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(In)Equality and (In)Justice

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ABSTRACT

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Understanding the exact connection between inequality and justice is important because justice is classically regarded as the first line of defense against self-interest and inequality. Absent a strong and clear link between inequality and justice, the sense of justice would not awaken to exert its moral suasion, no matter how great the inequality or how fast its increase. We obtain exact links between economic inequality and three parameters of the justice evaluation distribution–the mean, median, and variance–across a comprehensive set of inequality measures and a substantial starter set of just reward scenarios. This work shows that there is no general necessary connection between inequality and justice–inequality effects can be nonexistent, or can occur in opposite directions. There is, however, a striking pattern in some justice situations: As economic inequality increases, the average of the justice evaluations moves leftward, deeper into the territory of unjust underreward, and the distribution stretches outward, increasing the gulf between underrewarded and overrewarded and hollowing out the middle class. Further work specifying and strengthening the logical foundation will help guide development of sharp new empirical strategies for deeper understanding of the inequality-justice connection in all its manifestations.

JEL Classification: C02, C65, D31, D6, I3

Keywords: inequality, justice, fairness, lognormal distribution, Pareto distribution, power-function distribution, Gini coefficient, Atkinson inequality, Theil MLD, general inequality parameter, justice evaluation, justice evaluation function, justice evaluation distribution

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1. INTRODUCTION

Sociology, with roots in classical philosophy, proposes an unbroken line from inequality to the sense of justice to a variety of social outcomes such as cohesion, discontent, and revolution (Aristotle, Politics, Book II, Chapter 7; Aquinas, Commentary on Aristotle's Politics, Book II, Chapter 9; Alwin 1987):

$$\text{inequality} \rightarrow \text{justice} \rightarrow \text{social outcomes}$$

But despite a seemingly impeccable foundation, inequality effects can be difficult to find, as Jencks observes (Porter 2014). The search for inequality effects can be approached in a number of complementary ways, theoretical and empirical, focusing on one or the other link in the line or jumping from inequality to social outcomes. This paper addresses the first link, analyzing it theoretically.

Is there an exact, logically necessary, connection between inequality and justice? And if so, what is it? Understanding the link between inequality and justice is important because the sense of justice is classically regarded as the first line of defense against self-interest and inequality (Anselm, De Casu Diaboli, Chapter 14, De Concordia, Part III, Chapter 11; Duns Scotus, De Peccato Luciferi). But absent a strong and clear link between inequality and justice, the sense of justice would not awaken to exert its moral suasion, no matter how great the inequality or how fast its increase. Thus, establishing an a priori connection between inequality and manifestations of the sense of justice is an important task.

A related classical theme pertains to the virtues of the middle class and its importance to a healthy society, as well as its links to inequality and polarization (Aristotle, Politics, Book IV; Blau and Duncan 1967; Gornick and Jäntti 2014). Fortuitously, our analysis of the inequality-justice connection yields inequality effects on the gulf between underrewarded and overrewarded, enabling seamless integration of the shrinking of the middle class and societal polarization within a single inequality-justice framework.

Our strategy is simple. First, we focus on differences across persons in any material resources or possessions–not only money variables like wages, earnings, income, and wealth but also land and livestock–using the general term reward to cover all material holdings of interest and the term rewardee to designate the owner or recipient. Second, we rely on the justice evaluation function (JEF), a basic relation from justice theory which links three elements: (1) the rewardee's actual reward; (2) the observer's idea of the just reward for the rewardee (which could be self or other); and (3) the justice evaluation, which expresses the observer's assessment that the rewardee is justly or unjustly rewarded, and, if unjustly rewarded, whether underrewarded or overrewarded, and to what degree. Third, because all three elements have a distribution–respectively, the actual reward distribution (ARD), the just reward distribution (JRD), and the justice evaluation distribution (JED)–and hence all the usual distributional properties, such as mean and dispersion, it is straightforward to analyze the connection between inequality in the ARD and parameters of the JED, such as mean and variance. Finally, for rigorous assessment, the inequality-justice links are represented by the first (partial) derivatives of the JED parameters with respect to ARD inequality.

The JEF has proved useful both theoretically and empirically, and its properties (summarized in Jasso 2015:442-443) are well understood (Alwin 1987; Liebig and Sauer 2016; Osberg and Smeeding 2006; Resh and Sabbagh 2016; Shamon and Dülmer 2014; Turner and Stets 2006; Whitmeyer 2004). Though all three embedded distributions have dispersion, the focal inequality in this paper–the inequality to be linked to justice–is inequality in the actual reward distribution, and thus, for simplicity and convenience, the term inequality is restricted to dispersion in actual rewards, such as the quintessential income inequality. Dispersion in just rewards or justice evaluations will not be termed inequality, but simply dispersion.

To measure inequality, we use classic measures (e.g., Gini), which have been extensively discussed and analyzed–together with the general inequality parameter which appears to govern all measures of inequality in mathematically specified continuous univariate two-parameter distributions, as well as measures of inequality expressed as functions of the general inequality parameter within probability distributions (Jasso and Kotz 2008:37-41).

Thus, one could think of the work reported in this paper as an effort to populate with inequality effects the cells in a large matrix of inequality measures and parameters of the justice evaluation distribution. Yet even that large matrix is at best a skeleton, for each cell of the matrix must accommodate a multiplicity of justice situations and, in particular, a multiplicity of ideas of the just reward for self or others (Diekmann 2004; Dubins and Spanier 1961; Evans, Kelley, and Peoples 2010; Jasso, Törnblom, and Sabbagh 2016; Liebig and Sauer 2016). The just reward may be an amount directly selected (envisioned, desired, etc.), or it may be a previous or current own actual or just reward (or a function thereof) or the actual or just reward of someone else (or a function thereof), or it may be a parameter of the actual reward distribution or a subdistribution thereof (mean, median, minimum, maximum, etc.), or it may be the solution of a compensation function with multiple inputs (such as performance and tenure), or it may be the outcome of a procedure to divide a fixed amount, or it may arise via comparison to the actual rewards of everyone else. Further, the actual and just rewards may be positively or negatively associated or independent.

These many and varied sources of the just reward shape the justice situation–and the inequality-justice connection–in fundamental ways. A simple intuitive example will suffice to make this point in advance of the systematic work reported below. Suppose the just reward is a constant multiple of the actual reward, as would be the case if everyone thinks they deserve one-and-a-quarter times the actual reward. The actual and just rewards would cancel each other out, and all rewardees would have identical justice evaluations. Intuitively, if all the justice evaluations are identical, all the parameters of the justice evaluation distribution are constant, and there is no effect of inequality. Put bluntly, if everyone thinks they deserve the same proportional increase in pay, then whatever moral suasion the sense of justice might muster would be neutralized. For example, the average of the justice evaluations would stay the same, whether inequality increased, decreased, or stayed the same.

To represent the range of justice situations, we construct a set of scenarios for selection of the just reward. The scenarios cover, among others, the well-known equality scenario and the compare-to-all scenario in which each person's just reward reflects comparison to the actual rewards of every other person in the group or population (Evans et al. 2010; Diekmann 2004:492; Liebig and Sauer 2016).

The new matrix subsumes and substantially extends early results, which were based on a small set of combinations of justice parameters and justice scenarios and limited to work that did not report derivatives (Jasso 1999). For example, that early work suggested that as economic inequality increases, the average of the justice evaluations decreases. However, analysis of the enlarged set of justice situations shows that those early results, now confirmed by first (partial) derivatives, occur only within certain justice situations. In other justice situations, such as that described above in which everyone's idea of the just reward is a constant multiple of the actual reward, results may differ. Overall, the paper provides a starter set of comprehensive and fine-grained theoretical results, which show how the inequality-justice connection varies across justice scenarios and which can be used to guide and motivate further theoretical and empirical research and to interpret seemingly inconsistent results.

2. INEQUALITY AND JUSTICE: BUILDING BLOCKS AND PROCEDURES

2.1. Building Blocks: Inequality Measures

Classic inequality measures. We use five inequality measures–the Gini coefficient, the Atkinson measure defined as 1 minus the ratio of the geometric mean to the arithmetic mean (here called simply Atkinson), two Theil measures, and the coefficient of variation (CV). Formulas for these measures are well-known (see, e.g., Cowell 2011); four of them are collected in Jasso and Kotz (2008:37-41).

As the formulas show, the inequality measures incorporate one or more of three distributional parameters: the arithmetic mean, the geometric mean, and the standard deviation. Thus, justice parameters whose formulas include these terms will be ready to link to inequality.

General inequality parameter in continuous univariate two-parameter distributions of a positive quantity. Since the pioneering work of Pareto (1897), it has been known that in the Pareto variate the second (non-location) parameter operates as a general inequality parameter, governing all measures of inequality (Cowell 1977:95; Cramer 1971:51-58; Kleiber and Kotz 2003:78). Building on that tradition, Jasso and Kotz (2008) propose that in all two-parameter continuous univariate distributions, the second parameter, denoted *c*, operates as a general inequality parameter, and show that in the lognormal, Pareto, and power-function variates, all measures of relative inequality are monotonic functions of *c*. Accordingly, our set of inequality measures also includes *c*.

2.2. Building Blocks: Justice Measures

2.2.1. The Justice Evaluation Variable and the Justice Evaluation Function

People form ideas about what is just, for themselves and others. When actual situations differ from their ideas of justice, people assess the magnitude of the unfairness. This assessment is represented by the justice evaluation. Zero represents the point of perfect justice, and negative and positive numbers represent unjust underreward and overreward, respectively. Thus, a justice evaluation of zero indicates that the observer judges the rewardee (who can be self or other) to be perfectly justly rewarded. The closer a value of the justice evaluation to zero, the milder the injustice it indicates; and the farther away from zero, the greater the injustice.

The justice evaluation, denoted *J*, arises from the observer's comparison of the rewardee's actual amount of the reward, denoted *X*, to the amount the observer thinks just for the rewardee, denoted *X**. The function that links them takes the logarithm of the ratio of the actual reward to the just reward:

$$J = \ln\left(\frac{X}{X^*}\right) . \tag{1}$$

The log-ratio specification of the JEF has proved useful both theoretically and empirically, and its properties (summarized in Jasso 2015:442-443) are well understood (Alwin 1987; Jasso 1978, 1999; Liebig and Sauer 2016; Osberg and Smeeding 2006; Resh and Sabbagh 2016; Shamon and Dülmer 2014; Turner and Stets 2006; Whitmeyer 2004). For example, it integrates the two main rival conceptions, the ratio favored by Homans (1961) and others and the difference favored by Berger et al. (1972)--accomplished via the property of logarithms that the logarithm of a ratio equals the difference between two logarithms. And it embeds the property that deficiency is felt more keenly than comparable excess (now more commonly known as loss aversion), long considered central to matters of justice (Adams 1963:426; Brown 1986:78; Homans 1961:75-76); for example, if the just reward is 10, an actual reward of 5 will have a larger absolute magnitude than an actual reward of 15 (-.693 versus .405).[1, 2, 3]

2.2.2. The Justice Evaluation Distribution

In general, three justice evaluation distributions arise in the study of justice. First, the observer-specific JED collects all the justice evaluations about all rewardees in the eyes of one observer. Second, the rewardee-specific JED collects all the justice evaluations about one rewardee in the eyes of all observers. Third, the reflexive JED collects each observer's justice evaluation about self. If everyone in a population is both an observer and a rewardee, all the justice evaluations can be arrayed in a square matrix, each row representing an observer and each column representing a rewardee. Accordingly, each observer-specific JED occupies a row, and each rewardee-specific JED occupies a column; the reflexive JED occupies the main diagonal.

For the question addressed in this paper two of the three kinds of JEDs are relevant, namely, the observer-specific JED and the reflexive JED. The rewardee-specific JED is not relevant as it pertains to a single rewardee, and hence there is no inequality in the actual rewards (except, in some situations and in some research designs, due to observer perceptual distortion).

From (1) and the study of change-of-variable and convolution procedures in probability theory, it is clear that the justice evaluation function, besides linking the actual reward, the just reward, and the justice evaluation, also links the three corresponding distributions–the ARD,

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[1] It is also useful to note that the justice evaluation function originated in empirical work, where it was found to provide the best fit to data on the justice of earnings (Jasso 1978).

[2] Note also that the JEF, by explicitly incorporating observers' ideas of justice, avoids a main problem with inequality analysis, namely, that the implicit benchmark is perfect equality, which may in fact violate individuals' sense of justice.

[3] As per the notation in equation (1), we will use two equivalent sets of terms, not only the generic ARD, JRD, and JED but also the X, X^*, and J distributions.

JRD, and JED.

Accordingly, the objective in this paper is to link ARD inequality to parameters of the JED.

2.2.3. Parameters of the Justice Evaluation Distribution

2.2.3.1. Mean of the Justice Evaluation Distribution

Taking the mean of the justice evaluation in (1) yields:

$$\begin{aligned} E(J) &= E\left[\ln\left(\frac{X}{X^*}\right)\right] \\ &= E[\ln(X)] - E[\ln(X^*)] \, . \end{aligned} \tag{2}$$

By the well-known fact that the arithmetic mean of the log of a variable equals the log of the geometric mean of the variable, it is straightforward to write:

$$\begin{aligned} E(J) &= \ln\left[\frac{G(X)}{G(X^*)}\right] \\ &= \ln[G(X)] - \ln[G(X^*)] \, , \end{aligned} \tag{3}$$

where the operator G denotes the geometric mean. The presence of the geometric mean in both the formula for the mean of J and two inequality measures, Atkinson and Theil MLD (Jasso and Kotz 2008:38), will make it possible to establish a link between inequality and justice.

The mean of J may assume positive, negative, or zero values, as vividly seen in formula (3), which shows that its sign depends on the relative magnitude of the geometric means of the X and X^* distributions (or equivalently the ARD and JRD). Its value is interpreted as the center of gravity of the JED. Thus, a mean of zero indicates that the center of gravity of the JED lies at perfect justice; a negative value indicates that the center of gravity lies in the underreward region, and a positive value indicates that the center of gravity lies in the overreward region of the justice line.

The mean of J has two strengths and one limitation. The strengths follow from the property of the JEF that it can be expressed as the log of the actual reward minus the log of the just reward. As visible in expression (2), the combination of this property and the basic statistical theorem on expectation–that the mean of the sum(difference) of two variables equals the sum(difference) of their means–immediately yields the mean of J: $E(J)$ equals the mean of $\ln(X)$ minus the mean of $\ln(X^*)$. Thus, the first strength is that it is not necessary to find the distribution of J in order to find its mean. The second strength is that the mean of J is impervious to the relation between X and X^*.

The limitation of the justice mean is the ambiguity of a zero reading (Jasso 1999:148). Because the justice evaluations may be negative or positive (representing underreward and overreward, respectively), the two may offset each other, yielding a mean of zero. A mean of zero may as easily indicate that everyone is perfectly justly rewarded or that the underrewarded and overrewarded offset each other. Accordingly, it is useful to have another measure of location, and for that purpose we use the median.

2.2.3.2. <u>Median of the Justice Evaluation Distribution</u>

The median of the JED, denoted $M(J)$, is most easily and, for the work carried out here, most usefully expressed in terms of the quantile function evaluated at the probability .5:

$$M(J) \;=\; Q_J(.5)\,. \tag{4}$$

Though the median does not appear in any of the formulas for inequality measures in Jasso and Kotz (2008:38), it will be straightforward to express it in terms of the general inequality parameter c in the probability distributions to be analyzed below.

2.2.3.3. <u>Variance of the Justice Evaluation Distribution</u>

Taking the variance of the distribution of the justice evaluations in (1) yields:

$$\begin{aligned} Var(J) \;&=\; Var\left[\ln\left(\frac{X}{X^*}\right)\right] \\ &=\; Var[\ln(X)] + Var[\ln(X^*)] - 2\,Cov[\ln(X), \ln(X^*)]\,. \end{aligned} \tag{5}$$

The goal is to link $Var(J)$ to one or more of the inequality measures, including $Var(X)$ and the general inequality parameter c.

Note that, like the mean, the variance can be obtained without first obtaining the distribution of J. However, unlike the mean, the variance is sensitive to the correlation between X

and X^*, as visible in (5).

2.2.4. Two Remarks about the Justice Measures in Preparation for Linking Them to Inequality

Remark 1: Levels, changes, and effects. The justice evaluation ranges over both negative and positive numbers, and thus the mean and median of the JED can assume any value on the real line. Accordingly, a negative effect of inequality indicates that as inequality increases, the mean or median moves leftward; however, this move leftward can occur anywhere along the J continuum. It can be from greater overreward to lower overreward or from overreward to zero or from zero to underreward or from mild underreward to severe underreward.

Moreover, inequality in the ARD is not the only factor affecting the JED and its parameters. The mean of the ARD and both the mean and dispersion of the JRD are also at work. Equations (2), (3), and (5) already show the operation of the just rewards on the mean and variance of the JED, and the new expressions to be obtained below will show all the relevant terms. Thus, to assess changes in the outcome justice parameter of interest, it is necessary to assess changes in all the terms. However, to assess the effect of one term on the outcome justice parameters–as in the goal of this paper–it is only necessary to take the first partial derivative of each outcome justice parameter with respect to that one term, here X inequality.

Hiding in equations (2) and (3) for the mean but visible in equation (5) for the variance is an important feature of the justice situation and one that is critically important for assessing the effects of X inequality on at least some of the justice parameters–viz., whether the actual reward X and the just reward X^* are related. This in turn depends on the process by which individuals form ideas of the just reward.

Remark 2: Sources of ideas of the just reward. Where does X^* come from? This is a classic question in the study of justice as in all processes involving a reference individual or reference group. As discussed in Section 1, there is a wide range of sources of the just reward, and a useful approach must accommodate them (Evans et al. 2010; Diekmann 2004; Liebig and Sauer 2016).

If the actual reward and the just reward are related, then the X inequality may be present in multiple terms shaping the parameter of interest, and a full accounting of the effect of X inequality must incorporate its operation via those other terms. As noted, this is already visible in equation (5) for the variance of the JED.

The relation between X and X^* can be approached in two ways: (1) focusing on the individual's choice of the just reward, which could be, say, the arithmetic mean $E(X)$, the geometric mean $G(X)$, or the multiplicative function kX, or any other amount (Evans et al. 2010; Diekmann 2004; Liebig and Sauer 2016); or (2) focusing on the ARD and the JRD. In this paper, we follow the first approach, leaving the second approach to future work. Thus, for each of the three focal justice parameters, we assess the inequality-justice connection in a set of scenarios representing the individual's choice of the just reward.

2.3. Building Blocks: Probability Distributions

For the subset of analyses in which inequality is represented by the general inequality parameter in continuous univariate two-parameter distributions, we use three variates whose properties are well-known and make them appealing models for distributions of wages, income, wealth, and other economic variables–the lognormal, Pareto, and power-function (Johnson, Kotz, and Balakrishnan 1994, 1995; Kleiber and Kotz 2003). The lognormal and the power-function have their origin at zero and thus have no safety net, while the Pareto, in contrast, does. The lognormal and Pareto have no maximum, while the power-function has an upper ceiling, making it useful for modeling scarcity. Thus, these three families of distributions are useful for approximating different types of economic situations.

Formulas and graphs for the three main associated functions of the three variates–the cumulative distribution function (CDF), the probability density function (PDF), and the quantile function (QF)–are provided in Jasso and Kotz (2008:36-37). The formulas are expressed in terms of the mean μ and the general inequality parameter c.

For visualizing operation of the general inequality parameter, see the formulas for four inequality measures in the three distributions (Jasso and Kotz 2008:39). As shown, all the inequality measures are functions exclusively of the general inequality parameter *c*, and taking derivatives shows that they are monotonic functions. Moreover, Jasso and Kotz (2008:41-43) show that as *c* approaches its lower-inequality end, the distributions collapse onto the arithmetic mean–the point of perfect equality.

To obtain expressions for the mean and variance of the JED it will be useful to have formulas for the mean and variance of ln(*X*) in the three variates, and Appendix Table A provides them.

The distribution-specific formulas begin a subplot about the links between the Pareto and power-function distributions that will gather intensity as our work progresses. There are strong similarities and symmetries in the formulas, and these will extend to the *J* distribution when *X* (and *X**) are Pareto or power-function. To illustrate the initial links between the Pareto and the power-function: (1) the reciprocal of a Pareto variate is distributed as a power-function; and (2) the Pareto and power-function (of *c* greater than 1) are almost mirror images of each other, the Pareto with its single mode at the bottom of the range and a long right tail and the power-function with its single mode at the top of the range and a left tail extending to zero.

2.4. Building Blocks: Scenarios for the Just Reward *X** and Its Relation to the Actual Reward *X*

Just Reward Scenarios. The sources of the just reward based on the observer's selection can be grouped into three general types. The first is completely unconstrained; the second and third, each with subtypes, constrain the individual's choice of the just reward.

Scenario 1: *X** can assume any positive value. The individual has free rein, can choose any positive number.

Scenario 2: *X** is a positive constant. This scenario, which can be expressed in the general form $X^* = k$, lets the just reward assume any value, and this value is constant in the group. Though in principle this constant could be any value, such as the minimum or the maximum, two main subtypes may be discerned.

Scenario 2.1: *X** is the arithmetic mean of *X*. This is the equality scenario. For at least some individuals, for example, the only just division of a fixed pie is equality (Evans et al. 2010; Liebig and Sauer 2016; van den Bos et al. 2015).

Scenario 2.2: X^* is the geometric mean of X. This scenario has an interesting substantive origin. Students of fairness, justice, and relative deprivation observe that individuals may compare their own or others' actual rewards to the actual rewards of every other person in the group or population (Diekmann 2004:492). It can be shown that if the focal person's justice evaluation is obtained by averaging the justice evaluations arising from comparison to everyone else, then, as the population size increases, the just reward in the emergent justice evaluation approaches the geometric mean of X.

Scenario 3: X^* equals kX, k a positive number. In this scenario, the just reward is a constant multiple of X. This scenario has two main subtypes.

Scenario 3.1: X^* equals kX, k=1. In this scenario, for each rewardee the just reward equals the actual reward. Substantively, this is a rich case, corresponding to a variety of situations, ranging from the Stoic prescription to want only what one has to Homans' (1976:244) observation that "what is, is always becoming what ought to be." It follows that the justice evaluation equals zero, the point of perfect justice. Thus, the J distribution is an Equal distribution (sometimes called "degenerate" when defined as discrete, and "Dirac's delta" when defined as continuous), located at zero.

Scenario 3.2: X^* equals kX, k positive and not equal to one. In this scenario, for each rewardee the just reward is a multiple of the actual reward, but not one, which is covered by Scenario 3.1. Then for each rewardee the justice evaluation is $-\ln(k)$. If k is less than one, the justice evaluation is positive, and the result is overreward; if k is greater than one, the justice evaluation is negative, resulting in underreward. Thus, the J distribution is an Equal distribution, as in Scenario 3.1, but it is located at a negative or positive value of J, not at zero.

For handy reference, Table 1 summarizes the scenarios.

Table 1 about here

As noted, the J distribution in Scenarios 3.1 and 3.2 is Equal. The distributional form of J

in Scenario 2 is also in the literature for some special cases (Jasso 2015:455). For example, if the actual reward *X* is distributed as lognormal, Pareto, or power-function and if *X** is constant, *J* is distributed, respectively, as normal, negative exponential, and positive exponential. Given the links between the Pareto and power-function discussed in Section 2.3, it is not surprising that they both lead to exponential *J* distributions, the Pareto to a right-skewed negative exponential and the power-function to a left-skewed positive exponential. Thus, the form of the ARD combines with the justice scenario to generate a distinctive form for the JED.

2.5. Procedures

We analyze the effects of each of a set of inequality measures on three parameters of the JED, in several just reward scenarios, in both distribution-independent and distribution-specific contexts. Thus, we obtain, for each justice parameter, a broad array of expressions, together with the first (partial) derivatives with respect to the inequality measures.

Though the literature does not provide formulas for the median or variance of the *J* distribution or for the first partial derivatives (save one or two), it does provides a foundation of formulas for the mean of the *J* distribution in some special cases, such as the justice mean in Scenario 2.1 (*X** equals the mean of *X*) in a distribution-independent version and when *X* is lognormal or Pareto (Jasso 1999). We build on and extend those early results to a larger set of justice parameters, inequality measures, just reward scenarios, and distributions, and thence obtain the inequality effects on the three justice parameters across scenarios and distributions. Considering this extensive set of justice situations shows that the early results pertain to special cases. The larger set yields not only the early results but also the absence of inequality effects and even an opposite result.

3. INITIAL GENERAL RESULTS

Section 2.4 shows that the *J* distribution is Equal in Scenario 3. Thus, the justice parameters are immediate in this case, as are the associated inequality effects. Specifically, the *J* mean is zero in Scenario 3.1 ($X^* = X$), and a nonzero constant [-ln(*k*)] in Scenario 3.2. The *J* variance is zero in Scenario 3.

Substantively, these results help illuminate the rise and course of discontent with inequality. They may provide a partial answer to the abiding question why there is not more discontent when there is high and/or increasing economic inequality. How sharp the sting of inequality depends not only on *X* inequality but also on each individual's ideas of the just reward.

4. INEQUALITY AND THE MEAN OF THE DISTRIBUTION OF JUSTICE EVALUATIONS

4.1. Linking *X* Inequality and *E*(*J*): Scenario 1

From expressions (2) and (3), we construct a hybrid expression in which the first term on the righthand side of (2) is expressed in terms of the geometric mean:

$$E(J) = \ln[G(X)] - E[\ln(X^*)] . \tag{6}$$

Expressions (2) and (6) will enable statement of general formulas for the *J* mean in terms of the Atkinson and Theil MLD measures, as well as distribution-specific formulas in terms of the general inequality parameter *c* and the inequality measures in the lognormal, Pareto, and power-function distributions.

Inequality represented by the Atkinson and Theil MLD measures. Re-arranging terms in the Atkinson measure, denoted *A(X)*, we obtain an expression for the geometric mean of *X*:

$$G(X) = [1 - A(X)]\,[E(X)] . \tag{7}$$

Substituting expression (7) into formula (6), we see that the *J* mean can be written:

$$E(J) = \ln[1 - A(X)] + \ln[E(X)] - E[\ln(X^*)] . \tag{8}$$

Expression (8) shows that as any of three terms on the righthand side changes, the mean of the justice evaluation distribution *E(J)* also changes, as discussed in Section 2.2.4. The total change in *E(J)* is formally represented by the total differential:

$$dE(J) = \frac{\partial E(J)}{\partial A(X)} dA(X) + \frac{\partial E(J)}{\partial E(X)} dE(X) - \frac{\partial E(J)}{\partial E[\ln(X^*)]} dE[\ln(X^*)] . \tag{9}$$

This paper, however, focuses on the effect of economic inequality on the *J* parameters rather than on their total change. Accordingly, we take the first partial derivative of *E(J)* with respect to the Atkinson inequality,

$$\frac{\partial E(J)}{\partial A(X)} = -\frac{1}{1 - A(X)}, \qquad \textbf{(10)}$$

which provides the effect of the Atkinson inequality, holding constant the other factors. Given that the Atkinson inequality lies between zero and one, the derivative in (10) is always negative. Therefore, as Atkinson inequality increases, the *J* mean decreases. The *J* mean and the inequality effect are collected in Table 2, panel A.[4]

Table 2 about here

The formula for the *J* mean when inequality is represented by the Theil MLD and the associated first partial derivative are obtained in exactly the same way (obtain an expression for the geometric mean in terms of the MLD, and plug it into expression (6) for the *J* mean, then take the first partial derivative). These results appear in Table 2, panel B. As with the Atkinson measure, as inequality increases, the *J* mean moves leftward.

Inequality represented by the general inequality parameter *c*. To obtain the effect of inequality on the *J* mean, with inequality represented by *c*, we replace $E[\ln(X)]$ in (2), or, equivalently, $\ln[G(X)]$ in (6), with the corresponding expression in Appendix Table A for each of the three distributions.

Table 2, panel C, reports the expressions for the *J* mean and its first partial derivatives with respect to *c* in the three variates. Because *c* increases with inequality in the lognormal but

[4] Table 2 and the table that follows report formulas for the justice parameters together with formulas and/or directions of the inequality effects. Some of the justice parameters depend only on *X* inequality, others depend as well on *X** dispersion and the arithmetic means of the *X* and *X** distributions. Accordingly, some of the inequality effects are represented by first derivatives and others by first partial derivatives. For simplicity, table captions use notation for first partial derivatives, generically. Further, tables that report effects of two or more inequality measures (such as Table 2) or both ordinary and partial derivatives use a centered dot as a placeholder for the inequality measure.

decreases with c in the Pareto and power-function, the correct inequality interpretation is provided by the derivatives in the rightmost column.

Whether inequality is represented by the Atkinson measure, the MLD, or the general inequality parameter c, the inequality effects are the same. As inequality increases, the justice mean moves leftward.

Inequality represented by measures expressed as functions of the general inequality parameter *c*. It is straightforward to express *E(J)* as a function of any of the five measures and show that as inequality increases, the *J* mean decreases. Perhaps the easiest, almost immediate, result of this kind pertains to the two Theil measures in the lognormal case, for they both equal $c^2/2$ and are thus the negative of *E(J)* – which is $-c^2/2$.

Other results are equally straightforward albeit less immediate. We illustrate with one such result, based on the Gini coefficient when *X* is Pareto.

E(J) as a function of the Gini when *X* is Pareto. Algebraic manipulation of the formula for the Gini in the Pareto (Jasso and Kotz 2008:39) yields the inverse expression, namely, of c as a function of the Gini, denoted *G* (plain *G*, not *G* as an operator, which denotes the geometric mean):

$$c = \frac{G+1}{2G} . \qquad \textbf{(11)}$$

Plugging expression (11) into the formula for the *J* mean when *X* is Pareto (Table 2, panel C) yields:

$$E(J) = \frac{2G}{1+G} + \ln\left(\frac{1-G}{1+G}\right) + \ln[E(X)] - E[\ln(X^*)] . \qquad \textbf{(12)}$$

Taking the first partial derivative of *E(J)* with respect to the Gini coefficient yields a negative quantity, indicating that as the Gini coefficient increases, the justice mean moves leftward.

4.2. Linking *X* Inequality and *E*(*J*): Scenario 2.1

Now consider Scenario 2.1, in which the just reward is the arithmetic mean of *X*.

Replacing X^* by the arithmetic mean of X in the ratio in expression (2) yields:

$$\begin{aligned} E(J)^* &= E\left[\ln\left(\frac{X}{E(X)}\right)\right] \\ &= E[\ln(X)] - \ln[E(X)] \\ &= \ln[G(X)] - \ln[E(X)] \,, \end{aligned} \qquad \textbf{(13)}$$

where $E(J)^*$ denotes the mean in Scenario 2.1. Thus, $E[\ln(X^*)]$ in expression (2) reduces to $\ln[E(X)]$ in expression (13).

Inequality represented by the Atkinson and Theil MLD measures. In the expression for the J mean when inequality is represented by the Atkinson measure (Table 2, panel A) the rightmost term is replaced by $\ln[E(X)]$, which cancels out the second term, yielding:

$$E(J)^* = \ln[1 - A(X)] \,, \qquad \textbf{(14)}$$

whose first derivative with respect to the Atkinson measure is the same as in (10) and Table 2, panel A.

Doing exactly the same thing when inequality is represented by the Theil MLD leaves the single term–the negative of the MLD. Again, as inequality increases, the J mean moves leftward.

Inequality represented by the general inequality parameter c. Inspection of Table 2, panel C, makes plain that when the just reward X^* is fixed at the arithmetic mean of X, the two rightmost terms in the expressions for the justice means cancel out, exactly as they did with the Atkinson and Theil measures in panels A and B. Thus, the substantive result is the same as in Scenario 1, namely, as inequality increases, the J mean moves leftward.

Inequality represented by measures expressed as functions of the general inequality parameter c. Look at formula (12) for the J mean when X is Pareto and c is replaced by its expression as a function of the Gini. It is evident that the last two terms cancel out, leaving only the terms with the Gini. Taking first derivatives yields the same results as above.

4.3. Linking X Inequality and $E(J)$: Scenario 2.2

Scenario 2.2, in which the just reward is the geometric mean of X, leads to a different result. Replacing X^* in expression (2) by $G(X)$ yields:

$$\begin{aligned} E(J)^{**} &= E\left[\ln\left(\frac{X}{G(X)}\right)\right] \\ &= E[\ln(X)] - \ln[G(X)] \\ &= \ln[G(X)] - \ln[G(X)] = 0 , \end{aligned} \quad \textbf{(15)}$$

where $E(J)^{**}$ denotes the mean in Scenario 2.2.

Thus, the J mean in Scenario 2.2, in which the just reward is fixed at the geometric mean of X, equals zero, and there are no inequality effects.

5. INEQUALITY AND THE MEDIAN OF THE DISTRIBUTION OF JUSTICE EVALUATIONS

5.1. Linking *X* Inequality and *M*(*J*): Scenario 2

To obtain an expression for the median of the JED, we first re-write equation (4) in terms of the X and X^* components:

$$Q_J(.5) = Q_{\ln(X)}(.5) - \ln(X^*) . \quad \textbf{(16)}$$

Thus, the J median is the median of the ln(X) distribution minus the log of the constant just reward.

In each distributional family, we then plug the median of ln(X)–the distribution's quantile function in Jasso and Kotz (2008:38), evaluated at .5--into expression (16). Table 3, panel A, reports the J median when X is lognormal, Pareto, or power-function.

Table 3 about here

Table 3, panel A, also reports the first partial derivative of each distribution-specific median with respect to the general inequality parameter c. As shown, the inequality effects are negative. As economic inequality increases, the J median moves leftward toward the territory of underreward or its more severe regions.

5.2. Linking *X* Inequality and *M*(*J*): Scenario 2.1

In the equality scenario the rightmost term in each of the three distribution-specific expressions for the median becomes $\ln[E(X)]$, which is the same as the penultimate term, so that the two rightmost terms cancel out, leaving only the terms with the general inequality parameter c (Table 3, panel B). The negative inequality effects are preserved intact.

5.3. Linking *X* Inequality and *M*(*J*): Scenario 2.2

In the compare-to-all scenario, the rightmost term in each of the three distribution-specific expressions for the median (Table 3, panel A) becomes $\ln[G(X)]$. From the inequality formulas, it is evident that the two rightmost terms combine to form the Theil MLD. Replacing the two rightmost terms by the distribution-specific formulas for the MLD in terms of c (Jasso and Kotz 2008:39) yields expressions for the J median (Table 3, panel C).

Substantively, the results are striking. The inequality effects run the gamut--the effect of X inequality is zero when X is lognormal, negative when X is Pareto, and positive when X is power-function--showing the long reach of the shape of the income distribution.

At this juncture we establish that the inequality effects on justice can be nonexistent and, if present, can occur in both positive and negative directions, depending on the configuration of justice parameter, just reward scenario, and distributional form of income.

6. INEQUALITY AND THE VARIANCE OF THE DISTRIBUTION OF JUSTICE EVALUATIONS

6.1. Linking *X* Inequality and *Var*(*J*): Scenario 1

Look at expression (5) for $Var(J)$. It is evident that if the sum of the variances of $\ln(X)$ and $\ln(X^*)$ equals twice their covariance–as would be the case if X and X^* are identical--$Var(J)$ equals zero. Thus, the inequality effect on $Var(J)$ can be zero.

If X and X^* are independent, the variance of the J distribution reduces to:

$$Var(J) \;=\; Var[\ln(X)] + Var[\ln(X^*)] \,. \tag{17}$$

The formulas for the variance of $\ln(X)$ when X is distributed as lognormal, Pareto, or power-function show that in all three variates the variance of $\ln(X)$ is a function of the general inequality parameter c. Accordingly, it is straightforward to establish a link between inequality and $Var(J)$.

The inequality effects will be exactly as in Scenario 2 to which we turn.

6.2. Linking *X* Inequality and *Var*(*J*): Scenario 2

By a fundamental theorem, the variance of the sum or difference of a random variable and a constant equals the variance of the random variable. Thus, expression (5) leads to:

$$Var(J)^* = Var[\ln(X)] , \tag{18}$$

where $Var(J)^*$ denotes the J variance in Scenario 2.

The distribution-specific variance of $\ln(X)$ is already in Appendix Table A, and thus $Var(J)^*$ is immediate. Table 4 reports $Var(J)^*$ and the first derivatives with respect to c. Substantively, the effect of economic inequality on the J variance is positive, stretching the J distribution outward in both the underreward and overreward directions.

Table 4 about here

As seen in Sections 4.1 and 4.2 above, formulas expressed in terms of c can be expressed in terms of any of the inequality measures, which are themselves expressed in terms of c. To illustrate, consider $Var(J)^*$ when X is Pareto. Using the J variance in Table 4 and formula (11) for the inverse of the Gini in the Pareto, we obtain the Pareto-specific variance of J expressed in terms of the Gini:

$$Var(J)^* = \left(\frac{2G}{G+1} \right)^2 . \tag{19}$$

Taking the first derivative of (19) with respect to the Gini coefficient shows again that as X inequality increases, the J distribution stretches outward, increasing the gulf between the underrewarded and the overrewarded and hollowing out the middle class.

7. SUMMARY AND VISUALIZATION OF THE EFFECTS OF INEQUALITY ON THE DISTRIBUTION OF JUSTICE EVALUATIONS

7.1. Summary of Inequality Effects on Justice

Table 5 summarizes the inequality effects, for each justice parameter and each just reward scenario. As shown, there is a nontrivial number of cells with zero inequality effects. Thus, there is no general connection between inequality and justice. There is, however, in some scenarios, a striking pattern in which as inequality increases, the average of the justice evaluations moves leftward, deeper into the territory of unjust underreward, and the distribution stretches outward, increasing the gulf between the underrewarded and the overrewarded and hollowing out the middle class.

Table 5 about here

These results underscore Liebig and Sauer's (2016) observation that progress in understanding distributive justice requires progress in understanding ideas of the just reward, whether and how individuals' ideas of justice coalesce into collectivistic ideas of justice, and how groups and societies come to have particular distributional forms for income and wealth. The formulas for the justice parameters in special configurations obtained in this work provide a useful set of tools for advancing that effort.

7.2. Visualization of Inequality Effects on Justice

Each just reward scenario–each row in Table 5–generates a pattern of inequality effects (or their absence). To deepen understanding, we provide visualization of two scenarios–Scenarios 2.1 and 2.2, in which the just rewards are fixed at the mean and geometric mean of the actual rewards, respectively.

To visualize the inequality effects in Scenario 2.1, Figure 1 provides graphs of the JED arising from three members of each of the three modeling distributions. The *J* mean is represented by a vertical line from the horizontal axis to the distribution, with circles at both ends. As inequality in the actual reward distribution increases, the distributions of justice evaluations get shorter and stretch outward in both the underreward and overreward directions; the mean moves leftward, deeper into the territory of unjust underreward. Thus, there is a dual effect of inequality–pushing the center of gravity of the justice evaluations ever leftward and widening the gulf between the underrewarded and the overrewarded.[5]

[5] The stretching effect is less easy to see in the tails of the normal (panel A), the right tail of the negative exponential (panel B), and the left tail of the positive exponential (panel C). But,

Figure 1 about here

Graphs of the *J* distributions in Scenario 2.2 (X^* at the geometric mean of X) , shown in Figure 2, both resemble and differ from those in Figure 1. Like the graphs in Figure 1, as economic inequality increases, the graphs get shorter and stretch outward, the *J* variance increasing. However, unlike the graphs in Figure 1, the arithmetic means are all zero.

Figure 2 about here

8. CONCLUDING NOTE

Understanding the exact connection between inequality and justice is important because justice is classically regarded as the first line of defense against self-interest and inequality. Absent a strong and clear link between inequality and justice, the sense of justice would not awaken to exert its moral suasion, no matter how great the inequality or how fast its increase. We obtained exact links between economic inequality and three parameters of the justice evaluation distribution–the mean, median, and variance–across a comprehensive set of inequality measures and a substantial starter set of just reward scenarios. This work shows that there is no general necessary connection between inequality and justice–inequality effects can be nonexistent, or can occur in opposite directions. Important factors shaping the inequality-justice connection include ideas of the just reward, relations between the actual rewards and just rewards, and the distributional forms of income and wealth.

The fact that inequality effects can be zero helps illuminate the common observation that sometimes inequality seems not to matter. On the other hand, there is in some justice situations a striking pattern: as economic inequality increases, the average of the justice evaluations moves leftward, deeper into the territory of unjust underreward, and the distribution stretches outward, increasing the gulf between underrewarded and overrewarded and hollowing out the middle class.

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besides having the derivatives at hand (Table 4), it is clear that as the height of the curves diminishes, there is growth in the tails; moreover, in the two exponentials, the non-tail end is clearly stretching outward as inequality increases.

Of course, much further work is needed to deepen understanding of the exact inequality-justice connection. In particular:

First, it will be useful to enlarge the sets of just reward scenarios and parameters of the distribution of justice evaluations, exploring, for example, further scenarios for selecting the just reward and inequality effects on the proportion underrewarded.

Second, it will be useful to explore the connection between inequality in the actual rewards and the individual's own justice evaluation–as opposed to parameters of the justice evaluation distribution. While it seems reasonable to focus, as a first step, on links from one distributional parameter to another distributional parameter, there may be inequality effects directly on individuals. Indeed, there is a rich empirical literature on which to build (Alwin 1987).

Third, it will be useful to explore the connection between inequality in the actual rewards and the justice evaluation subdistribution in important subgroups, such as subgroups defined by race, sex, or religion. There are already clues in the literature, for example, that in certain specified kinds of distributions, inequality between persons and inequality between subgroups go hand in hand, as well as a rich literature on subgroup gaps (Jasso and Kotz 2008).

Further work specifying and strengthening the logical foundation will help guide development of sharp new empirical strategies for deeper understanding of the inequality-justice connection in all its manifestations.

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Table 1. Scenarios for the Just Reward X^* and Its Relation to the Actual Reward X

Scenario 1	X^* can assume any positive value
Scenario 2	X^* is a positive constant k
Scenario 2.1	X^* is the arithmetic mean of X (comparison to equality)
Scenario 2.2	X^* is the geometric mean of X (comparison to all)
Scenario 3	X^* equals kX, k positive
Scenario 3.1	X^* equals X (k positive and equal to one), so that $J = 0$
Scenario 3.2	X^* equals kX, k positive and not equal to one, so that $J = -\ln(k)$

Note: The justice evaluation J is specified as the logarithm of the ratio of the actual reward X to the just reward X^*: $J = \ln(X/X^*)$.

Table 2. Average of the Distribution of Justice Evaluations and Effect of Economic Inequality

X Distribution	Average of the Distribution of Justice Evaluations $E(J) = E[\ln(X)] - E[\ln(X^*)]$	Effect of Economic Inequality	
		$\frac{\partial E(J)}{\partial(\cdot)}$	$\frac{\partial E(J)}{\partial I(X)}$
A. Inequality in X represented by Atkinson's measure A			
General	$\ln[1 - A(X)] + \ln[E(X)] - E[\ln(X^*)]$	$-\frac{1}{1 - A(X)}$	< 0
B. Inequality in X represented by Theil's MLD			
General	$-MLD(X) + \ln[E(X)] - E[\ln(X^*)]$	-1	< 0
C. Inequality in X represented by general inequality parameter c in continuous univariate two-parameter probability distributions			
Lognormal ($c > 0$)	$-\frac{c^2}{2} + \ln[E(X)] - E[\ln(X^*)]$	$-c < 0$	< 0
Pareto ($c > 1$)	$\frac{1}{c} + \ln\left(\frac{c-1}{c}\right) + \ln[E(X)] - E[\ln(X^*)]$	$\frac{1}{c^2(c-1)} > 0$	< 0
Power-Function ($c > 0$)	$-\frac{1}{c} + \ln\left(\frac{c+1}{c}\right) + \ln[E(X)] - E[\ln(X^*)]$	$\frac{1}{c^2(c+1)} > 0$	< 0

Notes: The justice evaluation J is represented by the logarithm of the ratio of the actual reward X to the just reward X^*. The mean of the distribution of justice evaluations may thus be expressed as $E[\ln(X)] - E[\ln(X^*)]$. Overall inequality in X is denoted $I(X)$ and represented in three ways. The modeling distributions for X are specified in terms of two parameters, the mean μ and the general inequality parameter c. In the lognormal distribution, inequality is an increasing function of c; in the Pareto and power-function distributions, inequality is a decreasing function of c.

Table 3. Median of the Distribution of Justice Evaluations and Effect of Economic Inequality

X Distribution	Median of the Distribution of Justice Evaluations $M(J) = Q_J(.5)$	Effect of Economic Inequality $\frac{\partial M(J)}{\partial c}$	$\frac{\partial M(J)}{\partial I(X)}$
A. Scenario 2: X^* is a positive constant. Thus, $Q_J(.5) = Q_{\ln(X)}(.5) - \ln(X^*)$.			
Lognormal ($c > 0$)	$-\frac{c^2}{2} + \ln[E(X)] - \ln(X^*)$	$-c < 0$	< 0
Pareto ($c > 1$)	$\frac{\ln(2)}{c} + \ln\left(\frac{c-1}{c}\right) + \ln[E(X)] - \ln(X^*)$	$\frac{1}{c(c-1)} - \frac{\ln(2)}{c^2} > 0$	< 0
Power-Function ($c > 0$)	$-\frac{\ln(2)}{c} + \ln\left(\frac{c+1}{c}\right) + \ln[E(X)] - \ln(X^*)$	$\frac{\ln(2)}{c^2} - \frac{1}{c(c+1)} > 0$	< 0
B. Scenario 2.1: $X^* = E(X)$. Thus, $Q_J(.5) = Q_{\ln(X)}(.5) - \ln[E(X)]$.			
Lognormal ($c > 0$)	$-\frac{c^2}{2}$	$-c < 0$	< 0
Pareto ($c > 1$)	$\frac{\ln(2)}{c} + \ln\left(\frac{c-1}{c}\right)$	$\frac{1}{c(c-1)} - \frac{\ln(2)}{c^2} > 0$	< 0
Power-Function ($c > 0$)	$-\frac{\ln(2)}{c} + \ln\left(\frac{c+1}{c}\right)$	$\frac{ln(2)}{c^2} - \frac{1}{c(c+1)} > 0$	< 0

C. Scenario 2.2: $X^* = G(X)$. Thus, $Q_J(.5) = Q_{\ln(X)}(.5) - \ln[G(X)]$.

Lognormal ($c > 0$)	0	0	0
Pareto ($c > 1$)	$\frac{\ln(2) - 1}{c}$	$\frac{1 - \ln(2)}{c^2} > 0$	< 0
Power-Function ($c > 0$)	$\frac{1 - \ln(2)}{c}$	$\frac{\ln(2) - 1}{c^2} < 0$	> 0

Notes: The justice evaluation J is represented by the logarithm of the ratio of the actual amount X to the just reward X^*. The median of the distribution of justice evaluations is expressed as the quantile function evaluated at .5. Overall inequality in X is denoted $I(X)$. The modeling distributions for X are specified in terms of two parameters, the mean μ and the general inequality parameter c. In the lognormal distribution, inequality is an increasing function of the general inequality parameter c; in the Pareto and power-function distributions, inequality is a decreasing function of the general inequality parameter c.

Table 4. Variance of the Distribution of Justice Evaluations and Effect of Economic Inequality

X Distribution	Variance of the Distribution of Justice Evaluations $Var(J) = Var[\ln(X)] + Var[\ln(X^*)] - 2Cov[\ln(X), \ln(X^*)]$	Effect of Economic Inequality	
		$\frac{\partial Var(J)}{\partial c}$	$\frac{\partial Var(J)}{\partial I(X)}$
Scenario 2: X^* is a constant. Thus, $Var(J)^* = Var[\ln(X)]$.			
Lognormal ($c > 0$)	c^2	$2c > 0$	> 0
Pareto ($c > 1$)	$\frac{1}{c^2}$	$-\frac{1}{c^3} < 0$	> 0
Power-Function ($c > 0$)	$\frac{1}{c^2}$	$-\frac{1}{c^3} < 0$	> 0

Notes: The justice evaluation *J* is represented by the logarithm of the ratio of the actual amount *X* to the comparison amount X^*. Overall inequality in *X* is denoted *I(X)*. The modeling distributions for *X* are specified in terms of two parameters, the mean μ and the general inequality parameter *c*. In the lognormal distribution, inequality is an increasing function of the general inequality parameter *c*; in the Pareto and power-function distributions, inequality is a decreasing function of the general inequality parameter *c*.

Table 5. Inequality Effects on the Justice Parameters, by Scenarios for the Just Reward X^* and Its Relation to the Actual Reward X

	Parameters of the Justice Evaluation Distribution		
	$E(J)$	$M(J)$	$Var(J)$
Scenario 1	- 0	- 0 +	0 +
Scenario 2	- 0	- 0 +	+
Scenario 2.1	-	-	+
Scenario 2.2	0	- 0 +	+
Scenario 3	0	0	0
Scenario 3.1	0	0	0
Scenario 3.2	0	0	0

Note: The justice evaluation J is specified as the logarithm of the ratio of the actual reward X to the just reward X^*: $J = \ln(X/X^*)$.

Appendix Table A.1. Mean and Variance of ln(*X*)

X Distribution	Mean of ln(*X*) $E[\ln(X)] = \ln[G(X)]$	Variance of ln(*X*) $Var[\ln(X)]$
Lognormal ($c > 0$)	$-\frac{c^2}{2} + \ln[E(X)]$	c^2
Pareto ($c > 1$)	$\frac{1}{c} + \ln\left(\frac{c-1}{c}\right) + \ln[E(X)]$	$\frac{1}{c^2}$
Power-Function ($c > 0$)	$-\frac{1}{c} + \ln\left(\frac{c+1}{c}\right) + \ln[E(X)]$	$\frac{1}{c^2}$

Notes: The modeling distributions for *X* are specified in terms of two parameters, the mean *E*(*X*) and the general inequality parameter *c*. In the lognormal distribution, inequality is an increasing function of the general inequality parameter *c*; in the Pareto and power-function distributions, inequality is a decreasing function of the general inequality parameter *c*.

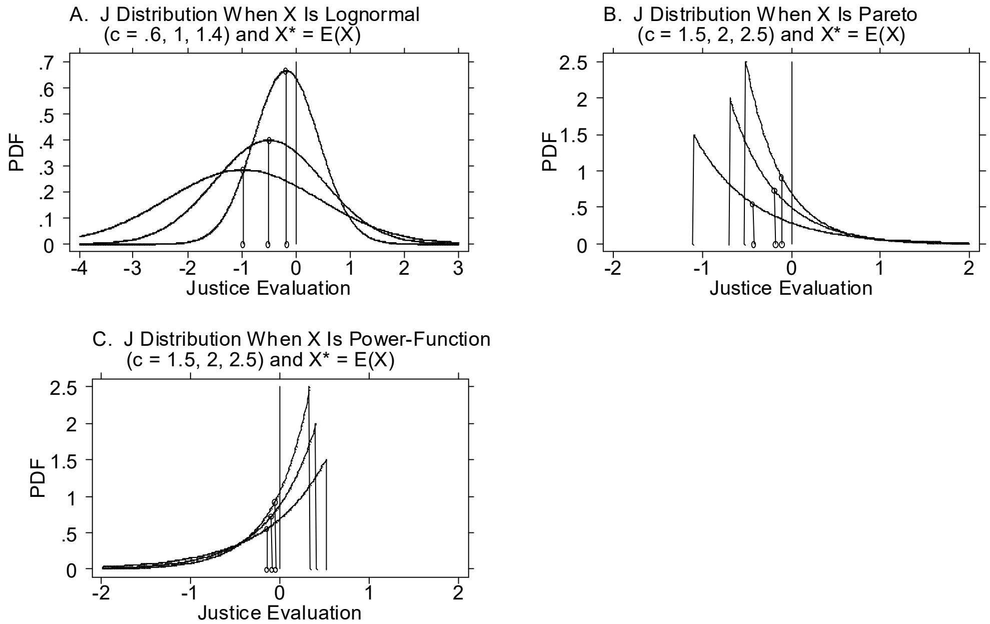

Figure 1. Justice Evaluation Distributions When the Just Reward X^* Is Fixed at the Arithmetic Mean of the Actual Rewards $E(X)$: Scenario 2.1, X Lognormal, Pareto, and Power-Function, J Normal, Negative Exponential, and Positive Exponential, Respectively. The arithmetic mean of the justice evaluations $E(J)$ is represented by vertical lines from the horizontal axis to the distribution, with circles at both ends. As economic inequality increases, $E(J)$ moves leftward, deeper into the territory of unjust underreward, and the distribution stretches outward in both underreward and overreward directions.

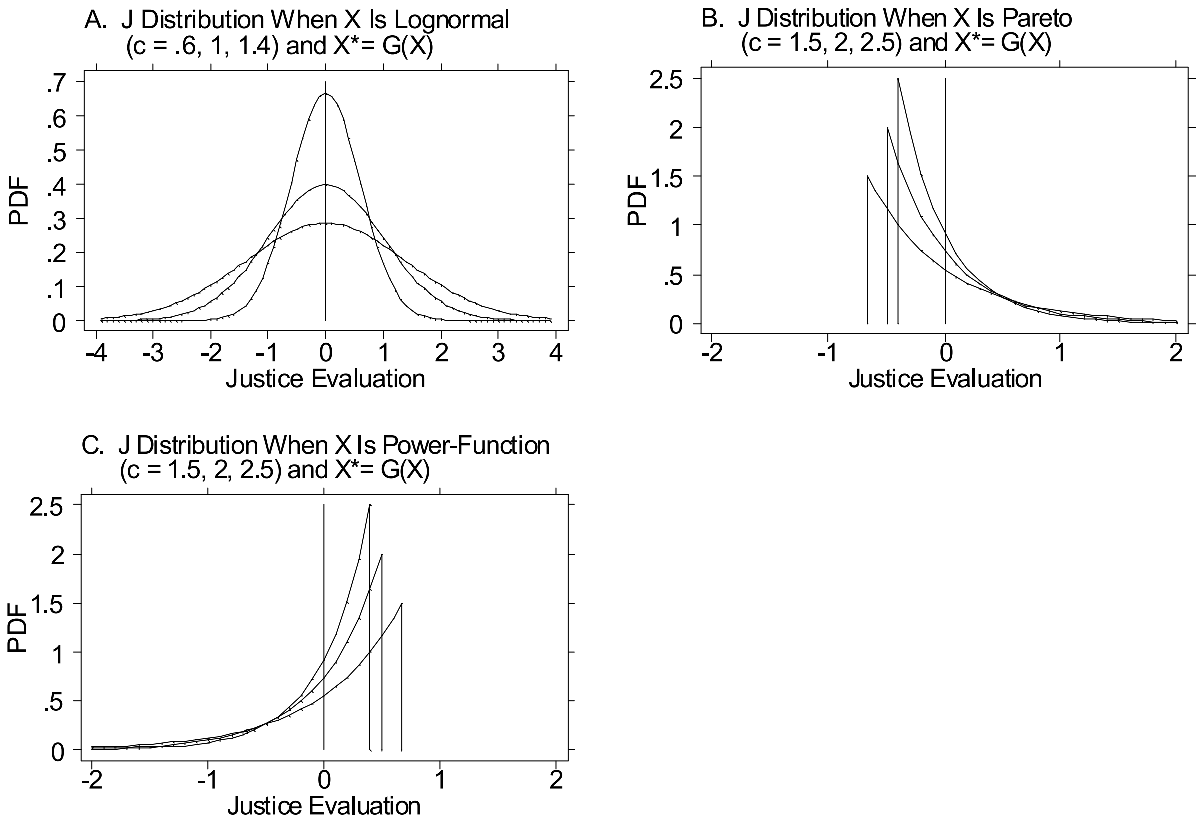

Figure 2. Justice Evaluation Distributions When the Just Reward *X** Is Fixed at the Geometric Mean of the Actual Rewards *G*(*X*): Scenario 2.2, *X* Lognormal, Pareto, and Power-Function, *J* Normal, Negative Exponential, and Positive Exponential, Respectively. As economic inequality increases, *E*(*J*) remains fixed at zero, while the distribution stretches outward in both underreward and overreward directions.