Trends in Labour Supply and Economic Growth*

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September 2005

Abstract

We study long-run changes in aggregate labour supply and shifts in employment across economic sectors within the context of balanced aggregate growth. We show that a model of many goods and uneven TFP growth in market and home production can rationalize balanced growth with a falling or U-shaped aggregate labour supply, structural change within market sectors and constant share of leisure time. The dynamics of labour supply are driven by substitutions between home and market production and depend critically on the existence of many market sectors. Conversely, the existence of home production improves the performance of predictions about structural change.

*We have benefited from suggestions and comments from Robert Lucas, Valerie Ramey and Randy Wright. We are especially grateful to Valerie Ramey for making available some of the data reported in Ramey and Francis (2005). An earlier version of this paper was presented in a keynote address at the CEPR/IZA annual labour economics conference at Ammersee in September 2005.

A feature of modern economic growth is the changing trend in aggregate labour supply. In the early stages of economic development the total supply of labour (measured in hours of work) typically falls. In later stages, trends become less clear-cut, with no clear overall dynamic pattern. In the United States the trend over the last century appears to be a distorted U-shape, a steep decline followed by a small rise. Other patterns, however, are also observed: the only undisputed "stylized fact" of low-frequency fluctuations in labour supply is that industrialization causes a long-lasting decline, which eventually dissipates.[1] The changing trends in aggregate labour supply that one finds in long runs of data are usually neglected by modern growth theory, which typically assumes a constant rate of labour force growth.

A seemingly unrelated feature of modern growth is "structural change," the continuous reallocation of labour across sectors of economic activities. In this paper we propose a unified framework for the study of these two phenomena and posit that they are part of the same economic process: the response of employment to the uneven distribution of technological change across production sectors located in the market and the home. The introduction of home production is critical in the explanation of changes in overall labour supply; perhaps surprisingly, structural change within market sectors also turns out to be critical in the explanation of labour supply, and home production turns out to be important in the explanation of labour reallocations within market sectors.[2] We show that our "technological" explanation of labour reallocations can explain the initial fall in aggregate labour supply and several different types of dynamics in later stages of economic development, depending on parameters and the number of distinct sectors, whilst at the same time satisfying Kaldor's facts of balanced aggregate growth.

The model that we use to demonstrate these claims has three general uses of time, market work, home work and leisure. Market work produces consumer and capital goods and we refer to it as the supply of labour. Home work produces consumption goods for

[1] Several contributory forces have been identified in the historical literature. The decline of agriculture and primitive industrial production at the initial stages of economic development leads to the withdrawal of many workers, especially women, from the labour force. Parallel to this, hours per worker in industry and services fall. In later stages the rise of a service sector attracts women back to market production. Hours of work per worker either stabilize or continue to fall at a decreasing rate, through, e.g., the growth of part-time employment. See Durand (1975, esp. ch. 4) and Maddison (1995) for cross-country evidence and Goldin (1995) for female labour supply in the US time series and in other countries. Evidence for the United States and the biggest European economies is further discussed in section 1 below.

[2] Home production has been studied extensively in a partial context, starting with Becker (1965), Gronau (1977) and many others. More recently it has been studied in the context of equilibrium business cycles and to some extent in the context of growth (see Gronau, 1997 for a survey, and Parente et al. (2000) and Gollin et al. (2000) for growth-related work). Structural change within the market economy has been studied by many authors, but to our knowledge our paper is the first that studies structural change with home production. See Kuznets (1966), Baumol (1967) and Fuchs (1980) for early contributions and Echevarria (1997), Kongsamut et al.(2001), Caselli and Coleman (2001) and Ngai and Pissarides (2004) for more recent work.

the individual's own use and the time allocated to it is like market work, but it is not part of the conventional definition of labour supply. We show that because of the uneven distribution of technological change the time allocated to home production is likely to change during the course of economic development. Under plausible assumptions the time allocated to home production is likely to increase in the early stages of economic development - implying a falling overall labour supply - but eventually it will decrease. By how much it decreases and what happens after the decrease depends on a number of factors. We devote most of the paper to a benchmark model with only one home-produced good, and show that the decrease in home production time continues indefinitely. But small generalizations to the model, such as the introduction of a second home-produced good, can give richer dynamics and imply more than one turning point in labour supply.

We emphasize that although our model has both home production and leisure time, the driving force for the long shifts in labour supply is the home production sector and not leisure. The existence of a steady state that satisfies Kaldor's aggregate facts requires constant leisure time, but not constant home production time. During periods of transition to an aggregate balanced-growth equilibrium - following for example war or some other major event that disturbs the initial growth equilibrium - changes in leisure also contribute to changes in aggregate labour supply, but these periods cannot explain the long swings in labour supply that is the topic of this paper. Our theoretical model is consistent with the class of real business cycle models discussed by King et al. (1988), which are characterized by an underlying balanced-growth path.

The intuition behind our results is that although market activities at the not-too-disaggregate level of agriculture, manufacturing and services are poor substitutes for each other, a lot of home production is a good substitute for services produced in the market. Over time, if the composite of service sectors (in the market and the home) has lower mean TFP growth rate than other market sectors, it attracts labour from agriculture and industry. This is a force for an increase in the employment share of both sectors. But if the market service sector has higher TFP growth than the home sector, within the services composite there is a movement of labour from the home to the market. This explains why the share of agriculture and manufacturing employment may decline faster when there is a home service sector. Labour employed in agriculture and manufacturing sectors has two potential destinations, both with low TFP growth rates. It can also explain why the share of market services may rise faster with home production. It receives labour from two sources, market sectors and the home sector. Although in the first stages of economic development, when agriculture employs a large fraction of the labour force, the movement of labour from agriculture to home production offsets the movement from home production to market services, eventually the movement from home production to market services takes over, bringing a decline of the home production sector. Reflecting this reversal, although in the first stages of economic development aggregate labour supply may decrease, eventually it increases. But this increase need not continue indefinitely. If both the market and home sectors produce in addition goods which do not experience growth in total factor productivity (or experience low common growth), both sectors attract labour indefinitely, implying that eventually overall home production time increases. If, for reasons not specified in our model, women are more likely to be engaged in home production than men are, our model can explain the fall in male labour supply and the eventual rise in female labour supply as part of a unified process of economic growth.

Support for the predictions of the model can be found in the evidence of Ramey and Francis (2005), who showed that leisure time in the United States has been on average constant since the beginning of the 20th century, but home production time and hours of market work have changed. We also report the results of some numerical illustrations which match fairly well the aggregate dynamics of labour supply and broad market sectors in the United States, as further support of the links between home production and aggregate labour supply.

Although we choose to emphasize the implications of our model for low-frequency changes in aggregate labour supply, our model has strong implications for the time devoted to home production, and by extension to female labour supply. We point out two implications here. First, we argue that at the first stages of economic development the decline of agricultural labour leads to an increase in home production time. The evidence of Mokyr (2000) and others is consistent with this claim. As incomes grow and labour is released from agriculture there is an increase in the demand for home production services: cleaner homes, better-prepared food, more care over child upbringing and so on. Second, our explanation of the decline of the share of the home production sector in later stages of economic development is different from the one put forward by Greenwood et al. (2005). Home production in our model is "marketized" (Freeman and Schettkat, 2005, Rogerson, 2004): both employment and output in the home sector fall, because similar goods can be produced more efficiently in the market. Greenwood et al. claim that although employment in the home sector falls, output rises because the sector employs more capital as the prices of durable goods fall. In our model the price of durable goods also falls because of higher TFP growth in manufacturing than in services, but the substitution of capital for labour is not the driving force for the decline in home production time.

Section 1 examines aggregate labour supply data in five countries, the United States and the four biggest European economies. It establishes the "stylized fact" of a downward trend in the share of working time in all economies, followed by more diverse dynamics. Section 2 outlines our benchmark model of multi-sector growth when there is home production and leisure. The dynamic properties of employment shares, the existence of an aggregate balanced growth path and a numerical illustration based on US data are derived and discussed in this section. Section 3 discusses an extension that gives more general results about the dynamic behaviour of home production and aggregate labour supply.

1 Long-run trends in hours

We divide total time into three groups, market work, home production and leisure. Market work includes all hours of work supplied to the market for remuneration. In a model without unemployment it is equal to the supply of labour and employment. Home work is time worked at home to produce goods for one's own consumption, and includes such activities as child care, cooking and cleaning. Leisure is time not used for production, such as watching television and spending time with friends.

In a representative agent model we are interested in the share of each use of time out of the total time available to the population for the three uses. If we had a time series for each time use computation of the shares would be straightforward. But although countries with rich statistical resources usually have some data on total hours of market work, there are very few observations of home production time, and even fewer of leisure time. So usually non-work time is calculated as a residual, which requires knowledge of the total amount of time allocated to the three activities of the model. We call this aggregate the denominator of our model.[3]

In general, the denominator can be calculated by starting from the total number of hours available to the entire population and removing components that are unavailable for the uses of time of the model, either because of exogenous constraints or because of other decisions made in the past.[4] The first such component is the time spent by a typical individual on sleep and other essential activities. Time use surveys show remarkable regularity across time and countries in this amount of time. We take the number used by Ramey and Francis (2005), 74 hours per week, and so assume that the basic week length available for work and leisure is 94 hours.[5]

Second, many individuals need to be removed from our denominator because they are unable to choose between work and leisure, either for health reasons or because they are institutionalized. Third, and related to this, a large number of individuals are in school, and so are again unable to choose between work and leisure. Schooling may be compulsory or voluntary, depending on age, but since our model does not include schooling choices, all these individuals should also be removed from our denominator.

Calculating precisely a denominator with currently available statistical resources is not possible for most countries. Because of this, many studies use the working-age population as their denominator (usually, ages 15-64), on the implicit assumption that

[3]For recent papers on the measurement and analysis of hours of work and leisure, see Juster and Stafford (1991), Maddison (1995), Ramey and Francis (2005) and Gali (2005).

[4]The approach and discussion that follow are influenced mainly by the approach of Ramey and Francis (2005). See also Ramey and Francis (2004).

[5]Of course, the important property of this number is that it is a constant, not whether it is a few hours bigger or smaller. Juster and Stafford (1991) report 68-72 hours for essential activities for the age group 24-64 years. Ramey and Francis justify the number 74 by a reference to Robinson and Godbey (1999) and the BLS Time Use Survey.

those in school are approximately given by the age groups under 15 and those not active for health reasons are given by the group over 65. For the United States, however, Ramey and Francis (2005) have recently calculated the non-institutionalized population over the age of 10, the full-time equivalent number of people in education up to the age of 24 and the fraction of people over 65 who are healthy enough to work. Making use of their data we calculated a denominator for the United States, which we call the available labour force.

Figure 1 shows the shares of time allocated to market work, home work and leisure. The first two series are from Ramey and Francis (2005) and the third is calculated as a residual. The trend shown is quadratic. The fraction of leisure time has remained remarkably stable for 100 years, a point emphasized by Ramey and Francis as well. Neither coefficient of the quadratic trend is statistically significant. Home production time follows a hump shape and market hours a U-shape, with both coefficients of the quadratic significant at the 1% level.[6] The highest point of home production is reached in the late 1960s and the lowest of market work in the mid 1970s. Although the rising part of the U-shape of market work is fairly flat there has been a substantial rise in the fraction of time devoted to work in the three decades since the lowest point in the mid 1970s. By exactly how much the trend has risen depends on the sample used to calculate it, but a figure of about 7-8% appears the most plausible.

Turning now to experience elsewhere, we do not have a series for the "available labour force" for the European countries, so we use the population of working age as the denominator. Figure 2 shows the share of working hours in the population of working age in the four major European countries and for comparison in the United States based on the same definition.[7] It is encouraging that the dynamics of work in the United States with the population of working age as denominator are very similar to the dynamics with the available labour force as denominator.[8]

Despite the obvious and often-cited differences between "Europe" and the United States evident in figure 2, the dynamics of hours of work in the European countries bear remarkable resemblance to the dynamics of the same series in the United States. We showed in figure 1 that the share of work hours first declined at a decreasing rate over a long period of time, and eventually reversed. The Italian share in figure 2 exhibits the same dynamic pattern, but the turning point comes about two decades after the US one.

[6] Ramey and Francis (2004) fit a quartic trend to market hours of work and also find a U-shape for the period since 1900.

[7] The data sources for hours are from the website of the Groningen Growth and Development Centre and for the working age population from the OECD website.

[8] Comparison between the available labour force series and the population of working age for the United States shows that in the early period the available series exceeds the population of working age by 6-8% because many 10-14 year olds who are excluded from the population of working age were in the labour force. In later years the difference becomes smaller, as the number of healthy older people added in the total available series offsets the number of young people removed because of school. Since about 1990 the two series have been virtually identical.

The United Kingdom also exhibits a similar pattern. The decline bottoms out fast, and the actual data series turns up in the early 1990s, although the trend does not reach a minimum by 2000. But the French and especially German experiences are still ones of fairly fast, although flattening out, decline.

The US series in figure 1 and the cross-country comparisons in figure 2 lead to the conclusion that the share of total time allocated to market work initially declines during economic growth. The decline slows down and at least in two or three of our countries it reverses. The reversal is due to the "marketization" of home production, with leisure time showing virtually no trend. Alongside these dynamics, as is well-known and documented elsewhere, the aggregate economy is on a balanced growth path and there is sectoral structural change. The main features of structural change are a rapid decline in agricultural employment, a rapid rise in service employment and a smaller hump-shaped change in manufacturing employment. The model of the next section shows that all these facts can have a common technological explanation.

2 Home production and leisure in a growth model

We simplify our exposition by assuming that market work takes place in three differentiated sectors and home production takes place in only one sector.[9] Each of the three market sectors captures a distinct feature of production. Sector 1, labelled agriculture, produces a consumption good that does not have close substitutes elsewhere; sector 2, labelled manufacturing, produces the economy's capital stock and another consumption good that also does not have close substitutes in other sectors; sector 3, labelled services, produces only a consumption good that has a close substitute produced in the home. Our labels are obviously not accidental, but we emphasize more the nature of the good produced in each sector rather than the accuracy of their description as agriculture, manufacturing and services.

We derive the equilibrium of our economy from a social planning problem that maximizes the utility function of a representative agent. Equilibrium is defined as a set of dynamic paths for the allocation of capital and time to the three market sectors, home production and leisure, and the allocation of the output of each sector to consumption

[9] The three-sector economy of this paper can easily be generalized to many sectors along the lines of Ngai and Pissarides (2004). It is also possible to extend the model to one of more than one home sector (see section 3).

and capital. The utility function of the representative agent is

$$
\begin{aligned}
U &= \int_0^\infty e^{-\rho t}\left[\ln\phi(.) + \theta v(1-l)\right]dt && (1)\\
\phi(.) &= \left(\sum \omega_j c_j^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \qquad j = a, m, sh && (2)\\
c_{sh} &= \left[\psi c_s^{(\sigma-1)/\sigma} + (1-\psi)\, c_h^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} && (3)\\
\rho, \theta, \omega_i, &> 0, \sum \omega_j = 1; \varepsilon, \psi \in (0,1), \sigma > 1; l \in (0,1), c_j \geq 0.
\end{aligned}
$$

where $v(.)$ is the utility of leisure, with $v' > 0, v'' < 0$, and $v' \to \infty$ as $l \to 1$, $\phi(.)$ is a CES utility function over final consumption goods and c_{sh} is a composite service good, which is the outcome of a CES combination of market and home goods. Subscript a stands for agriculture, m for manufacturing, s for market-produced service goods and h for home-produced service goods. l is the fraction of total time allocated to market and home work. The restrictions on ε and σ imply that market and home-produced services are close substitutes for each other but the outputs of the agricultural, manufacturing and composite service sectors are not close substitutes.

The restrictions on the utility function are a combination of sufficient restrictions previously derived by King et al. (1988) and Ngai and Pissarides (2004). King et al. (1988) show that in a one-sector model with consumption c and hours of work l the utility function $U(c,l) = \frac{c^{1-\theta}}{1-\theta} v(1-l)$ is necessary for the existence of a balanced-growth path. Our utility function in (1) is equivalent to setting $\theta = 1$, a restriction that was shown by Ngai and Pissarides (2004) to be necessary for the existence of a balanced growth path when there are many sectors with unequal TFP growth rates and $\varepsilon \neq 1$.

Our measure of total time is the total time available to the population who can work.[10] We let l_i denote the share of that time allocated to each of the four production activities ($i = a, m, s, h$) and write

$$\sum l_i = l. \tag{4}$$

Total market employment is $l_a + l_m + l_s \equiv q$, which, in the absence of unemployment, is also the conventional definition of the aggregate supply of labour. Market employment shares are then defined by l_i/q, for $i = a, m, s$. The U-shape fact about the aggregate labour supply is a statement about the evolution of q, whereas structural change refers to changes in the market shares l_i/q.

Production functions are identical in all activities except for their TFP parameters A_i,

$$F^i = A_i F(l_i k_i, l_i); \quad \dot{A}_i/A_i = \gamma_i \quad i = a, m, s, h, \tag{5}$$

[10] When we say in the theoretical model "population" we mean the "denominator" as defined in section 1, the total number of hours available to the entire population who can work. Similarly, when we express a variable in "per capita" terms we mean the ratio of that variable to our measure of population.

where the production function F is constant returns to scale, has positive and diminishing returns to inputs, and satisfies the Inada conditions, k_i is the capital-labour ratio of each sector and A_i is TFP in each sector i, with growth rate γ_i. We impose the quantitative restrictions $\gamma_a \geq \gamma_m > \gamma_s > \gamma_h$, which are justified later in the paper.

All sectors produce consumption goods but only manufacturing produces capital goods:

$$\begin{aligned} c_i &= A_i l_i F(k_i, 1) \qquad\qquad i = a, s, h && (6) \\ \dot{k} &= A_m l_m F(k_m, 1) - c_m - (\delta + \nu) k && (7) \\ \sum l_i k_i &= k, && (8) \end{aligned}$$

where δ is the capital depreciation rate, ν is the population growth rate and k is the ratio of the capital stock to the population.[11]

We obtain optimal allocations by maximizing the utility function in (1) subject to (4)-(8). We distinguish between the "static" conditions that give optimal allocations across sectors and the "dynamic" ones that give optimal allocations over time.

2.1 Optimal sector allocations

The optimal allocation of resources across industrial sectors is obtained from the first-order maximization conditions for c_i, l_i, k_i and l. Free factor mobility implies that both the value of marginal product of factors and the marginal rate of technical substitution between capital and labour are equalized across sectors. These imply equality of the capital-labour ratios across sectors and equality between relative prices and relative TFP levels:

$$\begin{aligned} k_i &= k_m = k/l \quad i = a, s, h, && (9) \\ \frac{\phi_i}{\phi_m} &= p_i = \frac{A_m}{A_i} \quad i = a, s, h. && (10) \end{aligned}$$

We use manufacturing as our numeraire throughout the analysis.

We can immediately derive one strong result about structural change and home production in this economy. From the conditions for optimal choice of c_s and c_h in (10), we obtain,

$$\varphi \equiv \frac{p_h c_h}{p_s c_s} = \left(\frac{1-\psi}{\psi}\right)^{\sigma} \left(\frac{A_s}{A_h}\right)^{1-\sigma}; \qquad (11)$$

[11] Our assumption of a single capital good used in the market and home does not necessarily imply the transfer of capital goods between the two uses after installation. Although we argue later that the home sector is declining, empirically the capital depreciation rate and population growth rate far exceed the rate of decline of the home sector, so gross investment in the home is always positive.

and from the production function we derive

$$\frac{l_h}{l_s} = \varphi. \tag{12}$$

So

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_h}{l_h} = (\sigma - 1)(\gamma_s - \gamma_h). \tag{13}$$

With $\sigma > 1$, if TFP in the market sector is rising faster than in the home sector, the home sector will be losing labour to the market sector. It implies that if the TFP growth rate of the market sector remains indefinitely above the TFP growth rate of the home sector, eventually the home sector will vanish and all services will be produced in the market. Thus, our claims about the eventual marketization of all home production come from two quantitative restrictions: $\sigma > 1$ and $\gamma_s > \gamma_h$. We return later in the paper to a discussion of these conditions and to the question whether the model can be generalized to yield a home sector in its asymptotic state.

In order to derive the direction of structural change in other sectors, we define a new variable to represent the ratio of expenditure on the outputs of agriculture and services to expenditure on the consumption of the manufacturing good, $x_i \equiv p_i c_i / c_m$. By definition, $x_m = 1$. Using (10) and (11), we derive in the Appendix,

$$x_a \equiv \frac{p_a c_a}{c_m} = \left(\frac{\omega_a}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_a}\right)^{1-\varepsilon} \tag{14}$$

$$x_s \equiv \frac{p_s c_s}{c_m} = \left(\frac{\omega_s}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_s}\right)^{1-\varepsilon} (1+\varphi)^{(\sigma-\varepsilon)/(1-\sigma)} \tag{15}$$

$$x_h \equiv \frac{p_h c_h}{c_m} = \varphi x_s \tag{16}$$

where $\omega_s \equiv \omega_{sh} \psi^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}$, which is the weight of the market service good in the utility function when home production is close to zero. Note also that if home production is not present in the utility function (i.e. $\psi \to 1$), we have $\varphi \to 0$ and $x_h \to 0$, and x_s has the same form as x_a.

The total value of consumption per capita, including the consumption of home produced goods, is

$$c = \sum p_i c_i = X c_m \tag{17}$$

where $X = \sum x_i$. Making use of (9) and (10), the total value of aggregate output per capita is given by:

$$y \equiv \sum p_i F^i = l A_m F(k_m, 1). \tag{18}$$

Using (14)-(18), we obtain

$$\frac{l_i}{l} = \frac{x_i}{X}\frac{c}{y}, \qquad i = a, s, h, \tag{19}$$

$$\frac{l_m}{l} = \frac{1}{X}\frac{c}{y} + \frac{s}{y}, \tag{20}$$

where $s \equiv y - c$ are savings. These equations show that the employment share used in the production of consumption good i is a fraction x_i/X of the aggregate consumption rate, whereas the manufacturing employment share has two parts, one that obeys the same law as the share of other consumption and another that is equal to the savings rate. The first component is employment required to produce the manufacturing consumption good and the second is the employment required to produce the economy's investment goods.

The results for employment shares in (19) and (20) imply (using the expression for x_i),

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_a}{l_a} = (1-\varepsilon)(\gamma_a - \gamma_s) + \frac{\varphi}{1+\varphi}(\sigma - \varepsilon)(\gamma_s - \gamma_h), \tag{21}$$

$$\frac{\dot{l}_h}{l_h} - \frac{\dot{l}_a}{l_a} = (1-\varepsilon)(\gamma_a - \gamma_h) - \frac{1}{1+\varphi}(\sigma - \varepsilon)(\gamma_s - \gamma_h). \tag{22}$$

Conditions (21) and (22) give an important result. In the absence of home production, the second term of (21) on the right vanishes, and we obtain the familiar result that for $\varepsilon < 1$ employment moves from agriculture, the high TFP-growth sector, to services (see Ngai-Pissarides, 2004). But since $\sigma > \varepsilon$ and $\gamma_s > \gamma_h$, the second term in (21) is also positive, and so the speed at which market services attract labour from agriculture is faster. Equation (22) gives a contrasting result. The first term on the right shows a movement of labour from agriculture to the home sector, because as with market services the home sector produces a service good that is a poor substitute for agricultural output. But the second term shows that (at least for $\sigma > \varepsilon$ and $\gamma_s > \gamma_h$) the movement is either mitigated or reversed, because service goods are more efficiently produced in the market. Intuitively, the introduction of a home production sector with small TFP growth rate accelerates the move out of agriculture because the gap between TFP in agriculture and the composite of the destination sectors is now bigger. We can also see that home production has its biggest impact on the decline of agriculture early on in the stage of economic development. From (11) the second term in (21) becomes progressively smaller over time and vanishes as $t \to \infty$, whereas the second term in (22) becomes progressively larger. The opposite signs in (22) are the reason behind the changing trends in aggregate labour supply as we now show more formally.

Using (19) and (20), the dynamics of l_i depend on the sector-specific component x_i/X and the aggregate components l and c/y. In the remainder of this section we study the dynamics of l_i and the implied aggregate labour supply due to the sector-specific components. The additional dynamics due to changes in the aggregate l and c/y can easily be obtained from (19) and (20) and we postpone discussion to the next section.

Using (11), and (14)-(16), we obtain,

$$\frac{\dot{x}_a}{x_a} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_a) \tag{23}$$
$$\frac{\dot{x}_s}{x_s} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) + (\sigma - 1)(\gamma_s - \gamma_{sh}) \tag{24}$$
$$\frac{\dot{x}_h}{x_h} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) - (\sigma - 1)(\gamma_{sh} - \gamma_h) \tag{25}$$
$$-\frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_m) \tag{26}$$

where $\bar{\gamma}$ is a weighted average of the TFP growth rates in all sectors, with the weight on γ_i equal to x_i/X. Because x_i/X is proportional to the share of employment used to produce consumption goods we call $\bar{\gamma}$ the consumption-weighted TFP growth rate of the economy. On the other hand, γ_{sh} can be interpreted as the TFP growth rate for the service composite (c_{sh}). It is given by the weighted average of TFP growth rates in the home and market service sectors, $\gamma_{sh} = (\gamma_s + \varphi\gamma_h)/(1+\varphi)$. Note that as $t \to \infty$, both $\bar{\gamma}$ and γ_{sh} converge to γ_s.

Given the ranking $\gamma_a \geq \gamma_m > \gamma_s > \gamma_h$, we have $\gamma_a > \bar{\gamma} > \gamma_{sh}$, so (23) implies that l_a is falling, (24) implies that l_s is rising, and (25) implies that the dynamics for l_h are not likely to be monotonic, because of the two opposing forces at work already discussed. We show in the Appendix that the growth rate of l_h is falling over time. Therefore, l_h is either hump-shaped, rising at first and falling later, or falling monotonically. It is hump-shaped if and only if the initial $\varphi(t_0)$ and $\bar{\gamma}(t_0)$ satisfy $H(t_0) > 0$ where

$$H(t) \equiv \frac{\bar{\gamma}(t) - \gamma_{sh}}{\gamma_{sh} - \gamma_h} - \frac{\sigma - \varepsilon}{1 - \varepsilon}.$$

This is more likely to be satisfied if the market and home service goods are not very close substitutes (small $\sigma - 1$), if service goods are very poor substitutes with other market goods (high $1 - \varepsilon$) and if the TFP growth rate of the home sector is not very small (high γ_h). All these are intuitive. When they are satisfied the home sector is not too inferior to the market sector and so loses labour at a slower rate in the transition to the asymptotic steady state. Moreover, given the other parameters, $H(t) > 0$ is more likely to be satisfied in the initial stages of economic development, when a large agricultural sector with a high TFP growth rate makes the initial $\bar{\gamma}(t_0)$ a high number.

The dynamics of manufacturing employment l_m are also non-monotonic if γ_m is below the initial $\bar{\gamma}$. But since $\bar{\gamma}$ converges to γ_s, the only remaining consumption sector in the asymptotic steady state, $\bar{\gamma}$ eventually falls below γ_m and so l_m also eventually falls until it converges to the savings ratio.

Overall labour supply is equal to $q = l - l_h$, so its dynamics parallel the dynamics of home production:

$$\frac{\dot{q}}{q} = -\frac{l_h}{l - l_h}\left(\frac{\dot{l}_h}{l_h}\right) \tag{27}$$

Just as the time devoted to home production eventually has to fall, aggregate labour supply eventually has to rise over time. But reflecting the likely hump-shaped evolution of home production, initially labour supply is likely to fall, giving a U-shaped aggregate labour supply.

2.2 Aggregate balanced growth

We now turn to the optimal intertemporal allocations. Note first that the marginal utility of manufacturing goods is ϕ_m/ϕ. Given that both ϕ and c_{sh} are homogenous of degree one, together with (10) and the definition of c, we obtain,

$$\phi = \sum_{i=a,m,sh} \phi_i c_i = \phi_m c, \tag{28}$$

which implies that the marginal utility of c_m is equal to $1/c$. So the optimal choice of leisure satisfies

$$\frac{\theta v'(1-l)}{1/c} = (1-\alpha_m) A_m F(k_m, 1), \tag{29}$$

where $\alpha_m = k_m F_K(k_m, 1)/F(k_m, 1)$ is the capital share in sector m. This, of course, is a restatement of the familiar condition that the marginal rate of substitution between leisure and consumption (in this case c_m, the numeraire) is equal to wages. Combining (29) with (18), leisure satisfies

$$\frac{c}{y} = \frac{1-\alpha_m}{\theta v'(1-l)\, l} \tag{30}$$

Therefore, as in one sector models, there is a close relationship between the dynamics of l and the dynamics of c/y.

In order to obtain these dynamics we restrict our production functions to be Cobb-Douglas, $F(k_i, 1) = k_i^{\alpha}$, $\alpha \in (0,1)$. This implies that our Hicks-neutral technology of the preceding sections is also labour-augmenting, which is required for the existence of a balanced growth path. Of course, under the Cobb-Douglas restriction, the α_m in (29) and (30) is equal to the constant α.

We show in the Appendix that the following two dynamic equations hold:

$$\frac{\dot{k}_e}{k_e} = \left[1 - \frac{1-\alpha}{\theta v'(1-l)\, l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} - \left(\delta + \nu + \frac{\gamma_m}{1-\alpha}\right) \tag{31}$$

and

$$\left[1+\frac{-v''(1-l)\,l}{\alpha v'(1-l)}\right]\frac{\dot{l}}{l}=\frac{\gamma_m+(1-\alpha)(\delta+\nu)+\rho}{\alpha}-\left(\frac{1-\alpha}{\theta v'(1-l)\,l}\right)\left(\frac{k_e}{l}\right)^{\alpha-1}. \tag{32}$$

where $k_e \equiv kA_m^{-1/(1-\alpha)}$ is the capital stock per capita in efficiency units. The economy converges to a unique steady state where l and k_e are constant. The unique steady state is saddle-path stable and the saddle path is downward-sloping, as shown in Figure 3.[12] It is clear from the figure that if the initial k_e is smaller than its steady-state level, then l is falling (leisure is rising) along the transition and the capital stock is rising, and from (30), c/y is also rising. So the transitional dynamics of the aggregate model parallel the transitional dynamics of the one-sector Ramsey model, except that our aggregates include the output and consumption of home production.

On the steady state path leisure time is constant but the supply of labour is not constant because of changes in the amount of time allocated to home production. But the constancy of both k_e and l implies that the capital-labour ratio in the economy as a whole grows at the rate of labour-augmenting technological growth in manufacturing, $\gamma_m/(1-\alpha)$. From (18) we also see that output per hour in the economy as a whole, y/l, is growing at the same rate. The aggregate capital stock in the market sector is given by

$$k_{GDP}=\sum_{i\neq h} l_i k_i = q k_m, \tag{33}$$

and so the market capital-labour ratio, k_{GDP}/q is simply k_m, which is equal to k/l and grows at rate $\gamma_m/(1-\alpha)$. Market output in this economy is

$$y_{GDP}=\sum_{i\neq h} p_i A_i k_i^{\alpha} l_i = q A_m k_m^{\alpha} \tag{34}$$

and so market output per hour, y_{GDP}/q is growing at the same constant rate and the capital-output ratio in the market economy is constant. This confirms our claim that our economy satisfies Kaldor's stylized facts of aggregate balanced growth, despite the changes in labour supply.

Since the only additional restriction needed to derive the balanced growth path is that the production functions be Cobb-Douglas, the dynamics derived in the preceding section for employment shares hold also on this balanced growth path. So labour supply is likely to exhibit a U-shaped evolution even when the economy's capital-output ratio is constant, and labour supply does not have to reach constancy or near constancy in finite time, even if the economy's aggregate ratios do. Away from the steady state the employment shares are characterized by some additional dynamics, due to changes in

[12] We note that the existence of a steady state requires constant TFP growth rate in the capital-producing sector only, i.e. γ_m must be constant. The other TFP growth rates appear only in the dynamics of relative prices and sectoral time allocations.

l and c/y. Usually, the transitional dynamics will be driven by the economy needing to accumulate more capital per worker than it has at some initial state, and we briefly discuss the implications of these transitions for employment shares and aggregate labour supply.

In a transition with rising capital-labour ratio the adjustment dynamics are characterized by rising leisure time, and so by a falling labour supply. The transitional dynamics are superimposed on the U-shaped steady-state evolution of labour supply. Therefore, on the downward-sloping branch of the U the transitional dynamics reinforce the fall in labour supply whereas on the upward branch of the U they mitigate the rise. This implies that the transitional dynamics have an asymmetric effect on the steady-state U-shape, increasing the steepness of the falling branch but making the rising branch flatter.

The saving rate falls along the transition. Equations (19) and (20) immediately yield that the impact on the employment shares of consumption sectors is positive. So when there are transitional dynamics the share of time allocated to market services is rising at an even faster rate but the decline of the share of agriculture and home production may slow down.

2.3 Numerical Illustration

Suppose now the economy is on the balanced growth path that solves (31) and (32). How do the overall labour supply and employment shares evolve in an economy with parameters that match US data? From (16), (19) and (20) we see that any initial distribution of time, (l_h, l_a, l_s, l_m), can be matched by choice of initial values for the vector (φ, x_a, x_s). Recall that this vector depends on preferences and technology, in particular on the weights of each good in the utility function, the elasticities of substitution between consumption goods, and the ratio of TFP levels of the goods involved. Once we choose these parameters to match the initial allocation of time we can trace the dynamics of time allocations by letting TFP levels evolve over time, updating the vector (φ, x_a, x_s).

Our assumption that the US economy has been on a balanced growth path in the 20th century is justified by the Kaldor facts and by the fact that the share of time allocated to leisure has been constant (see figure 1). We first match the time allocations in 1900 using data on market and home hours from Ramey and Francis (2005), and on employment shares from *Historical Statistics.*[13] Given reasonable preference and technology parameters (as fully explained in the Appendix), we then ask whether the model can match the evolutions of aggregate labour supply and sectoral market shares found in the data. Our main interest is to uncover the role of home production in these dynamics.

[13] We followed Kuznets (1966) and divided the economy into three sectors: (1) agriculture includes agriculture, forestry, and fisheries, (2) industry includes mining, manufacturing, construction, utilities, transportation and communication, and (3) services are the rest of the economy.

Figure 4 plots the data against the model's prediction for the ratio of home to market service work (the φ in (11) and (12)). The solid straight line is the model's prediction. In the model the rate of growth of this ratio is equal to $(1-\sigma)(\gamma_s - \gamma_h)$, so the downward-sloping log-linear line is due to $\sigma > 1$ and $\gamma_s > \gamma_h$. The fact that the data points appear to trace a straight line also justifies our assumption that the difference between the two growth rates has been approximately constant. Figure 5 shows the model's prediction for the time allocation into home and market production. Noting that with constant leisure time (of about 40 per cent of total time) the time allocated to home production is a mirror image of the time allocated to market work, we show only the fraction allocated to market work. The model's prediction of this time (the supply of labour) turns out to be U-shaped. The model's U tracks reasonably well the trend in the data and predicts that market hours will continue to grow in the United States. Figure 6 plots the prediction for market employment shares with and without home production. The thick solid line is the prediction of the model with home production, and the thin solid line is the prediction without home production. It is clear from the figure that as we argued intuitively, the predictions of structural change with home production improve. The decline of the share of agriculture and the rise of the share of services are closer to the data.

3 Is the eventual increase in labour supply inevitable?

Labour supply in our model inevitably increases monotonically after a certain point because the home sector eventually disappears. It disappears for two reasons, because it produces a good that is a close substitute to a market good and because the rate of TFP growth in the market sector is always bigger than it is in the home sector. The numerical illustration has shown that the decline of the home sector can be very slow, taking hundreds of years.[14] However, the model can be easily generalized to avoid this conclusion altogether, without affecting the main results derived so far.

The generalization requires the introduction of more than one home good. A trivial example is when there are home goods that do not have a market substitute, such as some aspects of child care. The production of these goods at home will survive irrespective of TFP growth rates. But some home production can also survive in the more general case when the home sector produces goods that have substitutes in the market. From (11) and (12) we obtain that l_h does not indefinitely fall in relation to l_s either if $\sigma = 1$, or if the ratio of TFP in the market and the home is constant (i.e., if their growth rates are equal). Previous quantitative literature has argued for $\sigma > 1$ at the aggregate level, but this does not exclude the possibility of some home goods having unit elasticity with

[14] For example, it takes about 300 years for the share of the home sector out of total work (l_h/l) to go down to 10%.

respect to some market goods. Nevertheless, we now assume $\sigma > 1$ and consider the implications of equal TFP growth rates.

More specifically, we briefly discuss the implications of dividing the service sector into a progressive component and a stagnant one. The progressive component includes sub-sectors such as business services, trade and catering, which have positive TFP growth rates. The stagnant component includes education, health care and the arts, which have zero or near zero TFP growth rates. We can now define two types of home production sectors, one that produces services that are close substitutes for progressive service goods, such as food preparation, and one that produces substitutes for stagnant services, such as caring for the elderly. The TFP growth rate in the progressive home sector is postulated to be below the one in the market sector, and this sector behaves like the one that we modelled in our benchmark case. It eventually vanishes through the marketization of all its output. The stagnant home sector has the same low TFP growth rate as the stagnant market sector, and the ratio of time employed in each remains constant over time. From (11) this ratio depends on preferences and the relative TFP level for the two goods, but from (13) and as long as the growth rates of their TFP levels are equal, their relative employment shares do not change.

In accordance with our technological explanation of structural change, the stagnant service composite will be attracting labour over time from all other sectors, which will be shared equally between the market and home sub-sectors. As before, the progressive market sector will be continually gaining labour from the progressive home sector. Consider the implications for the dynamics of overall labour supply. The employment share of the progressive home sector will be hump-shaped, as in the benchmark model. The share of the stagnant sector will be rising monotonically. So in the early stages of economic development, when both components are rising, the overall share of the home sector will be rising, and so labour supply falling. Labour supply will also be falling in the very distant stages of economic development, as it approaches the asymptotic steady state, because by that time the progressive home sector will have vanished but the share of the stagnant home sector will still be rising. Between the two extremes overall labour supply may be rising or falling, and may have more than one turning point.

The model can be further extended with the introduction of more sectors to give more complicated dynamics. It is obvious from the discussion with the two home sectors, however, that it is possible to have a monotonically falling labour supply, or one that falls at first, rises at some point but then falls again. The stylized fact of a falling labour supply is, if anything, reinforced by the introduction of a stagnant service sector. The main impact on the dynamics of labour supply take place in later stages, where, as we argued in section 1, cross-country data suggest less clear-cut dynamics patterns.

4 Conclusions

Our objective of showing that a unified framework can simultaneously account for balanced aggregate growth, structural change between agriculture, industry and services and diverse aggregate labour supply dynamics has been accomplished. Our prediction of the coexistence of a changing trend in aggregate labour supply on the one hand and balanced aggregate growth on the other is new to a model of economic growth and is the result of studying structural change in a model with home production. The assumptions that drive our results are (a) market goods are poor substitutes with each other but home-produced goods have close substitutes in the market, and (b) agriculture and industry have higher rates of total factor productivity growth than services, but within the services group market services have higher rates of TFP growth than home services. On the aggregate economy's balanced growth path the dynamics of aggregate labour supply are driven by the dynamics of home production, but off the steady state there are transitional dynamics with leisure time rising and the supply of labour falling.

Quantitative analysis shows that our model matches well the dynamics of US labour supply and market shares since 1900. Comparable European data show the same general patterns as in the United States but more recently with some delay in the marketization of home production. We did not discuss in any detail reasons for these differences, which we hope to explore in future work. We mention briefly here the implications of one factor, the existence of more administrative and tax barriers to market production in Europe than in the United States.

In our model, barriers - modelled for example along the lines of Ngai (2004) or Rogerson (2004) - reduce market TFP relative to home-production TFP. In response, agents shift more production to the home, and because home production is in services, market employment shares move in favour of manufacturing. Preliminary investigations with post-war data show that these predictions hold true: European economies that have lower overall labour supply than the United States also have much lower service share and higher manufacturing share. In addition, because the barriers make home production relatively more efficient, in the early stages of economic development, when TFP is low in the market, the market employment share might actually be higher with barriers than in an economy without barriers, for similar reasons that stagnant sectors attract more labour. But as TFP levels grow, eventually the market share of the economy with no barriers overtakes the share in the economy with barriers. The implication in our benchmark model is that although an economy with barriers still has a U-shaped aggregate labour supply, the lowest point of the U is reached later and there may be a crossing point before the minimum is reached. This would imply that the dynamic evolution of an economy can explain how one country can have higher labour supply at some point in its course of economic development and a lower one later on, even in the absence of exogenous policy shocks (as shown in figure 2).

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Appendix

Claim 1 *Derivation of x_i, $i = a, s, h$.*

Proof. The utility function and (10) immediately give the expressions for x_a and x_h. To derive x_s, first derive c_{sh}/c_s using the utility function and (10),

$$\begin{aligned} c_{sh}/c_s &= \psi^{\sigma/(\sigma-1)} \left[1 + \left[(1-\psi)/\psi\right](c_h/c_s)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ &= \psi^{\sigma/(\sigma-1)} \left[1 + \left[(1-\psi)/\psi\right]^{\sigma} (A_h/A_s)^{\sigma-1}\right]^{\sigma/(\sigma-1)} \\ &= \psi^{\sigma/(\sigma-1)} (1+\varphi)^{\sigma/(\sigma-1)} \end{aligned}$$

and so from (10) again

$$p_s = \frac{\omega_{sh}}{\omega_m}\left(\frac{c_m}{c_{sh}}\right)^{1/\varepsilon} \psi \left(\frac{c_{sh}}{c_s}\right)^{1/\sigma} = \frac{\omega_{sh}}{\omega_m}\left(\frac{c_m}{c_s}\right)^{1/\varepsilon} \psi \left(\frac{c_{sh}}{c_s}\right)^{(1/\sigma-1/\varepsilon)},$$

which together with c_{sh}/c_s implies,

$$\begin{aligned} x_s &\equiv (p_s c_s/c_m) = (\omega_{sh}/\omega_m)^{\varepsilon} (A_m/A_s)^{1-\varepsilon} \psi^{\varepsilon} (c_{sh}/c_s)^{(\varepsilon-\sigma)/\sigma} \\ &= (\omega_{sh}/\omega_m)^{\varepsilon} (A_m/A_s)^{1-\varepsilon} \psi^{\sigma(\varepsilon-1)/(\sigma-1)} (1+\varphi)^{(\varepsilon-\sigma)/(\sigma-1)}. \end{aligned}$$

■

Claim 2 *The equilibrium l and $k_e \equiv kA_m^{-1/(1-\alpha)}$ converge to a unique steady state.*

Proof. Using (9) and (10), the feasibility condition can be rewritten as follows:

$$\begin{aligned} \dot{k} &= l_m A_m k_m^{\alpha} - c_m - (\delta+\nu) k \\ &= \left(l - \textstyle\sum_{i=a,s,h} l_i\right) A_m k_m^{\alpha} - c_m - (\delta+\nu) k \\ &= l A_m k_m^{\alpha} - \textstyle\sum_{i=a,s,h} p_i A_i k_i^{\alpha} l_i - c_m - (\delta+\nu) k \\ &= l^{1-\alpha} A_m k^{\alpha} - c - (\delta+\nu) k \end{aligned}$$

From (29),

$$\frac{c}{k} = \left(\frac{1-\alpha}{\theta v'(1-l)\, l}\right)\left(\frac{k_e}{l}\right)^{\alpha-1},$$

so we have,

$$\frac{\dot{k}_e}{k_e} = \left[1 - \frac{1-\alpha}{\theta v'(1-l)\, l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} - D_k, \tag{35}$$

where $D_k = \delta + \nu + \gamma_m / (1-\alpha)$. Given that the marginal utility of c_m is equal to $1/c$, optimal saving implies,

$$\dot{c}/c = \alpha A_m k_m^{\alpha-1} - (\delta + \rho + \nu).$$

Finally, differentiation of (29) w.r.t. time yields,

$$\frac{\dot{c}}{c} + \frac{-v''(1-l)\,l}{v'(1-l)}\left(\frac{\dot{l}}{l}\right) = \gamma_m + \alpha\left(\frac{\dot{k}}{k} - \frac{\dot{l}}{l}\right)$$

which simplifies to,

$$\left[1 + \frac{-v''(1-l)\,l}{\alpha v'(1-l)}\right]\frac{\dot{l}}{l} = D_l - \frac{1-\alpha}{\theta v'(1-l)\,l}\left(\frac{k_e}{l}\right)^{\alpha-1}, \tag{36}$$

where $D_l = [\gamma_m + (1-\alpha)(\delta+\nu) + \rho]/\alpha$. There exists steady state l and k_e satisfying:

$$\begin{aligned} \dot{l} &= 0 : \frac{1-\alpha}{\theta v'(1-l)\,l}\left(\frac{k_e}{l}\right)^{\alpha-1} = D_l \\ \dot{k}_e &= 0 : \left[1 - \frac{1-\alpha}{\theta v'(1-l)\,l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} = D_k \end{aligned}$$

Solving the two equations yields,

$$\left(\frac{c}{y}\right)^* = \frac{1-\alpha}{\theta v'(1-l)\,l} = \frac{D_l}{D_l + D_k}$$

Substitution back to $\dot{l} = 0$ gives the unique steady state

$$\begin{aligned} l^* &: \theta v'(1-l)\,l = (1-\alpha)(D_l + D_k)/D_l \\ k_e^* &= l^*(D_l + D_k)^{1/(\alpha-1)}. \end{aligned} \tag{37}$$

To show convergence, note that $\dot{l} = 0$ is downward sloping and $\dot{k}_e = 0$ is upward sloping in $k_e - l$ space. Also, as $k_e \to 0$, we have $l \longrightarrow 1$ along $\dot{l} = 0$, and $l \to \bar{l} < 1$ along $\dot{k}_e = 0$, where $\bar{l}$ satisfies $v'(1-l)\,l = \theta/(1-\alpha)$. Finally, using (35) and (36) increasing k_e implies higher $\dot{l}$ and lower $\dot{k}_e$. We can now construct a phase diagram in $k_e - l$ space. The $\dot{l} = 0$ is downward sloping and unstable and the $\dot{k}_e = 0$ is upward sloping and stable, so there is a unique convergent downward-sloping saddle path (see Figure 4). ■

Claim 3 *If $\gamma_h < \gamma_s < \gamma_a$, then in the asymptotic steady state,*

$$l_m/l = s/y, \quad l_s/l = c/y, \quad l_a/l = l_h/l = 0.$$

Proof. From (19) and (20), the proof is completed if $\lim_{t\to\infty} x_s/X = 1$. Given $\gamma_s > \gamma_h$, (11) implies $\lim_{t\to\infty} \varphi = 0$. Given $\gamma_s < \gamma_a$, (14) and (15) imply

$$\lim_{t\to\infty} (x_a/x_s) = (\omega_a/\omega_s)^{\varepsilon} (A_s/A_a)^{1-\varepsilon} (1+\varphi)^{(\sigma-\varepsilon)/(\sigma-1)} = 0.$$

Finally, using (16) and $X = \sum x_i$,

$$\lim_{t\to\infty} (x_s/X) = \lim_{t\to\infty} [(1+x_a)/x_s + 1 + \varphi]^{-1} = 1.$$

■

Claim 4 *The growth rate of l_h is decreasing over time. Define*

$$H(t) \equiv (\bar{\gamma}(t) - \gamma_{sh}) / (\gamma_{sh} - \gamma_h) - (\sigma - 1) / (1 - \varepsilon).$$

If $H(t_0) > 0$, then l_h is hump-shaped. Otherwise, l_h falls monotonically.

Proof. Given

$$\dot{l}_h/l_h = (1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) - (\sigma - 1)(\gamma_{sh} - \gamma_h),$$

first note that $\lim_{t\to\infty} x_s/X = 1$ implies $\lim_{t\to\infty} \bar{\gamma} = \gamma_s$, and $\lim_{t\to\infty} \varphi = 0$ implies $\lim_{t\to\infty} \gamma_{sh} = \gamma_s$, so

$$\lim_{t\to\infty} \dot{l}_h/l_h = (1-\sigma)(\gamma_s - \gamma_h) < 0.$$

Therefore, eventually l_h starts to fall towards zero.

$$d\left(\dot{l}_h/l_h\right)/dt = (1-\varepsilon)\, d\bar{\gamma}/dt - (\sigma - \varepsilon)\, d\gamma_{sh}/dt.$$

Let $x_{sh} = x_s + x_h = (1+\varphi)x_s$, using the definition of $\bar{\gamma}$, equations (23) to (26) imply

$$\begin{aligned} d\bar{\gamma}/dt &= \textstyle\sum_{i=a,m,sh} (x_i\gamma_i/X)\left(\dot{x}_i/x_i - \dot{X}/X\right) + (x_{sh}/X)\, d\gamma_{sh}/dt \\ &= (1-\varepsilon)\textstyle\sum_{i=a,m,sh} (x_i\gamma_i/X)(\bar{\gamma} - \gamma_i) + (x_{sh}/X)\, d\gamma_{sh}/dt \\ &= -(1-\varepsilon)\textstyle\sum_{i=a,m,sh} (x_i/X)(\gamma_i - \bar{\gamma})^2 + (x_{sh}/X)\, d\gamma_{sh}/dt \end{aligned}$$

and given $\gamma_{sh} = (\gamma_s + \varphi\gamma_h)/(1+\varphi)$, using the definition of φ,

$$d\gamma_{sh}/dt = (\sigma - 1)(\gamma_s - \gamma_h)^2 \varphi (1+\varphi)^{-2} > 0,$$

so,

$$d\left(\dot{l}_h/l_h\right)/dt = (\varepsilon - 1)\textstyle\sum_{i=a,m,sh} (x_i/X)(\gamma_i - \bar{\gamma})^2 + [(1-\varepsilon)(x_{sh}/X) - (\sigma - \varepsilon)]\, d\gamma_{sh}/dt$$

which is negative since $(1-\varepsilon)x_{sh}/X < 1 - \varepsilon < \sigma - \varepsilon$, and $d\gamma_{sh}/dt > 0$. So l_h is hump-shaped over time if $\dot{l}_h(t_0) > 0$, i.e. $H(t_0) > 0$. If not, l_h is falling monotonically.

■

Parameters for the numerical illustration

We first explain how to match the initial employment shares, n_a, n_m, and n_s. By definition, $n_i = l_i/q$, and given $q = l - l_h = l - \varphi l_s$ we obtain $l_i/l = n_i/(1 + n_s\varphi)$ for $i = a, s, m$. We then use (19) and (20) to derive $x_i = (l_i/l)(l_m/l - \eta)^{-1}$ for $i = a, s$, where η is the steady-state savings rate in terms of total (market and home) output. In other words, we can set the initial x_i to match initial employment shares in the data given η and φ.

The growth rates of φ, x_a and x_s can be computed from (12), (14) and (15), so with knowledge of the preference parameters (ε, σ) and the differences in the TFP growth rates we can compute φ, x_a and x_s at any point in time. Given the path of φ, x_a and x_s, the model predicts the path of overall labour supply and the employment shares.

In order to obtain the paths of time allocation we calibrate seven parameters: $\gamma_m - \gamma_a, \gamma_m - \gamma_s, \gamma_s - \gamma_h, \varepsilon, \sigma, \varphi, \eta$. As in Ngai and Pissarides (2004), we set $\gamma_m - \gamma_a = -0.01$ and $\gamma_m - \gamma_s = 0.01$ to match the changes in the prices of agriculture and service goods relative to manufacturing goods, and choose $\varepsilon = 0.1$ as our benchmark. The historical price data are from *Historical Statistics of the United States* and *Economic Report of the President*. The price data for services start in 1929. The average annual growth rate for the relative price of services in terms of manufacturing is 0.98% for the period 1929-98. For the same period, the relative price of agriculture in terms of manufacturing price is falling at an average rate of 1.03%. In Ngai and Pissarides (2004), we obtained an estimate of $\varepsilon = 0.3$ using thirteen 2-digit consumption goods sectors. For our three sectors it is more appropriate to use a smaller parameter, so we set $\varepsilon = 0.1$. Our main results, however, are not altered when $\varepsilon = 0.2$ and $\varepsilon = 0.3$. The remaining four parameters are all related to home production. The key home production variable φ is the ratio of time allocated to market services to the time allocated to the home sector. The *Historical Statistics* provide service employment for the years 1899, 1919, and 1929-2003. Ramey and Francis (2005) provide annual data for home hours and market hours for the period 1900-2003. We use their ratio of home to market hours in 1900 to obtain $l_h/q = 0.76$. The service employment share in 1899 was 0.28, so $l_s = 0.28q$, which yields $\varphi = l_h/l_s = 2.73$ in 1900. Since both the home and market hours and employment share series are available for the period 1929-2003, and for 1899 and 1919, we can compute φ for the entire period (see Figure 5). Ideally, we should compute φ in 1900 using service hour shares but hour shares are only available from the year 1948. The computed series for φ for 1948-2003 are virtually identical using hours or employment shares. The average growth rate for the computed series for 1900-2003 is -0.007 and, as assumed by the model, it is linear in the logs. From (12), matching the growth rate of φ requires $(1 - \sigma)(\gamma_s - \gamma_h) = -0.007$. An explicit price series is lacking for home production, so we do not have direct information on $\gamma_s - \gamma_h$. The parameter σ is the elasticity of substitution between service and home goods. McGrattan et al. (1997) estimate that the elasticity of substitution between all market and all home goods is around 2. Since we have three types of market goods with elasticity of substitution $\varepsilon < 1$, our σ,which is the elasticity of substitution between services only and home goods, must be at least as large as the elasticity of substitution between all market and home goods. Setting $\sigma = 2$, implies $\gamma_s - \gamma_h = 0.007$, which we use as our benchmark. Finally, $\eta = \eta^0 (1 - p_h c_h / y)$, where η^0 is the observable total saving as a fraction of market production only. Our model yields $p_h c_h / y = l_h / l$, so $\eta = \eta^0 / (1 + l_h / q)$. Using Maddison's (1992) data, the gross saving rate was 0.19 in 1900. This gives $\eta = 0.11$. Collecting all our calibrated values, we have,

Table 1. Baseline Parameters

ε	$\gamma_m - \gamma_a$	$\gamma_m - \gamma_s$	$\gamma_s - \gamma_h$	σ	$\varphi(t_0)$	η
0.1	-0.01	0.01	0.007	2.0	2.73	0.11

To compute the path in Figure 7 without home production, we make use of the calibrated market parameters $(\varepsilon, \gamma_m - \gamma_a, \gamma_m - \gamma_s, \eta^0)$, with $\eta^0 = 0.18$ which matches the average gross saving rate between 1900-2000. Note that in this case $\varphi = 0$.

Figure 1
The allocation of time, USA, 1900-2003

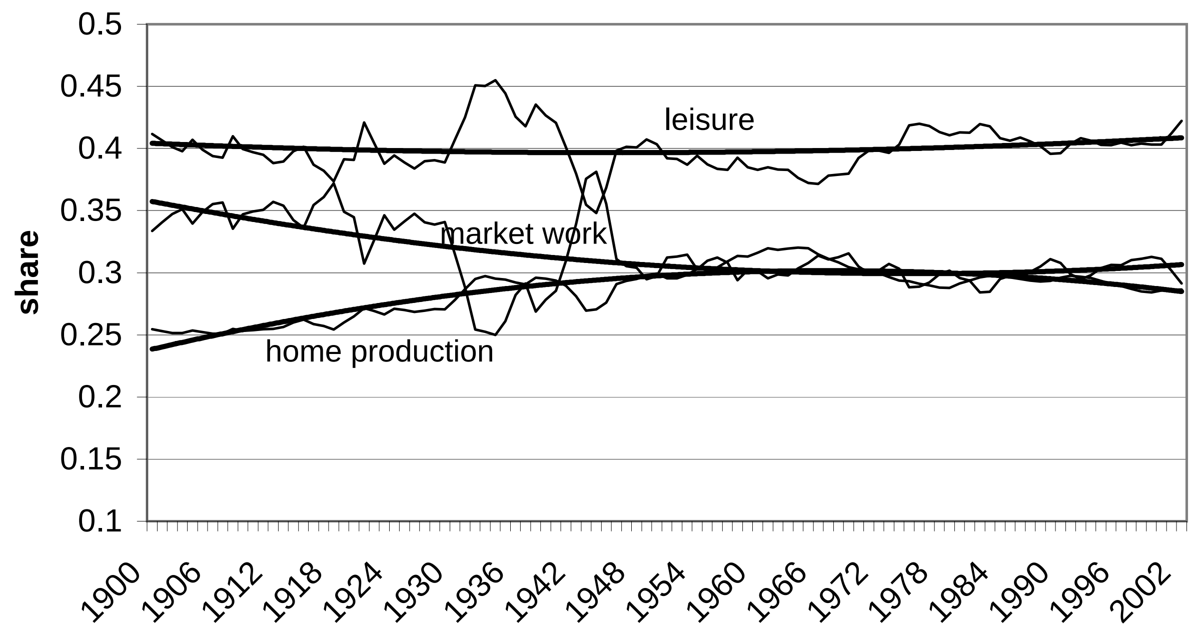


Figure 2
Hours of work in the population of working age, five countries 1960-2000

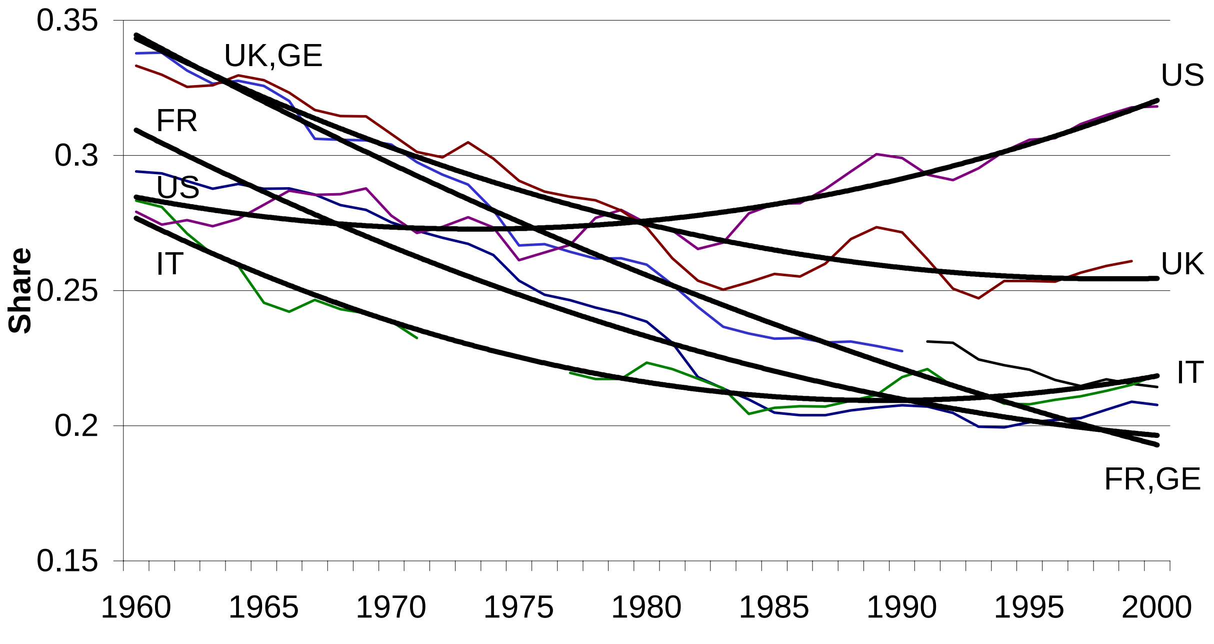


Figure 3

Aggregate Equilibrium

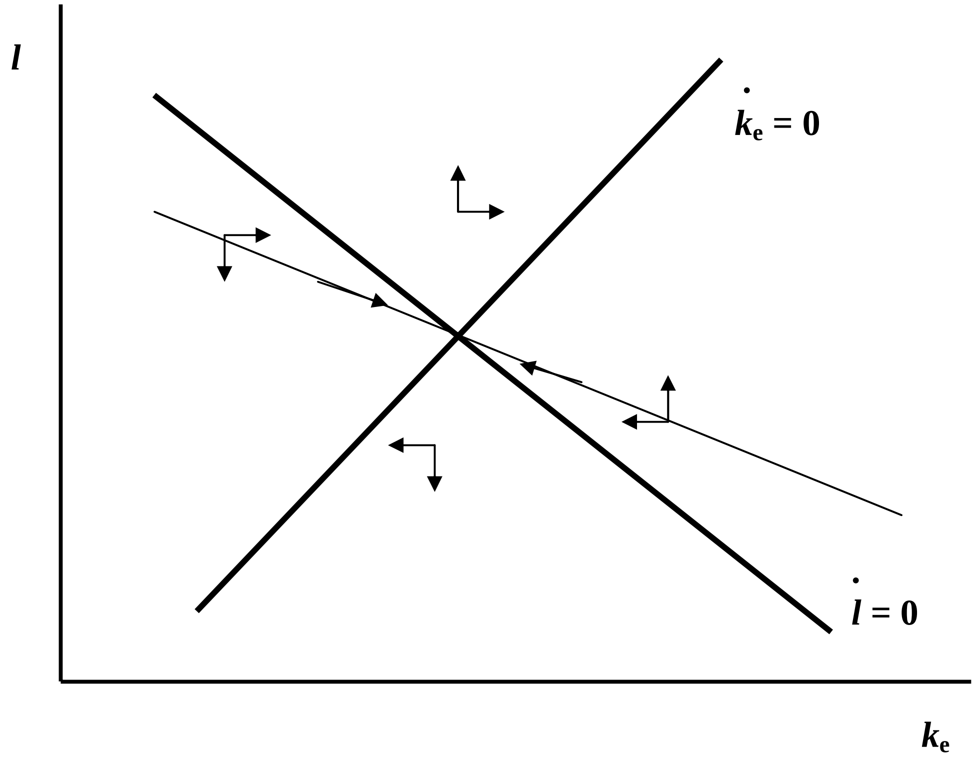


Figure 4
Allocation to home and market service work (logs), United States

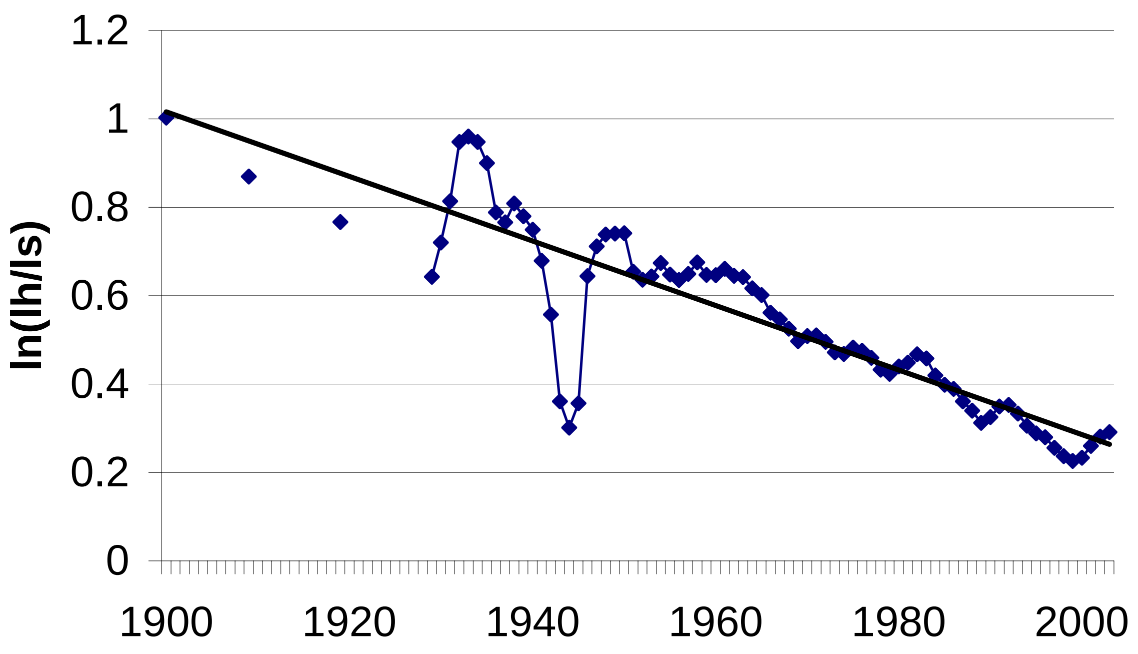
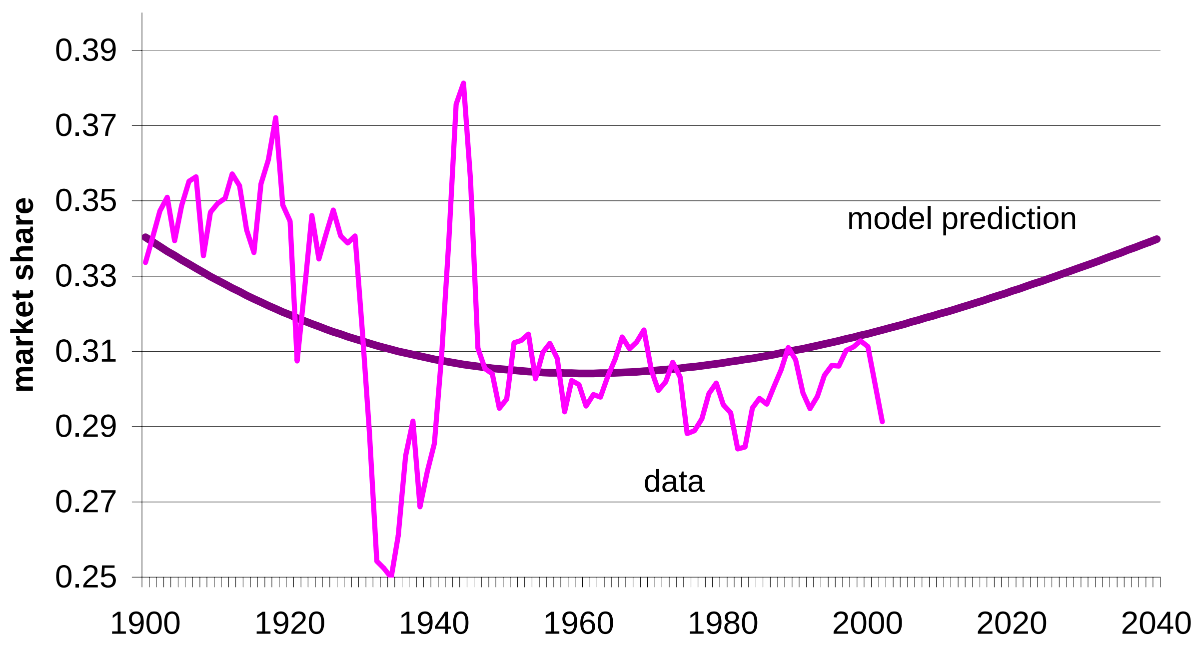


Figure 5
Market work, United States



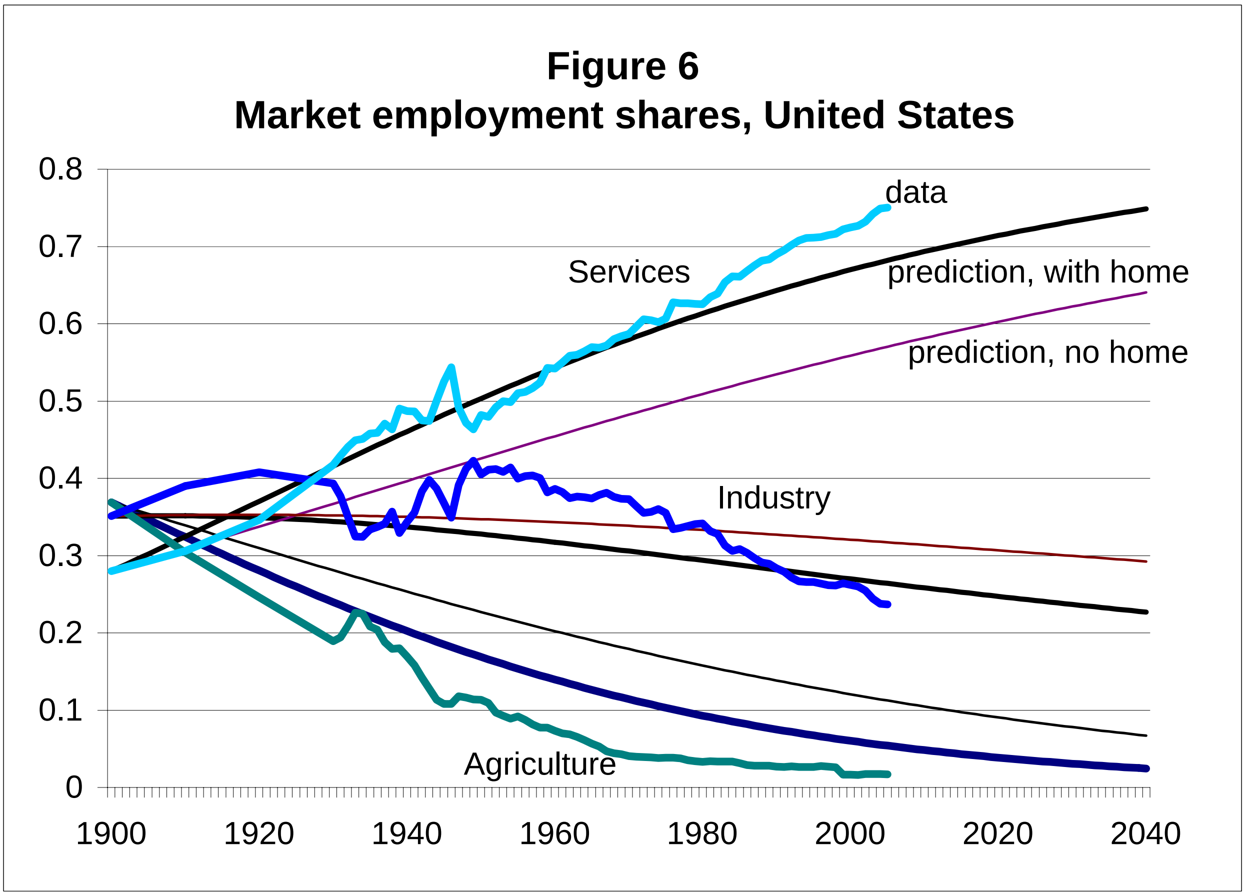


Figure 6
Market employment shares, United States
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1900
1920
1940
1960
1980
2000
2020
2040
data
Services
prediction, with home
prediction, no home
Industry
Agriculture