Technological Change, Firm Heterogeneity and Wage Inequality*

Guido Matias Cortes[†] Adrian Lerche[‡]

Uta Schönberg[§] Jeanne Tschopp[¶]

October 26, 2020

Abstract

We show that task-biased technological change is an important driver of the rise in between-establishment wage inequality in Germany. Using rich administrative social security data, we document that, within industries, establishments have become more heterogeneous in terms of their labor productivity, the share of abstract task workers that they hire, and the wages that they pay to workers in a given task. Moreover, within industries, more productive workplaces experience larger employment growth, larger increases in their abstract task share, and larger wage growth for both abstract and routine workers. We show that, within the context of a heterogeneous firm framework where firms differ in their technology of production, an aggregate task-biased shock is able to generate these patterns. In line with the predictions of the model, we find that industries with more ICT and robot adoption have experienced larger increases in worker sorting and between-establishment wage inequality within tasks.

*Cortes and Tschopp are grateful to the Social Sciences and Humanities Research Council of Canada for its financial support. We thank the Research Data Centre of the German Federal Employment Agency at the Institute for Employment Research for providing data access.

[†]Department of Economics, York University, 4700 Keele Street, Toronto, ON, M3J 1P3, Canada; gm-cortes@yorku.ca; +1 (416) 736-2100 (ext 33987).

[‡]Institut für Arbeitsmarkt- und Berufsforschung (IAB) der Bundesagentur für Arbeit (BA), Regensburger Strasse 104, 90478 Nürnberg, Germany; Adrian.Lerche@iab.de; +49 (911) 179 9213.

[§]Department of Economics, University College London, Gower Street, London, WC1E 6BT, United Kingdom; u.schoenberg@ucl.ac.uk, +44 203 549 5351.

[¶]Department of Economics, University of Bern, Schanzeneckstrasse 1, 3001 Bern, BE, Switzerland; jeanne.tschopp@vwi.unibe.ch; +41 31 631 80 81.

1 Introduction

Income inequality has risen dramatically over the past decades in most developed countries. Recent evidence has shown that a major part of this rise in inequality can be attributed to increasing differences in wages paid by different employers to workers who are similar in terms of their observable characteristics (Card et al., 2013; Song et al., 2018; Barth et al., 2016; Helpman et al., 2017). These findings imply that it is crucial to understand the role of firms in driving the increase in wage inequality. But why have firm-level wages become more unequal? In this paper we show that task-biased technological change is an important driver of the increase in wage inequality between establishments within industries in Germany.

The pioneering work of Autor et al. (2003) discusses how progress in terms of automation and digitization technologies can substitute for labor in specific occupations, namely those that are intensive in routine, easily codifiable tasks. An extensive subsequent literature has documented the decline of these types of occupations in many developed countries (Autor et al., 2006; Goos et al., 2014; Jaimovich & Siu, 2020; Green & Sand, 2015). The role of firms, however, has been absent from this literature. By reducing demand for routine workers, and increasing the demand for workers in abstract tasks, automation technologies have been argued to impact wage inequality due to changes in the relative wage between these two groups. As mentioned, however, the empirical evidence shows that inequality has primarily risen due to widening differences across workplaces for workers in a given task, i.e. due to *within-group* wage inequality. This paper shows that task-biased technological change can also account for these empirically crucial within-group changes.

Our analysis uses administrative social security data from the Institute for Employment Research (IAB), covering the universe of private sector workers in West Germany between 1990 and 2010. Given the availability of establishment identifiers, we are able to aggregate the data to the universe of private-sector establishments with at least one full-time employee. We supplement the data from the social security records with information from the IAB Establishment Panel, which provides measures of establishment-level sales and allows us to construct a measure of labor productivity for the establishments covered by the survey.

We begin by verifying the important role of workplaces in driving wage inequality in Germany. Between-establishment inequality accounts for about 75% of overall wage inequality in any given year, and most of the large increase in wage inequality observed between the mid 1990s and 2010 occurs between establishments. This rise in between-establishment wage inequality is not primarily a between-industry phenomenon, but rather reflects growing wage differences across establishments within industries. Importantly, this increased wage dispersion is observed among workers in the same task group, and hence is driven by increasing within-task inequality, rather than by between-task changes. In fact, the aggregate wage differential between workers in abstract and routine tasks in Germany grew only very slightly over our sample period.

We also document an empirical link between an establishment's relative productivity (within its industry-year) and the task input mix of its workforce. In particular, we show that more productive establishments within an industry employ a systematically higher share of workers in abstract (rather than routine) tasks.

Motivated by these facts, we set up a theoretical framework that features heterogeneous firms in order to analyze the impacts of task-biased technological change. In particular, we consider the model of Helpman et al. (2010), which introduces search and matching frictions (Diamond, 1982a,b; Mortensen & Pissarides, 1994) and match-specific abilities to the heterogeneous firm setting of Melitz (2003), thus generating wage differences between firms for workers who are ex-ante identical.

Guided by our motivating evidence, we allow firms to differ in their technology of production, with more productive firms having an incentive to disproportionately hire abstract workers. The model predicts that, in equilibrium, more productive firms will find it optimal to employ more workers of both types (but particularly abstract workers), and will pay higher wages to both types of workers.

We then introduce an aggregate task-biased technological shock in the spirit of Autor et al. (2003), whereby there is an increase in the relative productivity of abstract workers, motivating a shift away from routine tasks towards abstract tasks. Although this is an aggregate shock that is common across all firms, we show that this shock leads to an increase in between-firm wage inequality. This occurs through four distinct channels. First, there is an increase in the productivity threshold for production, which results in the exit of low-productivity firms. Although this reduces the range of productivities of operating firms, the assumption of Pareto-distributed productivity implies that this change in selection increases the variance of productivity among operating firms. The second channel is differential employment growth, whereby the most productive, highest-paying firms, are predicted to grow more, thus contributing to an increase in worker-weighted measures of between-firm inequality. The third channel is increased sorting: As a reaction to the shock, the most productive firms will disproportionately increase the share of abstract workers that they employ. Since abstract workers earn higher wages, this will exacerbate the differences in average wages between firms. Finally, the model also generates endogenous wage changes conditional on task, with more productive firms disproportionately increasing the wage that they pay to workers of each type, thus further contributing to the increase in between-firm wage inequality.

Guided by these predictions, we return to the IAB data. We first verify that, as predicted by the model, more productive firms within an industry-year pay higher average wages overall and conditional on worker tasks and worker observable characteristics. We also find that the correlation between productivity and various establishment characteristics (namely size, wage and abstract share) has become stronger over time. More productive establishments within an industry-year are found to tend to pull away from other establishments, by growing more and disproportionately increasing their wages. We also find that, over time, establishments within industries have become increasingly heterogeneous in terms of their task mix, rather than converging to more similar technologies of production. We also find that, in line with the model predictions, the sorting of abstract workers towards the most productive establishments in an industry has increased.

As a final verification of the importance of task-biased technological change in driving the patterns that we observe, we exploit variation across industries in the adoption of task-biased technologies. In particular, we focus on industry-level robot adoption data from the International Federation of Robotics, and ICT capital usage from the EUKLEMS data. We find that industries that have adopted more technology have experienced disproportionate increases in between-establishment wage inequality (overall and conditional on task), in between-establishment abstract share heterogeneity, and in the sorting of abstract workers to high-wage firms.

Our findings make important contributions to two strands of the literature. We contribute to the literature on between-firm wage inequality (Card et al., 2013; Song et al., 2018; Barth et al., 2016; Helpman et al., 2017) by analyzing technological change as a driving force behind the important patterns that have been documented. We also contribute to the literature on task-biased technological change (Autor et al., 2003, 2006; Goos et al., 2014; Michaels et al., 2014; Acemoglu & Restrepo, 2020) by showing that technological change not only impacts the relative demand for workers in different tasks (and thus impacts inequality through changing wage differentials between tasks), but also has important impacts in terms of worker sorting and firm-level wages for workers in a given task (and thus also impacts inequality through changing wage differentials between firms conditional on task).

2 Data

2.1 Social Security Records (BEH)

Our main data are drawn from social security records provided by the Institute for Employment Research in Nuremberg (IAB) – the so-called Beschäftigtenhistorik (BEH). While social security records are in principle available for the years 1975 to 2014 (1992 to 2014 for

East Germany), we focus on developments after 1990 when overall wage inequality started to increase sharply in Germany (see for example Dustmann et al. (2014); Card et al. (2013)). Due to structural breaks in key variables such as occupations (used to classify workers into routine and abstract ones) and workers' full-time status after 2010, we end the analysis in 2010. The data source comprises all men and women covered by the social security system, roughly 80% of the German workforce. Not included are civil servants, the self-employed, and military personnel.

Our data source offers some key advantages. A first advantage is its large sample size, allowing us to accurately capture trends in, for example, wage inequality even within detailed industries. Second, our data contain detailed and accurate information on a number of worker and establishment characteristics that are not always included in other administrative data sources, such as, for example, workers' occupation, education, employment status and wages (which always refer to a single establishment and are never averaged across establishments) and establishment's industry affiliation. Importantly, unique establishment identifiers allow us not only to decompose overall wage inequality into a within and between establishment component, but also to study (changes in) establishment heterogeneity within industries more broadly. Establishment identifiers also allow us to paint an accurate picture of entry and exit across industries and time.

From this data source, we select all full- and part-time employment spells that refer to June 30 of each year. We restrict the sample to workers who are currently not in an apprenticeship, are aged between 16 and 65, and are employed in West Germany. We exclude industries in the primary sector and some small industries such as private households and international organizations. We further drop workers with missing occupations, missing employment status, and implausibly low wages below the limit for which social security contributions have to be paid, as well as establishments with missing industry affiliation and establishments employing only part-time workers. These sample restrictions affect less then 1% of all observations.

As is common in administrative data sources, wages are censored at the highest social security limit, affecting on average about 8% of observations. We follow Dustmann et al. (2009) and Card et al. (2013) and impute censored wages, assuming that (log) wages are normally distributed with heterogeneous variances that vary by year, age, education and sex. We deflate wages using 1995 as the base year. Since we do not observe hours worked, we restrict the wage analysis to full-time workers. To compute total employment in establishments and industries (overall or by task), we include part-time workers with a weight of 0.5.

We follow Acemoglu & Autor (2011) to classify individuals as working in routine or abstract tasks based on their broad occupation codes. Our industry classification refers to 3-digit NAICS codes.

2.2 The IAB Establishment Panel (IABEP)

Since the social security records do not contain information on establishments's labor productivity, we augment the social security records with data from the IAB Establishment Survey (IABEP). The IABEP survey was first administered in 1993 to 4,265 West German firms, and was extended to East German firms in 1996. By 2010, the number of surveyed firms had increased to over 16,000. From this database, we select all West German establishments with at least one full-time employee that participated in the IABEP at least once. Adopting the same sample selection criteria as in the social security records (BEH), we drop establishments with missing industry affiliation as well as establishments in the primary sector and some smaller sectors such as private households and international organizations. Using the unique establishment identifiers, we then merge information from the IABEP to the BEH social security records. We compute an establishment's labor productivity as total sales (obtained from the IABEP), divided by the number of full-time equivalent workers (obtained from the BEH).

2.3 Industry-Level Technology Adoption Measures

We supplement these two main data sources with industry-level data on technology adoption. First, following Graetz & Michaels (2018) and Acemoglu & Restrepo (2020), we use data on robot usage from the International Federation of Robotics (IFR).[1] We use data prepared by Dauth et al. (2019), which matches IFR industry codes to the official industrial classification for the German labor market. The matched data covers 53 manufacturing industries.

Second, we use data on the adoption of capital related to information and communication technologies (ICT) from the EUKLEMS data set. We use data from the November 2009 release, which uses ISIC revision 3 industry codes which can be matched to the 2-digit industry codes in the social security data. Our measure of ICT assets is based on the real fixed capital stock of computing and communication equipment, and computer software.

[1] A robot is defined as an "automatically controlled, re-programmable, and multipurpose machine" and as "fully autonomous machines that do not need a human operator and that can be programmed to perform several manual tasks such as welding, painting, assembling, handling materials, or packaging."

3 Motivating Evidence

Overall and Between-Establishment Wage Inequality. Figure 1 provides a first overview of the evolution of wage inequality in West Germany between 1990 and 2010. Panel A shows that the overall standard deviation of individual log-wages rose sharply from the mid-1990s onwards, from 0.45 in 1995 to 0.53 in 2010, a 25% increase (see also Dustmann et al., 2009, 2014). The figure further highlights that, consistent with existing findings in the literature (e.g. Card et al., 2013), between-establishment wage inequality represents an important fraction of overall wage inequality in any given year (about 75%). Moreover, the employment-weighted standard deviation of average establishment log-wages (or alternatively, the log of average establishment wages) rises in tandem with the standard deviation of individual log-wages, indicating that nearly all of the increase in overall wage inequality occurs between establishments.

Overall vs Within-Industry Between-Establishment Wage Inequality. Panel B of Figure 1 contrasts the evolution of overall and within-industry between-establishment wage inequality, measured as the employment-weighted standard deviation of average establishment log-wages overall and within 3-digit industries.

Specifically, overall between-establishment wage inequality is measured as

$$\sqrt{\sum_f \frac{E_{ft}}{E_t} \left(\ln w_{ft} - \overline{\ln w_t}\right)^2},$$

where the subscripts f and t index establishments and time, respectively. E represents employment, $\ln w_{ft}$ is the average of individual log-wages in establishment f at time t, and $\overline{\ln w_t}$ is the overall employment-weighted mean of average establishment log-wages (which is equal to the overall mean of individual log wages) at time t.

Within-industry between-establishment wage inequality is measured as

$$\sqrt{\sum_k \frac{E_{kt}}{E_t} \sum_{f \in k} \frac{E_{f(k)t}}{E_{kt}} \left(\ln w_{f(k)t} - \overline{\ln w_{kt}}\right)^2},$$

where $\ln w_{f(k)t}$ is the average of individual log-wages in establishment f belonging to industry k and $\overline{\ln w_{kt}}$ is the employment-weighted mean of average establishment log-wages in industry k (which is equal to the overall mean of individual log wages in the industry).

The figure highlights that between-establishment wage inequality also rose within industries throughout the sample period, albeit at a slightly lower pace than overall between-establishment wage inequality. The increase in overall between-establishment wage inequality is therefore primarily a within-industry phenomenon, and not solely driven by wages in some industries rising relative to wages in other industries.

Panel B of Figure 1 further plots the evolution of "counterfactual" within-industry between-establishment wage inequality, where within-industry variances of average establishment log wages are weighted by the 1990 rather than the current industry employment structure. That is, counterfactual within-industry between-establishment wage inequality is measured as

$$\sqrt{\sum_k \frac{E_{k1990}}{E_{1990}} \sum_{f \in k} \frac{E_{f(k)t}}{E_{kt}} \left(\ln w_{f(k)t} - \overline{\ln w}_{kt}\right)^2}.$$

Counterfactual within-industry between-establishment wage inequality rose only slightly less than factual within-industry between-establishment wage inequality, implying that the increase in within-industry between-establishment wage inequality is not driven by industries with high between-establishment wage inequality growing at a disproportionately higher rate.

Within-Industry Between-Establishment Wage vs Productivity Inequality. Does the increase in within-industry between-establishment wage inequality reflect an increase in the dispersion of establishments' labor productivities? We investigate this in Panel C of Figure 1, drawing on the IAB Establishment Panel (IABEP). As expected, the BEH and the IABEP show a similar trend in within-industry between-establishment wage inequality, even though the time series based on the IABEP is, due to the much smaller sample size, more jumpy. The within-industry standard deviation of establishments' log labor productivities is, while somewhat jumpy, increasing over the sample period at a roughly similar pace as the within-industry standard deviation of establishments' average log wages.

Task Usage and the Abstract Wage Premium. A second key development in the German labor market over the past two to three decades, in addition to the rise in wage inequality, is a substantial shift in the occupational structure towards abstract occupations – a pattern that has been observed across many countries (see Acemoglu & Autor, 2011; Goos et al., 2009). Panel A of Figure 2 shows that the employment share of abstract workers steadily rose from about 18% in 1990 to more than 26% in 2010 – a rise of 44% over two decades. This increase was in part driven by industries which employ a larger share of abstract workers growing at a faster rate than industries which employ primarily routine workers. Yet, even when keeping the industry structure constant at 1990 levels (the grey dashed line), the employment share of abstract workers rose substantially by about 27%.

The increase in the abstract employment share was accompanied by a small increase in the abstract wage premium over the same period, as shown in Panel B of Figure 2. To compute the abstract wage premium, we first obtain the residual from an individual log wage regression that controls for a cubic in age, gender and foreign status and is estimated separately for each year. The actual abstract wage premium (the black circles in Panel B of Figure 2) is then computed as

$$\sum_k \frac{E_{kt}}{E_t} \left(\overline{\widetilde{\ln w}_{kt}^{A}} - \overline{\widetilde{\ln w}_{kt}^{R}} \right),$$

while the within-industry abstract wage premium holding the industry structure constant at its 1990 level (the grey diamonds in Panel B of Figure 2) is given by:

$$\sum_k \frac{E_{k1990}}{E_{1990}} \left(\overline{\widetilde{\ln w}_{kt}^{A}} - \overline{\widetilde{\ln w}_{kt}^{R}} \right)$$

where $\overline{\widetilde{\ln w}_{kt}^{A}}$ and $\overline{\widetilde{\ln w}_{kt}^{R}}$ denote the averages of the residual for abstract and routine workers in industry k and year t. Both the actual and counterfactual abstract wage premium evolve in a similar manner, increasing from about 29.5% to 32%.

Taken together, Panels A and B of Figure 2 indicate that firms' demand for abstract workers has increased relative to that for routine workers between 1990 and 2010. However, large supply shifts from routine to abstract employment over the same period appear to have prevented substantial increases in the abstract wage premium. The figures further highlight that the rise in the abstract (or skill) wage premium – the key channel highlighted in the literature on routine (or skill) biased technological change – plays only a minor role in accounting for the overall rise in wage inequality in Germany between 1990 and 2010. As shown in Panels C and D, there have been substantial increases in within-group wage inequality, conditional on worker tasks.

Establishment Productivity and Task Usage. A final piece of motivating evidence is related to the link between productivity and task usage at the establishment level. Column (1) of Table 1 shows the correlation between establishments' log productivity and their abstract employment share. The regressions include a set of fully interacted 3-digit industry and year fixed effects, so that identification is limited to variation within industry-year cells. The estimated coefficient shows that there is a positive and statistically significant correlation (at the 5% level) between productivity and abstract employment shares at the establishment level. In order to analyze this relationship using the much larger social security dataset, Column (2) shows that there is a significant positive correlation between establishment size and productivity, such that establishment size can be used as a proxy for productivity. In Column (3) we confirm, using the social security records data, that larger firms employ a higher share of abstract workers. Hence, there is an empirical link between establishment productivity and task usage, which will guide our modelling choices in the following section.

4 Theoretical Framework

In this section we set up a theoretical framework that helps guide our analysis of the link between task-biased technological change and between-firm wage inequality. Guided by the evidence in the previous section, we set up a model that allows for wage heterogeneity between firms, with a production structure that distinguishes between two tasks, and where firm productivity and task usage are linked. Specifically, we consider the framework of Helpman et al. (2010), a rich, yet tractable model of firm heterogeneity that allows for wage differences across firms within industries. Helpman et al. (2010) introduce standard Diamond–Mortensen–Pissarides (Diamond, 1982a,b; Mortensen & Pissarides, 1994) search and matching frictions into a Melitz (2003) model. We focus on the extension of the model which allows for two types of labor inputs (Section 5.2 of their paper) which, in our setting, we think of as two different tasks (abstract and routine).[2]

In what follows, we briefly outline the key components of the model and the equilibrium conditions, as derived by Helpman et al. (2010).[3] We then consider the implications of an aggregate routine-replacing technological change shock in the spirit of Autor et al. (2003) and Acemoglu & Autor (2011). We model this as an exogenous aggregate change in the factor-augmenting parameter associated with abstract workers, and study its implications for various establishment-level and industry-level outcomes.

4.1 Overview of the Helpman et al. (2010) Framework

Consumption

Within each sector, consumers demand a continuum of differentiated varieties. The aggregate consumption index is:

[2]Another framework that generates between-firm wage heterogeneity is the fair wage framework of Egger & Kreickemeier (2009, 2012). That setting requires assumptions about the way in which workers' fairness considerations relate to firm outcomes, which are somewhat ad-hoc. It is worth emphasizing that merely adding search and matching frictions to the Melitz (2003) model does not generate between-firm wage heterogeneity. This is because in equilibrium firm wages are equal to the replacement cost of a worker for the firm, which is common across firms in the presence of common search costs. Match-specific abilities and screening technologies are therefore crucial elements in order to generate wage heterogeneity in the Helpman et al. (2010) framework.

[3]For full details, we refer the reader to the Helpman et al. (2010) paper.

$$Q = \left[\int_{j \in J} q(j)^{\beta} dj \right]^{1/\beta}, \tag{1}$$

where j indexes varieties, J is the set of varieties within the sector, $q(j)$ denotes consumption of variety j, and $0 < \beta < 1$. The demand function for variety j is given by:

$$q(j) = A^{1/(1-\beta)} p(j)^{-1/(1-\beta)} \tag{2}$$

where A is a sectoral demand shifter and $p(j)$ is the price of variety j.

Production

There is a competitive fringe of potential firms that can choose to enter the market by paying an entry cost $f_e > 0$. Once a firm incurs the sunk entry cost, it observes its idiosyncratic value of θ, a parameter that is related to its productivity and its optimal production structure (as discussed below). θ is drawn from a Pareto distribution with scale parameter θ_{min} and shape parameter z, i.e. $G_\theta(\theta) = 1 - (\theta_{min}/\theta)^z$ for $\theta \geq \theta_{min} > 0$ and $z > 2$.[4] Once firms observe θ, they decide whether to exit or produce. Production involves a fixed cost of $f_d > 0$ units of the numeraire. Since in equilibrium all firms with the same value of θ behave symmetrically, firms can be indexed by θ.

Firms produce using a Constant Elasticity of Substitution (CES) technology using two types of labor inputs: abstract and routine workers (indexed by s and r, respectively). A firm's output depends on its value of θ, as well as its choice of how many workers of each type to hire (h_s and h_r), and the average match-specific ability of these workers ($\overline{a}_s$ and $\overline{a}_r$). Specifically, the production function is given by:

$$y = \left[\lambda_s \left(\theta \mu_s \overline{a}_s h_s^{\gamma}\right)^{\nu} + \lambda_r \left(\mu_r \overline{a}_r h_r^{\gamma}\right)^{\nu}\right]^{1/\nu} \tag{3}$$

where $0 < \nu < \beta$, $\lambda_s + \lambda_r = 1$, and μ_s and μ_r are aggregate task-augmenting technology parameters.[5] For simplicity, we normalize $\mu_r = 1$. μ_s can therefore be interpreted in relative terms, as the relative aggregate task-bias of technology in favor of abstract tasks. The firm-specific parameter θ enters into the production function as an abstract task-augmenting parameter (rather than a total factor productivity term). Firms that draw higher values of θ will be more productive overall (absolute advantage), but productivity will be particularly high for their abstract workers (comparative advantage). Hence, θ reflects both productivity

[4] The assumption that $z > 2$ ensures that the variance of θ is finite.

[5] The assumption that $\nu < \beta$ ensures that employment and wages of both types of workers are increasing in θ, as discussed below.

and the task-bias of production (in favor of abstract workers) of each firm. The model therefore assumes a link between firm productivity and firm's technological task bias, which is consistent with the empirical pattern documented in Table 1. Given this link, we refer to the parameter θ interchangeably as both firm-specific technology and firm-specific productivity.

Search, Screening and Wage Bargaining

The firm must pay a search cost of b_ℓ in order to be matched with n_ℓ workers, $\ell = \{s, r\}$.[6] Workers of a given task type are ex-ante identical but, upon matching with a firm, draw match-specific abilities from a Pareto distribution with shape parameter k and scale parameter a_{min}: $G_a(a) = 1 - (a_{min}/a)^k$; $a \geq a_{min} > 0$ and $k > 1$.[7] Ability is not observable by the firm or the worker, but a screening technology is available. By paying a screening cost $c\widetilde{a}_\ell^\delta/\delta$, firms are able to identify whether a worker's match-specific ability is above or below an (endogenously chosen) cutoff $\widetilde{a}_\ell$, where $\ell = \{s, r\}$, $c > 0$, $\delta > 0$.

Wages are determined through Stole & Zwiebel (1996a,b) bargaining, under conditions of symmetric information. Since the screening technology only reveals whether a worker's match-specific ability is above or below $\widetilde{a}_\ell$, but not the specific ability of any individual worker, the expected ability of all hired workers of a given type is the same, and equal to $\overline{a}_\ell$, the expected value of a conditional on being above the threshold $\widetilde{a}_\ell$. Therefore, all workers of a given type within a given firm receive the same wage.

4.2 Key Equilibrium Properties

As is standard in heterogeneous firm models, the presence of a fixed production cost implies that there is a zero-profit cutoff for productivity, θ_d, such that a firm that draws a productivity below this threshold exits without producing. Appendix A.1.2 shows how this productivity threshold can be pinned down using the Zero-Cutoff Profit condition, which requires the firm at the cutoff θ_d to make zero profits, along with the Free Entry condition, which states that the expected profits for a potential entrant should equal the fixed entry cost.

Closed-form solutions can be obtained for the equilibrium values of firm-level employment, wages, revenues, and profits for operating firms. Here we highlight the key properties of interest for our purposes.

[6] b_ℓ is determined endogenously by labor market tightness.

[7] This distribution is assumed to be common across both types of workers.

Firm-level employment for routine workers is given by:

$$h_r(\theta) = h_{dr}\left[1+\varphi(\theta)\right]^{\left(\frac{\beta\Lambda}{\nu\Gamma}-1\right)\left(1-\frac{k}{\delta}\right)} \tag{4}$$

where:

$$\varphi(\theta) = \mu_s^{\nu/\Lambda}\left(\frac{\lambda_s}{\lambda_r}\right)^{1/\Lambda}\left(\frac{b_s}{b_r}\right)^{-\gamma\nu/\Lambda}\theta^{\nu/\Lambda} \tag{5}$$

Here, $\Lambda \equiv 1-\nu\gamma-\nu(1-\gamma k)/\delta > 0$, $\Gamma \equiv 1-\beta\gamma-\beta(1-\gamma k)/\delta > 0$, and $\Lambda > \Gamma$ due to the assumption that $\nu < \beta$. The derivation of this result and the definition of h_{dr} are detailed in Appendix A.1.

Employment of abstract workers for a firm with productivity level θ is given by:

$$h_s(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-k/\delta}h_r(\theta) \tag{6}$$

And the firm's abstract share is given by:

$$\frac{h_s(\theta)}{h(\theta)} = \frac{b_r\varphi(\theta)^{1-k/\delta}}{b_s+b_r\varphi(\theta)^{1-k/\delta}}, \tag{7}$$

where $h(\theta) = h_s(\theta) + h_r(\theta)$.

As shown in Appendix A.2, these equilibrium equations imply:

$$\frac{\partial h_r(\theta)}{\partial\theta} > 0, \qquad \frac{\partial h_s(\theta)}{\partial\theta} > 0, \qquad \frac{\partial h_s(\theta)/h(\theta)}{\partial\theta} > 0. \tag{8}$$

The model therefore predicts that more productive firms will be larger than less productive firms, and will employ a larger number of both abstract and routine workers. More productive firms will also have a higher abstract employment share, implying that abstract workers disproportionately sort towards high-productivity firms.

Firm-level wages for routine workers can be written as:

$$w_r(\theta) = w_{dr}\left[1+\varphi(\theta)\right]^{\left(\frac{\beta\Lambda}{\nu\Gamma}-1\right)\frac{k}{\delta}} \tag{9}$$

The derivation of this result and the definition of w_{dr} are also detailed in Appendix A.1.

Wages for abstract workers are given by:

$$w_s(\theta) = \frac{b_s}{b_r}\varphi(\theta)^{k/\delta}w_r(\theta) \tag{10}$$

As shown in Appendix A.2, these equilibrium wage equations imply:

$$\frac{\partial w_r(\theta)}{\partial \theta} > 0, \qquad \frac{\partial w_s(\theta)}{\partial \theta} > 0 \tag{11}$$

The model therefore generates wage differences between firms, with more productive firms paying higher wages to both types of workers. Intuitively, this arises due to the complementarity between worker abilities and firm productivity, which gives an incentive for more productive firms to screen more intensively (i.e., choose a higher ability threshold). In equilibrium, wages are bargained down to the replacement cost of a worker, and given that more productive firms are only willing to hire workers with higher match-specific abilities, their workers are costlier to replace, and hence are paid a higher wage. Note that from the perspective of the worker, the expected wage conditional on being sampled is the same across all firms.

To summarize, the cross-sectional predictions of the model are that more productive firms are larger, have a higher abstract share, and pay higher average wages (overall and conditional on worker task).

4.3 Impacts of Task-Biased Technological Change

We model task-biased technological change (TBTC) as an exogenous increase in μ_s, the aggregate task-augmenting parameter for the abstract labor input in the production function in Equation (3). The literature on task-biased technological change (e.g. Autor et al., 2003; Acemoglu & Autor, 2011) argues that new automation technologies tend to replace labor in performing routine tasks, while complementing labor in abstract (or non-routine cognitive) tasks. Our modeling assumption captures the essence of this idea, given that an increase in μ_s increases the relative demand for abstract (relative to routine) workers. Note that this is an aggregate shock which impacts all firms in the economy. However, as we show below, the impacts of this common shock are very heterogeneous across firms with different productivity levels.

The key implications of an increase in μ_s within the context of the model outlined above are the following:[8]

Prediction 1: *Selection* – Task-biased technological change increases the productivity threshold for production θ_d.

Proof: *See Appendix A.3.*

Implications: By increasing the productivity threshold θ_d, TBTC leads to the exit of firms at the bottom of the productivity distribution. Although this reduces the range of productivity types among operating firms, the variance of productivity among operating firms increases. This is because the distribution of productivity among operating firms is a truncated Pareto distribution with scale parameter θ_d and shape parameter z. The variance of productivity among operating firms is increasing in the scale parameter θ_d.[9] Intuitively, the increase in θ_d entails the exit of a mass of relatively homogeneous unproductive firms, leading to an increased variance of productivity among the firms that remain in operation. Given the increase in the variance of productivity, wage inequality among operating firms increases. Moreover, given that low-θ firms have the highest routine-to-abstract ratios, the firms that will exit will be relatively routine-intensive, and hence their exit would contribute to the rise in the economy's overall abstract share.

Prediction 2: *Differential Employment Growth* – TBTC shifts employment towards more productive firms.

Proof: *As shown in Appendix A.3:*

$$\frac{\partial^2 h_r(\theta)}{\partial\theta\partial\mu_s} > 0 \qquad \text{and} \qquad \frac{\partial^2 h_s(\theta)}{\partial\theta\partial\mu_s} > 0. \tag{12}$$

Implications: This prediction implies that employment grows disproportionately for both types of workers in more productive firms. Hence, the cross-sectional employment-productivity relationship becomes stronger as a result of TBTC. This employment shift leads to an increase in (worker-weighted) between-firm wage inequality (by task and overall). Moreover,

[8]In what follows, we assume that the search costs b_s and b_r are not affected by technological change. The search costs are proportional to workers' expected income outside the sector (outside option). Helpman et al. (2010) discuss conditions under which the outside options can be assumed to be constant, even when there are shocks with aggregate implications (such as trade opening, in the setting analyzed in their paper).

[9]The variance of productivity among operating firms is given by $\frac{z\theta_d^2}{(z-1)^2(z-2)}$. Note that scale-invariant measures of inequality, such as the coefficient of variation, the Gini coefficient, or the Theil index, are decreasing in the shape parameter and are independent of the scale parameter, and hence would not be predicted to be affected by the change in θ_d.

given that more productive firms have a higher abstract employment share, this shift also contributes to the rise in the economy's overall abstract share.

Prediction 3: *Increased Sorting* – TBTC increases the abstract employment share within all firms, and particularly so within more productive firms (as long as the baseline number of routine workers within firms exceeds the baseline number of routine workers, which is the empirically relevant case).

Proof: *As shown in Appendix A.3:*

$$\frac{\partial\left[h_s(\theta)/h(\theta)\right]}{\partial\mu_s} > 0, \qquad \text{and} \qquad \frac{\partial^2\left[h_s(\theta)/h(\theta)\right]}{\partial\theta\partial\mu_s} > 0 \quad \text{if} \quad h_s(\theta) < h_r(\theta). \tag{13}$$

Implications: By increasing the share of abstract workers for all firms, task-biased technological change contributes to an increase in the share of abstract employment in the economy. In addition, firms become more heterogeneous in their task input mix: Since more productive firms disproportionately increase their abstract employment share, the dispersion of abstract shares across firms within industries will increase. This differential change also implies that the sorting of abstract workers towards high-wage firms will become stronger. Average wages in high-paying firms will increase relative to lower-paying firms (because of the disproportionate increase in their asbtract employment share), contributing to the increase in between-firm wage inequality.

Prediction 4: Task-biased technological change increases wage differentials by task across firms.

Proof: *As shown in Appendix A.3:*

$$\frac{\partial^2 w_r(\theta)}{\partial\theta\partial\mu_s} > 0 \qquad \text{and} \qquad \frac{\partial^2 w_s(\theta)}{\partial\theta\partial\mu_s} > 0. \tag{14}$$

Implications: Wages for both types of workers disproportionately increase within more productive firms, therefore leading to an increase in both weighted and unweighted wage inequality by task across firms within industries.

To summarize, the model predicts that task-biased technological change unambiguously leads to an increase in between-firm wage inequality. This operates through various distinct forces: selection, differential employment growth, sorting, and differential wage growth, all of which contribute to the rise in between-firm wage inequality. Intuitively, aggregate task-biased technological change exacerbates the comparative advantage of firms that are more productive in abstract tasks, which are initially the more productive firms overall. This leads to an increase in their relative size and relative wages. The model also predicts that selection, differential employment growth and sorting all contribute to the rise in the overall industry share of abstract employment.

5 Empirical Evidence

5.1 Cross-Sectional Correlations

We begin by verifying the empirical relevance of the cross-sectional equilibrium predictions discussed in Section 4.2. The model predicts that more productive firms are larger and employ a higher share of abstract workers. These predictions were verified in Table 1 and motivated our model setup. The model also predicts that more productive firms pay higher average wages, overall, and conditional on worker task. We verify these predictions in Table 2. As before, all regressions include 3-digit industry x year fixed effects, hence exploiting only variation across establishments within 3-digit industries at a given point in time. Observations are weighted by establishment size, and standard errors are clustered at the establishment level. The top panel draws on data from the IAB Establishment Survey linked to social security records, and uses log productivity (revenue per worker) as the key regressor of interest. The bottom panel draws on the full social security records, and uses log establishment size (which as shown above is positively and significantly correlated with establishment productivity) as the key regressor of interest. The dependent variable is the average establishment log wage in Column (1) and the establishment wage premium (conditional on observable worker characteristics, namely age, gender and nationality) in Column (2). The remaining columns show analogous wage measures separately for routine and abstract workers, respectively. The results consistently show that, within industries, more productive firms on average pay higher wages, both overall and conditional on task and/or worker characteristics.

5.2 Correlations over Time and Longitudinal Changes within Establishments

Predictions 2, 3, and 4 imply that the relationship between establishment productivity and establishment size, abstract share, and wage should become stronger over time if there is ongoing task-biased technological change. To test this, we estimate the correlations from Tables 1 and 2 separately for each year, controlling for 3-digit industry fixed effects. Figure 3 plots the coefficients from these yearly regressions. Coefficients have more than tripled in size throughout the sample period. For example, while in the early 1990s a 1% increase in the firm's labor productivity was associated with an increase in firm size of about 0.1%, the association had increased to nearly 0.4% by 2010. Similarly, the coefficient of log labor productivity on average firm log wages increased from about 0.05 in the early 1990s to 0.15 by 2010.

In line with this evidence, Table 3 shows a set of regressions that consider longitudinal changes within-establishments in various outcomes over 5-year windows, and their relationship with establishments' baseline productivity or size. As before, the regressions include interacted 3-digit industry and time fixed effects. The results show that, within industries, establishments that were more productive at baseline grew faster and experienced higher overall and residual wage growth, for both routine and abstract workers. This is not driven by selection, given that these more productive establishments are also less likely to exit the market. Similarly, drawing on data from the larger BEH rather than the IAB Establishment Survey, firms that are larger at baseline exhibit a larger increase in labor productivity, a larger increase in the employment share of abstract workers, higher overall and residual wage growth for both routine and abstract workers, and are less likely to exit the market. Thus, establishments that perform "better" at baseline—in terms of their labor productivity or size—pull away even further from other establishments in their industry. This is in line with the predictions of the model.

5.3 Abstract Share Heterogeneity and Sorting

Prediction 3 from the theoretical framework implies that workplaces within industries become more heterogeneous in terms of their abstract employment shares, with disproportionate sorting of abstract workers towards high-wage establishments.

As shown in Panel A of Figure 4, and in line with the prediction of the model, the within-industry standard deviation of establishments' abstract employment share, averaged across industries using either the contemporaneous or the 1990 industry structure, has increased over time. Hence, rather than having converged towards a single mode of production over time, establishments within the same industry have become increasingly heterogeneous in terms of the production technology that they use.

Panel B of Figure 4 shows that the co-variance between establishments' abstract employment share and their wage premium has increased over time. That is, abstract workers increasingly sort into establishments that pay higher wage premia even within detailed industries, a pattern that is in line with the prediction of the model and is consistent with the finding by Card et al. (2013) and Song et al. (2018) that high-wage workers increasingly sort into high-wage firms and that high-wage workers are increasingly likely to work with each other.

5.4 Technology Adoption: Industry-Level Analysis

We now test the implications of the model using more direct measures of technology adoption by exploiting variation at the industry level. In particular, we analyze whether industries with more technology adoption have experienced larger increases in between-establishment wage inequality, the standard deviation of abstract employment shares across establishments, and the sorting of abstract workers to high-wage establishments.

We first consider industry-level variation in the overall change in the employment share of abstract workers between 1990 and 2010. Since our focus is on task-biased technological change, we can think of industries that experience larger changes in their abstract employment share as being more exposed to this type of technological innovation. For simplicity, we divide industries into two groups, based on whether they experience above-median or below-median changes in the abstract employment share over the entire period.

Panel A of Figure 5 shows the evolution over time of the standard deviation of establishment wages for these two groups of industries. In line with our prediction, we find that industries that experience larger changes in their abstract employment share also experience a stronger increase in wage inequality between establishments. As shown in Panel B, these industries also experience a stronger increase in the between-establishment standard deviation of the employment share of abstract workers. Hence, in industries that experience an increased prevalence of abstract tasks among their workers, we do not observe that low-abstract establishments are catching up or disappearing from the industry. Instead, we observe increased heterogeneity in the labor input mix across establishments.

Figure 6 uses a more direct measure of technology exposure based on the change in robots per worker within industries over the 1993-2010 time period. Once again we divide industries into two groups, according to whether they experience above or below median changes during this time period.

Panel A first confirms that we can think of robot adoption as a task-biased technological change: Industries with above-median robot adoption experience a much larger increase in their abstract employment shares. The remaining panels show that these industries experience larger increases in between-establishment wage inequality, in the standard deviation of abstract shares across establishments, and in the covariance between establishment wage premia and establishment abstract shares.

Finally, Figure 7 shows that we obtain consistent results is we use a measure of technology adoption based on the industry's change in ICT capital stock per worker between 1991 and 2007 from the EUKLEMS data. Industries with more technology adoption experience larger increases in their abstract share, larger increases in wage inequality, and more sorting of abstract workers towards high wage establishments.

6 Conclusions

Establishments have become increasingly heterogeneous in terms of the wages that they pay to workers with similar observable characteristics. In this paper we contribute to our understanding of the driving forces behind this pattern by showing the important role played by task-biased technological change. While a large literature has considered the role of this type of technological change for wage inequality, it has focused on its implications in terms of wage differentials *between* workers in different task groups. Empirically, however, the increase in wage inequality is primarily driven by increased wage differentials *within* task groups, across establishments. Our paper is the first to show that an aggregate task-biased shock has differential impacts across workplaces, with more productive abstract-intensive establishments disproportionately benefiting from this shock. Rather than leading to a catching-up of routine-intensive establishments, an aggregate shock in favor of abstract workers increases the heterogeneity in task usage across establishments within industries, with more sorting of abstract workers to high wage establishments, and larger increases in inequality, overall and conditional on worker task. Understanding what type of policies can mitigate the increase in inequality remains a promising avenue for future work.

References

Acemoglu, D. & Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. *Handbook of Labor Economics*, 4, 1043–1171.

Acemoglu, D. & Restrepo, P. (2020). Robots and jobs: Evidence from US labor markets. *Journal of Political Economy*, 128(6), 2188–2244.

Autor, D. H., Katz, L. F., & Kearney, M. S. (2006). The polarization of the US labor market. *The American Economic Review*, 96(2), 189–194.

Autor, D. H., Levy, F., & Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics*, 118(4), 1279–1333.

Barth, E., Bryson, A., Davis, J. C., & Freeman, R. (2016). It's where you work: Increases in the dispersion of earnings across establishments and individuals in the United States. *Journal of Labor Economics*, 34(S2), S67–S97.

Card, D., Heining, J., & Kline, P. (2013). Workplace heterogeneity and the rise of West German wage inequality. *The Quarterly Journal of Economics*, 128(3), 967–1015.

Dauth, W., Findeisen, S., Suedekum, J., & Woessner, N. (2019). The adjustment of labor markets to robots. *Working Paper, University of Würzburg.*

Diamond, P. A. (1982a). Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5), 881–894.

Diamond, P. A. (1982b). Wage determination and efficiency in search equilibrium. *The Review of Economic Studies*, 49(2), 217–227.

Dustmann, C., Fitzenberger, B., Schönberg, U., & Spitz-Oener, A. (2014). From sick man of Europe to economic superstar: Germany's resurgent economy. *Journal of Economic Perspectives*, 28(1), 167–88.

Dustmann, C., Ludsteck, J., & Schönberg, U. (2009). Revisiting the German wage structure. *Quarterly Journal of Economics*, 124(2), 843–881.

Egger, H. & Kreickemeier, U. (2009). Firm heterogeneity and the labor market effects of trade liberalization. *International Economic Review*, 50(1), 187–216.

Egger, H. & Kreickemeier, U. (2012). Fairness, trade, and inequality. *Journal of International Economics*, 86(2), 184–196.

Goos, M., Manning, A., & Salomons, A. (2009). Job polarization in Europe. *American Economic Review*, 99(2), 58–63.

Goos, M., Manning, A., & Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review*, 104(8), 2509–2526.

Graetz, G. & Michaels, G. (2018). Robots at work. *The Review of Economics and Statistics*, 100(5), 753–768.

Green, D. A. & Sand, B. M. (2015). Has the Canadian labour market polarized? *Canadian Journal of Economics*, 48(2), 612–646.

Helpman, E., Itskhoki, O., Muendler, M.-A., & Redding, S. J. (2017). Trade and inequality: From theory to estimation. *Review of Economic Studies*, 84(1), 357–405.

Helpman, E., Itskhoki, O., & Redding, S. (2010). Inequality and unemployment in a global economy. *Econometrica*, 78(4), 1239–1283.

Jaimovich, N. & Siu, H. E. (2020). Job polarization and jobless recoveries. *The Review of Economics and Statistics*, 102(1), 129–147.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695–1725.

Michaels, G., Natraj, A., & Van Reenen, J. (2014). Has ICT polarized skill demand? Evidence from eleven countries over twenty-five years. *Review of Economics and Statistics*, 96(1), 60–77.

Mortensen, D. T. & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3), 397–415.

Song, J., Price, D. J., Guvenen, F., Bloom, N., & von Wachter, T. (2018). Firming up inequality. *The Quarterly Journal of Economics*, 134(1), 1–50.

Stole, L. A. & Zwiebel, J. (1996a). Intra-firm bargaining under non-binding contracts. *The Review of Economic Studies*, 63(3), 375–410.

Stole, L. A. & Zwiebel, J. (1996b). Organizational design and technology choice under intrafirm bargaining. *The American Economic Review*, (pp. 195–222).

Figure 1: Wage Inequality, 1990-2010

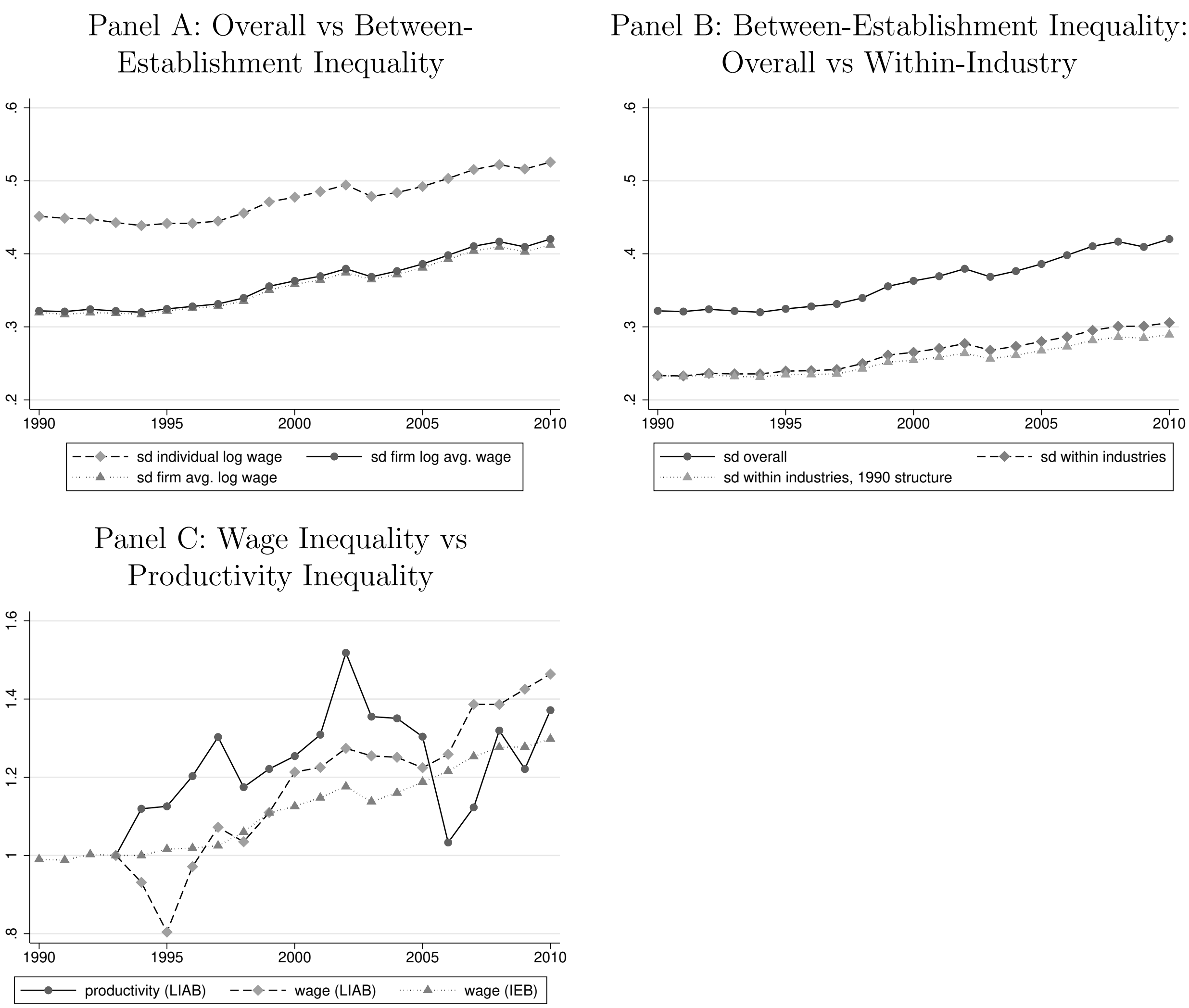


Figure 2: Task Usage and the Abstract Wage Premium

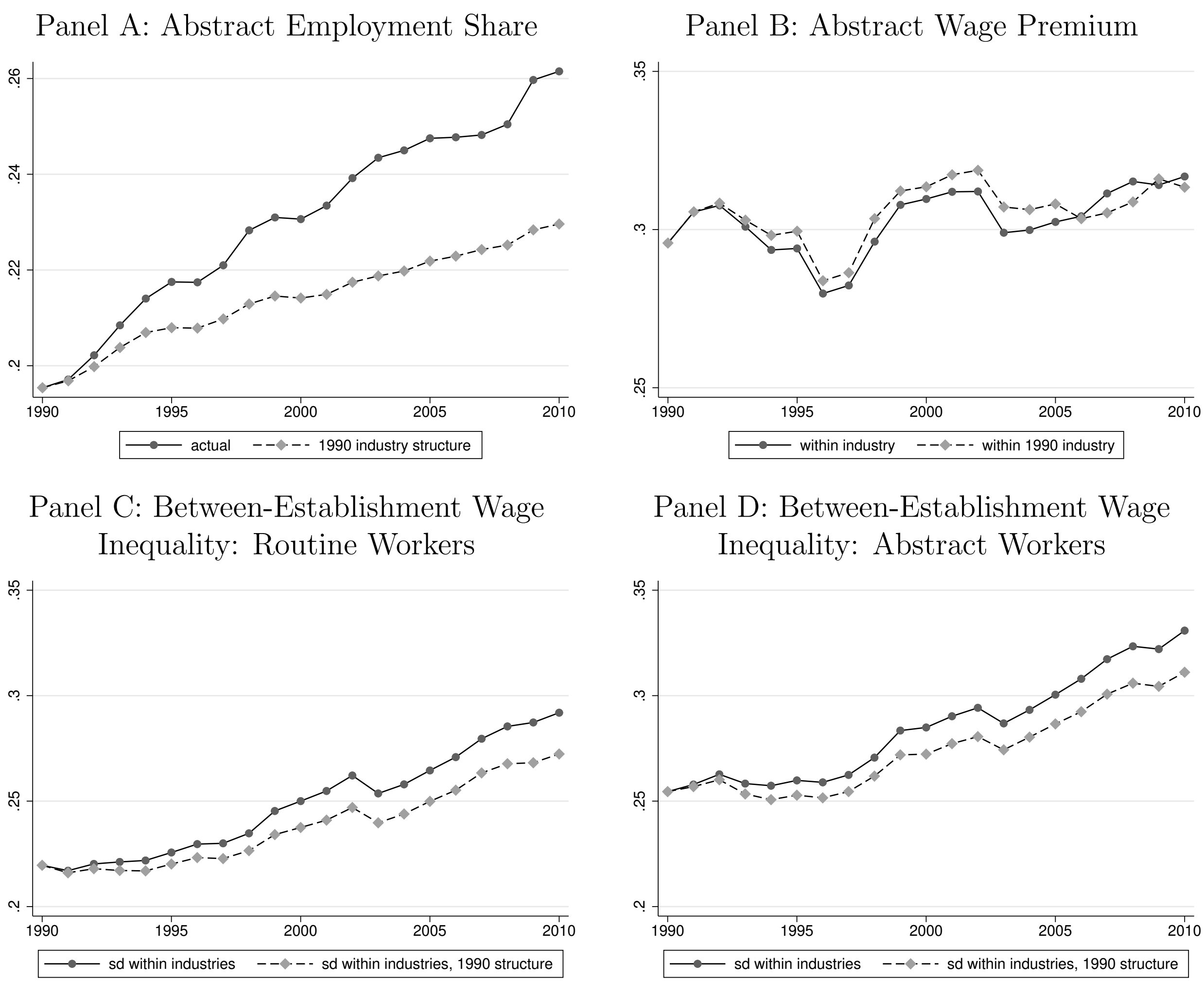


Figure 3: Year-by-Year Correlations with Establishment Productivity

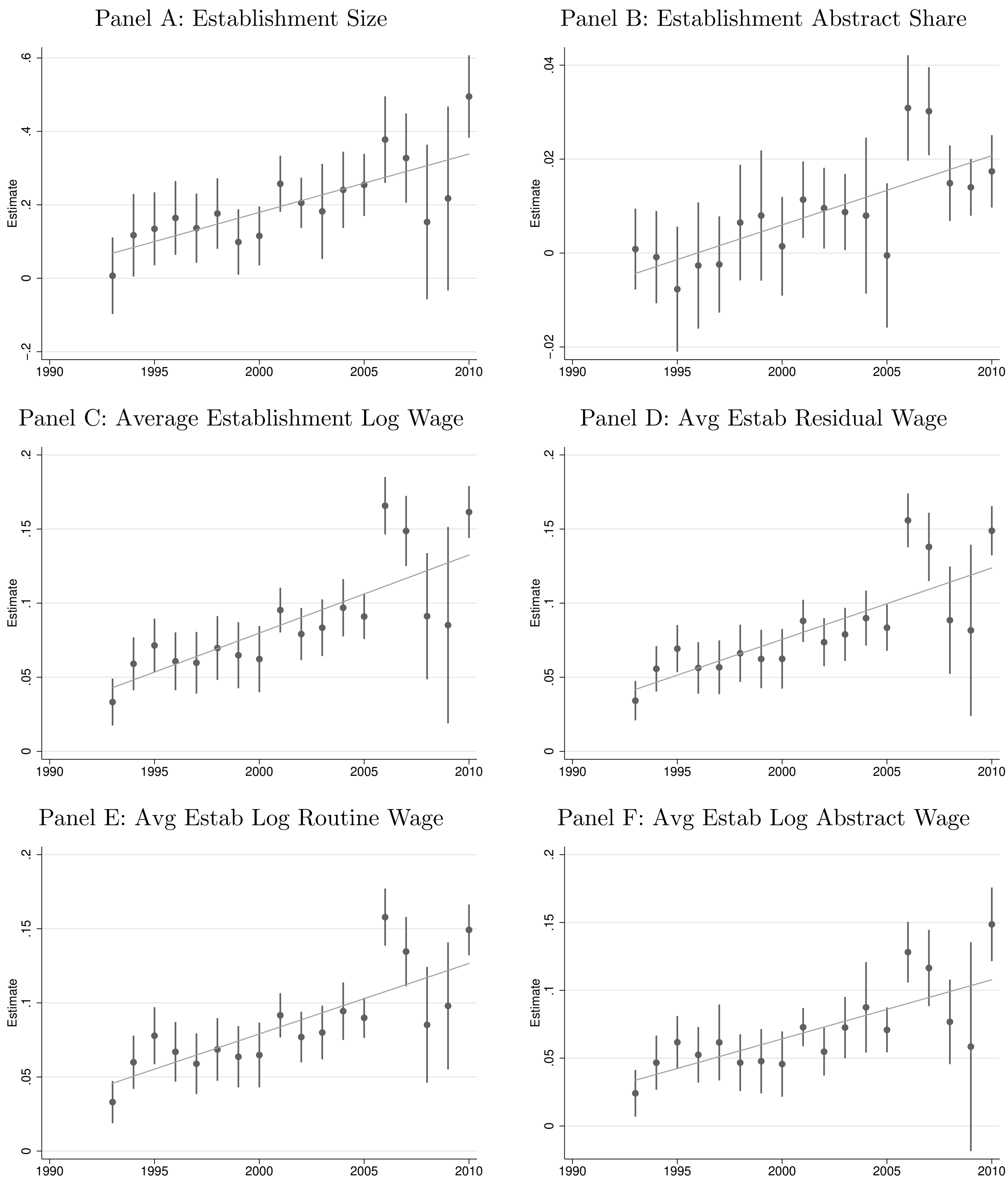


Figure 4: Abstract Share Heterogeneity and Sorting

Panel A: Evolution of the Standard Deviation of Abstract Employment Shares

Panel C: Co-variance between Abstract Share and Wage Premium

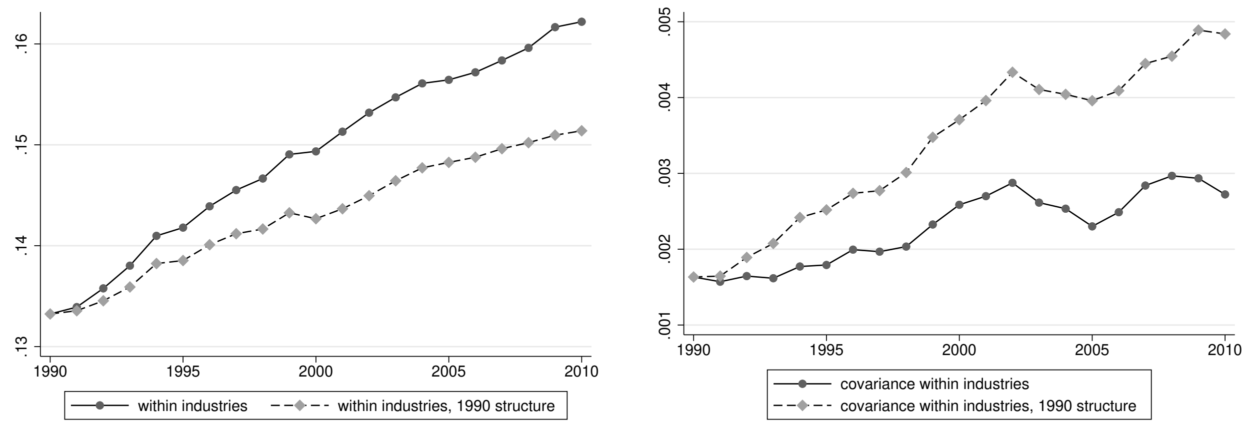
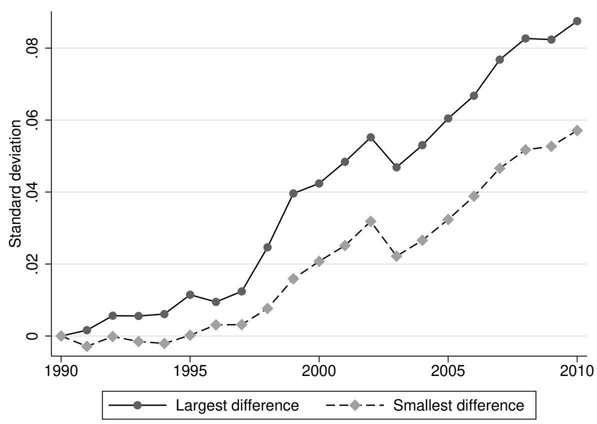
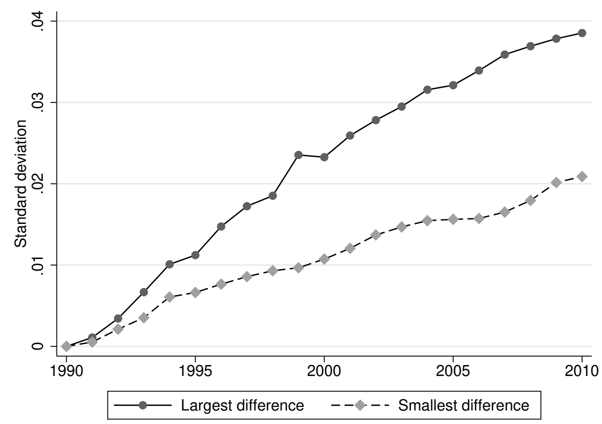


Figure 5: Technology Adoption: Changes in Abstract Share, 1990-2010

Panel A: Change in Standard Deviation of Establishment Wages



Panel B: Change in Standard Deviation of Establishment Abstract Share



Panel C: Covariance of Establishment Wage Premium and Abstract Share

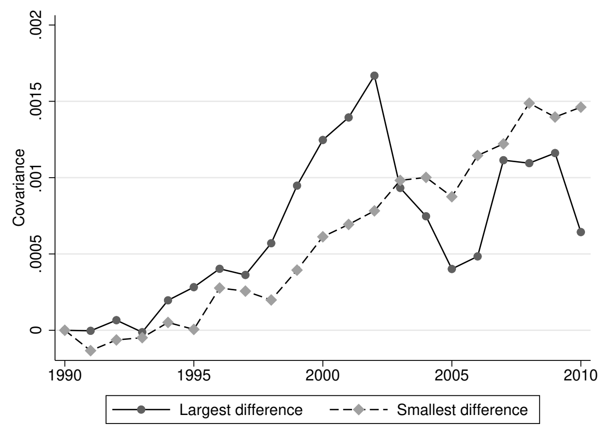


Figure 6: Technology Adoption: Changes in Robot Adoption, 1993-2010

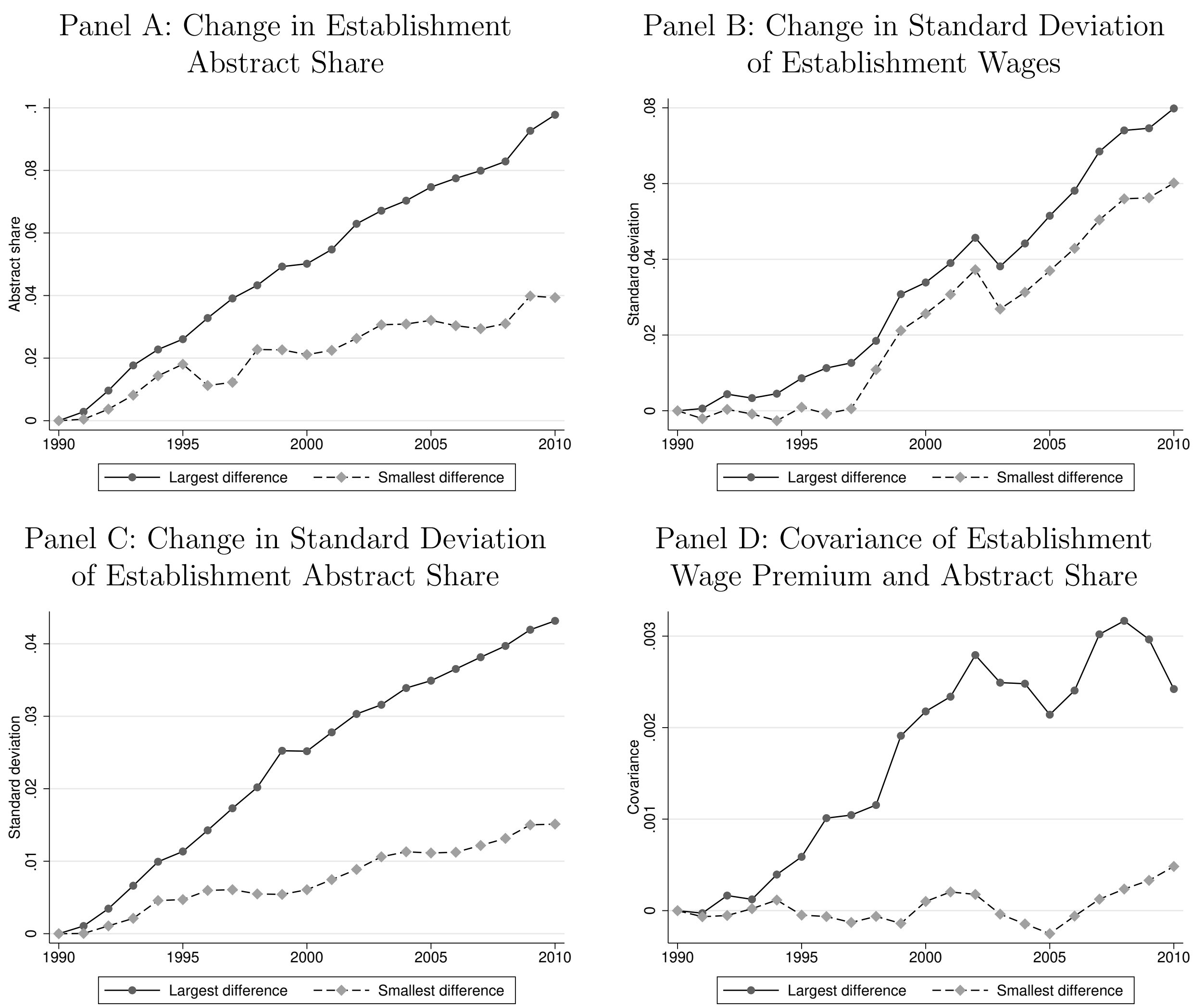


Figure 7: Technology Adoption: Changes in ICT Capital Adoption, 1991-2007

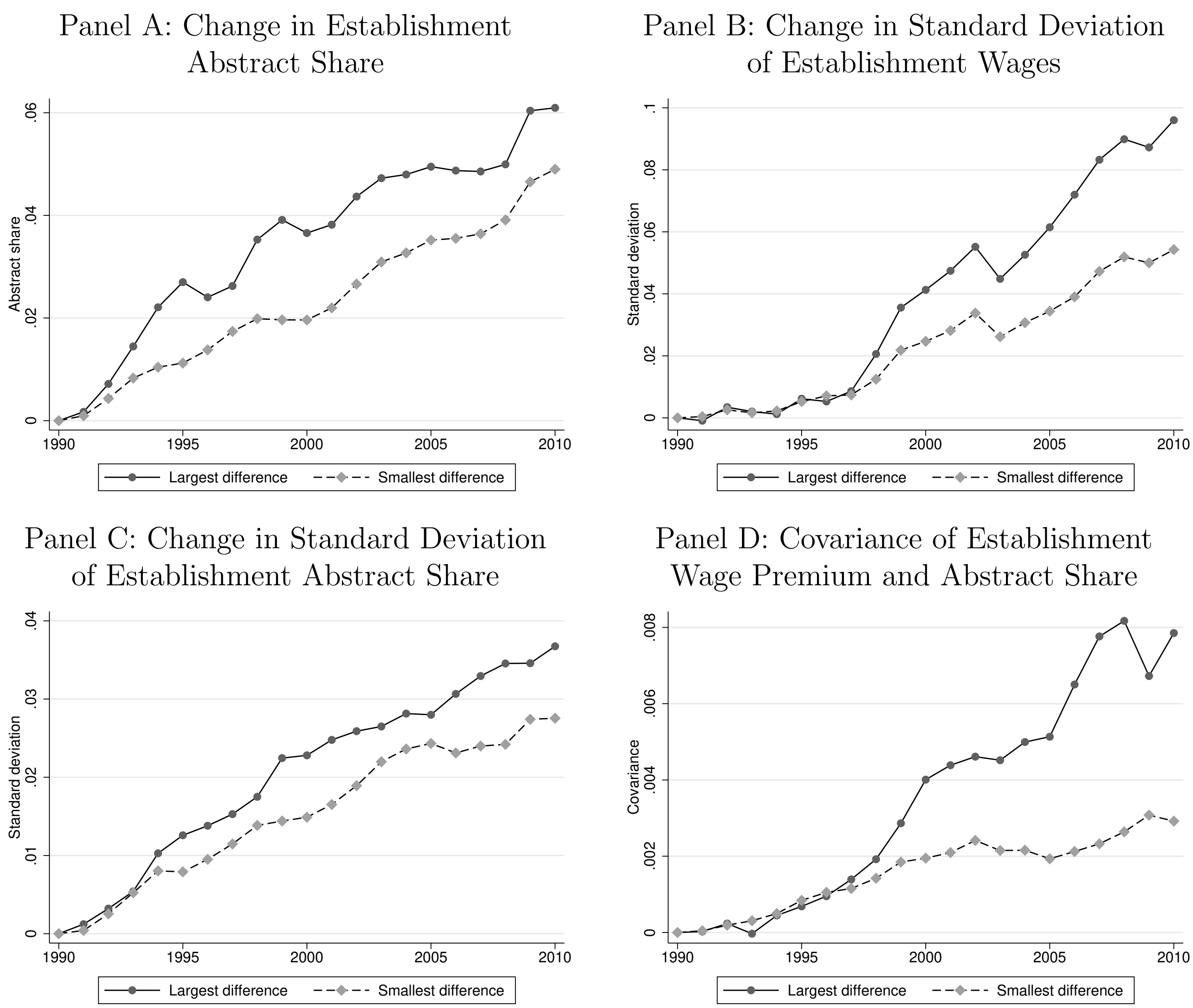


Table 1: Cross-Sectional Correlations Between Establishment Productivity and Abstract Share (Within Industry)

	Abstract Share		Productivity	Abstract Share
	(1)		(2)	(3)
Log Productivity (Rev. p. Worker)	0.0065** (0.003)	Log Estab Size (# of employees)	0.047*** (0.006)	0.0043*** (0.001)
N	86,883		86,883	26,814,744

Note: All regressions include a set of fully interacted 3-digit industry and year fixed effects. Observations are weighted by establishment size, and standard errors are clustered at the establishment level.

Table 2: Cross-Sectional Wage-Productivity-Size Correlations (Within Industry)

	Avg. Log wage	Estab Wage Premium	Avg. Log Routine Wage	Avg. Log Abstract Wage	Estab Routine Wage Premium	Estab Abstract Wage Premium
	(1)	(2)	(3)	(4)	(5)	(6)
Log Productivity	0.081^{***}	0.077^{***}	0.080^{***}	0.064^{***}	0.077^{***}	0.060^{***}
(Rev. p. Worker)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)
N	86,883	86,883	81,055	57,441	81,055	57,441

	Avg. Log wage	Estab Wage Premium	Avg. Log Routine Wage	Avg. Log Abstract Wage	Estab Routine Wage Premium	Estab Abstract Wage Premium
	(1)	(2)	(3)	(4)	(5)	(6)
Log Estab Size	0.081^{***}	0.070^{***}	0.072^{***}	0.078^{***}	0.065^{***}	0.066^{***}
(# of employees)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	26,814,744	26,814,744	22,560,227	10,039,987	22,560,227	10,039,987

Note: All regressions include a set of fully interacted 3-digit industry and year fixed effects. Observations are weighted by establishment size, and standard errors are clustered at the establishment level.

Table 3: Within-Establishment Changes

	Δ Estab Size	Δ Abstract Share	Δ Avg. Log Wage	Δ Avg. Log Rout. Wage	Δ Avg. Log Abs. Wage
	(1)	(2)	(3)	(4)	(5)
Estab prod. at baseline	0.027**	0.00047	0.0032*	0.0045**	-0.000044
	(0.011)	(0.0013)	(0.0017)	(0.0020)	(0.0037)
N	13,191	13,191	13,191	12,242	8,935

	Δ Estab Wage Premium	Δ Estab Routine Wage Premium	Δ Estab Abstract Wage Premium	P(exit)
	(1)	(2)	(3)	(4)
Estab prod. at baseline	0.0050***	0.0048**	0.0049	-0.012***
	(0.0017)	(0.0020)	(0.0033)	(0.0047)
N	13,191	12,242	8,935	20,899

	Δ Estab Productivity	Δ Abstract Share	Δ Avg. Log Wage	Δ Avg. Log Rout. Wage	Δ Avg. Log Abs. Wage
	(1)	(2)	(3)	(4)	(5)
Estab size at baseline	0.032***	0.0030***	0.0086***	0.0079***	0.0067***
	(0.0096)	(0.00025)	(0.00035)	(0.00036)	(0.00037)
N	5,460	3,452,385	3,452,385	2,827,173	1,284,459

	Δ Estab Wage Premium	Δ Estab Routine Wage Premium	Δ Estab Abstract Wage Premium	P(exit)
	(1)	(2)	(3)	(4)
Estab size at baseline	0.0089***	0.0083***	0.0077***	-0.042***
	(0.00032)	(0.00034)	(0.00036)	(0.00058)
N	3,452,385	2,827,173	1,284,459	6,349,800

Appendix A Model Appendix

This section contains details of the model and of derivations that were omitted in the main text. The presentation is not necessarily self-contained but rather complementary with Section 4 of the paper. We refer to Section 5.4 of the technical appendix of Helpman et al. (2010) for more details on the model with a CES production function and two types of workers. Note that in Helpman et al. (2010) the production function takes the form

$$y = \left[\lambda_1(\theta_1\overline{a}_1 h_1^{\gamma})^{\nu} + \lambda_2(\theta_2\overline{a}_2 h_2^{\gamma})^{\nu}\right]^{1/\nu}, \tag{A.1}$$

where the subscripts 1 and 2 denote two types of labor. In this paper, we assume

$$y = \left[\lambda_s(\theta_s\mu_s\overline{a}_s h_s^{\gamma})^{\nu} + \lambda_r(\overline{a}_r h_r^{\gamma})^{\nu}\right]^{1/\nu}, \tag{A.2}$$

where the subscripts s and r denote abstract and routine labor, respectively, and where μ_s is an abstract-augmenting technology parameter common to all firms. Hence, in this paper $\lambda_1 = \lambda_s\mu_s^{\nu}$, $\lambda_2 = \lambda_r$ and $\theta_2 = 1$.

In addition, for the derivations below, it is useful to note that $\varphi(\theta)$ and ϕ_ℓ, where $\ell \in \{s, r\}$ are defined as follows:

$$\varphi \equiv \frac{\lambda_s\mu_s^{\nu}\left(\theta\overline{a}_s h_s^{\gamma}\right)^{\nu}}{\lambda_r\left(\overline{a}_r h_r^{\gamma}\right)^{\nu}}, \qquad \phi_s \equiv \frac{\varphi}{1+\varphi}, \qquad \phi_r \equiv \frac{1}{1+\varphi}. \tag{A.3}$$

Appendix A.1 Derivations of the Key Equilibrium Relationships

This section derives the equilibrium relationships for the variables which play a crucial role when examining the impact of task-biased technological change on wage inequality.

Appendix A.1.1 Firm-level Equilibrium Variables

Below we use the following first-order conditions from the profit maximization problem to derive firm-level equilibrium revenues, employment and wages by tasks:

$$\frac{\beta\gamma}{1+\beta\gamma}\phi_\ell r(\theta) = b_\ell n_\ell(\theta) \tag{A.4}$$

$$\frac{\beta(1-\gamma k)}{1+\beta\gamma}\phi_\ell r(\theta) = c\widetilde{a}_\ell(\theta)^{\delta}, \tag{A.5}$$

where $\phi_r = [1+\varphi(\theta)]^{-1}$.

Revenues As Helpman et al. (2010) mention in Appendix 5.4 footnote 1, revenues can be expressed as:

$$r(\theta) = \kappa_y^{\beta} A \left[1+\varphi(\theta)\right]^{\beta/\nu} \left[\lambda_r^{1/\nu} \widetilde{a}_r(\theta)^{1-k\gamma} n_r(\theta)^{\gamma}\right]^{\beta}, \tag{A.6}$$

where $\kappa_y \equiv \frac{k a_{min}^{\gamma k}}{k-1}$. Using the first-order conditions along with equation (A.6) and the definition of ϕ_r, one obtains the revenue equation:

$$r(\theta) = \kappa_r \left[1+\varphi(\theta)\right]^{\frac{\beta\Lambda}{\nu\Gamma}}, \tag{A.7}$$

where κ_r is equivalent to:

$$\kappa_r \equiv A^{1/\Gamma} \left[\kappa_y \left(\frac{\beta}{1+\beta\gamma}\right)^{\frac{1-k\gamma}{\delta}+\gamma} \lambda_r^{1/\nu} \left(\frac{1-\gamma k}{c}\right)^{\frac{1-k\gamma}{\delta}} \left(\frac{\gamma}{b_r}\right)^{\gamma}\right]^{\beta/\Gamma}. \tag{A.8}$$

Employment by task and abstract employment share To obtain firm-level employment, note that from equation (A.4):

$$\begin{aligned} n_r(\theta) &= \frac{\beta\gamma}{1+\beta\gamma} \left[1+\varphi(\theta)\right]^{-1} b_r^{-1} r(\theta) \\ &= \left(\frac{\beta\gamma}{1+\beta\gamma}\right) b_r^{-1} \kappa_r \left[1+\varphi(\theta)\right]^{\frac{\beta-\nu}{\nu\Gamma}}, \end{aligned} \tag{A.9}$$

where $\frac{\beta-\nu}{\nu\Gamma} = \frac{\beta\Lambda}{\nu\Gamma} - 1 > 0$, and from equation (A.5):

$$\begin{aligned} \widetilde{a}_r(\theta) &= \left\{\frac{\beta(1-\gamma k)}{1+\beta\gamma} \left[1+\varphi(\theta)\right]^{-1} c^{-1} r(\theta)\right\}^{1/\delta} \\ &= \left[\frac{\beta(1-\gamma k)}{1+\beta\gamma}\right]^{1/\delta} c^{-1/\delta} \kappa_r^{1/\delta} \left[1+\varphi(\theta)\right]^{\frac{\beta-\nu}{\delta\nu\Gamma}}. \end{aligned} \tag{A.10}$$

Using expression $h_\ell(\theta) = n_\ell(\theta) \left(\frac{a_{min}}{\widetilde{a}_\ell(\theta)}\right)^k$, along with (A.9) and (A.10), we have that:

$$\begin{aligned} h_r(\theta) &= n_r(\theta) \left(\frac{a_{min}}{\widetilde{a}_r(\theta)}\right)^k \\ &= \left(\frac{\beta\kappa_r}{1+\beta\gamma}\right)^{1-k/\delta} \left(\frac{c}{1-\gamma k}\right)^{k/\delta} b_r^{-1} a_{min}^k \left[1+\varphi(\theta)\right]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)} && \text{(A.11)} \\ &= h_{dr} \left[1+\varphi(\theta)\right]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)}, && \text{(A.12)} \end{aligned}$$

where:

$$h_{dr} \equiv \left(\frac{\beta\kappa_r}{1+\beta\gamma}\right)^{1-k/\delta}\left(\frac{c}{1-\gamma k}\right)^{k/\delta} b_r^{-1} a_{min}^k. \tag{A.13}$$

Proceeding in a similar way for firm-level employment of abstract workers, we obtain:

$$h_s(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-k/\delta} h_r(\theta), \tag{A.14}$$

and it follows that the firm's employment share of abstract workers is given by:

$$\frac{h_s(\theta)}{h(\theta)} = \frac{b_r\varphi(\theta)^{1-k/\delta}}{b_s + b_r\varphi(\theta)^{1-k/\delta}}, \tag{A.15}$$

where $h(\theta) = h_s(\theta) + h_r(\theta)$.

Wages by task To derive equilibrium firm-level wages by task, it is useful to note that the solution of the Stole and Zwiebel bargaining game takes the following form:

$$w_\ell = \frac{\beta\gamma}{1+\beta\gamma}\frac{\phi_\ell r}{h_\ell} \tag{A.16}$$

Using (A.16) along with (A.6) and (A.12), we have that firm wages of routine workers are given by:

$$\begin{aligned} w_r(\theta) &= \frac{\beta\gamma}{1+\beta\gamma}\phi_r(\theta)\frac{r(\theta)}{h_r(\theta)} \\ &= \left(\frac{\beta\gamma}{1+\beta\gamma}\right)\left(\frac{\kappa_r}{h_{dr}}\right)[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}} && \text{(A.17)} \\ &= w_{dr}\,[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}}, && \text{(A.18)} \end{aligned}$$

where:

$$w_{dr} \equiv \left(\frac{\beta\gamma}{1+\beta\gamma}\right)\left(\frac{\kappa_r}{h_{dr}}\right). \tag{A.19}$$

Proceeding in a similar way for firm-level wages of abstract workers, we obtain:

$$w_s(\theta) = \frac{b_s}{b_r}\varphi(\theta)^{k/\delta} w_r(\theta). \tag{A.20}$$

Finally, combining the definition of $\varphi(\theta)$ together with the first-order conditions of the profit maximization problem, we obtain:

$$\varphi(\theta) = \mu_s^{\nu/\Lambda} \left(\frac{\lambda_s}{\lambda_r}\right)^{1/\Lambda} \left(\frac{b_s}{b_r}\right)^{-\gamma\nu/\Lambda} \theta^{\nu/\Lambda}. \tag{A.21}$$

Appendix A.1.2 Determination of the Productivity Threshold

As is standard in Melitz-type heterogeneous firm models, the productivity threshold for production, θ_d, is pined down by both the Zero-Cutoff Profit (ZCP) and the Free Entry (FE) conditions.

The ZCP condition, which requires that the firm at the cutoff θ_d makes zero profits, implies:[10]

$$\frac{\Gamma}{1+\beta\gamma} r(\theta_d) = f_d. \tag{A.22}$$

Moreover, given equation (A.7), relative revenues across two firms with productivities θ_1 and θ_2 can be written as:

$$\frac{r(\theta_1)}{r(\theta_2)} = \left[\frac{1+\varphi(\theta_1)}{1+\varphi(\theta_2)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}}. \tag{A.23}$$

Combining equation (A.23) along with the ZCP condition (A.22) we obtain:

$$r(\theta) = f_d \left(\frac{\Gamma}{1+\beta\gamma}\right)^{-1} \left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}}. \tag{A.24}$$

The FE condition states that the expected profits for a potential entrant should equal the fixed entry cost:

$$\int_{\theta_d}^{\infty} \pi(\theta) dG(\theta) = f_e. \tag{A.25}$$

Therefore, combining equations (A.24) and (A.25) implies:

$$f_d \int_{\theta_d}^{\infty} \left(\left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}} - 1 \right) dG(\theta) = f_e. \tag{A.26}$$

Equation (A.26) pins down the equilibrium threshold θ_d as a function of the parameters of the model and the search costs b_s and b_r.

[10] This is obtained by noting that profits can be written as:

$$\pi(\theta) = \frac{\Gamma}{1+\beta\gamma} r(\theta) - f_d.$$

Appendix A.2 The Relationship between Firm-specific Equilibrium Outcomes and Productivity

This section presents the proofs for the results in Equations (8) and (11) on the relationship between firm-level employment and wages and firm productivity.

First, note that:

$$\frac{\partial\varphi(\theta)}{\partial\theta} = \frac{\nu}{\Lambda}\mu_s^{\frac{\nu}{\Lambda}}\left(\frac{\lambda_s}{\lambda_r}\right)^{\frac{1}{\Lambda}}\left(\frac{b_s}{b_r}\right)^{-\frac{\gamma\nu}{\Lambda}}\theta^{\frac{\nu}{\Lambda}-1} \quad > \quad 0, \tag{A.27}$$

and recall that $\beta > \nu$, $\Lambda > \Gamma$ and $\delta > k$ such that $\frac{\beta-\nu}{\nu\Gamma} > 0$ and $1 - \frac{k}{\delta} > 0$.

Prediction 1: More productive firms are larger, employing more of both types of workers.
Proof: Taking the derivative of equations (A.12) and (A.14), we obtain:

$$\begin{aligned}
\frac{\partial h_r(\theta)}{\partial\theta} &= h_{dr}\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)-1}\cdot\frac{\partial\varphi(\theta)}{\partial\theta} &&> \quad 0\\
\frac{\partial h_s(\theta)}{\partial\theta} &= \frac{b_r}{b_s}\left[\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\cdot\frac{\partial\varphi(\theta)}{\partial\theta}\cdot h_r(\theta)+\varphi(\theta)^{1-\frac{k}{\delta}}\cdot\frac{\partial h_r(\theta)}{\partial\theta}\right] &&> \quad 0
\end{aligned} \tag{A.28}$$

Prediction 2: More productive firms have a higher employment share of abstract workers.
Proof: Taking the derivative of equation (A.15), we have that:

$$\frac{\partial}{\partial\theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{b_s b_r\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\cdot\frac{\partial\varphi(\theta)}{\partial\theta}}{\left[b_s+b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]^2} \quad > \quad 0 \tag{A.29}$$

Prediction 3: More productive firms pay higher wages to both types of workers.
Proof: Taking the derivative of equations (A.18) and (A.20), we obtain:

$$\begin{aligned}
\frac{\partial w_r(\theta)}{\partial\theta} &= w_{dr}\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}-1}\cdot\frac{\partial\varphi(\theta)}{\partial\theta} &&> \quad 0\\
\frac{\partial w_s(\theta)}{\partial\theta} &= \frac{b_s}{b_r}\left[\frac{k}{\delta}\varphi(\theta)^{\frac{k}{\delta}-1}\cdot\frac{\partial\varphi(\theta)}{\partial\theta}\cdot w_r(\theta)+\varphi(\theta)^{\frac{k}{\delta}}\cdot\frac{\partial w_r(\theta)}{\partial\theta}\right] &&> \quad 0
\end{aligned} \tag{A.30}$$

This result, combined with the prediction that more productive firms employ a higher share of abstract workers, unambiguously imply that firm average wages are increasing in firm productivity.

Appendix A.3 Impact of Task-Biased Technological Change

We model task-biased technological change (TBTC) as an increase in the parameter μ_s, i.e. as a factor-augmenting shock favoring abstract workers.

In order to evaluate how this task shock affects firms differentially across the productivity distribution we examine the second-order derivative of firm outcome variables, with respect to both the common abstract-augmenting technology parameter μ_s and firm productivity. To this end note that:

$$\frac{\partial\varphi(\theta)}{\partial\mu_s} = \frac{\nu}{\Lambda}\mu_s^{-1}\varphi(\theta) \ > \ 0 \ , \qquad \frac{\partial^2\varphi(\theta)}{\partial\mu_s\partial\theta} = \left(\frac{\nu}{\Lambda}\right)^2 \mu_s^{-1}\theta^{-1}\varphi(\theta) \ > \ 0 \ , \tag{A.31}$$

and

$$\frac{\partial\varphi(\theta)}{\partial\mu_s}\frac{\partial\varphi(\theta)}{\partial\theta} = \varphi(\theta)\cdot\frac{\partial^2\varphi(\theta)}{\partial\mu_s\partial\theta} > 0 \ . \tag{A.32}$$

Prediction 1: *Selection* – TBTC increases the productivity threshold for production θ_d.
Proof: We prove Prediction 1 by contradiction. Consider equation (A.26), which pins down the equilibrium threshold as a function of parameters of the model:

$$f_d\int_{\theta_d}^{\infty}\left(\left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}} - 1\right)dG(\theta) = f_e \tag{A.33}$$

Suppose first that TBTC has no effect on θ_d. Holding θ_d fixed, the increase in $[1+\varphi(\theta)]/[1+\varphi(\theta_d)]$ induced by the increase in μ_s would imply an increase in the term in the square brackets for all relevant values of θ evaluated in the integral. Hence, with a fixed θ_d the LHS of equation (A.33) would increase while the RHS would remain fixed. This implies that θ_d cannot remain constant if μ_s increases.

Suppose now that θ_d falls as a reaction to the increase in μ_s. This would lead to a further increase in the value of the term in the square brackets for all relevant values of θ (as there would now be a larger gap between θ and θ_d). At the same time, a fall of θ_d would increase the range of values of θ that are integrated over. Hence, a decrease in θ_d would unambiguously increase the LHS of equation (A.33) while the RHS would remain fixed. This implies that θ_d cannot decrease either.

This proves that the only change in θ_d consistent with condition (A.33) is an increase in

θ_d when μ_s increases. Therefore:

$$\frac{\partial \theta_d}{\partial \mu_s} \quad > \quad 0 \tag{A.34}$$

leading to the exit of the least productive firms.

Recall that productivity among operating firms follows a Pareto distribution with scale parameter θ_d. Thus, even if the range of productivity of operating firms decreases, TBTC increases the variance of productivity among operating firms. Given the dependence of firm wages on productivity, this results in an increase in the variance of firm wages and therefore wage inequality. In addition, since low-productivity firms have a lower share of abstract workers, their exit contributes to the increase in the economy's overall abstract share.

Prediction 2: *Differential Employment Growth* – TBTC shifts employment by task towards more productive firms.

Proof: Taking the first- and second-order derivatives of (A.12), we obtain:

$$\begin{aligned}
\frac{\partial h_r(\theta)}{\partial \mu_s} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right) h_r(\theta)\left[1+\varphi(\theta)\right]^{-1}\frac{\nu}{\Lambda}\mu_s^{-1}\varphi(\theta) \quad > \quad 0 \\
\frac{\partial^2 h_r(\theta)}{\partial \mu_s \partial \theta} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right) h_r(\theta)\left[1+\varphi(\theta)\right]^{-2}\cdot\frac{\partial^2 \varphi(\theta)}{\partial \mu_s \partial \theta}\left[1+\varphi(\theta)\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)\right] \quad > \quad 0
\end{aligned}$$

Hence, TBTC increases routine employment for all firms, but more so for more productive firms. Similarly, taking the derivatives of (A.14):

$$\begin{aligned}
\frac{\partial h_s(\theta)}{\partial \mu_s} &= \frac{\nu}{\Lambda}\mu_s^{-1}\left(1-\frac{k}{\delta}\right)\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\cdot\frac{\varphi(\theta)}{1+\varphi(\theta)}\right] h_s(\theta) \quad > \quad 0 \\
\frac{\partial^2 h_s(\theta)}{\partial \mu_s \partial \theta} &= \frac{\nu}{\Lambda}\mu_s^{-1}\left(1-\frac{k}{\delta}\right)\left\{\frac{\beta-\nu}{\nu\Gamma}\left[1+\varphi(\theta)\right]^{-2}\frac{\partial \varphi(\theta)}{\partial \theta} h_s(\theta)+\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\cdot\frac{\varphi(\theta)}{1+\varphi(\theta)}\right]\frac{\partial h_s(\theta)}{\partial \theta}\right\} \quad >
\end{aligned}$$

Hence, TBTC also increases abstract employment for all firms, but more so for more productive firms.

Prediction 3: *Increased Sorting* – Following TBTC, the increase in firm abstract employment share is relatively larger for more productive firms if the number of routine workers exceeds that of abstract workers at baseline. In such a case, abstract workers become disproportionately concentrated in more productive firms.

Proof: Taking the first-order derivative of (A.15) we get:

$$\frac{\partial}{\partial \mu_s}\left[\frac{h_s(\theta)}{h(\theta)}\right] = b_s\left(1-\frac{k}{\delta}\right)\frac{\nu}{\Lambda}\mu_s^{-1}\cdot\frac{1}{b_s+b_r\varphi(\theta)^{1-\frac{k}{\delta}}}\cdot\frac{h_s(\theta)}{h(\theta)} \quad > \quad 0$$

Hence, TBTC increases the share of abstract workers for all firms. Taking the second-order derivative of firm abstract employment share yields:

$$\frac{\partial^2}{\partial \mu_s \partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{1}{\left[b_s+b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]^3}\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\frac{\partial\varphi(\theta)}{\partial\theta}\left[b_s-b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]$$

Given that the ratio of abstract to routine workers is $h_s(\theta)/h_r(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-\frac{k}{\delta}}$, the term $\left[b_s-b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]$ is positive if $h_s(\theta)/h_r(\theta) < 1$ and negative if $h_s(\theta)/h_r(\theta) > 1$. Therefore,

$$\frac{\partial^2}{\partial \mu_s \partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] \quad > \quad 0 \quad \text{if} \quad \frac{h_s(\theta)}{h_r(\theta)} < 1, \tag{A.35}$$

implying that the increase in the share of abstract workers will be larger for more productive firms if the number of abstract workers outweighs the number of routine workers at baseline. Starting from a situation with relatively more routine workers, TBTC will therefore increase sorting of abstract workers into firms with relatively higher productivity levels.

Prediction 4: *Differential Wage Growth* – Following TBTC, more productive firms increase their wages for workers of both types relative to less productive firms.
Proof: Taking the first- and second-order derivatives of (A.18) we obtain:

$$\begin{aligned}
\frac{\partial w_r(\theta)}{\partial \mu_s} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\nu}{\delta\Lambda}\mu_s^{-1}w_r(\theta)\frac{\varphi(\theta)}{1+\varphi(\theta)} \quad > \quad 0 \\
\frac{\partial^2 w_r(\theta)}{\partial \mu_s \partial\theta} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\nu}{\delta\Lambda}\mu_s^{-1}\frac{1}{1+\varphi(\theta)}\left[\varphi(\theta)\frac{\partial w_r(\theta)}{\partial\theta}+\frac{w_r(\theta)}{1+\varphi(\theta)}\cdot\frac{\partial\varphi(\theta)}{\partial\theta}\right] \quad > \quad 0
\end{aligned}$$

Hence, TBTC increases firm wages of routine workers, and more so for more productive firms. Similarly, taking the derivatives of (A.20):

$$\begin{aligned}
\frac{\partial w_s(\theta)}{\partial \mu_s} &= \frac{k\nu}{\delta\Lambda}\mu_s^{-1}w_s(\theta)\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right] \quad > \quad 0 \\
\frac{\partial^2 w_s(\theta)}{\partial \mu_s \partial\theta} &= \frac{k\nu}{\delta\Lambda}\mu_s^{-1}\left\{\frac{\partial w_s(\theta)}{\partial\theta}\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right]+w_s(\theta)\left(\frac{\beta-\nu}{\nu\Gamma}\right)[1+\varphi(\theta)]^{-2}\frac{\partial\varphi(\theta)}{\partial\theta}\right\} \quad > \quad 0
\end{aligned}$$

Thus, the increase in firm wages of abstract workers is disproportionately larger for more productive firms. Given the rise in firm abstract share and wages for both types of workers, TBTC also leads to an increase in firm average wages for all firms. In addition, if TBTC increases sorting, then $[\partial^2 w_\ell(\theta)/\partial\mu_s\partial\theta] > 0$, where $\ell = \{s, r\}$, unambiguously implies that the rise of firm average wages is monotonically increasing in firm productivity.