hosts all intermediates.*

The key conclusion that we can draw from this section is that a larger and/or technologically lagging location need not have a lower wage. The range under which this statement holds true is crucially affected by the relative strength of the agglomeration economies. The more important intermediates are in the production of final outputs (high β) the wider the interval (12).

3.3.2 The general case. We now turn to the case where intermediates are tradable though at an iceberg cost. In contrast to the two extreme cases of non-tradability (section 3.2.1) and costless tradability (section 3.3.1) the trade equilibrium is no longer unique when $0 < \phi_s < 1$. We address this general case by starting with a situation where both locations have an equal share of labor and where their technology levels are also identical. We obtain:

Proposition 3: *Assume $0 < \phi_s < 1$, $\lambda = 1/2$ and $T = T^*$. Then the model described in eqs. (6) – (10) has at least three equilibria, a symmetric equilibrium where both locations produce the same quantitative range of final outputs and the same number of intermediates and two asymmetric equilibria where all intermediates are produced in one of the two locations.*

Proof: The symmetric equilibrium is immediate. The two asymmetric equilibria are verified by showing that the solution $n^* = 0$ (equivalently, the solution: $n = 0$) is consistent with the equation system (6) – (10). ■

The existence of multiple equilibria under increasing returns has been discovered in a related literature for different settings (e.g. Ethier 1982; Helpman and Krugman 1985; Van Marrewijk et al. 1997). From that perspective the emergence of multiple equilibria in our model is not a surprise. Moreover, in view of this literature it is also to be expected that this multiplicity of equilibria vanishes when we account for stability. To show this, we draw on (9) and (10) and define the net profit in the intermediate sector in home and foreign as the excess of expenditures over the revenue needed to break even. We consider the following dynamics, where the change in the number of firms producing intermediates is proportional to the net profit in each location:

$$
\begin{aligned}
\frac{dn}{dt} &= \omega(n,n^*)L - (1-\beta)[\bar{z}\omega(n,n^*)L + \bar{z}^* L^*] - f\sigma n\omega(n,n^*) \\
\frac{dn^*}{dt} &= L^* - (1-\beta)[(1-\bar{z})\omega(n,n^*)L + (1-\bar{z}^*)L^*] - f\sigma n^*
\end{aligned}
\qquad (13)
$$

Checking asymptotic stability by numerical analysis yields that of the three equilibria, only the symmetric one is stable.[11] Hence, we can focus on the symmetric equilibrium where the two locations have identical wages and where they also produce the same quantitative range of final and intermediate outputs, i.e. $\omega = 1$, $n = n^*$ and $1 - \bar{z}^* = \bar{z}$. It follows immediately that

[11] In the geography section (section 4) we identify a condition where symmetry breaks and stable asymmetric equilibria emerge.

increasing the level of trade freeness for final outputs ϕ_f reduces the range produced in home ($\bar{z}$ falls) and in the foreign location ($\bar{z}^*$ rises). Moreover, increasing θ raises the range of final outputs produced by home and foreign (i.e. $\bar{z}$ rises and $\bar{z}^*$ falls). Both locations are symmetrically affected by these two shocks, so there is no effect on the wage ratio. Notice that these results accord with our findings for the baseline case.

It is also possible to analytically explore the effects of asymmetries in labor endowments and in technology levels by looking at changes around the symmetric equilibrium. We consider changes in the relative labor endowments first and obtain:

Proposition 4 (population size). *Consider the symmetric equilibrium* $(\lambda,\ \omega, T/T^*) = (1/2\,, 1, 1)$. *An increase in home's share of labor* λ *has the following effect on the wage ratio:*

(i) *If* $\gamma < 1/\theta$ *and* $\phi_f > \hat{\phi}_f$, *then* $d\omega/d\lambda < 0$. *Otherwise* $d\omega/d\lambda \geq 0$.

(ii) *If* $2\gamma/(1+\phi_f^\theta) < 1/\theta$ *and* $\phi_s < \hat{\phi}_s$, *then* $d\omega/d\lambda < 0$. *Otherwise* $d\omega/d\lambda \geq 0$.

(iii) *If* β *is close to 0, then* $d\omega/d\lambda < 0$.

(iv) *If* θ *is close to infinity, then* $d\omega/d\lambda > 0$.

Proof: See appendix C.

The crucial lesson from proposition 4 is that the basic tension that comparative advantage and increasing returns exert on the wage ratio also prevails in the case where both final outputs and intermediates are costly to trade. The impact of the two types of trade costs is systematic. Proposition 4-*(i)* shows that an increase in home's share of labor has a positive effect on the wage ratio, if agglomeration economies (γ) are strong relative to comparative advantage ($1/\theta$) just as in the simple solvable case of section 3.2.1. The only amendments implied by 4-*(i) and* 4-*(ii)* concern the way trade costs for final outputs and for intermediates affect this balance. As is clear from these two conditions the balance shifts in favor of the wage increasing effect associated with increasing returns when trade freeness for final outputs is low, reinforcing our finding from section 3.2.2. The novel finding is that high levels of trade freeness for intermediates strengthen the impact of agglomeration economies so that it becomes more likely that the wage ratio moves in favor of the location whose labor force increases. Propositions 4-*(iii)* and 4-*(iv)* cover the limiting cases where the cost share of intermediates β becomes infinitesimally small and comparative advantage is approximately flat. In the first cases the wage ratio falls whereas it rises in the second. This is intuitive as agglomeration economies are basically eliminated when β is close to zero whereas comparative advantage is basically eliminated when θ is close to infinity.

Turning to differences in technologies we find:

Proposition 5 (technology differences). *Consider the symmetric equilibrium* $(\lambda, \omega, T/T^*) = (1/2, 1, 1)$ *. An improvement in home's technology level implies* $d\omega/d(T/T^*) > 0$, $d\bar{z}/d(T/T^*) > 0$ *and* $d\overline{z^*}/d(T/T^*) > 0$.

Proof: The same proof as for proposition 4 can be applied.

The implication of a technology improvement in final outputs in home is straightforward and accords with previous results. The range of final outputs produced by home (foreign) rises (shrinks) and this expansion (contraction) goes along with higher (lower) demand for home (foreign) labor which rationalizes the increase in the wage ratio ω. It is then immediate that the consumers in home face lower prices and have higher indirect utility. It is also immediate that the number of intermediates produced in home (foreign) shrinks (rises).

3.4 Summary and applications

Our model exhibits a fundamental tension between agglomeration economies and comparative advantage. The key lesson to be learned for international trade concerns the wage ratio between trading partners. Across all subcases studied in sections 3.2 and 3.3 we consistently find that the wage differential between large and small economies is not only shaped by the primitives that determine agglomeration economies and comparative advantage but also by the levels of trade costs across sectors. A reduction of trade costs for final outputs strengthens comparative advantage whilst the impact of agglomeration economies is enhanced by reductions in trade costs in the increasing returns sector. The former force shifts the wage ratio against larger economies whilst the latter force works in favor of large economies.

Our analysis speaks to various issues in international trade. Whereas trade models based on (Ricardian) comparative advantage imply that economies with a larger labor force have lower wages, just the opposite holds true in models based purely on increasing returns to scale. Allowing for the interaction of the two and taking the differential effect of trade costs into account provides richer implications. For example, the wages of advanced economies of different size and similar technology levels, such as Germany, Luxemburg and Belgium in Europe, are comparable, a finding that can neither be rationalized by a purely traditional nor a purely new trade model, but which may be the outcome of the interaction of comparative advantage and increasing returns. Importantly, this interplay between of comparative advantage and agglomeration economies also provides an explanation why it is empirically so difficult to identify the home market effect of new trade theory (Head and Mayer 2004).

If we apply the framework to trade between highly developed and developing economies, i.e. to a North-South trade context, we obtain the following implications. The participation of labor-rich countries in world trade (say China and India) implies that the labor force of the emerging economies rises and, hence, their range of production and exports. The model implies that this effect should be more pronounced the flatter is comparative advantage (the higher is θ). Intuitively, under these circumstances small cost differences suffice for a relocation of an industry producing a final output. Such a flattening of the the distribution of technologies across countries and regions has indeed been observed in recent decades. Bhagwati (1995) has christened this phenomenon 'kaleidoscopic comparative advantage'. Recent research suggests that it may have its origins in particular in the rapid transmission of knowledge and the convergence of institutions (Baumol 1989; Levchenko and Zhang 2015; Nunn and Trefler 2014). Trade costs have also undergone secular changes.[12] The reductions in the costs of shipping final goods and services, what Baldwin (2016) has termed the 'first great unbundling', imply that international specialization increases, i.e. the range of goods produced by home and by foreign falls and their imports increase ($\bar{z}$ falls, $\bar{z}^*$ rises). Hence there is more trade of final goods and services. The growth in world trade is reinforced by the reductions in transport, information and communication costs which raise trade in intermediates, Baldwin's (2006) 'second great unbundling'. The interplay of comparative advantage and increasing returns has important implications for the evolution of the international wage ratio $\omega = w/w^*$ between North and South. A larger economy benefits from agglomeration economies. Moreover, if comparative advantage becomes flatter, any conceivable negative effect of labor force size on the international wage ratio is dampened. Finally, if emerging economies manage to catch-up in terms of technology, then the positive effect on their relative wages is dampened if comparative advantage is weaker (high θ).

4 Geography

4.1 Spatial equilibrium and symmetry breaking

Spatial equilibrium. We now apply the model to an economic environment where labor is mobile across locations to work out the implications for economic geography. For this undertaking we

[12] Trade costs for final goods and services have dramatically fallen due to rapid falls of transport costs as a consequence of major improvements in transport technologies and trade liberalizations and there is also a recent trend of increasing trade of intermediate goods and services due to ground-breaking innovations in transport, information and communication technologies (Baldwin 2006).

abstract from any first-nature differences between locations from now on. Hence, home and foreign are characterized by identical preferences and identical technology levels, $T = T^*$. Moreover, home and foreign technologies are assumed to be perfect mirror images, guided by the parameter θ, so that the technology ratio becomes $A(z) = [(1-z)/z]^{\frac{1}{\theta}}$. Workers are perfectly mobile (no mobility costs) so that they are attracted to the location which offers the highest indirect utility. In our model indirect utilities correspond with real wages $V = w/P$ and $V^* = w^*/P^*$ for domestic and foreign workers, respectively. The ratio of indirect utilities is

$$\frac{V}{V^*} = \omega \frac{P^*}{P} \tag{14}$$

where the wage ratio ω is determined by the system (6) – (10) for a given allocation of workers across locations. The ratio of price levels is also straightforwardly determined (see appendix D). Symmetry ($\lambda = 1/2$) is a spatial equilibrium, by construction. However, due to the agglomeration economies in the sector of intermediates, this equilibrium need not be stable.

Before turning to the analysis of stability we note that an agglomeration of economic activity ($\lambda \neq 1/2$), if ever it turns out to be a spatial equilibrium in the model, can never involve full agglomeration ($\lambda = 1$ or $\lambda = 0$). This is because of comparative advantage which implies that each location is competitive in the production of at least some final outputs. *Crucially, comparative advantage is a fundamental dispersion force on a par with the centripetal forces famously listed by Fujita et al. (1999).*[13] We summarize this finding in:

Proposition 6: *The spatial equilibrium never involves full agglomeration in one location.*

Proof: See appendix E.

Analysis of symmetry breaking. We now check stability of the symmetric equilibrium. To do so, we consider the migration equation $d\lambda/dt = V - V^*$ in addition to firm entry dynamics specified in (13). Computing the eigenvalues of the Jacobian of this system of differential equations we obtain:

Proposition 7: *The condition for symmetry breaking is given by*

$$g \equiv (1-\phi_s)\left\{\phi_s\left[(1-\beta)(2\sigma - 2\beta\sigma + \beta - 2)\phi_f^{2\theta} + \beta(4\sigma - 4\beta\sigma + 2\beta - 2\theta - 3)\phi_f^{\theta} + \beta^2(2\sigma - 1)\right] + \phi_f^{\theta}\left[(2 - 2\sigma - \beta)\phi_f^{\theta} + \beta(2\theta + 1)\right]\right\} > 0 \tag{15}$$

[13] The list of Fujita et al. (1999) comprises immobile factors, land rent, the cost of commuting, congestion and other negative externalities (pure diseconomies) but not comparative advantage.

Two analytical results are straightforwardly implied by the symmetry break condition:

Corollary 1: *Consider the condition for symmetry breaking (15). Then we have:*
(i) If $\phi_s = 1$*, then* $g = 0$.
(ii) If $\gamma \geq 1/\theta$*, then* $g > 0$ *for all* $0 \leq \phi_s < 1$ *and* $0 < \phi_f < 1$.

The first part of the corollary implies that the location of workers is indeterminate when intermediates can be traded at no cost. This result is intuitive if seen against the backdrop of our trade analysis of section 3.3.1. There we have shown that for $\phi_s = 1$ there is a range of labor allocations that imply general equilibrium at a wage ratio of unity. Since $P_s = P_s^*$ and $\omega = 1$ when intermediates are traded at no cost, the implied ratio of producer prices for final outputs is also unity, $P/P^* = 1$. Hence, by (14) we have a spatial equilibrium. The second part of the corollary expresses, also quite intuitively, that for the mentioned ranges of trade costs symmetry is *always* broken if agglomeration economies (captured by γ) completely dominate our key measure of comparative advantage ($1/\theta$). To exclude these uninteresting cases, from now on we focus on parameter constellations specified in[14]

Assumption 1: Assume that $0 \leq \phi_s < 1$ and that $\gamma < 1/\theta$.

The non-linearity of the condition for symmetry breaking with respect to most of the parameters makes it impossible to obtain further general results. Analytical results can be derived for special cases, however. Moreover, due to the simplicity of our model which comprises only few parameters, agglomeration and dispersion forces are easy to disentangle numerically. We organize next section paralleling our analysis of trade (section 3). We start with the baseline case where intermediates are non-tradable. Thereafter we turn to the general case where these intermediates can be traded at a cost. This separation allows us to transparently work out the intuition of the effects of parameter changes.

4.2 The baseline case: non-tradable intermediates

When intermediates are purely localized eq. (15) simplifies dramatically. Imposing $\phi_s = 0$ the symmetry breaking condition can be rewritten as $\gamma = \beta/(\sigma - 1) > 2/[(2\theta + 1)\phi_f^{-\theta} - 1]$. The left-hand side of this inequality comprises agglomeration economies, γ. The right-hand side can be understood as an expression for the strength of comparative advantage which is positively

[14] The reason for the second restriction in assumption 1 parallels the imposition of the 'no-black-hole condition' in the new economic geography (e.g. Fujita et al. 1999).

influenced by the level of trade freeness of final outputs, ϕ_f. Intuitively, the lower the costs of trading final goods and services, the stronger the impact of comparative advantage inherent in technological differences across locations. In fact, the effect of comparative advantage is completely thwarted if trade costs are prohibitive ($\phi_f = 0$) so that the right-hand side is at its minimum (-2) whereas the impact of comparative advantage is maximal when trade is completely free ($\phi_f = 1$), so that the right-hand side is at its maximum ($1/\theta$). With lower costs of trading final outputs the dispersive force associated with comparative advantage plays out stronger which stabilizes dispersion. A fall in the costs of trading final outputs acts in favor of a more dispersed allocation of economic activity in space, however.

4.3 Tradable intermediates

We now turn to the case where all intermediates are tradable at a cost ($0 < \phi_s \leq 1$). Clear-cut comparative static results can be derived analytically for the cases covered in

Lemma 1: *Under Assumption 1 we have the following: (i) $g > 0$ holds if $\phi_f = 0$ and $0 < \phi_s < 1$. By continuity, this is true if ϕ_f is close to 0. (ii) $g < 0$ holds if $\phi_f = 1$. By continuity, this holds true if ϕ_f is close to 1. (iii) $g > 0$ holds if $\phi_s \to 1$.*

The first part of this lemma says that agglomeration emerges when the costs of trading final outputs are prohibitive or close to prohibitive, whilst dispersion prevails when trade is completely (or almost) costless. This generalizes our findings of section 4.2 to the case where intermediates are tradable. The intuition is the same: the dispersive effect of comparative advantage is completely thwarted if trade costs are (close to) prohibitive, and comparative advantage plays out most strongly when trade is (almost) completely free. The third part of the lemma implies that agglomeration emerges if trading intermediates is (close to) costless. Intuitively, agglomeration economies then play out most strongly.

We now complement these analytical results with a numerical exploration of the model. Our choice of the parameters is as follows. A reasonable value for β is 0.525 according to German and Japanese data for the last decades. The research inspired by Eaton and Kortum (2002) suggests an intermediate value of $\theta = 5.8$ for the parameter of comparative advantage. At the model's level of aggregation, a plausible value for the elasticity of substitution is $\sigma = 8$ (see Head and Mayer 2004).

Using these values we can portray the symmetry breaking condition with ϕ_s on the horizontal axis and ϕ_f on the vertical axis in Fig.1. We vary the parameter of trade freeness for final outputs between $\phi_f = 0.842$ and $\phi_f = 1$.[15] The blue curve depicts the boundary between symmetry and partial agglomeration. The symmetric equilibrium is stable above and unstable below this curve. The blue curve has a positive slope which clarifies the effect of ϕ_s. Increasing the level of trade freeness for intermediates brings about or tightens agglomeration since the economy is either kept in a state of agglomeration (initial positions below the blue curve) or brought into a state of agglomeration (initial positions above the blue curve) if the increase in ϕ_s is strong enough. Hence, increasing the trade freeness of intermediates strengthens agglomeration.

Fig. 1 Symmetry breaking

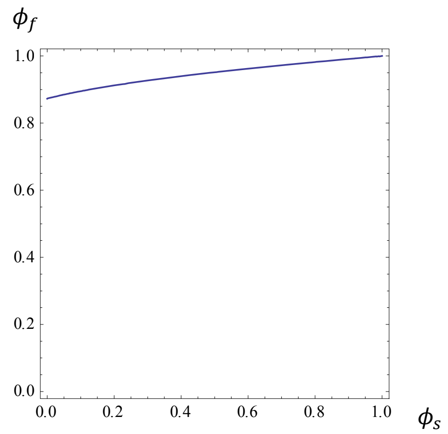


Fig.1 also shows that, starting at any given ϕ_s, a continuous increase of trade costs of final outputs ($\phi_f \downarrow$) induces or keeps agglomeration. This reinforces our analytical findings: as trade freeness for final outputs rises, comparative advantage plays out more strongly which works in favor of dispersion (symmetry).

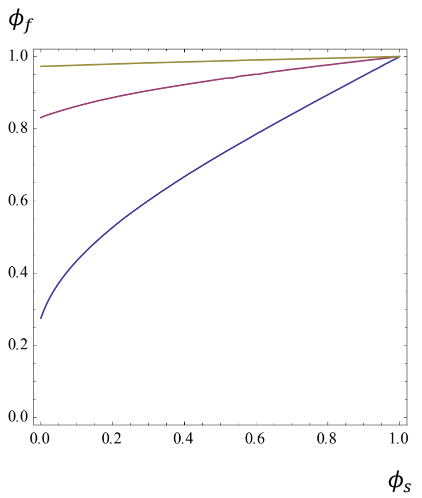
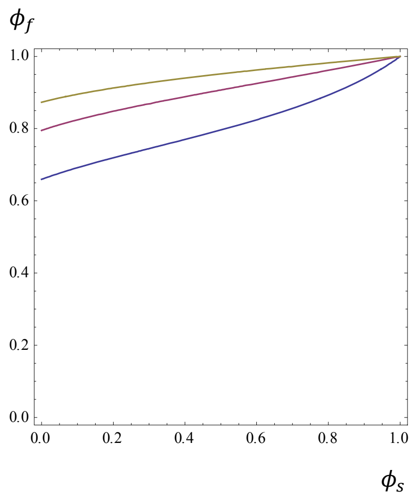
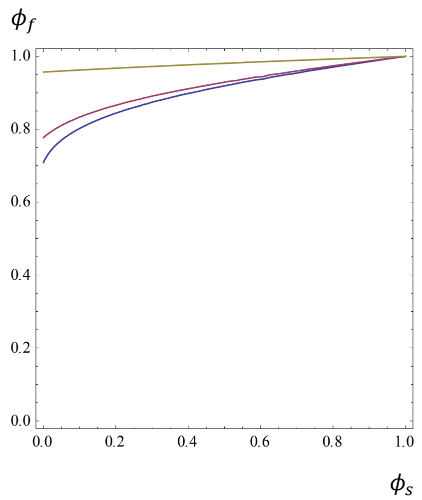
Robustness. It is clear from Fig. 1 that we are able to exhaust the parameter space for the two trade cost parameters. We have performed an extensive set of numerical exercises to check the robustness of the depicted relationship and also to explore the effects of variations of the other three parameters. We obtain the following qualitative findings (cf. Fig. 2).

First and foremost, the positive relationship between the trade cost parameter for final outputs ϕ_f and the trade cost parameter for intermediates ϕ_s holds true consistently. Hence, the fundamental result that increasing trade freeness for final outputs brings about or stabilizes dispersion whilst increasing trade freeness for intermediates strengthens or induces agglomeration holds true.

[15] The associated level of trade costs for final outputs is $\tau_f - 1 = 18.8\%$ or lower.

Second, a flattening of comparative advantage (an increase in θ) makes agglomeration more likely, as shown in Panel a of Fig.2. Third, it follows straightforwardly from (15) that $\partial g/\partial\beta > 0$. Panel b of Fig. 2 shows this result. An increase in the importance (cost share) of intermediates β works in favor of agglomeration.[16] Fourth, panel c in Fig.2 depicts that a fall in σ also strengthens agglomeration. The unambiguous theoretical results that we could derive with respect to the impact of change of the latter three parameters in the baseline case of non-tradable intermediates thus consistently hold true in the case when intermediates are tradable.

Fig. 2 Robustness: Symmetry breaking

<table>
<tr><td>a. Comparative advantage θ</td><td>b. Cost share of intermediates β</td><td>c. Elasticity of substitution σ</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>The symmetry breaking condition shifts up as θ is increased from 1.5 (blue) to 5 (red) to 10 (yellow)</td><td>The symmetry breaking condition shifts up as β is increased from 0.1 (yellow) to 0.3 (red) to 0.525 (blue)</td><td>The symmetry breaking condition shifts up as σ is lowered from 25 (yellow) to 15 (red) to 5 (blue)</td></tr>
</table>

Properties of the agglomeration equilibria. We already know that a spatial equilibrium never involves full agglomeration in one location because of comparative advantage (proposition 6). It is now straightforward to show that two asymmetric equilibria with partial agglomeration emerge, when symmetry is broken. These equilibria are mirror images of each other. Both final and intermediate goods are produced in the agglomerated location, whereas the periphery produces only final goods and no intermediates. Moreover, in the equilibria with partial agglomeration there exists a wage differential. The wage is lower in the more agglomerated location which produces both final outputs and intermediates, whereas the other location with a higher wage specializes in producing final goods. The lower wage in the agglomerated location is compensated

[16] Empirical evidence for Germany and Japan shows that β has been fairly constant (see also Becker and Muendler 2015 for Germany).

by the lower price index of final goods to equalize utility. Since all intermediates are produced in the agglomerated location, the price index of intermediates is lower there. We summarize these findings in:

Proposition 8 (partial agglomeration): *There exists a partially agglomerated equilibrium up to permutation:* $\lambda \in (1/2,1)$ *with* $n^* = 0$*, where* $w < w^*$, $P < P^*$*, and* $P_s < P_s^*$.

Proof: See appendix F.

Returning to Fig. 1 we find numerically that the symmetric equilibrium is stable in the domain above the blue curve and the asymmetric equilibria are stable below that curve. Moreover, starting on the blue boundary line, we numerically find at slightly lower levels of trade freeness for final outputs or a slightly higher levels of trade freeness for intermediates that there is a thin range of partial agglomeration where both locations produce both final outputs and intermediates. However, the periphery quickly loses all intermediates when trade freeness for final outputs is continuously lowered or trade freeness for intermediates is continuously increased. The more agglomerated region produces both a greater range of intermediates and a greater range of final outputs. Hence, our model implies that the larger location is more diversified than the small location.

4.4 Summary and applications

In this section we have applied the model to an economic geography context where labor is assumed to be mobile across locations. Our fundamental finding is that partial agglomeration emerges when agglomeration economies are strong relative to comparative advantage and that this, crucially, is more likely the lower are trade costs in increasing returns industries and the higher are trade costs in industries governed by comparative advantage. The novel insight that we derive from our framework is that the impact of a reduction of trade costs is far more complex than implied by new trade and New Economic Geography as it depends on the differential effect of trade costs in sectors governed by comparative advantage versus sectors governed by increasing returns.

We have worked out this general message by assuming that trade in final outputs is governed by comparative advantage whereas agglomeration economies prevail in the sector of intermediates. If we are willing to follow this strict interpretation, than we can apply the model to secular trends affecting trade costs (cf. section 3.4). The second unbundling, trade cost reduction for intermediate goods and services ($\phi_s \uparrow$), works in favor of agglomeration, whereas the first unbundling, i.e. increasing trade freeness for final goods and services ($\phi_f \uparrow$) strengthens the dispersive force of comparative advantage and, hence, fosters dispersion. If progress in information and communication technologies is faster than technological progress in in traditional transport technologies then the second unbundling may be more important and this would imply agglomerative tendencies for the locations involved. Clearly, the evolution of these trade costs, and their determinants is an empirical issue which merits to be looked at in further research.

Moreover a flattening of comparative advantage in the sense of an increase in θ contributes to agglomerative tendencies. Such a flattening may have different sources, as we have already noted. One such source is the transmission of knowledge among the locations involved. A further important source is the convergence of institutions. For an economic union such as the European Union, the institutional convergence may thus have contributed to the agglomerative tendencies borne out in the empirical literature (e.g. Combes and Overman 2004: Midelfart-Knarvik and Overman 2002). Hence, our framework suggests a novel channel to look at in addition to the NEG focus on trade costs.

Clearly, our model does neither feature an urban extension as considered in the monocentric city model nor a regional housing stock as in Helpman (1998). It may nonetheless serve as a foundation for a model of an urban system as we now explain. Building on the seminal analysis of Henderson (1974), the urban systems literature highlights the fundamental trade-off between agglomeration economies which drive up productivities and wages in cities and crowding economies associated with urban costs which grow with city size. A canonical version of this model also features Ethier's (1982) mechanism of input sharing (Duranton and Puga 2004; 2014). A well-known drawback of this canonical model is that each city is specialized in a single industry since intermediates are assumed to be sector-specific and non-tradable and final outputs are assumed to be traded without any cost between cities.[17] Our model relaxes these assumptions and, crucially, exhibits partial agglomeration for a wide range of parameters, since, driven by comparative advantage, both locations specialize on a range of final outputs in addition to producing intermediate goods and services. Hence, if we interpret the two locations of our model as being cities, we have an urban system with diversified cities. Moreover, our analysis then implies that the endogenously larger city is more diversified than the small city, a basic empirical fact, well-documented in urban economics (Duranton and Puga 2000).

[17] There is a literature which relaxes these two assumptions, but it is still thin (Duranton and Puga 2001; 2014). Moreover, Helsley and Strange (2014) recently developed a model which covers intermediate cases between complete specialization and complete diversity.

5 Conclusion

The division of labor between and within countries is driven by two key forces, comparative advantage and increasing returns. This analysis studies a simple unified Ricardo-Marshall framework to address the interplay between comparative advantage and increasing returns. Our analysis yields the key insight that the interaction between agglomeration economies and comparative advantage involves a fundamental tension which is crucially and differentially affected by trade costs. Whilst a reduction of trade costs fosters the impact of comparative advantage in industries governed by this force the impact of agglomeration economies is strengthened by trade cost reductions in the increasing returns sector.

The basic implication for international trade is that the wage ratio between large and small economies is not only shaped by the primitives that determine agglomeration economies and comparative advantage but also by the levels of trade costs across sectors. Applied to an economic geography context where labor is assumed to be mobile across locations, the key message is that partial agglomeration emerges when agglomeration economies are strong relative to comparative advantage and that this is more likely the lower are trade costs for increasing returns industries and the higher are trade costs for industries governed by comparative advantage. We have also seen that this model may serve as a foundation for an urban system where the endogenously emerging larger city exhibits more diversity in production.

Whilst we have worked out these implications within a specific model where comparative advantage prevails in sectors producing final outputs whereas sectors producing intermediates are driven by agglomeration economies, the mechanisms that we uncover are fundamental and should apply generally in models that exhibit both comparative advantage and increasing returns.

The tractability of the model should not only allow further generalizations (e.g. to many regions), it should in particular make it a useful tool for policy analysis and empirical work. Particular intriguing for future empirical work are the differential effects of trade costs and the pro-agglomeration effect of the flattening of comparative advantage implied by the model.

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Appendix A: Existence and uniqueness of equilibrium relative wage in the general case of the baseline scenario (where $\phi_s = 0$ and $0 < \phi_f < 1$)

With $\phi_s = 0$, we have $n/n^* = L/L^*$, so that the system of equations (6) – (10) can be reduced to the three equations:

(i) $\frac{1-\bar{z}}{\bar{z}} = \frac{T^*}{T}\phi_f^{\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}$ cutoff for home

(ii) $\frac{1-\overline{z^*}}{\overline{z^*}} = \frac{T^*}{T}\phi_f^{-\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}$ cutoff for foreign

(iii) $\omega = \frac{\overline{z^*}}{1-\bar{z}}\left(\frac{L}{L^*}\right)^{-1}$ trade balance condition

Solving (i) for $\bar{z}$ and (ii) for $\overline{z^*}$ and plugging them into (iii), we get a single equation in ω:

$$F(\omega) \equiv LL^{*2\gamma\theta}T^{*2}\omega^{1+2\theta} + (L\omega - L^*)L^{\gamma\theta}L^{*\gamma\theta}TT^*\phi_f^{\theta}\omega^{\theta} - L^{2\gamma\theta}L^*T^2 = 0$$

It can be easily shown that $F'(\omega) \lesseqgtr 0$ for $\omega \lesseqgtr \omega_0$ and that $F(0) < 0 < F(\infty)$, where ω_0 is a unique solution of $F'(\omega) = 0$. Hence, there exists a unique ω in the interval of $(0, \infty)$.

Appendix B: Comparative statics of the general case of the baseline scenario: Proposition 1

Log-linearization of the equation system (i), (ii) and (iii) yields:

(i)' $\left(\frac{-1}{1-\bar{z}}\right)\hat{\bar{z}} = \widehat{\left(\frac{T^*}{T}\right)} + \theta\,\hat{\phi}_f + \theta\,\hat{\omega} + \ln\left[\phi_f^{\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}\right]\hat{\theta} - \gamma\theta\,\widehat{\left(\frac{L}{L^*}\right)} - \gamma\theta\ln\left(\frac{L}{L^*}\right)\hat{\gamma}$

(ii)' $\left(\frac{-1}{1-\overline{z^*}}\right)\widehat{\overline{z^*}} = \widehat{\left(\frac{T^*}{T}\right)} - \theta\,\hat{\phi}_f + \theta\,\hat{\omega} + \ln\left[\phi_f^{-\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}\right]\hat{\theta} - \gamma\theta\,\widehat{\left(\frac{L}{L^*}\right)} - \gamma\theta\ln\left(\frac{L}{L^*}\right)\hat{\gamma}$

(iii)' $\hat{\omega} = \widehat{\overline{z^*}} + \left(\frac{\bar{z}}{1-\bar{z}}\right)\hat{\bar{z}} - \widehat{\left(\frac{L}{L^*}\right)}$

where we use hat-notation: e.g. $d\ln\omega = \frac{d\omega}{\omega} \equiv \hat{\omega}$.

Note that from (i): $\frac{1-\bar{z}}{\bar{z}}\frac{T}{T^*} = \phi_f^{\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}$ and from (ii): $\frac{1-\overline{z^*}}{\overline{z^*}}\frac{T}{T^*} = \phi_f^{-\theta}\omega^{\theta}\left(\frac{L}{L^*}\right)^{-\gamma\theta}$.

Using this, we can rewrite (i)' and (ii)' in a more informative / better interpretable way, so that the three equation system becomes:

(i)' $\left(\frac{-1}{1-\bar{z}}\right)\hat{\bar{z}} = \widehat{\left(\frac{T^*}{T}\right)} + \theta\,\hat{\phi}_f + \theta\,\hat{\omega} + \ln\left(\frac{1-\bar{z}}{\bar{z}}\frac{T}{T^*}\right)\hat{\theta} - \gamma\theta\widehat{\left(\frac{L}{L^*}\right)} - \gamma\theta\ln\left(\frac{L}{L^*}\right)\hat{\gamma}$

(ii)' $\left(\frac{-1}{1-\overline{z^*}}\right)\widehat{\overline{z^*}} = \widehat{\left(\frac{T^*}{T}\right)} - \theta\,\hat{\phi}_f + \theta\,\hat{\omega} + \ln\left(\frac{1-\overline{z^*}}{\overline{z^*}}\frac{T}{T^*}\right)\hat{\theta} - \gamma\theta\widehat{\left(\frac{L}{L^*}\right)} - \gamma\theta\ln\left(\frac{L}{L^*}\right)\hat{\gamma}$

(iii)' $\hat{\omega} = \widehat{\overline{z^*}} + \left(\frac{\bar{z}}{1-\bar{z}}\right)\hat{\bar{z}} - \widehat{\left(\frac{L}{L^*}\right)}$

Solving for $\hat{\omega}$, $\hat{\bar{z}}$ and $\widehat{\overline{z^*}}$ yields:

$$
\widehat{\omega}=\frac{-(1-\overline{z^*}+\bar{z})}{1+\theta(1-\overline{z^*}+\bar{z})}\widehat{\left(\frac{T^*}{T}\right)}+\frac{\theta(1-\overline{z^*}-\bar{z})}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\phi}_f-\frac{1-\gamma\theta(1-\overline{z^*}+\bar{z})}{1+\theta(1-\overline{z^*}+\bar{z})}\widehat{\left(\frac{L}{L^*}\right)}+\frac{(1-\overline{z^*}+\bar{z})\gamma\theta\ln\left(\frac{L}{L^*}\right)}{1+\theta(1-\overline{z^*}+\bar{z})}\hat{\gamma}
$$

$$
-\frac{\left[\bar{z}\ln\left(\frac{T}{T^*}\frac{1-\bar{z}}{\bar{z}}\right)+(1-\overline{z^*})\ln\left(\frac{T}{T^*}\frac{1-\overline{z^*}}{\overline{z^*}}\right)\right]}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\theta}
$$

$$
\hat{\bar{z}}=\frac{-(1-\bar{z})}{1+\theta(1-\overline{z^*}+\bar{z})}\widehat{\left(\frac{T^*}{T}\right)}-\frac{(1-\bar{z})\theta[1+2\theta(1-\overline{z^*})]}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\phi}_f+\frac{(1-\bar{z})\theta(\gamma+1)}{1+\theta(1-\overline{z^*}+\bar{z})}\widehat{\left(\frac{L}{L^*}\right)}+\frac{(1-\bar{z})\gamma\theta\ln\left(\frac{L}{L^*}\right)}{1+\theta(1-\overline{z^*}+\bar{z})}\hat{\gamma}
$$

$$
+\frac{(1-\bar{z})\left\{(1-\overline{z^*})\,\theta\ln\left(\frac{T}{T^*}\frac{1-\overline{z^*}}{\overline{z^*}}\right)-[1+\theta(1-\overline{z^*})]\ln\left(\frac{T}{T^*}\frac{1-\bar{z}}{\bar{z}}\right)\right\}}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\theta}
$$

$$
\widehat{\overline{z^*}}=\frac{-(1-\overline{z^*})}{1+\theta(1-\overline{z^*}+\bar{z})}\widehat{\left(\frac{T^*}{T}\right)}+\frac{(1-\overline{z^*})\theta[1+2\theta\bar{z}]}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\phi}_f+\frac{(1-\overline{z^*})\theta(\gamma+1)}{1+\theta(1-\overline{z^*}+\bar{z})}\ \widehat{\left(\frac{L}{L^*}\right)}+\frac{(1-\overline{z^*})\gamma\theta\ln\left(\frac{L}{L^*}\right)}{1+\theta(1-\overline{z^*}+\bar{z})}\hat{\gamma}
$$

$$
+\frac{(1-\overline{z^*})\left\{\bar{z}\theta\ln\left(\frac{T}{T^*}\frac{1-\bar{z}}{\bar{z}}\right)-(1-\bar{z}\theta)\ln\left(\frac{T}{T^*}\frac{1-\overline{z^*}}{\overline{z^*}}\right)\right\}}{1+\theta(1-\overline{z^*}+\bar{z})}\ \hat{\theta},
$$

where $\bar{z}$ and $\overline{z^*}$ in the above equations are the initial levels around which the respective parameter change takes place. We can summarize the results as follows:

(i) sgn $(d\omega/d\lambda)$ = sgn $[\gamma\theta Z-1]$ where $Z\equiv 1-\bar{z}^*+\bar{z}>1$; $d\bar{z}/d\,\lambda>0$; $d\bar{z}^*/d\,\lambda>0$.

(ii) $d\omega/d\,(T/T^*)>0$; $d\bar{z}/d\,(T/T^*)>0$; $d\bar{z}^*/d\,(T/T^*)>0$.

(iii) sgn $(d\omega/d\gamma)$ = sgn $(d\bar{z}/d\gamma)$ = sgn $(d\bar{z}^*/d\gamma)$ = sgn$(\lambda-1/2)$.

(iv) sgn $(d\omega/d\theta)$ = sgn $-\left\{[(1-\bar{z}^*)+\bar{z}]\ln\left(\frac{T}{T^*}\right)+\bar{z}\ln\left(\frac{1-\bar{z}}{\bar{z}}\right)+(1-\bar{z}^*)\ln\left(\frac{1-\bar{z}^*}{\bar{z}^*}\right)\right\}$
sgn $(d\bar{z}/d\theta)$ = sgn $\left\{(1-\bar{z}^*)\,\theta\ln\left(\frac{T}{T^*}\frac{1-\bar{z}^*}{\bar{z}^*}\right)-[1+\theta(1-\bar{z}^*)]\ln\left(\frac{T}{T^*}\frac{1-\bar{z}}{\bar{z}}\right)\right\}$;
sgn $(d\bar{z}^*/d\theta)$ = sgn $\left\{\bar{z}\theta\ln\left(\frac{T}{T^*}\frac{1-\bar{z}}{\bar{z}}\right)-(1-\bar{z}\theta)\ln\left(\frac{T}{T^*}\frac{1-\bar{z}^*}{\bar{z}^*}\right)\right\}$.

(v) sgn $(d\omega/d\phi_f)$ = sgn $(1-\bar{z}^*-\bar{z})$; $d\bar{z}/d\phi_f<0$; $d\bar{z}^*/d\phi_f>0$.

Appendix C: Proof of proposition 4

Solving eqs. (9), (10), and $\phi_f^{2\theta}=\left(\frac{1}{\bar{z}}-1\right)/\left(\frac{1}{\bar{z}^*}-1\right)$ with respect to n, n^* and $\overline{z^*}$ and substituting them into eqs. (6) and (8), we can reduce the five equilibrium conditions to two. Applying the implicit function theorem to the two, we can compute $d\omega/d\lambda$.

(i) Then, it can be readily shown that there exists at most one solution $\phi_f=\hat{\phi}_f$ of $G(\phi_f)\equiv d\omega/d\lambda|_{(\lambda,\omega)=(1/2,1)}=0$ in the interval of [0,1]. More precisely, (a) if $\gamma<1/\theta$, there exists $\hat{\phi}_f\in(0,1)$ such that $G(\phi_f)\gtreqless 0$ for $\phi_f\lesseqgtr\hat{\phi}_f$, and (b) if $\gamma>1/\theta$, $G(\phi_f)>0$ always holds.

(ii) It can also be shown that there exists at most one solution $\phi_s=\hat{\phi}_s$ of $G(\phi_s)\equiv d\omega/d\lambda|_{(\lambda,\omega)=(1/2,1)}=0$ in the interval of [0,1]. More precisely, (a) if $2\gamma/(1+\phi_f^{\theta})<1/\theta$, there exists $\hat{\phi}_s \in (0,1)$ such that $G(\phi_s) \gtreqless 0$ for $\phi_s \gtreqless \hat{\phi}_s$, and (b) if $2\gamma/(1+\phi_f^{\theta}) > 1/\theta$, $G(\phi_s) > 0$ always holds for all $0 \leq \phi_s < 1$.

(iii) We have

$$\lim_{\beta\to 0} \frac{d\omega}{d\lambda}\bigg|_{(\lambda,\omega)=(1/2,1)} = -\frac{4\left(1+\phi_f^{\theta}\right)}{1+2\theta+\phi_f^{\theta}} < 0$$

By continuity, the sign does not change for β sufficiently close to 0.

(iv) We have

$$\lim_{\theta\to\infty} \frac{d\omega}{d\lambda}\bigg|_{(\lambda,\omega)=(1/2,1)} = \frac{4(1-\phi_s)}{(\sigma-1)(1-\phi_s)+2\beta(1-\beta)(2\sigma-1)\phi_s} > 0$$

By continuity, the sign does not change for θ sufficiently close to infinity.

Appendix D: The ratio of price levels

Making use of the definition of price levels (1), unit costs (2) and the technology specification $A(z) = a^*(z)/a(z) = [T(1-z)/T^*z]^{\frac{1}{\theta}}$ we have:

$$\begin{aligned}
\frac{P}{P^*} &= \exp\left[\int_0^{\bar{z}^*} \log\kappa(z)\,dz + \int_{\bar{z}^*}^{\bar{z}} \log\kappa(z)\,dz + \int_{\bar{z}}^{1} \log[\tau\kappa^*(z)]\,dz - \int_0^{\bar{z}^*} \log[\tau\kappa(z)]\,dz \right.\\
&\qquad \left. - \int_{\bar{z}^*}^{\bar{z}} \log\kappa^*(z)dz - \int_{\bar{z}}^{1} \log\kappa^*(z)dz\right]\\
&= \exp\left[\int_0^{\bar{z}^*} \log\frac{1}{\tau}\,dz + \int_{\bar{z}^*}^{\bar{z}} \log\frac{\kappa(z)}{\kappa^*(z)}\,dz + \int_{\bar{z}}^{1}\log\tau\,dz\right]\\
&= \exp\left[\bar{z}^*\log\frac{1}{\tau} + \int_{\bar{z}^*}^{\bar{z}} \log\left[\left(\frac{T^*}{T}\frac{z}{1-z}\right)^{1/\theta} \omega^{1-\beta}\left(\frac{P_s}{P_s^*}\right)^{\beta}\right]dz + (1-\bar{z})\log\tau\right]\\
&= \exp\left[(1-\bar{z}-\bar{z}^*)\log\tau \right.\\
&\qquad \left. + \frac{1}{\theta}\log\frac{\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}}{\bar{z}^{*\bar{z}^*}(1-\bar{z}^*)^{(1-\bar{z}^*)}} + \log\left[\left(\frac{T^*}{T}\right)^{\frac{\bar{z}-\bar{z}^*}{\theta}} \omega^{(1-\beta)(\bar{z}-\bar{z}^*)}\left(\frac{P_s}{P_s^*}\right)^{\beta(\bar{z}-\bar{z}^*)}\right]\right]\\
&= \phi_f^{\bar{z}+\bar{z}^*-1}\left[\frac{\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}}{\bar{z}^{*\bar{z}^*}(1-\bar{z}^*)^{(1-\bar{z}^*)}}\right]^{1/\theta}\left(\frac{T^*}{T}\right)^{\frac{\bar{z}-\bar{z}^*}{\theta}} \omega^{(1-\beta)(\bar{z}-\bar{z}^*)}\left(\frac{P_s}{P_s^*}\right)^{\beta(\bar{z}-\bar{z}^*)}
\end{aligned}$$

In the geography analysis of section 4 we additionally impose $T = T^*$ so that we have:

$$\frac{P}{P^*} = \phi_f^{\bar{z}+\bar{z}^*-1}\left[\frac{\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}}{\bar{z}^{*\bar{z}^*}(1-\bar{z}^*)^{(1-\bar{z}^*)}}\right]^{1/\theta} \omega^{(1-\beta)(\bar{z}-\bar{z}^*)}\left(\frac{P_s}{P_s^*}\right)^{\beta(\bar{z}-\bar{z}^*)}$$

Appendix E: Proof of proposition 6 (no full agglomeration)

Suppose $\lambda = 1$. Since there is no production of intermediate goods in foreign, $n^* = 0$. Since all the final goods should be produced owing to the Cobb-Douglas utility, $\bar{z} = 1$.

Of the five equation system, eq. (7) for $\overline{z^*}$ irrelevant because there is no production of the final goods, eq. (8) for trade balance is irrelevant because there is no trade, and eq. (10) for n^* is irrelevant because there is no production of the intermediate. On the other hand, eq. (9) for n is relevant and is computed as $n = \beta L / f\sigma$. Finally, eq. (6) $\bar{z} = 1$ is relevant and is satisfied only if $\omega = 0$.

Then, eq. (13) is

$$\left.\frac{V}{V^*}\right|_{\omega=0} = \phi_f^{1-\bar{z}-\bar{z}^*} \left[\frac{\bar{z}^{*\bar{z}^*}(1-\bar{z}^*)^{(1-\bar{z}^*)}}{\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}}\right]^{1/\theta} \left.\omega^{1-(1-\beta)(\bar{z}-\bar{z}^*)} \left(\frac{P_S^*}{P_S}\right)^{\beta(\bar{z}-\bar{z}^*)}\right|_{\omega=0} = 0$$

Hence, full agglomeration $\lambda = 1$ is not a spatial equilibrium. The case of $\lambda = 0$ is similarly shown.

Appendix F: Proof of proposition 8

First, we show the uniqueness of the partial equilibrium given $n^* = 0$.

The utility differential is given by

$$\frac{V}{V^*} = \frac{w/P}{w^*/P^*} = H\omega^{1-(1-\beta)(\bar{z}-\bar{z}^*)} \left(\frac{P_S^*}{P_S}\right)^{\beta(\bar{z}-\bar{z}^*)}, \tag{16}$$

where

$$H \equiv \phi_f^{1-\bar{z}-\bar{z}^*} \left[\frac{\bar{z}^{*\bar{z}^*}(1-\bar{z}^*)^{(1-\bar{z}^*)}}{\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}}\right]^{1/\theta}$$

Using $\bar{z}^* = \frac{1}{1+\phi_f^{-\theta}\phi_S^{\frac{\beta\theta}{\sigma-1}}\omega^{(1-\beta)\theta}}$ which is implied by (6) when $n^* = 0$, and $\phi_f = [(1/\bar{z}-1)/(1/\overline{z^*}-1)]^{1/2\theta}$ and plugging ω and ϕ_f into the utility differential along with $P_S^*/P_S = \phi_S^{\frac{1}{1-\sigma}}$ which is implied when $n^* = 0$, we have

$$\frac{V}{V^*} = \left(\frac{1-\bar{z}}{\bar{z}^*}\right)^{\frac{\beta}{2\theta(1-\beta)}} \left(\frac{1-\bar{z}^*}{\bar{z}}\right)^{\frac{2-\beta}{2\theta(1-\beta)}} \phi_S^{\frac{-\beta}{(1-\beta)(\sigma-1)}} = 1$$

or

$$\left(\frac{1-\bar{z}}{\bar{z}^*}\right)^{\beta} \left(\frac{1-\bar{z}^*}{\bar{z}}\right)^{2-\beta} \phi_S^{\frac{-2\beta\theta}{\sigma-1}} = 1 \tag{17}$$

Let $\bar{z}^* = h(\bar{z})$ be the implicit function of (17). Let $\bar{z}^* = j(\bar{z}) \equiv \frac{\phi_f^{2\theta}\bar{z}}{1-\left(1-\phi_f^{2\theta}\right)\bar{z}}$ from (6) and (7). We show that $\bar{z}^* = h(\bar{z})$ and $\bar{z}^* = j(\bar{z})$ intersects once in the triangular domain of $\bar{z}^* < \bar{z} < 1$ and $\bar{z}^* > 1 - \bar{z}$.

We can easily show that $h'(\bar{z}) < 0$ and $j'(\bar{z}) > 0$. The curve $\bar{z}^* = h(\bar{z})$ passes through the side of the triangle $\bar{z}^* = \bar{z}$ at $(\bar{z}, \bar{z}^*) = \left(\frac{1}{1+\phi_s^{\frac{\beta\theta}{\sigma-1}}}, \frac{1}{1+\phi_s^{\frac{\beta\theta}{\sigma-1}}}\right)$ and approaches the vertex (1,0). On the other hand, the curve $\bar{z}^* = j(\bar{z})$ passes through the side of the triangle $\bar{z}^* = 1 - \bar{z}$ at $(\bar{z}, \bar{z}^*) = \left(\frac{1}{1+\phi_f^{\theta}}, 1 - \frac{1}{1+\phi_f^{\theta}}\right)$ and approaches the vertex (1,1). Hence, there exists a unique intersection in the triangle.

Second, we show that $\omega < 1$.

$$H|_{\bar{z}^*=j(\bar{z})} = \left(\frac{j(\bar{z})}{1-\bar{z}}\right)^{\frac{1+\bar{z}-j(\bar{z})}{2\theta}} \left(\frac{1-j(\bar{z})}{\bar{z}}\right)^{\frac{1-\bar{z}+j(\bar{z})}{2\theta}}.$$

We can verify that

$$\mathrm{sgn}\frac{d\left(H|_{\bar{z}^*=j(\bar{z})}\right)}{d\bar{z}} = \mathrm{sgn}\{(1-\bar{z}-\bar{z}^*)\ln[(1/\bar{z}-1)(1/\bar{z}^*-1)]\}$$

is positive and that $H|_{\bar{z}^*=j(\bar{z})} = 1$ at $\bar{z} = \frac{1}{1+\phi_f^{\theta}}$. Hence, $H > 1$ for all $\bar{z} \in \left(\frac{1}{1+\phi_f^{\theta}}, 1\right]$.

Rearranging $V/V^* = 1$ in (16), we get

$$\omega = H^{\frac{-1}{1-(1-\beta)(\bar{z}-\bar{z}^*)}} \phi_s^{\frac{\beta}{(1-\beta)(\sigma-1)[1-(1-\beta)(\bar{z}-\bar{z}^*)]}}.$$

Because the exponent of H is negative and $H > 1$, and because the exponent of ϕ_s is positive and $0 \leq \phi_s \leq 1$, it must be that $\omega < 1$.

Third, we show that $\lambda > 1/2$. Solving the trade balance equation (8) with $n^* = 0$ yields

$$\lambda = \frac{(1-\beta)\bar{z}^* + \beta}{(1-\beta)\bar{z}^* + \beta + (1-\beta)(1-\bar{z})\omega}$$

Since this is decreasing in ω,

$$\lambda - \frac{1}{2} > \frac{(1-\beta)\bar{z}^* + \beta}{(1-\beta)\bar{z}^* + \beta + (1-\beta)(1-\bar{z})} - \frac{1}{2} = \frac{\bar{z} + \bar{z}^* - 1 + \beta(2 - \bar{z} - \bar{z}^*)}{1 - (1-\beta)(\bar{z} - \bar{z}^*)} > 0$$

Fourth, $P < P^*$ is obvious from $\omega < 1$ and $V = V^*$. Finally, in obtaining (17), we have computed $P_s^*/P_s = \phi_s^{\frac{1}{1-\sigma}}$, which implies $P_s < P_s^*$.