Shadow Sorting

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Abstract

This paper investigates the border between formal employment, shadow employment, and unemployment in an equilibrium model of the labor market with market frictions. From the labor demand side, firms optimally create legal or shadow employment through a mechanism that is akin to tax evasion. From the labor supply side, heterogeneous workers sort across the two sectors, with high productivity workers entering the legal sector. Such worker sorting appears fully consistent with most empirical evidence on shadow employment. The model sheds also light on the "shadow puzzle", the increasing size of the shadow economy in OECD countries in spite of improvements in technologies detecting tax and social security evasion. Shadow employment is correlated with unemployment, and it is tolerated because the repression of shadow activity increases unemployment. The model implies that shadow wage gaps should be lower in depressed labor markets and that deregulation of labor markets is accompanied by a decline in the average skills of the workforce in both legal and shadow sectors. Based on micro data on two countries with a sizeable shadow economy, Italy and Braziil, we find empirical support to these implications of the model. The paper suggests also that policies aimed at reducing the shadow economy are likely to increase unemployment.

- Key Words: Unemployment, Matching, Shadow Activity.
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1 Introduction

Modern information technologies allowing to cross-check information coming from different administrative sources and to quickly buildup and update inventories of bank accounts, make it relatively easy to detect and repress shadow activity. However, this is not done and Governments' statements of "tolerance zero" vis-à-vis the informal sector do not seem to be taken too seriously by firms and workers who continue to go underground. Indeed, the informal sector is flourishing: available estimates point to an upward trend in the size of shadow economy in OECD countries from high levels. The shadow share of GDP ranges from a low 10 per cent of GDP in the Nordics, UK and Switzerland to peaks of 20 to 30 per cent in Southern Europe and Ireland and 40 per cent in transitional economies of Eastern Europe and Asia.

Why is the informal sector so much tolerated? How do borders between shadow employment, legal employment and unemployment evolve under different macroeconomic conditions and institutional configurations? What does the reduction of the shadow sector imply in terms of labor productivity?

In this paper we address these issues theoretically and empirically, and we offer a simple explanation of the "shadow puzzle": shadow employment and unemployment are two faces of the same coin. Shadow employment is indeed correlated with unemployment. Based on macro, regional as well as microdata in Italy and Brazil we find clear evidence for this claim. Following this result, we argue that shadow employment is tolerated because its repression increases unemployment, with undesirable political consequences.

Our theory endogenises the choice of both, workers *and* firms, to go idle in an equilibrium model of the labor market with market frictions. From the labor demand side, firms optimally create legal or shadow employment through a mechanism that is akin to tax evasion. Being shadow means not paying taxes (including social security contributions) and not being liable to severance pay in case of a breakup of the employment relationship. However, there is a positive probability that irregular employment is detected, in which case the match is immediately dissolved. From the labor supply side, heterogeneous workers sort across the two sectors, with high productivity workers entering the legal sector. Such worker sorting appears fully consistent with most empirical evidence on shadow employment.

Repressing shadow employment, that is, increasing the detection probability, means increasing job destruction and reducing job creation in the shadow segment. While this repression tends to increase total employment in the legal sector, it also increases unemployment. Available theories of the informal sector – recently reviewed by Schneider and Enste (2000) – do not capture these trade-offs. This is because such theories take a partial equilibrium approach, focus either on labor demand or on labor supply, and do not consider sorting of workers with varying productivity levels in the two pools. Another distinguishing feature of our model is indeed that it self-selects workers in the two pool endogenously, by determining the productivity threshold demarcating the two pools.

The model implies a positive correlation between unemployment and shadow employment that is evident in cross country data as well as in regional data from Brazil and Italy, two countries with large shadow employment. To ensure that such correlation is not a statistical artifact we use a unique Brazilian data set where unemployment and shadow employment are two mutually exclusive states, and we find strong support for the positive correlation.

The model also implies that shadow wage gaps should be lower in depressed labor markets. We find empirical support also for this implication.

The paper proceeds as follows. Section 1 presents few empirical regularities on shadow employment. Section 2 introduces and solves the model, obtaining the various equilibrium configurations. Section 3 evaluates the comparative statics properties of the equilibria and provides some numerical simulations of the model. Section 4 assesses the empirical relevance of the model, drawing on micro data from two countries with a large shadow pool, namely Brazil and Italy. Finally, Section 5 briefly summarizes and concludes.

2 Shadow Facts

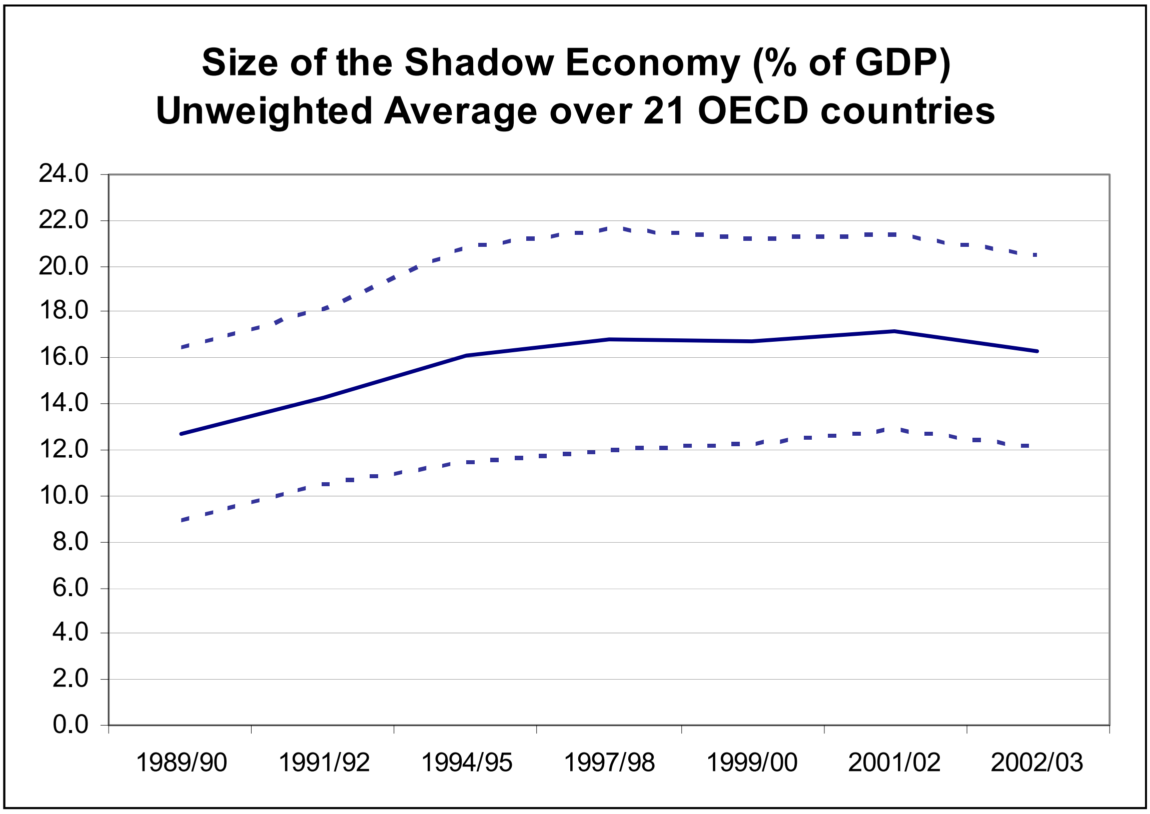
The consensus definition of the shadow economy is "all economic activities which contribute to the officially calculated (or observed) gross national product, but escape detection in the official estimates of GDP" [Feige, 1989 and 1994; Lubell 1991 and Schneider 1994]. This definition encompasses not only legal, but also illegal activities, such as trade in stolen goods, drug dealing, gambling, smuggling, etc.. In this paper we confine our attention to a subset of the shadow economy, namely to legal activities. As is apparent from the above, our notion of shadow employment is one of a lawful activity were it reported to tax authorities and subject to work regulations. We focus on this (large) subset of the shadow economy as our aim is to contribute to the literature on the enforcement of labor regulations and to complement research on tax evasion, which has so far overlooked the effects of tax evasion and shadow employment on unemployment[1].

Unfortunately, available estimates of the shadow economy do *not* disentangle legal from illegal shadow economy and rarely provide measures of shadow employment. The methods being used to measure the shadow economy either draw from *direct* inferences, that is surveys trying to elicit involvement of respondents in unregistered activities or estimates based on tax audits, or from *indirect* methods, which basically draw on the inconsistencies between different statistical sources in order to gauge the size of the underground economy. Among the latter methods, discrepancies between national income and expenditure statistics or between physical input (mainly electricity consumption) indicators of economic activity and official GDP statistics or between changes in the volumes of transactions and official GDP-GNP growth or in terms of "excess" currency demand (basically the residuals of a standard currency demand function), are the most frequently used. All the above methods have pros and cons, and the wide variance of estimates being provided is an indication of the limitations of these techniques. With these caveats in mind, let us briefly review the evidence on the size of the shadow economy, as also repeatedly summarized by Schneider (2002,2003,2004).

There are two key findings which are confirmed by all studies we are aware of.

The first common denominator of these *"consensus guesses"* is a marked upward trend in the size of the shadow economy. Figure 1 reproduces the (unweighted) average "shadow share" of GDP in all OECD countries for which estimates, *based on the same methodology,* are available for a relatively long-series. As revealed by the dotted lines (plotting one standard deviation above and below the unweighted cross-country average), there is no sign that this trend has increased the cross-country dispersion in the size of the shadow economy. The coefficient of variation of the shadow shares actually decreased from 1989-2000 to 2002-3 and there is not a single country with a declining shadow share. The upward trend in the shadow share is consistent across methods: it is found to hold not only in estimates based on currency demand, but also on the so-called DYMIMIC method (dynamic multiple indicators multiple causes, Giles, 1999) which estimates a set of structural equations within which the size of the shadow economy cannot be measured directly and then uses this predicted structural dependence in estimating the size of the shadow economy. Also estimates of the shadow economy in terms of headcounts point to an upward trend: Schneider

[1]See Burdett, Lagos and Wright (2000) for an analysis of the relationship between crime and unemployment.



Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, USA
Sources: Currency demand approach, Friedrich Schneider

Figure 1: The upward trend of the shadow economy

(2000) estimated that in the European area the number of persons working in the unofficial economy doubled within the two decades from 1978 to 1998.

The second fact is the relatively low productivity of shadow jobs documented by studies relying on micro-level data. In particular, Gonzaga (2003), Hoek (2004), Almeida and Carneira (2005), drawing on data on the informal sector in Brazil, Lacko (2000), Bernabe (2005) and Commander and Rodionova (2005), focusing on transitional economies, as well as Boeri and Garibaldi (2002) and Brandolini and D'Alessio (2004), drawing on Italian data consistently document that workers engaged in shadow employment have, on average, lower educational attainments than regular workers and or hold jobs requiring unskilled workers. The way in which shadow jobs are identified in these studies may not be neutral with respect to the productivity content of jobs in the two pools. However, the fact that low-skilled workers (or occupations) are represented in shadow employment is consistent across alternative measures of shadow employment.

Table 2 displays the distribution of employment by educational attainment for shadow and non-shadow segments of the labor force in Italy, according to different data sources and definitions. In particular, the top panel draws on Bank of Italy data and identifies shadow employment by looking at self-reported social security records: shadow employees are those who either reported to have never paid social security contributions throughout their career (definition 1) or who report the same number of months of contributions (definition 2) during the same employment spell two years apart (which implies that they have not been paying contributions in between the two interviews) [2]. Clearly definition 1 is more restrictive than definition 2. The mid-panel of Table 2 draws on labor Force Survey data and identifies as shadow employees those individuals who are employed according to internationally agreed, objective, definitions, but who define themselves as non-employed. Finally, the bottom panel draws on data collected by an ad-hoc Istat-Fondazione Curella survey carried out in Sicily in 1995 (Busetta and Giovannini, 1998). In this context, shadow employement is identified in the individuals reporting to hold an irregular job, where irregular means not paying social security contributions, understating the actual pay in order to pay lower taxes and contributions or being altogether without a labor contract.

All data sources and measures of shadow employment suggest that workers with lower educational attainments are over-represented in the shadow pool.

Overall, shadow employment has mainly the characteristics of "marginal shadow employment', that is, employment in low productivity jobs, rather than "development shadow employment', i.e., new jobs having the potential to become highly productive after some gestation period. In other words, "infant industry" arguments cannot be applied to justify tolerance vis-á-vis the informal sector. We are looking for deeper and empirically more relevant ("development shadow employment" seems to involves a tiny fraction of unregistered employment) explanations for the weak repression of shadow employment.

[2] Clearly this second definition requires exploiting the longitudinal features of the Bank of Italy Survey. For a description see Boeri and Brandolini (2004).

Shadow Employment by Educational Attainment of the Workforce
a) Bank of Italy survey, average 1995-2002

Education	Shadow (Def.1)	Shadow (Def. 2)	Control (Def.1 and 2)	Shadow (Def. 3)	Control (Def.3)
	Δcontrib=0	Δcontrib=0 + Δcontrib <0	Δcontrib=2	No contribution at all	At least 1 year of contribution
Primary or lower	13.5	14.7	7.5	32.1	30.5
Lower secondary	35.4	33.6	27.8	31.5	27.7
Lower vocational (3 years)	6.8	6.5	9.1	4.0	6.3
Secondary school	33.8	32.0	40.8	23.9	26.4
Tertiary education	10.5	13.1	14.8	8.5	9.1

b)LFS data, Italy average 1995-2002

Education	Shadow	Regular employment
Primary or lower	38.4	15.0
Lower secondary	25.6	36.1
Lower vocational (3 years)	4.3	7.8
Secondary school	24.5	29.9
Tertiary education	7.2	11.2

c) Istat-Fondazione Curella, Sicily 1995

Education	Main job		Secondary job	
	Shadow	Regular employment	Shadow employment	Regular employment
Primary or lower	24.0	13.5	19.5	8.8
Lower secondary	27.3	26.1	20.7	17.6
Secondary school	40.3	41.9	39.0	44.1
Tertiary education	8.4	18.4	20.7	29.4

Figure 2: Shadow Employment by Educational Attainment of the Labor Force

3 A Two Sectors Model with Sorting

3.1 Shadow Employment and Worker's Sorting

We consider an economy with a measure one of heterogenous workers and two sectors. The worker type is indicated by x, where x refers to labor market productivity and its value is drawn from a continuous cumulative distribution function F with support $[x_{\min,} x_{\max}]$. x is a fixed time invariant worker characteristic, with $x_{\min} > 0$.

There are two sectors in the labor market: the regular sector and the shadow sector. The gross value of production of each worker is indicated with px where p is a productivity component common to all jobs and x is an idiosyncratic component. To keep the notation simple, we initially assume that $p = 1$, and we consider changes in p in the numerical simulations. In the regular sector firms pay a production tax τ in every period in which they employ a worker. In the shadow sector the tax is evaded and there is an instantaneous monitoring rate equal to ρ. Conditional on being monitored in the shadow sector, the shadow job is destroyed. Both regular and shadow jobs are exogenously destroyed at rate λ[3].

Firms can freely post a vacancy in either sector. We focus on single jobs, and each firm is made of one job. Posting a vacancy in the regular sector costs k_g per period while in the shadow sector costs k_b. There is free entry of firms in both sectors and the equilibrium value of a vacancy is driven down to zero. Job creation characterizes the labor demand side of the model.

The labor supply is governed by the workers' sorting behavior. Workers are endowed with a unit of time and freely decide whether it is optimal to search and work in the shadow sector or in the legal sector. Entering a sector is a full time activity, and workers can not simultaneously work and/or search in both sectors. In the legal sector there is a specific unemployed income (the unemployment benefits) which is not available in the shadow sector.

Labor markets are imperfect, and there are market frictions in each sector. We follow the main matching literature (Pissarides, 2000) and assume that the meeting of vacant jobs and unemployed workers is regulated by a matching function with constant returns to scale. Different matching functions exist in different sectors. In what follows we let with v_g and v_b the number of vacancies in both sectors, and u_g and u_b the number of unemployed job seekers. The matching function in

[3] In the simulations we also assume that conditional on λ striking, regular jobs need to pay a firing tax T.

each sector is indicated with

$$m^i(u^i, v^i) \qquad i = g, b$$

with positive first derivative and negative second derivative. As in the traditional matching models with constant returns to scale, the transition rate depends on the relative number of traders and it is indicated with $\theta^i = \frac{v^i}{u^i}$. Specifically, the transition rate for firm is indicated with $q^i(\theta^i) = \frac{m(u^i,v^i)}{v^i}$ with $q'(\theta^i) < 0$, while the transition rate for workers is indicated with $\alpha'^i(\theta^i) = \theta^i q(\theta^i)$ with $\alpha' > 0$.

Successful matches in each sector enjoy a pure economic rent, and we let wages be the outcome of a Nash bargaining problem, with workers getting a fraction β of the total surplus. We assume, for simplicity, that β is identical in the two sectors.

We solve the model in three steps. First we present the value functions and the asset equations, and define the key equilibrium conditions. Next, we solve the workers' sorting behaviour in partial equilibrium, taking as given job creation (the labor demand side of the model) and the transition rate in each market. We then focus on job creation taking worker behaviour as given. Finally we discuss the general equilibrium of the model, and we perform a set of numerical simulations.

3.2 Discussion

Before proceeding to the solution of the model, few important issues need to be discussed. Our theory does not deal with the optimal enforcement of legal activity. Within the model, enforcement takes place through the combination of random detection (the monitoring rate ρ) and finite punishment (in the form of job destruction). The influential analysis of Becker (1968) has shown that, from the social welfare standpoint, it is always optimal to substitute a higher fine for a lower probability of detection, and that fines should be optimally set at their maximum level. In such optimal enforcement setting, shadow employment would not be observed in equilibrium. While the Becker argument is clear and convincing, we rarely observe such harsh punishment, possibly because important market imperfections reduce the size of the optimal fine. Davidson et al. (2004) have recently shown that with capital market imperfections and/or asymmetric information, the optimal fine lies below the maximum level. Even though we do not explicitly take into account these features, we believe that our realistic enforcement rule can be rationalized in such more complex models, which are nevertheless left to further research.

The difference between legal and shadow jobs considered in the model focuses only on tax compliance, and does not consider the possibility that jobs in the two sectors differ along other important dimensions, such as capital intensity, health insurance, and firm sponsored training. In reality, workers' sorting decision takes probably into account of various job characteristics, and there is evidence that legal jobs provide more training. We believe that it is technically possible to provide such key extensions, without affecting the main results of the paper.

Our model considers shadow employment as a full time activity and does not allow workers to hold multiple jobs (i.e. a regular job alongside a shadow job). In terms of flows, the model ignores on the job search and direct transitions from shadow to legal employment without intervening unemployment spells. Some of these features were considered by Boeri and Garibaldi (2002) in a matching model with fixed labor supply, without any scope for worker sorting, the key feature of this paper.

3.3 Value Functions

The value of a filled job in the legal sector with productivity x reads

$$rJ^g(x) = x - w^g(x) - \tau + \lambda[V^g - J^g(x)]$$

where τ is the tax rate, V^g is the value of a vacancy and r is the pure discount rate. Jobs are destroyed at the exogenous rate λ, and $w_g(x)$ is the wage rate.

Unemployment is a full time activity, and workers can not work in the shadow sector during an unemployment spell. The value of unemployment in the legal sector for a worker of type x is

$$rU^g(x) = b + \alpha^g(\theta)[W^g(x) - U^g(x)]$$

where b is the specific unemployed income (the unemployment benefits), and $W^g(x)$ is the value of the job for a type x. The value of a job in the legal sector is

$$rW^g(x) = w^g(x) + \lambda[U^g(x) - W^g(x)].$$

Posting vacancies in the legal sector is costly, and yields a per period return equal to $-k_g$. Conditional on meeting a worker, at rate $q^g(\theta^g)$, the firms gets the expected value of a job. In formula, its expression reads

$$rV = -k_g + q^g(\theta^g)\left[E\left[J(z) \mid z \in \Omega\right] - V\right]$$

where the expectation is taken with respect to the productivity of workers that search in the legal sector. The expression Ω refers to the support of workers that search in the legal sector.

The value functions for jobs in the shadow sector are similarly defined. The main differences is that in the shadow sectors firms do not pay the production tax τ and the job is monitored and destroyed at rate ρ. Further, there is no specific unemployed income b. The four value functions read

$$\begin{aligned}
rJ^b(x) &= x - w^b(x) + (\lambda + \rho)[V^b - J^b(x)] \\
rW^b(x) &= w^b(x) + (\lambda + \rho)[U^b(x) - W^b(x)] \\
rU^b(x) &= \alpha^b(\theta^b)[W^b(x) - U^b(x)] \\
rV^b &= -k_b + q^b(\theta^b)\left[E\left[J^b(z) \mid z \in \Omega^c\right] - V^b\right]
\end{aligned}$$

where Ω^c is support of workers that search in the shadow sector.

Wages in each sector and in each job are the outcome of a bilateral matching problem and workers get a fraction β of the total surplus so that

$$[W^i(x) - U^i(x)] = \beta[W^i(x) - U^i(x) + J^i(x) - V^i] \qquad i = b, g$$

for simplicity we have assumed that the fraction of the surplus is the same in both sectors.

3.4 Equilibrium Conditions

There are three key equilibrium conditions

- Free entry and job creation in the legal sector (JC^g), which implies that the value of a vacancy be zero

  $$V^g = 0$$

  This equation will determine market tightness in the legal sector θ^g

- Free entry and job creation in the shadow sector (JC^b), which implies that the value of a vacancy be zero

  $$V^b = o$$

  This equation will determine market tightness in the shadow sector θ^b

- Workers' sorting (*Sort*). If we assume that workers' sorting satisfies the reservation property, (a feature that holds in equilibrium) the labor supply is described by the marginal worker with productivity R, where R is the productivity level for which the worker is indifferent between the two sectors, so that

$$U^g(R) = U^b(R)$$

Using the reservation property, the three key conditions are

$$\alpha^b(\theta^b)[W^b(R) - U^b(R)] = b + \alpha^g(\theta^g)[W^g(R) - U^g(R)] \tag{Sort}$$

$$\frac{k_g}{q^b(\theta^b)} = \frac{\int_R^{x^u} J^g(z)dF(z)}{1 - F(R)} \tag{JC^g}$$

and

$$\frac{k_b}{q^b(\theta^b)} = \frac{\int_{x_l}^R J^b(z)dF(z)}{F(R)} \tag{JC^b}$$

The first condition says that the marginal worker is indifferent between searching for a job in the legal or the shadow sector. The second condition says that the total search costs in the legal sector are identical to the expected value of a job. The last condition has a similar interpretation, but refers to the shadow sector. The system determines the three endogenous variables θ^g, θ^b and R

3.5 Stocks

The model is closed by determining the stock of workers into the four possible labor market states: unemployment and employment in each of the two sectors. If we indicate with u^i the stock of unemployed in each sector and with n^i the stock of employed, we have

$$u^g + u^b + n^g + n^b = 1$$

Workers' sorting implies that the share of workers in the shadow sectors is $F(R)$ while the remaining $1-F(R)$ workers search in the legal sector. Employed workers in the shadow sector lose their job at rate $\lambda+\rho$ while they find jobs at a rate $\alpha^b(\theta^b)$ so that the balance flow condition for unemployment in the shadow sector is

$$\alpha^b(\theta^b)u^b = (\lambda + \rho)(F(R) - u^b)$$

where $n^b = F(R) - u^b$. Unemployment and employment in the shadow sector read respectively

$$u^b = \frac{(\lambda + \rho)F(R)}{\lambda + \rho + \alpha^b(\theta^b)}$$

$$n^b = \frac{\alpha^b(\theta^b)F(R)}{\lambda + \rho + \alpha^b(\theta^b)}$$

In the legal sector, the unemployment and the employment rate are respectively

$$u^g = \frac{\lambda(1-F(R))}{\lambda+\alpha^b(\theta^b)}$$

$$n^g = \frac{\alpha^b(\theta^b)(1-F(R))}{\lambda+\alpha^b(\theta^b)}$$

We are now in a position to formally define the equilibrium of the model.

Definition 1 *Equilibrium. The equilibrium is obtained by a triple R, θ^g and θ^b and a vector of stock variables that satisfy the value functions J^i, W^i, U^i, V^i $(i = g, b)$, Nash Bargaining, and i) Workers' sorting, ii) Job Creation in the legal sector, iii) Job Creation in the shadow sector, iv) balance flow conditions.*

3.6 Solving the worker's sorting behavior

The surplus of a job in each sector is defined as the sum of the worker's and firm value of being on the job, net of the respective outside options, so that

$$S^i(x) = J^i(x) - V^i + W^i(x) - U^i(x)$$

Using the value functions previously defined, as well as the free entry condition (which drives the value of a vacancy down to zero), the surplus of a match for a legal job with productivity x is

$$(r+\lambda)S^g(x) = x - \tau - b - \alpha^g(\theta^g)[W^g(x) - U^g(x)]$$

Recalling that wages get a fraction β of the total surplus, the previous expression reads

$$S^g(x) = \frac{x-\tau-b}{r+\lambda+\beta\alpha^g(\theta^g)}$$

with $S' = \frac{1}{r+\lambda+\beta\theta q(\theta)}$. Proceeding similarly, the surplus in the shadow sector is

$$S^b(x) = \frac{x}{r+\lambda+\rho+\beta\alpha^b(\theta^b)}$$

In partial equilibrium, the job finding rates a^i are constant, and the surplus from the job is an increasing linear function of the match specific productivity x.

The surplus from the job can be used to obtain an expression for the value of unemployment, whose expression is given by

$$U^b(x) = \frac{\alpha^b(\theta^b)\beta x}{r+\lambda+\rho+\beta\alpha^b(\theta^b)}$$

$$U^g(x) = b + \frac{\alpha^g(\theta^g)\beta[x-\tau-b]}{r+\lambda+\beta\alpha^g(\theta^g)}$$

Figure 3 shows the two value functions in partial equilibrium. The differences in the two curves are driven by the intercept (which is negative in the legal sector) and the slope. We make two key assumptions in this respect:

- **Taxation is large enough relative to unemployment benefits**. We formally assume that $b(r+\lambda) < \tau\alpha^g\beta$. This implies that the intercept of U^g is negative in Figure 3.
- **Monitoring is large enough**. We formally assume that $\alpha^g\rho\beta + (r+\lambda)\beta(\alpha^g - \alpha^b) > 0$. This implies that the value function of U^g is steeper than U^b.

From the value functions, we can get an expression for the reservation productivity. The reservation value R, if it exists, is the crossing point of the two lines. Its formal expression, when considering α^g and α^b exogenous and constant is

$$R = \frac{[\tau\alpha^g\beta - b(r+\lambda)](r+\lambda+\rho+\beta\alpha^b)}{\alpha^g\rho\beta + (r+\lambda)\beta(\alpha^g - \alpha^b)}$$

Existence in partial equilibrium requires $R > 0$, and the two key assumptions above ensure that R is positive. The equilibrium we are considering implies that shadow jobs are occupied by workers with low skills, in line with the evidence discussed in Section 2 of this paper This is a key premise of our theoretical analysis

Remark 2 *Shadow jobs are occupied by relatively low skilled workers.*

There are several results in the partial equilibrium setting, and are graphically obtained by shifts and movements of the two lines

- An *increase in unemployment benefits* reduces the reservation productivity R, so that more people search in the legal market. At given job finding rates, an increase in unemployment benefits increases legal employment. This is the standard entitlement effects of unemployment benefits, a labor supply phenomenon that was first noted by Burdett and Mortensen (1982) and Atkinson (1991) and recently received a lot of attention (Fredrikson and Holmlund, 2002; Garibaldi Wasmer,2005; Boeri, 2000). Formally, it is obtained by noting that

$$\frac{\partial R}{\partial b} = -\frac{(r+\lambda)(r+\lambda+\rho+\beta\alpha^b)}{\alpha^g\rho\beta + (r+\lambda)\beta(\alpha^g - \alpha^b)} < 0$$

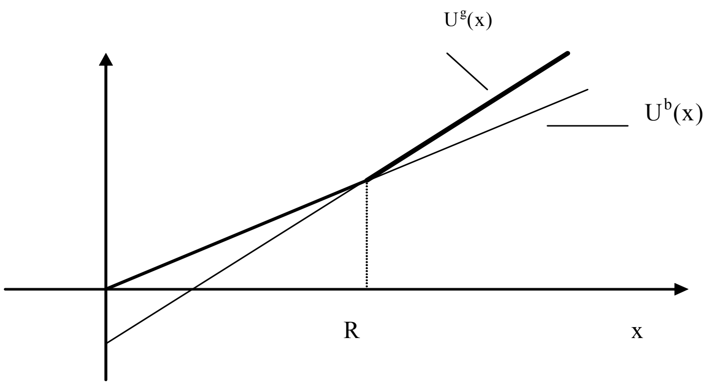


Figure 3: Workers' sorting in partial equilibrium (with constant job finding rate)

- An *increase in taxation* increases shadow employment. This is the standard mechanism that taxation moves away people from the regular sector into the shadow employment, as noted by the work of Schneider (2002) and recently by Davis and Henrekson (2004). Formally, it is obtained by observing that

$$\frac{\partial R}{\partial \tau} = \frac{\alpha^g \beta (r + \lambda + \rho + \beta \alpha^b)}{\alpha^g \rho \beta + (r + \lambda)\beta(\alpha^g - \alpha^b)} > 0$$

- An *increase in the monitoring rate* reduces shadow employment. An increase in the monitoring rate reduces the return from shadow employment and induces people to search in the legal market. Formally, this result is obtained by noting that

$$\frac{\partial R}{\partial \rho} = \frac{[b(r + \lambda) - \tau \alpha^g \beta]\alpha^b \beta (r + \lambda + \beta \alpha^g)}{[(r + \lambda)\beta(\alpha^b - \alpha^g) - \alpha^g \rho \beta]^2} > 0$$

3.7 Labor Demand and Job Creation

To solve for job creation we need to evaluate the expected value of a job. We first focus on legal jobs. After an integration by parts, and making use of the sharing rule, the integral in equation

JC^g can be written as

$$\begin{aligned}\int_R^{x^u} S(z)dF(z) &= S(x^u) - S(R) + (1-F(R))S(R) - S'(R)\int_R^{x^u} F(z)dz \\ &\quad \frac{\int_R^{x^u}(1-F(z))dz}{r+\lambda+\beta\theta^g q(\theta^g)} + \frac{(1-F(z))[R-\tau-b]}{r+\lambda+\beta\theta^g q(\theta^g)}\end{aligned}$$

so that the job creation condition is

$$\frac{k_g[r+\lambda+\beta\alpha^g(\theta^g)]}{q(\theta^g)(1-\beta)} = \frac{\int_R^{x^u}(1-F(z))dz}{1-F(R)} + [R-\tau-b] \tag{1}$$

Proceeding similarly for the expected value of bad jobs, the free entry condition reads

$$\frac{k_b[r+\lambda+\beta\alpha^b(\theta^b)]}{q(\theta^b)(1-\beta)} = R - \frac{\int_{x_l}^R F(z)dz}{F(R)} \tag{2}$$

Market tightness θ^i and the associated job finding rates α_i depend on the various parameters, as well as on the workers' sorting behavior. Most parameters have a direct effect on job creation, plus an indirect effect via the reservation productivity R. Formally, we can write

$$\begin{aligned}\alpha^g(\theta^g) &= \alpha^g(R(.), b, r, \lambda, \beta)) \\ \alpha^b(\theta^b) &= \alpha^b(R(.), \rho, \lambda, \beta))\end{aligned}$$

where the symbol $R()$ suggests that R is itself an endogenous variable. Some important comparative static results follows

- An *increase in the reservation productivity* R increases market tightness and the job finding rates in both sectors. An increase in R increases the average quality of the workforce in both sectors, so that firms naturally respond by posting more vacancies per unemployed. This result is important, and shows how sorting affects job creation. Formally, it is obtained by noting that $\frac{\partial\theta^g}{\partial R} > 0$ and $\frac{\partial\theta b}{\partial R} > 0$ since

$$\begin{aligned}\frac{k_g}{(1-\beta)}\frac{\beta\alpha'^g(\theta^g)q^g(\theta^g) - q'^g(\theta^g)(r+\lambda+\beta\alpha^g(\theta^g))}{q^g(\theta^g)^2}\frac{\partial\theta^g}{\partial R} &= \frac{f(R)\int_R^{x^u}F(z)dz}{(1-F(R))^2} \\ \frac{k_b}{(1-\beta)}\frac{\beta\alpha'^b(\theta^b)q^b(\theta^b) - q'^b(\theta^b)(r+\lambda+\beta\alpha^b(\theta^b))}{q^b(\theta^b)^2}\frac{\partial\theta^b}{\partial R} &= \frac{f(R)\int_{x^l}^R F(z)dz}{F(R)^2}\end{aligned}$$

  where the LHS is positive since $q' < 0$.

- An *increase in unemployment benefits* b, at given reservation productivity R, reduces job creation in the legal sector. This is the standard adverse effect of unemployment income on job creation, an effect that works mainly through the wage rule.

- An *increase in taxation*, at given reservation productivity R, reduces job creation in the legal sector. This is also a textbook adverse labor demand effect of taxation

- An *increase in the monitoring rate* ρ, at given reservation productivity R, reduces job creation in the shadow sector. Higher monitoring rate acts as an increase in the destruction rate on shadow jobs.

3.8 General Equilibrium

The general equilibrium of the model is obtained by solving for the triple $R\,,\theta^g,\theta^b$ that simultaneously satisfy Sort JC^b and JC^g. One way to solve for the general equilibrium result is to consider the workers' sorting condition by explicitly considering the relationship between the job finding rates and the reservation productivity. This is equivalent to solving

$$\frac{\alpha^b(R,.)\beta R}{r+\lambda+\rho+\beta\alpha^b(R,.)} = b + \frac{\alpha^g(R,.)\beta[R-\tau-b]}{r+\lambda+\beta\alpha^g(R,.)} \tag{3}$$

where the expression $\alpha^b(R,.)$ and $\alpha^g(R,.)$ are consistent with the job creation conditions. Both sides of the expression are increasing functions of R. The difference with respect to the partial equilibrium result is that the expressions for the value of unemployment in equations (3) are no longer simple linear function, but they are both increasing functions of R. To understand this, consider the effects of an increase in R on the value of unemployment in both sectors, there are two effects at work.

- First, there is a positive *surplus effect.* This is analogous to the effect analysed in partial equilibrium. An increase in R increases the value of unemployment in both sectors, but has a larger effect on the legal sector in light of the difference in the slope and the presence of ρ in the shadow sector.

- Second, there a is a *job creation* effect. An increase in R increases the job finding rate in both sector, since the average value of the workforce increases.

As both effects reinforce each other in a non linear fashion, multiple equilibria can not be ruled out ex-ante. This should not be surprising, since multiple equilibria in matching models with double heterogeneity are a standard feature (Albrecth and Vroman, 2002).

Remark 3 *Multiple equilibria can not be ruled out, and depend on the distribution of productivity.*

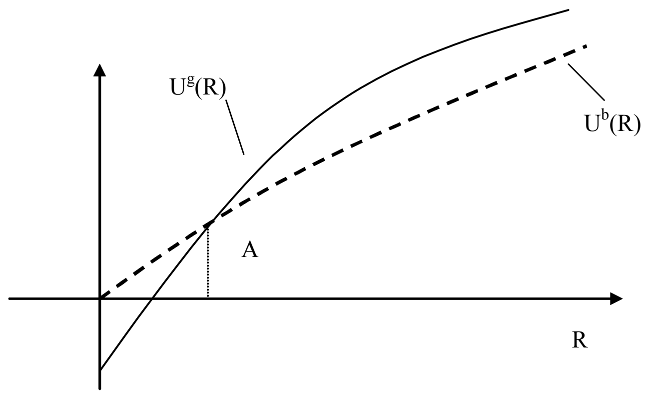


Figure 4: The General Equilibrium

Since both sides are increasing and non linear functions of R, there is no guarantee that the equilibrium is unique.

In the simulations that follow, where we use a distribution for the productivity x that is negative exponential, there is a unique equilibrium. In any case, if there were two equilibria, there would be different implications for the distribution of skills across the two sectors, with a perverse equilibrium that implies that high productivity workers enter the shadow sector. In figure 4, the equilibrium of point A is consistent with the skilled distribution that we highlighted in the comparative static section. The feature of such an equilibrium can be described as follows

$$\begin{aligned} U^g(R^*) &= U^b(R^*) \\ U'^g(R^*) &> U'^b(R^*) \end{aligned}$$

where the second condition ensures that the value function of the legal sector is the steepest one in the equilibrium point.

4 Simulations and Comparative Static

The comparative static results in the general equilibrium are not straightforward, since they combine the effect of each parameter on the labor demand and the labor supply of the model.

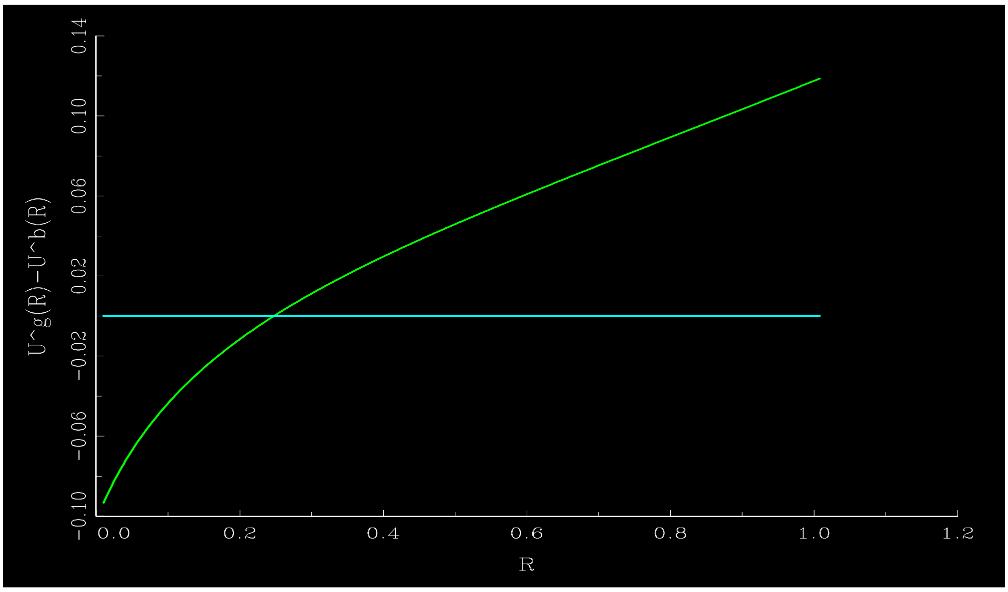


Figure 5: General Equilibrium with Exponential Distribution

Consider the effects of taxation. An increase in taxes tend to push jobs into the shadow sector, and to decrease the value of each job. This is a standard result that reduces job creation. Yet, the resulting increase in R improves the average quality of the workforce in the legal sector, with a positive effect on job creation. As a result, the total effect on job creation may be ambiguous.

Consider an increase in the monitoring rate. On the one hand, it reduces R from the labor supply standpoint and reduces job creation in the shadow sector. Both effects reinforce each other, and tend to reduce R. On the other hand, the reduction in R, by increasing the average productivity of workers in the legal sector, feeds back on job creation in the legal sector, and tends to reduce R. This suggests that an increase in the monitoring rate can reduce job creation in the good sector.

Similar logical arguments follow for the other comparative static exercise. The increase in unemployment benefits reduces (in partial equilibrium) the number of people in the shadow sector by reducing R. The fall in R induces a feed back effect on the average quality of the workforce in the legal sector and, from the labor demand side, a reduction in job creation.

4.0.1 Baseline Specification

The baseline specification of the model is described in Table 1. With respect to the model presented in the equations, the empirical specification of the productivity is px, where x is the idiosyncratic component of productivity and p is an aggregate component. Further, in addition to a production tax τ, the simulations consider also a firing tax T, to be paid only in the legal sector conditionally on a job separation (when the shock λ strikes).

The distribution is negative exponential. Figure 5 reports the difference between $U^g(R)-U^b(R)$ for different values of the reservation productivity. The general equilibrium is described by the crossing of such difference with the zero line. The figure clearly shows that there is a single crossing and that the equilibrium is unique. The baseline parameterization is described and reported in Table 1. Most parameters are standard in the literature (notably a 0.5 value for the bargaining share and the matching elasticity). The search costs correspond to 25 percent of the value of the labor product, a value that is roughly consistent with the structural estimates provided by Yashiv (2000).

The shadow rate, defined as the ratio between employment in the shadow sector and total employment (including both n^g and n^b at the denominator) is around 14 percent. We perform various comparative static exercises

Table 1: Calibration

Parameters	Notation	Legal	Shadow
Discount Rate	r	0.03	
Separation Rate	λ	0.15	0.15
Unemployed Income	b	0.10	0.00
Firing Tax	F	0.10	0.00
Matching Elasticity	η^i	0.50	0.50
Monitoring Rate	ρ	0.00	0.06
Production Tax	τ	0.20	0.00
Matching Function Constant	A^i	0.50	0.50
Workers' Surplus Share	β	0.50	0.50
Common Productivity	p	1.50	1.50
Search Costs	k^i	0.40	0.40
Equilibrium Values			
Sorting Productivity	R	0.24	
Market Tightness	θ^i	2.70	0.16
Job Finding Rate	α^i	0.82	0.28
Aggregate Statistics			
Unemployment	u^i	12.10	7.52
Employment	n^i	66.23	14.15
Shadow Rate	s	17.60	
Average Wage	w^i	1.37	0.12
(a), Distribution is Exponential with parameter B =1.00			
Source: Authors' calculation			

4.0.2 Changes in Aggregate Conditions

We study the effects of the increase in p on the general equilibrium of the model. The results are reported in Table 2. With the exception of p, all the other parameters are identical to those of Table 1.

An increase in aggregate productivity increases employment and reduces unemployment in the legal sector. Further, it reduces employment in the shadow sector. This is one of the key macroeconomic results of the paper. Unemployment and shadow employment are positively correlated across different states of the macroeconomy

- **Remark 4** *Unemployment and shadow employment are two faces of the same coin. Worse aggregate conditions induce an increase in both unemployment and shadow employment (as well as its shadow rate)*

Table 2: Changes in Aggregate Conditions

p	R	θ_b	θ_g	u_b	u_g	n_b	n_g	s	x_g	x_b	$\overline{w}_g$	$\overline{w}_b$
1.50	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.37	0.12
1.61	0.23	0.16	2.92	7.03	11.90	13.22	67.85	16.31	1.23	0.11	1.46	0.12
1.73	0.21	0.16	3.15	6.60	11.70	12.40	69.30	15.18	1.21	0.10	1.55	0.12
1.84	0.20	0.16	3.39	6.22	11.51	11.68	70.60	14.19	1.20	0.10	1.64	0.12
1.95	0.19	0.16	3.62	5.88	11.32	11.03	71.77	13.33	1.19	0.09	1.73	0.12

u_g and u_b are the unemployment rates respectively in the legal and shadow sector

n_g, E_{n_b}, are respectively legal and shadow employment.

x_g and x_b are the average idyosincratic productivity in the legal and shadow employment

$\overline{w}_g$ and $\overline{w}_b$ are the average wages legal and shadow employment

Source: Authors' calculation

The logic of this result can be expressed as follows. The increase in p tends to increase job creation and market tightness. Simultaneously, the increase in p induces a fall in the marginal productivity R, so that average quality worsens in both sectors. This tends to reduce job creation. The second effect appears to be quantitatively more important in the legal sector, since the productivity is proportional to x.

Table 2 shows that wage differentials between the legal and the shadow sector (the shadow wage gap) are quantitatively more important when aggregate business conditions are good.

- **Remark 5** *Wage differentials should be larger in less depressed regions.*

There are two adjustment mechanisms behind this result. First, a larger p directly affects match productivity inducing an increase in wages per any given x. Second, the rise in aggregate productivity involves a reduction of the productivity threshold so that the average quality of matches in both sectors decline. This tend to depress average wages in both sectors. As the aggregate shock is multiplicative, its direct (positive) effects on wages are quantitatively more important in the legal sector than in the shadow sector, whilst the indirect effects are nearly symmetric due to the common threshold, R.

Table 3: Changes in Total Taxation Conditions

τ	R	θ_b	θ_g	u_b	u_g	n_b	n_g	s	x_g	x_b	$\overline{w}_g$	$\overline{w}_b$
0.200	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.205	0.26	0.18	2.73	7.70	11.87	15.08	65.35	18.75	1.26	0.12	1.380	0.128
0.210	0.27	0.19	2.77	7.88	11.62	16.05	64.44	19.94	1.27	0.13	1.395	0.136
0.215	0.29	0.21	2.80	8.06	11.38	17.06	63.50	21.18	1.29	0.14	1.411	0.144
0.220	0.31	0.22	2.84	8.24	11.12	18.10	62.53	22.45	1.31	0.15	1.427	0.153

u_g and u_b are the unemployment rates respectively in the legal and shadow sector

n_g, En_b, are respectively legal and shadow employment.

$\overline{w}_g$ and $\overline{w}_b$ are the average wages legal and shadow employment

Source: Authors' calculation

Table 4: Changes in Firing Taxes

T	R	θ_b	θ_g	u_b	u_g	n_b	n_g	s	x_g	x_b	$\overline{w}_g$	$\overline{w}_b$
0.100	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.113	0.25	0.17	2.71	7.59	12.01	14.50	65.90	18.03	1.25	0.12	1.372	0.123
0.125	0.25	0.17	2.72	7.66	11.92	14.85	65.57	18.46	1.25	0.12	1.378	0.126
0.138	0.26	0.18	2.73	7.72	11.84	15.20	65.24	18.90	1.26	0.12	1.384	0.129
0.150	0.27	0.18	2.75	7.79	11.75	15.56	64.90	19.34	1.27	0.13	1.390	0.132

u_g and u_b are the unemployment rates respectively in the legal and shadow sector

n_g, En_b, are respectively legal and shadow employment.

$\overline{w}_g$ and $\overline{w}_b$ are the average wages legal and shadow employment

Source: Authors' calculation

4.0.3 Changes in Taxation and Regulations

We study the effects of the increase in τ on the general equilibrium of the model. The results are reported in Table 3. All the other parameters are identical to those of Table 1. More taxes and regulations increase shadow employment and reduce legal employment. This is the standard result of Schneider (2002). It is also consistent with the work of Davis and Henrekson (2005).

The effect of taxation on unemployment is quantitatively very modest, since there are two countervailing effects at work. There is the indirect effect on job creation via the increase in the reservation productivity (reducing unemployment) plus the direct effect of taxes on market tightness in the legal sector (increasing unemployment).

Changes in regulation (through the firing tax) are qualitatively analogous to the effects of taxation.

Table 5: Changes in Monitoring Intensity

ρ	R	θ_b	θ_g	u_b	u_g	n_b	n_g	s	x_g	x_b	$\overline{w}_g$	$\overline{w}_b$
0.06	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.37	0.12
0.07	0.22	0.13	2.62	7.31	12.53	12.48	67.67	15.57	1.22	0.11	1.33	0.11
0.09	0.20	0.11	2.57	7.17	12.86	11.24	68.73	14.05	1.20	0.10	1.31	0.10
0.10	0.19	0.10	2.53	7.08	13.11	10.27	69.54	12.87	1.19	0.09	1.29	0.09
0.11	0.18	0.08	2.50	7.01	13.31	9.50	70.18	11.92	1.18	0.09	1.28	0.08

u_g and u_b are the unemployment rates respectively in the legal and shadow sector

n_g, E_{n_b}, are respectively legal and shadow employment.

$\overline{w}_g$ and $\overline{w}_b$ are the average wages legal and shadow employment

Source: Authors' calculation

4.0.4 Changes in the Monitoring Rate

We study the effects of the increase in ρ on the general equilibrium of the model. The results are reported in Table 5. An increase in monitoring intensity reduces the shadow rate, but it increases unemployment

We view this result as extremely important, since it highlights one of the key reasons why governments may be reluctant to repress the shadow sector. The associated increase in unemployment is politically costly and thus avoided by utility maximizing politicians.

4.0.5 Changes in unemployed income

We now consider the effects of an increase in b. An increase in unemployed income reduces the shadow rate, and increases unemployment. Yet, the increase in participation in the legal sector increases legal employment and reduces shadow employment. Note that market tightness falls in both sectors.

The increase in unemployed income can be considered as a policy for uncovering (as opposed to repression) shadow activities. Various difficulties are likely to exist in reality in enforcing this policy (unemployment income requires larger taxation and very good monitoring). Yet, it can be quite effective.

Table 6: Changes in Unemployed Income

b	R	θ_b	θ_g	u_b	u_g	n_b	n_g	s	x_g	x_b	$\overline{w}_g$	$\overline{w}_b$
0.100	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.104	0.24	0.16	2.67	7.46	12.20	13.85	66.49	17.24	1.24	0.12	1.360	0.118
0.108	0.24	0.15	2.65	7.40	12.29	13.56	66.75	16.88	1.24	0.11	1.355	0.116
0.111	0.23	0.15	2.63	7.34	12.39	13.27	67.01	16.53	1.23	0.11	1.349	0.113
0.115	0.23	0.15	2.61	7.27	12.49	12.97	67.27	16.17	1.23	0.11	1.343	0.111

u_g and u_b are the unemployment rates respectively in the legal and shadow sector

n_g, En_b, are respectively legal and shadow employment.

$\overline{w}_g$ and $\overline{w}_b$ are the average wages legal and shadow employment

Source: Authors' calculation

5 Empirical Relevance

Our model implies: i) a positive cross-sectional and time-series correlation between the size of the shadow sector and unemployment (the two phenomena are just two faces of the same coin), ii) a "shadow wage gap" that is larger in countries-regions and years in which unemployment is lower, iii) a shadow employment that is increasing in taxation and labor market regulations, and iv) that tighter monitoring increases unemployment. From a political economy perspective, the latter result implies a lax enforcement of regulations in high-unemployment regions.

The purpose of this section is to evaluate the empirical relevance of i), ii) and iv). Implication iii) is common to other models of the shadow economy and holds in many cross-sectional studies, as reviewed by Schneider (2002).

5.1 Two faces of the same coin?

Figure 6 documents the correlation between the size of the shadow economy and the non-employment rate across countries and Figure 7 across Italian regions, in both cases over average period data. In particular, Figure 6 displays, on the vertical axis, the cross-country comparable estimates of the shadow economy over GDP provided by Schneider (2004) and, on the horizontal axis non-employment rates (unemployed and inactive as a fraction of the working age population) obtained from harmonised Labour Force Survey (LFS) data. Regional non-employment rates are also obtained from the (Italian) LFS, while the regional estimates of shadow employment are drawn from Istat. The latter are provided in terms of full-time equivalents (ULA, "unità di lavoro equivalenti") and are estimated building on the difference between survey-based employment and

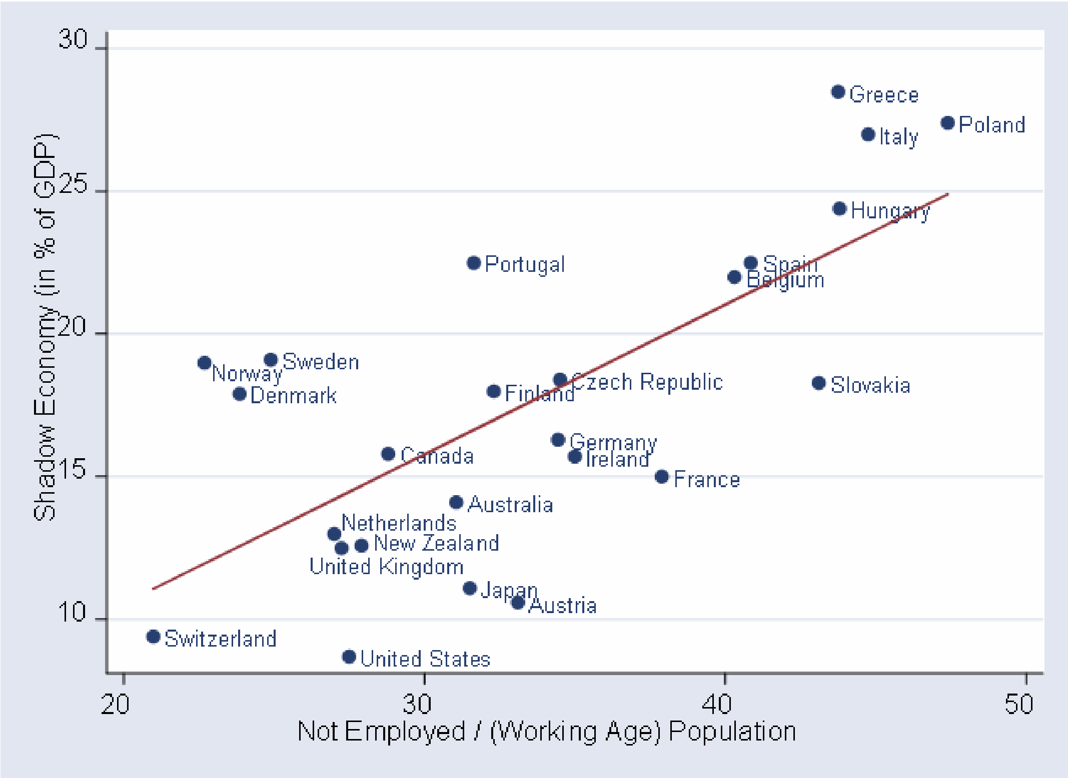


Figure 6: The Size of the Shadow Economy and Non Employment

employment levels, as computed on the basis of administrative (social security records) as well as estimates of illegal employment of foreign workers[4].

The correlation is striking in both cases: the cross-country correlation is .7 with a t-statistics of 4.76; the cross-regional correlation is .94 with a t-statistics of 11.79. It holds also when shadow employment is broken down by broad sectors, e.g., it is not a byproduct of the specialisation of Southern regions in sectors (e.g., agriculture) where shadow employment is larger. There is also no tendency over time to a reduction in regional differentials in shadow rates: they were in 1995 roughly as large as 10 years earlier.

Unfortunately, there are no long series of shadow employment and unemployment enabling to assess their pairwise correlation over time. Figure 8 hints at co-movements between the shadow rate and unemployment in Italy. The shadow rate initially rose with unemployment and then, more recently declined together with unemployment.

[4]See Calzaroni and Pascarella (1998) for details on the estimates of shadow employment in Italian macro-regions.

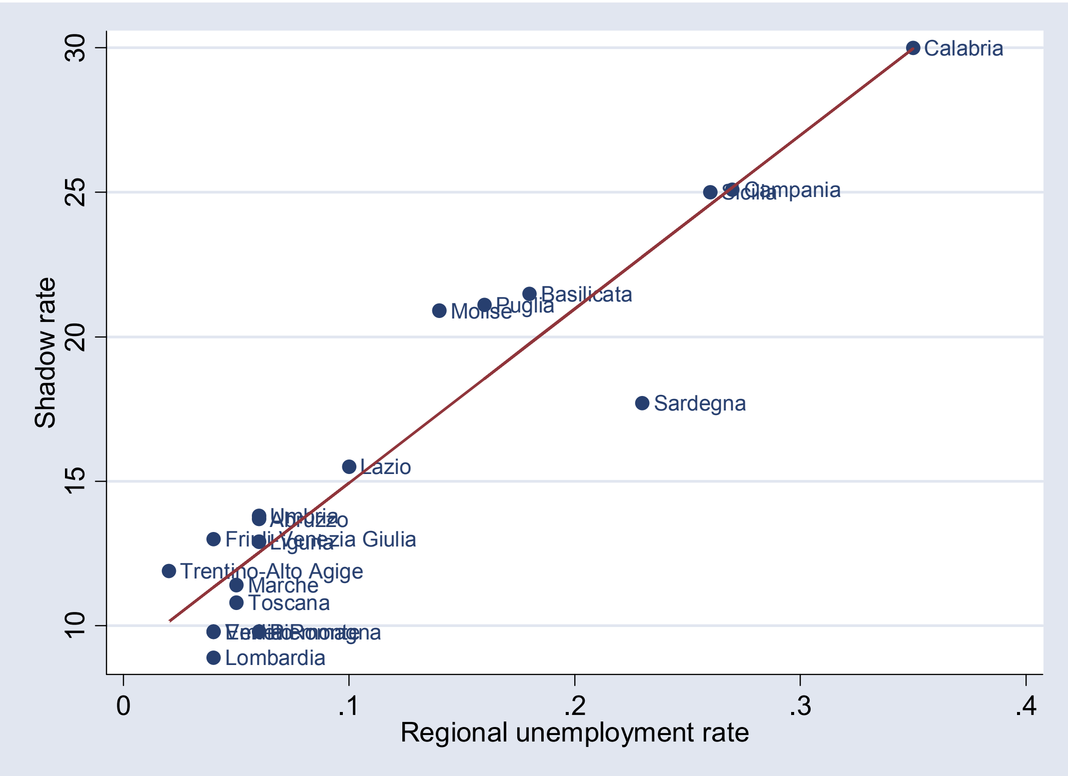


Figure 7: Shadow employment and unemployment across Italian regions

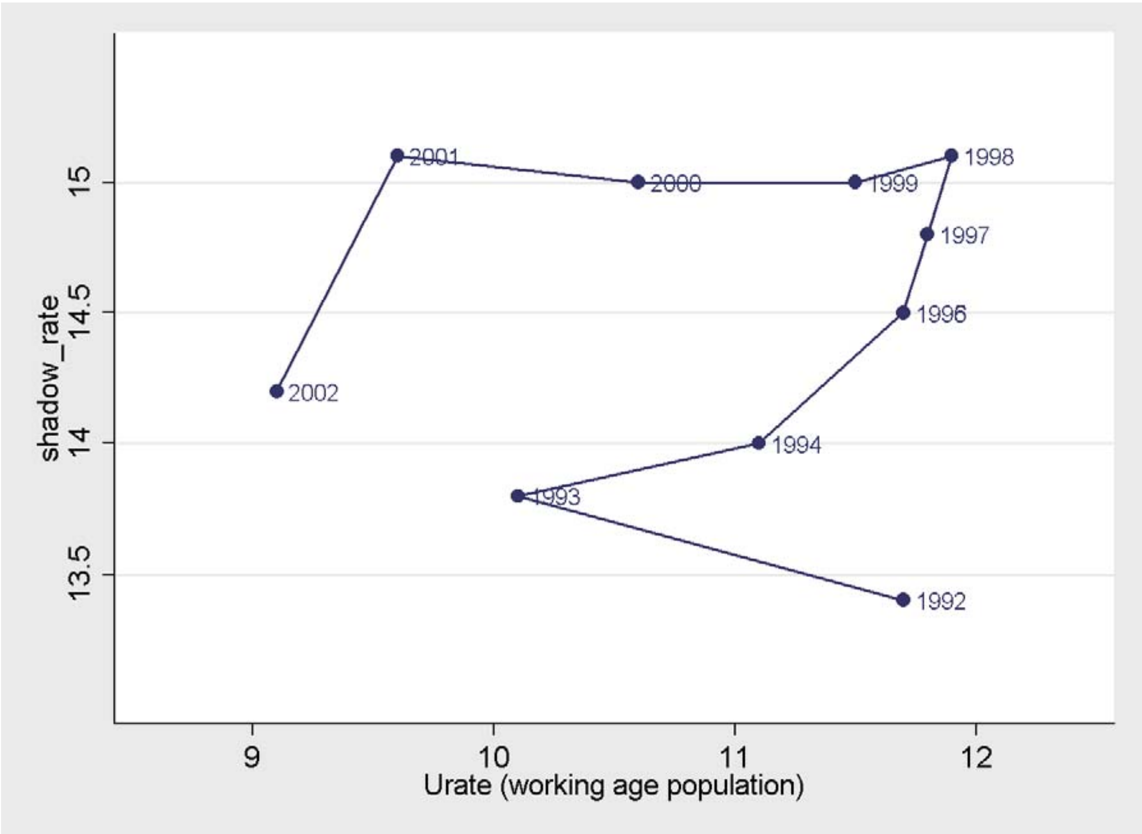


Figure 8: Unemployment and Shadow Employment Over time in Italy

All these correlations are consistent with the implications of our model and can be rationalized by the fact that positive macroeconomic shocks or greater efficiency in a region increases job creation and reduces the reservation productivity level at which jobs turn into formal jobs. However, given the size and statistical significance of correlations, one may think that they are a mere statistical artifact, related to the way in which the two measures are defined. As discussed in the Annex, a spurious correlation may be induced between shadow employment and the unemployment rate, when shadow employment is wrongly classified as unemployment by Labor Force Statistics. The large unemployment rates observed also among prime-age men in Southern Italian regions suggest that LFS data may indeed mis-classify jobs in the shadow sector. Unfortunately, estimates of the shadow economy generally come from statistical sources which are silent on labor market aggregates. When LFS data are used to measure shadow employment (e.g., as done in Table 2), they either just scrap the surface of the phenomenon (the number are too small to achieve regional representation) or concentrate only on the subset of shadow employment which is not mis-classified by LFS statistics. Hence, there is no way to map shadow employment into the different LFS aggregates.

An important exception is the PME (Monthly Employment) survey carried out in six Brazilian metropolitan areas since 1982. The survey design is similar to the CPS in the US and includes a question on the payment of social security contributions. Following Almeida and Carneiro (2005), Gonzaga (2003) and Hoeke (2005), we identify shadow workers as those individuals reporting to work but stating that they do not have a social security card. It is a relatively large component of the labor force: the shadow rate can be as high as roughly 1/3. By construction, these shadow workers cannot be classified as unemployed. Figure 9 displays the yearly shadow and unemployment rates in six Brazilian metropolitan areas since the inception of the survey. There is a remarkable positive correlation (ranging from .31 in Rio to .82 in Salvador with t-statistics in the range 3.4 to 6.1). This correlation cannot be a statistical artifact, and provides genuine evidence of our empirical implications.

5.2 The Shadow Wage Gap

Our model predicts that improvements in aggregate conditions increase the shadow wage gap.

Table 10 displays the shadow wage gap and a simple Oaxaca decomposition of this gap in Italy over time and across two macro-regions characterised by very different aggregate condition, such

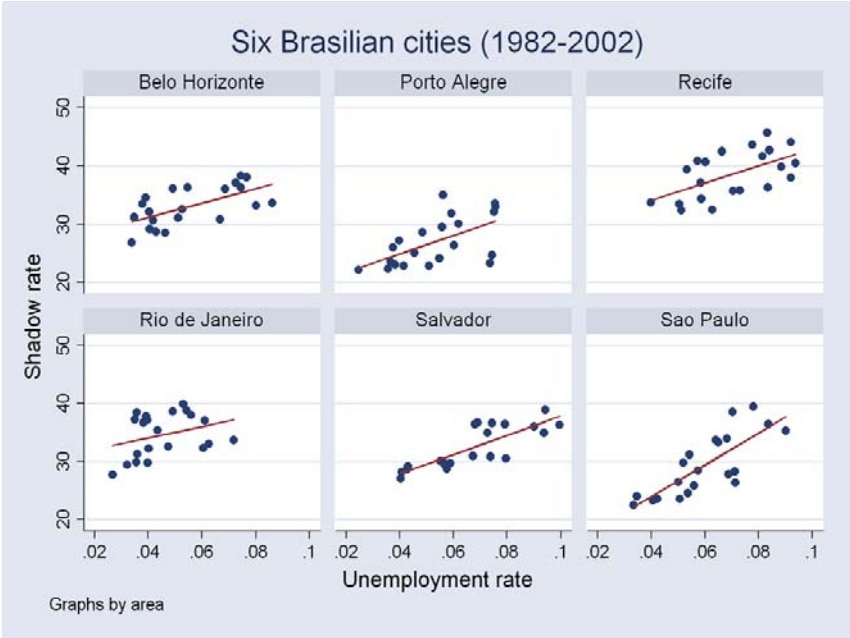


Figure 9: Shadow employment and unemployment in six Brazilian cities

as the North and the Mezzogiorno. In particular, drawing on the Bank of Italy SHIW we run two standard wage regressions for the legal and the shadow sector (individuals stating that they are working but they never paid social security contributions)

$$\bar{w}^g = \bar{X}^g \beta^g$$

and

$$\bar{w}^b = \bar{X}^b \beta^b$$

where $\bar{X}^i$ denotes average "personal-demographic" characteristics (educational attainments, gender, age, family status, etc.) of sector "i" and β^i the returns to these characteristics. Then we can decompose the shadow wage gap as the sum of a difference in quantities (explained part) and differences in returns (unexplained part), e.g.:

$$\bar{w}^g - \bar{w}^b = (\bar{X}^g - \bar{X}^b)\frac{1}{2}(\beta^g + \beta^b) + (\beta^g - \beta^b)\frac{1}{2}(\bar{X}^g + \bar{X}^b) \tag{4}$$

An advantage of this decomposition is that it isolates the component which drives the changes

		Oaxaca Decomposition of the Shadow Wage Gap		
		Shadow wage gap	**Explained**	**Unexplained**
All sample	1995	0.94	0.24	0.70
	1998	0.79	0.40	0.39
	2000	0.92	0.26	0.66
	2002	1.04	0.23	0.81
North	all years	0.95	0.30	0.65
South	all years	0.78	0.31	0.48

Notes: Controls include age,gender,family status and educational attainments
Source: Bank of Italy SHIW various years

Figure 10:

in the shadow wage gap according to our model: it is the unexplained (or difference in returns) component, that is, the second term in equation (4). The decomposition is akin to the partial equilibrium comparative statics exercise above, in that it assumes that differences in returns are uncorrelated with changes in the characteristics of the two pools. It should be interpreted as an approximation of the first-round effects of changes in the aggregate shock. Our exercise suggests that the shadow gap has been widening since 1998, at times in which unemployment was declining, and that it is larger in the dynamic North than in the depressed Southern labor markets. The key factor behind these differences is the unexplained (returns) component of the gap.

Hoeke (2005) also reports an increase in the shadow wage gap in Brazil during cyclical upturns.

5.3 Enforcement

Modern information technologies allow tax administrations to easily collect and cross-check information from a variety of source. For instance, the Spanish tax administration built-up an inventory of bank accounts which is particularly useful in tracking the shadow sector. The Italian "Agenzie delle Entrate" is developing an inventory of electricity, gas, telephone and water bills of contributors, which can be readily cross-checked with tax records.

There are plenty of anecdotes about poor enforcement in high-unemployment regions, although it is very hard to document this. There are documents of the Italian Agenzia delle Entrate stating that enforcement should be milder in small units and in agriculture, where shadow employment is over-represented. Almeida et al. (2005) report a negative correlation between unemployment and worksite inspections in Brazil. Broadly similar is the conclusions of the Osservatorio Veneto, although shadow employment in Veneto is very much related to immigration. A negative relationship between shadow employment and monitoring is driven in our model by the effects of controls on job creation in the shadow sector. But there can also be political economy argument for observing less repression of the shadow sector in high unemployment regions.

6 Final Remarks

An equilibrium search model of the labor market, with workers' sorting, contributes to explain the "shadow puzzle", the increasing size of the shadow economy in OECD countries in spite of improvements in technologies detecting tax and social security evasion. Our model has implications which are broadly supported by the, admittedly scant, evidence on shadow labor markets. In particular, we consistently find a positive cross-sectional and time-series correlation between the shadow rate and unemployment, and this correlation cannot be attributed to a statistical artifact.

Our model delivers also some policy implications. The most important is quite simple: *in order to reduce shadow employment, it is necessary to deregulate the labor market. Deregulation reduces unemployment, and shadow employment is reduced as a by-product.* In this context, the model confirms the traditional wisdom on labor market reforms, and suggests that any policy that fosters job creation and enhances aggregate productivity will induce a reduction in shadow employment. What about specific policies, aimed at discouraging the emergence of shadow activity? Our simple theory suggests that a very cautious approach in this area is warranted, since an increase in the monitoring rate may backfire: in equilibrium, higher monitoring reduces job creation, and increase unemployment. Tight enforcement of entitlement rules to unemployment benefits can be a better option acting on the supply side (when unemployment benefits are collected only by workers with a regular employment history, and cannot be cumulated to income from shadow jobs, the workers' incentive to enter the shadow sector are reduced) and hence has better job creation properties.

In further work we plan to investigate combinations of shadow and regular jobs, both in labor demand and supply. Although this extension will significantly increase the complexity of our model, we are aware that the choice to go shadow is not merely a dichotomic choice. Multiple job holding allows workers, for instance, to allocate hours across the two sectors. And firms can react to idyosincratic productivity shocks by crossing borders between shadow and regular jobs.

7 References

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7.1 Annex

7.2 A statistical artifact?

According to the labor force statistics, the working age population is classified as E^{lf}, U^{lf} , and N^{lf} where the values refer respectively to labor force employment, unemployment and out of the labor force. If the labor force is indicated with wap the function reads

$$E^{lf} + U^{lf} + N^{lf} = wap$$

The unemployment rate is than defined as

$$u^{lf} = \frac{U^{lf}}{E^{lf} + U^{lf}}$$

The official istat definition of the shadow rate, s, is given by the estimate of shadow employment (lavoro irregolare) over the sum of regular employment E^{r}, and shadow employment E^{s}

$$s = \frac{E^{r}}{E^{s} + E^{r}}$$

The key issue concerns the relationship between E^{s} and E^{lf} or whether shadow employment is part of the labor force employment. The answer depends on various assumptions regarding the position of shadow employment in the labor force statistics

Assumption 1: shadow employment within the employment measured in the labor force surveys.

This implies that

$$E^{lf} = E^{s} + E^{r}$$

Therefore

$$u^{lf} = \frac{U^{lf}}{E^{s} + E^{r} + U^{lf}}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^{s}} &< 0 \\ \frac{\partial s}{\partial E^{s}} &> 0 \end{aligned}$$

In other words, an increase in shadow employment E^{s} leads to an increase in the shadow rate and to a decrease in the unemployment rate. The empirical correlation, in this case is *not* a statistical artifact

Remark 6 *If shadow employment is part of labor force employment, the correlation between s and u is not a statistical artifact*

Assumption 2: shadow employment is within the out of the labor force measured in the labor force surveys.

This implies that

$$N^{lf} = \tilde{N} + E^s$$

where $\tilde{N}$ is a pure measure of out of the labor force (not observed in labor force statistics)

Therefore

$$u^{lf} = \frac{U^{lf}}{E^{lf} + U^{lf}}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^s} &= 0 \\ \frac{\partial s}{\partial E^s} &> 0 \end{aligned}$$

In other words, an increase in shadow employment leads to an increase in the shadow rate and has no impact on the unemployment rate. Also in this case, the empirical correlation is *not* a stastical artifact.

Remark 7 *If shadow employment is part of out of the labor force in labor force surveys, the correlation between s and u is not a statistical artifict*

Assumption 3: shadow employment is within unemployment measured in labor force surveys

This implies that

$$U^{lf} = \tilde{U} + E^s$$

where $\tilde{U}$ is a pure unemployment rate while E^s is shadow employment. In this case the unemployment rate derived from labor force statistics is

$$u^{lf} = \frac{\tilde{U} + E^s}{E^{lf} + \tilde{U} + E^s}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^s} &> 0 \\ \frac{\partial s}{\partial E^s} &> 0 \end{aligned}$$

Remark 8 *If shadow employment is part of labor force unemployment, the correlation between s and u is a statistical artifict*

In this latter scenario one should try to correct the official unemployment statistics. Is there a fraction of unemployed people that looks suspicios? Unfortunately there is no mapping from estimates of shadow employment to LFS definitions of employment, unemployment and inactivity. In order to device some method to track the labor market status of shadow employment we need to introduce some identifying restrictions. This requires some theoretical guidance.

Preferences for Rigid versus Individualized Wage Setting in Search Economies with Frictions

Tito Boeri and Michael C. Burda[*]

February 26, 2005

Abstract

Firing, startup and wage adjustment costs affect worker and firm preferences for rigid wages versus individualized Nash bargaining in a standard model of equilibrium unemployment in which workers vary by observable skill. Rigid wages permit savings on startup and wage adjustment costs and prevent workers from exploiting the firing friction in Nash-bargaining. For standard calibrations, the model can account for substantial political support for wage rigidity by both workers and firms, especially in labor markets for intermediate skill levels. The firing friction is necessary for this effect, whilst wage adjustment costs are not. Turbulence and startup costs also increase support for rigid wages.

JEL: J5, J6, D7

Keywords: Wage rigidities, employment protection, startup costs, equilibrium unemployment.

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1 Introduction

Although it is received wisdom among economists that wage rigidity reduces labor market e¢ciency and welfare, it is puzzling that a broad majority of industrial democracies - especially those in Europe - continue to retain institutions which set wages without reference to local productivity and labor market conditions. The coverage of such rigid wage agreements spans significantly beyond the presence of organised labor at the workplace: in France this "excess coverage" extends to almost 90 per cent of the workforce.[1] According to a 2001 Eurobarometer survey, 65% of EU citizens agree with the statement that "workers need strong unions to protect their interests". Similarly, a recent on-line survey on perceptions of institutions conducted by the McKinsey Company revealed that 60% of Germans consider labor unions to be important for society - compared with 66% for Greenpeace and 46% for ADAC, the German automobile club.

A number of economists, most notably St.-Paul (2000), have grappled with explaining how majorities can arise which favor the retention of institutional arrangements obstructing employment adjustment, such as employment protection legislation. This paper contributes to the literature by focusing on wage rigidity. It demonstrates, under fairly general conditions, that the presence of frictions in equilibrium models of unemployment can generate worker and …rm preferences for rigidities in wage setting. The equilibrium search and matching model made popular by Mortensen and Pissarides (1994, 1999a,b) (henceforth MP) delivers the framework for analysis. We distinguish between jobs with match-surplus shared between employers and workers in a decentralised fashion (‡exible-wage regime), and those with wages determined outside the parameters of the individual employment relationship (rigid-wage regime).

For our results to obtain, frictions are necessary. We consider three in the MP framework. The …rst is a …ring tax imposed on separations. While the impact of …ring taxes has been studied extensively, their interaction with other labor market institutions is less well-understood.[2] The second friction

[1] See Boeri, et al. (2001), and Ebbinghaus and Visser (2000) for more details on excess coverage.

[2] See Ljungqvist (2002) for a survey of theoretical work on the e¤ects of severance regulation on labor market outcomes. Ljungqvist and Sargent (2002) have studied interactions between …ring taxes and unemployment bene…ts. Similar interactions in a di¤erent model setting have been studied by Coe and Snower (1997).

is a cost which is incurred when setting up a new business or job. These costs are the main source of the rents which are shared between employers and employees when a job is created. The third friction is that worker-…rm matches are subject to continuation costs when productivity changes, which may be interpreted as unavoidable wage adjustment costs. These wage adjustment costs occur in the individually bargained wage regime and are seen as an inherent aspect of decentralized labor markets.

Flexible wage setting in a competitive search market is compared with a rigid-wage labor market, in which pay is determined without reference to individual match productivity. We show that workers of di¤erent skill levels have di¤erent preferences over rigid versus ‡exible wage setting. Using a calibrated version of the model, we can identify signi…cant regions of the skill distribution for which a …xed wage regime is preferred to one in which wages are determined by individual Nash-bargaining. The model predicts that support for the rigid-wage regime is stronger at intermediate rather than low skills levels, contrary to conventional wisdom. This is because rigid wages tend to increase job destruction and reduce job creation for the least skilled. Another …nding is that while wage adjustment costs can increase the value of rigid wage regime to workers, this e¤ect requires the existence of …ring frictions. Only …ring taxes are indispensable for a rigid wage regime to arise at the equilibrium. Thus, the friction interfering most directly with wage setting, notably wage adjustment costs, is not essential for rigid wages to be preferred by workers. Firing taxes are also found to interact with turbulence as well as startup costs in that the latter increase support for rigid wages in the presence of …ring frictions. These complementarities can explain "clusters of institutional rigidities" observed in some OECD countries (St.-Paul 2004). Finally, we …nd that preferences for the rigid wage regime increase when there is more turbulence in the labor market, notably at the upper end of the skill distribution.

In the next section, we analyse the benchmark MP model of equilibrium unemployment in the presence of the frictions discussed above, and contrast its behavior with an alternative in which wages are determined by systematic productivity according to an exogenous, rigid rule. Section 3 considers preferences of workers and …rms for the two regimes in the context of a calibrated version of the model. Section 4 evaluates how preferences for rigid wages vary in response to changes in underlying institutions. Section 5 concludes.

2 Flexible versus Rigid Wage Determination in Equilibrium Unemployment

2.1 General description

Consider a continuum of labor markets indexed by s 2 $(0,1]$ where s represents skill, a deterministic and observable component of worker productivity. Workers cannot change their skill level, and supply their labor inelastically to labor market s; they are either unemployed or employed. Firms either produce with one worker, or search with an open vacancy. They can enter freely and at zero cost any labor market and search for a worker, but to do so must pay a cost of sk per unit period. These costs can be thought of alternatively as startup expenses or recruitment costs. Firms can work with all types of workers but only one at any given point in time, and cannot search while employing a worker. When matched, a …rm and a worker generate periodic productivity sx, where x 2 $(0,1]$ is a match-speci…c component referred to as a "shock." For production to occur, a worker must be matched with a job. All new matches (i.e. …lled jobs) begin at the highest possible value of x (x = 1). Immediately thereafter, match productivity can change at Poisson frequency λ, in which case it is a random draw with a …xed, known cumulative distribution $F(x)$.

In both regimes, an exogenous …ring tax sT is levied on termination of job-worker matches, with $T < \frac{1}{r+\lambda}$. T represents a payment to a third party and should be considered pure deadweight loss, induced either by natural aspects of the employment relationship or by government regulation. These include legal fees paid to lawyers and other third parties when severance is contested, as well severance-related strikes, sabotage, or court-initiated delays in termination of labor contracts. This …ring tax is to be distinguished from severance compensation (a lump-sum transfer from employer to employee upon severance), which in principle can be o¤set by a compensating wage adjustment (see Lazear 1990, Burda 1992, Garibaldi and Violante 2004).

2.2 Steady-State Equilibrium State Valuations in a Labor Market of Skill s

Flexible wage regime We first define steady-state, equilibrium valuations of unemployment and employment in a market for labor of arbitrary skill s, when wages are perfectly flexible.[3] Given our assumptions, the valuation by workers of unemployment (U), and employment ($W(x)$), and by firms of an open vacancy (V) versus a job ($J(x)$) is given by the following four functional equations given x:

$$rU = b + \theta q(\theta)[W(1) - U] \quad (1)$$

$$rV = -sk + q(\theta)[J(1) - V] \quad (2)$$

$$rW(x) = w(x) + \lambda \int_R^1 (W(z) - W(x) - \rho)dF(z) + \lambda F(R)(U - W(x)) \quad (3)$$

$$rJ(x) = sx - w(x) + \lambda \int_R^1 ((J(z) - J(x))dF(z) + \lambda F(R)(V - sT - J(x)). \quad (4)$$

Equations (1) through (4) set normal returns on capitalized valuations of labor market states to their periodic payouts. In equation (1), the flow yield from the valuation of the state of unemployment at interest rate r is equated to income in unemployment or leisure equivalent b, plus an expected "capital gain" stemming from finding new employment at $x = 1$. The ratio of vacancies to unemployment $\theta \equiv v/u$ is a sufficient statistic for labor market market tightness and arises from a constant-returns-to-scale matching function $m = m(u, v)$; the probability of a vacancy matching with an unemployed worker is $q = \frac{m(u,v)}{v} = m(\theta, 1)$, with $q'(\theta) < 0, q''(\theta) > 0$, and $\lim_{\theta \to 0} q(\theta) = \infty$; the probability of an unemployed worker meeting a vacancy is thus $\frac{m(u,v)}{u} = \frac{\theta m(u,v)}{v} = \theta q(\theta)$. Equation (2) determines the valuation of an unfilled vacancy. Given an assumed common startup productivity level for all worker-job matches ($x = 1$), it follows that all vacancies in a given labor market are identical ex-ante.

The function $W(x)$ in (3) returns the value of employment in a job-worker match with current productivity x. Given x, the implicit rate of return on

[3] Where it is understood to hold for any arbitrary skill group, the subscript for s is suppressed for notational convenience.

the asset W is equal to the current wage plus the expected capital gain on the employment relationship.

We incorporate in the standard MP model an additional friction, ρ, which is paid whenever the Poisson match-speci…c productivity shock occurs and the match is not dissolved. Formally, the worker pays this cost, but since wages are bargained, both parties will ultimately share this cost in equilibrium. For simplicity, we assume that this cost is unavoidable and represents frictions inherent to the continuation of a ‡exible wage contract. One interpretation of ρ is a non-deferrable cost of renegotiation; more generally it represents any unavoidable investment necessary to maintain the existing match, given that the shock has occurred. Employers' surveys suggest that renegotiation, bargaining, information and organisational costs associated with wage adjustments to plant-level productivity changes can be substantial and discourage the adoption of productivity-related pay structures, even in non-unionised …rms.[4]

The lower bound of the de…nite integral, $R,$ is the endogenous cuto¤ or threshold value of productivity. If idiosyncratic productivity x falls below R, the match is no longer pro…table and the job/worker pair is destroyed. Because match dissolution allows the worker-…rm pair to avoid paying ρ, the equilibrium value of R will re‡ect savings on the wage adjustment cost realized when the match is destroyed. A similar arbitrage argument determines the valuation to a …rm of a …lled job in (4), given the current realization of x and for a worker of skill level s.

Rigid wage regime By assumption, wage adjustment costs can only be avoided by match dissolution. In contrast, they are not incurred at all in the rigid-wage regime, which we now describe. A rigid (or collective) wage labor market is one where labor compensation is independent of local or idiosyncratic in‡uences; i.e. match productivity or market tightness in the particular skill category. It will, however, depend on skill s. We will denote this rigid-wage as w^r. Wage adjustment costs can be avoided in the rigid-wage regime, where, by construction, the equilibrium state valuations by workers U^r and W^r in a labor market of skill s are independent of idiosyncratic

[4] In a survey conducted in September 2000 among 300 small and medium-sized Italian …rms, about a third of the respondents cited wage negotiation costs as a major drawack of plant-level bargaining. Signi…cantly, the employers most concerned about wage negotiation costs were …rms in which unions were not represented at the workplace. See www.frdb.org for more details.

productivity x:

$$rU^r = b + \theta^r q(\theta^r)[W^r - U^r] \tag{5}$$

$$rV^r = -sk + q(\theta^r)[J^r(1) - V^r] \tag{6}$$

$$rW^r = w^r + \lambda F(R^r)(U^r - W^r) \tag{7}$$

$$rJ^r(x) = sx - w^r + \lambda \int_{R^r}^{1} ((J^r(z) - J^r(x))dF(z) + \lambda F(R^r)(V^r - J^r(x) - sT) \tag{8}$$

The interpretation of equations (5) through (8) is similar to those of the previous section. Here R^r is the reservation productivity from the employer's perspective, which applies to a match in the rigid-wage regime; the job is destroyed for realizations of x lower than R^r. As in the flexible wage case, R^r will take different values for different skill levels and depend on w^r, ϕ, T and other parameters. Below, we will specify w^r in more detail. At this point, it is natural to impose a participation constraint on employment $W^r \geq U^r$, where U^r denotes the value of unemployment for a worker in the rigid wage segment.

Valuation of vacancies in equilibrium There are no restrictions on the entry of firms in any skill segment. Hence, in both regimes the equilibrium value of vacant jobs will satisfy the free entry condition $V = V^r = 0$. In the flexible wage regime, (2) becomes

$$J(1) = \frac{sk}{q(\theta)}, \tag{9}$$

and in the rigid-wage regime

$$J^r(1) = \frac{sk}{q(\theta^r)}. \tag{10}$$

2.3 Wage Determination

Flexible wage regime In the individualized wage setting regime, workers' remuneration is determined by a Nash sharing rule.[5] For an existing match

[5] Here we follow MP (1999a,b) and Pissarides (2000); for details see the Appendix. Detailed derivations of these and other results in this paper are available in a longer Appendix which can be obtained from the authors upon request.

in the competitive labor market, the Nash-bargained wage is given by

$$w(x) = \arg\max_{w} [W(x) - U]^{\beta} [J(x) + sT - V]^{(1-\beta)}$$

yielding the first order condition

$$W(x) - U = \beta \left[J(x) + W(x) + sT - V - U \right]. \tag{11}$$

It is convenient to solve first for the steady-state valuation of unemployment U. Combining (9) and $V = 0$ with (11) evaluated at $x = 1$ and inserting the result into (1) yields

$$rU = b + \frac{\beta s k \theta}{1 - \beta}. \tag{12}$$

The equilibrium state valuation of unemployment is linear in θ, which is a sufficient statistic for tightness in labor markets. Following Mortensen and Pissarides (1999a,b) we use (12) to obtain the equilibrium wage rule:

$$w(x) = (1 - \beta) \left[b + \lambda(1 - F(R))\rho \right] + \beta s \left(k\theta + x + rT \right). \tag{13}$$

Notice that the equilibrium wage depends not only on familiar parameters such as b (the monetary value of unemployment or leisure), θ (labor market tightness), x (match productivity), and T (severance cost), the startup cost k, but also on λ, the shock probability, and the wage adjustment cost ρ. These latter two factors are more important, the more likely a job is to survive $(1 - F(R))$. Idiosyncratic productivity shocks which do not lead to match dissolution make the worker partially liable for wage adjustment costs. By abandoning the match and passing into unemployment, wage adjustment costs can be avoided; consequently, a higher wage is needed to indemnify for this contingency. Effectively, the fallback of the worker is increased by the savings on future wage adjustment costs that is implied by a breakdown of negotiations and spell of unemployment.[6] The more power the employer has, the more likely will the wage reflect this "compensating differential" as opposed to insider rents. In contrast, bargaining power of workers links wages more tightly with idiosyncratic productivity, local market conditions, as well as the lock-in effect of the firing tax.

[6] Notice that the hold-up problem (Malcomson, 1997) does not arise in this context because the incidence of ρ is, by assumption, not subject to negotiation. An obvious extension which allows for postponement of the renegotiation would be more realistic, but is mathematically more cumbersome and goes well beyond the scope of this paper.

Rigid wage regime Rigid wages are assumed to depend positively on observable productivity s, so $w^r = w^r(s)$ with $dw^r/ds > 0$. We parametrize the wage schedule by the linear form $w^r = \overline{w} + \phi s$ with $0 < \phi < 1$, where $\overline{w} \geq b$ is a social minimum or minimum wage, while ϕ reflects skill-dependence of compensation independent of match productivity. Low values of ϕ suggest "egalitarian" wage structures, with higher values linking pay more tightly to systematic (deterministic) productivity.

2.4 Job Creation, Destruction and Equilibrium

2.4.1 Job Creation

Flexible wage regime The derivation of the job creation condition in the flexible regime follows Pissarides (2000).:

$$\frac{(1-\beta)(1-R)}{r+\lambda} - T = \frac{k}{q(\theta)} \tag{14}$$

This condition on R and θ is represented in the left panel of Figure 1 by the downward-sloping JC-curve (for job creation).[7] Note that neither s nor ρ affects the position of the JC curve. The intuition for this result is that wage adjustment costs do not affect the incentive to create a job at any given skill level, but rather influence the viability of the job via the surplus available to the match. Insofar as startup costs are proportional to skill in the particular labor market, there is no bias on the job creation margin in favour of a particular skill level.

Rigid wage regime The job creation condition for a job in the rigid wage labor market is shown in the Appendix to be given by

$$\frac{1-R^r}{r+\lambda} - T = \frac{k}{q(\theta^r)}. \tag{15}$$

The JC curve in the rigid labor market is plotted in the right panel of Figure 1. It remains strictly downward sloping in (θ^r, R^r)-space, since $q' < 0$, and lies everywhere above that of the competitive labor market. It is also independent of skill level s .

[7] Implicit differentiation of (14) gives $\frac{dR}{d\theta} = \frac{(r+\lambda)kq'}{(1-\beta)sq^2}$, where f is the density associated with F. Since $q'(\theta) < 0$, $\frac{dR}{d\theta} < 0$ unambiguously.

2.4.2 Job Destruction

Flexible wage regime As in the MP model, jobs are destroyed when productivity falls below its corresponding reservation or threshold level. In the individual-bargaining regime, R is implicitly defined for each skill s by the condition

$$J(R) = -sT. \tag{16}$$

At the same time, Nash bargaining (see below) also implies that R satisfies the zero match-surplus condition:

$$J(R) + sT - V + W(R) - U = 0 \tag{17}$$

and, given the free entry condition $V = 0$, it follows that

$$W(R) = U$$

that is, in this regime separations are privately, but not necessarily socially, efficient in the sense of Hosios (1990).

The reservation productivity level for the competitive search market, R, is determined implicitly by the job destruction condition[8]:

$$sR + \frac{s\lambda}{r+\lambda}\int_R^1 (z - R)dF(z) + rsT = b + \frac{\beta sk\theta}{1-\beta} + \lambda\left[1 - F(R)\right]\rho \tag{18}$$

The left-hand side is the flow benefit of a continuing match with productivity R; this is the current flow product plus the option value deriving from possible future improvements over the following time interval. The right-hand side represents the (opportunity) costs of maintaining the match at the threshold value of idiosyncratic productivity, plus the expected value of wage adjustment costs. This job destruction (JD) condition defines an upward-sloping curve in the (θ, R) space, which we show in the left panel of Figure 1.[9]

[8] The derivation of this condition is standard and can be found in Mortensen and Pissarides (1999b) or Pissarides (2000).

[9] Differentiate (18) and solve for $dR/d\theta$ to obtain $\frac{dR}{d\theta} = \frac{\frac{\beta k}{1-\beta}}{s\left[1 - \frac{\lambda}{r+\lambda}(1-F)\right] + \lambda f\rho} > 0$.

Rigid wage regime The hallmark of the rigid wage regime is that the value of a job to the employee is independent of match productivity. Hence, the set of idiosyncratic productivities for which the job is destroyed will not necessarily coincide with those for which the job has zero value to the worker at the assumed rigid wage. Rather, the participation constraint implies that for a given skill level, $W^r(R^r) = W^r > U^r$. In rigid-wage labor markets, the "consensual" dissolution of an employment relationship no longer applies, and there are always too many separations from the workers' perspective. Separations are inefficient in the sense that for some range of productivities workers will be fired, but at the given wage, they would prefer to continue working. Except on a set of measure zero, there are only involuntary layoffs in the rigid wage regime. In contrast, quits and layoffs are indistinguishable in competitive search labor markets.[10]

Because the rigid wage is not the outcome of individual level bargaining, surplus division obeys a rule of the residual claimant type. Let $S^r(x)$ be the total surplus resulting from a match for any s, so for any $x \in [R^r, 1]$

$$J^r(x) = \max(-sT, S^r(x) - (W^r - U^r)). \tag{19}$$

The firm obtains all surplus greater than $(W^r - U^r)$. The maximum operator applies since the firm can always close operation, here at cost sT. Unlike the individual-wage labor market, the decision to destroy a job is taken by employers unilaterally and given by $J^r < -sT$ for any s; yet in general at this point $W^r > U^r$. The reservation productivity R^r for a match for skill level s, that is, the reservation value for jobs under rigid wages is given by (see Appendix):

$$sR^r + \frac{\lambda s}{r + \lambda} \int_{R^r}^{1} (x - R^r) dF(x) = \overline{w} + \phi s - rsT \tag{20}$$

Unlike the flexible, individually bargained wage case, the component related to wage adjustment costs is absent. This expression represents the job destruction condition in the rigid search market, the JD-curve, which is plotted in (θ^r, R^r) space in Figure 2. The JD curve is horizontal, reflecting the independence of R^r of local labor market conditions. The unambiguous effect of increasing the firing tax T is evident from the figure: it reduces the job destruction threshold and raises the average duration of a job.

[10] Quits by workers cannot result in material gains, by assumption. Allowing for on-the-job search is a subject for future research.

In contrast to (18), neither labor market tightness (θ^r) nor individual worker bargaining strength (β) appear in the job destruction condition. The rigid wage influences the outcome via R^r, which is endogenously determined as the intersection of the JC and JD curves for every s. As in the flexible wage labor market, an increase in λ ceteris paribus shifts back the job destruction curve towards the origin.

2.4.3 Equilibrium

Flexible wage regime. The intersection of (18) with the job creation condition (14) defines a labor market equilibrium for submarket with skill s. For each skill level there exists a unique equilibrium reservation productivity and labor tightness pair (R^*, θ^*) given by the implicit functions of deterministic productivity s, the Poisson arrival rate λ, worker bargaining power β, startup costs k, wage adjustment costs ρ, firing tax T and income-equivalent in unemployment b:

$$\begin{aligned} R^* &= R^*(s, \lambda, \beta, k, \rho, b, T) \\ \theta^* &= \theta^*(s, \lambda, \beta, k, \rho, b, T). \end{aligned}$$

The result is depicted as the intersection points in both panels of Figure 1.

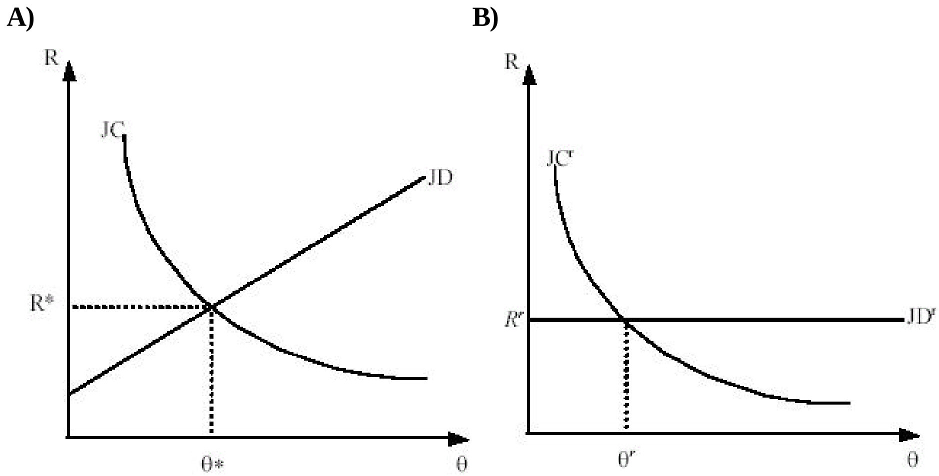
Given the equilibrium R^* and θ^*, the unemployment rate in the labor submarket for skill level s follows from the familiar flow condition for constant unemployment:

$$u^* \equiv u^*(s, \lambda, \beta, k, \rho, b, T) = \frac{\lambda F(R^*)}{\lambda F(R^*) + \theta^* q(\theta^*)}. \tag{21}$$

Rigid wage regime. Similarly, the intersection of the JD and the JC curves depicted in the right panel of Figure 1 gives unique equilibrium values of the reservation productivity and market tightness for the labor market under the rigid wage regime, which we call $R^r = R^r(s, k, \overline{w}, \phi, b, T)$ and $\theta^r = \theta^r(s, k, \overline{w}, \phi, b, T)$ respectively. Analogous to (21), the equilibrium unemployment rate u^r in a rigid-wage labor market with skill level s is given by .

$$u^r = \frac{\lambda F(R^r)}{\lambda F(R^r) + \theta^r q(\theta^r)} \equiv u^r(s, k, \overline{w}, \phi, b, T). \tag{22}$$

Figure 1: Equilibrium in labor markets of arbitrary skills with a) flexibly bargained wages and b) rigid wages



2.4.4 Comparative Statics

The dependence of the endogenous variables on the model parameters in the two regimes is described in the table below.

Table 1. Comparative Statics Results

Effect of ⇒ ....on ⇓		s	λ	ρ	b	β	T	$\overline{w}$	ϕ
Flexible wage regime	R^*	−	+	+	+	+	−		
	θ^*	+	+	−	−	−	−		
	u^*	−	+	+	+	+	?		
Rigid wage regime	R^r	−	+		+		−	+	+
	θ^r	+	+		−		−	−	−
	u^r	−	+		+		?	+	+

An increase in s shifts the JD curve downwards and the JC curve outwards from the origin, so an increase in skill unambiguously tightens the labor market and lowers the firing threshold in both regimes. An increase in the frequency of productivity shocks, wage adjustment costs and the value of leisure unambiguously increases unemployment in the flexible labor markets via their effects on wages. To the extent that a rigid wage does not depend on b, λ and ρ (and $\overline{w} > b$), job creation and destruction margins (hence unemployment) are unaffected by changes in these parameters. As noted above, increases in the minimum wage and in the slope of the wage-skill profile in the rigid segment have unambiguous effects on job duration (negative), market tightness (negative) and unemployment (positive). Finally the firing tax reduces both job creation and destruction while its effect on unemployment is ambiguous.

2.5 Closed Labor Markets

A market for labor may not exist for every skill level. It is useful to define $\underline{s}$ as the minimal skill class above which the labor market is open ($\theta > 0$); that is to say, in which positive vacancies are observed. If no vacancies are posted, the unemployment rate is 100% and the labor market is said to be closed.[11] Alternatively, a labor market is closed if there is no value of $x \in (0, 1]$ for which match surplus is positive. The value taken by $\underline{s}$ will generally depend on the wage setting regime. In the case of individualized, flexible wage-setting, as $\theta \rightarrow 0$, the JC condition (14) implies that R^* approaches $1 - (r + \lambda)T$ from below. Consequently there are no open labor markets for which $R^* \in [1 - (r + \lambda)T, 1]$. The JC condition for the rigid wage regime (20) contains the same implication.

In the flexible wage case, the value for $\underline{s}$, say $\underline{s}^*$ is implicitly given by $s : R^* = 1 - (r + \lambda)T$. Now consider the job destruction condition (18), substitute the inverse of q for θ, and then let the threshold R approach $1 - (r + \lambda)T$ from below:

$$\lim_{R \rightarrow 1-(r+\lambda)T} \underline{s}^* = \frac{b + \lambda[1 - F(1 - (r + \lambda)T)]\rho}{1 - (r + \lambda)T + \frac{\lambda}{r+\lambda}\int_{1-(r+\lambda)T}^{1}(z - [1 - (r + \lambda)T])dF(z) + rT}$$

$$= \left[\frac{r + \lambda}{r + \lambda F(1 - (r + \lambda)T)}\right][b + \lambda[1 - F(1 - (r + \lambda)T)]\rho] > b.$$

Note that when $\rho = T = 0$, then b is the lower bound for match productivity, below which labor markets will be closed; match productivity at the

[11] Since there is no gain from keeping a worker with a productivity which does not cover the opportunities costs of both parties, no worker should ever be observed working at a wage lower than b.

outset must strictly exceed the ‡ow bene…t from leisure. This will not be the case however, when $T > 0$, a fact that has been neglected by the literature; here labor markets can be closed, although workers in these skill classes have productivity strictly greater than b and would be willing to work if matched.

When wages are rigid, the same analysis as above can be applied to (15), taking the limit as R approaches $1 - (r + \lambda)T$ from below:

$$\begin{aligned} \lim_{R \to 1-(r+\lambda)T} \underline{s}^r &= \frac{\overline{w}}{1 - \lambda T + \frac{\lambda}{r+\lambda}\int_{1-(r+\lambda)T}^{1} (x - [1 - (r+\lambda)T])dF(x) - \phi} \\ &= \frac{(r+\lambda)}{r(1-\phi) + \lambda(1 - \phi - F(1-(r+\lambda)T))}\overline{w} > \overline{w} \end{aligned} \quad (23)$$

By inspection, labor markets are closed not only be the direct e¤ect of the minimum wage $\overline{w}$ exceeding maximal match productivity, but also by ϕ and T, which can create a band of productivities exceeding $\overline{w}$ for which workers cannot be productively employed.

3 Worker and Firm Preferences for Labor Market Regimes: A Calibration

3.1 Model Speci…cation

The objective of this section is to evaluate the properties of a calibrated version of the model with particular functional forms. In particular, we are interested in evaluating preferences of workers and …rms in markets for di¤erent skill classes for the two regimes. We thus follow a tradition initiated by Millard and Mortensen (1997) and Mortensen and Pissarides (1999) in analyzing the e¤ects of labor market institutions. We consider an economy with matching success probabilities given by the power functional form $q(\theta) = A\theta^{-\alpha}$ with $A > 0$, $0 < \alpha < 1$. The idiosyncratic shock is distributed uniformly over the interval (0,1]. Under these conditions, the job creation condition for market of skill s in the ‡exible search labor market is given by

$$\theta^* = \left[\frac{A}{k}\left(\frac{(1-\beta)(1-R)}{r+\lambda}\right) - T\right]^{1/\alpha} \quad (24)$$

while in the rigid search market it is characterized by the condition

$$\theta^r = \left[\frac{A}{k} \left(\frac{1 - R^r}{r + \lambda} \right) - T \right]^{1/\alpha}. \tag{25}$$

The job destruction conditions are respectively

$$sR^* + \frac{s\lambda(1 - R^*)^2}{2\,(r + \lambda)} = b + \frac{\beta s k \theta^*}{1 - \beta} + \lambda\,(1 - R^*)\,\rho - rsT \tag{26}$$

and

$$sR^r + \frac{s\lambda(1 - R^r)^2}{2\,(r + \lambda)} = \overline{w} + \phi s - rsT. \tag{27}$$

Equilibria exist when $R \in (0, 1]$ and $0 < \theta < \infty$. The critical value of skill for which markets are closed in the flexible wage economy is given by

$$\lim_{R \to 1-(r+\lambda)T} \underline{s}^* = \frac{b}{1 - \lambda T}$$

and in the rigid wage economy by

$$\lim_{R \to 1-(r+\lambda)T} \underline{s}^r = \frac{(r + \lambda)}{r\,(1 - \phi) + \lambda\,[(r + \lambda)\,T - \phi]}\overline{w} \tag{28}$$

3.2 Numerical values and characteristics of the model economy

Figure 2 displays the two value functions under the assumptions outlined above for each of the two regimes, calibrated using parameter values given in Table 2. Values chosen for λ and k are close to those used by Yashiv (2000) for a calibration of the MP model to Israeli data, as well as by Mortensen and Pissarides (1999a) in their evaluation of labor market policies. The minimum wage $\overline{w}$ is arbitrarily set just at unemployment income, which is itself established at a level leaving about one-sixth of the skill distribution out of employment under competitive conditions. It is difficult to find empirical counterparts of the pay scale parameter ϕ, as econometric estimates of union wage premia also reflect rent capture in particular industires and do not control for actual coverage (beyond membership) of collective bargaining. Estimates reported by Booth (1995, Chapter 6) imply an egalitarian wage-skill profile with higher premia for low and intermediate skill levels. Accordingly, we set ϕ at 0.55,which implies that the top decile of the skill ladder for which a labor market is operating is paid about 3 times more than the bottom decile. We impose the Hosios condition on the matching function (Hosios 1990), so an undistorted decentralized equilibrium would achieve the (restricted) social optimum. The value of the …ring tax is set at three months of output, which means roughly 4-5 months of pay. Finally, the value for the wage adjustment costs is admittedly arbitrary. In the baseline simulation it is set equal to 20% of one quarter's output. Recall that the wage adjustment costs represent the monetary valuation of all (including intangible) costs of rewriting the contract for the entire workforce of a …rm upon the realization of a idiosyncratic productivity shock.

Table 2. Parameter Values for Baseline Calibration

A (matching function e¤ectiveness)	0.60
$\alpha = \beta$ (elasticity of $q(\theta)$ and labor bargaining power)	0.50
b (income in unemployment)	0.15
λ (frequency of the match-speci…c shock)	0.10
r (real interest rate per quarter)	0.05
ρ (wage adjustment costs)	0.20
k (startup costs, proportional to productivity)	0.15
$\overline{w}$ (base or minimum wage)	0.15
ϕ (pay scale parameter)	0.55
T (…ring tax, proportional to productivity)	1.00

Assuming a uniform distribution of workers across skill classes, the equilibrium in the decentralized economy with the baseline calibration has a mean unemployment rate of 6.4% with a mean and median completed steady state unemployment duration of 1.2 and 1.0 quarters, respectively. In the rigid wage economy, the unemployment rate is 6.2% with a mean (median) duration of 3.3 (0.4) quarters. The striking deviation of median from mean in the rigid case results from wage rigidity, and is re‡ected in the wide range of unemployment rates from a low of 1.4% to a high of 93%. This range of equilibrium unemployment rates is absent from the ‡exible wage economy, since bargained wages decline when labor markets are soft (θ is low). As a result, labor markets are open for 81% of the productivity classes, compared with only 60% in the rigid wage economy.[12]

[12] Note that the unemployment rate is the average rate of unemployment of those labor markets which are operating, so in the rigid wage regime, the unemployment rate will signi…cantly understate the nonemployment rate, which includes closed markets.

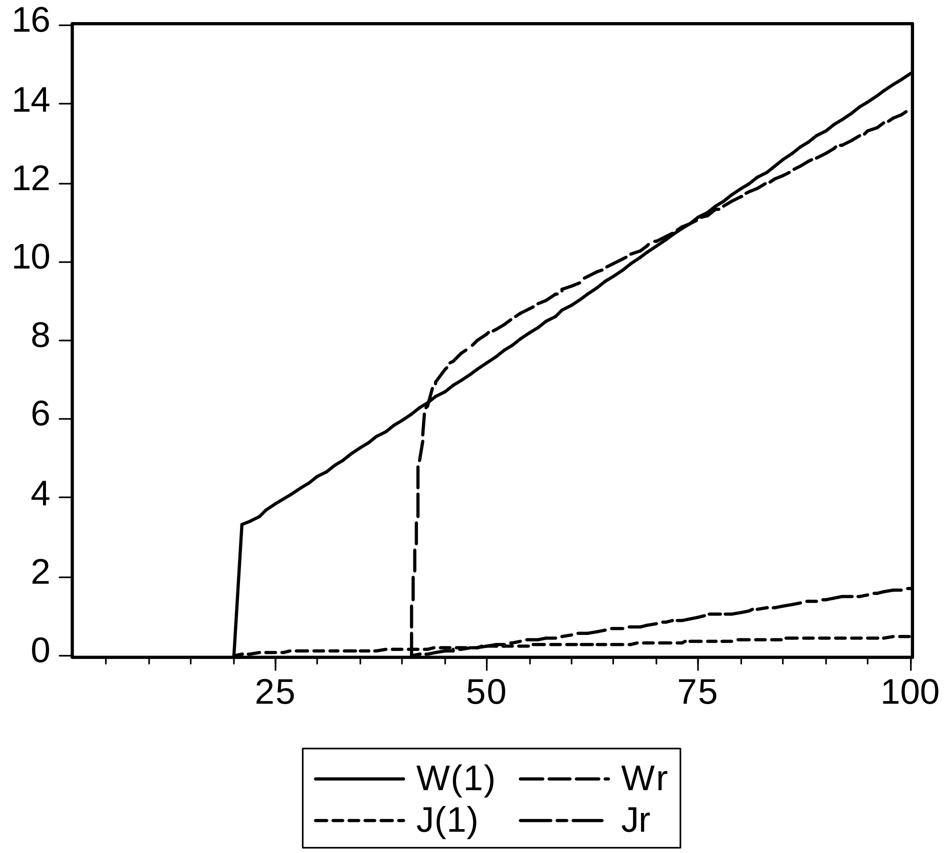
3.3 Results

The valuations of the state of employment and of a …lled job under both regimes are derived in the Appendix. In Figure 2, these valuations are plotted by skill s for the baseline calibration. As can be seen, the labor market is shut down for the lowest skill levels in both segments. Intermediate skill levels opt for the rigid regime. Higher-skill workers prefer to have their wages set under competitive conditions, while low-skill workers tend prefer the rigid regime - assuming their labor market is open. Local sensitivity analysis of the calibration around the baseline reveals that the fraction of those working in rigid labor markets which prefer them to competitive markets is positively related to the interest rate, the Poisson incidence parameter λ, the productivity pay parameter ϕ, the base wage $\overline{w}$, the wage adjustment cost ρ and the …ring tax T.

In Figure 2, preferences of employed workers for the regime clearly depend on skill, so the distribution of skill in the economy will play a pivotal role in determining aggregate preferences. In our base calibration depicted above, workers with skill levels s = $[0.45, 0.74]$ will prefer the rigid wage regime, while workers with skill in the intervals $[0.21, 0.44]$ and $[0.75, 1.00]$ prefer the competitive regime. With uniform distribution of skill, if the two alternatives were subjected to a vote among those with open labor markets in both regimes (with skills in the interval $[0.42, 1.00]$ the rigid regime would defeat the ‡exible economy in a one-on-one election, and this result would be even more likely with signi…cant mass in the middle of the skill distribution. Furthermore, if the agenda is such that workers with skills for which the labor market is closed vote their most preferred ‡exible wage, the voter of median productivity will also prefer a rigid wage regime.

It is noteworthy that …rms working with 50% of all potential skill levels and about two-thirds of all …rms in operation in the rigid wage regime will favor maintaining the status quo. In this model, "capitalists" - or those …rms with …lled vacancies - are likely to represent a conservative force in favor of rigid wage regimes. This is because for reasonable values of ϕ, …rms pro…t more from high productivity workers in the rigid regime. With free entry and zero pro…ts in equilibrium, the emergence of a lobby for a ‡exible, low-wage labor markets appears highly unlikely.

Figure 2: Equilibrium State Valuations in the two Regimes, Baseline Calibration



4 Interactions between Labor Market Institutions and Preferences for Rigid Wages

4.1 Renegotiation Costs versus Firing Taxes

Figure 3 displays valuation of filled jobs ($W(1)$ and W^r) and filled jobs ($J(1)$ and $J^r(1)$) for the baseline case, with variation across the wage adjustment cost $\rho \in \{0.0, 0.25, 0.5\}$ and the firing tax $T \in \{0.0, 0.75, 1.25\}$. Taken alone, levels of wage adjustment costs ρ necessary to induce significant support for a rigid wage regime are too large to be realistic. The results suggest that - at least for models obeying the Hosios condition in the absence of significant frictions - the Nash sharing rule delivers outcomes in the MP model that are unlikely to be improved upon. This is the panel in the upper left-hand side of the figure.

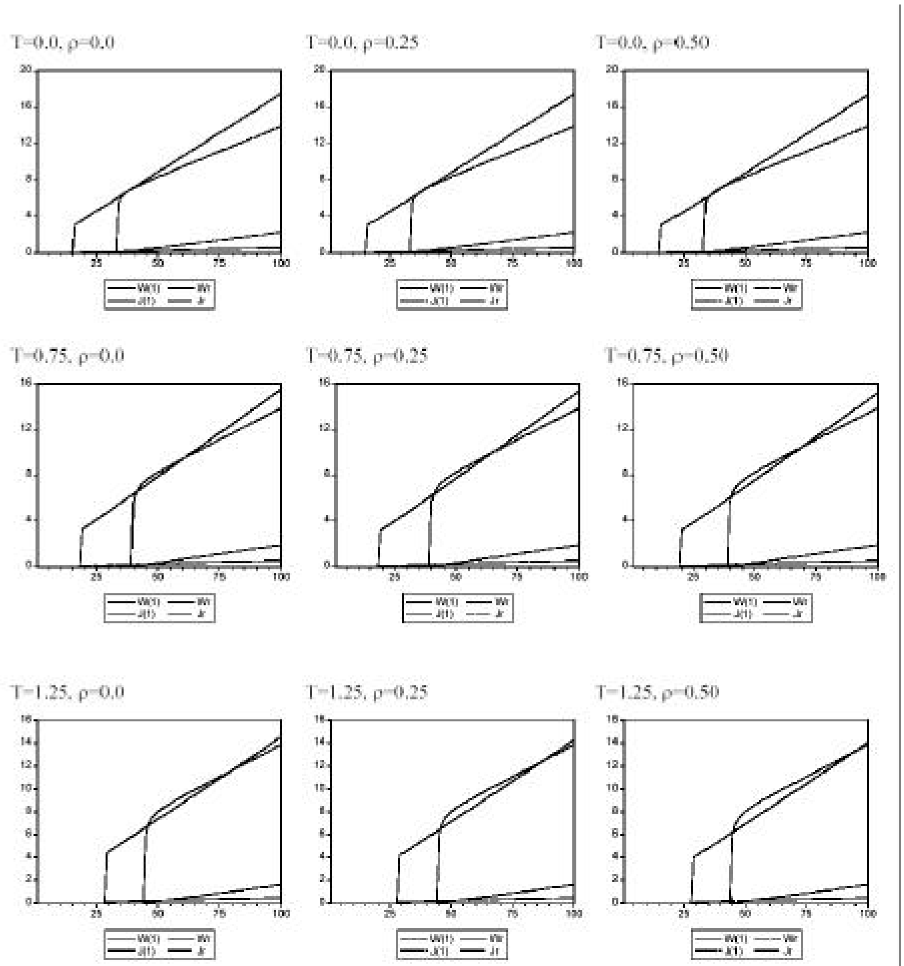
Figure 3 does show clearly, however, that the relative attractiveness of the rigid wage regime is enhanced as the firing tax increases ($T > 0$). The greater the firing friction, the more popular rigid wages are likely to be. This prediction is consistent with the observation of union support for labor market regulations, as well as the correlation of employment protection legislation with union organization (e.g., Boeri, Brugiavini and Calmfors, 2001; Checchi and Lucifora, 2002). Employment protection legislation appears to be a complement to wage rigidities, rather than a substitute, as in the models predicting that support for unions should decline when employment protection is a public good (Booth 1995). Our result is related to Bruegemann's (2004) finding that when wages are rigid, workers are more likely to prefer job protection.

The intuition for the interaction may be obtained from the wage equation (13) The fact that sT enters with a positive sign (and multiplied by the interest rate parameter and the bargaining power) means that regimes with higher firing taxes will have higher wages under Nash bargaining, ceteris paribus. By definition, a rigid wage regime prevents the wage from responding to higher firing taxes, and thereby ameliorate the negative effects on labor market outcomes.

4.2 Labor Market Frictions and Turbulence

Ljungqvist and Sargent (1998, 2002) have stressed the role of idiosyncratic variance or turbulence in the rise of European unemployment. In their analy-

Figure 3: Regime Valuation for Baseline with Alternative Renegotiation Costs (ρ) and Firing Taxes (T)



sis, increasing rates of job destruction interacting with labor market policies - unemployment benefits and/or severance regulation - have led to lower turnover and increased unemployment durations. The empirical stylized fact which has motivated this strand of the literature is an increasing fraction of earnings variance in the United States which cannot be accounted for by observable factors. Our model should have something to say about these issues, in particular: under what conditions could turbulence - the rate at which jobs are subjected to productivity changes - induce workers and firms to prefer rigid to individualized Nash-bargained wages?

To address this question, we consider the regime valuations of workers and firms for different combinations of shock incidence parameter λ and the wage adjustment costs ρ while maintaining otherwise model parameter settings at benchmark values. In addition we consider variation of λ juxtaposed against variation in the firing tax (T). The results are displayed in the panels of Figures 4 and 5. Both diagrams show increasing support for a rigid wage regime in the middle of the productivity distribution when turbulence increases in the labor market, and that this support is increasing in the levels of the two frictions. Figure 4 suggests that support for rigid wages among the workers with the highest skill levels is stronger when λ is larger. This result can only partly be attributed to the savings on wage adjustment costs: support for wage rigidity is also increasing in λ at low levels of ρ. Figure 5 displays the regime valuations when ρ is set at its (negligible) default value; evidently, positive levels of the firing tax are also sufficient for the effect to go through. For values of the frictions considered to be plausible in this paper, only the firing tax is a necessary condition for increasing turbulence to translate into increasing support for wage rigidity.

4.3 Firing Taxes and Startup Costs

Finally, we turn to another important friction in the MP model of equilibrium unemployment, the cost to a firm of posting and maintaining a vacancy while searching for a worker. Broadly interpreted, these costs are a stand-in for all startup costs related to a firm's entering a market and are often linked to product market regulation. Product market regulation has gained prominence in recent discussions of European unemployment, as well as in

Figure 4: Regime Valuation for Baseline with Alternative Renegotiation Costs (ρ) and Rates of Turbulence (λ)

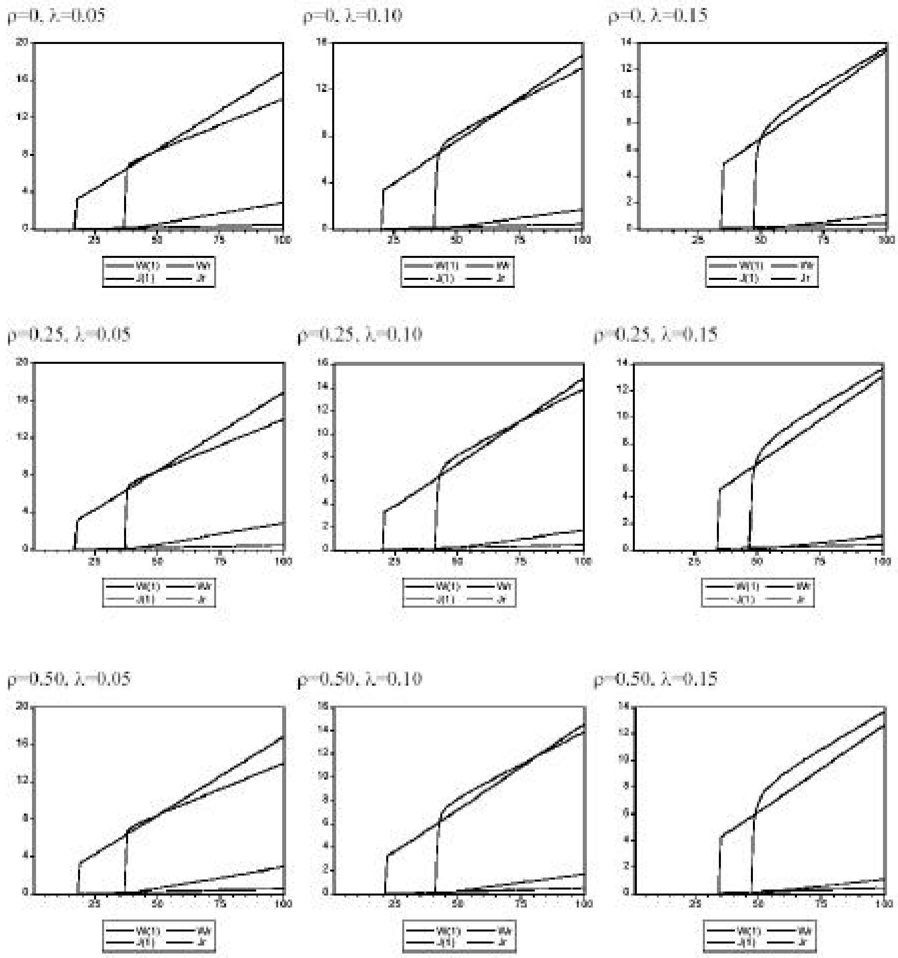
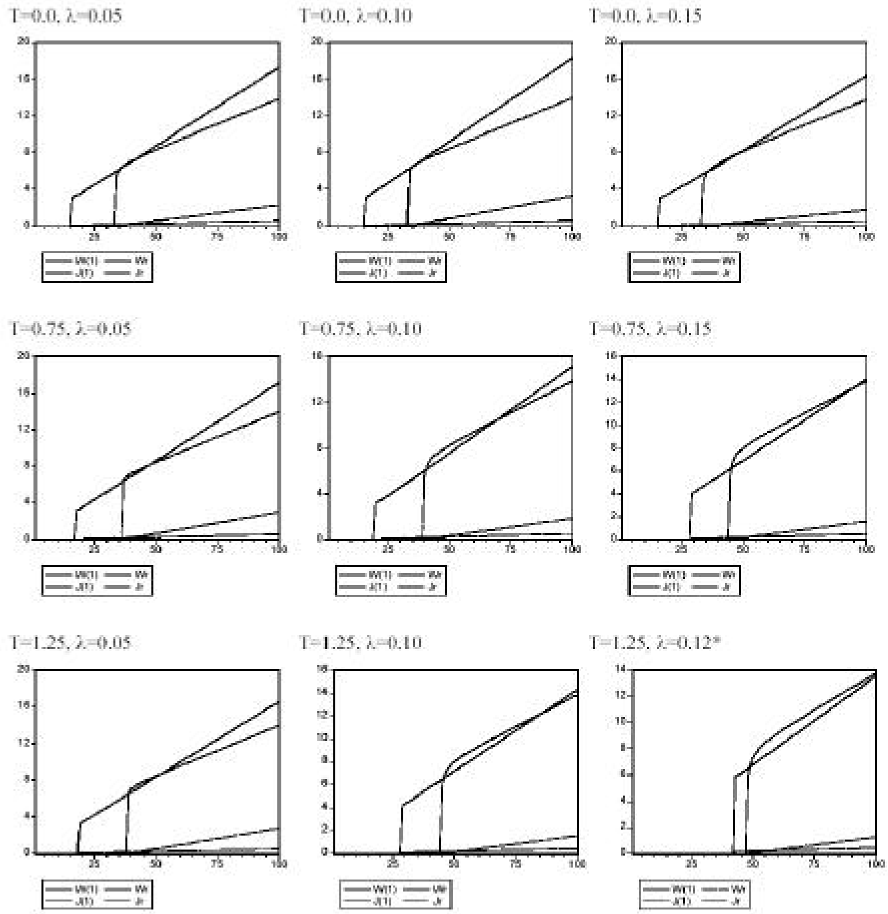


Figure 5: Regime Valuation for Baseline with Alternative Firing Taxes (T) and Rates of Turbulence (λ)



the determinants of economic backwardness and development.[13] According to the World Bank (Djankov, et al. (2002)) large di¤erences can be observed across European economies in this regard: for example, it is estimated that the number of days required to start a company varies in the European Union from 4 days in Denmark, 11 days in the Netherlands and 18 days in the United Kingdom, to 45, 49, and 56 days in Germany, France and Belgium respectively.

Figure 6 displays state valuations for workers in employment and …lled jobs in the benchmark case, which result from variation in values of the …ring tax (T) and startup costs k. A similar result emerges to that with wage adjustment costs: when $T = 0$ employed workers never prefer rigid over individually bargained wages, regardless of the value of k. When $T > 0$, on the other hand, middle-level skill segments emerges which prefer the rigid wage regime, and their mass is evidently increasing in k. Complementarity between the two frictions emerge for similar reasons as before: in the rigid wage regime, workers are prevented from exploiting advantages in bargaining which arise in tight markets.[14]

5 Conclusions

Equilibrium unemployment theory can shed new light on the preferences of workers and …rms for di¤erent regimes of wage determination. Our main results can be summarised as follows.

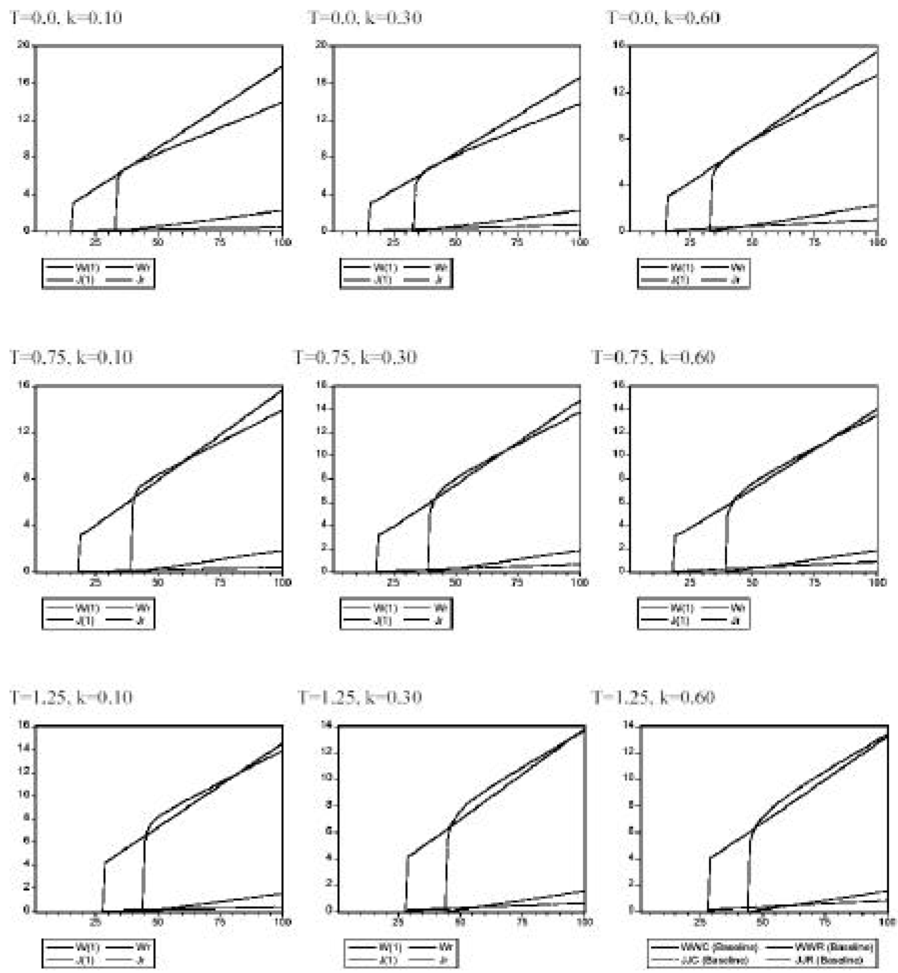
Support for rigid wage regimes may be expressed as membership in a labor union, but also as political endorsement of rigid wage policies, such as minimum wages and the extension of contract wages to nonunionized workers. Interestingly, the lowest and the highest skilled workers will prefer the competitive search market: the former because they are otherwise frozen out of access to a job, the latter because they can do better in competitive search markets. This is consistent with the observation of higher coverage rates around the mode of the distribution of workforce educational attainment.

A central …nding is that, as far the di¤erent frictions are concerned, employment protection is essential for the rigid wage regime to …nd signi…cant

[13] See Djankov et al. (2002), Nickell (1999), Boeri et al. (1999), Blanchard and Giavazzi (2003), Haefke and Ebell (2003) and St-Paul (2004).

[14] Note in (8) that the bargained wage is increasing in both labor market tightness as well as the startup cost.

Figure 6: Regime Valuation for Baseline with Alternative Firing Taxes (T) and Vacancy/setup costs (k)



support in equilibrium. Startup costs also make a rigid wage regime more attractive, notably when there is higher turbulence. Surprisingly enough, the friction interfering most directly with wage setting, wage adjustment costs, is not essential for the rigid wage regime to be attractive to a nontrivial segment of the working population.

Severance protection, in the form of a deadweight …ring tax, increases the relative attractiveness of rigid wage policies, because it further increases utility of rigid wage workers who keep their jobs, measured relative to the competitive search equilibrium. Although severance taxation is a deadweight loss for the labor market, it can increase the relative appeal of rigid wage policies for the segment of low to medium skills for which a rigid labor market exists. Signi…cantly and perhaps not surprisingly, the support of rigid wages from …rms is signi…cant, especially in higher skill labor segments.

Our results point to complementarity between various types of labor market rigidities, a theme that has been studied elsewhere.[15] In particular, …ring taxes play a central role in our results. Not only do they increase the attractiveness of wage rigidity in the …rst instance, they also work in conjunction with other frictions to accentuate the attractiveness of rigid wages. Most strikingly, the presence of the …ring friction increases the appeal of a rigid wage regime in times of higher turbulence. Workers located at the upper end of the skill distribution are more likely to endorse a rigid-wage regime when productivity shocks are more frequent, even when wage adjustment costs are negligible.

It is easy to see how these issues are related to collective bargaining and union coverage. Although the wage rule was taken as given, it could be chosen optimally from the perpective of some agent or set of agents. For example, an economy choosing among linear wage-productivity rules might select the value of ϕ most preferred by the median voter of all employed workers (employed worker of median productivity); one might alternatively give some weight to the unemployed in each labor market as well as those workers in closed labor markets. Indeed, one might consider a value of ϕ collectively determined in an economy-wide Nash bargain between workers and …rms. All these variations represent vistas for future research.

[15] See Coe and Snower (1997), for example. St.-Paul (2004) has recently stressed that the most successful reforms in Europe were made by changing several policies at once.

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7 Appendix

7.1 Derivation of job creation conditions in the rigid wage regime

Use the equilibrium valuation equation (8) to value a job under the rigid wage regime at $x = R^r$, imposing $V^r = 0$ and the fact that at the destruction margin by definition $J^r(R^r) + sT = 0$, to solve for $\lambda \int_{R^r}^{1} J^r(z)dF(z)$:

$$\lambda \int_{R^r}^{1} J^r(z)dF(z) = w^r - sR^r - [r + \lambda(1 - F(R^r))]\, sT$$

Substitute this into (8) with $V^r = 0$ and obtain

$$(r + \lambda)\, J^r(x) = s\,(x - R^r) - (r + \lambda)\, sT.$$

Now set $x = 1$ and use the zero profit condition in the rigid wage regime (10) to obtain the JC-condition:

$$\begin{aligned} (r + \lambda)\, J^r(1) &= s\,(1 - R^r - (r + \lambda)T) = (r + \lambda) \left[\frac{sk}{q(\theta^r)} \right] \\ &\Rightarrow \frac{(1 - R^r)}{(r + \lambda)} - T = \frac{k}{q(\theta^r)} \end{aligned} \tag{29}$$

The derivation for the flexible wage case (e.g. Pissarides 2000) simply replaces w^r with the competitive analog (13), and similar manipulations lead to the JC-condition for the individually negotiated wage case:

$$(1 - \beta) \left(\frac{1 - R}{r + \lambda} - T \right) = \frac{k}{q(\theta)}. \tag{30}$$

7.2 Derivation of job destruction condition in the rigid wage regime

Rewrite the job valuation equation (8) and impose $V^r = 0$ to obtain

$$(r + \lambda)\, J^r(x) = sx - w^r + \lambda \int_{R^r}^{1} J^r(z)dF(z) - \lambda F(R^r) sT. \tag{31}$$

Substitute $J^r(z) = \frac{s(z - R^r)}{(r+\lambda)} - sT$ in the integral on the right hand side:

$$
\begin{aligned}
(r+\lambda)\, J^r(x) &= sx - w^r + \lambda \int_{R^r}^{1} \frac{s\,(z - R^r)}{r+\lambda} dF(z) \\
&\quad - \lambda [1 - F(R^r)]\, sT - \lambda F(R^r) sT \\
&= sx - w^r + \frac{\lambda s}{r+\lambda} \int_{R^r}^{1} (z - R^r)\, dF(z) - \lambda s T. \qquad (32)
\end{aligned}
$$

Imposing $x = R^r$ and $J^r(R^r) = -sT$ yields the condition for job destruction

$$w^r = sR^r + \frac{\lambda s}{(r+\lambda)} \int_{R^r}^{1} (z - R^r)\, dF(z) + rsT$$

Now substitute $w^r = \overline{w} + \phi s$ and rearrange:

$$sR^r + \frac{\lambda s}{r+\lambda} \int_{R^r}^{1} (z - R^r) dF(z) = \overline{w} + \phi s - rsT$$

The flexible wage case is solved analogously to the rigid case (See Mortensen and Pissarides (1999a,b)

$$sR + \frac{s\lambda}{r+\lambda} \int_{R}^{1} (z - R) dF(z) = b + \frac{\beta s k \theta}{1-\beta} + \lambda [1 - F(R)]\, \rho - rsT.$$

7.3 Comparative statics in the rigid wage regime

The two equations for the job destruction and creation conditions under rigid wages, rewritten slightly:

$$R^r + \frac{\lambda}{r+\lambda} \int_{R^r}^{1} (z - R^r) dF(z) = \frac{\overline{w}}{s} + \phi - rT \qquad (33)$$

$$\frac{k}{q\,(\theta^r)} = \frac{(1 - R^r)}{(r+\lambda)} - T$$

Differentiation and rearrangement leads to the form $\mathrm{A}^r \begin{bmatrix} dR^r \\ d\theta^r \end{bmatrix} = \mathrm{b}^r$, with

$$\mathrm{A} = \begin{bmatrix} 1 - \frac{\lambda}{r+\lambda}(1 - R^r) & 0 \\ \frac{1}{r+\lambda} & -\frac{kq'}{q^2} \end{bmatrix}, \text{ and } \mathrm{b} = \begin{bmatrix} \frac{d\overline{w}}{s} - \frac{\overline{w}}{s^2} ds + d\phi - r dT - T dr \\ -\frac{1 - R^r}{(r+\lambda)^2}(dr + d\lambda) - dT - \frac{dk}{q} \end{bmatrix}.$$

Inspection reveals that the determinant of A is now given by $\Delta^r \equiv \left[1 - \frac{\lambda}{r+\lambda}(1-R^r)\right]\left[-\frac{kq'}{q^2}\right]$ which is also unambiguously positive. The comparative statics results are

$$\frac{\partial R^r}{\partial s} = \frac{\frac{\overline{w}}{s^3}\frac{kq'}{q^2}}{\Delta^r} < 0; \; \frac{\partial \theta^r}{\partial s} = \frac{\frac{\overline{w}}{s^2(r+\lambda)}}{\Delta^r} > 0$$

$$\frac{\partial R^r}{\partial \overline{w}} = \frac{-\frac{kq'}{sq^2}}{\Delta^r} > 0; \; \frac{\partial \theta^r}{\partial \overline{w}} = \frac{-\frac{1}{s(r+\lambda)}}{\Delta^r} < 0$$

$$\frac{\partial R^r}{\partial \phi} = \frac{-\frac{kq'}{q^2}}{\Delta^r} > 0; \; \frac{\partial \theta^r}{\partial \phi} = \frac{-\frac{1}{(r+\lambda)}}{\Delta^r} < 0$$

$$\frac{\partial R^r}{\partial b} = \frac{\partial R^r}{\partial \beta} = \frac{\partial R^r}{\partial \rho} = 0; \; \frac{\partial \theta^r}{\partial b} = \frac{\partial \theta^r}{\partial \beta} = \frac{\partial \theta^r}{\partial \rho} = 0$$

$$\frac{\partial R^r}{\partial T} = \frac{r\left(\frac{kq'}{q^2}\right)}{\Delta^r} < 0; \; \frac{\partial \theta^r}{\partial T} = \frac{-\frac{\lambda}{r+\lambda}R^r}{\Delta^r} < 0$$

$$\frac{\partial R^r}{\partial r} = \frac{-T\left(\frac{kq'}{q^2}\right)}{\Delta^r} > 0; \; \frac{\partial \theta^r}{\partial r} = \frac{\frac{T}{(r+\lambda)}}{\Delta^r} > 0$$

7.4 Derivation of the Equilibrium Labor Market State Valuations

In this section we derive expressions for the equilibrium capital asset values of the state of employment at full initial productivity ($W(1)$ or W^r), and the value of a newly-filled vacancy ($J(1)$ or $J^r(1)$).

7.4.1 Flexible Wage Regime

Rearrange the first order condition or sharing rule (11) using (14), and (12) to get

$$W(1) = \frac{b}{r} + \frac{\beta s(1-R^*)}{(r+\lambda)} + \frac{\beta s k\theta^*}{r(1-\beta)} \tag{34}$$

so

$$\frac{\partial W(1)}{\partial s} = \frac{\beta(1-R^*)}{(r+\lambda)} + \frac{\beta k\theta^*}{r(1-\beta)} - \frac{\beta s}{(r+\lambda)}\frac{\partial R^*}{\partial s} + \frac{\beta sk}{r(1-\beta)}\frac{\partial \theta^*}{\partial s}$$

$$= \frac{\beta}{(r+\lambda)r}\left[r\,(1-R^*) + \frac{(r+\lambda)\beta k\theta^*}{(1-\beta)} - rs\frac{\partial R^*}{\partial s} + \frac{k\,(r+\lambda)}{(1-\beta)}\frac{\partial \theta^*}{\partial s}\right] > 0. \quad (35)$$

so in (s, W) space, the valuation of the competitive employment state is strictly increasing in skill s. Intuitively, s has three effects on the valuation of a job. First it increases the flow payoff in all cases that the job survives. Second it lowers the threshold value of productivity, holding all else constant, and thereby increases the expected duration of the job. Finally, it raises equilibrium job tightness in the local labor market, raising the probability of finding a job in that labor market, given that one is unemployed.The sign of the second derivative involves the curvature of response of R^* and θ^* respectively to s :

$$\frac{\partial W^2(1)}{\partial s^2} = -\frac{\beta}{(r+\lambda)}\left[\frac{\partial R^*}{\partial s} + s\frac{\partial^2 R^*}{\partial s^2}\right] + \frac{\beta k}{r\,(1-\beta)}\left[\frac{\partial \theta^*}{\partial s} + s\frac{\partial^2 \theta^*}{\partial s^2}\right] \quad (36)$$

and is ambiguous. One sufficient condition for convexity of the value of competitive segment employment is that R^* and θ^* are not too responsive to s: $\frac{s\frac{\partial^2 R^*}{\partial s^2}}{\frac{\partial R^*}{\partial s}} < 1$ and $\frac{s\frac{\partial^2 \theta^*}{\partial s^2}}{\frac{\partial \theta^*}{\partial s}} > -1$.

For the valuation of firms, we have

$$J(1) = \frac{sk}{q(\theta)}$$

and differentiate, obtaining:

$$\frac{\partial J(1)}{\partial s} = \frac{k}{q\,(\theta)} - \frac{skq'}{q^2}\frac{\partial \theta}{\partial s} > 0 \quad (37)$$

so that an increase in skills unambiguously increases the value of the firm (filled job). For the same reasons as above, the sign of the second derivative of $J(1)$ with respect to s cannot be determined unambiguously.

The derivatives of the equilibrium valuations of employment and filled jobs can be obtained in a straightforward way and their signs are summarized in the Table A1.

Table A1

Effect of ...	b	ρ	λ	β	k
...on					
$W(1)$	?	−	−	+	−
$J(1)$	−	−	−	−	?

The valuation of unemployment U is straightforward:

$$rU = b + \theta q(\theta)[W(1) - U] \tag{38}$$

$$[r + \theta q(\theta)]U = b + \theta q(\theta)W(1)$$

$$U = \frac{b + \theta q(\theta)W(1)}{r + \theta q(\theta)} \tag{39}$$

$$W(1) - U = \frac{rW(1) - b}{r + \theta q(\theta)} \tag{40}$$

but $rW(1) = b + \frac{r\beta s(1-R^*)}{r+\lambda} + \frac{\beta s k \theta^*}{1-\beta}$ so

$$W(1) - U = \frac{\frac{r\beta s(1-R^*)}{(r+\lambda)} + \frac{\beta s k\theta^*}{(1-\beta)}}{r + \theta q(\theta)}$$

$$W(1) - U + J(1) = \frac{\frac{r\beta s(1-R^*)}{(r+\lambda)} + \frac{\beta s k\theta^*}{(1-\beta)}}{r + \theta q(\theta)} + \frac{sk}{q(\theta)}$$

$$S(1) = \left[\frac{\frac{r\beta(1-R^*)}{(r+\lambda)} + \frac{\beta k\theta^*}{(1-\beta)}}{r + \theta q(\theta)} + \frac{k}{q(\theta)}\right] s$$

7.4.2 Rigid Wage Regime

Combining (7) and (5) and solving we obtain:

$$W^r = \frac{b}{r} + \frac{1}{1 + \frac{\lambda F(R^r)}{r+\theta^r q(\theta^r)}}\left[\frac{(\overline{w} + \phi s) - b}{r}\right].$$

Note that if $\overline{w} = b$,

$$W^r = \frac{1}{r}\left[b + \frac{1}{1 + \frac{\lambda F(R^r)}{r+\theta^r q(\theta^r)}}\phi s\right].$$

As in the individualized-wage segment, it is possible to show that W^r is unambiguously increasing in skill s. Intuitively, raising s raises the value of employment because it increases pay directly, as well as equilibrium job tightness in the local labor market, raising the probability of finding a job. It also increases the duration of a job. As long as ϕ is strictly positive, higher skills will be associated with a higher ‡ow payo¤ in the continuation region. As in the competitive case, the sign of the second derivative of W^r is ambiguous.

For the valuation of …rms we have

$$J^r(1) = \frac{sk}{q(\theta^r)}$$

Di¤erentiate to obtain

$$\frac{\partial J^r(1)}{\partial s} = \frac{k}{q(\theta^r)} - \frac{skq'}{q^2}\frac{\partial \theta^r}{\partial s} > 0 \qquad (41)$$

unambiguously. As in the individualized wage regime, the second derivative of $J^r(1)$ is ambiguous.

Table A2 summarizes the e¤ects of other changes on $W^r(1)$ and $J^r(1)$ conditional on the match surviving:

Table A2

E¤ect of ...	$\overline{w}$	ϕ	λ	T
...on				
$W^r(1)$	+	+	$-$	$-$
$J^r(1)$	$-$	$-$	$-$	+

As for the match surplus, use as before the fact that $W^r - U = \frac{rW^r - b}{r+\theta^r q(\theta^r)}$ so that we have

$$\begin{aligned} W^r - U^r &= \frac{\overline{w} + \phi s - b}{r + \theta^r q(\theta^r) + \lambda F(R^r)} \\ S^r(1) &= W^r - U^r + J^r(1) = \frac{\overline{w} + \phi s - b}{r + \theta^r q(\theta^r) + \lambda F(R^r)} + \frac{sk}{q(\theta^r)}. \end{aligned} \qquad (42)$$

If $\overline{w} = b$, we have:

$$S^r(1) = \left[\frac{\phi}{r + \theta^r q(\theta^r) + \lambda F(R^r)} + \frac{k}{q(\theta^r)}\right] s$$