Equal treatment, asymmetric information and wage rigidity

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Abstract

We adapt the model of Menzio and Moen (2010) to consider a labour market with directed search in which firms can commit to wage contracts but cannot commit not to replace incumbent workers. Workers are risk averse, so that there exists an incentive for firms to smooth wages over time and in the face of shocks to labour productivity. To avoid worker replacement (which saves on the ex ante wage bill), they may choose a wage for new hires that is equally unresponsive to shocks. This leads to a large degree of downward rigidity in the wages of new hires, and magnifies the response of unemployment and vacancies to negative shocks. Our version of the Menzio-Moen model allows for the analysis of positive probability shocks in a tractable way. Moreover, we argue that the model provides a useful framework for analyzing other sources of wage rigidity; for example the interplay between asymmetric information can substantially enhance wage rigidity and increase the responsiveness of unemployment and vacancies to productivity shocks.

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1 Introduction

In this paper we develop a model of "equal treatment", i.e., in which wages of new hires are tied to wages of those of ongoing workers. The implication is that if there is a reason for ongoing wages to be rigid—here, risk-aversion—this will be transmitted to the wages of new hires. And it is the that latter that is important for employment fluctuations.[1]

We adapt the model of Menzio & Moen (2010), henceforth MM. In their paper overlapping generations of two-period lived firms interact with infinitely lived workers. We simplify the model to a two-period version that is more tractable for our purposes, but the basic ideas are as in their paper. Firms can commit to wage contracts, current and future, but not to employment. That is, they cannot commit not to layoff a worker. In particular, if the wage for new hires is below that of incumbents, the firm will have an incentive to replace its incumbents if it can find suitable applicants. Anticipating this, workers will have a preference for a contract in which wages of future hires are never below their own wages, so that the firm will have no incentive to attempt to replace them. It may then be that firms offer such contracts as the ex ante costs of hiring are lower by a sufficient amount to offset having to forgo the potential benefit of a lower wage for new hires in some future states. That is, it may be optimal to satisfy a "no replacement constraint" that requires that the wage for new hires is never below that of incumbents.[2]

In adverse future states, because of the no replacement constraint, the firm will trade-off a desire to smooth the wages of workers in ongoing employment, with the benefits from cutting the wage for new entrants. Treated on their own merit, the latter would receive a lower wage, but this would take it below the optimal wage to be paid to incumbents. The upshot then is that there is a degree of downward wage rigidity. The opposite is not true however. In particularly good states there is no problem in paying a higher wage to new entrants than to incumbents, so the rigidity only operates in a downward direction.

Because the wage for new entrants is allocational, the downwardly rigid wage affects hiring, and increases the variability of both unemployment and vacancies in

[1] A recent paper which analyses this idea within the search-matching model is Gertler & Trigari (2009). An earlier model with similar implications in a competitive labour market is Thomas (2005).

[2] This type of argument was also made in Snell & Thomas (2010) in the context of a perfectly competitive labour market. MM's model however concerns a frictional labour market, and we follow their approach.

response to productivity shocks, a point made also by MM.[3]

We extend the model to incorporate asymmetric information about the state of nature, specifically that firms are better informed. In this case we show that wages may be fully rigid downwards, thus further amplifying the variability of unemployment and vacancies. Such simple non-contingent labour contracts are well documented (e.g., Oswald (1986), Blinder & Choi (1990), and see Malcomson (1997) for an excellent overview). We show that it is the interplay between the equal treatment and the asymmetric information that leads to this result; without equal treatment introducing asymmetric information has no impact on allocations.

This mechanism is most closely related to Menzio (2005) who showed that firms, who post wages, may not respond to better states of the world by raising wages for new hires given that improves the position of incumbents, and this rigidity amplifies employment variability. Likewise Kennan (2010) develops a model of procyclical information rents to firms which has similar implications; in his model if a privately observed (to firms) component of match surplus has more dispersion when the aggregate state of the economy is better, and bargaining leads to an outcome in which firms capture the informational rent, again wages are relatively rigid and procyclical rents to employer mean that employment fluctuations are magnified.

Recent evidence from a study of 15 European Union countries by Galuscak et al. (Galuscak, Keeney, Nicolitsas, Smets, Strzelecki & Vodopivec (2012)) suggests that new hire wages are intimately related to wages structures already existing in the firm; moreover this relationship is stronger in periods of labour market slack, which is a feature of the equilibrium we derive here. Galuscak et al. argue that fairness and incentive issues are important in leading to this linkage. This is consistent with evidence collected by Bewley (1999) who argued that internal equity considerations make it difficult for firms to employ new hires at a wage below that paid to incumbents. Gertler & Trigari (2009) estimate the cyclicality of hiring wages in the U.S. by using Survey of Income and Program Participation data and argue that wages of new hires do not appear to be more procyclical than those of ongoing employees.

[3]Similarly in models, which has competitive labour markets, equal treatment can lead to amplified unemployment fluctuations due to firms optimally damping wage fluctuations. See for an earlier model with these features.

2 The Model

We adapt the model of MM and adopt their notation where possible. There are two periods $t = 1, 2$. We assume that each firm and worker lives for both periods with K firms and $S \cdot K$ workers. Both K and S are large. We identify each firm with an entrepreneur who owns it. In each period a representative firm operates a decreasing returns technology producing a perishable good, with production function $f(n; x)$, where n is the current number of workers employed at the firm, $x \in X$ is a productivity shock observable at the start of the period, and $f' > 0$, $f'' < 0$. (Hours per worker are not variable.) Current profits, not including job creation costs, are given by $f(n; x) - wn$, where w denotes the (real) wage paid in the current period (assuming a uniform wage, which need not hold in period 2). We assume that $x = x_0$ is fixed at $t = 1$, but at $t = 1$, x is a random variable, common across firms, with finite support. Henceforth x without a 0 subscript will refer to the second period productivity shock. . Each worker has a per-period utility of consumption function $v(c)$, $v' > 0$ and $v'' < 0$. Workers cannot borrow or save, so consume all their current income; we assume there is no discounting of the future by workers. Entrepreneurs on the other hand are risk-neutral, but they also have a zero discount rate.

A firm has a wage policy $\sigma = \left(w_1, (w_{2i})_{i=1,2}\right)$ to which it commits, where i is the length of the worker's tenure and w_{2i} may be random (state contingent); so at $t = 1$ workers are offered a wage contract (w_1, w_{22}) and period 2 hires are offered w_{21}. (We also consider the case where there is no commitment to w_{21} later in the paper.) A worker who accepts a contract at $t = 1$ suffers exogenous separation from the firm at the end of the first period, with probability δ. In this case he will be in the same position as a worker who failed to gain employment in the first period; in the second period such unattached workers seek work.[4] As in MM, employment is assumed to be "at will", so during the matching stage of the second period (after observing x) the firm can dismiss a worker without compensation, and a worker can quit without penalty. We assume that such workers remain unemployed in their second period. A worker who is unemployed in any period receives an income of b.

At the start of each period (in period 2,after x is observed), search and matching occur. We assume directed search (see Moen (1997) for the seminal paper in this area, and also Acemoglu & Shimer (1999), and Rudanko (2009)). We follow MM in the following. Briefly, an unemployed worker can apply for one job at a single firm

[4]MM assume that separated workers cannot work in the period immediately following separation.

each period. We rule out on-the-job search, so that at $t = 2$ a worker cannot apply for a job if he is already employed. We identify the 'type' of a job with the utility V a successful applicant gets from it. The application succeeds with probability $p(\theta(V))$, where $\theta(V)$, 'the expected queue length for the job,' is the ratio of applicants to jobs of type V, that is, the inverse of labor market tightness. (The determination of $\theta(V)$ is discussed below.) The function $p(\cdot)$ is assumed to be strictly decreasing, differentiable and such that $p(0) = 1$, $p(\infty) = 0$. Correspondingly the firm fills a job of type V with probability $q(\theta(V))$ where $q(\cdot)$ is strictly increasing, and satisfies $q(\theta) = p(\theta)\theta$, $q(0) = 0$ and $q(\infty) = 1$. Moreover, denoting the elasticity of q wrt θ by $\epsilon_q(\theta)$, $q(\theta)\epsilon_q(\theta)/(1 - \epsilon_q(\theta))$ is assumed to be a decreasing function of θ.[5] At $t = 2$, unemployed workers can apply for jobs that are already filled; if there is a successful applicant, the firm can, by at will contracting, choose whether to replace the incumbent or not. If $w_{21} \geq w_{22}$ firms will have no incentive to do this, but for $w_{21} < w_{22}$ the incentive exists and in this case a filled job is as attractive as an unfilled one from the point of view of an applicant. In the latter case, then, to the extent that the matching process succeeds in selecting a successful applicant, the incumbent is at risk of losing her position.

Simultaneously with committing to a wage policy at the start of $t = 1$, firms choose how many new jobs $\overline{n}_i$ to create in period $i = 1, 2$, at a cost of $k > 0$ per job; $\overline{n}_2$ depends on the shock x. There is no cost associated with receiving applicants for filled jobs. Unfilled jobs from the first period 'die' at the end of the period, along with filled jobs in which exogenous separation occurred (little depends on this). The implication is that employment at the firm in period i will increase by $q(\theta(V))\,\overline{n}_i$.

Our model differs from MM in the following principal respects. First, our workers are two-period lived rather than infinitely lived (firms in MM are two-period lived), and we have a two-period horizon. Secondly, rather than having firms of fixed size (number of jobs) with constant productivity per filled job and free entry of firms, we suppose that there are a fixed number of firms, each with a decreasing returns to scale technology. The supply of jobs then varies not with variations in the number of firms entering the market, but with the choice of firms about how many jobs (or "vacancies") to create each period. The fixed cost per job created replaces MM's assumption of a fixed cost incurred per firm that enters.

Let Z_1 be the lifetime utility of a worker at the search stage, and $Z_2(x)$ that of a worker at $t = 2$ searching for work in state x. (Z_1 and Z_2 are the endoge-

[5] MM, who assume this, point out that many standard matching processes satisfy these assumptions.

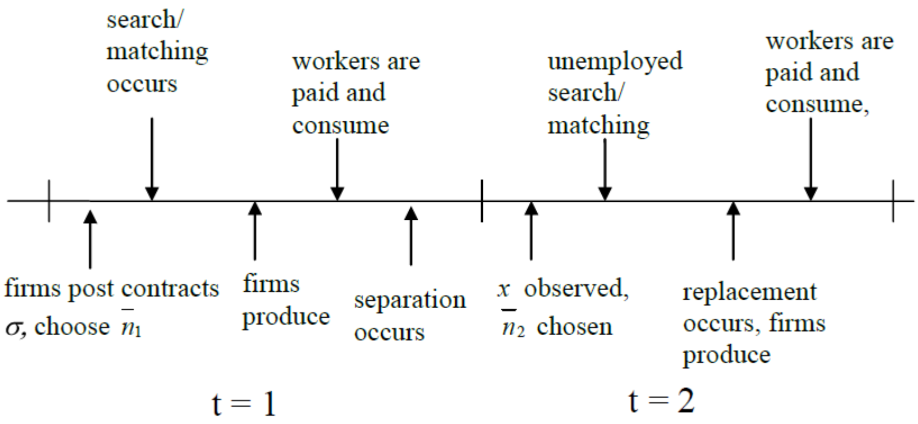

Figure 1: Timeline

nous variables determining the economic environment facing the firm.) Define $Z = \left(Z_1, (Z_2(x))_{x \in X}\right)$. The value to a worker at $t = 1$ from being employed by a firm with wage policy σ then is

$$V_1(\sigma; Z) := v(w_1) + E[\delta Z_2(x) + (1-\delta)v(w_{22}(x))]$$

if the worker only faces a separation risk, where E denotes expectation. On the other hand, if replacement occurs in some states, that is, if $w_{21} < w_{22}$, then in such states the term inside the square brackets must be replaced by

$$\delta Z_2(x) + (1-\delta)q(\theta_2)v(b) + (1-\delta)(1-q(\theta_2))v(w_{22}(x)),$$

where $\theta_2 = \theta_2(w_{21}, Z_2(x))$ (defined below) is the queue length in that state for a firm offering w_{21}. This reflects the additional risk $q(\theta_2)$ to a surviving worker of being replaced by a successful applicant.[6]

Let U_1 be the lifetime utility of a worker at $t = 1$ who fails to get a job:

$$U_1(Z) = v(b) + E[Z_2(x)],$$

as currently the worker receives b and is able to search next period. Given U_1 and Z_1, the expected queue length for a job offering V_1 is assumed to satisfy:

$$\theta_1(V_1, Z_1, U_1) = \begin{cases} \theta : p(\theta)V_1 + (1-p(\theta))U_1 = Z_1, & \text{if } V_1 > Z_1 \\ 0, & \text{if } V_1 \leq Z_1 \end{cases} \tag{1}$$

[6] To avoid complicating the exposition, we shall ignore the possibility that at the optimal period 2 wage, the firm would prefer to dismiss some of its incumbents. This would arise if $w_{22} > f'((1-\delta)n_1; x)$. Likewise, we assume that $w_{22} \geq b$, or otherwise it would be in the interests of the worker to quit. In our simulations, parameters are chosen such that neither scenario arises.

The idea is that if the value of the job to a successful applicant, V_1, is greater than the value of search, Z_1, the expected queue length is driven up to the point where workers are indifferent between applying for the job and searching somewhere else, and vice versa. The expected queue length for the job will be zero if the value of the job is less than (or equal to) the value of search.

For a worker at $t = 2$ the value from being employed at the wage w_{21} is $v(w_{21})$, so the expected queue length for period 2 firms and workers for a job with wage w_{21} is

$$\theta_2(w_{21}, Z_2) = \begin{cases} \theta : p(\theta)v(w_{21}) + (1 - p(\theta))v(b) = Z_2, & \text{if } v(w_{21}) > Z_2 \\ 0, & \text{if } v(w_{21}) \leq Z_2 \end{cases} \tag{2}$$

Assuming that incumbents are not replaced in period 2, a firm's profit is:

$$\begin{aligned} F(\sigma; \overline{n}_1, (\overline{n}_2(x))_{x\in X}; Z) &= (f(n_1; x_0) - w_1 n_1 - k\overline{n}_1) + \\ &\quad E[(f((1-\delta)n_1 + n_2; x) - w_{22}(1-\delta)n_1 - w_{21}n_2 - k\overline{n}_2)] \end{aligned}$$

where n_i is the number of new hires in period i, and is given by $n_i = q(\theta_i)\overline{n}_i$, $i = 1, 2$, where θ_i depends on σ as given by $\theta_1(V_1(\sigma, Z), Z_1, U_1(Z))$ in (1) and $\theta_2(w_{21}, Z_2(x))$ in (2) above. Otherwise, in any state where replacement occurs, the expression for second period profit is replaced by

$$f((1-\delta)n_1 + n_2; x) - w_{22}(1 - q(\theta_2))(1-\delta)n_1 - w_{21}(n_2 + q(\theta_2)(1-\delta)n_1) - k\overline{n}_2,$$

where $q(\theta_2)(1-\delta)n_1$ is the number of incumbents who are replaced by new hires, and $n_2 = q(\theta_2)\overline{n}_2$ is the number of new hires *into newly created jobs.*

Competitive Search Equilibrium

We define an equilibrium:

Definition 1 *A symmetric stationary competitive search equilibrium consists of search values* $Z = \left(Z_1, (Z_2(x))_{x\in X}\right)$*, and a wage policy* σ *and job creation plan* $\left(\overline{n}_1, (\overline{n}_2(x))_{x\in X}\right)$ *with the following properties:*

(i) *Profit maximization*: For all $(\sigma'; \overline{n}_1', (\overline{n}_2'(x))_{x\in X})$,

$$F((\sigma; \overline{n}_1, (\overline{n}_2(x))_{x\in X}); Z) \geq F(\sigma'; \overline{n}_1', (\overline{n}_2'(x))_{x\in X}; Z);$$

and (ii) *Consistency*: $\theta_1(V_1(\sigma, Z), Z_1, U_1) = S/\overline{n}_1$, and, for all x, if $w_{21} \geq w_{22}$ (no replacement occurs), $\theta_2(w_{21}, Z_2(x)) = S_2/\overline{n}_2(x)$ where $S_2 := ((1 - p(S/\overline{n}_1)) + \delta p(S/\overline{n}_1))S$ is the number of old workers (per firm) seeking work in period 2, while if $w_{21} < w_{22}$ (replacement occurs) $\theta_2(w_{21}(x), Z_2(x)) = S_2/(\overline{n}_2(x) + (1-\delta)q(S/\overline{n}_1)\overline{n}_1)$.

2.1 No replacement in state x

We start by characterizing an optimal policy assuming that in state x, $w_{21} \geq w_{22}$. We will deal with the issue of whether this is optimal below, that is whether a policy with $w_{21} < w_{22}$ might yield higher profits. We proceed heuristically.[7] In period 2 in any state x, given n_1 and w_1, following MM it can be shown that the firm must locally maximize profits plus weighted incumbent utility.[8] In particular, given it is optimal *not* to replace, it must maximize

$$\begin{aligned} f\left((1-\delta)n_1+n_2;x\right) \quad & -w_{22}(1-\delta)n_1 - w_{21}n_2 - k\overline{n}_2 + \\ & \left(1/v'\left(w_1\right)\right)n_1\left((1-\delta)\,v\left(w_{22}\right)+\delta Z_2\left(x\right)\right), \end{aligned} \tag{3}$$

with respect to $\overline{n}_2, w_{21}, w_{22}, w_{21} \geq w_{22}$, where $n_2 = q\left(\theta\left(w_{21}, Z_2\left(x\right)\right)\right)\overline{n}_2 =: \tilde{q}\left(w_{21}, x\right)\overline{n}_2$. We write $\tilde{q}' \equiv \partial\tilde{q}/\partial w_{21}$. Note that the last term in (3) includes the continuation utility of an incumbent, taking into account the separation possibility, and multiplied by the number of incumbents. The intuition here is that any change which affects the utility of the firm's old workers can be offset by a change in the first period wage, leaving V_1 unchanged (and hence n_1). Multiplying the utility change through by the inverse of first period marginal utility then converts it (for a small change) to the first period wage saving per worker. If this was not satisfied then profits can be increased.

There are two cases to consider:

(A) If the "no replacement constraint" $w_{21} \geq w_{22}$ is not binding, then differentiating (3) with respect to w_{22},

$$(1-\delta)n_1 = n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\,v'\left(w_{22}\right)\right), \tag{4}$$

so that $w_1 = w_{22}$. Intuitively the firm should stabilize the wages of the first period hires if there is no cost to doing this. In this case, also differentiating with respect to w_{21}, we get

$$f'\left((1-\delta)n_1+n_2;x\right)q'\overline{n}_2 - w_{21}q'\overline{n}_2 - q\overline{n}_2 = 0, \tag{5}$$

[7] The following necessary conditions are derived formally in the Appendix by considering the two-period problem. Alternatively, it can be directly established that (3) below must hold at a local maximum subject to $w_{21} \geq w_{22}$.

[8] MM introduce a sunspot into their model, and this allows the firm to randomize between replacement and no-replacement. They can then show that an equivalent of (3) must be maximized across replacement/no replacement regimes and derive analytical sufficient conditions for no-replacement to be optimal. We could follow a similar approach here, but as we are able to compute numerical solutions straightforwardly the solution can be checked directly. Moreover the restriction to contracts dependent only on the productivity shock simplifies the presentation.

and simplifying:

$$f'(n)\,\tilde{q}' - w_{21}\tilde{q}' - q = 0,$$

where we write $n \equiv (1-\delta)n_1 + n_2$ for total period 2 employment. Finally, differentiating with respect to $\overline{n}_2$,

$$f'(n) = w_{21} + k/q. \tag{6}$$

We can combine these latter two to get

$$q^2\left(\tilde{q}'\right)^{-1} = k. \tag{7}$$

Intuitively, in order to increase employment by one unit, the firm could open $1/q$ jobs at a cost of k/q. Alternatively a wage increase of $1/\left(\overline{n}_2\tilde{q}'\right)$, holding the number of jobs constant, accomplishes the same thing by increasing the probability each existing job is filled, at a cost of $q\overline{n}_2 \times 1/\left(\overline{n}_2\tilde{q}'\right) = q/\tilde{q}'$. The two must be equal in equilibrium.

In the proof of Proposition 1 it is shown that (7) can be solved to give a positively sloped locus of values for n_2 and w_{21} compatible with equilibrium. This locus defines an upward sloping 'quasi-supply' curve of labor: when equilibrium n_2 is higher, it is harder to fill each job because the labor market is tighter (θ_2 is lower, so $k/q(\theta_2)$ is higher); this makes wage increases more attractive as a way of filling jobs than creating jobs, so w_{21} rises until the two methods cost the same. This locus is independent of the profitability of filling a job. We refer to this as the *commitment quasi-supply curve.* It corresponds to the solution to the first-order conditions in the case where firms can commit not to replace incumbent workers, and thus ignores the no-replacement constraint $w_{21} \geq w_{22}$. (The two coincide in this case because the constraint is not binding by assumption.) Combining this with the downward sloping (6), which is a standard labor demand equation, where the unit cost of increasing employment $k/q(\theta_2)$ is added to the wage (itself increasing as n_2 increases),[9] yields a unique equilibrium for each productivity shock whenever the no-replacement constraint does not bind.[10] As x varies, only the labor demand curve shifts. Denote the solution of (6) and (7) by $\left(w_{2i}^C(x, w_1, n_1), n_2^C(x, w_1, n_1)\right)$, where the $C-$superscript indicates that this is the solution to the FOCs in the case of commitment.

Since in this case, $w_{21} \geq w_{22} = w_1$, we conclude that the intersection of (6) and (7) occurs at or above w_1.

[9] As n_2 increases, we must have $p(\theta)$ increasing as $n_2 = p(\theta)S_2$ and hence θ has fallen as $p' < 0$; thus $q(\theta)$ falls given that $q' > 0$.

[10] The position of these two curves depend only on n_1, which implies the value of S_2.

(B) If on the other hand $w_{21} \geq w_{22}$ is binding at the optimum, the intersection of (6) and (7) occurs at a wage below w_1 but the wage can be shown to be above $w_{21}^C(x, w_1, n_1)$, while employment is below $n_2^C(x, w_1, n_1)$. In the proof it is shown that $k < q^2/\tilde{q}'$. The unit cost of increasing employment through creating extra jobs, k/q, is lower than that through increasing wages, $q^2/\tilde{q}'$, but it would not pay to cut wages and increase jobs as the wage cut has a negative externality on incumbents' wage smoothing. More intuitively, if productivity is low enough that the equilibrium hiring wage under commitment w_{21}^C would be below w_1, then the no-replacement constraint would be violated (recall that $w_{22}^C = w_1$). To satisfy the constraint w_{22} must be cut, which is costly as it reduces wage smoothing so firms are less willing to let wages fall. The quasi-labor-supply curve is thus flatter below w_1.

Consequently, taking as given w_1, we can plot a *no-commitment quasi-supply* curve in $w_{21} - n_2$ space, which coincides with the commitment one above w_1, but below w_1 the curve lies above the commitment curve. Equilibrium occurs at the intersection with the labor demand curve. As x varies, the latter curve is shifted. In Figure 2, a situation where the crossing point occurs below w_1 is illustrated. The equilibrium values are at point A, rather than at the commitment solution.[11] If x is sufficiently high that the intersection occurs above w_1, then the equilibrium will be at the commitment solution, $\left(w_{2i}^C(x, w_1, n_1), n_2^C(x, w_1, n_1)\right)$. The proposition summarizes the discussion.

Proposition 1 *Suppose replacement does not occur in state x. Then (a) if equilibrium hiring wages in period 2 are below period 1 wages, $w_{21} < w_1$, the wage is higher and employment is lower than they would be in that state if firms were able to commit, that is, $w_{21} > w_{21}^C(x; w_1, n_1)$ and $n_2 < n_2^C(x; w_1, n_1)$; moreover $w_{22} = w_{21} < w_1$. Otherwise (b) wages and employment are at the commitment levels: $w_{21}^{NC}(x; w_1, n_1) = w_{21}^C(x; w_1, n_1)$ and $n_2^{NC}(x; w_1, n_1) = n_2^C(x; w_1, n_1)$, with $w_{22}^{NC}(x; w_1, n_1) = w_1$. Case (a) occurs when the labor demand curve intersects the commitment quasi-supply curve below w_1; otherwise case (b) occurs.*

[11] If commitment was allowed in such a state, unless the state has negligible probability, then the equilibrium two-period contract may be different, that is, w_1 and n_1 may differ. The proposition concerns the implied values of w_{21}^C and n_2^C in a hypothetical equilibrium which has the same period 1 values.

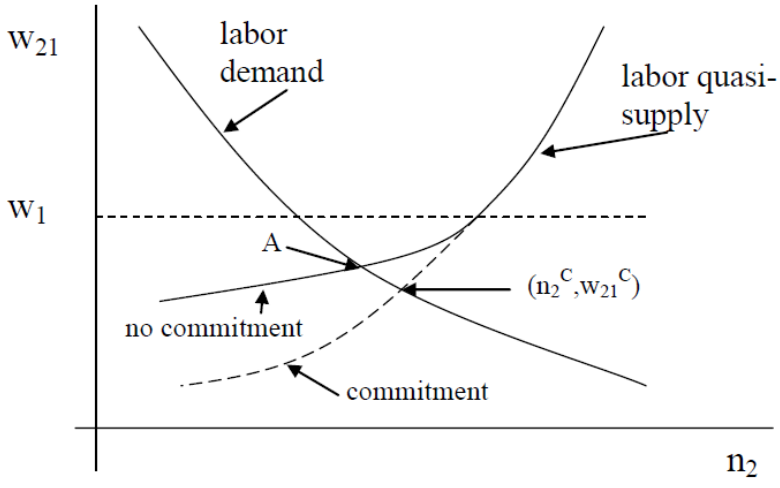

Figure 2: No-commitment quasi-supply

2.2 Replacement in state x

If replacement occurs, again the firm must locally maximize profits plus weighted incumbent utility:

$$f\left((1-\delta)n_1+n_2;x\right)-w_{22}(1-\delta)(1-q)n_1-w_{21}\left(q(1-\delta)n_1+n_2\right)-k\overline{n}_2$$
$$+n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v\left(w_{22}\right)+\delta Z_2+(1-\delta)\,qv\left(b\right)\right),$$

where $\overline{n}_2$ is again the number of new jobs created, and $n_2=q\left(\theta\left(w_{21},Z_2\left(x\right)\right)\right)\overline{n}_2$. Then differentiating with respect to w_{22},

$$(1-\delta)(1-q)n_1=n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v'\left(w_{22}\right)\right), \tag{8}$$

so that $w_1=w_{22}$, as expected. Intuitively the firm should stabilize the wages of the first period hires as there is no cost to doing this—given the replacement probability is independent of w_{22}. Differentiating with respect to w_{21} we get

$$f'\left(n;x\right)q'\overline{n}_2 \quad -w_{21}q'\left((1-\delta)n_1+\overline{n}_2\right)-q\left((1-\delta)n_1+\overline{n}_2\right)+ \tag{9}$$
$$n_1\left(1/v'\left(w_1\right)\right)\left(1-\delta\right)\left(q'\right)\left(v\left(b\right)-v\left(w_{22}\right)\right)=0 \tag{10}$$

where the latter term is the extra cost of compensating more replaced workers for their loss of utility whereas previously we got

$$f'\left(n;x\right)q'\overline{n}_2-w_{21}q'\overline{n}_2-q\overline{n}_2=0$$

and differentiating with respect to $\overline{n}_2$,

$$f'\left(n;x\right)q=w_{21}q+k. \tag{11}$$

We can combine these latter two to get

$$(k/q)\, q'\overline{n}_2 - w_{21} q' \left((1-\delta)n_1\right) + n_1 \left(1/v'\left(w_1\right)\right)(1-\delta)\left(q'\right)\left(v\left(b\right) - v\left(w_{22}\right)\right) = q\left((1-\delta)n_1 + \overline{n}_2\right) \tag{12}$$

instead of

$$k\tilde{q}'/q = q. \tag{13}$$

Note then that the RHS of (12), if we divide through by $\overline{n}_2$, is bigger, while the LHS is smaller. Recall that $q^2/\tilde{q}'$ is increasing in θ and w_{21}.Thus to reestablish equality we need to decrease $q^2/\tilde{q}'$, that is at fixed θ we reduce w_{21}, so in $w_{21} - \theta$ space, the downward sloping locus must be shifted downward.

Given the tractability of the model, we proceed in our simulations by computing an equilibrium under the assumption that replacement is not optimal in any state. We then check whether replacement can improve profits. If this is true, we have an equilibrium but this does not logically rule out the possibility of an equilibrium with replacement existing at the same time.[12]

3 Asymmetric Information

So far we have seen that equal treatment leads to a measure of downward real rigidity. We now consider adding asymmetric information about the period 2 state x, and we argue that this may lead to a completely rigid period 2 wage for incumbents, and more importantly, also for new hires for a range of adverse shocks. We will assume that in period 2 ongoing hires in a firm can only observe wages w_{21} and w_{22}, but cannot observe x (nor z_2 so they cannot infer x). (Nor can they observe the total employment or vacancies at the firm.) The resultant incentive compatibility constraints on the contract imply that the equilibrium contract exhibits a much higher degree of wage rigidity and employment and vacancy fluctuations.

This contrasts with early models in the asymmetric information implicit contracting literature in which labour supply is observable to workers (Chari 1983, Green & Kahn 1983, Grossman & Hart 1981) —in a single worker model as considered in such models this is inevitable of course. In practice, however, the level of employment in a firm can be difficult to define precisely. For example, if the relevant employment level is at the plant, the firm may be able to move production to other companies or plants within the same company, making it difficult to condition on employment (as argued by Stiglitz (1986)).

[12] Although in none of the simulations carried out has this occurred.

As before, assuming that incumbents are not replaced in period 2, a firm's profit is:

$$F\left(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X};Z\right)=\left(f\left(n_1\right)-w_1n_1-k\overline{n}_1\right)+E[F_x]$$

where F_x is period 2 profits in state x and is given by

$$F_x\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right):=\left(f\left((1-\delta)n_1+n_2;x\right)-w_{22}(1-\delta)n_1-w_{21}n_2-k\overline{n}_2\right)$$

(again n_i is the number of new hires in period i, and is given by $n_i=q\left(\theta_i\right)\overline{n}_i$, $i=1,\ 2$, where θ_i depends on σ as given by $\theta_1(V_1\left(\sigma,Z\right),Z_1,U_1\left(Z\right))$ in (1) and $\theta_2(w_{21},Z_2\left(x\right))$ in (2) above). We now have the firm's maximization problem as:

$(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X})$ maximizes $F((\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X});Z)$ subject to the incentive compatibility constraints[13]

$$\begin{aligned}F_x\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right) &\geq \\ &\max_{x',\overline{n}_2'}\left\{\left(f\left((1-\delta)n_1+n_2';x\right)-w_{22}(x')(1-\delta)n_1-w_{21}(x')n_2'-k\overline{n}_2'\right)\right\}\end{aligned}$$

where $n_2'=q\left(\theta_2\right)\overline{n}_2'$ and $\theta_2=\theta_2(w_{21}(x'),Z_2\left(x\right))$.

We can establish the following. Suppose that under certainty the no replacement constraint strictly binds. Then there is a perturbation of this model with two different states such that equilibrium period 2 wages are constant. This is illustrated below:

In general simulations suggest that these assumptions lead to a form of contract that has a fixed period 2 wage for a wide range of shocks. To see the intuition, consider the no-commitment solution, suppose there are two states x_1 and x_2 at $t=2$ and that we are in the region where the no replacement constraint is binding in both states, $w_{12}\left(x\right)=w_{22}\left(x\right)$, $x=x_1,\ x_2$. If the wage varies with the state, say if $w_{12}\left(x_1\right)=w_{22}\left(x_1\right)<w_{12}\left(x_2\right)=w_{22}\left(x_2\right)$, in state x_2 the firm will prefer to "announce" state x_1: It benefits from paying a lower wage to its existing employees. In addition because the no-replacement constraint is binding, the wage for new hires would optimally be set lower, and the firm would benefit from a lower wage just considering this group. So for both reasons period 2 profits increase. Consequently the no-commitment solution would violate incentive compatibility. This argument

[13] These are ex post (after the period 2 state is observed) constraints; for simplicitly we assume that n_1 is contractible. Otherwise the IC constraints should be expressed in terms of an ex ante constraint which requires that should the firm deviate at date 1 and in any period 2 states it cannot increase its discounted profit. Since in the latter case the ex post constraints would also hold, the results would be very similar.

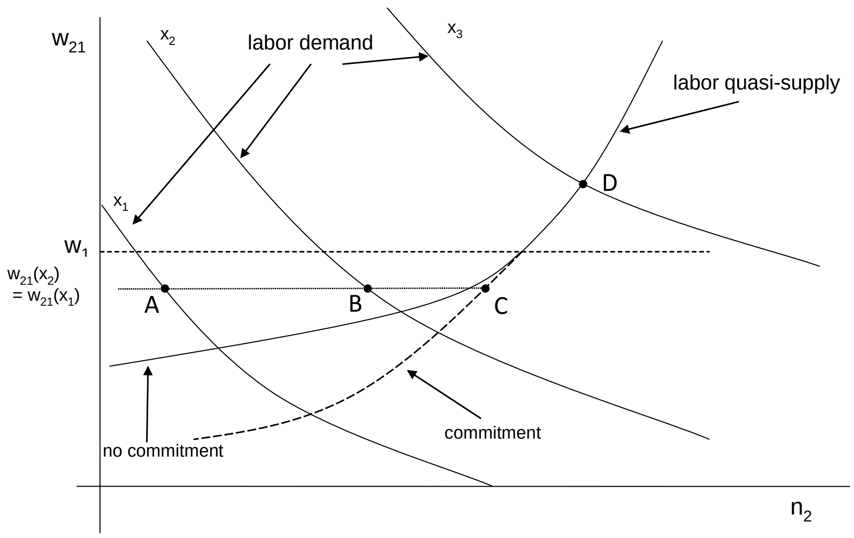

Figure 3: A rigid wage under asymmetric information

works for small wage variations across states; however, the lower w_{12} might be so low—below the optimal level in the other state—that switching to it reduces profits on new hires. This is unlikely to outweigh the gains from cutting w_{22} though, as these are first-order and large, while around the optimal hiring wage the change in profits on cutting w_{21} would be second-order.[14] The incentive compatible contract is illustrated by the points A and B in Figure 3.

Note also that this logic may not apply if there are states in which the no replacement constraint does not bind. In this case the firm will prefer to have the new hire wage above that of the incumbent, so that the new hire wage is flexible upwards. Specifically as the state of nature improves, the new hire (equilibrium) wage that is optimal taking w_1 as given, but ignoring the no replacement and incentive compatibility constraints, rises above the constant wage for the lower states. At this point w_{21} then is at this unconstrained level (see point D in Figure 3), while w_{22} will be slightly higher than the constant wage. The latter is due to the fact that there

[14]To be clear, for very high rates of turnover and wages that are not very close together, the no-commitment solution *will* satisfy IC. However in our simulations even for negative shocks up to 50% below the better shock, IC fails unless turnover rates are extremely high, around 90% or higher.

would now be a cost in deviating by announcing a lower state given that the new hire wage would fall below the optimal level; this allows there to be a potential gain on the reduced wagebill of incumbents, without violating incentive compatibility, so the incumbent wage can be increased towards w_1 (recall that $w_{22} = w_1$ is optimal). Initially this is a comparison between a second-order cost and a first-order gain, so the increase in w_{22} is itself second-order to avoid violating the incentive constraints.

In contrast to the earlier analysis, the no-commitment quasi-supply curve now coincides with the commitment one below w_1. So the region of "flexibility" extends further. The reason for this is that incentive compatibility lowers the incumbent wage even in relatively good states, and the no replacement constraint only first binds at lower levels of the new hire wage. (See point C in Figure 3.)

Summary 2 *For a wide range of parameters we find a downwardly rigid period* 2 *new hire wage (but at a level below* w_1*), while new hire wages are flexible upwards: specifically for shocks such that the intersection of the labour demand and quasi-supply curves lies above the downwardly rigid wage, new hire wages will be at the intersection. Wages are allocational. Incumbent wages are also constant for the range of shocks where new hire wages are rigid downwards, and mildly procyclical otherwise.*

Consider instead the nature of the contract with these informational assumptions but with commitment (not to replace) on the part of the firm. The firm then will offer a non-contingent period 2 contract wage to period 1 hires (equal to w_1), but would be unrestricted in offering the optimal hiring wage to period 2 workers. Since a stable wage for incumbents is optimal, and incentive compatible, the solution will be identical to the commitment solution considered earlier. Without commitment, though, as just argued, if the solution satisfies the no replacement constraint, then the fact that wages are non-contingent has direct implications for hires.

3.1 Simulation

We report the following simulation. Again we suppose that the matching technology is given by $p(\theta) = M\theta^{\eta-1}$, $q(\theta) = M\theta^{\eta}$, where $M = 1/10$ and $\eta = 1/2$ (this is the same specification used in MM's example). Further $v(c) = c^{0.5}$, $f(n; x) = x \log(n)$, $k = 0.012$; $S = 10$; the period 1 deterministic state is $x_0 = 11$,while there are three equiprobable states at $t = 2 : x_l = 8$, $x_m = 9$, $x_h = 15$. These results are indicative;

we are not attempting a full calibration exercise given the two period nature of the model.

Wage variability is as predicted across the three scenarios. For $\delta = 0.4$, $b = 0.5$, under commitment we find that $w_{21}(x) < w_1$ in both lower states, while $w_{21}(x_h) > w_1$. Wages are procyclical and vary by 5% across the two lower states (expressed relative to $w_{21}(x_l)$). Replacement is not optimal. Without commitment the no replacement constraint binds in both lower states. Wage variation across the two states is under 3.5%, hence lower, in line with Proposition 1. With asymmetric information added to no commitment, the optimal contract specifies a constant wage in these two states ($w_{21}(x_l) = w_{22}(x_l) = w_{21}(x_m) = w_{22}(x_m)$).

To examine unemployment variability, we compute the change in period 2 unemployment across states l and m under the three model scenarios, and express the no commitment and no commitment + asymmetric information changes relative to the commitment solution (as percentage increases). We consider a range of values for δ and b (b varies between 0.25 and 5, but in the table we give the period 1 replacement ratio, denoted r). Results are mostly fairly insensitive to parameter changes, and across the ones reported it can be seen that a higher turnover rate diminishes the increase in variability due to no commitment; this is intuitive, as higher turnover implies that the incentive to insure incumbents becomes smaller relative to the desire to take advantage of a slack labour market, so wages are more flexible and unemployment correspondingly less variable. Correspondingly, the additional effect of adding asymmetric information to no commitment is larger.

Table 1

Additional unemployment variability relative to commitment model

Turnover δ/Replacement rate r	no commitment (%)	asymmetric information (%)
$\delta = 0.3$, $r = 0.3$	25	46
$\delta = 0.3$, $r = 0.4$	22	46
$\delta = 0.3$, $r = 0.5$	20	45
$\delta = 0.4$, $r = 0.3$	16	41
$\delta = 0.4$, $r = 0.5$	12	39

Note.

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4 Appendix: Proof of Proposition 1

We derive necessary conditions by considering the following Lagrangian, assuming an interior solution and assuming that there is no replacement in state x. We give the appropriate expression if there is no undercutting in period 2 in *any* state; otherwise an analogous argument applies (if there is replacement in some state $x' \neq x$ it modifies the expectation term in (14) and (17) but they cancel).

$$\begin{aligned}&(f(\tilde{q}_1(V_1)\overline{n}_1) - w_1\tilde{q}_1(V_1)\overline{n}_1 - k\overline{n}_1) + \\ &E_{x'}[(f((1-\delta)\tilde{q}_1(V_1)\overline{n}_1 + \tilde{q}(w_{21},x')\overline{n}_2; x') - w_{22}(1-\delta)\tilde{q}_1(V_1)\overline{n}_1 - w_{21}\tilde{q}(w_{21},x')\overline{n}_2 - k\overline{n}_2)] \\ &+E_{x'}[\lambda_{x'}(w_{21} - w_{22})],\end{aligned}$$

where $\tilde{q}_1(V_1)$ is defined analogously to $\tilde{q}(w_{21},x)$, $\lambda_{x'}$ is the multiplier on the $w_{21} \geq w_{22}$ constraint in state x' and recall $V_1 = v(w_1) + E[\delta Z_2(x') + (1-\delta)v(w_{22}(x'))]$. This leads to the FOCs:

$$\tilde{q}_1' v'(w_1)\overline{n}_1(f'(n_1) - w_1 + E_{x'}[f'(n;x')(1-\delta) - w_{22}(x')(1-\delta)]) - \tilde{q}_1(V_1)\overline{n}_1 = 0 \quad (14)$$

$$f'(n;x)\tilde{q}(w_{21},x) - w_{21}\tilde{q}(w_{21},x) - k = 0 \quad (15)$$

$$f'(n;x)\tilde{q}'\overline{n}_2 - \tilde{q}(w_{21},x)\overline{n}_2 - w_{21}\tilde{q}'\overline{n}_2 + \lambda_x = 0 \quad (16)$$

$$\begin{aligned}&\tilde{q}_1' v'(w_{22}(x))(1-\delta)\overline{n}_1(f'(n_1) - w_1 + \\ E_{x'}[\quad &f'(n;x')(1-\delta) - w_{22}(x')(1-\delta)]) - \lambda_x - (1-\delta)\tilde{q}_1(V_1)\overline{n}_1 = 0\end{aligned} \quad (17)$$

together with the complementary slackness conditions. Note that (15) implies (6) in the text.

From (14) and (17),

$$\frac{v'(w_1)}{v'(w_{22})}\left(q_1+\frac{\lambda_x}{\overline{n}_1(1-\delta)}\right)=q_1. \tag{18}$$

Using this to eliminate λ_x in (16):

$$f'(n;x)\,\tilde{q}'\overline{n}_2-\tilde{q}(w_{21},x)\,\overline{n}_2-w_{21}\tilde{q}'\overline{n}_2+q_1\overline{n}_1(1-\delta)\left(\frac{v'(w_{22})}{v'(w_1)}-1\right)=0. \tag{19}$$

There are two cases.

A. If $\lambda_x=0$, then from (18) $w_1=w_{22}$, and (19) implies (5) in the text and hence (7). We characterize points which satisfy (7). For clarity, we let $\tilde{w}_{21}$ and $\tilde{\theta}_2$ denote the individual firm's values. Then

$$\tilde{q}'=\frac{dq}{d\theta_2}\frac{d\tilde{\theta}_2}{d\tilde{w}_{21}}\,|_{Z_2\text{ constant}}\,.$$

From (2),

$$\frac{d\tilde{\theta}_2}{d\tilde{w}_{21}}\,|_{Z_2\text{ constant}}=-\frac{pv'(w_{21})}{\frac{dp}{d\theta_2}(v(w_{21})-v(b))},$$

and differentiating $q=p\cdot\theta_2$ to eliminate $\frac{dp}{d\theta_2}$, we get

$$\tilde{q}'=-\frac{dq}{d\theta_2}\frac{p\theta_2 v'(w_{21})}{\left(\frac{dq}{d\theta_2}-p\right)(v(w_{21})-v(b))}.$$

After rearrangement,

$$\frac{q^2}{\tilde{q}'}=q^2\frac{\left(1-\frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)}{\theta_2\frac{dq}{d\theta_2}}\frac{v(w_{21})-v(b)}{v'(w_{21})}.$$

From our assumption on q, q^2 is increasing in θ_2, and the second term in the product is also increasing in θ_2 by assumption (it is the inverse of $q(\theta)\,\epsilon_q(\theta)\,/\,(1-\epsilon_q(\theta)))$ while the final term is increasing in w_{21}. Thus the locus of values of θ_2 and w_{21} such that (7) holds is negatively sloped. Recall that $n_2=p(\theta_2)\,S_2$, and as $p'<0$, there is a one-to-one negative relationship between n_2 and θ_2. So (7) can be solved to give a positively sloped locus of values for n_2 and w_{21} compatible with equilibrium.

Next, (15) is negatively sloped in n_2-w_{21} space by $f''<0$ and $q(\theta_2)=q(p^{-1}(n_2/S_2))$, $q'>0$, $p'<0$. Therefore (w_{21},n_2) is at the unique intersection point, denoted by $\left(w_{21}^C(x;w_1,n_1)\,,n_2^C(x;w_1,n_1)\right)$ in the text. Since $w_{21}\geq w_1$ implies $\lambda_x=0$ (see next line), this establishes claim (b).

B. If $\lambda_x > 0$, then $w_{22} = w_{21}$ and from (18) $w_1 > w_{22} = w_{21}$, and (19) implies

$$(1-\delta)n_1 - \left(f'(n)\,\tilde{q}'\overline{n}_2 - w_{21}\tilde{q}'\overline{n}_2 - q\overline{n}_2\right) = n_1\left(1/v'(w_1)\right)\left((1-\delta)\,v'(w_{21})\right). \quad (20)$$

(This also follows from differentiating (3) with respect to w_{21} after setting $w_{21} = w_{22}$.) Thus, eliminating f' using (15), and using $n_2 = q\overline{n}_2$,

$$1 + \frac{\left(1 - k\tilde{q}'/q^2\right) n_2}{n_1\,(1-\delta)} = \frac{v'(w_{21})}{v'(w_1)}, \quad (21)$$

so that as $w_{21} < w_1$, $k\tilde{q}'/q^2 < 1$, i.e., $k < q^2/\tilde{q}'$. Holding n_2 (and hence θ_2) constant, $q^2/\tilde{q}'$ is increasing in w_{21}, so the locus of points (n_2, w_{21}) satisfying (21) must lie above—w_{21} is higher—that defined by (7). At $w_{21} = w_1$ we have $k\tilde{q}'/q^2 = 1$, so the two loci coincide. Thus the downward sloping (15) must intersect (21) at a higher wage and a lower value for n_2 than it would intersect (7). This establishes claim (a).

Since $\lambda_x > 0$ if and only if $w_{21} < w_1$, the final claim of the proposition follows.