# Estimation of a Collective Model of Labor Supply with Female Nonparticipation


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Abstract :

*In this paper we estimate a collective model of labor supply on the population of French couples, using the model developed by Chiappori, Fortin and Lacroix. We extend this model to female nonparticipation, following the theoretical approach of Donni. We propose a whole parametric specification, taking into account the change of gradient of the sharing rule along the female participation frontier. According to empirical results, there is effectively a net switch of regime when the wife stops to work : she becomes then unable to obtain a better personal allocation if household resources grow. Distribution factors like the sex-ratio are used to test the model. The direction of the sex-ratio effect is opposite to what expected. That lead us to reject the separability assumption of individual utilities : housework probably has an important role in nonparticipating couples.*



Keywords : *collective models ; labor supply ; nonparticipation ; intrahousehold allocation ; corner solutions*
JEL classification : *D13 ; J22*



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## 1. Introduction

Contrary to the usual unitary approach where household behaviour is the result of the maximization of a single utility function at the household level, the collective framework introduces a utility function for each individual of the family and assumes that household decisions are Pareto-efficient. Thus household behaviour depends both on individual preferences and on the intrahousehold balance of power which influences an unspecified bargaining process. We can then explain some empirical features incompatible with the unitary approach (for example, consumption depends not only on household income but also on relative earnings of each family member, cf. Lundberg and Pollak, 1996). From a theoretical point of view, the collective approach is also in agreement with the principles of methodological individualism. Moreover, it allows to recover individual intrahousehold welfare, so that it should have multiple applications in normative economics.

Initially developed by Chiappori (1988, 1992), household collective models have been recently the subject of many empirical works on household labor supply. They generally use two concepts. First, the sharing rule. Under a separability assumption of individual utilities, household decisions are a two-step process (1) household members share their nonlabor income, following a sharing rule (2) each member chooses independently his (her) labor supply by maximizing his (her) utility subject to his (her) personal allocation of nonlabor income. The sharing rule summarizes the result of the intrahousehold negotiation and is identifiable up to a constant. Second, the distribution factors. Distribution factors are variables which influence the intrahousehold balance of power and enter as arguments of the sharing rule function. They make the identification of the sharing rule more robust. Moreover, they generate testable restrictions on labor supply functions.

Chiappori, Fortin and Lacroix (2002) have estimated and tested a collective couple labor supply model, where two distribution factors are used : divorce legislation, and the sex-ratio. Both are related to the "marriage market" : the bargaining position of each spouse depends on the consequences of a possible divorce which is used as a threat point in the negotiation process. Factors which make divorce easier for women, like a legislation favourable to women (more alimonies, for example), or a high sex-ratio (there are more men than women in the local population, so that a divorced wive has higher opportunities of remarriage), lead to a better female nonlabor income allocation and then reduce female labor supply ; symmetrically, the male allocation becomes lower, so that male labor supply increases. This model works well on American data : the testable restrictions implied by the theoretical model are accepted, and empirical results make sense.

A similar model has been estimated and tested on French data (Moreau, 2000 and 2002) and on British data (Clark, Couprie and Sofer, 2004). The testable restrictions are accepted, although partially in the French case[1]. But the effect of the sex-ratio on labor supply is not as straightforward as in the American case : it can be insignificant (in United Kingdom, and also on French data with some variants of the model) ; and according to French estimations, male leisure is not a normal good, so that the sex-ratio effect on male labor supply is inverted.

Most often, the estimates were obtained on the subpopulation of working couples : both members are employed, so that labor supply decisions are limited to the choice of weekly or annual hours of work. Rectricting to a subpopulation may lead to selection bias. And above all the decision to participate or not in the labor market is not studied. Hence recent work has adapted the collective model to male or female nonparticipation. Further theoretical developments have been needed to extend notions of reservation wage and sharing rule. Taking into account nonparticipation leads also to econometric

[1] Slutsky's condition on male Marshallian labor supply is not satisfied.

difficulties : specifications must be adapted to corner solutions and the econometric model is not linear anymore. Donni (2001, 2003) has considered the case of continous labor supply functions (each spouse chooses to participate or not, and the number of hours if participating). Blundell, Chiappori, Magnac and Meghir (2001) have considered the case of male discrete choices (the husband has only two possibilities : nonparticipation or full-time employment)[2]. Both articles have proved that, under separability assumption, there still exists a sharing rule function which is identifiable up to a constant everywhere, whether the spouses participate or not[3]. Donni indicates that the sharing rule function is probably not differentiable along the participation frontiers : there is probably a change of gradient. This change of gradient takes an important place in our article.

First empirical applications have followed. Blundell et alii (2001) themselves have studied the British labor market. A feature of this labor market is that numerous husbands do not parcipate (20% of their sample[4]). But when participating, men are generally full-time employed, so that male labor supply can not be treated as a continous variable. Hence they focus on male discrete choice[5]. Collective labor supply models with male and female discrete choices[6] have also been used by Vermeulen to study a Belgian fiscal reform (Vermeulen, 2004) and retirement decisions in the Nederlands (Michaud & Vermeulen, 2004). To the best of our knowledge, the first application of Donni's model with male and female continous choices has been made by Bloemen (2004) on Dutch data : male labor supply has sufficient variability in the Nederlands to be treated as continous. Using a specific functional form, Bloemen develops a complete parametric model with two changes of gradient (one along the male participation frontier, one along the female participation frontier), and deals with problems of statistical coherency that arise when two changes of gradient are simultaneously introduced. According to his estimations, the only effective change of gradient is the female's one.

In the present paper we complete another application of Donni's model, using French data. Two points characterize our approach. First, we consider that men always participate, and focus on female nonparticipation. Indeed, male nonparticipation can be approached in a different way in the French labor market. In France, the age of retirement (or early retirement) depends more on the status of the worker and on employer's decisions than on individual choices. Once he perceives a retirement pension, a man is generally not allowed to work anymore (or he faces strong restrictions). Anyway, state pensions are higher than in Britain and probably exceed the reservation wage. Removing (early) retired persons from the field of a labor supply study appears then to be sensible. Once retired and students[7] have been removed from the sample, the male nonparticipation rate is low : only 6% in our sample. Moreover, this nonparticipation looks involuntary, due to disease or long term unemployment. Hence male nonparticipation does not appear to be a personal choice, and studying it is beyond the scope of a labor supply model. Male labor supply choice is then limited to the annual number of hours worked, which is continous and almost normally distributed around the legal hours[8]. On the contrary, female nonparticipation is frequent (even after students and retired are removed) and looks mostly

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[2] They also allow for female nonparticipation with continous female hours, but they do not focus on modelling female nonparticipation.

[3] Except inside the set of joint nonparticipation.

[4] Couples with children are excluded, so that husbands young men or men aged 50-60 are numerous in the sample. This contributes to explain this high figure.

[5] They also allow for female nonparticipation with continous female hours, but they do not focus on modelling female nonparticipation.

[6] His model allows for multiple choices, for example nonparticipation/part-time/full-time.

[7] As retired, students rarely participate in France.

[8] Had we allowed for male nonparticipation, we should have adopted Blundell et alii's discrete approach, because male labor supply would not have been continous in this case : there is a gap between nonparticipation and hours made by participating men (who are almost always full time employed).

voluntary[9] : one wife in five presents herself as a housewife. Because a lot of wives have part-time jobs, female labor supply is continously distributed from nonparticipation to full-time employment.

Hence our model allows only for female nonparticipation. There is only one participation frontier, with a change of gradient along this frontier. This simplifies greatly the model, while preserving its realism. It seems to us that simplicity is an advantage for further development of collective models : ideally a collective model should take into account simultaneously nonparticipation and other issues like public goods, domestic production, children... Moreover, by focussing on the unique change of gradient, we specify bounds between which the associated parameter must evolve, and we obtain a testable restriction.

The second specific point of our approach is the introduction of distribution factors in the collective model with nonparticipation. Distribution factors generate testable restrictions and allow us to test the relevance of the collective model. Although the collective model has been accepted on participating couples (cf. Chiappori et alii, Clark et alii, Moreau), it is not proved that it is still valid when we study participation decisions. Indeed, predictions of collective model with separable utilities are counter-intuitive : according to the model, an increase in women's bargaining power leads to a better wife's nonlabor income allocation and discourages her to participate ; yet an improvement of the women's position in the society is generally correlated with a higher female participation rate. Hence we test the model by introducing distribution factors. Such a test was not included in previous work on nonparticipation. Our findings question the model. In fact, what is rejected is not the collective approach, but the separability assumption of individual utilities. This assumption is very strong because it ignores domestic production (among others).

We use two distribution factors as Chiappori-Fortin-Lacroix (2002) : a divorce indicator, and the sex-ratio. So our model is an direct extension of their model to female nonparticipation. The divorce indicator can not be the divorce legislation, because divorce laws are homogeneous on the French territory. We use a substitute : the divorce rate. Divorce is more or less easy according to the region because of religious attitudes, local culture, or judicial practices[10]. Because the majority of divorce procedures are initiated by women, an easy divorce (measured by a high divorce rate) may be deemed favourable to women.

The paper is structured as follows. We recall the collective model principles and extend the parametric model of Chiappori-Fortin-Lacroix to female nonparticipation (section 2). Using the French data of the European Panel (section 3), we estimate this model without and with distribution factors (section 4). Finally, we conclude (section 5).

## 2. The model

### 2.1. The collective labor supply framework

We consider a couple whose members (*f* and *m*) choose their labor supply (denoted by $L_f$ and $L_m$ belonging in [0,T]) in a budget set delimited by a linear constraint :

$C \leq Y + w_f L_f + w_m L_m$ ,

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[9] With a model mixing labor supply and demand, Laroque and Salanié (2000) have estimated that female non employment is mostly voluntary whereas male non employment is rather involuntary.

[10] For example, Sofer and Solloghoub (1992) have that alimonies ordered by judges for similar cases of divorce are different across towns.

where Y, $L_f$ and $L_m$ are respectively the nonlabor income of the household and the female and male wages. There is a unique Hicksian composite good with a price set to one. This good is supposed to be private, so that the aggregate consumption of the household is divided in two individual parts $C_f$ and $C_m$ which are not observable.

The classical unitary model would then suppose that labor supplies and private consumptions are determined by the maximisation of a single household utility function $U(L_f,L_m,C_f,C_m)$ subject to the budget constraint. We reject this approach and adopt the collective representation of household behaviour (Chiappori, 1988 and 1992). The general collective framework introduces specific utility functions for each spouse, denoted $U_f$ and $U_m$, and assumes that decisions result in Pareto-efficient outcomes :

*Assumption A1 : $(L_f,L_m,C_f,C_m)$ is chosen to be Pareto-efficient.*

The Pareto-efficiency hypothesis on which the collective approach is founded seems not restrictive, because family members know them each other very well and have to make repeated games for a long time. Under this assumption, there exists a weighting factor µ belonging to [0,1] such that $(L_f,L_m,C_f,C_m)$ solves the following program :

(P1) *max* $\mu U_f + (1-\mu)U_m$
*subject to*
$C_f + C_m$ £ $Y + w_f L_f + w_m L_m$ ,
$0$ £ $L_f$ £ $T$ *and* $0$ £ $L_m$ £ $T$

The weighting factor vary freely across households, depending on the respective bargaining position of both spouses within each couple. So it depends on their relative incomes and can be specified as a function of $(w_f,w_m,Y)$, as in the basic collective model. But µ can also depend on any other variable which affects the intrahousehold balance power. Such variables are called *distribution factors*. Example of distribution factors are the parameters of the local marriage market, like the sex-ratio or divorce laws. They will influence the bargaining positions of the spouses if divorce or separation is used as a threat in the negotiatiin Process. Thus µ is specified as a function of $(w_f,w_m,Y,s)$, where s is a vector of distribution factors. In order to obtain well-behaved labor supply and consumption functions, we have to make a regularity assumption :

*Assumption A2 : the weighting factor is a single-valued and infinitely differentiable function* $\mu(w_f,w_m,Y,s)$.

So we do not specify the bargaining process and suppose simply that it gives a regular function as a result, which will probably be the case if the threat points are continuous and regular functions of $(w_f,w_m,Y,s)$. An underlying assumption of A2 is that µ does not depend on the final choices of the household. For example, if female nonparticipation in the labor market weakens directly the bargaining power of the woman, then female job status has to be introduced as an argument of function µ, and assumption A2 is not satisfied anymore[11].

Under A1 and A2, the derivatives of labor supplies verify a testable restriction (Bourguignon, Browning and Chiappori, 1995) :

[11]On the contrary, assumption A2 can easily be extended to the case where µ depends on a vector of exogeneous control variables Z. It will not change the following results if Z is also introduced as control variables for $U_f$ and $U_m$. But it will be impossible to identify separately the effect of Z on µ and on $U_i$.

(R1) $$\frac{\partial L_f / \partial s_1}{\partial L_m / \partial s_1} = \frac{\partial L_f / \partial s_2}{\partial L_m / \partial s_2} = \ldots = \frac{\partial L_f / \partial s_p}{\partial L_m / \partial s_p}$$

where $s=(s_1,s_2,\ldots,s_p)$.

In the most general version of the collective model, individual utility functions $U_f$ and $U_m$ would depend both on $(L_f,L_m,C_f,C_m)$, which would allow every kind of altruism, externalities and sharing. But in empirical versions of the collective model, a separability assumption is generally added, in order to obtain more testable restrictions and to identify the model :

*Assumption A3 : individual utilities $U_i$ (i=f,m) have the form $U_i(T-L_i,C_i)$, where the fonctions $U_i$ are strictly increasing, strictly concave and infinitely differentiable in their arguments.*

Under A3, program (P1) is equivalent to the existence of some function $\phi(w_f,w_m,Y,s)$, called *sharing rule*, such that each spouse solves the following program (Chiappori, 1992 ; Chiappori, Fortin and Lacroix, 2002) :

(P2) $\max U_i(1-L_i,C_i)$
subject to
$C_i \leq \phi_i + w_i L_i$ , *where* $\phi_f = \phi(w_f,w_m,Y,s)$ *and* $\phi_m = Y - \phi(w_f,w_m,Y,s)$
$0 \leq L_i \leq T$

So the choice of $(L_f,L_m,C_f,C_m)$ is a two-stage decision process :
(1) the couple arranges the sharing rule, which is the result of an unspecified bargaining process,
(2) each spouse separately chooses her (his) labor supply given her (his) personal allocation of nonlabor income.

Labor supplies have then a particular structure :

(1) $$L_f = L_f[w_f, \phi(w_f,w_m,Y,s)]$$
$$L_m = L_m[w_m, Y - \phi(w_f,w_m,Y,s)] ,$$

where $L_i$ is member *i*'s Marshallian labor supply function stemming from $U_i$. This particular structure imposes testable restrictions on the first and second order partial derivatives of the observed labor supplies $L_f(w_f,w_m,Y,s)$ and $L_m(w_f,w_m,Y,s)$. These restrictions take the form of six equations denoted R2 (besides equations R1), plus two inequations which are the equivalent of Slutsky's restrictions in the collective framework. Moreover, the first order derivatives of the sharing rule are identified, so that the sharing rule is defined up to a constant (for more details see Chiappori, Fortin and Lacroix, 2002).

The relevance of assumption A3 is questionable. First, it formally assumes that preferences are egotistic (the individual welfare does not depend on the situation of the spouse), whereas family members generally give support to each other. In fact this is not a problem because the results are extended to "caring" preferences, where the welfare of each member is a function of $U_f$ and $U_m$. What is more embarrassing is that assumption A3 implies that we ignore household public goods and housework. Public goods lead to positive externalities between $C_f$ and $C_m$ ; this is incompatible with A3. And housework made by the wife (husband) during her (his) nonlabor time increases the utility of the husband (the wife), so that $U_m$ may depend on $L_f$ and vice versa. In fact, housework (like public goods) is compatible with assumption A3 if we suppose that it is fixed exogenously before the bargaining of the sharing rule ; it is then embedded in individual preferences. For example, if time spent in housework and its division between spouses are determined by homogeneous cultural rules, individual labor supply choices have no influence on the amount of housework made and on the spouse's utility, so that A3 holds. It is the same if housework and its division depend only on exogenous parameters, like number of children or housing conditions[12]. Hence we exclude couples with young children from the analysis : time spent on childcaring is important and clearly endogenous, depending on the wife's choice to work or to look herself after children.

## 2.2 female nonparticipation in the collective approach

We now examine how the collective model works if we allow for female nonparticipation, while men always participate in the labor market. The set of possible values of $(w_f,w_m,Y,s)$ is partitioned in two sets of female nonparticipation and participation, denoted respectively NP and P, and delimited by a participation frontier. The female wage $w_f$ is not observed in NP, it becomes a potential wage which is supposed to be known by both members of the couple.

Since we have made no assumptions in Participation in A1-A2-A3, the main results of the collective model remain valid if we allow for female (or male) nonparticipation : final decisions $(L_f,L_m,C_f,C_m)$ are the solution of maximizatiin Program (P1) or (P2), a sharing rule $\phi(w_f,w_m,Y,s)$ is defined and the reduced labor supplies take the form (1).

A first consequence of (1) is that $w_f$ influences male labor supply not only in P but also in NP. This is a testable result of the collective model. If the unitary model were used, $w_f$ would influence $L_m$ only in P, because male labor supply would depend on male wage and other household income sources (that is $w_f$ and Y in P, but only Y in NP). In the collective framework, $w_f$ has an impact through the bargaining process : $w_f$ is an argument of $\mu$ in (P1). This impact works everywhere and adds to the direct income effect which works only in P.

Nonparticipation leads to corner solutions of (P1) or (P2). Labor supply functions are generally not differentiable along the participation frontier. Then the testable restrictions (R1) and (R2) and the formulas identifying the derivatives of the sharing rules (cf. Chiappori, Fortin, Lacroix, 2002) become inapplicable on the participation frontier, because all these expressions use the derivatives of labor supplies $L_f$ and $L_m$ ; moreover the derivatives of $L_f$ are set to zero in NP. Further theoretical work is necessary to prove that the sharing rule is identifiable across the participation frontier.

Donni (2003) has examined female and male nonparticipation and established that the sharing rule is identifiable up to an additive constant on the whole space of $(w_f,w_m,Y,s)$ values[13], and that the preferences represented by $U_f$ and $U_m$ are uniquely identified. Thus the collective model is identified (up to a constant) in P and NP, while restrictions (R1) and (R2) remain valid in P. The proof uses a technical assumption :

*Assumption A4 :* $\max_{i=f,m} \left( \mid t_i(w^*_f,w^*_m,Y,s) - t_i(w^\circ_f,w^\circ_m,Y,s) \mid \right) \text{ £ } d.\max_{i=f,m} \left( \mid w^*_i - w^\circ_i \mid \right)$

*for any* $(w^*_f,w^*_m,Y,s)$ *and* $(w^\circ_f,w^\circ_m,Y,s)$,
*with* $d < 1$,

where $t_i(w_f,w_m,Y,s)$ is the marginal rate of substitution between leisure and consumption issued from $U_i$ computed for $L_i=0$ and $C_i=\phi_i(w_f,w_m,Y,s)$ :

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[12] These exogenous variables are then introduced as control variables in individual Marshallian demands.

[13] Distribution factors s are not introduced explicitely, but they would not change the demonstration.

$$t_i(w_f, w_m, Y, s) = -\frac{\partial U_i / \partial L \left(T, \Phi_i(w_f, w_m, Y, s)\right)}{\partial U_i / \partial C \left(T, \Phi_i(w_f, w_m, Y, s)\right)}$$

Assumption A4 is not very restrictive and garantees that there exists a single female reservation wage $\gamma(w_m,y,s)$ for each $(w_m,y,s)$ and that the participation set takes a regular form :

$P = \{ (w_f,w_m,y,s) \,/\, w_f \geq \gamma(w_m,y,s) \}$

In effect, the uniqueness of the reservation wage is not automatically deduced from the collective framework.

Blundell et alii (2001) have examined discrete participation (a member - here the man - chooses to participate or not, without choosing the number of hours worked). They also conclude that the sharing rules and individual utilities are identifiable on the whole space up to an additive constant, provided that technical assumptions are satisfied.

Once the identifiability of the sharing rule established, we now examine how it must be specified in empirical work. Three results arise and will guide parametric specification in §1.3 below :

• function $\phi(w_f,w_m,Y,s)$ is continuous and infinitely differentiable inside P and inside NP. This is the consequence of regularity assumptions included in A2 and A3. These conditions assure that the optimum $(L_f,L_m,C_f,C_m)$ issued from program (P1) is a continuous and infinitely differentiable function of $(w_f,w_m,Y,s)$, at least outside corner solutions. Then by writing :

(2) in NP : $\phi(w_f,w_m,Y,s) = C_f(w_f,w_m,Y,s)$
in NP : $\phi(w_f,w_m,Y,s) = C_f(w_f,w_m,Y,s) + w_f.L_f(w_f,w_m,Y,s)$

it is clear that $\phi(w_f,w_m,Y,s)$ is regular inside each set.

• function $\phi(w_f,w_m,Y,s)$ is continuous along the participation frontier. As $\mu$ is a continous function of $(w_f,w_m,Y,s)$ (assumption A2), like $U_f$ and $U_m$ (cf. A3), and because of the strict concavity of $U_f$ and $U_m$ (cf. A3), it is easy shown that optimum $(L_f,L_m,C_f,C_m)$ issued from program P1 is continuous even for corner solutions. So $C_f(w_f,w_m,Y,s)$ and $L_f(w_f,w_m,Y,s)$ are continuous along the frontier, with of course $L_f(w_f,w_m,Y,s)=0$ on the frontier. Then according to (2), $\phi(w_f,w_m,Y,s)$ is continous along the frontier.

• function $\phi(w_f,w_m,Y,s)$ is generally not differentiable along the participation frontier, so that a possible change of gradient has to be specified. Simple counter-examples can be drawn from the maximization of (P1) when direct utilities $U_i(T\text{-}L_i,C_i)$ are specified[14]. Intuitively, this change of gradient may be interpreted as follows. When she receives an higher amount $\phi$, a working wife increases her welfare both by increasing her consumption (labor supply being unchanged) and by optimizing her consumption-leisure trade-off, while a nonworking wife can only increase her consumption. The welfare gain

[14] A Cobb-Douglas form $U_i(C_i,1\text{-}L_i) = a_i.\log(C_i\text{-}c_i°)+(1\text{-}a_i).\log(1\text{-}L_i)$ leads to the following sharing rule :
*in NP :* $f(w_f,w_m,Y,s) = \mu.Y + \mu.[w_m\text{-}c_m°] - (1\text{-}\mu).[w_f\text{-}c_f°]$
*in NP :* $f(w_f,w_m,Y,s) = c_f° + a_f.[l/(1+ a_f.l)].[Y+ w_m\text{-} c_f°\text{-}c_m°]$ *with* $l=\mu/(1\text{-}\mu)$
so that the derivatives $\partial\phi/\partial Y$, $\partial\phi/\partial w_f$, $\partial\phi/\partial w_m$ are clearly different on both sides of the participation frontier. If $\mu$ is a constant (unitary case), these partial derivatives are constants which take different values in P and NP (provided $a_f<1$). In the collective framework $\mu$ is a function $\mu(w_f,w_m,Y,s)$, so that terms in $\partial\mu/\partial Y$, $\partial\mu/\partial w_f$, or $\partial\mu/\partial w_m$ are added to theses constants ; according to (A2) these terms are continuous across the frontier.

associated to an increase of $\phi$ is then expected to become lower in NP than in NP[15]. According to (P1) and (P2), the couple chooses $\phi$ in order to maximize :

$V=\mu V_f(w_f, f) + (1-\mu)V_m(w_m, Y-f)$ ,

where $V_i(w_i, f_i)$ is the indirect utility function linked to preferences $U_i$. When the wife stops to work, the fall of the welfare gain $\partial V_f/\partial\phi$ means that it becomes suddenly less efficient to increase the wife's allocation $\phi$ (and relatively more efficient to increase the husband's allocation Y-$\phi$). Thus, when crossing the frontier from (P) to (NP), the derivatives $\partial\phi/\partial Y$ or $\partial\phi/\partial w_m$ are expected to fall.

### 2.3 Parametric specification of the model

*Assumptions on functional forms*

We choose a semilog specification for the Marshallian labor supplies and the sharing rule, following Chiappori, Fortin and Lacroix (2001). First we assume that the sharing rule takes the form :

(3) in NP : $f = K_0 + K_1 \log(w_f) + K_2 \log(w_m) + K_3 \log(w_f).\log(w_m) + K_4 Y + s'.K_5 = X'K$ ,

in NP : $f = k_0 + k_1 \log(w_f) + k_2 \log(w_m) + k_3 \log(w_f).\log(w_m) + k_4 Y + s'.k_5 = X'k$ ,

*where we denote* $X' = (1, \log(w_f), \log(w_m), \log(w_f).\log(w_m), Y, s)$ *and* $s'=(s_1,s_2,\ldots,s_p)$.

As in Donni (2003), the sharing rule takes different values in NP and (NP), in order to allow for nondifferentiability along the participation frontier. The continuity of the sharing rule imposes further restrictions on K and k, as we will see below. Second, we assume that the Marshallian individual labor supplies associated with $U_f$ and $U_m$ take the form :

(4) $L_f = \max(0, L^*_f)$, *where*

$L^*_f = a_0 + a_1 \log(w_f) + a_2 f$

$L_m = b_0 + b_1 \log(w_m) + b_2 (Y-f)$

Observable and unobservable heterogeneity of preferences are included in parameters $a_0$ and $b_0$ :

(5) $a_0 = a^*_0 + Z'_f.a_3 + u_f$

$b_0 = b^*_0 + Z'_m.b_3 + u_m$

*where* $Z_i$ *is a vector of exogeneous variables*

*and* $\begin{pmatrix} u_f \\ u_m \end{pmatrix} \rightarrow N\left(0, \begin{pmatrix} s^2_f & r.s_f.s_m \\ r.s_f.s_m & s^2_m \end{pmatrix}\right)$

so that we correlation between unobservable factors influencing male and female preferences are correlated.

[15] In fact, $\partial V_f/\partial\phi$ is continous along the frontier. As $V_f$ is concave in $\phi$, $\partial V_f/\partial\phi$ decreases with $\phi$. What is expected is a discontinuity of $\partial^2 V_f/\partial^2\phi$: when we cross the frontier from (P) to (NP), the decrease of $\partial V_f/\partial\phi$ becomes suddenly faster.

*"Reduced form" of the model in (P)*

From (3) and (4) we obtain labor supplies on the participation set :

(6) $L_f = f_0 + f_1 \log(w_f) + f_2 \log(w_m) + f_3 \log(w_f).\log(w_m) + f_4 Y + s'.f_5$
$= X'.f$
$L_m = m_0 + m_1 \log(w_f) + m_2 \log(w_m) + m_3 \log(w_f).\log(w_m) + m_4 Y + s'.m_5$
$= X'.m$

where :

| | | |
|---|---|---|
| *(6.0)* | $f_0 = a_2K_0 + a_0$ | $m_0 = -b_2K_0 + b_0$ |
| *(6.1)* | $f_1 = a_2K_1 + a_1$ | $m_1 = -b_2K_1$ |
| *(6.2)* | $f_2 = a_2K_2$ | $m_2 = -b_2K_2 + b_1$ |
| *(6.3)* | $f_3 = a_2K_3$ | $m_3 = -b_2K_3$ |
| *(6.4)* | $f_4 = a_2K_4$ | $m_4 = b_2(1-K_4)$ |
| *(6.5)* | $f_5 = a_2K_5$ | $m_5 = -b_2K_5$ |

Reduced parameters f and m are identifiable from the observation of labor supplies in NP. Structural parameters a, b and K can then be recovered from estimated values of f and m : first we obtain $a_2/b_2$ by (6.3) ; then (6.4) gives us $K_4/(1-K_4)$, so that we obtain $K_4$, $a_2$ and $b_2$ ; finally $K_1$, $K_2$, $K_3$ and $K_5$ are recovered from (6.1) to (6.3) and (6.5). However, (6.0) allows only to recover $a_0$ and $b_0$ up to constant $K_0$ : we recognize that the sharing rule and the individual labor supplies are identified up to an additive constant. Note that the estimation of parameters $f_3$ and $m_3$ (terms in $\log(w_f).\log(w_m)$) and of parameters $f_4$ and $m_4$ (terms in Y) is crucial for the identification of structural parameters. On the contrary, the distributions factors appear only in (6.5) and are not indispensable for identification of $K_1$-$K_4$.

*Testable restrictions*

From (6.3) and (6.5), we also obtain testable restrictions on reduced parameters f and m :

*(7a)* $f_3/m_3 = f_5/m_5 = -a_2/b_2$

As s is a vector, $f_5$ and $m_5$ are vectors denoted $(f_{5,1},...,f_{5,p})$ and $(m_{5,1},...,m_{5,p})$, so that (7a) must be verified for each component and can be rewritten :

(7b) $f_3/m_3 = f_{5,1}/m_{5,1} = ... = f_{5,p}/m_{5,p}$

Each distribution factor generates an identity to test. In a model without distribution factor, there is no restriction to test. Restrictions (7) are equivalent to restrictions (R1) and (R2) in our parametric specification and allow to test the collective model with separable utilities. Note that, from the six equations (R2), only one equation subsists ; the other equations vanish because all cross derivatives are null in our parametric form, except $\partial^2 L_i/\partial w_f \partial w_m$.

*Slutsky's restrictions*

Slutsky's conditions on individual labor supplies :
$\partial\Lambda_i / \partial w_i$ - $L_i . \partial\Lambda_i / \partial\Phi_i$

take here the form :

$$\frac{a_1}{w_f} - a_2 L_f \geq 0 \quad \text{and} \quad \frac{b_1}{w_m} - b_2 L_m \geq 0$$

*The participation frontier*

From (6) we obtain the equation of the participation frontier :

*(8a)* $X'f = 0$

which can also be written :

$$(8b) \quad \log(w_f) = -\frac{f_0 + f_2 \log(w_m) + f_4 Y + s'.f_5}{f_1 + f_3 \log(w_m)}$$

Provided $f_1 + f_3.log(w_m)$ ¹ $0$ , the reservation wage is uniquely defined, so that assumption A4 is not needed.

*Continuity of the sharing rule along the participation frontier*

On the frontier, both expressions of the sharing rule in (3) are identical :

*(9)* $X'f = 0$ Þ $X'K = X'k$

(9) is verified for every X, so that the hyperplan orthogonal to f is included in the hyperplan orthogonal to (k-K). Vectors f and k-K are then colinear :

$k - K = a.f$

Thus taking into account female nonparticipation needs only one structural parameter $\alpha$ in addition to K, a and b.

Reciprocally, if the sharing rule in NP takes the form

*(10)* *in NP :* $f = X'K$
*in NP :* $f = X'(K+a.f)$,

then the continuity along the participation frontier $X'f = 0$ is assured, with a change of gradient if $\alpha \neq 0$.

*"Reduced form" of the model in NP and additional testable restriction*

From (4) and (10) we obtain labor supplies on the nonparticipation set :

$$(11)\quad L^*_f = X'.f\,.(1 + a_2.\mathbf{a}\ )$$
$$L_m = X'.m - X'f\,.(b_2.\mathbf{a}\ )$$

The latter equation introduces a parameter $\beta = b_2.\mathbf{a}$ , identifiable by the observation of male labor supply in NP.
The former equation imposes a further testable restriction. In effect, we must have $L^*_f \leq 0$ in NP. By (6), we also have X'f ≥ 0 in NP, so that X'f ≤ 0 in NP. Then we must have :

$$(12a)\qquad 1 + a_2.\mathbf{a}\ ³\ 0$$

or equivalently :

$$(12b)\qquad 1 + (a_2/b_2)\mathbf{b}\ ³\ 0$$

$$(12c)\qquad 1 - (f_3/m_3)\mathbf{b}\ ³\ 0$$

Restriction (12) is another consequence of the functional form (1) and has the same status as restrictions (7).

Consider now the limiting case : $1 + (a_2/b_2)\mathbf{b} = 0 \quad \Leftrightarrow \quad \mathbf{b} = -b_2/a_2 = m_3/f_3$
By (11) we have $L^*_f = 0$ in NP. By (4) it means that we have in NP :

$$(13)\quad f = -[a_0 + a_1 \log(w_f)] / a_2$$

The right side of expression (13) can be interpreted as the "reservation sharing rule" of the wife. In effect, given preferences $U_f$, it is possible to define a reservation sharing rule $f_0(w_f)$ : $f_0(w_f)$ is the minimal nonlabor allocation $\phi$ such that a wife earning $w_f$ stops to participate[16]. The right side of (13) corresponds to the reservation sharing rule associated with female preferences described by (4). In the general case $1 + (a_2/b_2)\mathbf{b} > 0$, the sharing rule is equal to the reservation sharing rule on the participation frontier, while it is greater inside (NP). In the particular case $\mathbf{b} = b_2/a_2$, the sharing rule remains equal to the reservation sharing rule in the whole set (NP). As a result, the sharing rule in NP does not depend on $w_m$, Y and s : once the wife stops to work, all additional household ressource $\partial Y$ or $\partial w_m$ is left to the husband, whatever environment factors s.

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[16] We suppose that leisure is a normal good to women ($a_2 < 0$).

*"Reduced form" of the model*

Finally, from (5), (6) and (11) we obtain the estimable model :

$$(14)\quad \text{if } L^*_f \geq 0 : \quad L_f = L^*_f, \quad L_m = L^*_m$$

$$\text{if } L^*_f \leq 0 : \quad L_f = 0, \quad L_m = L^*_m - b.L^*_f$$

*where :*

$$L^*_f = X'.f = f^*_0 + f_1 \log(w_f) + f_2 \log(w_m) + f_3 \log(w_f).\log(w_m) + f_4 Y + s'.f_5 + Z'_f.a_3 + u_f$$

$$L^*_m = X'.m = m^*_0 + m_1 \log(w_f) + m_2 \log(w_m) + m_3 \log(w_f).\log(w_m) + m_4 Y + s'.m_5 + Z'_m.b_3 + u_m$$

*and :*

$$\begin{pmatrix} u_f \\ u_m \end{pmatrix} \rightarrow N\left(0, \begin{pmatrix} s^2_f & r.s_f.s_m \\ r.s_f.s_m & s^2_m \end{pmatrix}\right)$$

## 3. Data and estimation

### 3.1 Source and sample

The data we use to estimate the model must include measures of labor supplies, wages, and nonlabor household incomes (variable Y). Indeed, as we see in equations (6.0)-(6.5), the estimation of parameters $f_4$ and $m_4$ (terms in Y) is crucial for the identification of structural parameters. Few data sources provide a good measure of Y. That is why we have chosen to use the French data of the European Community Household Panel. This source includes a comprehensive measure of household resources. Moreover it is a panel. Our static model needs only cross-section data, but the longitudinal dimension is useful to measure components of Y which are related to savings (see below).

The French panel has followed each year over the period 1994-2001 an initial sample of individuals in their successive households. 5953 different couples (married or not) have been observed at least once[17]. Our sample consists in 2841 couples entering at least once in our field (see definion below). Although each couple has been observed up to 8 times, we retain only one observation per couple, in order to obtain independent observations and to handle residuals ($u_f$,$u_m$) as independent and identically distributed. This observation is randomly selected, but if possible we eliminate observations corresponding to the year following the union of the couple or the year preceding its separation : assumption A1 is more likely to be satisfied in couples whose members have been living together for a long time (repeated games) and get along (willingness to cooperate).

[17] 121 individuals (61 men and 60 women) have been observed with two or even three successive spouses during the 8 years. The two or three resulting couples are treated as different couples. However, if a couple separate and later join together again, it is treated as the same couple.

*Definition of the field :*

A couple is selected if :

- both spouses may participate in the labor market, according to their age and status : people excluded are students (initial studies only) ; conscripts for national military service ; persons over 65 years old ; retired or early retired persons (they generally are not allowed – or only partially allowed - to draw simultaneously their pension and a salary, and they do it rarely).
- both spouses are salaried (if working). Self-employed people are excluded because it is difficult to identify separately male and female earnings when both spouses work together.
- the couple has no children under 6 years old. As discussed in §2.2, assumption A3 is less credible for couples with young children. The exclusion of couples with young children does not reduce much the size of the sample : thanks to longitudinal data, most couples have been interviewed at least once either before the birth of the children or after the sixth birthday of the youngest child.
- the husband is participating in the labor market (in the sense that he has worked at least once during the 12 last monthes). Without this last condition, we would observe in our field a male nonparticipation rate of 6,3%. Nonparticipating men are considered either as long term unemployed (two third of them) or inactive. Half of them are suffering from a long illness or a handicap, so that their ability to work is questionable.

The female nonparticipation rate reaches 23,0% in our sample (table 1). Most nonparticipating women are inactive without any handicap and consider themselves as “housewives”.

## 3.2 Definition of variables

*Labor supplies and wages*

Labor supplies $L_f$ and $L_m$ are defined as the annual hours of work, computed as follows[18] :
*(45/12) x (number of monthes in employment during the 12 last monthes) x (weekly hours of work)*
If the individual has a secondary occupation, hours in primary and secondary occupations are added. Because the questionnaire asks how many hours are actually worked, weekly hours declared can exceed greatly legal hours (35-39 hours per week) and legal overtime[19]. As shown in graph 1, male annual hours are continuously distributed around full-time employment (1750-1800 hours per year), with a sufficient variability to allow for estimations ; but few men work less than 1500 hours, whereas female labor supply takes all values between 0 and 1500.

The measure of the wage rate is the average hourly earnings, defined by dividing total annual labor income over annual hours of work.

*Nonlabor income*

The nonlabor income variable includes miscellaneous elements and the way we define and construct it may influence the results. Variable Y is here computed as $Y_1 + Y_2 - Y_3$, taking into account pensions ($Y_1$), income from household assets ($Y_2$), and household savings ($Y_3$). As in Chiappori, Fortin and Lacroix (2002), we remove savings in order to be consistent with an intertemporally separable life

[18] 12 monthes of employment correspond to 45 working weeks, once all holidays deducted.
[19] Extreme values of labor supply have been truncated at 3000 hours per year.

cycle model involving a two-stage budgeting process. In the first step, the couple optimally allocates life cycle wealth over each future period and chooses the amount of savings. This determines a budget constraint for the present period, where nonlabor income disposable for consumption is net of savings. In the second step, present consumptions and labor supplies are chosen conditional on this budget constraint.

We have not included any benefit in Y (like unemployment benefits, housing or family allowances, or social aid), because these benefits are generally means tested and directly depending on the labor supply choices. For the same reasons, taxes are not taken into account. Our empirical approach ignores then the tax and redistribution system. For this we would need to compute marginal tax rates for each individual in order to obtain net wages and simulate the nonlabor income in case of nonparticipation.

The term $Y_1$ includes all types of pensions which do not depend on the present choices of the household : retirement pensions[20], disability benefits, alimonies from ex-spouses, and sums received from other households[21].

The term $Y_2$ is obtained from a variable measuring total household assets[22]. To impute income from assets, we use an interest rate of 7,5%, which is the average rate of real capital gains observed during the period 1994-2001 in France (Insee, 2001). Alternative rates (5% and 10%) have been tested and does not change much the results.

The savings $Y_3$ are estimated for each couple from the total household assets variable A. The variable $Y_3$ is computed as $A_t - A_{t-1}$, by using the observation of A at the previous wave of the panel. An alternative computation has been tested, using all available observations of the couple during the period 1994-2001. We assume that household saves each year the same amount S, so that we write $A_t = A_{t°} + S.(t - t°) + \varepsilon_t$ . Provided at least two observations of total assets are available, this simple regression model (estimated separately for each couple) gives us an estimation of the savings of the household. This alternative computation leads to a less noisy savings variable, but this smoothing does not change much the final results because of the instrumentation (see below).

*Distribution factors*

Two distribution factors (vector s) have been introduced : the divorce rate and the sex-ratio, observed at the department level at the current year. The divorce rate comes from justice ministry statistics, while sex-ratio is computed from Insee demographic data “estimations localisées de population”. The divorce rate is the ratio “annual number of divorces in the departement” on “population of the department”. Its variability on the territory is sufficient, although lower than thirty years ago (Baillon et alii, 1981). It varies from less than 0.10% in some country areas (Massif Central) and in the West (traditionaly more catholic), to more than 0.25% in Paris and along the Mediterranean coast (graph 3).

The sex-ratio variable is computed for each household. We have tested two alternative formulas :

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[20] It concerns only some specific categories of workers who are allowed to perceive their retirement pension early in the life cycle and to take another job (ex-members of the army, civil servants who are mother of three children…)

[21] It concerns mainly young couples who are receive money from their parents.

[22] Total household assets includes housing for owner-occupiers. Because housing does not provide any monetary income, it is removed from total assets. To do this, total assets of owner-occupiers have been simply divided by 3, because specific sources on assets (*enquête Patrimoine, Insee*) show that the share of housing in total assets is around 2/3 on average (Insee, 1999). We should construct a better measure in later versions of this paper : due to lack of time, we have not used all information on assets contained in the data source.

$$SR_1 = \frac{N_m(age_m)}{N_f(age_f)} \quad \text{or} \quad SR_2 = \frac{N_m(age_f + 2)}{N_f(age_m - 2)}$$

where $age_f$ and $age_m$ are both spouses's ages, and $N_s(a)$ means the number of persons of sex $s$ and age $a$ in the department population. In both cases, the sex-ratio measures the relative scarcity of women in the local population. Formula $SR_1$ supposes implicitely that in case of separation the husband would search a woman who has the same age as his current wife, while the wife would search a man who has the same age as her current husband. Formula $SR_2$ is based on the fact that men generally choose a younger wife while women choose an older husband : the mean of variable $d=age_m-age_f$ is equal to 2 years. We then suppose that the husband would search for a woman who is 2 years younger than him, while the wife would search a man who is two years older than her. Both formulas seem relevant, but both have the drawback to be strongly correlated with d (when the husband is much older than his wife, $SR_1$ is low while $SR_2$ is high). Thus the sex-ratio effect may hide an effect of the spouses's relative age. We use then a mixed formula for the results presented in section 4 :

$$SR_c = \log\left(\sqrt{SR_1.SR_2}\right)$$

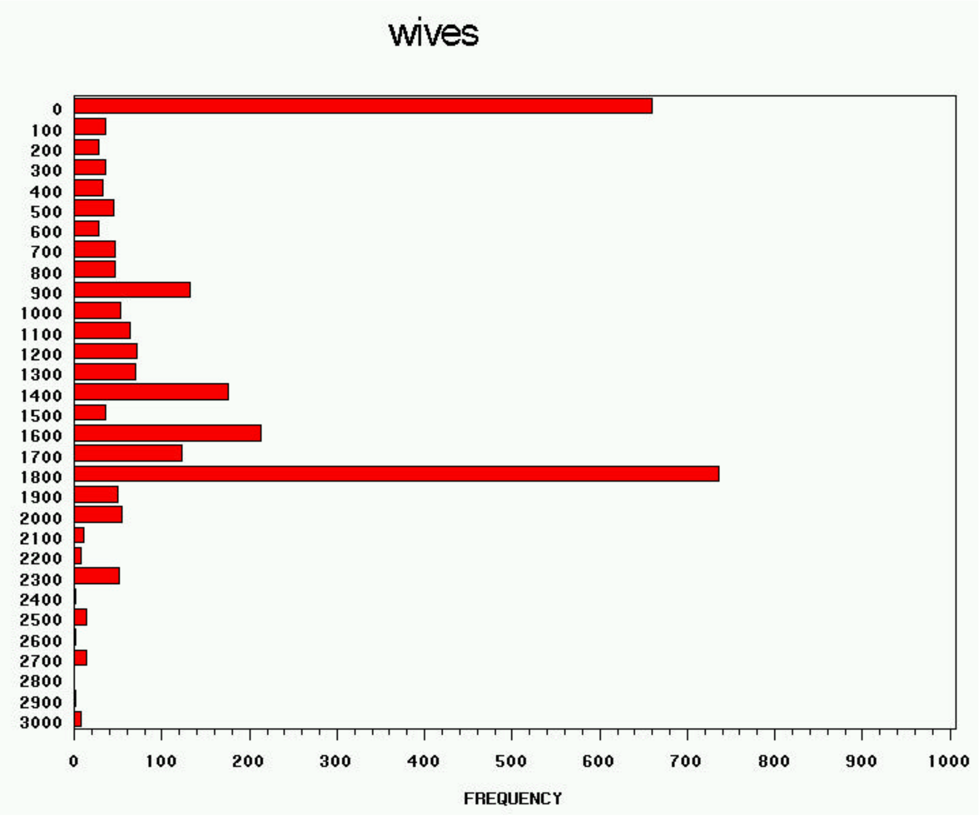
Using the log avoids dissymetric extreme values of the sex-ratio. Variable $SR_c$ is not correlated with d. It is still correlated with $age_f$ or $age_m$, but this is not a problem because the age is introduced in the model as a control variable influencing the individual labor supply.

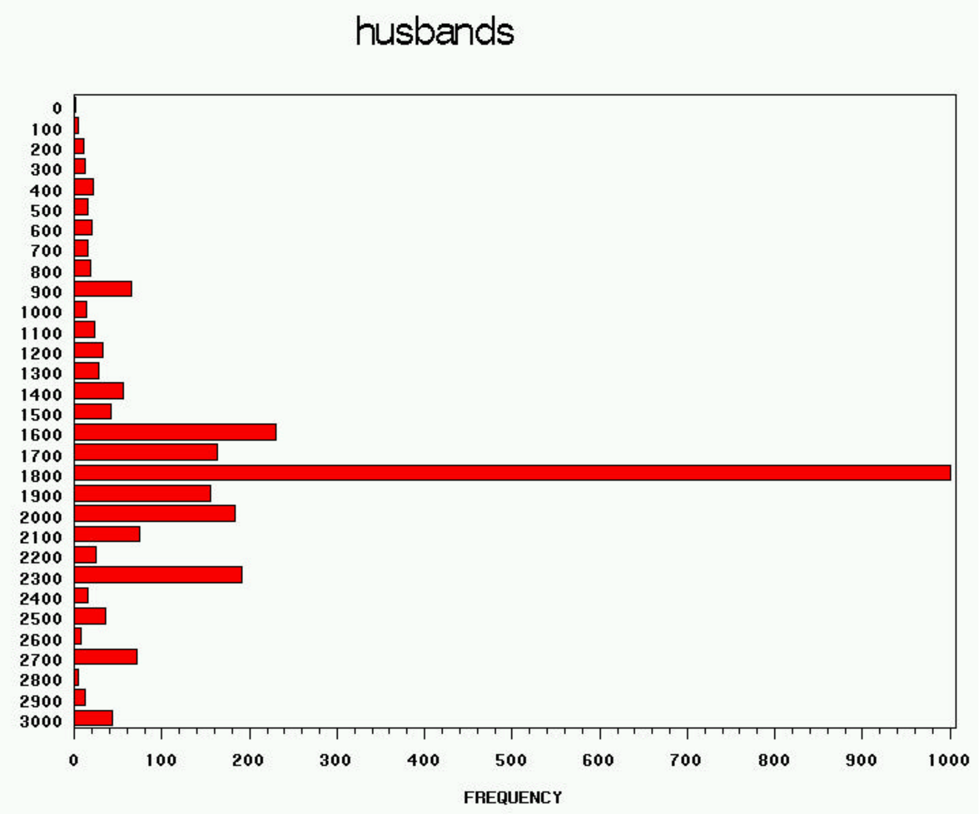
**Table 1 – Descriptive statistics**

| | Mean | Standard deviation | P01 | P10 | Median | P90 | P99 |
|---|---|---|---|---|---|---|---|
| | | | Percentiles | | | | |
| **Female participation rate** | 0.770 | | | | | | |
| **$L_f$** if participating*(annual hours)* | 1446 | 524 | 120 | 675 | 1620 | 1845 | 2700 |
| **$L_m$***(annual hours)* | 1778 | 454 | 330 | 1350 | 1755 | 2250 | 3150 |
| **$w_f$** if participating *(Francs per hour)* | 63 | 32 | 27 | 34 | 55 | 99 | 194 |
| **$\log(w_f)$** if participating | 4.05 | 0.42 | 3.30 | 3.53 | 4.01 | 4.60 | 5.27 |
| **$\log(w_f)$** instrumented | 4.09 | 0.28 | 3.61 | 3.79 | 4.04 | 4.49 | 4.84 |
| **$w_m$** *(Francs per hour)* | 75 | 41 | 29 | 40 | 64 | 123 | 230 |
| **$\log(w_m)$** | 4.21 | 0.45 | 3.37 | 3.69 | 4.16 | 4.81 | 5.44 |
| **$\log(w_m)$** instrumented | 4.25 | 0.32 | 3.66 | 3.89 | 4.18 | 4.73 | 5.11 |
| **Y** *(thousands Francs)* | -16.319 | 53.267 | -129.29 | -77.991 | -20.099 | 48.385 | 126.445 |
| **Y** instrumented *(thousands Francs)* | -16.580 | 26.834 | -71.052 | -48.059 | -17.453 | 13.183 | 55.221 |
| **Divorce rate** *(%)* | 0.19 | 0.03 | 0.12 | 0.15 | 0.19 | 0.22 | 0.26 |
| **sex-ratio** | -0.021 | 0.060 | -0.170 | -0.099 | -0.019 | 0.051 | 0.130 |
| **number of children** | 1.17 | 1.21 | 0 | 0 | 1 | 3 | 4 |
| **$age_f$** | 39 | 10 | 21 | 25 | 40 | 52 | 59 |
| **$age_m$** | 41 | 10 | 23 | 27 | 43 | 54 | 60 |

For variables $\log(w_f)$, $\log(w_m)$, and Y, two distributions are presented : the initial distribution (on the subsample excluding missing values) and the distribution of predicted values after instrumentation (see §3.3).

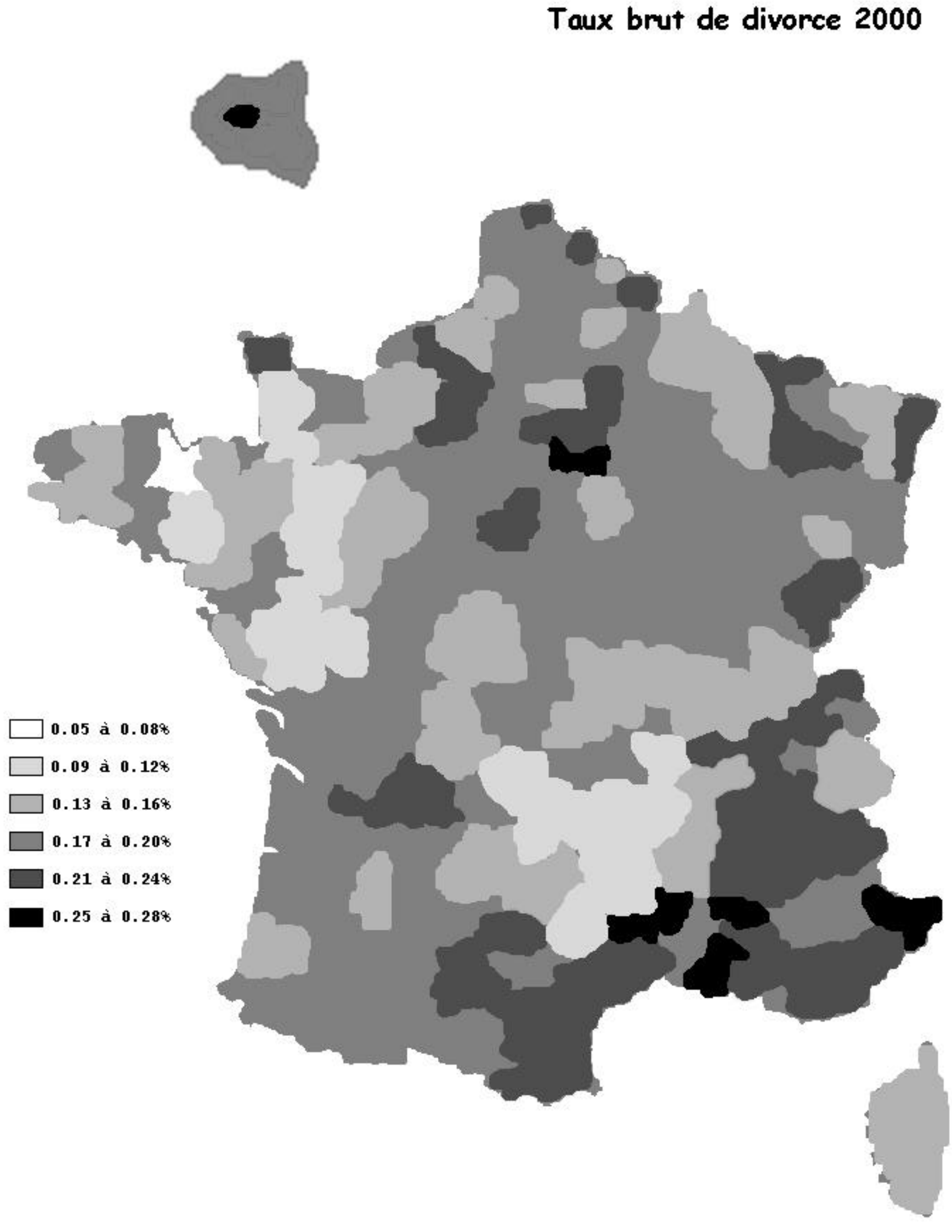
**Graph 1 – Female and male labor supply : distribution of annual hours**




Size of the sample : 2841 couples

**Graph 2 – Divorce rate by French department**




Divorce rate = annual number of divorces / population

### 3.3 Estimation method

As in Blundell et alii (2001), we use a two-step estimation method. First we impute instrumented values of $\log(w_f)$, $\log(w_m)$, and Y, for all couples in our sample. In the second step we include those values in the labor supply model (14), which is estimated by the method of maximum likelihood.

As female wage is concerned, the first step is indispensable because $w_f$ is not observed on the nonparticipation set. Female wages are then imputed using the Heckman's method to correct the selectivity bias due to nonparticipation : first we estimate a probit model of female participation on the instruments, then we construct the inverse Mill's ratios for each women, and we include it in the instrumental equation explaining $\log(w_f)$ by the instruments on the participation set.

We have three other reasons to instrument wages and nonlabor income. First, there are missing values that would reduce our sample if we did not make imputations. Second, the variables $w_f$, $w_m$, and Y are measured with error. The error is particularly important for savings components of nonlabor income. Third, from a theoretical point of view, these variables are endogenous : individuals who work a lot may have unobservable characteristics influencing also their hourly wage or their savings behaviour.

We use a large set of instruments : male and female characteristics (age, age², education level, age interacted with education level, father's occupation[23], variable mixing nationality and place of birth) ; household characteristics (number of children, dwelling status[24], type of geographic area[25]) ; distribution factors (divorce rate, sex-ratio) ; and finally the year of observation (see the complete list of instruments and their effects on endogeneous variables on table 2). All instruments refer either to permament individual characteristics (date of birth, education level, origins) or to choices generally made by the household before the current year (choice of geographic location, housing purchase, number of children). Hence our instruments are not caused by current labor supply choices. However this is not sufficient to garantee the exogeneity of all instruments : some unobservable factors may be correlated with labor supply and with some instruments. We then assume the exogeneity of our instruments, following the usual practices (Mroz, 1987) and previous work on collective models (Moreau, 2000 ; Chiappori, Fortin, Lacroix, 2002)[26].

Vector $Z_i$ (i=f,m) of exogeneous control variables in (14) includes $age_i$, $age_i^2$, number of children, and type of geographic area.

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[23] Mother's occupation was also tested and eliminated because it has no significant effect on $\log(w_f)$, $\log(w_m)$, and Y.

[24] owner versus tenant

[25] rural/small town/middle town/large town/Paris and suburb

[26] An instrument is debatable : the number of children. Although this variable seems exogeneous a priori, Moreau (2002) showed that some results are sensible to the inclusion of this variable in the list of instruments, and finally obtained better results by treating it as endogeneous. For the moment, we have included it in the list, but variants should be tested in later versions of this paper.

**Table 2 - Regressions on the instruments** (OLS) **: Significancy Fisher test**

| instruments | degrees of freedom | logWm | | LogWf | | Y | |
|---|---|---|---|---|---|---|---|
| | | F Value | sign. | F Value | sign. | F Value | sign. |
| **Dwelling status** | 1 | **45.64** | *** | **14.88** | *** | **298.17** | *** |
| **year** | 7 | **4.15** | *** | **2.13** | ** | **28.03** | *** |
| **$age_m$** | 1 | **37.02** | *** | **0.01** | | **35.54** | *** |
| **$age_m$*$age_m$** | 1 | **5.23** | ** | **0.57** | | **36.57** | *** |
| **$age_f$** | 1 | **13.02** | *** | **61.69** | *** | **18.09** | *** |
| **$age_f$*$age_f$** | 1 | **0.13** | | **0.54** | | **26.27** | *** |
| **$education_m$** | 12 | **30.26** | *** | **2.74** | *** | **1.00** | |
| **$education_f$** | 12 | **0.90** | | **9.86** | *** | **1.67** | * |
| **$age_m$*$education_m$** | 12 | **3.56** | *** | **1.05** | | **0.90** | |
| **$age_f$*$education_f$** | 12 | **1.36** | | **4.50** | *** | **1.90** | ** |
| **father's $occupation_m$** | 11 | **1.35** | | **1.65** | * | **0.88** | |
| **father's $occupation_f$** | 11 | **5.12** | *** | **1.11** | | **2.03** | ** |
| **nationality and place of $birth_m$** | 8 | **2.14** | ** | **0.70** | | **0.33** | |
| **nationality and place of $birth_f$** | 8 | **1.70** | * | **1.07** | | **0.69** | |
| **type of area** | 4 | **13.40** | *** | **7.19** | *** | **16.14** | *** |
| **number of children** | 4 | **3.91** | *** | **1.69** | | **1.12** | |
| **divorce rate** | 1 | **17.67** | *** | **0.35** | | **0.33** | |
| **sex-ratio** | 1 | **0.00** | | **1.70** | | **0.53** | |
| *Mills ratio* | | | | **0.90** | | | |
| $R^2$ | | 0.517 | | 0.468 | | 0.288 | |

global significancy of the instrument : *** at 1% level ; ** at 5% level ; * at 10% level

## 4. Empirical results

### 4.1 Model without distribution factors

First we estimate the model without distribution factors. As we noticed before, distribution factors are not necessary to identify the structural model : the interacted term in $\log(w_f).\log(w_m)$ is sufficient for identification[27]. But then restriction (7) vanishes so that we can not test the collective model.

The observable model has been estimated without imposing restriction (12) on parameter $\beta$. The estimated $\beta$=0.192 is smaller that $m_3/f_3$=136/504=0.270, so that (12) is satisfied. Results are presented in table 3. The wife's parameters are all very significant. Taking into account the woman'decision to participate or not helps us to obtain a precise estimation. On the contrary, most husband's parameters are not significant : male labor supply depends significantly on $w_f$, $w_m$, and Y ; but the interacted term $\log(w_f).\log(w_m)$ is too demanding and makes parameters in $w_f$ not very significant. The male parameter in Y is unexpectedly positive : leisure appears to be an inferior good to men. This feature of French data has already be noticed on participating couples by Moreau (2000). Residuals of the two labor supplies equations are positively correlated : men who works much live generally with women working much. This feature was already noticed on French participating couples by Fermanian and Lagarde (1998).

Structural parameters are then recovered, using Delta method in order to compute standard deviations (table 4). Estimates of the Marshallian labor supply confirm that leisure is an inferior good to men ($b_2>0$), whereas it is a normal good to women ($a_2<0$). Because $a_1>0$ and $a_2<0$, Slutsky's restrictions (see §2.3) are globally satisfied for women. For men they are only locally satisfied : $b_1>0$ but $b_2>0$. The male Slutsky'condition can be rewritten $L_m w_m \leq b_1/b_2$, so it is satisfied for all men whose annual salary is lower than a certain amount (here 192.000 F). In our sample 85% of men are under this threshold[28].

The parameters of the sharing rule are significantly different from zero. In the participation set, we obtain a quasi-symmetric sharing rule, with $K_4 \approx 0.5$. An increase in nonlabor income is equitably shared between working spouses. The effects of $w_f$ and $w_m$ for common values of wages are shown in graph 3. An increase in male earnings lead to a better wife's allocation, whereas an increase in female earnings can lead either to a better husband's allocation (case of low male wage) or to the contrary (case of high male wage). When spouses are altruistic (caring) to each other, they tend to share their ressources : an increase in one's wage lead to an increase in the other's allocation. But when bargaining is prevailing over altruism, an increase in her wage gives her a better bargaining position and allows her to obtain more nonlabor income from her husband (in addition to her growing salary).

Such effects had already appeared in previous work on participating couples. What is new here is the clear switch of regime when the wife stops to work. In the nonparticipation set, the sharing rule is dissymetric : an increase in nonlabor household income is now mainly attributed to the husband ($k_4$=0.14). An increase in his wage is almost entirely kept by the husband, while the wife's allocation increases only slightly (graph 3). Once the wife stops to work, it becomes more difficult for her to obtain a higher personal income when household resources (Y or $w_m$) grow. Hence the direction of the change of gradient is consistent with our theoretical discussion (section 1.2). The direction is also the same as in Bloemen's (2004) empirical work on Dutch couples.

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[27] In fact, identification is based on the second derivatives $\partial^2 L_i / \partial w_f \partial w_m$ of labor supply functions.

[28] The threshold is a little higher in the model with distribution factors presented in §4.2 (231000 F) ; Slutsky's conditions are then satisfied for 90% of men.

Parameter $\beta$ summarizes the change of gradient. $\beta$ is positive, which corresponds to the direction of change described above. We find $\beta$=0.192. This value is significantly different from zero (case where there would be no change of gradient), which confirms the importance of the change. But $\beta$ is not significantly different from the maximum authorized value $m_3/f_3$=0.270 (limiting case described in section 1.3, where $\phi$ would remain at the reservation sharing rule of the wife). When she does not work, the wife receives more than her reservation sharing rule, but not much more. Because her reservation sharing rule is an increasing function of her potential wage ($a_1/a_2<0$), the wife's allocation is increasing with her wage. This seems logical : when she does not work, she does not contribute to the household income, so that the female potential wage has a pure bargaining effect. We notice on graph 3 that the slope $\partial\phi/\partial w_f$ is always higher in the nonparticipation set (pure bargaining effect) than in the participation set (altruism and bargaining mixed).

### 4.2 Model with distribution factors

When distribution factors are used, we have the choice to estimate the reduced model with or without imposing restrictions (7), so that we can test the restrictions imposed by the collective model with separable utilities.

In the unrestricted model (table 5), we find that the divorce rate and the sex-ratio have significant effect on female labor supply, but they have no effect on male labor supply. Thus, although ratios $m_3/f_3$ (cross-wages parameters), $m_{5.1}/f_{5.1}$ (divorce rate parameters), and $m_{5.2}/f_{5.2}$ (sex-ratio parameters) are not of the same sign, restriction (7) is accepted because those ratios are not significantly different from zero. Note that the effect of the divorce rate is not very significant even for women, so that this distribution factor seems not very conclusive.

When we impose (7) by leaving parameter $\beta$ free, we find $\beta \approx 0.2$ (like in the model without distribution factors), but the common ratio $m_3/f_3=m_{5.1}/f_{5.1}=m_{5.2}/f_{5.2}$ is now estimated at round 0.05. Then restriction (12) is not satisfied anymore. We must then impose it. Results of the restricted model are presented in table 6. The parameter values that maximize the likelihood function subject to restrictions (7) and (12) verify then $1- (f_3/m_3)b = 0$ : we are in the limiting case where the sharing rule in NP is set on the wife's reservation sharing rule. The consequences appear clearly on table 7 and graph 3 : when the wife is not working, her allocation depends only on her potential wage.

Being in the limiting case $\beta=m_3/f_3$ question the relevance of the collective model with separable utilities. The restrictions implied by this model are not formally rejected. If we test the restricted model against the unrestricted model by the likelihood ratio test, restrictions (7) and (12) are accepted at usual thresholds[29]. Moreover, the limiting case is not totally incompatible with our theoretical model : if the female preferences are such that her marginal utility of consumption tends to zero in the nonparticipation set, it is inefficient to increase the wife's allocation when she is not working.

Yet the limiting case suggests another intrahousehold process, where individual utilities are not separable anymore. Assume the husband's utility increases with female nonlabor time (probably because wives make a lot of domestic production during her nonlabor time). While the wife is working, the husband's interest is rather to reduce female labor supply by increasing her allocation $\phi$ (up to a certain point : an increase in $\phi$ reduces his personal consumption too). Once the wife stops to work, it

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[29] Restrictions (7) and (12) are accepted up to the threshold 36%.

is not anymore in the husband's interest to increase her allocation : his interest is clearly to negotiate a low $\phi$ in order to maximize his consumption. This may explain the radical change of regime.

Hence two alternative explanations of the limiting case can be proposed. The first is our collective model (described in section 1), with a female marginal utility of consumption tending to zero if she does not work. In this model the sharing rule is determined independently of domestic work (assumption A3). The second would be an alternative model where both spouses bargain simultaneously the sharing rule $\phi$ and the quantity of housework made by the wife (measured by the time she spends, denoted D)[30]. The husband's utility is increasing in D, whereas the wife's utility is decreasing in D, depending on her pure leisure time $T-L_f-D$. There is always a two-step process : (1) both spouses negotiate $\phi$ and D (2) they choose their labor supply independently subject to their personal time and budget constraint. While the wife is working, the husband negotiate easily an higher $\phi$ against an higher D. But when she stops to work, D can not grow anymore[31], and the husband becomes reluctant to give an higher $\phi$. If his bargaining position is very strong, the husband keeps $\phi$ at the reservation sharing rule : we are in the limiting case. Both explanations are compatible with the limiting case, we can not decide between them for the moment. Moreover they are not exclusive : we may have a low female marginal utility of consumption and a powerful husband reluctant to increase $\phi$.

Now another striking result is the positive direction of the sex-ratio effect on female labor supply. According to our estimations, a high sex-ratio - that is a relative scarcity of women, which is deemed to be favourable to women - increases significantly female labor supply. In our theoric framework, this means that the wife's allocation decreases with the sex-ratio, which is paradoxical. This result is unexpected because previous work did not find a positive correlation between sex-ratio and female labor supply : on American data the correlation is negative (Chiappori et alii, 2002) ; on British data it is not significant (Clark, Couprie and Sofer, 2004) ; and on French data (issued from the same source) the effect is either negative or nonsignificant (Moreau, 2000 and 2002)[32]. The difference with our findings has two explanations. First, the definition of the sex-ratio influences the results : previous work had used formula $SR_1$, whereas we have used formula $SR_c$ (cf. §3.2). As we saw before, formula $SR_1$ mixes the pure effect of the sex-ratio and an effect of relative ages of the spouses. Had we used formula $SR_1$, the sex-ratio effect had be lower in absolute value (table 8). But anyway it had always be positive and very significant. The main explanation is the extension of the sample to nonparticipating wives. If we restrict our sample to participating wives, the sex-ratio effect is nonsignificant (table 8). Hence the paradoxical effect of the sex-ratio comes from introduction of female nonparticipation[33].

The paradoxical effect of the sex-ratio may be another consequence of domestic production. Consider the alternative model described above, where spouses bargain simultaneously D and $\phi$. Nonparticipation corresponds to a high D negotiated against a high $\phi$, and participation corresponds to

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[30] For the sake of simplicity, we ignore here the housework made by the husband.

[31] One can consider that a non working wife makes the maximum amount of domestic work possible (the needs of the household are saturated). Another idea is that the marginal utility of her pure leisure becomes higher than her wage when she stops to work : while she works, she accepts to make one more hour of housework if he accepts to increase $\phi$ to $\phi + w_f$ ; once she stops to work, she refuses to make one more hour against an increase in $\phi$ of $w_f$. It is the less interesting for th husband to increase $\phi$.

[32] It depends on the instruments. Moreau obtains a nonsignificant effect when the number of children is exogeneous (as we do here). But when the number of children is considered as endogeneous, the sex-ratio effect has a significant negative effect on female labor supply.

[33] More computations have to be done to confirm this result. Another interesting finding is that the sex-ratio effect is positive for low educated women (who participate less) whereas it tends to be negative for hign educated women (who almost always participate).

low D and a low $\phi$. When her bargaining power gets better (increase in the sex-ratio), a nonparticipating wife wants to negotiate a reduction in her housework D (she does not claim a higher $\phi$ because $\phi$ is already high). Once she has obtained this reduction, she has more available time, so that the probability to participate in the labor market increases. Now consider a participating wife. Because D is already low, she does not want to reduce D, but she claims and obtains an higher $\phi$ when her bargaining position gets better. In this case, D is almost constant, so that assumption A3 holds : the collective model with separable utilities is accepted. Hence the positive direction of the sex-ratio effect is unexplained by the collective model with separable utilities, whereas an alternative model taking into account domestic work is able to explain it.

**Table 3 – Model without distribution factors : Maximum Likelihood estimation**

| | Wives | | | Husbands | | |
|---|---|---|---|---|---|---|
| | **Est.** | *std.* | Sign. | **Est.** | *std.* | sign. |
| **$\log(w_f)$** | **3 175** | *802* | *** | **588** | *372* | |
| **$\log(w_m)$** | **1 885** | *750* | ** | **812** | *353* | ** |
| **$\log(w_f).\log(w_m)$** | **-504** | *179* | *** | **-136** | *84* | |
| **Y** *(thousands of french Francs)* | **-2.855** | *0.822* | *** | **0.809** | *0.400* | ** |
| Type of area : 0 – country | **-259** | *65* | *** | **-32** | *32* | |
| 1 – town under 10 000 inh. | **-168** | *63* | *** | **-27** | *33* | |
| 2 – town 10000 to 100 000 | **-287** | *66* | *** | **-64** | *34* | * |
| 3 – town over 100 000 inh. | **-205** | *59* | *** | **-50** | *30* | * |
| 4 – Paris and suburb | **0** | *ref.* | | **0** | *ref.* | |
| Number of children | **-214** | *17* | *** | **-8** | *9* | |
| Age of the individual | **115** | *18* | *** | **7** | *8* | |
| Age² | **-1.769** | *0.216* | *** | **-0.132** | *0.098* | |
| $s_i$ (residual standard deviation) | **872** | *17* | *** | **448** | *4* | *** |
| | **Global parameters** | | | | | |
| r (correlation between residuals) | **0.133** | *0.030* | *** | | | |
| b (change of gradient) | **0.192** | *0.067* | *** | | | |

significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

**Table 4 – Model without distribution factors : structural parameters**

| **Sharing rule** *(thousands french francs)* : | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Participation | | | | Nonparticipation | | |
| **$\log(w_f)$** | K1 | **-372** | *164* | ** | k1 | **13** | *257* | |
| **$\log(w_m)$** | K2 | **-322** | *146* | ** | k2 | **-93** | *168* | |
| **$\log(w_f).\log(w_m)$** | K3 | **86** | *37* | ** | k3 | **25** | *45* | |
| **Y** *(thousands francs)* | K4 | **0.488** | *0.197* | ** | k4 | **0.142** | *0.271* | |
| **Marshallian individual labor supplies :** | | | | | | | | |
| | | **Wives** | | | | **Husbands** | | |
| **$\mathrm{Log}(w_i)$** | a1 | **998** | *203* | *** | b1 | **303** | *58* | *** |
| $f_i$ (nonlabor allocation in thousands F) | a2 | **-5.848** | *1.938* | *** | b2 | **1.580** | *0.745* | ** |

significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

**Table 5 – Unrestricted model with distribution factors : Maximum Likelihood estimation**

| | Wives | | | Husbands | | |
|---|---|---|---|---|---|---|
| | **Est.** | *std.* | sign. | **Est.** | *std.* | sign. |
| **$\log(w_f)$** | **3182** | *800* | *** | **586** | *373* | |
| **$\log(w_m)$** | **1918** | *749* | ** | **812** | *354* | ** |
| **$\log(w_f).\log(w_m)$** | **-505** | *179* | *** | **-136** | *84* | * |
| **Y** *(thousands of french Francs)* | **-2.901** | *0.822* | *** | **0.801** | *0.401* | ** |
| **Divorce rate** | **-1007** | *614* | * | **-54** | *324* | |
| **Sex-ratio** | **1391** | *387* | *** | **20** | *202* | |
| Type of area : 0 – country | **-335** | *68* | *** | **-34** | *33* | |
| 1 – town under 10 000 inh. | **-229** | *64* | *** | **-29** | *34* | |
| 2 – town 10000 to 100 000 | **-362** | *68* | *** | **-66** | *36* | * |
| 3 – town over 100 000 inh. | **-225** | *60* | *** | **-51** | *30* | * |
| 4 – Paris and suburb | **0** | *ref.* | | **0** | *ref.* | |
| Number of children | **-219** | *17* | *** | **-8** | *9* | |
| Age of the individual | **120** | *18* | *** | **8** | *9* | |
| Age² | **-1.779** | *0.215* | *** | **-0.135** | *0.099* | |
| $s_i$ (residual standard deviation) | **869** | *17* | *** | **448** | *4* | *** |
| | Global parameters | | | | | |
| r (correlation between residuals) | **0.136** | *0.030* | *** | | | |
| b (change of gradient) | **0.197** | *0.068* | *** | | | |

significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

**Table 6 – Restricted model with distribution factors : Maximum Likelihood estimation**

| | Wives | | | Husbands | | |
|---|---|---|---|---|---|---|
| | **Est.** | *std.* | sign. | **Est.** | *std.* | sign. |
| **$\log(w_f)$** | **3251** | *796* | *** | **345** | *199* | * |
| **$\log(w_m)$** | **1985** | *746* | *** | **589** | *181* | *** |
| **$\log(w_f).\log(w_m)$** | **-520** | *178* | *** | **-81** | *42* | * |
| **Y** *(thousands of french Francs)* | **-2.884** | *0.820* | *** | **0.756** | *0.363* | ** |
| **Divorce rate** | **-1001** | *607* | * | **-157** | *114* | |
| **Sex-ratio** | **1327** | *378* | *** | **208** | *99* | ** |
| Type of area : 0 – country | **-331** | *68* | *** | **-44** | *32* | |
| 1 – town under 10 000 inh. | **-227** | *64* | *** | **-36** | *34* | |
| 2 – town 10000 to 100 000 | **-359** | *68* | *** | **-74** | *34* | ** |
| 3 – town over 100 000 inh. | **-225** | *60* | *** | **-52** | *30* | * |
| 4 – Paris and suburb | **0** | *ref.* | | **0** | *ref.* | |
| Number of children | **-219** | *17* | *** | **-8** | *9* | |
| Age of the individual | **120** | *18* | *** | **9** | *8* | |
| Age² | **-1.782** | *0.216* | *** | **-0.140** | *0.091* | |
| $s_i$ (residual standard deviation) | **869** | *17* | *** | **448** | *4* | *** |
| | Global parameters | | | | | |
| r (correlation between residuals) | **0.122** | *0.028* | *** | | | |
| b (change of gradient) | **0.156** | *0.062* | ** | | | |

significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

**Table 7 – Restricted model with distribution factors : structural parameters**

| **Sharing rule** *(thousands french francs)* **:** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **Participation** | | | | **Nonparticipation** | | |
| **log($w_f$)** | K1 | **-286** | *163* | * | k1 | **135** | *60* | ** |
| **log($w_m$)** | K2 | **-257** | *140* | * | k2 | **0** | | |
| **log($w_f$).log($w_m$)** | K3 | **67** | *25* | *** | k3 | **0** | | |
| **Y** *(thousands francs)* | K4 | **0.374** | *0.176* | ** | k4 | **0** | | |
| **Divorce rate** | K5 | **130** | *94* | | k5 | **0** | | |
| **sex-ratio** | K6 | **-172** | *82* | ** | k6 | **0** | | |
| **Marshallian individual labor supplies :** | | | | | | | | |
| | | **Wives** | | | | **Husbands** | | |
| **log($w_i$)** | a1 | **1045** | *335* | *** | b1 | **279** | *43* | *** |
| $f_i$ (nonlabor allocation in thousands F) | a2 | **-7.716** | *3.117* | ** | b2 | **1.208** | *0.383* | *** |

significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

**Table 8 - Effect of the sex-ratio on female labor supply : variants**[34]

| **sample used for estimation** | **formula used for the sex ratio** | | | | | |
|---|---|---|---|---|---|---|
| | | $SR_c$ | | | $SR_1$ | |
| | **Est.** | *std.* | sign. | **Est.** | *std.* | Sign. |
| all couples | **1391** | *387* | *** | **217** | *81* | *** |
| participating wives only | **391** | *334* | | **37** | *33* | |

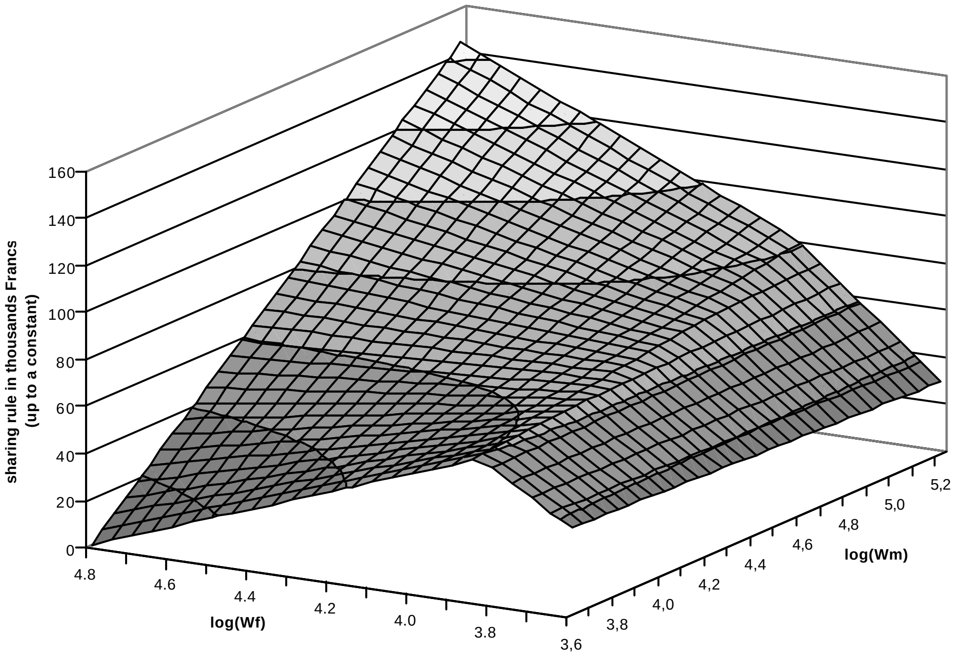
significancy of the parameters: *** at 1% level ; ** at 5% level ; * at 10% level

Note : the standard deviation of variable $SR_1$ is four times higher than $SR_c$. This explains partly the lower estimate with $SR_1$. However the standardized estimate stay lower with formula $SR_1$**.**

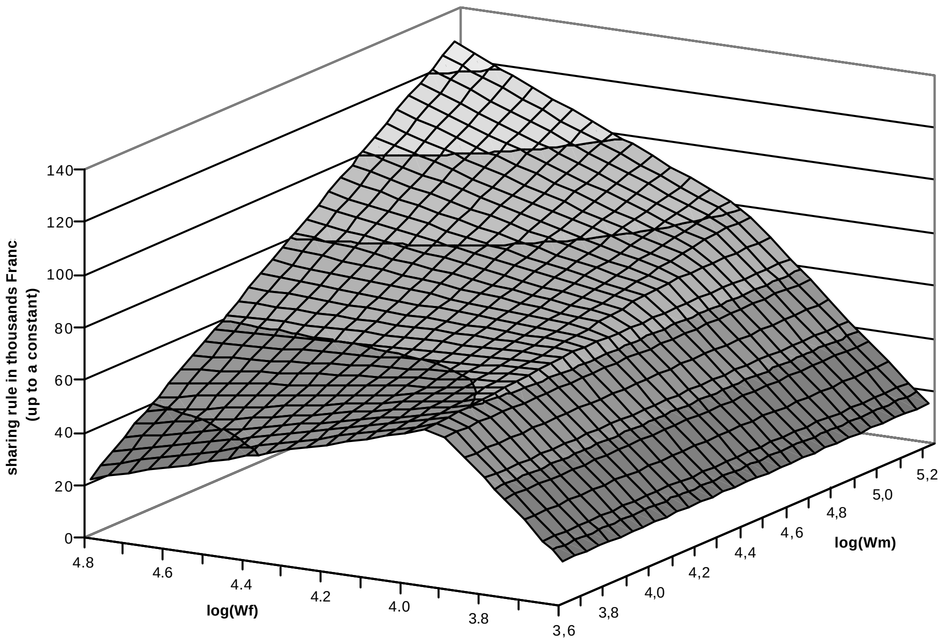
[34] Estimates of the unrestricted model. Due to lack of time, we have not yet computed the restricted model and the sharing rule.

**Graph 3 – Sharing rule (female allocation), function of female and male wages**

Model without distribution factors




Model with distribution factors




Note : Values of $w_f$ and $w_m$ vary from the first to the last percentile of wages distribution. Inside the participation set (left side of the surface) and the nonparticipation set (right side), the form of the sharing rule would be the same for any value of nonlabor income Y and distribution factors s. But the position of the frontier depends on household characteristics (Y, s, and control variables $Z_f$) : the frontier is here drawn for characteristics favourable to nonparticipation : the wife is 50 years old, with 3 children, living in a small town, with Y=0 ; the reservation wage (here around 55 F/hour, increasing in $w_m$) is then higher than for an average couple.

## 5. Conclusion

Using French data, we have extended the collective model of household labor supply (Chiappori, Fortin and Lacroix, 2002) to all couples, whether the wife participates or not in the labor market. Under a separability assumption of individual utilities, this model allows us to recover (up to a constant) a sharing rule function, which indicates how the couple's members share their resources given the intrahousehould balance of power. Following the theoretical work of Donni (2003), we show that the sharing rule is continous along the wife's participation frontier, but there is a possible change of gradient when the frontier is crossed : the parameters of the sharing rule function are different in the nonparticipation set and in the participation set, this difference being summarized by a unique parameter.

We show that this parameter can not exceed a certain bound. If we are on this bound (limiting case), the sharing rule is only depending on the wife's wage in the nonparticipation set. In couples where the wife does not work, all things happen then as if the husband gave his wife exactly what is needed to obtain her nonparticipation, but nothing more. If the male wage or the household nonlabor income grows, the husband takes all ; but if the female potential wage grows (and becomes higher than her reservation wage), the husband gives her more money, as if he wanted her to stay inactive.

According to our estimations, the change of gradient allowed in the sharing rule is very significant. A first striking result is that we are either near or in the limiting case. There is a radical switch of regime when the wife stops to work. When she participates, the wife's allocation grows with her husband's earnings and household nonlabor income is equitably shared. But when she is inactive, the wife's allocation stops to increase with the household income and tends to become insensitive to all factors except her potential wage. Although this does not lead to a rejection of the collective model with separable individual utilities, this result suggests that female nonparticipation may lead to positive externalities for the husband. We can then propose an alternative explanation, where utilities are not separable because of domestic work.

We have also introduced distribution factors like the sex-ratio, in order to test the collective model. Then we obtain a second striking result : the effect of the sex-ratio on women's participation is opposite to what expected. According to the collective model with separable utilities, an increase of the scarcity of women - which is deemed favorable to them - should increase the nonlabor allocation of the wife and then reduce her labor supply. On the contrary, according to our estimations, it increases female participation. Nonparticipation is then correlated with weak bargaining position[35]. This lead us to reject the collective model with separable utilities, whereas a model taking into account domestic work may explain the effect of the sex-ratio.

Had we restricted our sample to participating wives, the sex-ratio effect would have been insignificant, and the model would not have been rejected. While we consider only participating women, the collective model with separable utilities works, probably because housework is not very important. But when we consider nonparticipating women, the separability assumption does not hold anymore, probably because housewives make a lot of domestic production. So further research on collective labor supply models with nonparticipation should take into account housework.

[35] The form of the sharing rule in the nonparticipation set in another sign of this weakness.

**References :**


Baillon D., Costecalde N., Godin G., Munoz-Perez B. (1981), « Le Divorce en France », *Les collections de l'Insee,* série D, n°85-86.

Bloemen H. G. (2004), "An Empirical Model of Collective Household Labor Supply with Nonparticipation", *discussion paper,* TI 2004-010, Tinbergen Institute.

Blundell R., Chiappori P-A., Magnac T. et Meghir C. (2001), « Collective Labor Supply : Heterogeneity and Nonparticipation", *working paper,* n°01/19, Institute of Fiscal Studies.

Blundell R. and MaCurdy T., « Labor supply : a review of alternative approaches » in *Handbook of Econometrics*.

Bourguignon F., Browning M., Chiappori P.-A. "The Collective Approach to household behaviour", *working paper,* n°95-04, DELTA.

Chiappori P-A. (1988), « Rational Household Labor Supply », *Econometrica,* vol 56, n°1, pp. 63-90.

Chiappori P-A. (1992), « Collective Labor Supply and Welfare », *Journal of Political Economy,* vol 100, n°3, pp. 437-467.

Chiappori P-A., Fortin B. et Lacroix G. (2002), « Marriage Market, Divorce Legislation, and Household Labor Supply », *Journal of Political Economy,* vol 110, n°1, pp. 37-72.

Clark A., Couprie H. et Sofer C. (2004), « La modélisation collective de l'offre de travail : mise en perspective et application aux données britanniques », *Revue Economique,* vol 55, n° 4, pp. 767-789.

Donni O. (2000), « Collective Household Labor Supply : Theory », *working paper,* n°2000-08, DELTA.

Donni O. (2000), «Collective Household Labor Supply : Extensions », *working paper,* n°2000-08, DELTA.

Donni O. (2003), «Collective Household Labor Supply : Nonparticipation and Income Taxation », *Journal of PublicEconomics,* n°87, pp. 1179-1198.

Fermanian J.D. and Lagarde S. (1999), « Les horaires de travail des couples », *Economie et Statistique,* n°321-322.

Insee (1999), « Revenus et Patrimoine des Ménages », *collection Synthèses*, n°28, Insee.

Insee (2001), « Revenus et Patrimoine des Ménages », *collection Synthèses,* n°47, Insee.

Laroque G., Salanié B. (2000), « Une décomposition du non-emploi en France », *Economie et Statistique,* n°331.

Lundberg S. et Pollak R.A. (1996), « Bargaining and Distribution in Marriage », *Journal of Economic Perspectives*, n°10, pp. 139-158.

McElroy M.B. and Horney M.J. (1981), «Nash-Bargained Household Decisions », *International economic Review*, vol 22, n°2, pp. 333-349.

Michaud P.-C. and Vermeulen F. (2004), "Labor supply of elderly couples : a collective approach", *working paper*, CentER, Tilburg University.

Moreau N. (2000), "Une application d'un modèle collectif d'offre de travail sur données françaises", *Economie et Prévision*, n°146, 2000/5.

Moreau N. (2002) « Modèles collectifs d'offre de travail : analyses et tests sur données françaises », thèse de doctorat, GREQAM, Université d'Aix-Marseille.

Moreau N., Donni O. (2002) «Estimation d'un modèle collectif d'offre de travail avec taxation », *Annales d'économie et de statistique*, n°65, janvier-mars, Insee.

Sofer C. et Sollogoub M. (1992), « Une confrontation de trois modèles de mariage à partir de l'analyse des transferts ordonnées lors du divorce », *Economie & Prévision* n°102-103, 1992.

Vermeulen F. (2004) « A collective model for female labour supply with nonparticipation and taxation », *working paper*, CentER, Tilburg University.

## Appendix : Likelihood function

In this appendix we write the likelihood function associated with the parametric model (14) :

$$
\begin{aligned}
&\textit{if } L^*_f \geq 0 \textit{ (set P) :} && L_f = L^*_f \\
& && L_m = L^*_m \\
&\textit{if } L^*_f \leq 0 \textit{ (set NP) :} && L_f = 0 \\
& && L_m = L^*_m - b.L^*_f \\
&\textit{where :} \quad L^*_f = X'_f.f + u_f \\
&\qquad\qquad\;\; L^*_m = X'_m.m + u_m \\
&\textit{and} \quad \begin{pmatrix} u_f \\ u_m \end{pmatrix} \rightarrow N\left(0, \begin{pmatrix} \sigma^2_f & \rho.\sigma_f.\sigma_m \\ \rho.\sigma_f.\sigma_m & \sigma^2_m \end{pmatrix}\right)
\end{aligned}
$$

The log-likelihood function takes the form :

$$LV = \sum_{i \in P} \log\left[V_i(L_{if}, L_{im})\right] + \sum_{i \in NP} \log\left[V_i(L_{im})\right]$$

where $V_i$ is the elementary likelihood associated with observation i.

*Likelihood in (P) :*

In P we have simply :

$$V_i(L_{if}, L_{im}) = \frac{1}{\sigma_f \sigma_m} \varphi_2\left[\frac{L_{if} - X'_{if}.f}{\sigma_f}, \frac{L_{im} - X'_{im}.m}{\sigma_m}, \rho\right]$$

where $\varphi_2$ is the probability density function of the standardized bivariate normal distribution :

$$\varphi_2(u, v, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{u^2 + v^2 - 2\rho uv}{2(1-\rho^2)}\right]$$

*Likelihood in (NP) :*

First we write :

$L_{im} = L^*_{im} - b.L^*_{if} = ( X'_{im}.m - b. X'_{if}.f ) + v_i$ ,

with $v_i = u_{im} - b.u_{if}$ .

We have :

$$Var(v_i) = \sigma^2_m\left[1 + b^2\frac{\sigma^2_f}{\sigma^2_m} - 2\rho b\frac{\sigma_f}{\sigma_m}\right] = \sigma^2_v$$

$$Cov(u_{if}, v_i) = s_f s_m \left( r - b \frac{s_f}{s_m} \right),$$

$$Corr(u_{if}, v_i) = \frac{r s_m - b s_f}{s_v} = r.$$

So $v_i$ takes the form :

$$v_i = r \frac{s_v}{s_f} u_{if} + s_v \sqrt{(1-r^2)}\ w_i\ ,$$

where $w_i$ is a standardized normal variable, $w_i$ and $u_{if}$ being independant.

And $L_{im}$ takes the form :

$$L_{im} = \left( X'_{im}\, m - b.\, X'_{if}\ f \right) + r \frac{s_v}{s_f} u_{if} + s_v \sqrt{(1-r^2)}\ w_i$$

Hence the probability density of $L_{im}$ conditionnaly on $u_{if}$ is :

$$\frac{1}{s_v \sqrt{(1-r^2)}}\ j \left[ \frac{L_m - \left( X'_{im}\, m - b.\, X'_{if}\ f \right) - r \frac{s_v}{s_f} u_{if}}{s_v \sqrt{(1-r^2)}} \right],$$

where φ(x) is the density of the standardized normal distribution.

Because we are in NP, $u_{if}$ takes any value such that $u_{if} \leq -X'_{if}.f$ , and the likelihood function is :

$$V_i(L_{im}) = \int_{-\infty}^{-X'_{if}\ f} ¶u_{if}\ \frac{1}{s_f}\ j \left( \frac{u_{if}}{s_f} \right) \frac{1}{s_v \sqrt{(1-r^2)}}\ j \left[ \frac{L_{im} - \left( X'_{im}\, m - b.\, X'_{if}\ f \right) - r \frac{s_v}{s_f} u_{if}}{s_v \sqrt{(1-r^2)}} \right]$$

This expression can be simplified. First we note :

$$u^*_{if} = \frac{u_{if}}{s_f}$$

$$v^*_i = \frac{v_i}{s_v} = \frac{L_{im} - \left( X'_{im}\, m - b.\, X'_{if}\ f \right)}{s_v}$$

so that :

$$V_i(L_{im}) = \int_{-\infty}^{-\frac{X'_{if}\ f}{s_f}} \frac{1}{s_v \sqrt{(1-r^2)}}\ ¶u^*_{if}\ j \left( u^*_{if} \right) j \left[ \frac{n^*_i - r u^*_{if}}{\sqrt{(1-r^2)}} \right]$$

$$= \frac{1}{s_v} \int_{-\infty}^{-\frac{X'_{if} f}{s_f}} j_2\left(u^*_{if}, n^*_i, r\right) \P u^*_{if}$$

$$= \frac{1}{s_v} \int_{-\infty}^{-\frac{X'_{if} f}{s_f}} \frac{1}{\sqrt{(1-r^2)}} \P u^*_{if}\, j\left(n^*_i\right) j\left[\frac{u^*_{if} - r n^*_i}{\sqrt{(1-r^2)}}\right]$$

$$= \frac{1}{s_v} j\left(n^*_i\right) \int_{-\infty}^{-\frac{X'_{if} f}{s_f}} \frac{\P u^*_{if}}{\sqrt{(1-r^2)}} j\left[\frac{u^*_{if} - r n^*_i}{\sqrt{(1-r^2)}}\right]$$

$$= \frac{1}{s_v} j\left(n^*_i\right) \Phi\left[\frac{-\frac{X'_{if} f}{s_f} - r n^*_i}{\sqrt{(1-r^2)}}\right]$$

where $\Phi(x)$ is the cumulative distribution function of the standardized normal distribution.