R&D Networks and Industrial Strategy

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Introduction

- R&D partnerships have become a widespread phenomenon characterizing technological dynamics, especially in industries with rapid technological development (cf. Hagedoorn, 2002),[1] such as, for instance, the pharmaceutical, chemical and computer industries (see e.g. Ahuja, 2000; Powell et al., 2005).[2]
- Firms have become more specialized on specific domains of a technology and they tend to combine their knowledge with the knowledge of other firms that are specialized in different technological domains (Powell et al., 1996; Weitzman, 1998).[3]
- Despite the importance of R&D collaborations for technological change and economic growth, there is no comprehensive and applied study of R&D policy (network design, key players, subsidies vs. tax) in such networked markets.

[1] Hagedoorn, J., May 2002. *Inter-firm R&D partnerships: an overview of major trends and patterns since 1960.* Research Policy 31 (4), 477492.

[2] Ahuja, G., 2000. *Collaboration networks, structural holes, and innovation: A longitudinal study.* Administrative Science Quarterly 45, 425455

[3] Weitzman, M. L., 1998. *Recombinant growth.* The Quarterly Journal of Economics 113 (2), 331 360.

- We analyze R&D collaboration networks in industries where firms are competitors in the product market.
- We build on the R&D network model by Goyal and Moraga-Gonzalez (2001)[4] in which benefits from collaborations arise by sharing knowledge about a cost-reducing technology. By forming collaborations, however, firms also change their own competitive position in the market as well as the overall market structure.
- We derive the equilibrium quantity and R&D effort choices of firms when they are competing in different markets/sectors, while allowing for within and between sectoral R&D collaborations.

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[4] Goyal, S., Moraga-Gonzalez, J. L., 2001. *R&D Networks.* RAND Journal of Economics 32 (4), 686707.

- We then analyze welfare (producer and consumer surplus) in independent as well as interdependent markets, captured by varying degrees of substitutability between goods.
- We study key player firms, i.e. the firms whose exit reduces welfare the most.
- We then analyze R&D subsidy programs, either as a fixed share of R&D expenditures homogeneous across firms, or targeted towards individual firms.

Related Literature

- D'Aspremont & Jacquemin (1988)[5] analyze a Cournot duopoly model with and without R&D collaboration.
- Goyal & Moraga-Gonzalez (2001)[6] introduce a network of R&D collaborating firms (with both quantity and R&D effort choice).
- Westbrock (2010)[7] analyzes welfare in R&D collaboration networks.
- König et al. (2011)[8] study equilibria and welfare in R&D collaboration networks, but assume independent markets.

⇒ We provide a complete characterization of equilibrium output and R&D effort choices in multiple interdependent markets and analyze a range of policy instruments (network design, key player analysis, subsidy programs).

[5] D'Aspremont, C. & Jacquemin, A., *Cooperative and noncooperative R&D in duopoly with spillovers*, The American Economic Review, 1988, 78, 1133-1137.

[6] Goyal, S. & Moraga-Gonzalez, J. L., *R&D Networks*, RAND Journal of Economics, 2001, 32, 686-707.

[7] Westbrock, B., *Natural concentration in industrial research collaboration*, The RAND Journal of Economics, 2010, 41, 351-371.

[8] König, M. D.; Battiston, S.; Napoletano, M. & Schweitzer, F., *The Efficiency and Stability of R&D Networks*, Games and Economic Behavior, 2011, 75, 694-713.

- Bloom et al. (2012)[9] study empirically a production function with both technology spillovers and market competition effects.
- Cabrales et al. (2010)[10] study spillover effects in random graph-like networks.
- Calvo-Armengol et al. (2004),[11], and Liu et al. (2011)[12] estimate peer effects and apply the key player policy to education and crime.

⇒ We provide a micro-foundation for the technology spillover and market competition effects, and estimate it with a unique panel data set on R&D alliances matched to annual financial reports.

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[9] Bloom, N.; Schankerman, M. & Van Reenen, J., *Identifying technology spillovers and product market rivalry*, NBER Working Paper No. 13060, 2007.

[10] Cabrales, A.; Calvo-Armengol, A. & Zenou, Y., *Social interactions and spillovers*, Games and Economic Behavior, Elsevier, 2010.

[11] Calvo-Armengol, A.; Patacchini, E. & Zenou, Y., *Peer Effects and Social Networks in Education*, Review of Economic Studies, 2009, 76, 1239-1267.

[12] Liu, X.; Patacchini, E.; Zenou, Y. & Lee, L., *Criminal networks: Who is the key player?*, Mimeo, Stockholm University, Department of Economics, 2011.

The Model

- The demand q_i for the good produced by firm i in market $\mathcal{M}_m$, $m = 1, \ldots, M$. A representative consumer in market $\mathcal{M}_m$ obtains the following gross utility from consumption of the goods $(q_i)_{i \in \mathcal{M}_m}$[13]

$$\bar{U}_m((q_i)_{i \in \mathcal{M}_m}) = \alpha_m \sum_{i \in \mathcal{M}_m} q_i - \frac{1}{2} \sum_{i \in \mathcal{M}_m} q_i^2 - \varrho \sum_{i \in \mathcal{M}_m} \sum_{j \in \mathcal{M}_m, j \neq i} q_i q_j .$$

- The consumer maximizes net utility $U_m = \bar{U}_m - \sum_{i \in \mathcal{M}_m} p_i q_i$, where p_i is the price of good i. This gives the inverse demand function for firm i

$$p_i = \bar{\alpha}_i - q_i - \varrho \sum_{\substack{j \in \mathcal{M}_m, \\ j \neq i}} q_j , \tag{1}$$

  where we have denoted by $\bar{\alpha}_i = \sum_{m=1}^{M} \alpha_m \mathbb{1}_{\{i \in \mathcal{M}_m\}}$.

[13] The parameter α_m captures the market size or heterogeneity in products, whereas $\varrho \in (0, 1]$ measures the degree of substitutability between products. In particular, $\varrho = 1$ depicts a market of perfectly substitutable goods, while $\varrho \to 0$ represents the case of local monopolies.

- Firms can reduce their costs for production by investing into R&D as well as by establishing an R&D collaboration with another firm.
- The amount of this cost reduction depends on the effort e_i that a firm i and the effort e_j that its R&D collaboration partners $j \in \mathcal{N}_i$ invest into the collaboration.
- Given the effort level $e_i \in \mathbb{R}_+$, marginal cost c_i of firm i is given by

$$c_i = \bar{c}_i - e_i - \psi \sum_{j=1}^{n} a_{ij} e_j, \tag{2}$$

  where $a_{ij} = 1$ if firms i and j set up a collaboration (0 otherwise) and $a_{ii} = 0$.

- We assume that R&D effort is costly. In particular, the cost of R&D effort is an increasing function and given by $Z = \gamma e_i^2$, $\gamma > 0$ (similar to e.g. D'Aspremont, 1988). Firm i's profit π_i is then given by
$$\pi_i = (p_i - c_i)q_i - \gamma e_i^2. \tag{3}$$
- Inserting marginal cost from Equation (2) and inverse demand from Equation (1) into Equation (3) gives

$$\begin{aligned}\pi_i &= (p_i - \bar{c}_i + e_i + \psi \sum_{j=1}^{n} a_{ij} e_j) q_i - \gamma e_i^2 \\ &= (\bar{\alpha}_i - \bar{c}_i) q_i - q_i^2 - \varrho \sum_{j=1}^{n} b_{ij} q_i q_j + q_i e_i + \psi q_i \sum_{j=1}^{n} a_{ij} e_j - \gamma e_i^2,\end{aligned} \tag{4}$$

where $b_{ij} \in \{0,1\}$ is the ij-th element of the matrix $\mathbf{B}$ defined by $\mathbf{B} \equiv \sum_{m=1}^{M} (\mathbf{u}_m \mathbf{u}_m^\top - \mathbf{D}_m)$, and $\mathbf{u}_m$ is a zero-one vector with elements $u_{mi} = 1$ if $i \in \mathcal{M}_m$ and $u_{mi} = 0$ otherwise for all $i = 1, \ldots, n$, and $\mathbf{D}_m = \mathrm{diag}(\mathbf{u}_m)$ is the diagonal matrix with diagonal elements given by $\mathbf{u}_m$.

- The FOC of profits with respect to R&D effort e_i of firm i is given by

$$\frac{\partial \pi_i}{\partial e_i} = q_i - 2\gamma e_i = 0,$$

so that we obtain

$$e_i = \frac{1}{2\gamma} q_i.$$

- This proportional relationship between R&D effort levels and output has been confirmed in a number of empirical studies (see e.g. Cohen and Klepper, 1996).[14]

[14] Cohen, W., Klepper, S., 1996. *A reprise of size and R & D.* The Economic Journal 106 (437), 925951.

- The FOC with respect to quantity is given by

$$\frac{\partial \pi_i}{\partial q_i} = \bar{\alpha}_i - \bar{c}_i - 2q_i - \varrho \sum_{j=1}^{n} b_{ij} q_j + e_i + \psi \sum_{j=1}^{n} a_{ij} e_j .$$

- Inserting equilibrium efforts (assuming a simultaneous move game) and rearranging terms gives

$$q_i = \frac{2\gamma(\bar{\alpha}_i - \bar{c}_i)}{4\gamma - 1} - \frac{2\gamma\varrho}{4\gamma - 1} \sum_{j=1}^{n} b_{ij} q_j + \frac{\psi}{4\gamma - 1} \sum_{j=1}^{n} a_{ij} q_j .$$

- In the following we denote by

$$\mu_i \equiv \frac{2\gamma(\bar{\alpha}_i - \bar{c}_i)}{4\gamma - 1}, \quad \rho \equiv \frac{2\gamma\varrho}{4\gamma - 1}, \quad \lambda \equiv \frac{\psi}{4\gamma - 1}, \tag{5}$$

so that we obtain for equilibrium quantity

$$q_i = \mu_i - \rho \sum_{j=1}^{n} b_{ij} q_j + \lambda \sum_{j=1}^{n} a_{ij} q_j. \tag{6}$$

- This can be written in matrix-vector notation as follows

$$\mathbf{q} = \boldsymbol{\mu} - \rho\mathbf{B}\mathbf{q} + \lambda\mathbf{A}\mathbf{q}$$

or, equivalently,

$$(\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A})\mathbf{q} = \boldsymbol{\mu}.$$

- The matrix $\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A}$ is invertible if its determinant is not zero. This also guarantees the uniqueness and existence of the equilibrium. [15] A sufficient condition for invertibility is given by

$$\rho + \lambda < \left(\max \left\{ \lambda_{\mathrm{PF}}(\mathbf{A}), \max_{m=1,\ldots,M} \{ (|\mathcal{M}_m| - 1) \} \right\} \right)^{-1},$$

- When the inverse of $\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A}$ exists, we can write equilibrium quantities as

$$\mathbf{q} = (\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A})^{-1}\boldsymbol{\mu}.$$

[15] The determinant of $\mathbf{I}_n - \sum_{j=1}^{p} \lambda_j \mathbf{W}_j$ is strictly positive if $\sum_{j=1}^{p} |\lambda_j| < 1/\max_{j=1,\ldots,p} \|\mathbf{W}_j\|$, where $\|\mathbf{W}_j\|$ is any matrix norm, including the spectral norm (which is the largest eigenvalue of $\mathbf{W}_j$). We have that the largest eigenvalue of the matrix $\mathbf{B}$ is equal to the size of the largest market $|\mathcal{M}_m|$ minus one (as this is a block-diagonal matrix with all elements being one in each block and zero diagonal), and the largest eigenvalue of $\mathbf{A}$ is the Perron-Frobenius eigenvalue $\lambda_{\mathrm{PF}}(\mathbf{A})$.

- Profits in equilibrium can be written as

$$\pi_i = (\bar{\alpha}_i - \bar{c}_i)q_i - \varrho q_i \sum_{j=1}^{n} b_{ij} q_j + \frac{\psi}{2\gamma} q_i \sum_{j=1}^{n} a_{ij} q_j - \left(1 - \frac{1}{4\gamma}\right) q_i^2.$$

which can be simplified to

$$\pi_i = \left(1 - \frac{1}{4\gamma}\right) q_i^2. \tag{7}$$

- If there is only a single market, with $M = 1$, then $\rho\mathbf{B} = \rho(\mathbf{u}\mathbf{u}^\top - \mathbf{I}_n)$ where $\mathbf{u} = (1, \ldots, 1)^\top$ is an n-dimensional vector of ones.
- Equilibrium quantity is given by

$$\mathbf{q} = \frac{1}{1-\rho}\left(\mathbf{b}_{\boldsymbol{\mu}}(G,\phi) - \frac{\rho\|\mathbf{b}_{\boldsymbol{\mu}}(G,\phi)\|_1}{1+\rho(\|\mathbf{b}_{\boldsymbol{u}}(G,\phi)\|_1 - 1)}\mathbf{b}_{\mathbf{u}}(G,\phi)\right). \quad (8)$$

where $\phi = \frac{\lambda}{1-\rho}$ and $\mathbf{b}_{\boldsymbol{u}}(G,\phi)$ and $\mathbf{b}_{\boldsymbol{\mu}}(G,\phi)$ is the $\boldsymbol{\mu}$*-weighted Bonacich centrality* defined by[16]

$$\mathbf{b}_{\boldsymbol{\mu}}(G,\phi) = (\mathbf{I}_n - \phi\mathbf{A})^{-1}\boldsymbol{\mu}.$$

[16] Calvo-Armengol, A., Patacchini, E., Zenou, Y., 2009. *Peer effects and social networks in education.* Review of Economic Studies 76, 12391267.

- When also $\mu_i = \mu$ for all $i = 1, \ldots, n$ we can further simplify this to

$$\mathbf{q} = \frac{\mu}{1 + \rho(\|\mathbf{b_u}(G, \phi)\|_1 - 1)} \mathbf{b_u}(G, \phi), \tag{9}$$

  where $\phi = \frac{\lambda}{1-\rho}$ and

$$\mathbf{b_u}(G, \phi) = (\mathbf{I}_n - \phi \mathbf{A})^{-1} \mathbf{u}$$

  is the *Bonacich centrality* with parameter ϕ (Bonacich, 1987).[17]
- In the case of independent markets, when goods are non-substitutable $\rho = 0$, this further simplifies to $\boldsymbol{q} = \mu \mathbf{b}_{\boldsymbol{u}}(G, \phi)$.

[17] Bonacich, P., 1987. *Power and centrality: A family of measures.* American Journal of Sociology 92 (5), 1170.

Welfare

- Inserting the inverse demand from Equation (1) into net utility U_m of the consumer in market $\mathcal{M}_m$ shows that

$$U_m = \frac{1}{2} \sum_{i \in \mathcal{M}_m} q_i^2 + \frac{\varrho}{2} \sum_{i \in \mathcal{M}_m} \sum_{\substack{j \in \mathcal{M}_m, \\ j \neq i}} q_i q_j .$$

- In the special case of non-substitutable goods, when $\varrho = 0$, we obtain

$$U_m = \frac{1}{2} \sum_{i \in \mathcal{M}_m} q_i^2 ,$$

- while in the case of perfectly substitutable goods, when $\varrho = 1$, we get

$$U_m = \frac{1}{2} \left(\sum_{i \in \mathcal{M}_m} q_i \right)^2 .$$

- Total consumer surplus is then given by $U = \sum_{m=1}^{M} U_m$. Producer surplus is given by aggregate profits $\Pi = \sum_{i=1}^{n} \pi_i$. Welfare is then given by $W = U + \Pi$.

Welfare – Independent Markets

- When products are not substitutable then social welfare is given by producer and consumer surplus, which can then be written as

$$W(G) = \sum_{i=1}^{n} \left(\frac{q_i^2}{2} + \pi_i \right) = \frac{\omega}{2} \sum_{i=1}^{n} q_i^2,$$

  where we have denoted by $\omega \equiv 3 - \frac{1}{2\gamma}$.
- Assuming further that $\mu_i = \mu$ for all $i = 1, \ldots, n$, we have that $\mathbf{q} = \mu \mathbf{M}(G, \lambda)\mathbf{u}$, where we have denoted by $\mathbf{M}(G, \lambda) \equiv (\mathbf{I}_n - \lambda \mathbf{A})^{-1}$. We then obtain

$$W(G) = \frac{\omega}{2} \mathbf{q}^\top \mathbf{q} = \frac{\mu^2 \omega}{2} \mathbf{u}^\top \mathbf{M}(G, \lambda)^2 \mathbf{u}.$$

- Observe that the quantity $\mathbf{u}^\top \mathbf{M}(G,\phi)\mathbf{u}$ is the *walk generating function* $N_G(\lambda)$ of G (Cvetkovic, 1995).[18]
- Let N_k denote the number of walks of length k in G. Then we can write N_k as follows

$$N_k = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{[k]} = \mathbf{u}^\top \mathbf{A}^k \mathbf{u},$$

where $a_{ij}^{[k]}$ is the ij-th element of $\mathbf{A}^k$.

[18] Cvetkovic, D., Doob, M., Sachs, H., 1995. *Spectra of Graphs: Theory and Applications.* Johann Ambrosius Barth.

- The walk generating function is then defined as

$$N_G(\lambda) \equiv \sum_{k=0}^{\infty} N_k \lambda^k$$
$$= \mathbf{u}^\top \left(\sum_{k=0}^{\infty} \lambda^k \mathbf{A}^k \right) \mathbf{u} = \mathbf{u}^\top \left(\mathbf{I}_n - \lambda \mathbf{A} \right)^{-1} \mathbf{u} = \mathbf{u}^\top \mathbf{M}(G, \lambda) \mathbf{u}.$$

- For a k-regular graph G_k we obtain

$$N_{G_k}(\lambda) = \frac{n}{1 - k\lambda}.$$

It holds that $N_G(0) = n$, and one can show that $N_G(\lambda) \geq 0$.

- Using the fact that

$$N_k = \mathbf{u}^\top \mathbf{A}^k \mathbf{u} = \sum_{i=1}^{n} (\mathbf{u}^\top \mathbf{v}_i)^2 \lambda_i^k,$$

we can write the walk generating function as follows

$$N_G(\lambda) = \mathbf{u}^\top \mathbf{M}(G, \lambda)\mathbf{u} = \sum_{k=0}^{\infty} N_k \lambda^k = \sum_{i=1}^{n} \frac{(\mathbf{v}_i^\top \mathbf{u})^2}{1 - \lambda_i \lambda}.$$

- We further have that

$$\mathbf{u}^\top \mathbf{M}(G,\lambda)^2 \mathbf{u} = \frac{d}{d\lambda}(\lambda N_G(\lambda)) = N_G(\lambda) + \lambda \frac{d}{d\lambda} N_G(\lambda).$$

so that we can write

$$\begin{aligned}
\mathbf{u}^\top \mathbf{M}(G,\lambda)^2 \mathbf{u} &= \sum_{i=1}^{n} \frac{(\mathbf{v}_i^\top \mathbf{u})^2}{1-\lambda_i\lambda} + \sum_{i=1}^{n} (\mathbf{u}^\top \mathbf{v}_i)^2 \sum_{k=0}^{\infty} k\lambda^k \lambda_i^k \\
&= \sum_{i=1}^{n} \frac{(\mathbf{v}_i^\top \mathbf{u})^2}{1-\lambda_i\lambda} + \sum_{i=1}^{n} \frac{(\mathbf{u}^\top \mathbf{v}_i)^2 \lambda\lambda_i}{(1-\lambda\lambda_i)^2} \\
&= \sum_{i=1}^{n} \frac{(\mathbf{u}^\top \mathbf{v}_i)^2}{1-\lambda\lambda_i} \left(1 + \frac{\lambda\lambda_i}{1-\lambda\lambda_i}\right) \\
&= \sum_{i=1}^{n} \frac{(\mathbf{u}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2}.
\end{aligned}$$

- From the above it follows that welfare can also be written as

$$W(G) = \frac{\mu^2\omega}{2}\frac{d}{d\lambda}(\lambda N_G(\lambda)) = \frac{\mu^2\omega}{2}\sum_{i=1}^{n}\frac{(\mathbf{u}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2}.$$

- A similar calculation for the case of $\mu_i \neq \mu_j$ shows that

$$\boldsymbol{\mu}^\top \mathbf{M}\boldsymbol{\mu} = \sum_{i=1}^{n}\frac{(\boldsymbol{\mu}^\top \mathbf{v}_i)^2}{1-\lambda\lambda_i},$$

and similarly

$$\boldsymbol{\mu}^\top \mathbf{M}^2\boldsymbol{\mu} = \sum_{i=1}^{n}\frac{(\boldsymbol{\mu}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2}.$$

- Welfare can then be written as

$$W(G) = \frac{\mu^2\omega}{2}\boldsymbol{\mu}^\top \mathbf{M}^2 \boldsymbol{\mu} = \frac{\mu^2\omega}{2}\sum_{i=1}^{n}\frac{(\boldsymbol{\mu}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2}.$$

- In the limit of large λ the efficient graph $G^* = \text{argmax}_{G\in\mathcal{H}(n,m)} W(G)$ is a *nested split graph* in which the ordering of degrees $\{d_i\}_{i=1}^n$ follows the ordering of $\{\mu_i\}_{i=1}^n$.
- Note that similar results relating the largest eigenvalue to efficiency have been obtained in Corbo & Parkes (2006)[19] and König et al. (2011)[20].

[19] Corbo, J., Calvo-Armengol, A., Parkes, D., 2006. *A study of nash equilibrium in contribution games for peer-to-peer networks.* SIGOPS Operation Systems Review 40 (3), 6166.

[20] König, M. D., Battiston, S., Napoletano, M., Schweitzer, F., 2011. *The efficiency and stability of R&D networks.* Games and Economic Behavior 75, 694713.

- Since, in the k-regular graph G_k it holds that $N_G(\lambda) = \frac{n}{1-k\lambda}$ and $\frac{d}{d\lambda}(\lambda N_G(\lambda)) = N_G(\lambda) + \lambda \frac{d}{d\lambda} = N_G(\lambda) = \frac{n}{1-k\lambda} + \frac{nk\lambda}{(1-k\lambda)^2} = \frac{n}{1-k\lambda}\left(1 + \frac{k\lambda}{1-k\lambda}\right) = \frac{n}{(1-k\lambda)^2}$, we get a lower bound on welfare in the efficient graph $\frac{n}{(1-\frac{2m}{n}\lambda)^2} \leq W(G^*)$, where we have used the fact that the number of links in a k-regular graph is given by $m = \frac{nk}{2}$.

- In order to derive an upper bound, observe that

$$\mathbf{u}^\top \mathbf{M}(G,\lambda)^2\mathbf{u} = \sum_{i=1}^{n} \frac{(\mathbf{u}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2},$$

and we can write welfare as follows

$$\begin{aligned} W(G) &= \frac{\mu^2\omega}{2}\sum_{i=1}^{n}\frac{(\mathbf{u}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_i)^2} \\ &\le \frac{\mu^2\omega}{2}\frac{\sum_{i=1}^{n}(\mathbf{u}^\top \mathbf{v}_i)^2}{(1-\lambda\lambda_1)^2} \\ &\le \frac{\mu^2\omega}{2}\frac{n}{(1-\lambda\lambda_1)^2}, \end{aligned}$$

where we have used the fact that $N_G(0) = \sum_{i=1}^{n}(\mathbf{u}^\top \mathbf{v}_i)^2 = n$ so that $(\mathbf{u}^\top \mathbf{v}_1)^2 < n$.

- Moreover, the largest eigenvalue in a graph G with m links and n nodes is bounded from above by $\lambda_1 \leq \sqrt{\frac{2m(n-1)}{n}} \leq n-1$.[21] This gives us an upper bound on welfare according to

$$W(G^*) \leq \frac{\mu^2 \omega}{2} \frac{n}{\left(1 - \lambda\sqrt{2m(n-1)/n}\right)^2}.$$

- If the number of links m can be chosen freely, because the largest eigenvalue λ_1 is upper bounded by the largest eigenvalue of the complete graph K_n, which is the $(n-1)$-regular graph. In this case, upper and lower bounds coincide, and the efficient graph G^* is therefore complete, that is $K_n = \mathrm{argmax}_{G \in \mathcal{G}(n)} W(G)$.

[21] If we assume that G is connected then we can also use the bound $\lambda_1 \leq \sqrt{2m - n + 1} \leq n - 1$.

- **Proposition:** Assume that $\mu_i = \mu$ for all $i \in \mathcal{N}$. Then welfare in the efficient graph $G^* = \text{argmax}_{G \in \mathcal{H}(n,m)} W(G)$ can be bounded from above and from below as

$$\frac{\mu^2 \omega}{2} \frac{n}{(1-\lambda \bar{d})^2} \leq W(G^*) \leq \frac{\mu^2 \omega}{2} \frac{n}{\left(1-\lambda \sqrt{(n-1)\bar{d}}\right)^2},$$

where $\bar{d} = \frac{2m}{n}$ is the average degree in G.

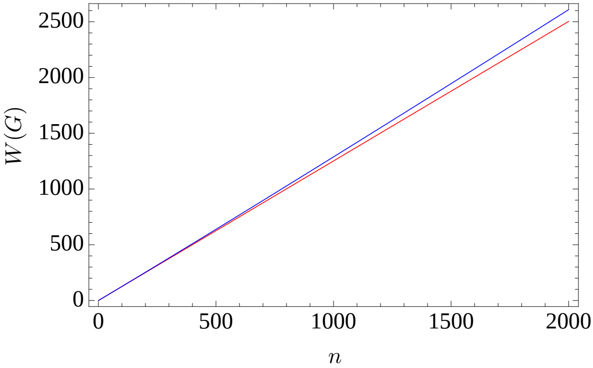

FIGURE: The two bounds from the above proposition for $\varrho = 0.1$, $\psi = 0.001$, $\mu = 1$, $m = n - 1$ and $\gamma = 1$ for varying values of n.

Welfare – Interdependent Markets

- In this section we allow for products to be substitutable, i.e. $\varrho > 0$. Then social welfare is given by

$$W(G) = \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + \varrho \sum_{i=1}^{n}\sum_{j\neq i}^{n} b_{ij}q_iq_j\right) + \sum_{i=1}^{n}\pi_i,$$

where equilibrium output and profit are given by Equations (8) and (7).

- Inserting profits as a function of output delivers

$$W(G) = \left(\frac{3}{2} - \frac{1}{4\gamma}\right)\sum_{i=1}^{n} q_i^2 + \frac{\varrho}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n} b_{ij}q_iq_j = \frac{\omega}{2}\mathbf{q}^\top\mathbf{q} + \frac{\varrho}{2}\mathbf{q}^\top\mathbf{B}\mathbf{q},$$

where we have denoted by $\omega \equiv 3 - \frac{1}{2\gamma}$.

- **Proposition:** Denoted by $\mathbf{C} = \mathbf{A} - \frac{\varrho}{\lambda}\mathbf{B}$, let $\{\nu_i\}_{i=1}^n$ be the eigenvalues of $\mathbf{C}$ and $\{\mathbf{v}_i\}_{i=1}^n$ the associated eigenvectors. Then welfare can be written as

$$W(G) = \frac{\omega - \varrho}{2} \frac{(\boldsymbol{\mu}^\top \mathbf{v}_1)^2}{(1-\lambda\nu_1)^2} \left(1 + \frac{\varrho}{\omega - \varrho} \mathbf{v}_1^\top \mathbf{B} \mathbf{v}_1\right) + o\left(\frac{1}{1-\lambda\nu_1}\right)^2.$$

- This shows that when spillover effects are strong such that the leading terms in $1/(1-\lambda\nu_1)$ dominate, then welfare is determined by the weighted sum of the eigenvector components $\boldsymbol{\mu}^\top \mathbf{v}_1 = \sum_{i=1}^n \mu_i v_{1,i}$ and the pairwise eigenvector complementarity effects in different markets $\mathbf{v}_1^\top \mathbf{B} \mathbf{v}_1 = \sum_{i=1}^n \sum_{j=1}^n v_{1,i} b_{ij} v_{1,j}$.

- To gain further insights, we will assume in the following that there is only a single market (with $M = 1$, $b_{ij} = 1$ for $i \neq j$ and $b_{ii} = 1$ for all $i, j \in \mathcal{N}$) and make the homogeneity assumption that $\mu_i = \mu$ for all $i \in \mathcal{N}$. Then welfare can be written as follows

$$W(G) = \frac{\omega - \varrho}{2} \|\mathbf{q}\|_2^2 + \frac{\varrho}{2} \|\mathbf{q}\|_1^2,$$

where $\|\mathbf{q}\|_p \equiv \left(\sum_{i=1}^n q_i^p\right)^{\frac{1}{p}}$ is the ℓ^p-norm of $\mathbf{q}$ and $\mathbf{u} = (1, \ldots, 1)^\top$ is a vector of ones. Using the fact that $\|\mathbf{q}\|_2 \leq \|\mathbf{q}\|_1 \leq \sqrt{n}\|\mathbf{q}\|_2$, we obtain an upper bound on welfare given by

$$W(G) \leq \frac{\omega + (n-1)\varrho}{2} \|\mathbf{q}\|_2^2 = \frac{2\gamma(\omega + (n-1)\varrho)}{4\gamma - 1} \Pi,$$

where aggregate profits are given by $\Pi = \sum_{i=1}^n \pi_i$. Hence, welfare is upper bounded by a proportionality factor times the total profits generated in the economy.

- **Proposition:** Consider a large market with substitutable goods where $\varrho > 0$. Further, assume that $\mu_i = \mu$ for all $i = 1, \ldots, n$. Denote by $\mathcal{G}(n)$ the class of graphs with n nodes and the class of graphs with n nodes and m links by $\mathcal{H}(n,m) \subset \mathcal{G}(n)$. Then for small values of ϕ, such that terms of the oder $O(\phi^3)$ can be neglected, welfare $W(G)$ is maximized in the graph $G \in \mathcal{H}(n,m)$ with the smallest degree variance σ_d^2.
- This contrasts to previous studies such as Westbrock (2010), where it is argued that welfare in R&D collaboration networks is increasing with the degree variance.

- **Proposition:** Consider a large market with substitutable goods where $\varrho > 0$, assume that $\mu_i = \mu$ for all $i = 1, \ldots, n$. Denote by $\mathcal{G}(n)$ the class of graphs with n nodes and the class of graphs with n nodes and m links by $\mathcal{H}(n, m) \subset \mathcal{G}(n)$. Then for small values of ϕ such that terms of the oder $O(\phi^4)$ can be neglected, welfare $W(G)$ for two graphs $G, G' \in \mathcal{H}(n, m)$ with the same degree variance σ_d^2 is higher for the one which is less degree assortative.[22]

[22] The assortativity coefficient $\rho_d(G) \in [-1, 1]$ is essentially the Pearson correlation coefficient of degree between nodes that are connected. Positive values of $\rho_d(G)$ indicate that nodes with similar degrees tend to be connected, while negative values indicate that nodes with different degrees tend to be connected. See Newman (2002) for further details.

- **Proposition:** Consider substitutable goods and assume that $\mu_i = \mu$ for all $i = 1, \ldots, n$, and define $\omega \equiv 3 - 1/(2\gamma)$. Denote by $\mathcal{G}(n)$ the class of graphs with n nodes and the class of graphs with n nodes and m links by $\mathcal{H}(n,m) \subset \mathcal{G}(n)$. Moreover, assume that $0 < \rho < 1$.
- Welfare in the efficient graph $G^* = \mathrm{argmax}_{G \in \mathcal{H}(n,m)} W(G)$ can be bounded from above and from below as

$$\frac{\mu^2}{2} \frac{n((n-1)\varrho + \omega)}{((n-1)(\rho-\lambda)+1)^2} \leq W(G^*) \leq \frac{\omega - \varrho}{2} \frac{\mu^2}{\rho^2} \left(\frac{\varrho}{\omega - \varrho} + \frac{1-\rho}{n(1-\rho-\lambda\sqrt{2m(n-1)/n})} \right).$$

- In the limit of weak spillovers and large population size the efficient graph in $\mathcal{G}(n)$ is the complete graph K_n, that is $\lim_{\lambda \to 0} \lim_{n \to \infty} W(K_n) = W(G^*)$.

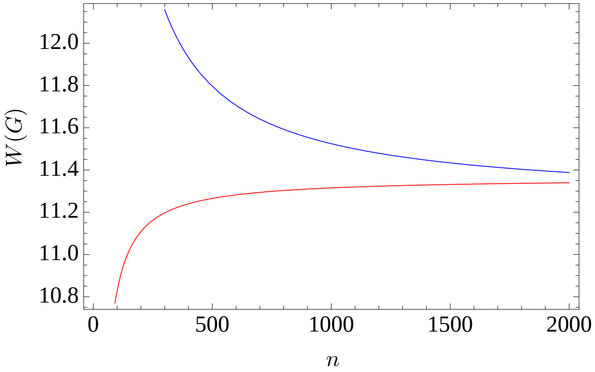

FIGURE: (Left panel) The two bounds from the above proposition for $\varrho = 0.1$, $\psi = 0.001$, $\mu = 1$, $m = n - 1$ and $\gamma = 1$ for varying values of n.

- **Proposition:** Consider substitutable goods and assume that $\mu_i = \mu$ for all $i = 1, \ldots, n$, and define $\omega \equiv 3 - 1/(2\gamma)$. Then in the limit of ϕ approaching the inverse of the largest eigenvalue λ_{PF} from below welfare can be written as

$$\lim_{\phi \uparrow 1/\lambda_{\mathrm{PF}}} W(G) = \frac{\omega - \varrho}{2} \frac{\mu^2}{\rho^2} \left(\frac{\varrho}{\omega - \varrho} + \frac{1}{\|\mathbf{v}_1\|_1^2} \right).$$

- Further, denote by $\mathcal{G}(n)$ the class of graphs with n nodes and the class of graphs with n nodes and m links by $\mathcal{H}(n,m) \subset \mathcal{G}(n)$. Consider the class $\mathcal{S}(n,m) \subset \mathcal{H}(n,m)$ of graphs with a large spectral gap, such that $\lambda_1 = \lambda_{\mathrm{PF}}$ is much larger than λ_j for all $j \geq 2$. Then the welfare maximizing graph $G^* = \mathrm{argmax}_{G \in \mathcal{S}(n,m)} W(G)$ in this class is the one that minimizes the ℓ^1-norm $\|\mathbf{v}_1\|_1$ of the principal eigenvector $\mathbf{v}_1$ associated with the largest eigenvalue λ_1.

- The quantity $\|\mathbf{v}_1\|_1^2 = \left(\sum_{i=1}^n v_{1i}\right)^2$ has been called *mixedness* of G by Rucker et al. (2002),[23] since it relates to the variance of the principal eigenvector components as follows

$$\sigma_{\mathbf{v}_1}^2 = \frac{1}{n-1}\left(\sum_{i=1}^n v_{1i}^2 - \frac{1}{n}\left(\sum_{i=1}^n v_{1i}\right)^2\right) = \frac{n - \|\mathbf{v}_1\|_1^2}{n(n-1)}.$$

[23] Rucker, G., Rucker, C., Gutman, I., 2002. *On kites, comets, and stars. sums of eigenvector coefficients in (molecular) graphs.* Zeitschrift f ür Naturforschung 57 (3/4), 143153.

- The variance $\sigma^2_{\mathbf{v}_1}$ is decreasing in $\|\mathbf{v}_1\|_1$, and it is minimal for the regular graph where $v_{1i} = 1/\sqrt{n}$ for all $i = 1, \ldots, n$, that is to say they are maximally mixed. Welfare can then be written as

$$\lim_{\phi \uparrow 1/\lambda_{\mathrm{PF}}} W(K_{1,n-1}) = \frac{\omega - \varrho}{2} \frac{\mu^2}{\rho^2} \left(\frac{\varrho}{\omega - \varrho} + \frac{1}{n(1 - (n-1)\sigma^2_{\mathbf{v}_1})} \right).$$

- The implication for heterogeneity resembles the results in Westbrock (2010) on the role of the degree variance in the welfare maximizing graph.
- This suggests that the welfare maximizing graph (among the graphs with a large spectral gap) is eigenvector heterogeneous, or minimally mixed. Rucker et al. (2002) have shown by means of numerical computations for all networks of size $n \leq 10$ that graphs called *k-kites* minimize the mixedness.

- A graph with a principal eigenvalue λ_1 contains the more walks, the larger is $\|\mathbf{v}_1\|_1^2$. Moreover, the reciprocal $1/\|\mathbf{v}_1\|_1^2$ measures the share of self returning walks among all walks. It follows that, a small value of $\|\mathbf{v}_1\|_1^2$ implies a large share of self returning walks, or a small probability that a randomly chosen walk ends at a vertex other than its origin.
- In terms of our model, where the network governs the way knowledge spillovers and diffusion are directed between firms, we thus find that the welfare maximizing graph has a large share of self returning walks, that is, knowledge originating in a firm passes through others before returning to its originator.
- This indicates that maximizing the cross-fertilization of knowledge and *knowledge recombination* between firms is welfare enhancing (cf. Weitzman, 1998).[24]

[24] Weitzman, M. L., 1998. *Recombinant growth.* The Quarterly Journal of Economics 113 (2), 331360.

The Key Player Policy

- When firms compete in independent markets ($\varrho = 0$) then social welfare is given by

$$W(G) = \sum_{i=1}^{n} \left(\frac{q_i^2}{2} + \pi_i \right) = \frac{\omega}{2} \sum_{i=1}^{n} q_i^2,$$

  where $\omega = 3 - \frac{1}{2\gamma}$
- In interdependent markets ($\varrho > 0$) it is given by

$$W(G) = \frac{1}{2} \left(\sum_{i=1}^{n} q_i^2 + \varrho \sum_{i=1}^{n} \sum_{j \neq i}^{n} b_{ij} q_i q_j \right) + \sum_{i=1}^{n} \pi_i,$$

  where equilibrium output and profit are given by Equations (8) and (7). Let G^{-i} be the network obtained from G by removing firm i.
- Then the key firm $i^* \in \mathcal{N} = \{1, \ldots, n\}$ is given by

$$i^* = \arg\max_{i \in \mathcal{N}} \{W(G) - W(G^{-i})\}.$$

Key Player – Independent Markets

- **Proposition:** Assume that goods are not substitutable, i.e. $\rho = 0$, let $\phi < 1/\lambda_{\mathrm{PF}}(G)$ and define $\mathbf{M} \equiv (\mathbf{I}_n - \phi\mathbf{A})^{-1}$. Moreover, let $N_G(\phi, i) = m_{ii}(G, \phi)$ denote the generating function of the number of closed walks that start and terminate at node i. Then the key firm is given by $i^* = \arg\max_{i \in \mathcal{N}} c_i(G, \phi)$, where the centrality of firm i is given by

$$c_i(G,\phi) = \frac{b_{\boldsymbol{\mu},i}(G,\phi)}{N_G(\phi,i)} \left[(\mathbf{M}(G,\phi)\mathbf{b}_{\boldsymbol{\mu}}(G,\phi))_i - \frac{1}{2} \frac{b_{\boldsymbol{\mu},i}(G,\phi)}{N_G(\phi,i)} (\mathbf{M}(G,\phi)^2)_{ii} \right].$$

- **Proposition:** Assume that goods are substitutable, i.e. $\rho > 0$, that the matrix $\mathbf{M}(G, \rho, \lambda) = (\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A})^{-1}$ exists, and let $\mathbf{b}_{\boldsymbol{\mu}}(G, \rho, \lambda) = \mathbf{M}(G, \rho, \lambda)\boldsymbol{\mu}$. Then the key firm is given by $i^* = \arg\max_{i \in \mathcal{N}} c_i(G, \rho, \lambda)$, where the centrality of firm i is given by

$$c_i(G, \rho, \lambda) = \frac{b_{\boldsymbol{\mu},i}(G, \rho, \lambda)}{m_{ii}(G, \rho, \lambda)} \Bigg((\mathbf{M}(G, \rho, \lambda)(\omega\mathbf{I}_n + \varrho\mathbf{B})\mathbf{b}_{\boldsymbol{\mu}}(G, \rho, \lambda))_i - \frac{1}{2} \frac{b_{\boldsymbol{\mu},i}(G, \rho, \lambda)}{m_{ii}(G, \rho, \lambda)} (\mathbf{M}(G, \rho, \lambda)(\omega\mathbf{I}_n + \varrho\mathbf{B})\mathbf{M}(G, \rho, \lambda))_{ii} \Bigg).$$

- Observe the difference in the weighted Bonacich cenralities $\mathbf{b}_{\boldsymbol{\mu}}(G, \cdot)$ in the two previous propositions: While the first is the standard weighted Bonacich centrality of the network G with firm specific weights μ_i (cf. Definition 1 in Ballester et al. , 2006), the Bonacich centrality we consider depends on both, the adjacency matrix $\mathbf{A}$ and the block diagonal matrix $\mathbf{B}$ indicating which firm is competing with which other firm.
- We further find that the centrality measures introduced here differ from the inter centrality introduced in Ballester et al. (2006), where the intercentrality of an agent i in network G is defined as $c_i(G, \phi) = b_i^2(G, \phi)/N_G(\phi, i)$.

The Homogeneous R&D Subsidy Program

- We assume that firms obtain a subsidy $s \geq 0$ per unit of R&D. The profit of firm i can then be written as (cf. e.g. Hinloopen, 2001, 2003)[25]

$$\pi_i = (\bar{\alpha} - \bar{c}_i)q_i - q_i^2 - \varrho q_i \sum_{j \neq i} b_{ij} q_j + q_i e_i + \psi q_i \sum_{j=1}^{n} a_{ij} e_j - \gamma e_i^2 + s e_i. \tag{10}$$

[25] Hinloopen, J., *Subsidizing R&D Cooperatives*, De Economist, Springer, 2001, 149, 313-345.; Hinloopen, J., *R&D efficiency gains due to cooperation*, Journal of Economics, Springer, 2003, 80, 107-125.

- Assume that the matrix $\mathbf{M} = (\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A})^{-1}$ exists, then unique interior Nash equilibrium is given by

$$\mathbf{q} = \bar{\mathbf{q}} + s\mathbf{r}, \tag{11}$$

- where we have denoted by

$$\bar{\mathbf{q}} = \mathbf{M}\boldsymbol{\mu}$$
$$\mathbf{r} = \lambda\mathbf{M}\left(\frac{1}{\psi}\mathbf{u} + \mathbf{d}\right),$$

- and the vector $\bar{\mathbf{q}}$ gives equilibrium quantities in the absence of the subsidy.
- Furthermore, equilibrium profits are given by

$$\pi_i = \left(1 - \frac{1}{4\gamma}\right) q_i^2 + \frac{1}{4\gamma} s^2, \tag{12}$$

Homogeneous R&D Subsidy – Independent Markets

- When firms operate in independent markets, where $\varrho = 0$, respectively $\rho = 0$, gross social welfare is given by

$$W(G, s) = \sum_{i=1}^{n} \left(\frac{q_i^2}{2} + \pi_i \right).$$

- The optimal R&D subsidy s^* is found by maximizing welfare $W(G, s)$ less the cost of the subsidy $s\mathbf{u}^\top \mathbf{e} = s \sum_{i=1}^{n} e_i$ (cf. Spencer, 1983).[26]

[26] Spencer, B. J. & Brander, J. A., *International R & D Rivalry and Industrial Strategy*, The Review of Economic Studies, Oxford University Press, 1983, 50, 707-722.

- The social planner's problem is then given by

$$s^* = \arg\max_{s \in \mathbb{R}_+} \overline{W}(G, s) = \left(W(G, s) - s\mathbf{u}^\top \mathbf{e} \right),$$

where equilibrium output and profit are given by (11) and (12).

- Net social welfare is given by

$$\begin{aligned} \overline{W}(G, s) = W(G, s) - s\sum_{i=1}^{n} e_i &= \sum_{i=1}^{n} \left(q_i^2 + \pi_i - se_i \right) \\ &= \frac{\omega}{2}\sum_{i=1}^{n} q_i^2 - s\frac{1}{2\gamma}\sum_{i=1}^{n} q_i - \frac{n}{4\gamma}s^2, \end{aligned}$$

where we have denoted by $\omega = 3 - \frac{1}{2\gamma}$.

- The FOC of net welfare $\overline{W}(G,s) \equiv W(G,s) - s\mathbf{u}^\top \mathbf{e}$ is given by

$$\frac{\partial \overline{W}(G,s)}{\partial s} = \omega \sum_{i=1}^{n} \bar{q}_i \left(\omega r_i - \frac{1}{2\gamma} \right) + s \sum_{i=1}^{n} \left(\omega r_i^2 - \frac{1}{\gamma} r_i - \frac{1}{2\gamma} \right) = 0.$$

- We then obtain the optimal subsidy level

$$s^* = \frac{\sum_{i=1}^{n} \bar{q}_i \left(\frac{1}{2\gamma} - \omega r_i \right)}{\sum_{i=1}^{n} \left(r_i \left(\omega r_i - \frac{1}{\gamma} \right) - \frac{1}{2\gamma} \right)},$$

where the equilibrium quantities are given by Equation (11).

- For the second-order derivative we obtain

$$\frac{\partial^2 \overline{W}(G,s)}{\partial s^2} = -\frac{1}{2\gamma} \sum_{i=1}^{n} \left(r_i^2 (1 - 6\gamma) + 2 r_i + 1 \right),$$

and we have an interior solution if the condition $\sum_{i=1}^{n} \left(r_i^2 (1 - 6\gamma) + 2 r_i + 1 \right) \geq 0$ is satisfied.

Homogeneous R&D Subsidy – Interdependent Markets

- For a given network G, social welfare $W(G,s)$ is given by the sum of consumer surplus and firms' profits. When firms compete in a homogeneous product oligopoly then social welfare is given by

$$W(G,s) = \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + \varrho \sum_{i=1}^{n} \sum_{j \neq i}^{n} b_{ij} q_i q_j\right) + \sum_{i=1}^{n} \pi_i,$$

- The optimal R&D subsidy s^* is found by maximizing welfare $W(G,s)$ less the cost of the subsidy $s\mathbf{u}^\top \mathbf{e} = s\sum_{i=1}^{n} e_i$ (Spencer, 1983).

- The social planner's problem is then given by

$$s^* = \arg\max_{s \in \mathbb{R}_+} \overline{W}(G, s) = \left(W(G, s) - s\mathbf{u}^\top \mathbf{e}\right),$$

  where equilibrium output and profit are given by (11) and (12).
- Net welfare can be written as

$$\begin{aligned}\overline{W}(G, s) &= \frac{1}{2}\sum_{i=1}^{n} q_i^2 + \frac{\varrho}{2}\sum_{i=1}^{n}\sum_{j \neq i}^{n} b_{ij} q_i q_j + \sum_{i=1}^{n} \pi_i - s\sum_{i=1}^{n} e_i \\ &= \frac{\omega}{2}\sum_{i=1}^{n} q_i^2 + \frac{n}{4\gamma}s^2 + \frac{\varrho}{2}\sum_{i=1}^{n}\sum_{j \neq i}^{n} b_{ij} q_i q_j - \frac{1}{2\gamma}\sum_{i=1}^{n}(q_i + s)s,\end{aligned}$$

  where we have denoted by $\omega = 3 - \frac{1}{2\gamma}$.

- The FOC of net welfare $\overline{W}(G,s)$ is given by

$$\frac{\partial \overline{W}(G,s)}{\partial s} = \sum_{i=1}^{n}\left(\omega\bar{q}_i r_i - \frac{1}{2\gamma}\bar{q}_i + \frac{\varrho}{2}b_{ij}(\bar{q}_i r_j + \bar{q}_j r_i)\right) + s\sum_{i=1}^{n}\left(\omega r_i^2 - \frac{1}{\gamma}r_i - \frac{1}{2\gamma} + \varrho\sum_{j=1}^{n} b_{ij} r_i r_j\right) = 0.$$

- The optimal subsidy level is then given by

$$s^* = \frac{\sum_{i=1}^{n}\left(\bar{q}_i(\omega r_i + \frac{1}{2\gamma}) + \frac{\varrho}{2}\sum_{j=1}^{n} b_{ij}(\bar{q}_i r_j + \bar{q}_j r_i)\right)}{\sum_{i=1}^{n}\left(\frac{1}{2\gamma} + r_i\left(\frac{1}{\gamma} - \omega r_i - \varrho\sum_{j=1}^{n} b_{ij} r_j\right)\right)},$$

where the equilibrium quantities are given by Equation (11).

- The second-order derivative is given by

$$\frac{\partial^2 \overline{W}(G,s)}{\partial s^2} = -\frac{1}{2\gamma}\sum_{i=1}^{n}\left(r_i^2(1-6\gamma)+2r_i+1-2\gamma\varrho\sum_{j=1}^{n}b_{ij}r_ir_j.\right).$$

Hence, the solution is interior if

$$\sum_{i=1}^{n}\left(r_i^2(1-6\gamma)+2r_i+1-2\gamma\varrho\sum_{j=1}^{n}b_{ij}r_ir_j\right)\geq 0.$$

Targeted R&D Subsidy

- In the following we assume that each firm obtains a subsidy $s_i \geq 0$ per unit of R&D for all $i = 1, \ldots, n$. The profit of firm i can then be written as (cf. e.g. Hinloopen, 2001)

$$\pi_i = (\bar{\alpha} - \bar{c}_i) q_i - q_i^2 - \varrho q_i \sum_{j \neq i} b_{ij} q_j + q_i e_i + \psi q_i \sum_{j=1}^{n} a_{ij} e_j - \gamma e_i^2 + s_i e_i .$$

- Assume that the matrix $\mathbf{M} = (\mathbf{I}_n + \rho\mathbf{B} - \lambda\mathbf{A})^{-1}$ exists, then the unique interior Nash equilibrium is given by

$$\mathbf{q} = \bar{\mathbf{q}} + \mathbf{R}\mathbf{s}, \tag{13}$$

- where we have denoted by

$$\bar{\mathbf{q}} = \mathbf{M}\boldsymbol{\mu}$$
$$\mathbf{R} = \lambda\mathbf{M}\left(\frac{1}{\psi}\mathbf{I}_n + \mathbf{A}\right),$$

- and equilibrium profits are given by

$$\pi_i = \left(1 - \frac{1}{4\gamma}\right) q_i^2 + \frac{1}{4\gamma} s_i^2. \tag{14}$$

Targeted R&D Subsidy – Independent Markets

- When firms operate in independent markets, where $\varrho = 0$, social welfare is given by

$$W(G, \mathbf{s}) = \sum_{i=1}^{n} \left(\frac{q_i^2}{2} + \pi_i \right).$$

- The optimal R&D subsidy $\mathbf{s}^* \in \mathbb{R}_+^n$ is found by maximizing welfare $W(G, \mathbf{s})$ less the cost of the subsidy $\mathbf{s}^\top \mathbf{e} = \sum_{i=1}^n s_i e_i$ (Spencer, 1983). The social planner's problem is then given by

$$\mathbf{s}^* = \arg\max_{\mathbf{s} \in \mathbb{R}_+^n} \overline{W}(G, \mathbf{s}) = \left(W(G, \mathbf{s}) - \mathbf{s}^\top \mathbf{e} \right),$$

where equilibrium output and profit are given by (13) and (14).

- Net welfare can be written as follows

$$\overline{W}(G,\mathbf{s}) = \sum_{i=1}^{n}\left(\frac{q_i^2}{2} + \pi_i - s_i e_i\right)$$
$$= \frac{\omega}{2}\sum_{i=1}^{n} q_i^2 - \frac{1}{2\gamma}\sum_{i=1}^{n} q_i s_i - \frac{1}{4\gamma}\sum_{i=1}^{n} s_i^2,$$

where we have denoted by $\omega = 3 - \frac{1}{2\gamma}$.

- The FOC for net welfare $\overline{W}(G,\mathbf{s})$ yields the following system of linear equations

$$\frac{\partial \overline{W}(G,\mathbf{s})}{\partial s_i} = -\frac{1}{2\gamma}\bar{q}_i - \frac{1}{2\gamma}s_i + \sum_{k=1}^{n} r_{ki}\left(\omega\bar{q}_k + \frac{\omega}{2}\sum_{j=1}^{n} r_{kj}s_j - \frac{1}{2\gamma}s_k\right)$$
$$+ \sum_{k=1}^{n}\left(\sum_{j=1}^{n} r_{kj}s_j\right)\left(\frac{1}{2}r_{ki} - \frac{1}{2\gamma}\delta_{ki}\right) = 0.$$

- In vector-matrix notation this can be written as

$$(\mathbf{I}_n + 2\mathbf{R} - 2\gamma\omega\mathbf{R}^2)\mathbf{s} = (2\gamma\omega\mathbf{R} - \mathbf{I}_n)\bar{\mathbf{q}}.$$

- When the conditions for invertibility are satisfied, it then follows that the optimal subsidy levels can be written as

$$\mathbf{s}^* = (\mathbf{I}_n + 2\mathbf{R} - 2\gamma\omega\mathbf{R}^2)^{-1}(2\gamma\omega\mathbf{R} - \mathbf{I}_n)\bar{\mathbf{q}},$$

with $\bar{\mathbf{q}} = (\mathbf{I}_n - \lambda\mathbf{A})^{-1}\boldsymbol{\mu} = \mathbf{b}_{\boldsymbol{\mu}}$.
- The second-order derivative is given by

$$\frac{\partial^2 \overline{W}(G, \mathbf{s})}{\partial \mathbf{s} \partial \mathbf{s}^\top} = -\frac{1}{2\gamma}\left(\mathbf{I}_n + 2\mathbf{R} - 2\gamma\omega\mathbf{R}^2\right).$$

- Hence, we obtain an interior solution if the matrix $\mathbf{I}_n + 2\mathbf{R} - 2\gamma\omega\mathbf{R}^2 = \mathbf{I}_n + (1 - 6\gamma)\mathbf{R}^2 + 2\mathbf{R}$ is positive definite, which means that it is also invertible and its inverse is also positive definite.

Targeted R&D Subsidy – Interdependent Markets

- For a given network G, social welfare $W(G, \mathbf{s})$ is given by the sum of consumer surplus and firms' profits

$$W(G) = \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + \varrho \sum_{i=1}^{n}\sum_{j\neq i}^{n} b_{ij} q_i q_j\right) + \sum_{i=1}^{n} \pi_i,$$

  where equiplibrium output is given by Equation (13) and profits are given by (14)
- The optimal R&D subsidy $\mathbf{s}^* \in \mathbb{R}_+^n$ is found by maximizing welfare $W(G, \mathbf{s})$ less the cost of the subsidy $\mathbf{s}^\top \mathbf{e} = \sum_{i=1}^{n} s_i e_i$ (Spencer, 1983).

- The social planner's problem is then given by

$$\mathbf{s}^* = \arg\max_{\mathbf{s}\in\mathbb{R}_+^n} \overline{W}(G,\mathbf{s}) = \left(W(G,\mathbf{s}) - \mathbf{s}^\top \mathbf{e}\right),$$

where equilibrium output and profit are given by (13) and (14).
- One can show that if the matrix $\mathbf{I}_n - 2\mathbf{R}^\top\left(\gamma(\omega\mathbf{I}_n + \varrho\mathbf{B})\,\mathbf{R} - \mathbf{I}_n\right)$ is positive definite, the optimal subsidy levels are given by

$$\mathbf{s}^* = \left(\mathbf{I}_n - 2\mathbf{R}^\top\left(\gamma(\omega\mathbf{I}_n + \varrho\mathbf{B})\,\mathbf{R} - \mathbf{I}_n\right)\right)^{-1}\left(2\gamma\mathbf{R}^\top(\omega\mathbf{I}_n + \varrho\mathbf{B}) - \mathbf{I}_n\right)\bar{\mathbf{q}}.$$

Empirical Implications – Data

- For the purpose of estimating our model we use the MERIT-CATI database.[27]
- This database contains information about strategic technology agreements, including any alliance that involves some arrangements for mutual transfer of technology or joint research, such as joint research pacts, joint development agreements, cross licensing, R&D contracts, joint ventures and research corporations.
- We used annual data about balance sheets and income statements from Standard & Poor's Compustat US and Global fundamental databases to match it with the firm names in the MERIT-CATI data.
- For this purpose we adopted and extended the name matching algorithm developed as part of the NBER patent data project.[28]

[27] Hagedoorn, J., May 2002. *Inter-firm R&D partnerships: an overview of major trends and patterns since 1960*. Research Policy 31 (4), 477492.
[28] See `https://sites.google.com/site/patentdataproject/Home`.

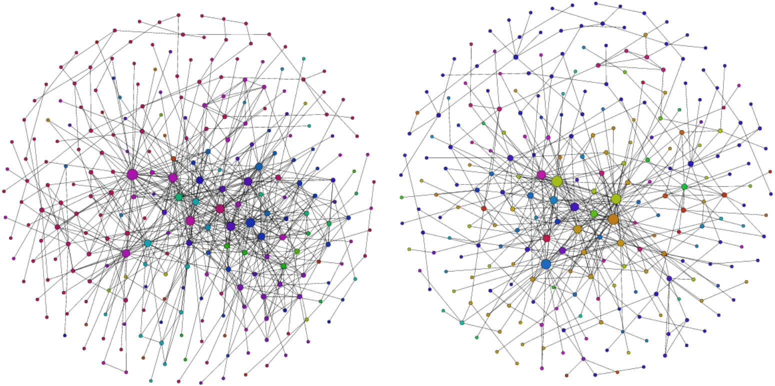

FIGURE: Network snapshots of the largest connected component for the years 1990 ($n = 259$, $m = 621$) and 1995 ($n = 256$, $m = 434$). A node's size indicates its eigenvector centrality. Node colors represent different industry SIC codes at the 4-digit level. The nodes' sizes indicate their degree.

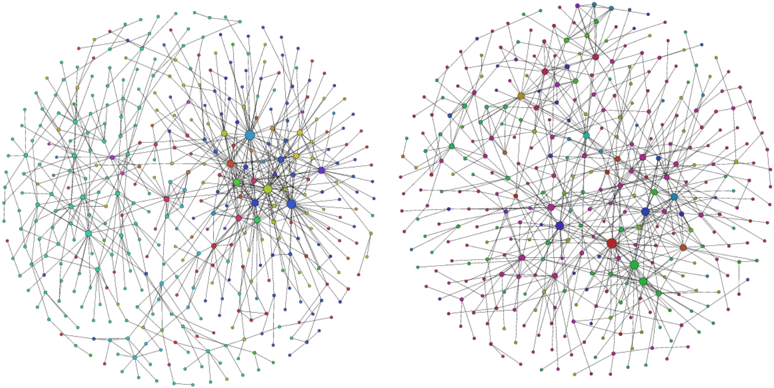

FIGURE: Network snapshots of the largest connected component for the years 2000 ($n = 403$, $m = 635$) and 2005 ($n = 358$, $m = 571$). A node's size indicates its eigenvector centrality. Node colors represent different industry SIC codes at the 4-digit level. The nodes' sizes indicate their degree.

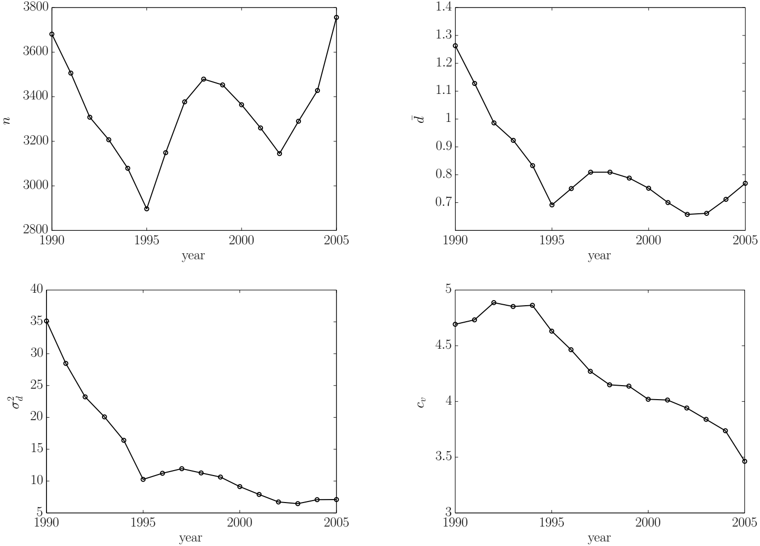

FIGURE: The number of firms n participating in an alliance, the average degree $\bar{d}$, the degree variance σ_d^2 and the degree coefficient of variation $c_v = \sigma_d/\bar{d}$.

- Given the effort level e_{it}, the empirical counterpart to the marginal cost c_{it} of firm i of Equation (2) at period t with $\bar{c}_{it} = \mathbf{x}_{it}'\boldsymbol{\delta} + \eta_i^* + \varepsilon_{it}$ is

$$c_{it} = \mathbf{x}_{it}'\boldsymbol{\delta} + \eta_i^* + \varepsilon_{it} - e_{it} - \psi \sum_{j=1}^{n} a_{ij,t} e_{jt}, \qquad (15)$$

where $\mathbf{x}_{it}$ is a k-dimensional vector of observed exogenous characteristics of firm i, η_i^* captures the unobserved (to the econometrician) firm-specific fixed effect, and ε_{it} captures the remaining unobserved (to econometricians) characteristics of the firms. We assume η_i^* and ε_{it} can be observed by other firms.

- At period t, firm i's profit is given by

$$\pi_{it} = (p_{it} - c_{it})q_{it} - \frac{1}{2\gamma}e_{it}^2. \tag{16}$$

- The inverse demand function for firm i is given by

$$p_{it} = \bar{\alpha}_m + \bar{\alpha}_t - q_{it} - \varrho \sum_{j=1}^{n} b_{ij} q_{jt}, \tag{17}$$

  where $b_{ij} = 1$ if i and j are in the same market and zero otherwise.

- $\bar{\alpha}_m$ captures the *market-specific fixed effect* and $\bar{\alpha}_t$ captures the *time fixed effect* due to exogenous demand shifters that affect consumer income, number of consumers (population), consumer taste and preferences, and expectations over future prices of complements and substitutes or future income.

- Inserting (15) and (17) into (16) gives

$$\begin{aligned}\pi_{it} &= (p_{it} - c_{it})q_{it} - \frac{1}{2\gamma}e_{it}^2 \\ &= (\bar{\alpha}_m + \bar{\alpha}_t - \varrho \sum_{j=1}^{n} b_{ij}q_{jt} - \mathbf{x}_{it}'\boldsymbol{\delta} - \eta_i^* - \varepsilon_{it} \\ &\quad + e_{it} + \psi \sum_{j=1}^{n} a_{ij,t}e_{jt})q_{it} - q_{it}^2 - \frac{1}{2\gamma}e_{it}^2. \end{aligned} \tag{18}$$

- The FOC with respect to effort in Equation (18) is given by

$$\frac{\partial \pi_{it}}{\partial e_{it}} = q_{it} - \frac{1}{\gamma}e_{it} = 0,$$

- which leads to the best response effort

$$e_{it} = \gamma q_{it}. \tag{19}$$

- The FOC with respect to output q_{it} in (18) is given by

$$\frac{\partial \pi_{it}}{\partial q_{it}} = \bar{\alpha}_m + \bar{\alpha}_t - \varrho \sum_{j=1}^{n} b_{ij} q_{jt} - \mathbf{x}'_{it}\boldsymbol{\delta} - \eta_i^* - \varepsilon_{it} + e_{it} + \psi \sum_{j=1}^{n} a_{ij,t} e_{jt} - 2q_{it} = 0,$$

- which leads to the best response output

$$2q_{it} = \bar{\alpha}_m + \bar{\alpha}_t - \varrho \sum_{j=1}^{n} b_{ij} q_{jt} - \mathbf{x}'_{it}\boldsymbol{\delta} - \eta_i^* - \varepsilon_{it} + e_{it} + \psi \sum_{j=1}^{n} a_{ij,t} e_{jt}, \tag{20}$$

- or equivalently

$$q_{it} = \frac{\bar{\alpha}_m - \eta_i^*}{2} + \frac{\bar{\alpha}_t}{2} - \frac{\varrho}{2} \sum_{j=1}^{n} b_{ij} q_{jt} - \frac{1}{2}\mathbf{x}'_{it}\boldsymbol{\delta} - \frac{1}{2}\varepsilon_{it} + \frac{1}{2}e_{it} + \frac{\psi}{2} \sum_{j=1}^{n} a_{ij,t} e_{jt}. \tag{21}$$

- We denote by $\kappa_t \equiv \frac{1}{2-\gamma}\bar{\alpha}_t$, $\eta_i \equiv \frac{1}{2-\gamma}(\bar{\alpha}_m - \eta_i^*)$, $\epsilon_{it} \equiv -\frac{1}{2-\gamma}\varepsilon_{it}$, $\vartheta \equiv -\frac{1}{2-\gamma}\varrho$, $\boldsymbol{\beta} \equiv -\frac{1}{2-\gamma}\boldsymbol{\delta}$ and $\varphi \equiv \frac{1}{2-\gamma}\psi\gamma$.
- Then we can write the best response output of firm i as

$$q_{it} = \varphi \sum_{j=1}^{n} a_{ij,t} q_{jt} + \vartheta \sum_{j=1}^{n} b_{ij} q_{jt} + \mathbf{x}_{it}' \boldsymbol{\beta} + \eta_i + \kappa_t + \epsilon_{it}, \qquad (22)$$

- while the empirical counterpart to Equation (19) is

$$e_{it} = \gamma q_{it} + u_{it}, \qquad (23)$$

  with an i.i.d. error term u_{it}.
- Observe that the econometric specification in Equation (22) is similar to the product competition and technology spillover production function estimation in Bloom et al. (2007).[29]
- However, differently to these authors, we explicitly take into account the technology spillovers stemming from R&D collaborations.

[29] Bloom, N., Schankerman, M., Van Reenen, J., 2007. *Identifying technology spillovers and product market rivalry*, NBER Working Paper No. 13060.

- In vector-matrix form we can write (22) and (23) as

$$\mathbf{q}_t = \varphi \mathbf{A}_t \mathbf{q}_t + \vartheta \mathbf{B} \mathbf{q}_t + \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\eta} + \kappa_t \mathbf{1}_n + \boldsymbol{\epsilon}_t, \tag{24}$$

$$\mathbf{e}_t = \gamma \mathbf{q}_t + \mathbf{u}_t, \tag{25}$$

  where $\mathbf{q}_t = (q_{1t}, \cdots, q_{nt})'$, $\mathbf{e}_t = (e_{1t}, \cdots, e_{nt})'$, $\mathbf{A}_t = [a_{ij,t}]$, $\mathbf{B} = [b_{ij}]$, $\mathbf{X}_t = (\mathbf{x}_{1t}, \cdots, \mathbf{x}_{nt})'$, $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_n)'$, $\boldsymbol{\epsilon}_t = (\epsilon_{1t}, \cdots, \epsilon_{nt})'$, $\mathbf{u}_t = (u_{1t}, \cdots, u_{nt})'$, and $\mathbf{1}_n$ is an n-dimensional vector of ones.

- For the T periods, equations (24) and (25) can be written as

$$\mathbf{q} = \varphi \mathrm{diag}\{\mathbf{A}_t\}\mathbf{q} + \vartheta(\mathbf{I}_T \otimes \mathbf{B})\mathbf{q} + \mathbf{X}\boldsymbol{\beta} + \mathbf{1}_T \otimes \boldsymbol{\eta} + \boldsymbol{\kappa} \otimes \mathbf{1}_n + \boldsymbol{\epsilon}, \tag{26}$$

$$\mathbf{e} = \gamma \mathbf{q} + \mathbf{u}, \tag{27}$$

where $\mathbf{q} = (\mathbf{q}_1', \cdots, \mathbf{q}_T')'$, $\mathbf{e} = (\mathbf{e}_1', \cdots, \mathbf{e}_T')'$, $\mathbf{X} = (\mathbf{X}_1', \cdots, \mathbf{X}_T')'$, $\boldsymbol{\kappa} = (\kappa_1, \cdots, \kappa_T)'$, $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1', \cdots, \boldsymbol{\epsilon}_T')'$, and $\mathbf{u} = (\mathbf{u}_1', \cdots, \mathbf{u}_T')'$.

- We allow fixed effects $\boldsymbol{\eta}$ and $\boldsymbol{\kappa}$ to depend on $\text{diag}\{\mathbf{A}_t\}$, $\mathbf{B}$ and $\mathbf{X}$ by treating them as vectors of unknown parameters. When the number of firms (or the number of time periods) is large, we may have the incidental parameter problem.
- To avoid this problem, we transform (26) using a within projector $\mathbf{J} = \mathbf{J}_T \otimes \mathbf{J}_n$ where $\mathbf{J}_T = \mathbf{I}_T - \frac{1}{T}\mathbf{1}_T\mathbf{1}_T'$ and $\mathbf{J}_n = \mathbf{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n'$. The transformed equation (26) is

$$\mathbf{J}\mathbf{q} = \varphi\mathbf{J}\text{diag}\{\mathbf{A}_t\}\mathbf{q} + \vartheta\mathbf{J}(\mathbf{I}_T \otimes \mathbf{B})\mathbf{q} + \mathbf{J}\mathbf{X}\boldsymbol{\beta} + \mathbf{J}\boldsymbol{\epsilon}. \qquad (28)$$

- To estimate (28), we consider the IV matrix $\mathbf{Q} = \mathbf{J}[\text{diag}\{\mathbf{A}_t\}\mathbf{X}, (\mathbf{I}_T \otimes \mathbf{B})\mathbf{X}, \mathbf{X}]$, where $\mathbf{J}\text{diag}\{\mathbf{A}_t\}\mathbf{X}$ are IVs for the collaboration effect and $\mathbf{J}(\mathbf{I}_T \otimes \mathbf{B})\mathbf{X}$ are IVs for the competition effect. Let $\mathbf{P}_1 = \mathbf{Q}(\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'$ and $\mathbf{Z} = [\text{diag}\{\mathbf{A}_t\}\mathbf{q}, (\mathbf{I}_T \otimes \mathbf{B})\mathbf{q}, \mathbf{X}]$. The 2SLS estimator for coefficients in (28) is given by $(\mathbf{Z}'\mathbf{P}_1\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{P}_1\mathbf{q}$.
- With the estimates of $\varphi, \vartheta, \boldsymbol{\beta}$, we can recover $\boldsymbol{\eta}$ and $\boldsymbol{\kappa}$ by the least squares dummy variable method.
- From (26), we can use $\mathbf{X}$ as an IV for the endogenous regressor $\mathbf{q}$ in equation (27). Let $\mathbf{P}_2 = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. The 2SLS estimator for γ is given by $(\mathbf{q}'\mathbf{P}_2\mathbf{q})^{-1}\mathbf{q}'\mathbf{P}_2\mathbf{e}$.

TABLE: Parameter estimates (with standard errors in parenthesis) from a panel regression with time dummies of Equations (24) and (25). Model A does not include firm fixed effects (f.e.), while Model B introduces also firm fixed effects.

	Model A		Model B	
time f.e.	yes		yes	
firm f.e.	no		yes	
φ	0.0278***	(0.0034)	0.0070***	(0.0026)
ϑ	-0.0036***	(0.0004)	-0.0019***	(0.0006)
β_1	0.0749***	(0.0056)	0.0463***	(0.0089)
β_2	0.8465***	(0.0480)	1.0523***	(0.0466)
ϕ	0.0329***	(0.0018)	0.0329***	(0.0018)

*** Statistically significant at 1% level.
** Statistically significant at 5% level.
* Statistically significant at 10% level.

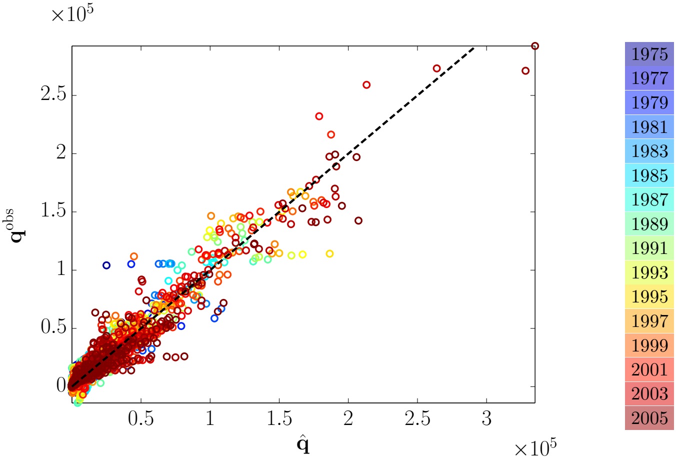

FIGURE: Predicted output $\hat{\mathbf{q}}$ vs. observed output $\mathbf{q}^{\mathrm{obs}}$ with the estimates from Table 1. The coefficient of determination is $R^2 = 0.9447$.

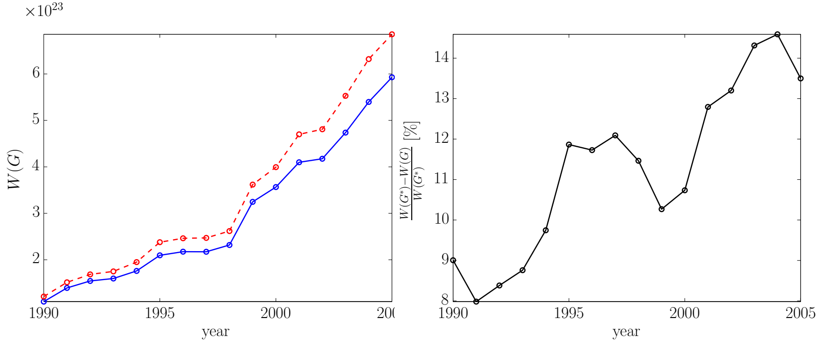

FIGURE: (Left panel) Welfare from a lower bound on welfare in the efficient graph (dashed line) and actual welfare (straight line). (Right panel) A lower bound on the relative percentage welfare loss in the observed network structure.

- The above figure compares a lower bound on welfare for the efficient graph with the actual value for each year of observation.
- For this lower bound we have evaluated welfare for the star network $K_{1,n-1}$ with the firm with the highest firm fixed effect μ_i among all $i = 1, \ldots, n$ in the center.
- Welfare in the star network is always higher than in the observed network.
- Moreover, we find that the welfare loss incurred from a non-optimal network structure can go up to at least 15%.
- This result indicates that industry concentration can be welfare improving (cf. Westbrock, 2010).[30]

[30] Westbrock, B., 2010. *Natural concentration in industrial research collaboration.* The RAND Journal of Economics 41 (2), 351371.

TABLE: Key player ranking for the year 1990 for the first 25 firms.

Firm	Share [%][a]	$\mathbf{d}$	$\boldsymbol{cor}$[b]	$\mathbf{v}_{\mathrm{PF}}$	Betweenness[c]	Closeness[d]	$q_i/\|\mathbf{q}\|_1$ [%][e]	$\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1}$ [%][f]	$\frac{W(G)-W(G^{-i})}{W(G)}$ [%]	Rank
General Motors Corp.	12.1445	71	13	0.0545	0.0436	451.4219	6.3868	6.9548	26.1895	1
Exxon Corp.	10.1151	20	12	0.0146	0.0023	352.7285	5.6310	5.6582	20.0114	2
DaimlerChrysler Corp	5.2310	14	11	0.0195	0.0017	330.0020	2.7845	2.7652	4.7664	3
Siemens A.G.	20.1008	142	14	0.1877	0.0911	518.0625	2.6481	3.0801	4.7141	4
Toyota Motor Corp.	6.2806	43	13	0.0549	0.0153	407.9688	2.3643	2.4894	3.6432	5
Chevron	3.7009	24	12	0.0141	0.0079	351.7266	2.3346	2.3785	3.6077	6
Fiat SpA.	4.7173	32	11	0.0408	0.0168	396.7344	2.2644	2.3316	3.4254	7
Texaco Inc.	3.9206	22	12	0.0158	0.0028	349.6562	2.0619	2.1155	2.8536	8
Hitachi Ltd.	37.6873	75	14	0.1289	0.0359	478.9062	2.0948	2.2311	2.8436	9
Volkswagen A.G.	4.1641	15	6	0.0096	0.0047	281.2852	2.0732	2.1032	2.7253	10
Altria Group	57.0787	0	0	0.0000	0.0000	0.0000	1.8096	1.8096	2.0193	11
Renault	2.9712	12	6	0.0042	0.0020	270.2812	1.4652	1.4655	1.3496	12
Toshiba Corp.	10.4548	78	14	0.1312	0.0313	460.5176	1.3695	1.4819	1.2849	13
Hoechst A.G.	13.8715	23	8	0.0115	0.0127	348.9766	1.3674	1.3890	1.1965	14
Unilever N.V./Plc.	8.2910	11	7	0.0068	0.0035	323.2695	1.3842	1.3815	1.1803	15
Elf Aquitaine	3.1007	7	3	0.0025	0.0049	259.8105	1.3961	1.3813	1.1778	16
Sony Corp.	32.0711	41	14	0.0883	0.0110	404.7207	1.2995	1.3867	1.1035	17
Bayer A.G.	12.8762	10	4	0.0016	0.0056	251.6250	1.2787	1.2797	1.0223	18
Alcatel-Lucent	31.0329	0	0	0.0000	0.0000	0.0000	1.2259	1.2030	0.9260	19
Boeing Company	37.1888	5	4	0.0086	0.0001	278.9453	1.2054	1.2029	0.9010	20
Procter & Gamble	58.8860	5	3	0.0002	0.0013	168.6270	1.1413	1.1567	0.8038	21
Metro AG	11.3765	0	0	0.0000	0.0000	0.0000	1.0519	1.0519	0.6823	22
Total SA	2.2696	0	0	0.0000	0.0000	0.0000	1.0404	1.0227	0.6463	23
Pepsico Inc.	52.5069	0	0	0.0000	0.0000	0.0000	1.0222	1.0202	0.6441	24
Thyssen A.G.	76.5099	2	1	0.0005	0.0009	161.7305	0.9655	0.9655	0.5749	25

[a] Market share in the primary 4-digit sector in which the firm is operating.
[b] The coreness of node i, cor_i, is k if and only if $i \in G_k$ and $i \notin G_{k+1}$. We have that $\mathrm{cor}_i \leq d_i$.
[c] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.
[d] The closeness centrality of node i is computed as $\sum_{j=1}^{n} 2^{-d_G(i,j)}$, where $d_G(i,j)$ is the length of the shortest path between i and j in the network G (Dangalchev, 2006).
[e] The relative output of a firm i is computed as $q_i/\|\mathbf{q}\|_1 = b_{\boldsymbol{\mu},i}/\|\mathbf{b}_{\boldsymbol{\mu}}\|_1$.
[f] The decrease in output due to the removal of firm i is computed as $\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1} = \frac{b_{\mathbf{u},i}(G)b_{\boldsymbol{\mu},i}(G)}{m_{ii}(G)}/\|\mathbf{b}_{\boldsymbol{\mu}}(G)\|_1$.

TABLE: Key player ranking for the year 2005 for the first 25 firms.

Firm	Share [%][a]	d	*cor*[b]	$\mathbf{v}_{PF}$	Betweenness[c]	Closeness[d]	$q_i/\|\mathbf{q}\|_1$ [%][e]	$\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1}$ [%][f]	$\frac{W(G)-W(G^{-i})}{W(G)}$ [%]	Rank
Exxon Corp.	7.8647	0	0	0.0000	0.0000	0.0000	3.7309	3.6956	16.7222	1
DaimlerChrysler Corp	7.5743	26	8	0.0086	0.0166	124.7754	2.8003	2.8927	9.7477	2
Toyota Motor Corp.	7.7760	10	8	0.0049	0.0010	103.9712	2.6744	2.6696	8.8657	3
General Motors Corp.	7.7341	17	7	0.0065	0.0086	119.6819	2.4176	2.4638	7.2635	4
Total SA	3.6544	0	0	0.0000	0.0000	0.0000	2.1774	2.1568	5.6712	5
Mitsubishi Corp	87.2569	11	10	0.1259	0.0004	168.5938	2.0457	2.1371	5.1913	6
Chevron	4.4312	6	6	0.0001	0.0000	44.0676	1.9724	1.9538	4.6487	7
Volkswagen A.G.	4.8178	11	8	0.0046	0.0051	104.1240	1.7631	1.7474	3.8583	8
Mitsui Group	30.0437	3	3	0.0008	0.0000	53.0688	1.7001	1.7056	3.4748	9
Itochu Corp.	21.1047	2	1	0.0000	0.0007	25.0889	1.3800	1.3842	2.2842	10
Hitachi Ltd.	27.8692	30	10	0.1718	0.0282	200.1504	1.2883	1.4023	2.1411	11
Sumitomo Corp	90.5320	1	1	0.0000	0.0000	1.5000	1.2806	1.2806	1.9770	12
RWE AG	3.5459	0	0	0.0000	0.0000	0.0000	1.2495	1.2262	1.8721	13
Marubeni Corp.	17.5319	0	0	0.0000	0.0000	0.0000	1.1755	1.1710	1.6550	14
Siemens A.G.	11.0608	13	5	0.0255	0.0059	140.9321	1.1065	1.1287	1.4847	15
UBS AG	66.4551	0	0	0.0000	0.0000	0.0000	0.9381	0.9381	1.0609	16
Sony Corp.	32.1340	33	10	0.2352	0.0171	212.3281	0.7980	0.8702	0.8779	17
NTT DoCoMo	4.3962	16	7	0.1035	0.0086	176.1514	0.8158	0.8445	0.8543	18
Altria Group	40.0416	0	0	0.0000	0.0000	0.0000	0.8349	0.8333	0.8390	19
Fiat SpA.	2.3538	17	9	0.0051	0.0044	97.6677	0.8024	0.7841	0.7784	20
Metro AG	17.6754	2	2	0.0171	0.0000	112.4143	0.7942	0.8048	0.7721	21
Toshiba Corp.	9.9939	40	10	0.2512	0.0215	214.1133	0.7217	0.8056	0.7381	22
Intel Corp.	9.8341	60	8	0.2462	0.0385	221.3911	0.7279	0.7321	0.7053	23
Endesa	1.5322	0	0	0.0000	0.0000	0.0000	0.7678	0.7535	0.7034	24
Renault	2.0905	6	4	0.0029	0.0013	91.4758	0.7420	0.7197	0.6349	25

[a] Market share in the primary 4-digit sector in which the firm is operating.
[b] The coreness of node i, cor_i, is k if and only if $i \in G_k$ and $i \notin G_{k+1}$. We have that $\text{cor}_i \leq d_i$.
[c] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.
[d] The closeness centrality of node i is computed as $\sum_{j=1}^{n} 2^{-d_G(i,j)}$, where $d_G(i,j)$ is the length of the shortest path between i and j in the network G (Dangalchev, 2006).
[e] The relative output of a firm i is computed as $q_i/\|\mathbf{q}\|_1 = b_{\boldsymbol{\mu},i}/\|\mathbf{b}_{\boldsymbol{\mu}}\|_1$.
[f] The decrease in output due to the removal of firm i is computed as $\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1} = \frac{b_{\mathbf{u},i}(G) b_{\boldsymbol{\mu},i}(G)}{m_{ii}(G)} / \|\mathbf{b}_{\boldsymbol{\mu}}(G)\|_1$.

- A ranking of the first 25 firms with the highest impact on welfare upon exit in the year 1990 can be found in Table 2 while the corresponding ranking in the year 2005 is shown in Table 3.
- We observe that the ranking of degree, or other centrality measures often used in the literature such as betweenness centrality or closeness centrality (cf. Wasserman & Faust, 1994)[31] can explain the ranking of firms that we find.
- The table also shows the relative decrease in output incurred by a removal of the respective firm. This quantity is closest to the intercentrality index introduced in Ballester et al. (2006).[32] However, we find that the ranking computed from a decline in welfare that we use here does not coincide with a ranking computed on the basis of a decline in aggregate production.

[31] Wasserman, S., Faust, K., 1994. *Social Network Analysis: Methods and Applications.* Cambridge University Press.

[32] Ballester, C., Calvo-Armengol, A., Zenou, Y., 2006. *Whos who in networks. wanted: The key player.* Econometrica 74 (5), 14031417.

- From the tables showing the key player rankings we find that the decline in welfare due to the removal of the highest ranked firm can amount to 26% in the year 1990 while in the year 2005 it is 17%.
- These tables also show the coreness of a firm. The coreness is a lower bound on the Bonacich centrality of a firm in the network.
- The coreness of networks of firms has also been studied empirically in Kitsak et al. (2010)[33], where it is found that the coreness of a firm correlates with its market value.
- We can easily explain this from our model because we know that firms in higher cores tend to have higher Bonacich centrality, and therefore higher sales and profits.

[33] Kitsak, M., Riccaboni, M., Havlin, S., Pammolli, F., Stanley, H., 2010. *Scale-free models for the structure of business firm networks.* Physical Review E 81, 036117.

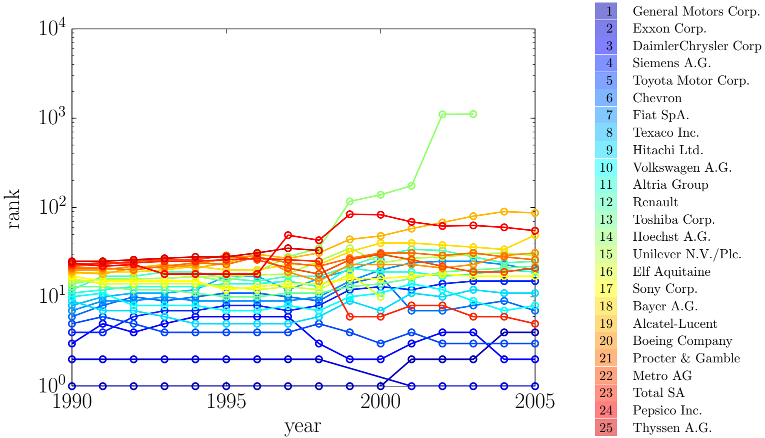

FIGURE: The change in the ranking of the 25 highest ranked firms in the year 1990 from Table 2 to the year 2005.

- Figure 8 shows the change in the ranking of the 25 highest ranked firms in the year 1990 from Table 2 over the years 1990 to 2005.
- The ranking of firms can be quite stable for some, while it is rather versatile for others.
- For example *Daimler Chrysler Corp.* (respectively, *Daimler Benz AG*) is among the three highest ranked firms in 1990, and in 2005 it is the second highest ranked firm.
- In contrast, *Hoechst A.G.*, which was among the 14th highest ranked firms in 1990, slipped down to rank 1112 in the year 2003.

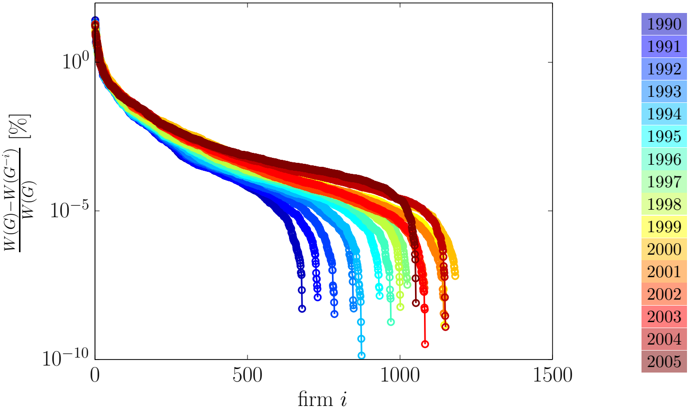

FIGURE: The ordered percentage decrease in welfare due to the removal of firm i over the years 1990 to 2005. The exit of most firms has only a minor impact on welfare, while the highest ranked ones can considerably affect welfare.

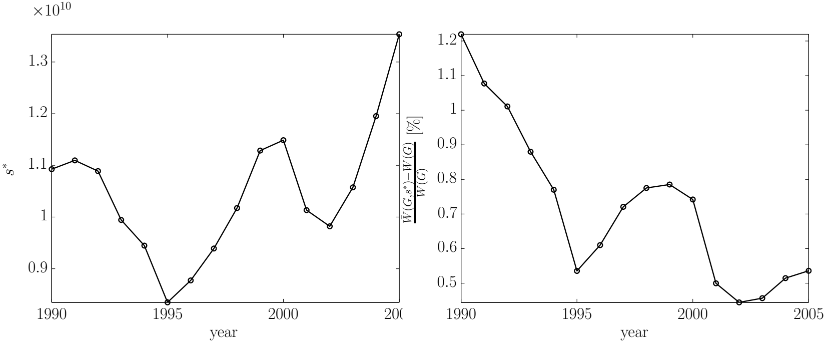

FIGURE: (Left panel) The optimal subsidy level s^* over time. (Right panel) The percentage increase in welfare due to the subsidy s^* over time.

TABLE: Subsidies ranking for the year 1990 for the first 25 firms.

Firm	Share [%][a]	$\mathbf{d}$	cor[b]	$\mathbf{v}_{\mathrm{PF}}$	Betweenness[c]	Closeness[d]	$q_i/\|\mathbf{q}\|_1$ [%][e]	$\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1}$ [%][f]	$\mathbf{s}^* \cdot 10^{-12}$	Rank
Intel Corp.	12.2966	66	14	0.1404	0.0222	458.6562	0.0757	0.0850	0.6835	1
Siemens A.G.	20.1008	142	14	0.1877	0.0911	518.0625	2.6481	3.0801	0.6296	2
General Motors Corp.	12.1445	71	13	0.0545	0.0436	451.4219	6.3868	6.9548	0.6161	3
Sun Microsystems	11.0880	88	14	0.1664	0.0222	434.1582	0.1983	0.2290	0.5603	4
Texas Instruments Inc.	20.5932	67	14	0.1217	0.0159	415.5879	0.3183	0.3528	0.5086	5
Motorola Inc.	18.5193	59	14	0.1340	0.0172	424.8301	0.6078	0.6790	0.5048	6
National Semiconductor Corp.	5.3366	42	14	0.1048	0.0045	422.4453	0.1260	0.1326	0.4912	7
Toyota Motor Corp.	6.2806	43	13	0.0549	0.0153	407.9688	2.3643	2.4894	0.4306	8
Toshiba Corp.	10.4548	78	14	0.1312	0.0313	460.5176	1.3695	1.4819	0.4208	9
Electronic Data Systems Corp.	6.8935	21	14	0.0711	0.0045	381.2832	0.4214	0.4393	0.4025	10
TRW Inc	7.0559	43	13	0.0515	0.0111	364.2559	0.4283	0.4569	0.4005	11
Honeywell Inc.	63.9769	51	14	0.1004	0.0117	416.0898	0.2564	0.2770	0.3924	12
McDonnell Douglas Corp.	21.8941	44	12	0.0338	0.0125	343.3789	0.7611	0.8368	0.3877	13
Hitachi Ltd.	37.6873	75	14	0.1289	0.0359	478.9062	2.0948	2.2311	0.3512	14
Fiat SpA.	4.7173	32	11	0.0408	0.0168	396.7344	2.2644	2.3316	0.3427	15
Harris Corp.	5.1937	31	14	0.0739	0.0050	388.8887	0.1532	0.1645	0.3241	16
Texaco Inc.	3.9206	22	12	0.0158	0.0028	349.6562	2.0619	2.1155	0.3082	17
Tektronix Inc.	17.5728	42	14	0.0909	0.0054	360.7246	0.0781	0.0845	0.2925	18
Sequent Computer Systems Inc.	1.1185	23	13	0.0632	0.0030	343.7422	0.0245	0.0253	0.2853	19
Novell Inc.	0.5695	37	14	0.0873	0.0061	366.3691	0.0236	0.0252	0.2812	20
Xerox Corp.	84.2264	30	14	0.0817	0.0045	385.7695	0.6497	0.6918	0.2793	21
Chevron	3.7009	24	12	0.0141	0.0079	351.7266	2.3346	2.3785	0.2737	22
Unisys Corp.	10.9318	38	14	0.0802	0.0181	366.0273	0.4398	0.4599	0.2727	23
Sony Corp.	32.0711	41	14	0.0883	0.0110	404.7207	1.2995	1.3867	0.2664	24
Exxon Corp.	10.1151	20	12	0.0146	0.0023	352.7285	5.6310	5.6582	0.2603	25

[a] Market share in the primary 4-digit sector in which the firm is operating.

[b] The coreness of node i, cor_i, is k if and only if $i \in G_k$ and $i \notin G_{k+1}$. We have that $\mathrm{cor}_i \leq d_i$.

[c] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.

[d] The closeness centrality of node i is computed as $\sum_{j=1}^{n} 2^{-d_G(i,j)}$, where $d_G(i,j)$ is the length of the shortest path between i and j in the network G (Dangalchev, 2006).

[e] The relative output of a firm i is computed as $q_i/\|\mathbf{q}\|_1 = b_{\boldsymbol{\mu},i}/\|\mathbf{b}_{\boldsymbol{\mu}}\|_1$.

[f] The decrease in output due to the removal of firm i is computed as $\frac{\|\mathbf{q}(G)\|_1-\|\mathbf{q}(G^{-i})\|_1}{\|\mathbf{q}(G)\|_1} = \frac{b_{\mathbf{u},i}(G) b_{\boldsymbol{\mu},i}(G)}{m_{ii}(G)} / \|\mathbf{b}_{\boldsymbol{\mu}}(G)\|_1$.

TABLE: Subsidies ranking for the year 2005 for the first 25 firms.

Firm	Share [%][a]	$\mathbf{d}$	$\boldsymbol{cor}$[b]	$\mathbf{v}_{\mathrm{PF}}$	Betweenness[c]	Closeness[d]	$q_i/\Vert\mathbf{q}\Vert_1$ [%][e]	$\frac{\Vert\mathbf{q}(G)\Vert_1-\Vert\mathbf{q}(G^{-i})\Vert_1}{\Vert\mathbf{q}(G)\Vert_1}$ [%][f]	$\mathbf{s}^* \cdot 10^{-12}$	Rank
Toshiba Corp.	9.9939	40	10	0.2512	0.0215	214.1133	0.7217	0.8056	0.7492	1
Fujitsu Ltd.	17.3622	30	10	0.1993	0.0159	204.4375	0.5489	0.5861	0.7061	2
Sony Corp.	32.1340	33	10	0.2352	0.0171	212.3281	0.7980	0.8702	0.6701	3
Microsoft Corp.	21.5980	53	8	0.1986	0.0856	245.1406	0.3186	0.3302	0.6051	4
Hitachi Ltd.	27.8692	30	10	0.1718	0.0282	200.1504	1.2883	1.4023	0.5715	5
DaimlerChrysler Corp	7.5743	26	8	0.0086	0.0166	124.7754	2.8003	2.8927	0.5409	6
Intel Corp.	9.8341	60	8	0.2462	0.0385	221.3911	0.7279	0.7321	0.5257	7
Sharp Corp.	8.5948	19	10	0.1325	0.0056	160.2207	0.3600	0.3670	0.4821	8
General Motors Corp.	7.7341	17	7	0.0065	0.0086	119.6819	2.4176	2.4638	0.4711	9
Toyota Motor Corp.	7.7760	10	8	0.0049	0.0010	103.9712	2.6744	2.6696	0.4686	10
Mitsubishi Corp	87.2569	11	10	0.1259	0.0004	168.5938	2.0457	2.1371	0.4509	11
NTT DoCoMo	4.3962	16	7	0.1035	0.0086	176.1514	0.8158	0.8445	0.3851	12
Motorola Inc.	12.4529	53	7	0.1643	0.0697	226.1182	0.2207	0.2409	0.3776	13
Mitsubishi Electric Corp	5.6782	13	8	0.1218	0.0054	189.0078	0.4231	0.4420	0.3755	14
Continental A.G.	4.3929	9	8	0.0046	0.0001	99.3442	0.2786	0.2777	0.3721	15
Volkswagen A.G.	4.8178	11	8	0.0046	0.0051	104.1240	1.7631	1.7474	0.3580	16
Cisco Systems Inc	63.1857	26	8	0.1322	0.0175	197.3105	0.2771	0.2984	0.3354	17
Lear Corp	26.7974	10	6	0.0130	0.0100	136.8804	0.2392	0.2473	0.3316	18
Infineon Technologies AG	2.1293	40	7	0.1879	0.0181	209.3833	0.1713	0.1682	0.3234	19
Sun Microsystems	7.3032	26	7	0.1003	0.0224	198.1792	0.1719	0.1822	0.3211	20
Johnson Controls Inc.	43.0902	10	6	0.0030	0.0027	93.0432	0.2931	0.3029	0.3039	21
Texas Instruments Inc.	3.3920	18	7	0.0814	0.0048	165.2466	0.1699	0.1591	0.2734	22
Oracle Corp.	7.8059	14	5	0.0358	0.0075	162.6182	0.1583	0.1478	0.2484	23
Omron Corp.	0.9875	8	3	0.0114	0.0054	103.6377	0.1137	0.1147	0.1678	24
Comcast Corp	16.9208	11	7	0.0585	0.0027	152.2764	0.5242	0.5389	0.1618	25

[a] Market share in the primary 4-digit sector in which the firm is operating.
[b] The coreness of node i, cor_i, is k if and only if $i \in G_k$ and $i \notin G_{k+1}$. We have that $\mathrm{cor}_i \leq d_i$.
[c] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.
[d] The closeness centrality of node i is computed as $\sum_{j=1}^{n} 2^{-d_G(i,j)}$, where $d_G(i,j)$ is the length of the shortest path between i and j in the network G (Dangalchev, 2006).
[e] The relative output of a firm i is computed as $q_i/\Vert\mathbf{q}\Vert_1 = b_{\boldsymbol{\mu},i}/\Vert\mathbf{b}_{\boldsymbol{\mu}}\Vert_1$.
[f] The decrease in output due to the removal of firm i is computed as $\frac{\Vert\mathbf{q}(G)\Vert_1-\Vert\mathbf{q}(G^{-i})\Vert_1}{\Vert\mathbf{q}(G)\Vert_1} = \frac{b_{\mathbf{u},i}(G) b_{\boldsymbol{\mu},i}(G)}{m_{ii}(G)} / \Vert\mathbf{b}_{\boldsymbol{\mu}}(G)\Vert_1$.

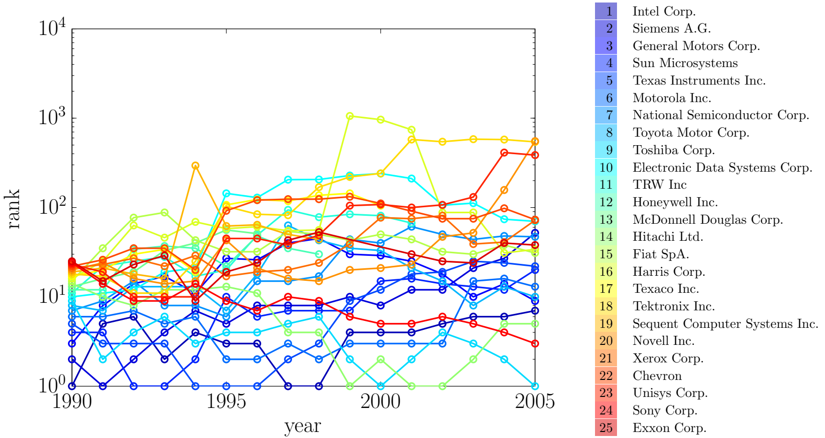

FIGURE: The change in the ranking of the 25 highest subsidized firms in the year 1990 from Table 4 to the year 2005.

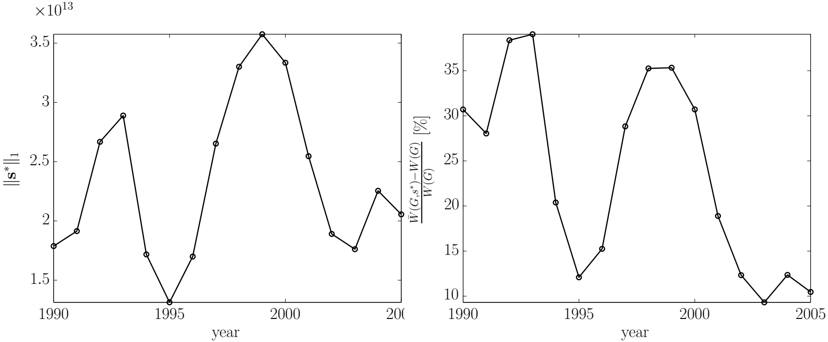

FIGURE: (Left panel) The total subsidy level $\|\mathbf{s}^*\|_1$ when the subsidies are targeted towards specific firms. (Right panel) The percentage increase in welfare due to the targeted subsidies $\mathbf{s}^*$ over time.

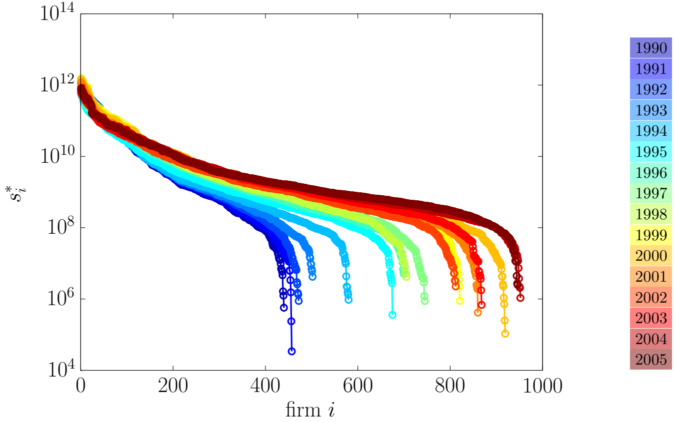

FIGURE: The ordered targeted subsidy level of firm i over the years 1990 to 2005.

- We find that a targeted subsidy program can improve welfare by up to 37%.
- Moreover, the optimal subsidy levels show a strong variation over time. Both, the homogeneous and the aggregate targeted subsidy seem to follow a cyclical trend that resembles the one we have observed for the number of firms with R&D collaborations and the average number of collaborations in a given year.
- This cyclical trend is also reminiscent of the R&D expenditures observed in the empirical literature on business cycles (cf. Barlevy, 2007; Gali, 1999).[34,35]

[34] Barlevy, G., 2007. *On the cyclicality of research and development.* The American Economic Review 97 (4), 11311164.

[35] Gali, J., 1999. *Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?*, American Economic Review 89 (1), 249271

- We can compare the optimal subsidy level predicted from our model with the R&D tax subsidies actually implemented in the United States and selected other countries between 1979 to 1997 (see Bloom et al, 2002; Imullitti, 2010).[36,37]
- While these time series typically show an increase of R&D subsidies over time, they do not seem to incorporate the cyclicality that we obtain for the optimal subsidy levels. Our analysis thus suggests that policy makers should adjust R&D subsidies to these cycles.

[36] Bloom, N., Griffith, R., Van Reenen, J., 2002. *Do R&D tax credits work? evidence from a panel of countries* 19791997. Journal of Public Economics 85 (1), 131.

[37] Impullitti, G., 2010. *International competition and U.S. R&D subsidies: A quantitative welfare analysis.* International Economic Review 51 (4), 11271158.

- While studies such as Spencer & Brander (1983)[38] and Acemoglu et al. (2012)[39] find that R&D often should be taxed rather than subsidized, we find in line with e.g. Hinloopen (2001)[40] that R&D subsidies can have a significantly positive effect on welfare.
- As argued by Hinloopen (2001) the reason why our results differ from Spencer & Brander (1983) is that we take into account consumer surplus when deriving the optimal R&D subsidy.
- Moreover, in contrast to Acemoglu et al. (2012) we do not focus on entry and exit but incorporate the network of R&D collaborating firms. We see our analysis as complementary to Acemoglu et al. (2012), and we show that R&D subsidies can trigger considerable welfare gains when technology spillovers through R&D alliances are incorporated.

[38] Spencer, B. J., Brander, J. A., 1983. *International R&D rivalry and industrial strategy.* The Review of Economic Studies 50 (4), 707722

[39] Acemoglu, D., Akcigit, U., Bloom, N., Kerr, W., 2012. *Innovation, reallocation and growth.* Stanford University Working Paper.

[40] Hinloopen, J., 2001. *Subsidizing R&D cooperatives.* De Economist 149 (3), 313345.

Extension: Bertrand Competition

- In the case of price setting firms we obtain from the profit function (3) the FOC with respect to price p_i for firm i

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - c_i)\frac{\partial q_i}{\partial p_i} - q_i = 0.$$

- When $i \in \mathcal{M}_m$, then observe that from the inverse demand in Equation (1) we find that

$$q_i = \frac{\alpha_m(1-\varrho_m) - (1-(n_m-2)\varrho_m)p_i + \varrho_m \sum_{j\in\mathcal{M}_m,\, j\neq i} p_j}{(1-\rho)(1+(n_m-1)\varrho_m)},$$

where $n_m \equiv |\mathcal{M}_m|$.

- The FOC with respect to R&D effort is the same as in the case of perfect competition, so that we get $e_i = \frac{1}{2\gamma} q_i$. Inserting equilibrium effort and rearranging terms gives

$$
\begin{aligned}
q_i = {} & \frac{2\gamma(1-(n_m-2)\varrho_m)(\alpha_m - \bar{c}_i)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m - \varrho_m) - 1(1-(n_m-2)\varrho_m)} \\
& - \frac{2\gamma\varrho_m(1-(n_m-2)\varrho_m)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m - \varrho_m) - 1(1-(n_m-2)\varrho_m)} \sum_{\substack{j \in \mathcal{M}_m, \\ j \neq i}} q_j \\
& + \frac{\psi(1-(n_m-2)\varrho_m)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m - \varrho_m) - 1(1-(n_m-2)\varrho_m)} \sum_{j=1}^{n} a_{ij} q_j .
\end{aligned}
$$

- If we denote by

$$
\mu_i \equiv \frac{2\gamma(1-(n_m-2)\varrho_m)(\alpha_m-\bar{c}_i)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m-\varrho_m)-1(1-(n_m-2)\varrho_m)},
$$
$$
\rho \equiv \frac{2\gamma\varrho_m(1-(n_m-2)\varrho_m)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m-\varrho_m)-1(1-(n_m-2)\varrho_m)},
$$
$$
\lambda \equiv \frac{\psi(1-(n_m-2)\varrho_m)}{2\gamma\varrho_m(4-(2-\varrho_m)n_m-\varrho_m)-1(1-(n_m-2)\varrho_m)}.
$$

- Then we can write equilibrium quantities as follows

$$
q_i = \mu_i - \rho \sum_{j=1}^{n} b_{ij} q_j + \lambda \sum_{j=1}^{n} a_{ij} q_j. \tag{29}
$$

- Observe that the reduced form Equation (29) is identical to the Cournot case in Equation (6).

Extension: Intra- vs. Interindustry Collaborations

- The marginal cost of production is

$$c_i = \bar{c}_i - e_i - \psi_1 \sum_{j=1}^{n} a_{ij}^{(1)} e_j - \psi_2 \sum_{j=1}^{n} a_{ij}^{(2)} e_j.$$

- If the matrix $\mathbf{I}_n + \rho \mathbf{B} + \lambda_1 \mathbf{A}^{(1)} + \lambda_2 \mathbf{A}^{(2)}$ is invertible, this gives us the equilibrium quantities

$$\mathbf{q} = (\mathbf{I}_n + \rho \mathbf{B} + \lambda_1 \mathbf{A}^{(1)} + \lambda_2 \mathbf{A}^{(2)})^{-1} \boldsymbol{\mu}.$$

- The econometric specification in vector-matrix form can be written as

$$\mathbf{q}_t = \vartheta \mathbf{B} \mathbf{q}_t + \xi \mathbf{A}_t^{(1)} \mathbf{q}_t + \varphi \mathbf{A}_t^{(2)} \mathbf{q}_t + \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\eta} + \boldsymbol{\kappa}_t + \boldsymbol{\epsilon}_t, \quad (30)$$

$$\mathbf{e}_t = \Phi \mathbf{q}_t + \mathbf{u}_t. \quad (31)$$

TABLE: Parameter estimates (with standard errors in parenthesis) from a fixed effects panel regression with time dummies of Equations (30) and (31). Model A does not include firm fixed effects (f.e.), while Model B introduces also firm fixed effects.

	Model C		Model D	
time f.e.	yes		yes	
firm f.e.	no		yes	
φ_1	0.0606***	(0.0100)	0.0242***	(0.0099)
φ_2	0.0231***	(0.0036)	0.0037*	(0.0021)
ϑ	-0.0042***	(0.0004)	-0.0021***	(0.0006)
β_1	0.0741***	(0.0056)	0.0495***	(0.0078)
β_2	0.8377***	(0.0502)	1.0413***	(0.0502)
ϕ	0.0329***	(0.0018)	0.0329***	(0.0018)

*** Statistically significant at 1% level.

** Statistically significant at 5% level.

* Statistically significant at 10% level.

Summary & Conclusion

- We analyze R&D collaboration networks in industries where firms are competitors in the product market.
- We provide a micro-foundation for the technology spillover and market competition effects, and estimate it with a unique panel data set on R&D alliances and firms' annual financial reports.
- We then analyze welfare (producer and consumer surplus) in independent as well as interdependent markets, captured by varying degrees of substitutability between goods.
- We study key player firms, i.e. the firms whose exit reduces welfare the most.
- We then analyze R&D subsidy programs, either as a fixed share of R&D expenditures homogeneous across firms, or targeted towards individual firms.