Constrained vs Unconstrained Labor Supply: The Economics of Dual Job Holding*

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Abstract

This paper develops a unified model of dual and unitary job holding based on a Stone-Geary utility function. The model incorporates both constrained and unconstrained labor supply. Panel data methods are adapted to accommodate unobserved heterogeneity and multinomial selection into 6 mutually exclusive labor supply regimes. We estimate the wage and income elasticities arising from selection and unobserved heterogeneity as well as from the Stone-Geary Slutsky equations. The labor supply model is estimated with data from the British Household Panel Survey 1991- 2008. Among dual job holders, our study finds that the Stone-Geary income and wage elasticities are much larger for labor supply to the second job compared with the main job. When the effects of selection and unobserved heterogeneity are taken account of, the magnitudes of these elasticities on the second job tend to be significantly reduced.

Keywords: dual job; labor supply; Stone-Geary

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1 Introduction

Researchers have progressively extended labor supply theory in both static and dynamic dimensions to account for a richer variety of labor supply behavior. One fruitful area for research on labor supply is that of multiple job holding. One of most interesting aspects of multiple job holding is the motivation behind the decision to hold more than one job. A number of studies show that this decision is not only motivated by an hours constraint on the main job (also known as moonlighting), but also by a desire to hold a portfolio of jobs. Mostly, this literature has focused either on the determinants of each decision or on the labor supply for only one of the possible regimes. Considerably less attention has been paid to the development of a general labor supply model that allows for moonlighting as a response to an hours constraint on the main job and the joint determination of the hours supplied to two jobs when the decision to hold two jobs is not dictated by a constraint on the main job. In this paper we develop such a labor supply model based on a Stone-Geary utility function which allows us to model the choices of an individual who can hold up to two jobs.

Dual job holding is a pervasive phenomenon in many economies. Between 1994 and 2002, the weekly rate of dual job holding in the U.K. was around 4.5 percent (Office for National Statistics, 2002), but when computed on a monthly basis, the rate was found to be almost twice as high (Panos et al., 2011). Although Amuedo-Dorantes and Kimmel (2009) concludes that dual job holding is pro-cyclical, the weekly rate of dual job holding has remained stable at about 5 percent in the U.S. during the 2000-2010 decade notwithstanding the recessions in 2001 and 2008 (Hipple, 2010). Dual job holding seems even more common in developing and transition economies, where the incidence of domestic production that takes place in the informal sector is typically higher than in developed countries. The rate of dual-job holding Russian males doubled from the early to the mid 90's and stayed around 12 percent for the remainder of the decade (Foley, 1997). A survey of Tanzanian workers with a regular job in the formal economy found that more than half of them also held a job in the informal sector (Theisen, 2009).

Dual job holding is typically associated with an hours constraint on the main job. Firms typically offer a fixed hours and wage employment package subject to labor market regulations or union contracts pertaining to overtime. If the number of hours that a firm offers falls short of the optimal number of hours that a utility maximizing worker would choose at the going wage, then a rational individual will take a second job under the condition that it pays more than his reservation wage on the second job. This is what we regard as the hours constraint motivation for holding multiple jobs, or "moonlighting". Moonlighting can be viewed as a substitute (perhaps temporary) for job search for a job with the optimal package of hours and wage rate. However, moonlighting alone cannot explain the behavior of all dual job holders. In fact, Allen (1998) concludes that unconstrained workers are more likely to have two jobs than are constrained workers. This result has led to a rich line of research on the motivation behind the decision to hold two jobs. Some individuals may decide to allocate their working time between two or more jobs because they have a personal preference for job differentiation. For example, some workers may hold two jobs because jobs are heterogeneous and they are not perfect substitutes (Kimmel and Conway, 2001). Others may hold a second job as a form of hedging against the risk of losing employment (Bell et al., 1997) or as a way to gradually transition to a new primary job, often self-employment (Panos et al., 2011). We group all reasons for holding two jobs that are not due to an hours constraint under the job portfolio label.

In this paper we expand on Kimmel and Conway (2001) by using a Stone-Geary utility function to motivate the empirical work in the context of a dual job holding model.[1] The estimation is carried out for a sample of male workers from the British Household Panel Survey (BHPS). We use recent development in econometrics to model unobserved heterogeneity and sample selection in a panel data setting. We derive the labor supply functions for unitary and dual job holders. For the latter, we distinguish whether or not they face an hours constraint on the main. If there is an hours constraint on the main job does it produce over-employment or under-employment? In addition we extend binary sample selection methods for panel data to multinomial selection into one of 6 mutually exclusive labor supply outcomes in the presence of unobserved heterogeneity. From our estimates, we compute the wage elasticities and income elasticities for each category of worker according to their constrained and dual-job status. We confirm the results in the literature that the labor supply for a unitary job holder is quite inelastic (Altonji and Paxson, 1988). Furthermore, the derived Slutsky elasticities are supplemented by wage and income effects associated with selection and unobserved heterogeneity. These total elasticities reveal how failure to control for selection and unobserved heterogeneity can lead to seriously biased conclusions, especially for dual job holders.

Section 2 reviews the literature on dual job holding; section 3 presents the theoretical frame-

[1] Typically the Stone-Geary utility function is used to estimate expenditure functions for multiple commodity groups. See Chung (1994) for a review of the main studies based on a Stone-Geary utility function.

work used to derive our labor supplies equations; section 4 describes the data; section 5 discusses the estimation strategies of our empirical models; section 6 presents the empirical findings; and section 7 is a summary and conclusion.

2 Literature Review

Early theoretical work focused only on the hours constraint aspect of moonlighting (Perlman, 1966). Shishko and Rostker (1976) and Frederiksen et al. (2008) found that labor supply becomes more elastic to changes in the wage rate after accounting for the decision to moonlight. Extending the moonlighting model to a household labor supply framework, Krishnan (1990) found that the husband's decision to hold a second job is a substitute for the wife's decision to enter the labor market. Paxson and Sicherman (1998) concluded that moonlighting is a short-run solution to a situation of under-employment, while searching for a job that offers the target hours of work. However, the latter result is not supported by other studies that found that dual job holding is quite persistent over time and not just a short-run decision, thus casting doubt on the hours constraint hypothesis (Böheim and Taylor, 2004; Panos et al., 2011).

A number of papers have tried to identify the determinants and hence the motives behind the decision to hold two jobs. Typically all studies conclude that while the probability of holding two jobs increases in the presence of hours or liquidity constraints (Abdukadir, 1992; Kimmel and Conway, 2001; Panos et al., 2011), unconstrained workers are actually more likely to hold two jobs than constrained workers, thus suggesting that job portfolio motives may be even more important than the hours constraint (Allen, 1998; Böheim and Taylor, 2004). Exploiting the information contained in the 1991 Current Population Survey, Averett (2001) can identify the motives for holding two jobs. She classified as moonlighters all individuals who report working on a second job (1) to meet regular household expenses, (2) to pay off debts, (3) to save for the future or (4) to buy something special. She identified as dual job holders with job portfolio motives all individuals who report working on a second job (1) to get experience in a different occupation or to build a business, (2) to help out a friend or relative, (3) because he/she enjoys the work on the second job, and (4) other reasons. She estimated the probability of being a moonlighter, conditional on being a dual job holder, but she is unable to identify any specific determinant that is consistently significant across alternative models.

Only a handful of papers have actually attempted to estimate labor supply models that include the hours constraint and the job portfolio motive as alternative motives to working on a second job. Wu et al. (2009) estimated a labor supply function on the second job that included an indicator for being satisfied with the hours worked on the main job, but failed to recognize that the specification of labor supply for moonlighters is different from that associated with the job portfolio hypothesis. In particular, the hours supplied on the first job should be included in the labor supply equation for the second job for moonlighters but not in the labor supply equation for the second job in the job portfolio model. To the best of our knowledge, Kimmel and Conway (2001) is the only attempt that recognizes this important distinction. However, their data does not allow them to identify whether the decision to work on a second is motivated by an hours constraint. Consequently, they first need to estimate the probability that a moonlighter faces an hours constraint on the main occupation using a disequilibrium model. They then use these predicted probabilities to estimate the alternative labor supply using a switching regression model. Although they work with panel data, no attempt is made to control for individual unobserved heterogeneity.

3 Conceptual Framework

In this section we introduce the theoretical labor supply functions obtained from utility maximization for a Stone-Geary Utility function. Our approach is motivated by a desire to understand dual-job labor supply from the perspective of a carefully articulated utility maximization framework. The objective is to base empirical analysis on an internally consistent analytical framework that can capture some salient features of labor supply in both constrained and unconstrained decision environments. While the choice of any specific utility function is inherently arbitrary, we seek a specification that is sufficiently flexible to accommodate agent heterogeneity in labor supply decisions and at the same time provides a unified treatment of several labor supply regimes encountered in practice. In a simple unitary job framework, there is no ambiguity about expressing the marginal utility of leisure as simply minus one times the marginal disutility of labor supply. The utility function is easily expressed either in terms of leisure (total time available for work or leisure - labor supply) or in terms of hours of work. When there is more than one job involved, there is a need to recognize that the marginal disutility of work can vary with the job. Hence, one needs to be able to write the utility function in a way that the marginal utility of leisure freed up from each job is clearly identified. To this end we uniquely adapt a Stone-Geary Utility function model to represent labor supply to more than one job. The advantage of a Stone-Geary specification is that it specifies upper bounds to labor supply to each job as well as a lower bound to income. The marginal utility of leisure (disutility of work) corresponding to each job is easily expressed. The derivations of the labor supply elasticities for the Stone-Geary utility are presented in a technical appendix available upon request of the authors.

Unconstrained dual job holder

Consider utility maximization for a multiple (dual) job holder who is not constrained in his choice of hours to work at two jobs:

$$U = (\gamma_1 - h_1^*)^{\alpha_1} \ (\gamma_2 \ - h_2^*)^{\alpha_2} \, (y^* - \gamma_3)^{1-\alpha_1-\alpha_2} \tag{1}$$

where $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \gamma_3 > 0$, h_m^* represents the time allocated to job m, and y^* is income. The parameters γ_1 and γ_2 represent the upper bounds on the time that can be expended on jobs 1 and 2, and still have the utility function defined. The total time available for work and leisure (T) is defined by

$$T = \sum_{m=1}^{2} \gamma_m.$$

The parameter γ_3 represents the lower bound on the amount of income necessary for the utility function to be defined. The terms $(\gamma_m - h_m^*)$, $m = 1, 2$ represent the times freed up by each job for leisure consumption. Total consumption of leisure time ℓ is residually obtained as

$$\begin{aligned}\ell &= T - h_1^* - h_2^* \\ &= \gamma_1 + \gamma_2 \ - h_1^* - h_2^* \\ &= (\gamma_1 \ - h_1^*) + (\gamma_2 \ - h_2^*)\,.\end{aligned}$$

The economic problem facing an unconstrained dual job holder can be stated as

$$\max_{h_1,h_2,y} U = (\gamma_1 - h_1^*)^{\alpha_1} \ (\gamma_2 - h_2^*)^{\alpha_2} \ (y^* - \gamma_3)^{1-\alpha_1-\alpha_2}$$

$$\text{s.t. } y^* = \sum_{m=1}^{2} w_m h_m^* + I,$$

$$0 < h_m^* < \gamma_m, \ m = 1, 2 \text{ and}$$

$$\sum_{m=1}^{2} h_m^* \leq T,$$

where w_m is the wage or pecuniary rewards to the mth job, and I is exogenous non-labor income. The utility maximizing dual labor supply functions are given by

$$h_1^* = (1-\alpha_1)\,\gamma_1 - \alpha_1\gamma_2\left(\frac{w_2}{w_1}\right) + \alpha_1\gamma_3\left(\frac{1}{w_1}\right) - \alpha_1\left(\frac{I}{w_1}\right) \tag{2}$$

$$h_2^* = (1-\alpha_2)\,\gamma_2 - \alpha_2\gamma_1\left(\frac{w_1}{w_2}\right) + \alpha_2\gamma_3\left(\frac{1}{w_2}\right) - \alpha_2\left(\frac{I}{w_2}\right). \tag{3}$$

Equivalently, the earnings versions of dual labor supply may be expressed as

$$w_1 h_1^* = \alpha_1\gamma_3 + (1-\alpha_1)\,\gamma_1 w_1 - \alpha_1\gamma_2 w_2 - \alpha_1 I \tag{4}$$

$$w_2 h_2^* = \alpha_2\gamma_3 + (1-\alpha_2)\,\gamma_2 w_2 - \alpha_2\gamma_1 w_2 - \alpha_2 I. \tag{5}$$

The α parameters of the utility function are of course literally the elasticities of the utility index with respect to the time released for leisure consumption from each job. A more useful interpretation of these parameters can be obtained from the equilibrium conditions. Note that the maximum amount of discretionary income may be defined as

$$y_{\max}^* = \gamma_1 w_1 + \gamma_2 w_2 + I - \gamma_3. \tag{6}$$

In equilibrium the values for h_1^* and h_2^* satisfy the following solutions for α_1 and α_2 obtained from equations (4) and (5):

$$\begin{aligned}\alpha_1 &= \frac{(\gamma_1 - h_1^*)\,w_1}{\gamma_1 w_1 + \gamma_2 w_2 + I - \gamma_3} \\ &= \frac{(\gamma_1 - h_1^*)\,w_1}{y_{\max}^*}\end{aligned}$$

and

$$\begin{aligned}\alpha_2 &= \frac{(\gamma_2 - h_2^*)\, w_2}{\gamma_1 w_1 + \gamma_2 w_2 + I - \gamma_3} \\ &= \frac{(\gamma_2 - h_2^*)\, w_2}{y^*_{\max}}.\end{aligned}$$

α_1 and α_2 represent the shares of maximum discretionary income that are expended on the consumption of leisure arising from working less than the maximum threshold hours. The remaining share is discretionary non-leisure consumption as a share of maximum discretionary income:

$$\begin{aligned}1 - \alpha_1 - \alpha_2 &= 1 - \frac{(\gamma_1 - h_1^*)\, w_1 + (\gamma_2 - h_2^*)\, w_2}{\gamma_1 w_1 + \gamma_2 w_2 + I - \gamma_3} \\ &= \frac{w_1 h_1^* + w_1 h_2^* + I - \gamma_3}{\gamma_1 w_1 + \gamma_2 w_2 + I - \gamma_3} \\ &= \frac{y^* - \gamma_3}{y^*_{\max}}.\end{aligned}$$

<u>Unconstrained unitary job holders</u>

We impose the restriction that unitary and dual job holders have the same utility function. This allows one to characterize individuals who move back and forth between unitary and dual job holding as responding to changes in their economic environments and avoids the awkwardness of having to assume that these individuals are responding to periodic ad hoc changes in preferences.

For individuals who hold only one job, we then condition on $h_2^* = 0$:

$$\begin{aligned}\max_{h_1, y} U \ &= (\gamma_1 - h_1^*)^{\alpha_1}\ (\gamma_2\ )^{\alpha_2}\,(y^* - \gamma_3)^{1-\alpha_1-\alpha_2} \\ \text{s.t. } & y = w_1 h_1^* + I, \\ & 0 < h_1^*\ < \gamma_1, \\ & h_1^* \leq T.\end{aligned}$$

labor supply to job 1 in this case can be shown to be

$$h_1^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}\right)\gamma_1 + \left(\frac{\alpha_1}{1-\alpha_2}\right)(\gamma_3)\left(\frac{1}{w_1}\right) - \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{I}{w_1}\right) \tag{7}$$

or in terms of earnings

$$w_1 h_1^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}\right)\gamma_1 w_1 + \left(\frac{\alpha_1}{1-\alpha_2}\right)(\gamma_3) - \left(\frac{\alpha_1}{1-\alpha_2}\right)I. \tag{8}$$

Constrained dual job holder

We assume that constraints on labor supply for dual job holders apply only to job 1, i.e. workers are constrained either because they desire more hours on job 1 (under-employed) or they desire fewer hours on job 1 (over-employed). Consequently, constrained dual job holders are assumed to be working their desired hours on job 2 conditional on their constrained hours in job 1. For an individual who is constrained at $h_1 = \ddot{h}_1$, the utility maximization problem becomes

$$\max_{h_2,y} U = \left(\gamma_1 - \ddot{h}_1\right)^{\alpha_1} (\gamma_2 - h_2^*)^{\alpha_2} (y^* - \gamma_3)^{1-\alpha_1-\alpha_2}$$

$$\text{s.t. } y = w_2 h_2^* + w_1 \ddot{h}_1 + I,$$

$$0 \leq h_2^* < \gamma_2, \; 0 \leq \ddot{h}_1 < \gamma_1, \text{ and}$$

$$\ddot{h}_1 + h_2^* \leq T,$$

While labor supply to job 1 is fixed at $\ddot{h}_1$, desired labor supply to job 2 is determined according to

$$h_2^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_1}\right)\gamma_2 + \left(\frac{\alpha_2\gamma_3}{1-\alpha_1}\right)\left(\frac{1}{w_2}\right) - \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1\ddot{h}_1 + I}{w_2}\right). \tag{9}$$

In terms of expected earnings, labor supply to job 2 would simply be

$$w_2 h_2^* = \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_1}\right)\gamma_2 w_2 + \left(\frac{\alpha_2\gamma_3}{1-\alpha_1}\right) - \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(w_1\ddot{h}_1 + I\right). \tag{10}$$

Constrained unitary job holder

For a constrained unitary job holder, the hours worked $(\ddot{h}_1)$ are treated as exogenous so there is no corresponding labor supply equation.

4 Data

The estimation of our model is conducted using data from the British Household Panel Survey (BHPS). The survey began in 1991 with a sample of some 5,500 household randomly drawn from all areas of Great Britain. To this initial sample, an over-sample of 1,500 households form Scotland and Wales was added in 1999 and a sample of 2,000 households from Northern Ireland was added in 2001. Individuals are followed over time through an annual questionnaire. The survey asked whether in the month preceding the interview the respondent had worked on a second job. The sample is restricted to prime age working men (age 18 to 65) who are not enrolled in school, to avoid standard selection problems associated with the labor supply decision of women or individuals eligible for retirement. We also exclude self-employed individuals on the main job. For the purposes of the BHPS, the main job for a dual job holder is the job in which one works the most hours.

Importantly for the scope of this study, BHPS contains information about the presence of an hours constraint on the main job. Specifically respondents were asked whether they would have liked to work more, less, or the same hours assuming that they would be paid the same amount per hour. Since this question was asked directly after respondents reported their hours of work on the main job, we interpret the answer to this question as an indicator for an hours constraint on the main job. Accordingly for each type of job holder (unitary or dual) we can identify if he is constrained on the main job. In the end we have 6 possible cases: 1) unitary job holders who work their desired amount of hours on the main job - unconstrained unitary job holders; 2) unitary job holders who would have liked to work more hours on the main job (under-employed); 3) unitary job holders who would have liked to work less hours on the main job (over-employed); 4) dual job holders who work their desired hours on the main job - unconstrained dual job holders; 5) dual job holders who are under-employed on their main job; 6) dual job holders who are over-employed on their main job.

While most of the variables are straightforward, some may require explanation about how they were constructed. The gross wage rate was calculated by dividing the monthly earnings by the usual hours worked on the relevant job times four. This procedure was preferred to the self-reported information on the hourly wage rate because it guarantees internal consistency between the estimation of the hours and the earnings equations. To mitigate the effect of outliers, we deleted from our sample individuals who earn less than £1/hour or more than £100/hr. Moreover we drop individuals with reported hours of work outside the 1 to 99 percentile of the distribution of hours worked in the sample. Non-labor income is the sum of all state benefits (including pension), money transfer, and income from rent, saving, and investment.

We determine an individual's marginal income tax rate (τ_{it}) based on the information we have on weekly earnings coupled with the personal tax allowances and tax rate bands in effect in the UK for each year in our sample period. If we let W_{mit} refer to nominal gross wage rates, the tax-rate adjusted nominal wage rate for each job is simply $(1-\tau_{it})\, W_{mit}$, $m=1,2$. Changes in British tax law after 1993 treat dividend and savings income differently from other forms of non-labor income. Since we are unable to identify the separate components of non-labor income, we apply the dividend and savings marginal tax rates to the entire amount of non-labor income. If we let NLI_{it} represent nominal gross non-labor income, the tax-rate adjusted nominal non-labor income is given by $(1-\tau_{it}^{I})NLI_{it}$. To make things concrete, we model weekly hours of labor supply with hourly wage rates corrected for inflation and marginal income tax rates.

Real non-labor and total income are measured on a weekly basis. Individual wage rates and non-labor income corrected for inflation and tax rates are calculated according to

$$\begin{aligned} w_{mit} &= \frac{W_{mit}\,(1-\tau_{it}^{w})}{P_t},\ m=1,2 \\ I_{it} &= \frac{(1-\tau_{it}^{I})NLI_{it}}{P_t} \end{aligned}$$

where P_t is a cost of living deflator for period t[2].

Table 1 reports means for variables in our analyses. After excluding observations with missing data for any variables in the models, we are left with a total of 44,921 observations. We have complete information on dual job holder observations in 2,785 cases, which account for about 6% of all the observations in our estimation sample. Almost 60% of dual job holding episodes are associated with no hours constraints on the main job; another 31% of the dual job holding episodes is associated with workers who are over-employed on the main job, and the remaining 9% of dual job holding episodes is associated with workers who are under-employed on the main job. This last result indicates that the usual explanation for holding two jobs, i.e. the need to fulfill an unmet hours target on the main job, does not seem to fit well with the stylized facts in the UK. Moreover, the under-employed hours (constrained) model cannot explain why

[2]Our adjustment for taxes is a simple approximation that treats marginal rates as proportional in order to obtain local after tax wage rates and income. Hence, our analysis does not attempt to incorporate the kinked budget arising from a graduated income tax system.

so many dual job holders actually desired to work less hours on the main job: if a worker is over-employed on the main job, why would he take a second job? The job portfolio model offers a reasonable explanation for this finding. Jobs are heterogeneous for a variety of reasons and that is why some workers prefer to allocate their desired hours of work over a portfolio of jobs. Long contractual hours on the main job may actually bring a worker above the desired equilibrium hours of work. Although the portfolio model seems to better serve the stylized facts in the UK, one cannot disregard that only 6 percent of the sample of unitary job episodes consider themselves under-employed. This is somewhat lower than the incidence of under-employment among dual job holders, thus suggesting that individuals do take a second job in response to a situation of under-employment on the main job.

Not surprisingly, we find that under-employed individuals work less hours on the main job than unconstrained workers while over-employed individuals work more hours on the main job than unconstrained workers. Conditional on being constrained, on average dual job holders work less hours on their main job, but after adding the hours supplied on the second job, dual job holders work more total hours per week. The average hourly wage on the main job for unitary job holders is higher than for dual job holders. For dual job holders, the hourly wage rate on the second job is higher than on the main job, although the weekly earnings on the main job are higher because individuals work more hours on the main job. Neither the portfolio nor the constrained labor supply regimes impose any restrictions on the relative magnitudes of the rates of pay between the two jobs. Compared with their unconstrained fellow workers, the average hourly wage of under-employed workers is lower and the average hourly wage of over-employed workers is higher.

Unitary and dual job holders differ on a number of socioeconomic dimensions. For example, dual job holders tend to be younger than unitary job holders and less likely to be married. Moreover, under-employed workers (both unitary and dual job holders) seem to be less educated than the other two classes of workers: only 19% of under-employed unitary job holders and 17% of under-employed dual job holders have some degree above the A level.[3] This rate is higher for over-employed workers (both unitary and dual job holders) at 29%. Hence, it could well be that some underlying selection process determines whether a workers falls into one of the six categories in a systematic way.

Table 2 reports an unconditional transition probability matrix over the period of our study.

[3]'A' level education represents 13 years of education/upper secondary school.

We add a row and column corresponding to moving into the sample from out of the sample and moving out of the sample from within the sample. For any given period, those with the highest probability of moving out of the sample the next period were over-employed dual job holders (over_dual) with a probability of 0.29. On the other hand, among those who move into the sample in any given period, the probability is the highest (0.55) that they would fall into the category of unconstrained unitary job holders (unc_unit). While the probability of remaining in one's current regime exceeds the probabilities of moving to any other labor supply regime for most regimes, two exceptions are found among under-employed workers (under_unit & under_dual). For example the probabilities that under-employed workers – under_unit and under_daul – would transition in the next period to being unconstrained unitary job holders (unc_unit) are 0.38 and 0.20 versus remaining in the same regimes with probabilities 0.22 and 0.12, respectively. Other noteworthy transitions are unconstrained unitary job holders (unc_unit) becoming over-employed unitary job holders (over_unit) with probability 0.17 and over-employed unitary job holders (under_unit) transitioning to unconstrained unitary job holders (unc_unit) with probability 0.26. Similarly, the probability is 0.20 that an unconstrained dual job holder (unc_dual) would transition to an unconstrained unitary job holder (unc_unit). These are of course raw transitions, but they point to the desirability of specifying a single unifying utility function for workers across all labor supply outcomes. In the empirical model below we treat the placement of workers into labor supply regimes as the result of a multinomial logit selection process with unobserved heterogeneity.

5 Empirical Model

To introduce the stochastic element in the model, one can think of $w_m h_m^*$ and y^* as planned earnings and income. The relationship between actual labor supply earnings, $w_m h_m$ from job m, and planned earnings, $w_m h_m^*$, is given by $w_m h_m = w_m h_m^* + v_m$, where v_m is a random error term. The relationship between actual income, y, and planned income, y^*, may be expressed as

$y^* = y - v_y$, where $y = \sum_{m=1}^{2} w_m h_m + I$. Note that we can solve for v_y from

$$\begin{aligned} v_y &= y - y^* \\ &= \left(\sum_{m=1}^{2} w_m h_m + I \right) - \left(\sum_{m=1}^{2} w_m h_m^* + I \right) \\ &= \sum_{m=1}^{2} w_m \left(h_m - h_m^* \right) \\ &= \sum_{m=1}^{2} v_m. \end{aligned}$$

Because constrained hours on job 1 are treated as exogenous, we do not estimate corresponding labor supply functions in these cases. This leaves us with five labor supply functions to estimate that span four selection regimes: h_1 and h_2 for unconstrained dual job holding - case (1), h_1 for unconstrained unitary job holding - case (2), h_2 for under-employed dual job holders on job 1 - case (3), and h_2 for over-employed dual hob holders on job 1 - case (4). Hours are measured as hours per week, wages are measured as hourly wage rates, and non-labor and total income are measured on a weekly basis. All monetary variables are expressed in terms of 2008 prices. Following Renna et al. (2013), our empirical estimation is conducted for the earnings version of the Stone-Geary labor supply model.

Our analysis extends the sample selection approaches of Lee (1983), Wooldridge(1995; 2010), and Dustmann and Rochina-Barrachina (2007) to multivariate selection in a panel data setting. The first stage of our panel data estimation of the dual labor supply model is estimation of a pooled multinomial logit. Let s_{it} represent a variable that assumes the values $0, 1, ..., 5$ corresponding to the six job holding outcomes. We can equivalently define indicator variables corresponding to these six labor supply outcomes: $s_{itj} = 1 [s_{it} = j]$. Following Wooldridge (2010, pp.653-654), we assume that $P(s_{it} = j \mid x_{it}, c_i) = P(s_{it} = j \mid x_i, c_i)$, $j = 0, 1, ..., 5$, where x_{it} is a vector of the explanatory variables (which can include time invariant variables), x_i is the vector of the means of the variables x_{it} for the ith individual, and c_i is unobserved heterogeneity. A simplifying assumption that permits averaging out of the c_i terms is that $P(s_{it} = j \mid x_i) = P(s_{it} = j \mid x_{it}, \bar{\omega}_i)$, where $\bar{\omega}_i$ is a vector of means of the variables in x_{it} that are either time-varying or time-invariant.

The universally available variables for our model are specified by

$$x_{it} = (1, w_{1it}, I_{it}, \text{Age}_{it}, \text{Educ}_{it}, \text{MS}_{it}, \text{DP}_{it}, \text{Year}_{it})$$
$$\bar{\omega}_i = \left(\bar{w}_{1i}, \bar{I}\,_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i, \overline{\text{Year}}_i\right),$$

where Age is the individual's age, Educ is a vector of educational attainment dummy variables, MS is marital status (= 1 if married), DP is the number of dependent children, and Year is a set of year indicator variables. Given the above assumptions, our multinomial logit selection model generates probabilities according to

$$P_{jit} = P\left(s_{it} = j \mid x_{it}, \bar{\omega}_i\right), j = 1, ..., 5$$
$$= \Lambda\left(x_{it}, \bar{\omega}_i, \beta_j\right)$$
$$P_{0it} = 1 -_{j=1}^{5} P_{ijt},$$

where β_j is the multinomial logit parameter vector for outcome j.

Let $z_{jit} = \Phi^{-1}\left(P_{jit}\right)$, where Φ^{-1} is the inverse standard normal CDF. It is clear that $\Phi\left(z_{jit}\right) = P_{jit} = \Lambda\left(x_{it}, \bar{\omega}_i, \beta_j\right)$. Accordingly, we construct the appropriate Inverse Mill Ratio (IMR) variables $\lambda_{jit} = \dfrac{\phi\left(z_{jit}\right)}{\Phi\left(z_{jit}\right)}$ that will be added as regressors in the five labor supply equations.

We estimate the Stone-Geary model's boundary parameters γ_1, γ_2, and γ_3 directly from our panel data sample. Let $\tilde{\gamma}_1$ be the highest integer value that satisfies $h_1^{\max} < \tilde{\gamma}_1 \leq 1 + h_1^{\max}$ for the combined samples for all workers who work job 1 over all periods; let $\tilde{\gamma}_2$ be the highest integer value that satisfies $h_2^{\max} < \tilde{\gamma}_2 \leq 1 + h_2^{\max}$ for the combined samples for all workers who work job 2 over all periods; and let $\tilde{\gamma}_3$ be the lowest integer value that satisfies $y^{\min} - 1 \leq \tilde{\gamma}_3 < y^{\min}$ for the combined samples for all workers over all periods, where $h_m^{\max}$ is the maximum observed hours of work for job m and $y^{\min}$ is the lowest observed income.

Let v_{mlit} represent the sum of an unobserved individual effect for labor supply and an idiosyncratic error term, where $m = 1, 2$ for job 1 or job 2, $l = 1, 2, 3, 4$ indexes the four labor supply selection regimes, $i = 1, ..., n$, and $t = 1, ..., T$. The error structure for each labor supply regime can be characterized by (see Wooldridge 2010, pp.832-837)

$$v_{mlit} = \theta_{ml}\lambda_{mlit} + \bar{Z}_{li}\pi_{ml} + u_{mlit}.$$

where $\bar{Z}_{li}$ is a vector of time averaged variable means for all individuals in regime l, π_{ml} is a conforming parameter vector, $u_{mlit} = v_{mli} - E\left(v_{mlit}|q_{mlit}, \lambda_{mlit}, \bar{Z}_{li}\right)$, and q_{mlit} is a labor supply variable (defined below) arising from maximization of the Stone-Geary utility function.

The labor supply equations are jointly estimated by pooled, non-linear Seemingly Unrelated Regressions (SUR) with cross-equation restrictions on the parameters α_1 and α_2. In practice the Inverse Mills Ratios (IMR's) are replaced by their estimated values $\hat{\lambda}_{mlit}$ obtained from the multinomial logit model and the standard errors are bootstrapped.[4] The labor supply elasticities based on the estimated model will include not only those arising from the Stone-Geary utility function but also those that involve the selection terms and unobserved heterogeneity. In order to focus only local period effects of marginal changes in wage rates and non-labor income, we hold constant the time averaged sample means of these variables.

The empirical labor supply functions and elasticities are specified below[5].

<u>Unconstrained dual job holders</u>

$$w_{1it}\left(h_{1it} - \tilde{\gamma}_1\right) = \alpha_1 q_{1it} + \theta_{11}\hat{\lambda}_{1it} + \bar{Z}_{1i}\pi_{11} + \ u_{11it} \tag{11}$$

$$w_{2it}\left(h_{2it} - \tilde{\gamma}_2\right) = \alpha_2 q_{1it} + \theta_{21}\hat{\lambda}_{1it} + \bar{Z}_{1i}\pi_{21} + \ u_{21it}. \tag{12}$$

where

$$\begin{aligned} q_{11it} = q_{21it} &= q_{1it} \\ &= \tilde{\gamma}_3 - \tilde{\gamma}_1 w_{1it} - \tilde{\gamma}_2 w_{2it} - I_{it}, \end{aligned}$$

$$\bar{Z}_{1i} = \left(\bar{w}_{1i}, \bar{w}_{2i}, \bar{I}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i, \overline{\text{Year}}_i\right),$$

and π_{11} and π_{21} are the corresponding parameter vectors.

The implied hours equation for job 1 from the estimated earnings equation (11) is given by

$$\hat{h}_{1it} = \left(1 - \hat{\alpha}_1\right)\tilde{\gamma}_1 + \hat{\alpha}_1\left(\frac{\tilde{\gamma}_3 - \tilde{\gamma}_2 w_{2it} - I_{it}}{w_{1it}}\right) + \hat{\theta}_{11}\frac{\hat{\lambda}_{1it}}{w_{1it}} + \frac{\bar{Z}_{1i}\hat{\pi}_{11}}{w_{1it}}. \tag{13}$$

[4]Estimated standard errors reported in the paper are bootstrap estimates from 200 replications that account for all estimation steps, including the estimation of multinomial logit regression and boundary parameters.

[5]For the sample mean values used to correct for unobserved heterogeneity in the labor supply equations, we average only over the time-series for which the individual was in the particular labor supply regime.

We express the total labor supply effect of w_{1it} as

$$\left(\frac{\partial \hat{h}_{1it}}{\partial w_{1it}}\right)^{T} = \left(\frac{\partial \hat{h}_{1it}}{\partial w_{1it}}\right)^{SG} + \hat{\theta}_{11}\left[\frac{1}{w_{1it}}\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}} - \frac{\hat{\lambda}_{1it}}{(w_{1it})^{2}}\right] - \frac{\bar{Z}_{1i}\hat{\pi}_{11}}{(w_{1it})^{2}}$$

where $\left(\frac{\partial \hat{h}_{1it}}{\partial w_{1it}}\right)^{SG}$ is the slope of the uncompensated labor supply curve obtained from the Stone-Geary utility function. Accordingly, the total labor supply elasticity with respect to w_{1it} is obtained from

$$\begin{aligned}\hat{\eta}^{T}_{11it} &= \left(\frac{w_{1it}}{\hat{h}_{1it}}\right)\left(\frac{\partial \hat{h}_{1it}}{\partial w_{1it}}\right)^{T} \\ &= \hat{\eta}_{11it} + \hat{\theta}_{11}\left[\frac{1}{\hat{h}_{1it}}\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}} - \frac{\hat{\lambda}_{1it}}{w_{1it}\hat{h}_{1it}}\right] - \frac{\bar{Z}_{1i}\hat{\pi}_{11}}{w_{1it}\hat{h}_{1it}}\end{aligned}$$

where $\hat{\eta}_{11it} = \hat{\eta}^{c}_{11it} + \hat{\epsilon}_{11\mathrm{I}}$ is the estimated Stone-Geary uncompensated labor supply elasticity with respect to w_{1it}, $\hat{\eta}^{c}_{11it}$ is the compensated labor supply elasticity with respect to w_{1it} (evaluated at $\hat{h}_{1it}$) , and $\hat{\epsilon}_{11\mathrm{I}}$ is the own wage income effect elasticity. The term $\hat{\theta}_{11}\left[\frac{1}{\hat{h}_{1it}}\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}} - \frac{\hat{\lambda}_{1it}}{w_{1it}\hat{h}_{1it}}\right]$ captures the job 1 labor supply elasticity effects of w_{1it} on the probability of being an unconstrained dual job holder. Although w_{1it} does not directly affect the controls for unobserved heterogeneity, it does impact the effect of unobserved heterogeneity on job 1 labor supply via the term $-\frac{\bar{Z}_{1i}\hat{\pi}_{11}}{w_{1it}\hat{h}_{1it}}$.

The total cross labor supply effect of w_{2it} is identical to the uncompensated cross wage effect of w_{2it} on labor supply to job 1 from the Stone-Geary utility function:

$$\left(\frac{\partial \hat{h}_{1it}}{\partial w_{2it}}\right)^{T} = \left(\frac{\partial \hat{h}_{1it}}{\partial w_{2it}}\right)^{SG}.$$

Therefore, the total labor supply elasticity for job 1 with respect to w_{2it} is the same as the estimated Stone-Geary uncompensated cross wage elasticity:

$$\begin{aligned}\hat{\eta}^{T}_{12it} &= \left(\frac{w_{2it}}{\hat{h}_{1it}}\right)\left(\frac{\partial \hat{h}_{1it}}{\partial w_{2it}}\right)^{T} \\ &= \hat{\eta}_{12it},\end{aligned}$$

where $\hat{\eta}_{12it} = \hat{\eta}^{c}_{12it} + \hat{\epsilon}_{12\mathrm{I}}$, $\hat{\eta}^{c}_{12it}$ is the compensated cross substitution effect elasticity, and $\hat{\epsilon}_{12\mathrm{I}}$ is the cross wage income effect elasticity.

The total pure income effect of non-labor income on labor supply to job 1 is obtained as

$$\left(\frac{\partial \hat{h}_{1it}}{\partial I_{it}}\right)^{T} = \left(\frac{\partial \hat{h}_{1it}}{\partial I_{it}}\right)^{SG} + \left(\frac{\hat{\theta}_{11}}{w_{1it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}}\right)$$

where $\left(\frac{\partial \hat{h}_{1it}}{\partial I_{it}}\right)^{SG}$ is the Stone-Geary pure income effect for labor supply to job 1. Accordingly, the total labor supply pure income effect elasticity is obtained from

$$\begin{aligned}\hat{\eta}^{T}_{1Iit} &= \left(\frac{I_{it}}{\hat{h}_{1it}}\right)\left(\frac{\partial \hat{h}_{1it}}{\partial I_{it}}\right)^{T} \\ &= \hat{\eta}_{1Iit} + \left(\frac{\hat{\theta}_{11} I_{it}}{w_{1it}\hat{h}_{1it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}}\right)\end{aligned}$$

where $\hat{\eta}_{1Iit}$ is the Stone-Geary pure income effect elasticity for job 1. The term $\left(\frac{\hat{\theta}_{11} I_{it}}{w_{1it}\hat{h}_{1it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}}\right)$ captures the job 1 labor supply elasticity effects of I_{it} on the probability of being an unconstrained dual job holder.

The implied hours equation for job 2 obtained from the estimated earnings equation (12) is given by

$$\hat{h}_{2it} = (1-\hat{\alpha}_2)\,\tilde{\gamma}_2 + \hat{\alpha}_2\left(\frac{\tilde{\gamma}_3 - \tilde{\gamma}_1 w_{1it} - I_{it}}{w_{2it}}\right) + \hat{\theta}_{21}\frac{\hat{\lambda}_{1it}}{w_{2it}} + \frac{\bar{Z}_{1i}\hat{\pi}_{21}}{w_{2it}}. \tag{14}$$

Similarly as in job 1, we express the total labor supply effect of w_{2it} on job 2 as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\right)^{T} = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\right)^{SG} - \frac{\hat{\theta}_{21}\hat{\lambda}_{1it}}{(w_{2it})^2} - \frac{\bar{Z}_{1i}\hat{\pi}_{21}}{(w_{2it})^2},$$

where $\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\right)^{SG}$ is the slope of the uncompensated labor supply curve for job 2 obtained from the Stone-Geary utility function. Accordingly, the total job 2 labor supply elasticity with respect to w_{2it} is obtained from

$$\begin{aligned}\hat{\eta}^{T}_{22it} &= \left(\frac{w_{2it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\right)^{T} \\ &= \hat{\eta}_{22it} - \frac{\hat{\theta}_{21}\hat{\lambda}_{1it}}{w_{2it}\hat{h}_{2it}} - \frac{\bar{Z}_{1i}\hat{\pi}_{21}}{w_{2it}\hat{h}_{2it}}\end{aligned}$$

where $\hat{\eta}_{22it} = \hat{\eta}^{c}_{22it} + \hat{\epsilon}_{22\mathrm{I}}$ is the estimated Stone-Geary uncompensated job 2 labor supply elasticity with respect to w_{2it}, $\hat{\eta}^{c}_{22it}$ is the compensated labor supply elasticity with respect to w_{2it} (evaluated at $\hat{h}_{2it}$) , and $\hat{\epsilon}_{22\mathrm{I}}$ is the own wage income effect elasticity. Although w_{2it} does not affect the probability of a worker being an unconstrained dual job holder, it does affect the impact of selection on job 2 labor supply via the term $-\dfrac{\hat{\theta}_{21}\hat{\lambda}_{1it}}{w_{2it}\hat{h}_{2it}}$. Similarly, w_{2it} impacts the effect of unobserved heterogeneity on job 2 labor supply via the term $\dfrac{\bar{Z}_{1i}\hat{\pi}_{21}}{w_{2it}\hat{h}_{2it}}$.

The total cross-labor supply effect of w_{1it} on job 2 labor supply is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\right)^{T} = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\right)^{SG} + \left(\frac{\hat{\theta}_{21}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}}\right),$$

where $\left(\dfrac{\partial \hat{h}_{2it}}{\partial w_{1it}}\right)^{SG}$ is the cross-wage effect for the uncompensated job 2 labor supply curve obtained from the Stone-Geary utility function. Therefore, the total cross-labor supply elasticity of w_{1it} on job 2 labor supply is obtained as

$$\begin{aligned}\hat{\eta}^{T}_{21it} &= \left(\frac{w_{1it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\right)^{T} \\ &= \hat{\eta}_{21it} + \left(\frac{\hat{\theta}_{21} w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}}\right),\end{aligned}$$

where $\hat{\eta}_{21it} = \hat{\eta}^{c}_{21it} + \hat{\epsilon}_{21\mathrm{I}}$, $\hat{\eta}^{c}_{21it}$ is the compensated cross substitution effect elasticity, and $\hat{\epsilon}_{21\mathrm{I}}$ is the cross wage income effect elasticity. The term $\left(\dfrac{\hat{\theta}_{21} w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\dfrac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}}\right)$ captures the job 2 labor supply elasticity effects of w_{1it} on the probability of being an unconstrained dual job holder.

The total pure income effect of non-labor income on labor supply to job 2 is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\right)^{T} = \left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\right)^{SG} + \left(\frac{\hat{\theta}_{21}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}}\right)$$

where $\left(\dfrac{\partial \hat{h}_{2it}}{\partial I_{it}}\right)^{SG}$ is the Stone-Geary pure income effect for labor supply to job 2. Accordingly, the total labor supply pure income effect elasticity is obtained from

$$\begin{aligned}\hat{\eta}^{T}_{2Iit} &= \left(\frac{I_{it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\right)^{T} \\ &= \hat{\eta}_{2Iit} + \left(\frac{\hat{\theta}_{21} I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}}\right)\end{aligned}$$

where $\hat{\eta}_{2Iit}$ is the Stone-Geary pure income effect elasticity for job 2. The term $\left(\frac{\hat{\theta}_{21}I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial\hat{\lambda}_{1it}}{\partial I_{it}}\right)$ captures the job 2 labor supply elasticity effects of I_{it} on the probability of being an unconstrained dual job holder.

Unconstrained unitary job holders

$$w_{1it}\left(h_{1it}-\tilde{\gamma}_1\right)=\left(\frac{\alpha_1}{1-\alpha_2}\right)q_{12it}+\theta_{12}\hat{\lambda}_{2it}+\bar{Z}_{2i}\pi_{12}+\ u_{12it} \tag{15}$$

where

$$q_{12it}=\tilde{\gamma}_3-\tilde{\gamma}_1 w_{1it}-I_{it},$$

$$\bar{Z}_{2i}=\left(\bar{w}_{1i},\bar{I}_i,\overline{\text{Age}}_i,\overline{\text{Educ}}_i,\overline{\text{MS}}_i,\overline{\text{DP}}_i,\overline{\text{Year}}_i\right),$$

and π_{12} is the corresponding parameter vector.

The implied hours equation for an unconstrained unitary job holder is obtained from the estimated earnings equation (15):

$$\hat{h}_{1it}\,|_{h_2=0}=\left(\frac{1-\hat{\alpha}_1-\hat{\alpha}_2}{1-\hat{\alpha}_2}\right)\tilde{\gamma}_1-\left(\frac{\hat{\alpha}_1}{1-\hat{\alpha}_2}\right)\left(\frac{I_{it}-\tilde{\gamma}_3}{w_{1it}}\right)+\hat{\theta}_{12}\frac{\hat{\lambda}_{2it}}{w_{1it}}+\frac{\bar{Z}_{2i}\hat{\pi}_{12}}{w_{1it}}. \tag{16}$$

We express the total labor supply effect of w_{1it} as

$$\left(\frac{\partial\hat{h}_{1it}}{\partial w_{1it}}\,|_{h_2=0}\right)^T=\left(\frac{\partial\hat{h}_{1it}}{\partial w_{1it}}\,|_{h_2=0}\right)^{SG}+\hat{\theta}_{12}\left[\frac{1}{w_{1it}}\frac{\partial\hat{\lambda}_{2it}}{\partial w_{1it}}-\frac{\hat{\lambda}_{2it}}{\left(w_{1it}\right)^2}\right]-\frac{\bar{Z}_{2i}\hat{\pi}_{12}}{\left(w_{1it}\right)^2},$$

where $\left(\frac{\partial\hat{h}_{1it}}{\partial w_{1it}}\,|_{h_2=0}\right)^{SG}$ is the slope of the uncompensated labor supply curve obtained from the Stone-Geary utility function. Accordingly, the total labor supply elasticity with respect to w_{1it} is obtained from

$$\begin{aligned}\hat{\eta}^T_{11it}\,|_{h_2=0} &= \left(\frac{w_{1it}}{\hat{h}_{1it}}\right)\left(\frac{\partial\hat{h}_{1it}}{\partial w_{1it}}\,|_{h_2=0}\right)^T\\ &= \hat{\eta}_{11it}\,|_{h_2=0}+\hat{\theta}_{12}\left[\frac{1}{\hat{h}_{1it}}\frac{\partial\hat{\lambda}_{2it}}{\partial w_{1it}}-\frac{\hat{\lambda}_{2it}}{w_{1it}\hat{h}_{1it}}\right]-\frac{\bar{Z}_{2i}\hat{\pi}_{12}}{w_{1it}\hat{h}_{1it}},\end{aligned}$$

where $\hat{\eta}_{11it}\,|_{h_2=0}=\hat{\eta}^c_{11it}\,|_{h_2=0}+\hat{\epsilon}_{11\text{I}}\,|_{h_2=0}$ is the estimated Stone-Geary uncompensated labor supply elasticity with respect to w_{1it}, $\hat{\eta}^c_{11it}\,|_{h_2=0}$ is the compensated labor supply elasticity with respect to w_{1it} (evaluated at $\hat{h}_{1it}$) , $\hat{\epsilon}_{11\text{I}}\,|_{h_2=0}$ is the own wage income effect elasticity. The term $\hat{\theta}_{12}\left[\frac{1}{\hat{h}_{1it}}\frac{\partial\hat{\lambda}_{2it}}{\partial w_{1it}}-\frac{\hat{\lambda}_{2it}}{w_{1it}\hat{h}_{1it}}\right]$ captures the job 1 labor supply elasticity effects of w_{1it} on the probability of being an unconstrained dual job holder. w_{1it} impacts the effect of unobserved heterogeneity on job 1 labor supply via the term $-\frac{\bar{Z}_{2i}\hat{\pi}_{12}}{w_{1it}\hat{h}_{1it}}$.

The total pure income effect of non-labor income on labor supply to job 1 is obtained as

$$\left(\frac{\partial\hat{h}_{1it}}{\partial I_{it}}\,|_{h_2=0}\right)^T=\left(\frac{\partial\hat{h}_{1it}}{\partial I_{it}}\,|_{h_2=0}\right)^{SG}+\left(\frac{\hat{\theta}_{12}}{w_{1it}}\right)\left(\frac{\partial\hat{\lambda}_{2it}}{\partial I_{it}}\right)$$

where $\left(\frac{\partial\hat{h}_{1it}}{\partial I_{it}}\,|_{h_2=0}\right)^{SG}$ is the Stone-Geary pure income effect for labor supply. Accordingly, the total labor supply pure income effect elasticity is obtained from

$$\begin{aligned}\hat{\eta}^T_{1Iit}\,|_{h_2=0}&=\left(\frac{I_{it}}{\hat{h}_{1it}}\right)\left(\frac{\partial\hat{h}_{1it}}{\partial I_{it}}\,|_{h_2=0}\right)^T\\&=\hat{\eta}_{1Iit}\,|_{h_2=0}+\left(\frac{\hat{\theta}_{12}I_{it}}{w_{1it}\hat{h}_{1it}}\right)\left(\frac{\partial\hat{\lambda}_{2it}}{\partial I_{it}}\right)\end{aligned}$$

where $\hat{\eta}_{1Iit}\,|_{h_2=0}$ is the Stone-Geary pure income effect elasticity. The term $\left(\frac{\hat{\theta}_{12}I_{it}}{w_{1it}\hat{h}_{1it}}\right)\left(\frac{\partial\hat{\lambda}_{2it}}{\partial I_{it}}\right)$ captures the labor supply elasticity effects of I_{it} on the probability of being an unconstrained unitary job holder.

Constrained dual job holders

Constrained dual job holders desiring either fewer or more hours:

$$w_{2it}\left(h_{2it}-\tilde{\gamma}_2\right)=\left(\frac{\alpha_2}{1-\alpha_1}\right)q_{2it}+\theta_{23}\hat{\lambda}_{23it}+\bar{Z}_{3i}\pi_{23}+u_{23it}\text{ (overemployed)}\tag{17}$$

$$w_{2it}\left(h_{2i}-\tilde{\gamma}_2\right)=\left(\frac{\alpha_2}{1-\alpha_1}\right)q_{2it}+\theta_{24}\hat{\lambda}_{24it}+\bar{Z}_{3i}\pi_{24}+u_{24it}\text{ (underemployed).}\tag{18}$$

where

$$\begin{aligned}q_{23it}&=q_{24it}=q_{2it}\\&=\tilde{\gamma}_3-\tilde{\gamma}_2w_{2it}-\left(w_{1it}\ddot{h}_{1it}+I_{it}\right),\end{aligned}$$

$$\bar{Z}_{3i}=\left(\bar{w}_{2i},\overline{w_{1i}\ddot{h}_{1i}},\bar{I}_i,\overline{\text{Age}}_i,\overline{\text{Educ}}_i,\overline{\text{MS}}_i,\overline{\text{DP}}_i,\overline{\text{Year}}_i\right)\,(\mathit{overemployed}),$$

$$\bar{Z}_{4i} = \left(\bar{w}_{2i}, \overline{w_{1i}\ddot{h}_{1i}}, \bar{I}_i, \overline{\text{Age}}_i, \overline{\text{Educ}}_i, \overline{\text{MS}}_i, \overline{\text{DP}}_i, \overline{\text{Year}}_i\right) (underemployed),$$

$\ddot{h}_{1it}$ is the constrained hours on job 1, and π_{23} and π_{24} are the corresponding parameter vectors.

For <u>overemployed</u> workers the implied hours equation for job 2 is obtained from the estimated earnings equation (17):

$$\hat{h}_{2it}\Big|_{h_1=\ddot{h}_1>h_1^*} = \left(\frac{1-\hat{\alpha}_1-\hat{\alpha}_2}{1-\hat{\alpha}_1}\right)\tilde{\gamma}_2 - \left(\frac{\hat{\alpha}_2}{1-\hat{\alpha}_1}\right)\left(\frac{w_{1it}\ddot{h}_{1it}+I_{it}-\tilde{\gamma}_3}{w_{2it}}\right) + \frac{\hat{\theta}_{23}\hat{\lambda}_{23it}}{w_{2it}} + \frac{\bar{Z}_{3i}\hat{\pi}_{23}}{w_{2it}}. \tag{19}$$

We express the total labor supply effect of w_{2it} on job 2 as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG} - \frac{\hat{\theta}_{23}\hat{\lambda}_{23it}}{(w_{2it})^2} - \frac{\bar{Z}_{3i}\hat{\pi}_{23}}{(w_{2it})^2},$$

where where $\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG}$ is the slope of the uncompensated labor supply curve for job 2 obtained from the Stone-Geary utility function. Accordingly, the total job 2 labor supply elasticity with respect to w_{2it} is obtained from

$$\begin{aligned}\hat{\eta}^T_{22it}\Big|_{h_1=\ddot{h}_1>h_1^*} &= \left(\frac{w_{2it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T \\ &= \hat{\eta}_{22it}\Big|_{h_1=\ddot{h}_1>h_1^*} - \frac{\hat{\theta}_{23}\hat{\lambda}_{23it}}{w_{2it}\hat{h}_{2it}} - \frac{\bar{Z}_{3i}\hat{\pi}_{23}}{w_{2it}\hat{h}_{2it}}\end{aligned}$$

where $\hat{\eta}_{22it}\Big|_{h_1=\ddot{h}_1>h_1^*} = \hat{\eta}^c_{22it}\Big|_{h_1=\ddot{h}_1>h_1^*} + \hat{\epsilon}_{22\text{I}}\Big|_{h_1=\ddot{h}_1>h_1^*}$ is the estimated Stone-Geary uncompensated job 2 labor supply elasticity with respect to w_{2it}, $\hat{\eta}^c_{22it}\Big|_{h_1=\ddot{h}_1>h_1^*}$ is the compensated labor supply elasticity with respect to w_{2it} (evaluated at $\hat{h}_{2it}\Big|_{h_1=\ddot{h}_1>h_1^*}$), and $\hat{\epsilon}_{22\text{I}}\Big|_{h_1=\ddot{h}_1>h_1^*}$ is the own wage income effect elasticity. The effects of selection and unobserved heterogeneity on job 2 labor supply are affected by changes in w_{2it} because the selection and unobserved heterogeneity terms in the job 2 labor supply function include w_{2it} in the denominator. After differentiating job 2 labor supply with respect to w_{2it} and converting to elasticities, the selection and unobserved heterogeneity effects are captured by $-\frac{\hat{\theta}_{23}\hat{\lambda}_{23it}}{w_{2it}\hat{h}_{2it}}$ and $-\frac{\bar{Z}_{3i}\hat{\pi}_{23}}{w_{2it}\hat{h}_{2it}}$, respectively.

Because hours are constrained for job 1, the wage rate for job 1 can only have income and selection effects on labor supply to job 2 but not substitution effects. The total cross-labor supply effect of w_{1it} on job 2 labor supply is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG} + \left(\frac{\hat{\theta}_{23}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial w_{1it}}\right),$$

where $\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG}$ is the cross-wage effect for the uncompensated job 2 labor supply curve obtained from the Stone-Geary utility function. Therefore, the total cross-labor supply elasticity of w_{1it} on job 2 labor supply is obtained as

$$\begin{aligned}\hat{\eta}^T_{21it}\Big|_{h_1=\ddot{h}_1>h_1^*} &= \left(\frac{w_{1it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T \\ &= \hat{\eta}_{21it}\Big|_{h_1=\ddot{h}_1>h_1^*} + \left(\frac{\hat{\theta}_{23}\ w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial w_{1it}}\right),\end{aligned}$$

where $\hat{\eta}_{21it}\Big|_{h_1=\ddot{h}_1>h_1^*} = \hat{\epsilon}_{21\mathrm{I}}\Big|_{h_1=\ddot{h}_1>h_1^*}$ is the estimated Stone-Geary uncompensated job 2 cross-labor supply elasticity, and $\left(\frac{\hat{\theta}_{23}\ w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial w_{1it}}\right)$ is the job 2 labor supply elasticity effects of w_{1it} on the probability of being a constrained over-employed dual job holder.

The total pure income effect of non-labor income on labor supply to job 2 is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG} + \left(\frac{\hat{\theta}_{23}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial I_{it}}\right)$$

where $\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^{SG}$ is the Stone-Geary pure income effect for labor supply to job 2. Accordingly, the total labor supply pure income effect elasticity is obtained from

$$\begin{aligned}\hat{\eta}^T_{2Iit}\Big|_{h_1=\ddot{h}_1>h_1^*} &= \left(\frac{I_{it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1>h_1^*}\right)^T \\ &= \hat{\eta}_{2Iit}\Big|_{h_1=\ddot{h}_1>h_1^*} + \left(\frac{\hat{\theta}_{23} I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial I_{it}}\right)\end{aligned}$$

where $\hat{\eta}_{2Iit}\Big|_{h_1=\ddot{h}_1>h_1^*}$ is the Stone-Geary pure income effect elasticity for job 2. The term $\left(\frac{\hat{\theta}_{23} I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{23it}}{\partial I_{it}}\right)$ captures the job 2 labor supply elasticity effects of I_{it} on the probability of being a constrained over-employed dual job holder.

For underemployed workers the implied hours equation for job 2 is obtained from the estimated earnings equation (18) as

$$\hat{h}_{2it}\Big|_{h_1=\ddot{h}_1<h_1^*} = \left(\frac{1-\hat{\alpha}_1-\hat{\alpha}_2}{1-\hat{\alpha}_1}\right)\tilde{\gamma}_2 - \left(\frac{\hat{\alpha}_2}{1-\hat{\alpha}_1}\right)\left(\frac{w_{1it}\ddot{h}_{1it}+I_{it}-\tilde{\gamma}_3}{w_{2it}}\right) + \hat{\theta}_{24}\hat{\lambda}_{24it} + \bar{Z}_{4i}\hat{\pi}_{24} \tag{20}$$

We express the total labor supply effect of w_{2it} on job 2 as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG} - \frac{\hat{\theta}_{24}\hat{\lambda}_{24it}}{(w_{2it})^2} - \frac{\bar{Z}_{4i}\hat{\pi}_{24}}{(w_{2it})^2},$$

where where $\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG}$ is the slope of the uncompensated labor supply curve for job 2 obtained from the Stone-Geary utility function. Accordingly, the total job 2 labor supply elasticity with respect to w_{2it} is obtained from

$$\begin{aligned}\hat{\eta}^T_{22it}\Big|_{h_1=\ddot{h}_1<h_1^*} &= \left(\frac{w_{2it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{2it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T \\ &= \hat{\eta}_{22it}\Big|_{h_1=\ddot{h}_1<h_1^*} - \frac{\hat{\theta}_{24}\hat{\lambda}_{24it}}{w_{2it}\hat{h}_{2it}} - \frac{\bar{Z}_{4i}\hat{\pi}_{24}}{w_{2it}\hat{h}_{2it}}\end{aligned}$$

where $\hat{\eta}_{22it}\Big|_{h_1=\ddot{h}_1<h_1^*} = \hat{\eta}^c_{22it}\Big|_{h_1=\ddot{h}_1<h_1^*} + \hat{\epsilon}_{22\mathrm{I}}\Big|_{h_1=\ddot{h}_1<h_1^*}$ is the estimated Stone-Geary uncompensated job 2 labor supply elasticity with respect to w_{2it}, $\hat{\eta}^c_{22it}\Big|_{h_1=\ddot{h}_1<h_1^*}$ is the compensated labor supply elasticity with respect to w_{2it} (evaluated at $\hat{h}_{2it}\Big|_{h_1=\ddot{h}_1<h_1^*}$), and $\hat{\epsilon}_{22\mathrm{I}}\Big|_{h_1=\ddot{h}_1<h_1^*}$ is the own wage income effect elasticity. The effects of selection and unobserved heterogeneity on job 2 labor supply are affected by changes in w_{2it} because the selection and unobserved heterogeneity terms in the job 2 labor supply function include w_{2it} in the denominator. After differentiating job 2 labor supply with respect to w_{2it} and converting to elasticities, the selection and unobserved heterogeneity effects are captured by $-\frac{\hat{\theta}_{24}\hat{\lambda}_{24it}}{w_{2it}\hat{h}_{2it}}$ and $-\frac{\bar{Z}_{4i}\hat{\pi}_{24}}{w_{2it}\hat{h}_{2it}}$, respectively.

Because hours are constrained for job 1, the wage rate for job 1 can only have income and selection effects on labor supply to job 2 but not substitution effects. The total cross-labor supply effect of w_{1it} on job 2 labor supply is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG} + \left(\frac{\hat{\theta}_{24}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial w_{1it}}\right),$$

where $\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG}$ is the cross-wage effect for the uncompensated job 2 labor supply curve obtained from the Stone-Geary utility function. Therefore, the total cross-labor supply

elasticity of w_{1it} on job 2 labor supply is obtained as

$$\hat{\eta}^T_{21it}\Big|_{h_1=\ddot{h}_1<h_1^*} = \left(\frac{w_{1it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial w_{1it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T$$
$$= \hat{\eta}_{21it}\Big|_{h_1=\ddot{h}_1<h_1^*} + \left(\frac{\hat{\theta}_{24}\ w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial w_{1it}}\right),$$

where $\hat{\eta}_{21it}\Big|_{h_1=\ddot{h}_1<h_1^*} = \hat{\epsilon}_{21\mathrm{I}}\Big|_{h_1=\ddot{h}_1<h_1^*}$ is the estimated Stone-Geary uncompensated job 2 cross-labor supply elasticity, and $\left(\frac{\hat{\theta}_{24}\ w_{1it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial w_{1it}}\right)$ is the job 2 labor supply elasticity effects of w_{1it} on the probability of being a constrained under-employed dual job holder.

The total pure income effect of non-labor income on labor supply to job 2 is obtained as

$$\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T = \left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG} + \left(\frac{\hat{\theta}_{24}}{w_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial I_{it}}\right)$$

where $\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^{SG}$ is the Stone-Geary pure income effect for labor supply to job 2. Accordingly, the total labor supply pure income effect elasticity is obtained from

$$\hat{\eta}^T_{2Iit}\Big|_{h_1=\ddot{h}_1<h_1^*} = \left(\frac{I_{it}}{\hat{h}_{2it}}\right)\left(\frac{\partial \hat{h}_{2it}}{\partial I_{it}}\Big|_{h_1=\ddot{h}_1<h_1^*}\right)^T$$
$$= \hat{\eta}_{2Iit}\Big|_{h_1=\ddot{h}_1<h_1^*} + \left(\frac{\hat{\theta}_{24}I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial I_{it}}\right)$$

where $\hat{\eta}_{2Iit}\Big|_{h_1=\ddot{h}_1<h_1^*}$ is the Stone-Geary pure income effect elasticity for job 2. The term $\left(\frac{\hat{\theta}_{24}I_{it}}{w_{2it}\hat{h}_{2it}}\right)\left(\frac{\partial \hat{\lambda}_{24it}}{\partial I_{it}}\right)$ captures the job 2 labor supply elasticity effects of I_{it} on the probability of being a constrained under-employed dual job holder.

6 Empirical Results

Table 3 reports the panel data multinomial logit model of selection into the six mutually exclusive labor supply regimes. Unconstrained unitary job holding is the reference labor supply outcome. Increases in the wage rate on job 1 lowers the odds that one would find themselves in any labor supply situation other than being an unconstrained unitary job holder, though only in the case of under-employed unitary job holders does this wage affect fail to achieve statistical significance. Age effects go in the same direction as the job 1 wage effects but is statistically significant only for being an under-employed unitary job holder. Number of children has a positive and statistically significant effect on the odds of finding oneself a constrained dual job holder, with the effect over twice as high for being under-employed compared with being over-employed. Both higher educational (1st degree) and lower educational (Certificate of Secondary Education) attainments reduce the odds of being an under-employed dual job holder relative to being an unconstrained unitary job holder. The main effects of being married is to raise the odds of being an over-employed unitary job holder and lower the odds of being an unconstrained dual job holder. Although not reported in Table 3, the time averaged covariates and year indicator variables were generally statistically significant in the estimated panel data multinomial logit model.

Estimates of the basic parameters of the labor supply model are reported in Table 4. As would be expected the boundary hours parameter for the second job (26 hours per week) is much less than on the main job (81 hours per week). The estimated values of α_1 and α_2 satisfy theoretical restrictions, i.e. they are positive and bounded on the unit interval. Furthermore, $\hat{\alpha}_2 > \hat{\alpha}_1$ implies that for a dual job holder utility is more responsive to changes in time not spent working on job 2 than to changes in time not spent working on job 1. In other words, leisure associated with reduced hours on the second job yields higher marginal utility than leisure associated with reduced hours on the main job. Only two of the five IMR θ parameter estimates are statistically significant. Individuals who are selected into working as unconstrained unitary job holders are types who have a propensity to work more hours. Those who are selected into working as unconstrained dual job holders are types who in job 2 have a propensity to earn more which means to work more hours given the assumption of exogeneity of wage rates for job 2. For the remaining labor supply regimes, we find no evidence of selection bias.

In Table 5 we report the estimated labor supply elasticities evaluated at the sample specific mean values of the variables corresponding to each of the labor supply regimes. Theoretical restrictions on the labor supply elasticities are satisfied in every case. There are no theoretical predictions for uncompensated own wage elasticities, but these turn out to be positive without exception. Because the substitution effects dominate the income effects, there is no incidence of backward bending Stone-Geary supply curves at the mean. In the case of the unconstrained unitary job holders, the income effect largely offsets the substitution effect so that the uncompensated labor supply elasticity is quite small. While allowing for selection and unobserved heterogeneity reverses the signs of the elasticities for unconstrained unitary job holders to be negative, they remain quite small and economically insignificant. Among dual job holders (both constrained and unconstrained), the labor supply elasticities for the second job are much larger in absolute value than those associated with the main job for unconstrained dual job holders. The total labor supply elasticities associated with job 2 are smaller, sometimes significantly so, than the inflated Stone-Geary labor supply elasticities. This finding follows from the fact that the sample selection and unobserved heterogeneity effects generally offset the Stone-Geary elasticities. For example, the Stone-Geary own wage labor supply elasticities on the second job for dual job holders range from 5.1 to 7.30 while the total own wage elasticities range from -0.071 to 0.914.

7 Summary and Conclusion

Using a Stone-Geary utility function we derive a more general model of labor supply that allows for workers to take on a second job. Our model is general in the sense that the reason for holding two jobs is not restricted to an hours constraint on the main job. We adopt the weekly earning version of our model because it consistently dominates the hours version of labor supply in our earlier investigations. For the estimation we use data from the BHPS, a unique data set that contains not only information about the second job, but also information about the hours constraint on the main job. We take advantage of the panel nature of this data set and seek to model unobserved heterogeneity by extending Wooldridge (2010) to a multinomial logit selection equation.

From the results of our earnings equations, we compute both the Stone-Geary labor supply elasticities and the total elasticities that incorporate the effects of sample selection and unobserved heterogeneity. Taking account of the labor supply effects of sample selection and unobserved heterogeneity yields total labor supply elasticities that are generally much smaller for job 2 compared with the Stone-Geary elasticities. When considering the job 2 elasticities versus those of job 1 elasticities among unconstrained dual job holders, our findings support the argument that job 2 is the marginal job and, as such, the hours supplied to job 2 should be more responsive to changes.

A significant generalization of our model of dual job labor labor supply would be to incorporate joint labor supply decisions for all adult members of the household. However, the data and modeling demands of a such an approach go well beyond the scope of our initial treatment of multiple job holding.

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Table 1: Summary Statistics (mean)

	Unitary job holders			Dual job holders		
Variable	**Unconstrained**	**Underemployed**	**Overemployed**	**Unconstrained**	**Underemployed**	**Overemployed**
Weekly hours worked on job 1	43.07	40.47	47.29	41.92	38.76	45.26
Weekly hours worked on job 2				7.04	7.00	6.75
Weekly non-labour income	52.83	54.30	51.22	55.22	55.47	49.55
Weekly earnings on job 1	341.38	272.61	387.75	301.41	230.45	350.49
Wage rate on job 1	8.01	6.76	8.32	7.27	5.97	7.86
Weekly earnings on job 2				54.02	51.58	64.99
Wage rate on job 2				10.31	8.96	12.69
Age	37.96	33.23	40.85	36.31	31.79	38.47
Educational attainment						
Higher degree (omitted)	0.04	0.02	0.04	0.04	0.01	0.06
1st degree	0.13	0.10	0.15	0.12	0.11	0.15
HND, HNC, teaching	0.09	0.07	0.10	0.08	0.05	0.08
A level	0.24	0.26	0.23	0.23	0.35	0.23
O level	0.26	0.28	0.25	0.31	0.32	0.27
CSE	0.07	0.09	0.06	0.08	0.06	0.06
None of these	0.17	0.18	0.19	0.14	0.10	0.15
Married (=1)	0.72	0.60	0.81	0.68	0.56	0.77
Number of children	0.67	0.72	0.71	0.73	0.64	0.84
Number of individuals	4446	564	2436	332	53	174
Number of observations	24672	2560	14904	1660	252	873

Notes: Based on British Household Panel Survey (1991-2008). All income variables are net of tax with prices in 2008.

Table 2: Transition Probability (%)

	Regime at T=t+1						
Regime at T=t	**Out of Sample**	**unc_unit**	**under_unit**	**over_unit**	**unc_dual**	**under_dual**	**over_dual**
Out Of Sample	0.00	54.47	6.17	30.36	5.26	0.62	3.11
unc_unit	21.21	56.62	3.48	16.98	1.25	0.17	0.29
under_unit	27.30	38.20	21.92	9.75	1.11	1.32	0.41
over_unit	20.20	26.45	1.61	50.21	0.63	0.05	0.86
unc_dual	28.36	19.80	2.81	6.12	31.42	1.81	9.68
under_dual	28.10	19.83	8.26	7.85	19.01	11.57	5.37
over_dual	29.45	7.96	0.95	16.75	16.75	1.19	26.96

Notes: Based on British Household Panel Survey (1991-2008).

Table 3: Multinomial Logit Model of Labor Supply Outcomes

	Unitary job holders			**Dual job holders**		
	Unconstrained	**Underemployed**	**Overemployed**	**Unconstrained**	**Underemployed**	**Overemployed**
Wage rate (job 1)	–	-0.003	-0.036∗	-0.027‡	-0.203∗	-0.037‡
		(0.013)	(0.005)	(0.015)	(0.043)	(0.019)
Age	–	-0.255‡	-0.056	-0.188	-0.325	0.227
		(0.131)	(0.066)	(0.159)	(0.394)	(0.211)
N of children	–	-0.043	0.001	-0.007	0.242‡	0.113‡
		(0.041)	(0.021)	(0.050)	(0.124)	(0.066)
1st degree	–	-0.318	0.126	-1.560∗	1.132	-0.345
		(0.486)	(0.216)	(0.501)	(1.210)	(0.629)
HND, HNC, teaching	–	0.026	0.075	-0.797	0.314	1.049
		(0.588)	(0.281)	(0.627)	(1.443)	(0.822)
A level	–	-0.311	-0.094	-0.579	1.355	-0.436
		(0.512)	(0.240)	(0.530)	(1.224)	(0.704)
O level	–	-0.223	0.091	-0.134	1.316	-0.643
		(0.550)	(0.259)	(0.573)	(1.338)	(0.772)
CSE	–	-0.610	0.158	-2.283∗	-1.468	-1.832
		(0.764)	(0.393)	(0.880)	(2.102)	(1.250)
None of these	–	-0.716	-0.461	-0.424	-1.389	-0.516
		(0.658)	(0.314)	(0.725)	(1.788)	(0.973)
Married (=1)	–	0.039	0.233∗	-0.137	-0.569†	0.000
		(0.096)	(0.054)	(0.121)	(0.270)	(0.176)
Weekly non-labour income/100	–	0.001†	-0.000	0.001	-0.000	-0.001
		(0.000)	(0.000)	(0.000)	(0.001)	(0.001)
Constant	–	-1.081∗	-1.635∗	-2.827∗	-3.980∗	-4.032∗
		(0.273)	(0.131)	(0.400)	(1.038)	(0.517)
Log pseudo-likelihood	-4.7e+04					
N	44921					

Notes: Based on British Household Panel Survey (1991-2008). ∗, † and ‡ indicate significance at 1, 5 and 10 percent levels respectively. Robust standard errors are in parentheses. Controls for current year time indicator variables and time averaged explanatory variables, including time averaged year dummy variables - complete results available from authors.

Table 4: Earnings Model Results

	Boundary Parameters
$\widehat{\gamma_1}$	81
$\widehat{\gamma_2}$	26
$\widehat{\gamma_3}$	31
	Earnings Model
$\widehat{\alpha_1}$	0.184$*$
	(0.023)
$\widehat{\alpha_2}$	0.621$*$
	(0.040)
$\widehat{\theta_{12}}$	49.487$*$
	(17.507)
$\widehat{\theta_{11}}$	-0.865
	(12.528)
$\widehat{\theta_{21}}$	25.259†
	(11.347)
$\widehat{\theta_{23}}$	-1.760
	(8.630)
$\widehat{\theta_{24}}$	-0.445
	(18.982)
Log likelihood	-9.3e+05
N	44921

Notes: Pooled data from BHPS 1991-2008; All income variables are expressed in 2008 prices; Standard errors in parentheses are bootstrap estimates from 200 replications; $*$, † and ‡ indicate significance at 1, 5 and 10 percent levels respectively; Time averaged explanatory variables are included - complete results available from authors.

Table 5: Labor Supply Elasticities (Evaluated at the Regime Specific Sample Means)

	Unitary Job (unconstrained)	**Dual Job (unconstrained)**		**Dual Job (overemployed)**	**Dual Job (underemployed)**
	Job 1	Job 1	Job 2	Job 2	Job 2
η_{mm}	0.027	0.170	7.301	6.245	5.122
η_{mk}		-0.158	-7.072	-6.036	-4.842
η_{mI}	-0.071	-0.030	-0.596	-0.699	-0.923
η^c_{mm}	0.515	0.354	7.923	7.007	5.884
η^c_{mk}		-0.127	-3.410		
ϵ_{mmI}	-0.487	-0.184	-0.621	-0.762	-0.762
ϵ_{mkI}		-0.031	-3.662	-6.036	-4.842
η^T_{mm}	-0.046	0.575	0.914	0.688	-0.071
η^T_{mk}		-0.158	-7.054	-6.038	-4.846
η^T_{mI}	-0.071	-0.030	-0.601	-0.699	-0.923

Notes: Based on BHPS 1991-2008.

Table 6: Changes in Weekly Hours (evaluated for 10% changes in w_1, w_2, I)

	Unitary Job(unconstrained)		**Dual Job (unconstrained)**			**Dual Job (overemployed)**			**Dual Job (underemployed)**		
	w_1	I	w_1	w_2	I	w_1	w_2	I	w_1	w_2	I
$\Delta\hat{h}_1^T$	-0.2	-0.3	2.4	0.50	-0.10						
$\Delta\hat{h}_2^T$			-3.6	-0.70	-0.30	-2.2	0.3	-0.3	-3.2	-0.0	-0.6
$\Delta\hat{h}^T$	-0.2	-0.3	-1.2	-0.20	-0.40	-2.2	0.3	-0.3	-3.2	-0.0	-0.6

Notes: $\hat{h}_{1t}^T\,|_{h_2=0} = 42.9$, $\hat{h}_1^T = 41.9$, $\hat{h}_2^T = 5.1$, $\hat{h}_{2t}\Big|_{h_1=\ddot{h}_1>h_1^*} = 3.7$, $\hat{h}_{2t}\Big|_{h_1=\ddot{h}_1<h_1^*} = 6.7$

A Technical Appendix

A.1 Theoretical Stone-Geary elasticities

Unconstrained dual job holder

The own wage Slutsky equation in elasticity form may be expressed by

$$\eta_{mm} = \eta^c_{mm} + \epsilon_{mm\mathrm{I}}\ ,$$

where

$$\begin{aligned}\eta_{mm} &= \frac{w_m}{h^*_m}\frac{\partial h^*_m}{\partial w_m}\\ &= \frac{\alpha_m}{w_m h^*_m}\left(\gamma_k w_k + \ I \ - \gamma_3\right) \gtreqless 0,\end{aligned}$$

is the uncompensated own wage elasticity for job m,

$$\begin{aligned}\eta^c_{mm} &= \frac{w_m}{h^*_m} S_{mm}\\ &= \frac{\alpha_m}{w_m h^*_m}\left(\gamma_k w_k + w_m h^*_m + \ I \ - \gamma_3\right) > 0,\end{aligned}$$

is the compensated own substitution effect elasticity for job m, S_{mm} is the compensated own substitution effect, and

$$\epsilon_{mm\mathrm{I}} \ = -\alpha_m \ < 0,$$

is the own wage income effect elasticity.

The pure income effect elasticity for job m is given by

$$\begin{aligned}\eta_{mI} \ &= \frac{I}{h_m}\frac{\partial h^*_m}{\partial I}\\ &= -\frac{\alpha_m I}{w_m h^*_m} < 0.\end{aligned}$$

The Slutsky equation for cross wage effects in elasticity form is given by

$$\eta_{mk} = \eta^c_{mk} + \epsilon_{mk\mathrm{I}},$$

where

$$\eta_{mk} = \frac{w_k}{h_m^*}\frac{\partial h_m^*}{\partial w_k}$$
$$= \frac{-\alpha_m \gamma_k w_k}{w_m h_m^*} < 0,$$

is the uncompensated cross wage effect elasticity of labor supply to job m from a change in the wage for job k,

$$\eta_{mk}^c = \frac{w_k}{h_m^*} S_{mk}$$
$$= \frac{-\alpha_m w_k}{w_m h_m^*} (\gamma_k - h_k^*) < 0,$$

is the compensated cross substitution effect elasticity, S_{mk} is the compensated cross substitution effect of a change in the wage on job k on labor supply to job m, and

$$\epsilon_{mkI} = \frac{-\alpha_m w_k h_k^*}{w_m h_m^*} < 0,$$

is cross-wage income effect elasticity. Observe that both uncompensated and compensated increases in the wage for job k lead to reductions in labor supply to job m.

Unconstrained unitary job holders

The own wage Slutsky equation in elasticity form for job 1 when not working a second job is expressed as

$$\eta_{11} \,|_{h_2=0} \; = \eta_{11}^c \,|_{h_2=0} + \epsilon_{11I} \,|_{h_2=0} \; ,$$

where

$$\eta_{11} \,|_{h_2=0} \; = \frac{w_1}{h_1^*}\frac{\partial h_1^*}{\partial w_1} \,|_{h_2=0}$$
$$= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1^*}\right)(\; I \; - \gamma_3) \gtreqless 0,$$

is the uncompensated own wage effect elasticity for job 1,

$$\eta_{11}^c \,|_{h_2=0} \; = \frac{w_1}{h_1^*} S_{11} \,|_{h_2=0}$$
$$= \left(\frac{\alpha_1}{1-\alpha_2}\right)\left(\frac{1}{w_1 h_1}\right)(w_1 h_1 + I - \gamma_3) > 0,$$

is the compensated own substitution effect elasticity for job 1, $S_{11}\,|_{h_2=0}$ is the own compensated substitution effect, and

$$\epsilon_{11I}\,|_{h_2=0} = \frac{-\alpha_1}{1-\alpha_2} < 0,$$

is the income effect elasticity from the own wage.

The pure income effect elasticity for h_1^* is determined by

$$\begin{aligned}\eta_{1I}\,|_{h_2=0} &= \frac{I}{h_1^*}\frac{\partial h_1^*}{\partial I}\,|_{h_2=0} \\ &= \left(\frac{-\alpha_1}{1-\alpha_2}\right)\left(\frac{I}{w_1 h_1^*}\right) < 0.\end{aligned}$$

<u>Constrained dual job holder</u>

The own wage Slutsky equation in elasticity form for job 2 when constrained on job 1 may be expressed as

$$\eta_{22}\Big|_{h_1=\ddot{h}_1} = \eta_{22}^c\Big|_{h_1=\ddot{h}_1} + \epsilon_{22I}\Big|_{h_1=\ddot{h}_1},$$

where

$$\begin{aligned}\eta_{22}\Big|_{h_1=\ddot{h}_1} &= \frac{w_2}{h_2^*}\frac{\partial h_2^*}{\partial w_2}\Big|_{h_1=\ddot{h}_1} \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2 h_2^*}\right)\Big(w_1\ddot{h}_1 + \ I \ - \gamma_3\Big) \gtreqless 0,\end{aligned}$$

is the uncompensated wage uncompensated own wage elasticity for job 2,

$$\begin{aligned}\eta_{22}^c\Big|_{h_1=\ddot{h}_1} &= \frac{w_2}{h_2^*}S_{22}\Big|_{h_1=\ddot{h}_1} \\ &= \left(\frac{\alpha_2}{1-\alpha_1}\right)\left(\frac{1}{w_2 h_2^*}\right)\Big(w_1\ddot{h}_1 + w_2 h_2^* + I - \gamma_3\Big) > 0,\end{aligned}$$

is the compensated own substitution elasticity, $S_{22}\Big|_{h_1=\ddot{h}_1}$ is the compensated own substitution effect, and

$$\begin{aligned}\epsilon_{22I}\Big|_{h_1=\ddot{h}_1} &= \eta_{22}\Big|_{h_1=\ddot{h}_1} - \eta_{22}^c\Big|_{h_1=\ddot{h}_1} \\ &= \frac{-\alpha_2}{1-\alpha_1} < 0,\end{aligned}$$

is the income effect elasticity from the own wage.

The pure income effect elasticity for h_2^* is determined by

$$\eta_{2I}\Big|_{h_1=\ddot{h}_1} = \frac{I}{h_2}\frac{\partial h_2^*}{\partial I}\Big|_{h_1=\ddot{h}_1}$$
$$= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{I}{w_2 h_2^*}\right) < 0.$$

Note that the compensated cross-substitution effect of wages on job 1 on labor supply to job 2 is necessarily zero when hours are constrained in job 1 because wages on job 1 can only have income effects. Hence the uncompensated cross-wage elasticity of w_1 on h_2^* is the same as the cross-wage income effect elasticity:

$$\eta_{21}\,|_{h_1=\ddot{h}_1} = \frac{w_1}{h_2}\frac{\partial h_2^*}{\partial w_1}|_{h_1=\ddot{h}_1} = \epsilon_{21I}\,|_{h_1=\ddot{h}_1}$$
$$= \left(\frac{-\alpha_2}{1-\alpha_1}\right)\left(\frac{w_1\ddot{h}_1}{w_2 h_2^*}\right) < 0.$$

Constrained unitary job holder

For a constrained unitary job holder, the hours worked $(\ddot{h}_1)$ are treated as exogenous so there is no corresponding labor supply equation.

A.2 Empirical elasticities based on the effects of key economic variables on the effects of selection and unobserved heterogeneity.

Unconstrained dual job holder

$$\frac{\partial \hat{\lambda}_{1it}}{\partial w_{1it}} = \left[\frac{-\hat{\lambda}_{1it}\left(z_{1it} + \hat{\lambda}_{1it}\right)}{\phi\left(z_{1it}\right)} \right] \left[\frac{\partial \hat{P}_{1it}}{\partial w_{1it}} \right], z_{1it} = \Phi^{-1}\left(\hat{P}_{1it}\right), \text{ and } \hat{P}_{1it} = \Lambda\left(x_{it}, \bar{\omega}_i, \tilde{\beta}_1\right).$$

$$\frac{\partial \hat{\lambda}_{1it}}{\partial I_{it}} = \left[\frac{-\hat{\lambda}_{1it}\left(z_{1it} + \hat{\lambda}_{1it}\right)}{\phi\left(z_{1it}\right)} \right] \left[\frac{\partial \hat{P}_{1it}}{\partial I_{it}} \right].$$

Unconstrained unitary job holders

$$\frac{\partial \hat{\lambda}_{2it}}{\partial w_{1it}} = \left[\frac{-\hat{\lambda}_{2it}\left(z_{2it} + \hat{\lambda}_{2it}\right)}{\phi\left(z_{2it}\right)} \right] \left[\frac{\partial \hat{P}_{2it}}{\partial w_{1it}} \right], z_{2it} = \Phi^{-1}\left(\hat{P}_{2it}\right), \text{ and } \hat{P}_{2it} = \Lambda\left(x_{it}, \bar{\omega}_i, \tilde{\beta}_2\right).$$

$$\frac{\partial \hat{\lambda}_{2it}}{\partial I_{it}} = \left[\frac{-\hat{\lambda}_{2it}\left(z_{2it} + \hat{\lambda}_{2it}\right)}{\phi\left(z_{2it}\right)} \right] \left[\frac{\partial \hat{P}_{2it}}{\partial I_{it}} \right].$$

Constrained dual job holders

For over-employed dual job holders:

$$\frac{\partial \hat{\lambda}_{23it}}{\partial w_{1it}} = \left[\frac{-\hat{\lambda}_{23it}\left(z_{3it} + \hat{\lambda}_{3it}\right)}{\phi\left(z_{3it}\right)} \right] \left[\frac{\partial \hat{P}_{3it}}{\partial w_{1it}} \right], z_{3it} = \Phi^{-1}\left(\hat{P}_{3it}\right), \text{ and } \hat{P}_{3it} = \Lambda\left(x_{it}, \bar{\omega}_i, \tilde{\beta}_3\right),$$

$$\frac{\partial \hat{\lambda}_{23it}}{\partial I_{it}} = \left[\frac{-\hat{\lambda}_{23it}\left(z_{3it} + \hat{\lambda}_{3it}\right)}{\phi\left(z_{3it}\right)} \right] \left[\frac{\partial \hat{P}_{3it}}{\partial I_{it}} \right].$$ For under-employed dual job holders:

$$\frac{\partial \hat{\lambda}_{24it}}{\partial w_{1it}} = \left[\frac{-\hat{\lambda}_{24it}\left(z_{4it} + \hat{\lambda}_{4it}\right)}{\phi\left(z_{4it}\right)} \right] \left[\frac{\partial \hat{P}_{4it}}{\partial w_{1it}} \right], z_{4it} = \Phi^{-1}\left(\hat{P}_{4it}\right), \text{ and } \hat{P}_{4it} = \Lambda\left(x_{it}, \bar{\omega}_i, \tilde{\beta}_4\right),$$

$$\frac{\partial \hat{\lambda}_{24it}}{\partial I_{it}} = \left[\frac{-\hat{\lambda}_{24it}\left(z_{4it} + \hat{\lambda}_{4it}\right)}{\phi\left(z_{4it}\right)} \right] \left[\frac{\partial \hat{P}_{4it}}{\partial I_{it}} \right].$$