Do Educational Vouchers Reduce Inequality and Inefficiency in Education?

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Abstract

Policy debates around the topic of educational vouchers as an approach to improve the educational system are still ongoing and a consensus on the potential benefits or drawbacks has not been reached yet. This paper uses an agent-based model to simulate the impact that educational vouchers have on a highly heterogeneous school district. It implements the Experienced Weighted Learning Algorithm (EWA) to model the optimal decision making processes of the participating schools. The simulations are performed under multiple income distributions to show how the effect of the vouchers on the educational outcomes of student groups varies with various income distributions. The results indicate an ambiguous effect on low income students, namely that the introduction of a choice based system has a negative effect on low income students in low performing schools, due to "*cream skimming*", that is highly motivated students leaving the schools but also a positive effect on the students exercising choice and switching to better schools. However, the negative effects are partly absorbed by low performing schools improving their educational services as a response to a decline in enrollment. In a further step, the paper shows that target vouchers allow to obtain the benefits of the increased competition while avoiding the deterioration of the peer group.

Keywords: school choice, educational vouchers, agent-based modeling
JEL-Classification: I24, I22, H52, C63

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1 Introduction

The school systems in elementary and secondary education in many developed countries show some room for improvement, especially as they exhibit great gaps between low-income or minority students and their respective counterparts. One possible strategy for improving school performance is to introduce competition, for instance through the provision of vouchers to all students or certain student groups (e.g. merit based vouchers). The vouchers would provide a large student body with the opportunity to attend any school of their choice rather than having to attend the public neighborhood school.

The idea is based on the paradigm that, analogous to other markets, schools having to compete for students, will enhance efficiency and the quality of their product, i.e. educational experience for the students attending.[1] Hoxby (2002) for instance argues, that the increased amount of competition will enhance efficiency and render distributional concerns less relevant, as an overall improved school system will help to improve all students' educational experience ("a rising tide that lift[s] all boats"). The argument does however not take the idiosyncrasies of the educational system into account, that would make this prediction not hold. In fact, MacLeod and Urquiola (2012) show that if one applies the theory of incomplete markets and incomplete contracts to the educational system, the predictions must be much more nuanced, and that there is no a priori reason to conclude that the introduction will in fact benefit all students.

The empirical research supports the argument that expectations of educational vouchers improving the educational outcomes throughout all student groups should be tempered (for a survey see Barrow and Rouse, 2008; MacLeod and Urquiola, 2012). While there are a number of educational vouchers programs to date that display small positive effects on student outcomes (see Neal, 2002), it is not quite clear, how far the estimations of empirical studies are applicable to a larger scale voucher program, since there is only very small evidence on large scale voucher programs (see Chakrabarti, 2013b,a, 2008, who describes evidence from Florida and Milwaukee Voucher Programs and finds a positive effect on public schools after the introduction of the voucher).

Therefore, the introduction of a choice based system for a large student body raises multiple questions regarding the effect of such vouchers on student allocations. In particular, the educational outcomes of those students who exercise this choice and those students who remain at the public schools would be of interest. Will the program actually benefit students from low income families or minorities who are attending low performing schools, i.e. will the prevalent inequality in education be reduced? Will the public schools actually get better and increase their efficiency or rather be gradually deprived of their resources and get worse?

In this paper, I propose an agent-based model of an educational voucher market which draws on the seminal contributions by Manski (1992) and Epple and Romano (1998, 2008) and use the model to evaluate the impact of a voucher program on inequality and inefficiency in the educational system in a heterogeneous school district. In particular, the goal of the paper is to take the heterogeneity of the student body and the school system into account, in order to allow for students to have varying characteristics in multiple dimensions that are orthogonal to each other. This allows to show distributional outcomes that are only based on the inherent characteristics of students and the resulting school choices. The model simulates how various students groups are affected by the introduction of the voucher program (*direct effect*) and whether the voucher actually leads to an increase in educational service offered by public schools (*indirect effect*). In addition, I show how the effect of the voucher varies with the type of the income distribution and the type of voucher that is used. In order to simulate the optimizing behavior of participating schools, I implement the 'Experienced Weighted Learning Algorithm' Camerer and Hua Ho (1999), to allow for schools to achieve optimal choices that are based on their own learning activity, and thereby avoid imposing restrictive assumptions on their optimizing behavior.

[1] The idea dates far back, Thomas Paine for instance proposed a voucher plan in 1792, in *The Rights of Man*, for a discussion see West (1967). The more recent awakening of interest is typically credited to Friedman (1955, 1962, 1997)

The outcomes resulting from the model show an ambiguous effect on low income students. While students actually exercising choice profit from a voucher system, the effect on students remaining in their original schools is less clear. Given that the majority of students exercising choice and leaving into better schools have mostly higher ability levels, the students staying behind observe a decline in the peer group quality (known as the "cream skimming effect"). However, the learning algorithm, allows for schools to react to a sudden decline in student enrollment by adjusting educational quality accordingly. Thus, the low performing schools actually react to increased competition and adjust their educational service to attract more students, that is, the "cream skimming" effect, is partly alleviated through the reaction of public schools. When the model is extended to allow so called target vouchers, which are a function of the skill level of the respective students, the observed deterioration effect is alleviated and public schools observe less of a decline in their mean ability. This result confirms the finding in Epple and Romano (2008).

The paper is structured as follows. In Section 2, I will lay out the details of the model that I employ. Section 3 and section 4 present results from the baseline case for universal vouchers and the case of target vouchers, respectively. Section 5 then concludes.

2 The Model

2.1 Model Specification

The model presented here draws on contributions by Manski (1992) and Epple and Romano (1998, 2008) and extends those models along several lines, in particular to account for the heterogeneity of the student body and includes a simultaneous and endogenous optimizing mechanism of the schools. The model represents a school district that is comprised of a homogeneous public sector and a homogeneous private sector. Both sectors have restricted knowledge of each others payoffs, i.e. there is no perfect information and both sides can merely observe the other sides' policies after they have been implemented, i.e. they move simultaneously. The model neglects informational aspects, that is all schools are entirely transparent for households regarding the achievement of the students enrolled and the expenses per student. This appears to be a realistic assumption given the information that is made publicly available in many school districts in the United States. The model does not address school finance policy, that is government expenses are not bounded.

2.2 Household Characteristics

Each household i has an income, denoted as y_i, and each household has one student who has an ability of b_i. Furthermore, each household has a private school preference p_i, which is a continuous variable measuring the idiosyncratic preference of a student for private school relative to public school, where p_i is distributed normal with variance one and mean μ, $p_i \sim \mathcal{N}(\mu,\sigma)$.[2] I assume that y, b, and p are distributed independently of one another, which is not a very realistic assumption but rather serves the purpose of warranting that any distributional outcome will be the result of the behavioral assumptions and not of any predetermined distributions. In particular, b and y have uniform distributions, f(b) and f(y), which are assumed to be continuous and positive on their support, $S_b = [0, b_{max}]$ and $S_y = [0, y_{max}]$.

Households choose to maximize their utility function, $U_{is} = U(\cdot)$, through their choice of school for the households' student. $U(\cdot)$ is increasing in the consumption of a numeraire good, the mean ability of the student body, m_s, and the instructional expenditure of the school, e_s, and is continuous and

[2]similar to Manski (1992) I use p_i to adjust initial enrollment in public/private schools to reflect more realistically the empirically observed ratio of students in private vs. public schools, it can reflect various forces: preference for religious relative to secular schooling, the time required to commute to a private school relative to public school etc., in an extended model, one could replace this with more detailed characteristics

twice differentiable in all three arguments. Educational achievement for student i, thus becomes $a_i = a(m_s, e_s)$, which is a nonnegative, and increasing function of the mean ability of the school attended and the expenditure per student at the school attended. By taking m_s into account, households consider the peer-group effect in their choice of school.[3] Let t_s be the tuition charged at school s, and v_{is} the voucher that student i receives for attending schools s, then we have $U_{is}(\cdot) = U(y_i - t_s + v_{is}, m_s, e_s, p_i)^4$, with $U'()$ positive for all arguments.

As mentioned above, the model abstracts away from informational issues, students are thus perfectly informed about school characteristics as they rank schools. In particular, households maximize utility, $U_i = U_i(\cdot)$, through their choice of school.

Utility for public schools is given by:

$$U_{i0} = \alpha_i log(e_s) + \beta_i m_s + \gamma log(y_i) \tag{3}$$

For private schools we have:

$$U_{i1} = \begin{cases} \alpha_i log(e_s) + \beta_i m_s + \gamma log(y_i - t_s + v_{is}) + p_i & \text{if } t_s \geq v_{is} \\ \alpha_i log(e_s) + \beta_i m_s + \gamma log(y_i) + p_i & \text{else .} \end{cases} \tag{4}$$

with $\alpha_i = \alpha(b_i)$, $\beta_i = \beta(b_i)$, and $\gamma > 0$, where $\alpha'()$ and $\beta'()$ are all positive, i.e. students with higher intrinsic ability, ceteris paribus, value the educational quality and peer quality of each respective school higher than students with lower intrinsic ability.[5] Notice the condition $t_s \geq v_{is}$, which is necessary to preclude kickbacks by schools, that is schools that enter the market at the lower spectrum of educational quality and attract particularly low income students by kicking back monies of the voucher. This does not exclude topping up by participating schools, thus schools are allowed to charge a higher tuition than the voucher amount if they deem it appropriate in order to maximize their objective function.[6]

2.3 School Characteristics

Schools are divided into public and private, with schools being homogeneous within each sector. Every student can choose between the public school sector (s = 0) and the private school sector (s = 1), whereas

[3] see Henderson et al. (1978); Summers and Wolfe (1977); Hoxby (2000) who provide empirical evidence for a peer group effect in educational achievement

[4] as Epple and Romano (1998, 2008) I assume that $U_i(\cdot)$ satisfies everywhere the "single crossing" condition in income y_i and and in motivation h_i, that is:

$$\partial(\frac{\partial U/\partial e}{\partial U/\partial t_s})/\partial y_n > 0 \tag{1}$$

and:

$$\partial(\frac{\partial U/\partial e}{\partial U/\partial t_s})/\partial h_n > 0 \tag{2}$$

where e represents educational quality, either as expenditure per student or the mean ability of students enrolled in that particular school. 1 implies that in the (e,t_s)-plane, the indifference curves of students with the same motivation are becoming steeper as income increases. This is equivalent to saying that income elasticity of demand for educational quality is positive, i.e. educational quality is a normal good. Analogously, 2 implies a positive motivation elasticity of demand for educational quality.

[5] see Rothstein (2006) for evidence of parents putting higher emphasis on peers rather than school quality. This can be used as justification to increase the beta coefficient in the utility function

[6] (Epple and Romano, 2008) exclude both, kickbacks and topping up

the former is available for free and preferred to no schooling, that is, tuition t_0 for public schools is set to zero, while tuition for private schools is endogenous. School finance policy warrants public and private schools a funding of v_{is} that is proportional to enrollment (and a function of student characteristics). Each school chooses their expenditure per student, e_s, and has a publicly known mean student ability, m_s, which is endogenous. Both, e_s and m_s constitute educational quality for the school. The amount of funding for the schools depends on the type of policy that is examined. For the sake of simplicity of notation, I will refer to all forms of funds from the government (state and federal) as a voucher, v_s. This voucher will be stable in size for public schools throughout all policy experiments, while it will be varied for private schools.

2.4 Public Sector Schools

Public Schools do not face within sector competition and offer free admission to all students.[7] They are local monopolists and as such are not restricted to have zero profits. Alternatively one can view them as more costly to operate and less effective in fulfilling their educational task compared to private schools. Thus I assume, that the public sector can set expenditure e_0 such that their maximization problem yields a surplus:

$$\max_{e_s} \sum_{i=0}^{n} (1 - c_i)(v_s - e_s) = 0 \tag{5}$$

where c_i is an indicator variable that takes the value of 1 when student i is enrolled in private school.

2.5 Private Sector Schools

Private schools act competitively as the typical schools district consists of numerous private schools and students who are able to to afford private school tuition are able to choose between these various options.[8] They set their tuition, t_s, and their expenditure, e_s, such that they can maximize their objective function:

$$\max_{t_s, e_s} \sum_{i=0}^{n} c_i(t_s + v_s - e_s) \tag{6}$$

Given the competition, I assume that they choose their expenditure, e_1, such that equation 2 becomes zero ('zero-surplus condition'), we thus have:

$$\sum_{i=0}^{n} c_i(t_s + v_s - e_s) = 0 \tag{7}$$

I assume that the private sector grows through existing schools increasing in size or new schools entering. Private schools then choose t_s and thereby set e_s. In doing so, they do not face any restrictions with the introduction of the voucher, i.e. they charge tuition above the voucher level. I assume that tuition is set in such a way, as to maximize private school sector enrollment, i.e. the maximization problem of the private sector becomes:

[7] given that about 90 percent of the student body attends public schools, and 70 percent attend their local neighborhood school, this assumption seems appropriate (USGovPrint., 2012)

[8] an alternative modeling strategy could be to take the reputation of schools into consideration which could hinder new entries

$$\max_{t_1} \sum_n c_n \tag{8}$$

Thus, private schools attempt to maximize their enrollment. One possible motivation for this modeling strategy is that one could think that every time profit possibilities arise a new school enters and offers the same quality at a lower tuition in order to attract new students.

2.6 Endogeneous Variables

The complexity of the presented model obviates a closed form solution.[9] Instead I employ a learning algorithm Experience-Weighted Attraction (EWA) as suggested by Camerer and Hua Ho (1999) which allows me to model the path towards a stable outcome of the model in which the schools get as close as possible to maximizing their objective functions.[10] This Quasi-Equilibrium is stable and satisfies household utility maximization, and an approximate school's profit maximization given the the respective other sector's choice.[11] That is, both sectors do not have an incentive to unilaterally deviate from the given outcome, thus the EWA-Model leads to stable Nash Equilibria in all parameter settings.[12]

The EWA Model that is employed, assigns attraction levels to all possible strategies based on previous payoffs or preconceived beliefs and maps those into probabilities for actual choices. Each period after one strategy is drawn from the distributions the attraction for that particular strategy is updated based on the current period's payoff and the discounted previous periods payoff. Then strategies with higher attraction levels are then chosen more frequently. In particular, the model consists of the law of motion for the experience weight which depreciates payoffs from previous periods:

$$N_t = \rho N_{t-1} + 1 \tag{9}$$

where ρ is a discount factor. The attraction for each respective strategy j at time t for sector s is given as:

$$A^j_{s,(t)} = \frac{\{\phi N_{t-1} A^j_{s,t-1} + [\delta + (1-\delta) * I(s^j_s = s_{s,t})]\pi_s(s^j_s, s_{-s,t})\}}{N_t} \tag{10}$$

with ϕ as a depreciation factor of previous attraction levels, and where δ represents the weight put on the fact whether a strategy has been employed before or not. $s_{s,t}$ denotes the strategy employed by school sector s at period t, $s_{-s,t}$ is the strategy employed by the respective other school sector at period t, $I(s^j_s, s_{s,t})$ is an indicator function which equals one if $s^j_s = s_s t$ and zero otherwise. Finally, $\pi_s(s^j_s, s_{-s,t})$ is a payoff that sector s obtains if strategy s^j_s is chosen and the other sector chooses s_{-s}. The model is initialized with $N(t)$ and A^j_t according to beliefs about what experience the actors might be able to draw from, given similar other situations. The attraction levels of each strategy are then mapped into a probability for each strategy j to be chosen by sector s, and each sector then draws from a logit formulation to make a choice for the next period which is given by:

[9] Manski (1992) employs a numerical simulation through all possible combinations of endogenous variables and then solves for a unique equilibrium using backward induction from the public school's perspective, given that they are dominant and the private sector consists only of small firms that take the actions of the public school sector as given

[10] this is somewhat similar to the "*epsilon equilibrium*" of Epple and Romano (1998)

[11] see Scotchmer and Wooders (1987); Scotchmer (1997), who show some of the difficulties of finding an equilibrium in club economies, with private schools being an example of clubs with "non-anonymous crowding"

[12] see Fowler (2011) who uses an Agent Based Model to replicate the results of the Core-Periphery Model and obtains a comparable Nash Equilibrium

$$P^j_{i,t+1} = \frac{e^{\lambda A^j_t}}{\sum_{k=1}^{m_i} e^{\lambda A^j_t}} \tag{11}$$

where λ is a parameter and m_i is the number of possible strategies that sector s can employ.

The EWA Model combines two fundamental types of learning learning processes, namely reinforcement learning and belief learning and treats both as border cases for the model's parameters (for $\delta = 0$ and $\delta = 1$). As such, it constitutes a general learning model which has experimental evidence supporting the algorithm and the choice of parameters (see Camerer and Hua Ho, 2008; Brenner, 2006).

2.7 Model Calibration

The parameters of the household utility function, $U(\cdot)$ are $\alpha_i = \alpha(b_i)$ and $\beta_i = \beta(b_i)$, which are assumed to be continuous and positive on their support, $S_\alpha = [2.5, 5]$ and $S_\beta = [1, 2]$, and $\gamma = 25$.[13] The private school preference parameter μ is set such that the enrollment ratio between public and private schools reflects the empirically observed ratio in the Unites States, which yields $\mu = -2.5$.[14]

The simulations are performed in varying income distributions, in particular the school district consists of families from five annual income groups, which are defined as:

• low:	• \$0 - \$20,000
• lower middle:	• \$20,001 - \$40,000
• middle:	• \$40,001 - \$60,000
• upper middle:	• \$60,001 - \$80,000
• high:	• \$80,001 - \$100,000

The income is distributed uniformly within these intervals. The simulations are performed with the following distributions:

- all families have an income of \$ 50,000. (Gini coefficient: 0.00)
- all classes contain 20% of families.(Gini coefficient: 0.33)

Note that throughout both distribution the average income remains unchanged at \$ 50,000, thus all the observed effects result only from the change in the distribution. The number of students is set such that the results obtained are stable, particularly to warrant that the learning mechanism of the schools receive enough feedback (for N very small, the inflow of new students each period does not guarantee a correct feedback for the payoff function). For this purpose $N \geq 100$ is appropriate.

The public schools receive funding proportional to enrollment, i.e. they receive a "public voucher" equal to \$ 6000 ($v_0 = 6$). Voucher levels for private schools vary from zero ($v_s = 0$), to \$ 2000 ($v_s = 2$), to \$ 4000 ($v_s = 4$) per student. The parameters of the EWA model are $\rho = 0.01$, $\phi = 0.9$, $\lambda = 5$, $N_0 = 1$, and $\delta = 1$.

[13] the current parameter values imply that students at the upper end of the ability spectrum would in the optimum spend 1/6 or 1/11 of their income on education. Epple and Romano (1998) point out that the current percentage of aggregate disposable personal income for the United States which is spent on education is close 5.6 %

[14] see Appendix A for more details on the calibration. For the empirical counterparts of the used parameters (see USGov-Print., 2012)

3 Universal Vouchers

3.1 Computational Results

Appendix A contains tables for all simulation results reported below. I first describe the results for the endogenous variables, private school tuition and public school expenditure as depicted in Figure 1, both panels. Throughout all distributions, the private school tuition (t_s) $\forall s = 1$ falls as the voucher increases, while the tuition in the unequal distribution remains well above the tuition of the equal distribution intially and then falls below the equal distribution. Notice however, that due to the fact that educational expenditure in private schools is the sum of private school expenditure and the voucher, in all three scenarios the educational expenditure in private schools remains constant. Meanwhile in the right panel of Figure 1 we can see that public schools increase their expenditure per student, e_0, as the voucher increases. This is the effect of increased competition, the so called *indirect effect* or *general equilibrium effect*. Notice that the spending of public schools is higher in the equal distribution at all times. This can be explained by the fact, that with more students in the middle income class, the number of students that default into public school (and thus do not have choice) is lower. Thus, public schools face a higher competitive pressure in the equal distribution. This explanation also corresponds with the observation above, namely that private schools have a higher expenditure per student in the unequal distributions initially. Given that more students default into public school, they are serving a student body that has a higher mean income and thus prefers higher educational expenses.

Note that there is no balanced-budget condition for the government, the resulting outcome after the introduction of vouchers can thus lead to an increase in government expenditure.

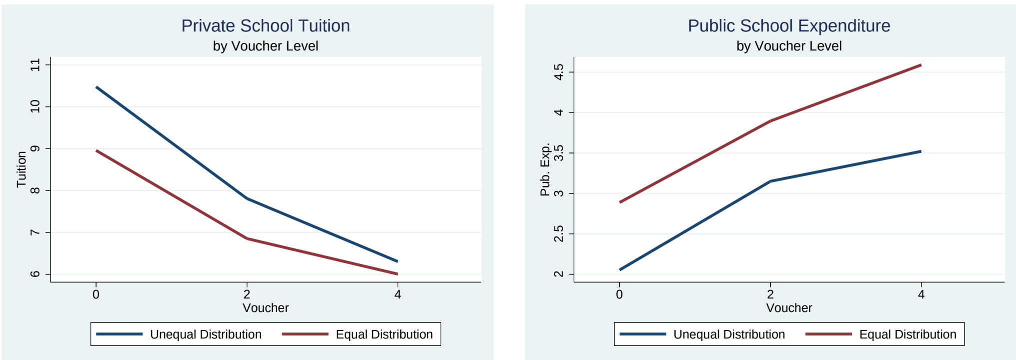


Figure 1: Endogeneous Variables

The response of both schools lead to an overall increase in the expenses valued by students as can be seen in both panels of Figure 2. In both distributions the expenditure per student increases with the voucher introduction, while the expenditure valued in the unequal distribution increases slower than the equal distribution. This can be explained by the fact that in the equal distribution there are more students close to the threshold of switching from public to private. This implies more competitive pressure for public schools and therefore their stronger reaction.

This increase is also present with low income families, as depicted in the lower right panel of Figure 2, which is partly due to the increased expenditure e_0 by public schools and partly due to the increased enrollment of students in private schools.[15]

[15] Both effects can be separated by switching the objective function of the public schools to one which maximizes enrollment

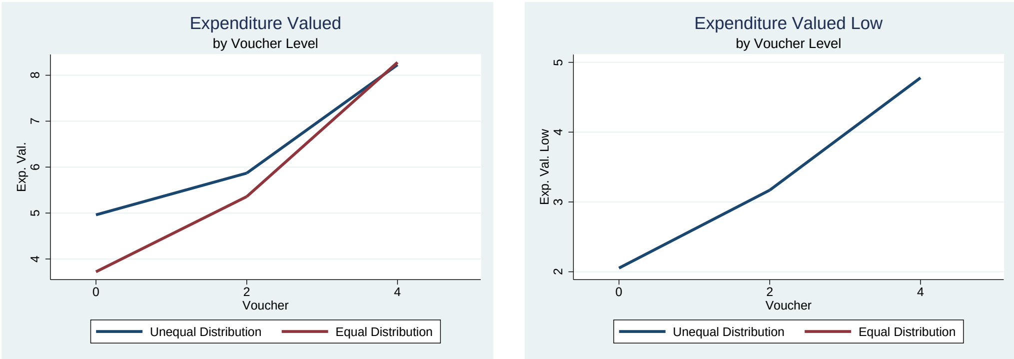
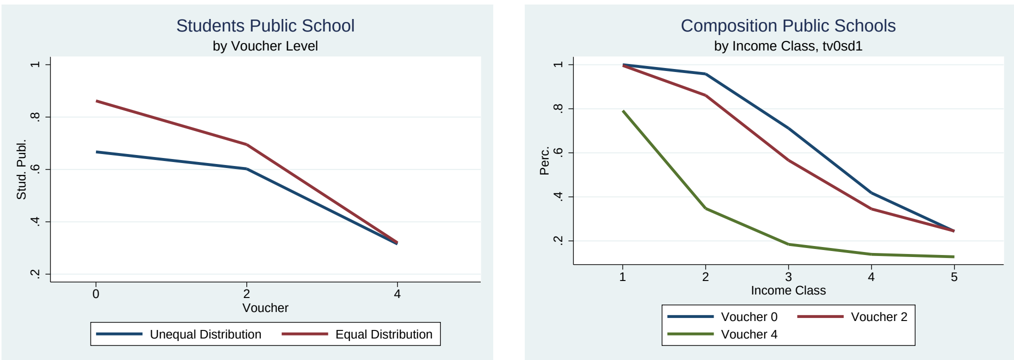


Figure 2: Expenditure Valued

Throughout all distributions we can observe that the fraction of students enrolling in private school increases with the introduction of the voucher, see Figure 3, left panel. Observe also, that in the initial case without a voucher, there are more students enrolled in public school in the equal distribution, due to the higher quality of public schools in this setting.

In the right panel of Figure 3 we can see the enrollment in the public school sector as a function of income class for all three voucher levels. You can see that with rising income, the fraction of students enrolled in public school decreases, which does not change with the introduction of the voucher. You can also see that the largest difference between the voucher levels is for the income classes two and three. These are the classes closest to the income threshold where switching to private school is most likely, holding all other household characteristics constant. These two classes are the ones with the highest proportion of school changers and thus profit from the voucher the most in this regard.

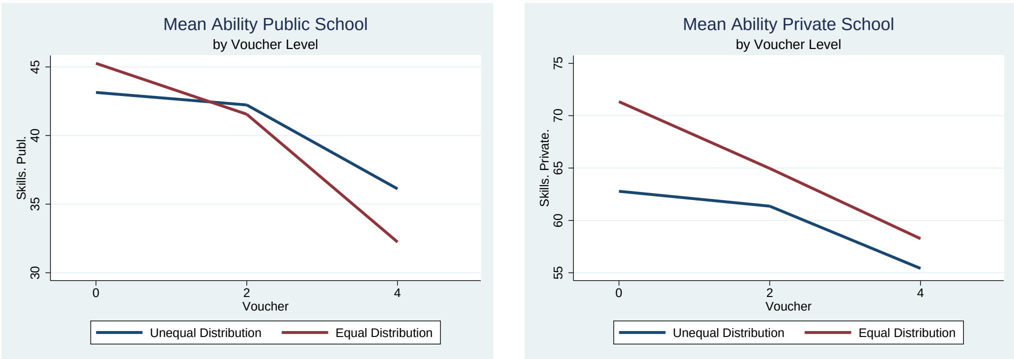


(a) (b)

Figure 3: (a) Enrollment in Public Schools and (b) Composition of public schools

In Figure 4, we can see the mean ability in the public sector (left panel) vs. private sector (right panel) in the equal and the unequal distribution. In both school systems we can observe a strong decline in mean ability. Notice also that the mean ability in the public schools is higher in the equal distribution before the voucher is introduced. Analogously to the higher educational quality in the equal distribution as explained

instead of surplus, then schools always set expenditure to the maximum possible amount of v_0 and thus one can measure the change in educational expenses solely attributed to an increase in private school enrollment above, this is simply the result of the public school sector being of higher quality in terms of educational expenditure and mean ability. However, due to the higher number of students switching schools in the equal distribution when the voucher is introduced, we can observe a stronger decline of skills in the public schools as it is mostly the high skilled students who are exercising choice. This also explains how in the private school sector we can observe a stronger decline in mean ability in the equal distribution case. As more students switch from public to private, this lowers the mean ability since the high ability students from public schools are on the lower end of the ability spectrum in the private schools. This implies that the students staying behind in the public school are observing a decline in their peer group.



(a) (b)

Figure 4: (a) Average Skills Public Schools and (b) Average Skills Private Schools

We can thus observe an ambigious effect on the educational attainment,$a_i = a(m_s, e_s))$, of students from the lower income groups. While the ones switching schools experience improved educational outcomes, the ones staying back are exposed to two effects. First, the educational expenses by public schools, e_0 increase and thus improve their educational experience. However, at the same time, as the peer group in public schools deteriorates, i.e. m_s declines, public school students have worse outcomes in this regard.[16]

4 Target Vouchers

In this section I analyze an alternative voucher program similar to Epple and Romano (2008) that allows to obtain some of the benefits of school competition without leading to detrimental outcomes for some students especially in the lower income groups. The main target of such an alternative voucher program would be to avoid *stratification* or *cream skimming*, that is the best students leaving the worst schools and leaving behind a group of students that is deprived of its better peers. For this purpose, I examine a targeted voucher system that allows the voucher to vary with household characteristics, i.e. $v = v(b, y)$, and extend the analysis in Epple and Romano (2008), who only examine a voucher that varies with ability.[17]

[16]this result is in line with findings from Manski (1992)

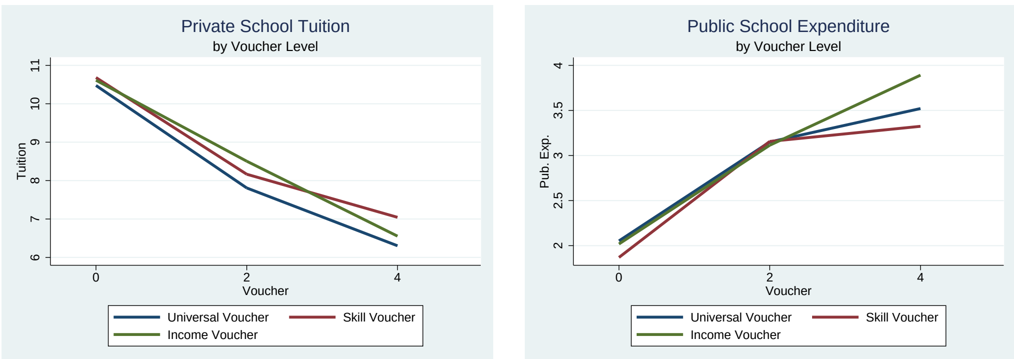
[17]Epple and Romano (2008) exclude the analysis of the income linked voucher on the premise that it would have to eliminate all income differences by being allowed to be negative in order to be able to decrease the gap between public and private schools

The targeted vouchers I examine are of the following form:

$$v_i(y_i) = \frac{v_0}{2} + \frac{v_1}{100}(\bar{y} - y_i) \tag{12a}$$
$$v_i(b_i) = \frac{v_0}{2} + \frac{v_1}{100}(\bar{b} - b_i) \tag{12b}$$

where v_0 can take three values twice in size as before, 0, 4, and 8. Notice that the latter two voucher levels have doubled in value, this ensures that with the functional form of equations $12a$ and $12b$, the average voucher given to students remains the same and only the distribution and the maximum voucher level changes. Both voucher programs allow to subsidize students that are below the overall mean ability, $\bar{b}$ or below the overall mean income, $\bar{y}$, that is, these students receive a voucher higher than the average voucher v_0, with the students at the lower end of the spectrum receiving the highest possible voucher (e.g. for $y_i = 0$ or $b_i = 0$, we have $v_1 = 4$ or $v_1 = 8$). Meanwhile students above $\bar{b}$ or $\bar{y}$ receive a voucher gradually lower than v_0 with the students at the upper end of the spectrum receiving no voucher (e.g. for $y_i = 100$ or $b_i = 100$, $v_0 = 0$). All of the policy experiments reported below are with an unequal distribution of income as used in the previous section, i.e. all classes contain 20% of families.(Gini coefficient: 0.33)

As before, I begin with the endogenous variables explaining school behaviors. The graphs below contain the universal voucher from the previous section for ease of comparison. The left panel in Figure 5 shows how private school tuition decreases very simimar in the two target voucher settings compared to the universal voucher system. Again, throughout all voucher levels, the expenditure in private schools remains constant. The public schools also exhibit a similar behavior to the universal case, i.e. an increase in expenditure follows the introduction of the voucher. Notice, that the income voucher causes a stronger increase than the universal voucher while the skill voucher has the weakest effect for $v_1 = 8$.



(a) (b)

Figure 5: Endogeneous Variabless

As we look to the results in Figure 6, we can see that expenditure valued develops similar to the universal voucher program, that is we can observe an increase in the expenditure valued by students. This also holds for the case of the expenditure valued for low income students. Notice that here, the effect is most pronounced in the case of $v_1 = 8$, as this specifically supports the low income students ability to switch schools.

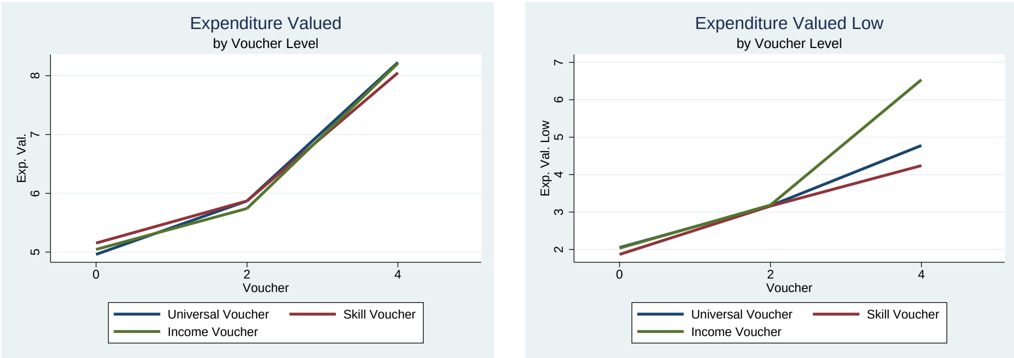


Figure 6: Expenditure Valued

Figure 7 plots a similarly decreasing enrollment in all voucher programs as the voucher level increases. The upper right panel and the lower panels display the switching pattern by income class as in the previous chapter. The upper right panel with the universal voucher reproduce for comparison. The lower left panel shows the composition for skill voucher which displays a very similar development as the universal voucher, i.e. again the lowest class is merely exercising choice wile the classes two and three are the ones mostly switching schools. The lower right panel displays the composition for the income voucher, which unlike the two other programs display a very large number of low income students exercising choice.

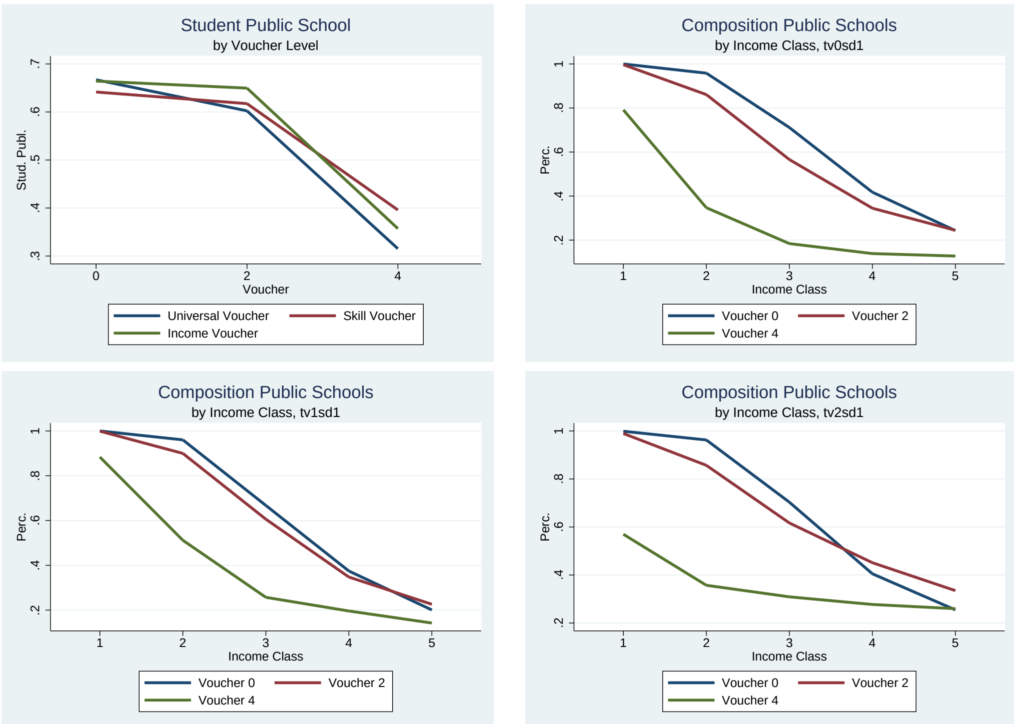
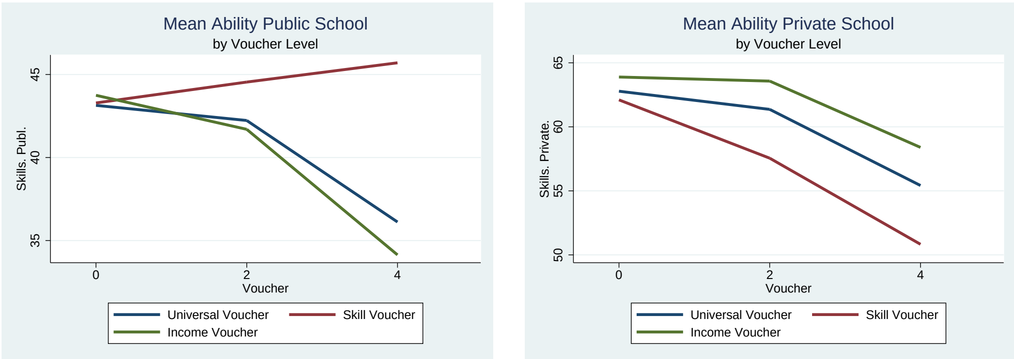


Figure 7: Public school enrollment and composition

Figure 8 plots the mean ability in public (left panel) and private schools (right panel). Mean ability falls in all voucher programs in the private school, for public schools we only observe a decrease in mean ability for the universal voucher and the income voucher. The skill voucher actually leads to an increase in the mean ability public schools. This is caused by the incentive scheme that the skill voucher entails. The voucher amount is negatively correlated with the students' ability, i.e higher skilled students obtain less voucher amounts than low skilled students. Unlike the universal voucher and the income voucher which do not consider the skill level, the skill voucher is able to ameliorate the deterioration of the peer group (even slightly reverses it).



(a) (b)

Figure 8: (a) Mean Ability Public Schools and (b) Mean Ability Private Schools

5 Conclusion

I developed an agent-based model drawing on Manski (1992) and Epple and Romano (1998, 2008) and extended those models to study the distributional effects of a voucher program in a heterogeneous school district. In particular the model allows to simulate the optimizing behavior of schools and households without having to include ad hoc behavioral assumptions. The model shows under which conditions the public school sector displays a positive response to the increased competition and thereby renders a positive educational outcome for students who remain in theses schools. In particular, the paper shows that public schools increase their expenditure per student (public school rents fall) under a voucher system, adding to an overall expenditure per student increases and thus leading to an increase in efficiency. However, students who remain in public school are exposed to a decline in peer group quality due to students with higher abilities or higher income are exercising choice with a higher probability. The model simulates this change in the student body, i.e. students exercising choice are either of high ability or from high income families. Thus the overall effect on inequality remains ambiguous.

In a next step, I have extended the model to allow for a targeted voucher program which allows to make the voucher a function as income or ability. Similar to Epple and Romano (2008) I examine the effects of these alternate voucher programs and find that only the skill targeted voucher is able to introduce competition into educational system without introducing *cream skimming* along with it. That is, the skill targeted voucher is able to increase the educational expenditure of public schools while at the same time avoiding a deterioration of peer quality. Thus, using such a voucher program would allow to avoid the negative effect of a voucher program on the students from low income families and the students with lower ability.

6 Appendix

6.1 Calibration

Given that for private schools ($s = 1$) expenditure equals, $e_s = v_s + t_s$ the derivative of the student utility function with respect to tuition is:

$$\frac{\partial U}{\partial t_s} = \frac{\alpha}{t_s + v_s} - \frac{\gamma}{y_n - t_s} \tag{13}$$

For $v_s = 0$, this implies that in the optimum, households want to spend $\frac{\alpha}{\alpha+\gamma}$ of their income on education, or utility rises with tuition until tuition is optimal, namely (solving the equation above for t_1, after setting it equal to zero):

$$t_1 = \frac{\alpha}{\alpha + \gamma} y_n \tag{14}$$

For the given calibration this would mean that high motivated students are willing to spend 1/6 of their family income in optimum, which seems very high. Thus, either α needs to be smaller, or γ needs to be higher.

One could also arrive at this result if one took the derivative of the optimal value for t_s with respect to v_s, i.e.:

$$t_1^* = \frac{\alpha}{\alpha + \gamma} y_n - \frac{\gamma}{\alpha + \gamma} v_1 \tag{15}$$

$$\frac{\partial t_1^*}{\partial v_1} = -\frac{\gamma}{\alpha + \gamma} \tag{16}$$

6.2 Simulation Results

Table 1: Add caption

	Equal Distribution			Unequal Distribution		
	v_1 = 0	v_1= 2	v_1 = 4	v_1 = 0	v_1= 2	v_1 = 4
private school tuition	8.9976	6.4671	5.1628	10.4077	7.8744	5.3277
public school expenditure	2.9146	2.9146	2.9146	2.2227	3.1205	4.0005
Low Inc Studs Public	.	.	.	0.3054	0.3414	0.3545
LowMid Inc Studs Public	.	.	.	0.2892	0.2921	0.1928
Mid Inc Studs Public	1	1	1	0.2186	0.1758	0.1603
UpMid Inc Studs Public	.	.	.	0.116	0.1109	0.1447
High Inc Studs Public	.	.	.	0.0708	0.0799	0.1478
Low Inc Studs Private	.	.	.	0	0.0022	0.1302
LowMid Inc Studs Private	.	.	.	0.0115	0.0719	0.2041
Mid Inc Studs Private	1	1	1	0.1452	0.2081	0.2159
UpMid Inc Studs Private	.	.	.	0.361	0.3397	0.2138
High Inc Studs Private	.	.	.	0.4824	0.3782	0.236
Students Enrol Public	0.8578	0.8578	0.8578	0.6868	0.6041	0.3095
Students Enrol Private	0.1422	0.1422	0.1422	0.3132	0.3959	0.6905
Public Studs Low Inc	.	.	.	1	0.996	0.5309
Public Studs LowMid Inc	.	.	.	0.9744	0.8438	0.3051
Public Studs Mid Inc	0.8578	0.8578	0.8578	0.7489	0.5628	0.25
Public Studs UpMid Inc	.	.	.	0.4222	0.3432	0.2417
Public Studs High Inc	.	.	.	0.2607	0.2598	0.2274
Expenditure Valued	3.7527	3.7527	3.7527	4.8584	5.8619	7.7327
Expenditure Valued Low	.	.	.	2.2225	3.1392	6.0559
Expenditure Valued Low Mid	.	.	.	2.453	4.1698	7.741
Expenditure Valued Mid	3.7527	3.7527	3.7527	4.3909	6.1678	8.1868
Expenditure Valued Up Mid	.	.	.	7.0892	7.7058	8.2787
Expenditure Valued High	.	.	.	8.3619	8.2421	8.3364
Cost Government	5.1468	5.1468	5.1468	4.1208	4.4164	4.619
Cost Families	1.2243	1.2243	1.2243	3.2702	3.1337	3.6981
Gini Coefficient	0.0626	0.0626	0.0626	0.3198	0.3185	0.3141
Average Inc Overall	5243.71	5243.71	5243.71	5196.19	5210.42	5301.72
Average Skills Overall	49.3963	49.3963	49.3963	49.6078	49.3397	49.6465
Average Skills Public	44.6245	44.6245	44.6245	43.3916	41.7766	35.6766
Average Skills Private	67.1855	67.1855	67.1855	64.0129	61.5075	55.749

Table 2: Add caption

	Skill Voucher			Income Voucher		
	v_1 = 0	v_1= 4	v_1 = 8	v_1 = 0	v_1= 4	v_1 = 8
private school tuition	10.6263	8.1117	7.2617	10.3174	8.4099	6.4699
public school expenditure	2.024	3.0801	2.9369	1.8437	3.4172	3.8034
Low Inc Studs Public	0.3127	0.3587	0.5016	0.3279	0.2734	0.3578
LowMid Inc Studs Public	0.3103	0.2881	0.2315	0.2968	0.2665	0.185
Mid Inc Studs Public	0.2019	0.1792	0.1029	0.2028	0.2055	0.1704
UpMid Inc Studs Public	0.1109	0.1048	0.0861	0.1073	0.1466	0.1506
High Inc Studs Public	0.0641	0.0692	0.0778	0.0652	0.108	0.1363
Low Inc Studs Private	0	0.0008	0.048	0	0.0059	0.1394
LowMid Inc Studs Private	0.0123	0.0466	0.1727	0.0215	0.0493	0.2106
Mid Inc Studs Private	0.169	0.219	0.2424	0.1729	0.1836	0.2132
UpMid Inc Studs Private	0.3431	0.3312	0.2715	0.3455	0.3432	0.2218
High Inc Studs Private	0.4756	0.4024	0.2654	0.4601	0.4181	0.215
Students Enrol Public	0.6577	0.6041	0.3637	0.6361	0.7187	0.3235
Students Enrol Private	0.3423	0.3959	0.6363	0.3639	0.2813	0.6765
Public Studs Low Inc	1	0.9979	0.8388	1	0.9921	0.5329
Public Studs LowMid Inc	0.9716	0.8841	0.4395	0.9443	0.9272	0.3001
Public Studs Mid Inc	0.6791	0.5545	0.21	0.6514	0.7324	0.2856
Public Studs UpMid Inc	0.3991	0.3416	0.1642	0.3742	0.527	0.2533
Public Studs High Inc	0.235	0.2251	0.1491	0.2229	0.4159	0.2472
Expenditure Valued	5.1029	5.9311	8.3559	5.0308	5.3618	8.3915
Expenditure Valued Low	2.0238	3.0926	4.2671	1.8433	3.463	6.7237
Expenditure Valued Low Mid	2.2876	3.9555	7.8094	2.3663	3.9005	8.5629
Expenditure Valued Mid	5.0298	6.3159	9.6755	4.9831	5.2389	8.7297
Expenditure Valued Up Mid	7.4299	7.7871	10.0228	7.3237	6.6934	8.9362
Expenditure Valued High	8.7763	8.5889	10.1366	8.5552	7.4938	8.9847
Cost Government	3.9462	4.4164	4.7274	3.8166	4.8748	4.647
Cost Families	3.6895	3.2238	4.6562	3.7672	2.3087	4.3814
Gini Coefficient	0.3174	0.3197	0.3202	0.3157	0.3147	0.3166
Average Inc Overall	5229.25	5208.9	5207.96	5268.49	5227.32	5220.54
Average Skills Overall	49.8193	49.0148	49.1216	49.4329	49.1605	49.2908
Average Skills Public	43.6749	43.7657	44.1282	42.8389	43.171	33.7339
Average Skills Private	62.3151	57.4718	51.648	62.161	65.2448	56.783

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