Unemployment Risk and the Distribution of Assets*

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Abstract

How does the distribution of assets affect a worker's job finding prospects? We analyze a labor market with uninsurable unemployment risk where the worker's asset holdings affect her job finding probability as well as the productivity of the jobs she applies for. In the absence of insurance, workers with low asset holdings have a *precautionary job search motive*, they direct their search to low productivity jobs because they offer a low wage and low risk. We show that sorting of low asset workers into low productivity jobs occurs under a condition closely related to Decreasing Relative Risk Aversion. We calibrate the infinite horizon economy and find that endogenously choosing the job finding probabilities is quantitatively important: the high asset holders find a job with a probability that is 18% lower. We also evaluate a tax financed unemployment insurance scheme and how it affects the allocation and value of unemployment. Higher benefits are beneficial for consumption smoothing but deter the entry of firms and change the probability of job finding. We show how these effects interact with each other across for different asset holders.

Keywords. Unemployment Risk. Precautionary Savings. Precautionary Job Search. Sorting. Unemployment Insurance.

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1 Introduction

Unemployment risk is arguably the biggest risk a worker faces in her lifetime. Even if there is no market for unemployment insurance, individuals or their families can nonetheless self insure by accumulating assets while employed, in order to run those assets down when unemployed. This allows them to smooth consumption while unemployed. But workers simultaneously also use the labor market to self insure by applying for jobs that have a higher job finding probability. We ask a basic question: what is the role of the distribution of assets in providing self insurance and how does it affect the optimal provision government mandated unemployment insurance (UI) benefits that are financed by income taxes? There is of course a tension between the unemployed who receive UI benefits and the employed who pay for them through taxes. But there is now also a tension within the pool of the unemployed. Those with high asset levels can easily smooth consumption without the need for high UI benefits and anticipate being employed soon enough when they have to pay for those benefits through taxes. That is why the value of unemployment decreases in benefit for high asset holders while it increases for workers on the left side of asset distribution.

We model the worker's savings and job search decision in a labor market where workers can direct their search towards jobs of different productivity, with firms posting wages to attract applicants. The worker's incentives are thus to trade off wages and job productivity against the probability of finding a job. Asset holdings crucially affect this tradeoff because the worker is less exposed to the consumption risk inherent in joblessness. In addition to the standard *precautionary savings motive* with asset-contingent consumption smoothing à la Bewley-Huggett-Aiyagari, workers now also counter unemployment risk by directing her search to jobs with a high matching probability, call it a *precautionary job search motive*. Key in our analysis is the non-degenerate distribution of assets that is determined endogenously. Our main objective is to analyze how the inequality inherent in a labor market with unemployment frictions interacts with the inequality that results from asset accumulation, i.e., how the two precautionary motives interplay.

The contribution of the paper is double. First, we show theoretically that under a condition related to decreasing relative risk aversion there is sorting of workers with heterogeneous asset holdings into firms with heterogeneous productivities. The sorting happens despite the fact that there is no technological complementarity (supermodularity) between job productivity and worker skill. There is nonetheless complementarity between firm productivity and worker assets because risk aversion generates different preferences for self insurance, with high asset holders trading off a lower insurance for a higher productivity job. To establish the sorting in this model, we solve this as an allocation problem with risk aversion and therefore imperfectly transferable utility (ITU) as well as search frictions. It is the selection or sorting of workers into different productivity jobs that is responsible for the different matching probabilities of different asset holders. While directed search in the presence of risk aversion has been analyzed in the literature – most notably Acemoglu and Shimer (1999) and more recently

Golosov, Maziero, and Menzio (2012) –, these are representative agent models without a non-degenerate distribution of assets.[1]

This interaction between the distribution of assets and the incentives to search for jobs as well as smooth consumption is not merely a theoretical artifact. We show that it is important quantitatively. We analyze the steady state of an infinite horizon version where workers and firms sort in each period. In this steady state, unemployed workers run down their assets, while at the same time moving their target from high to low productivity jobs. Employed workers run up their assets anticipating the eventual job loss and necessity to insure while unemployed. Workers continuously move up and down the asset distribution, but the aggregate distribution of assets is stationary. We derive the ergodic distribution in this steady state as well as wages, savings, jobs search decisions (and unemployment), and the vacancy posting decision for every asset and productivity level. Unlike most existing work on unemployment insurance, we are able to incorporate the endogenous saving of the employed.[2] This is novel in the literature, and we manage to do this computationally using a shooting algorithm to solve for the allocation of unemployed job searchers, and a brute force algorithm to solve for the consumption-savings decision of he employed.

We calibrate our model to the US economy and find that the two key features in our model are quantitatively important: the ability of a worker to direct her search towards jobs with different bundles of wages (productivities) and job finding probabilities, as well as the endogenous accumulation of assets. In our calibration, as predicted by the theory, there is indeed positive assortative matching of asset holders into high productivity jobs. And while job productivity differs substantially – the highest productivity job produces 29% more output than the lowest productivity job – only a very small fraction of the output differences is passed through into wage differences. Instead, we find that the asset holdings substantially affect the job finding probability. That of the low asset holders is 18% higher than that of the high asset holders. This establishes the important role of endogenous job finding rates and their interaction with the distribution of asset holdings.

In this setting we analyze the role of government mandated unemployment benefits. We have no pretense of analyzing a general mechanism design question where agents submit messages about their private asset holdings and receive a benefit in depending on their and all other agents' messages. This turns out to be a immensely complex problem with an infinite horizon and a continuum of heterogeneous agents. Rather, we analyze a realistic unemployment insurance institution where ex ante homogeneous workers with ex post heterogeneous (but time varying) asset holdings receive a constant benefit while unemployed and pay a constant tax rate on wage income while employed.

[1] Acemoglu and Shimer (1999) do consider a non-degenerate distribution when analyzing the case of CARA, which, as we show in this paper, is a knife-edge case with no sorting and where the distribution is indeterminate.

[2] The standard assumption in the literature is that employed workers value are constant (see for example Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008)). This is typically achieved by assuming that once employed, they do not face job separation, in conjunction with the assumption that discounting is exactly proportional to the return on assets. All this implies that workers in each period consume the return on their assets, keeping their asset holdings invariant.

There are multiple channels through which benefits affect the equilibrium allocation and therefore value of unemployment. We single out four equilibrium effects that result from an increase in benefits. 1. The unemployed worker is better insured and enjoys smoother consumption. 2. Because of better insurance, workers tend to apply for more productive jobs with higher wages. Both these affect value of unemployment positively. The next effects are negative. 3. Higher wages reduce the firm's benefits and therefore job creation. 4. Higher benefits uniformly lead to lower job finding probabilities and therefore higher unemployment. We are interested in what the net effect is of these countervailing forces. But there are conflicts of interest between different agents. Not only are the unemployed broadly speaking better off from higher benefits than the employed, benefits have higher welfare effects for those with low assets. Benefits always increase value of unemploymnet for those with low assets, and only for very low benefit levels for those with high assets.

Related Literature

We are intellectually indebted to earlier work that has shaped our thinking on this topic. This paper is related to a large literature on unemployment risk and risk averse agents. Danforth (1979) is one of the first to analyze search with risk averse workers in a partial equilibrium setting. Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008) analyze optimal unemployment insurance in a similar setting. Our paper is a general equilibrium search model with risk averse agents, closely related to Acemoglu and Shimer (1999). They analyze workers with identical asset holdings and focus on the incentives for firms to create jobs. Golosov, Maziero, and Menzio (2012) consider a similar setup to Acemoglu and Shimer (1999) with identical agents and analyze optimal taxation and benefits. Here, we focus on the distribution of assets and where the distribution of those assets is non-degenerate.

Our model follows in the footsteps of Krusell, Mukoyama, and Sahin (2010), who analyze the relation between asset dependent consumption-savings decisions and unemployment risk. Our focus is on directed rather than random search. This is not merely a semantic distinction. Directed search allows for the fact that the asset holdings affect the job finding probability. At the same time, from the block recursivity property of directed search (Menzio and Shi (2011)) we can separately solve for the distribution of types from the job search and savings decision. And from the combination of directed search with two-sided heterogeneity (Eeckhout and Kircher (2010)), we can actually solve an assignment problem with risk aversion. What is novel about the search problem here is that agents are risk averse, that it involves a consumption-savings decision, that there is sorting, and that the economy is dynamic (infinite horizon). The novelty of our approach allows us to analyze an economy where the asset distribution is endogenous and where both savings and job search decisions depend on the worker's asset holdings. We can thus analyze how unemployment benefits affect workers' asset holdings and in turn their job search decisions.

This paper is also related to large quantitative literature that looks at the welfare impact of a change in UI in search and matching models with risk averse agents. Merz (1995), Andolfatto (1996) and den Haan, Ramey, and Watson (2000) study the macroeconomic implications of search frictions in business cycle models, in an economy where a worker's idiosyncratic is are fully insured. Krusell, Mukoyama, and Sahin (2010) nests this Diamond-Mortensen-Pissarides framework with asset dependent consumption savings decisions as in Bewley (undated), Huggett (1993) and Aiyagari (1994). This allows them to analyze the interaction of search frictions with the precautionary savings motive. The contribution of our paper is to take this one step further. We introduce endogenous job search that allows workers to implicitly insure unemployment risk, the precautionary job search motive. We find that this is important quantitatively, and as a result, a change in unemployment insurance changes the workers' welfare by affecting their job search decision.

There is direct evidence in the literature for the main mechanism of our model, namely that higher asset holdings leads to prolonged job search. Card, Chetty, and Weber (2007) find that a lump sum transfer of two months of salary reduces the job finding rate by 8-12%. These numbers are in line with what we find for our benchmark economy.[3] Chetty (2008) shows that the elasticity of the job finding rate with respect to unemployment benefits decreases with liquid wealth. And Browning and Crossley (2001) show that unemployment insurance improves consumption smoothing for poor agents, but not for rich ones. Herkenhoff (2013) and Herkenhoff, Phillips, and Cohen-Cole (2015) provide evidence for the effect of better credit access on lower job finding rates. Herkenhoff (2013) shows that through this channel, increased credit access leads to longer recessions and slower recoveries. And Herkenhoff, Phillips, and Cohen-Cole (2015) exploit credit tightening over the business cycle which leads to an increase in employment and a decrease in output and productivity. We believe our model is novel in providing a theoretical framework where this observed relation between asset holdings and job finding rates stems from a precautionary job search motive.

Finally, in an interesting piece, Michelacci and Ruffo (2014) analyze a related question where workers are heterogeneous: how optimal unemployment insurance varies over the life cycle. Because workers accumulate human capital, young workers have strong incentives to find a job, yet they do not have the means to smooth consumption. Instead, older workers have less incentives and can smooth consumption better. They focus on role of human capital accumulation and in to that end, assume that matching probabilities are exogenous.

This paper is organized as follow. In section 2 we lay out the model. In section 3 we derive the equilibrium allocation and the conditions under which there exists positive (negative) assortative matching for two special case economies. In section 4, we compute and quantitatively analyze the full infinite horizon model. We also evaluate the impact of changes in UI on the equilibrium allocation and on value of unemployment. We conclude in section 5.

[3] See also Rendon (2006) and Lentz (2009) for related findings

2 The Model

Timing. This is a T-period economy in which agents make a joint consumption-savings and job search decision. Endowed with assets, in each period $t < T$, unemployed workers choose their consumption-savings level, as well as which job to search for. Our interest is in analyzing the infinite horizon setting $T \to \infty$ (Section 3.2). To gain insights into the mechanism and in order to derive analytical results, we first analyze the two-period model $T = 2$ (section 3.1), in which workers make decisions only once at $t = 1$.

Agents. There is a measure one of workers, indexed by their heterogeneous asset holdings in period t $a_t \in \mathcal{A} = [\underline{a}, \overline{a}] \subset \mathbb{R}_+$.[4] Let $G(a)$ denote the measure of unemployed workers with asset levels weakly below $a \in \mathcal{A}$. The distribution of asset holdings amongst unemployed and employed workers is endogenous. The objective of the infinite horizon model is precisely to derive the ergodic distribution of assets. We assume a is private information. Each worker supplies her labor and can only apply to one job at a time. Firms are heterogeneous in their productivities y and each have one job. Let $y \in \mathcal{Y} = \left[\underline{y}, \overline{y}\right] \subset \mathbb{R}_+$ and assume the firm type is observable. $H(y)$ denotes the measure of firms that post vacancies and with a type weakly below y. The total measure of firms is $H(\overline{y})$ =2. H and G are C^2 with strictly positive derivatives h and g. The distribution of asset holdings of unemployed and employed workers as well as the distribution of firms with vacancies are endogenous and denoted by $F_u(a), F_e(a)$ and $F_f(y)$. These distributions will in general depend on time t, but our interest to derive them will be for the stationary equilibrium in the infinite horizon economy, in which case we drop the time subscript.

Preferences and Technology. Workers are risk averse and their preferences are represented by the Von Neumann-Mergenstern utility function $u(c)$ over consumption level c, where $u : \mathbb{R}_+ \to \mathbb{R}_{++}$. We assume that u is increasing and concave: $u' > 0, u'' < 0$. Agents discount utility with factor $\beta < 1$. Savings can be invested in a risk free bond at a fixed rate $R = 1 + r > 1$. We assume that firms are owned by entrepreneurs who are risk neutral and who do not participate in the labor market.[5] Firms have one job and can post a vacancy at cost k. Output produced at a firm of type y is $f(y)$.

Search Technology. Job search is directed and there is a search technology that governs those frictions. The frictions crucially depend on the degree of competition for jobs, as captured by the ratio of vacancies to unemployed workers, denoted by $\theta \in [0, \infty]$. This represents the relative supply and demand for jobs, as it determines the probability of a match for an unemployed worker denoted by $m(\theta)$, where $m : [0, \infty] \to [0, 1]$: the higher the value of θ, the easier it is for a worker to find a job, so m is a strictly

[4]For much of the paper we will drop the subscript t and in the recursive (two-period) formulation we refer to $a_t = a$ ($a_1 = a$) and $a_{t+1} = a'$ ($a_2 = a'$).

[5]We could also have assumed that profits are distributed as the risk free dividend of a mutual fund owned by all workers and that has all firms in its portfolio as in Golosov, Maziero, and Menzio (2012). This approach does not affect any of the results since the dividend deterministically increases the workers' asset holdings and merely shifts the asset distribution.

increasing function: $m' > 0$. In contrast, the higher the ratio of firms to workers, the easier it is for a firm to fill its vacancy. We denote the probability that a firm gets matched by $q(\theta)$, where $q : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $q' < 0$. Since matching is always in pairs, the matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = \theta m(\theta)$. We also require the standard assumptions hold: m is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of firms to workers θ and not the number of unemployed workers and vacancies effectively means that we assume a matching technology that is constant returns. As the number of workers and firms doubles, the number of matches doubles, yet the matching probabilities remain unchanged.

As is inherent in the nature of directed search, there is a separate submarket for each firm and worker type pair. Heterogeneous firms and workers operate in different markets, while identical agents are in a common market. This permits workers to direct their search to those firms that offer the optimal terms (matching probability and wages) and it enables firms with vacancies to influence the search decision of workers by changing the terms of the wage offer.

Whenever unemployed, a worker searches to find a job, and once employed she holds the job until the match is separated with exogenous probability λ.

Unemployment Benefits. We assume that there is an unemployment benefit b received by all unemployed workers. The benefit b is financed by a budget balancing tax τ on wages. This requires that the sum of all benefits b over the unemployed agents is equal the sum of all taxes levied on wage income τw. We assume that UI is financed using taxes that are proportional to wages. We also assume that all the income for the unemployed comes from the UI. For a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int \omega(a) f_e(a) da$$

Actions. In period $t < T$, workers choose their consumption-savings bundle. Given assets a, each worker chooses the assets a' saved, and next period's consumption is contingent on the labor market outcome, $c'_e = Ra' + (1 - \tau)w$ when employed and $c'_u = Ra' + b$ when unemployed. Within the same period t, a two-stage labor search extensive form game determines the labor market outcome. Firms first simultaneously announce wages w' that will be paid starting in the next period: $w' \in \mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}_+$. The contract space is restricted to invariant wages. After observing wage-firm type pairs (w', y), the workers then choose which pair to apply to. Denote by $P(y, w)$ and $Q(a, a', y, w)$ the distribution of actions by firms and workers: $P(y, w)$ is the measure of firms that offers a productivity-wage pair below (y, w) and $Q(a, a', y, w)$ is the measure of workers with assets below a who save less than a' and who apply for productivity-wage pairs below (y, w). We impose that those distributions of actions are consistent with the initial distributions of types $H(y)$ and $G(a)$, i.e., that there is market clearing. In particular, it must be the case that $P_{\mathcal{Y}}(\cdot) = H(\cdot)$ and $Q_{\mathcal{A}} = G(\cdot)$, where $P_{\mathcal{Y}}$ and $Q_{\mathcal{A}}$ are the marginal distributions. This ensures that the allocation is measure preserving.

Value Functions and Equilibrium. Denote by $U(a)$ the value of being unemployed with asset level a and by $E(a)$ the value of being employed.[6] We can then write

$$U(a) = \max_{a',\theta} \left\{u(c_u) + \beta \left[m(\theta)E(a', y, w') + (1 - m(\theta))U(a')\right]\right\} \tag{1}$$

$$\text{s.t.} \quad c_u + a' = Ra + b \quad \text{and} \quad a' \geq 0$$

$$E(a, y, w) = \max_{a'} \left\{u(c_e) + \beta[\lambda U(a') + (1 - \lambda)E(a', y, w)]\right\} \tag{2}$$

$$\text{s.t.} \quad c_e + a' = Ra + (1 - \tau)w \quad \text{and} \quad a' \geq 0$$

A worker carries over last period's assets with return R. If unemployed, her income is thus $Ra + b$ and it is $Ra + (1 - \tau)w$ if employed. Her consumption is equal to this income net of here savings for next period a'. The unemployed worker's decision therefore is to optimally choose next period's assets as well as the submarket θ in which to apply for a job. The employed worker only chooses how much of her assets a' to save.

The value to the firm that posts a vacancy is:

$$V(y) = -k + \max_{w'} \beta[q(\theta)J(y, w') + (1 - q(\theta))\tilde{V}(y)]\}, \tag{3}$$

where $\tilde{V}(y)$ is the continuation value when the job was not filled. In the stationary, infinite horizon allocation, $\tilde{V}(y) = V(y)$. At a cost k, the firm announces and commits to a wage w' that it will pay in the next period in the case of a match. Firms also discount the future at rate β. $V(y)$ is the value of a vacancy for a firm with productivity y and $J(y, w)$ is the value of a filled job for a firm with productivity y when paying a wage w.

$$J(y, w) = f(y) - w + \beta[\lambda V(y) + (1 - \lambda)J(y, w)]. \tag{4}$$

We adopt the equilibrium concept used by Acemoglu and Shimer (1999). To accommodate the two sided heterogeneity of firm productivity and worker assets, we will use the version of their equilibrium adjusted by Eeckhout and Kircher (2010) to allow for two-sided heterogeneity and a continuum of agents. They consider the Acemoglu and Shimer (1999) setup as a large game where each individual's payoff is determined only by her own action and the distribution of actions in the economy, which consists of the optimal choices of each of the individuals in the distribution.[7]

[6] More precisely, $U(a, a', y, w, P, Q)$ is the value of an unemployed worker with assets a who saves a', who applies to a job y with wage w and who anticipates a distribution of offers P and a distribution of jobs Q. Likewise for $E(a, a', y, w, P, Q)$.

[7] The queue length θ is a function of the distribution of offers P and visiting decisions Q. Written explicitly, θ_{PQ} : $\mathcal{Y} \times \mathcal{W} \to [0, \infty]$ is the expected queue length at each productivity-wage combination (y, w). Then along the support of the

In line with the literature on directed search (see for example McAfee (1993), Acemoglu and Shimer (1999)), we impose restrictions on the beliefs about off equilibrium path behavior. In the current setup, beliefs about the queue length corresponding to firm or worker choices that do not occur in equilibrium are not defined. Therefore, we define those off equilibrium path beliefs corresponding to the notion of subgame perfection.[8] Firms expect workers to queue up for jobs as long as it is profitable for them to do so given the options they have on the equilibrium path. Formally, this defines the queue length over the entire domain as: $\theta(a,w) = \sup\{\theta \in \mathbb{R}_+ : \exists a, m(\theta)[y - w \geq \max_{y,w \in \text{supp} P} U(a,y,w,P,Q)]\}$. In all other cases, the queue length is zero.

This now permits us to define equilibrium. When time is finite, the equilibrium can be defined recursively starting from an initial asset distribution. In the infinite horizon economy, we solve for the stationary asset distribution. In each period, an equilibrium is a pair of distributions (P, Q) such that the following conditions hold: 1. Worker optimality: $(a, a', y, w) \in \text{supp } Q$ only if (y, w') maximizes (1) and (2) for a; 2. Firm optimality: $(y, w') \in \text{supp } P$ only if w' maximizes (6) and (4) for y.

This is a matching problem with a non-linear pairwise Pareto frontier. Existence is established in Legros and Newman (2007) and Kaneko (1982). Jerez (2012) establishes the existence of an equilibrium in a directed search model with a continuum of agents and a general matching technology.

The (measure preserving) market clearing condition is particularly transparent when matching is monotone. Then there is one-to-one matching of a to y, which we represent by a function $\mu : \mathcal{Y} \to \mathcal{A}$. Under positive assortative matching (PAM), $\mu'(y)$ is positive and it is negative under negative assortative matching (NAM). Under PAM high asset workers match with high productivity jobs, and the market clearing condition can be written as:

$$\int_a^{\overline{a}} \theta(a) g(a) da = \int_{\mu(a)}^{\overline{y}} h(y) dy.$$

3 The Equilibrium Allocation

We first analyze a simple two period model. The objective is to provide us with insights into how the per period allocation of asset holders to firms works. We then turn to the infinite horizon model, where we focus attention on the steady state and where we lay the ground for the calibration and policy exercise. For the purpose of the theory results in this section, we assume benefits and vacancy posting costs are zero: $b = 0, k = 0$. Benefits and vacancy posting costs are important for the calibration in the infinite horizon model, but do not add any insights in understanding the mechanism of the equilibrium allocation.

firms' wage setting distribution, $\theta_{PQ} = dQ_{\mathcal{Y}\mathcal{W}}/dP$ is given by the Radon-Nikodym derivative, where $Q_{\mathcal{Y}\mathcal{W}}$ is the marginal distribution of Q with respect to $\mathcal{Y}$ and $\mathcal{W}$.

[8] Peters (1997) and Peters (2000) provide micro foundations for a version of this model where this assumption is indeed justified as the limit of deviations in a finite game.

3.1 The two-period model

We first analyze the decentralized equilibrium allocation in the two period model where all workers are initially unemployed. Let there be an exogenously given initial distribution of assets $G(a)$. With $T = 2$, there is only a consumption-search (a, θ) decision in period 1. In the final period, consumption is determined by period's savings decision and the outcome of the job search. The value of both employment and unemployment are therefore equal to the utility of consumption in the respective states: $E(a', y, w') = u(Ra' + w')$ and $U(a') = u(Ra')$. We can then rewrite (1) (since all are unemployed in the first period, $E(a, y, w)$ is null) as:

$$U(a) = \max_{a',\theta} \left\{ u(a - a') + \beta \left[m(\theta) u(Ra' + w') + (1 - m(\theta)) u(Ra') \right] \right\}, \tag{5}$$

In other words, the consumption is completely pinned down by the choice of a' and the labor market choice θ which determines w' and the matching probability $m(\theta)$. The expected payoff to a firm y posting a vacancy is

$$V(y) = \max_{w'} \beta q(\theta) \left(f(y) - w' \right), \tag{6}$$

from (6) since $J(y, w') = f(y) - w'$, and the continuation value is zero.

The firm's problem is to set wages w to maximize expected profits $V(y)$. The consumer's problem is to maximize expected utility from consumption while simultaneously making an optimal search decision. We can therefore write the equilibrium worker and firm optimization as:

$$\max_{a',\theta} \left\{ u(a - a') + \beta \left[m u(Ra' + w') + (1 - m) u(Ra') \right] \right\}$$
$$\text{s.t. } V = \max_{w'} \beta q \left(f(y) - w' \right).$$

Given $w' = f(y) - \frac{V}{\beta q}$ and the optimal choice of wages follows from the optimal choice of queue length θ, we can write this problem as a matching problem with non-linear Pareto frontier denoted by $U(a, y, V)$:

$$U(a, y, V) = \max_{a',\theta} u(a - a') + \beta \left[m u \left(c'_e \right) + (1 - m) u(Ra') \right]$$

where $c'_e = Ra' + f(y) - \frac{V}{\beta q}$. Then the solution to the maximization problem is $a'^{\star}(a, y, V), \theta^{\star}(a, y, V)$ and satisfies:

$$-u'(a - a') + \beta R \left[m u'(c'_e) + (1 - m) u'(Ra') \right] = 0 \tag{7}$$

$$\beta m' \left[u \left(c'_e \right) - u(Ra') \right] + \beta u' \left(c'_e \right) \frac{\theta q' V}{\beta q} = 0 \tag{8}$$

The optimal savings behavior and optimal job search simultaneously implies a matching decision.

That is, a worker a effectively chooses a firm y. We can now analyze this allocation problem with a non-linear frontier $U(a,y,V)$, where a' and θ are chosen endogenously. We use the standard solution method for an assignment problem. The worker takes the firm payoff $V(y)$ as given (call it the hedonic price schedule) and chooses the firm type y that maximizes her expected utility. From the first order condition, the optimal y therefore satisfies $U_y + U_V \frac{\partial V}{\partial y} = 0$. This implies:

$$\beta m u'(c_e') \left(v_y - \frac{V'}{\beta q} \right) = 0. \tag{9}$$

where the effect of y and V on V through a' and θ is zero from the envelope theorem: $U_{a'} = 0, U_\theta = 0$. The details of the derivation of the partial derivatives are in the Appendix.

We want to ascertain under which circumstances there is monotone matching of asset holdings a in job productivities y. This is now a matching problem $U(a,y,V)$ where a type a chooses the optimal y, given optimizing behavior. The allocation is denoted by $a = \mu(y)$. Then the total cross derivative of U with respect to a and y is positive provided

$$\frac{d^2}{dady} U = U_{ay} + U_{Vy} \frac{\partial V}{\partial y} = U_{ay} - \frac{U_y}{U_V} U_{Va},$$

where we use the first order condition to substitute for $\frac{\partial V}{\partial y}$. Therefore, there will be Positive Assortative Matching in types a, y provided $U_{ay} > \frac{U_y}{U_V} U_{aV}$. The next proposition establishes under which conditions on the primitives (preferences and technology) this is satisfied:

Proposition 1 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$\frac{u'(c_e') - u'(Ra')}{u(c_e') - u(Ra')} < \frac{u''(c_e')}{u'(c_e')}. \tag{U}$$

Proof. In Appendix. ■

This Proposition establishes under what conditions of the utility function agents with higher levels of assets will choose more risky jobs. The does not immediately allow for a straightforward interpretation, and in the next two results we characterize the properties. First, we show that within the class of Hyperbolic Absolute Risk Aversion (HARA) utility functions, the condition is satisfied whenever absolute risk aversion is decreasing (DARA).

Proposition 2 *Consider the class of utility functions with Hyperbolic Absolute Risk Aversion (HARA):*

$$u(c) = \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^\gamma \qquad \text{where } \alpha > 0, \beta > \frac{\alpha c}{1-\gamma}.$$

*Then condition (**U**) holds whenever there is Decreasing Absolute Risk Aversion (DARA): $\gamma < 1$. It holds with opposite inequality when there is Increasing Absolute Risk Aversion (IARA): $\gamma > 1$.*

Proof. In Appendix. ■

A number of results for special cases of the HARA preferences immediately follow, including CRRA, logarithmic, CARA, risk neutrality and the quadratic.

Corollary 1 *Consider the class of HARA utility functions. Condition (***U***) holds:*

1. *under CRRA* $u(c) = \frac{1-\gamma}{\gamma} c^\gamma$ $(\alpha = 1-\gamma, \gamma < 1, \beta = 0)$ *and Log utility: :* $u(c) = \log c$ $(CRRA, \gamma \to 0)$*;*
2. *with equality under CARA* $u(c) = 1 - e^{-\alpha c}$ $(\beta = 1, \gamma \to -\infty)$ *and Risk Neutral* $u(c) = \alpha c$ $(\gamma = 1)$*;*
3. *with opposite inequality under Quadratic utility:* $u(c) = -\frac{1}{2}(-\alpha c + \beta)^2$ $(\gamma = 2)$*.*

Proof. In Appendix. ■

The results for HARA may indicate that condition (**U**) holds more generally. The answer is partially true. For small differences between the level of consumption when a job is obtained and the consumption of unemployment ($c_e - Ra' = w$ small), we can indeed completely generalize the characterization: when there is DARA, condition (**U**) is satisfied and high asset types choose high productivity jobs. This is proven in Proposition 3. However, for general utility functions beyond HARA and with wages w large, this characterization does not hold. In Example 1 in the Appendix, we show by counterexample that for w large, Decreasing Absolute Risk Aversion (DARA) is not sufficient for the condition to hold.

Proposition 3 *When* w *is small, condition (***U***) is satisfied for any utility function that exhibits Decreasing Absolute Risk Aversion (DARA),* $-\frac{u''}{u'} < 0$*, and thus has positive risk prudence,* $u''' > 0$*. Likewise, it holds with opposite inequality under IARA.*

Proof. In Appendix. ■

Condition (**U**) establishes that there are complementarities in the match value between a firm type y and a worker with assets a. In other words, the match value $U(a, y, V)$ between types a and y is supermodular, and therefore the equilibrium expected payoff for the worker is increasing in a, and so is the equilibrium expected payoff to the firm in y. While there are no technological complementarities (all workers are identically skilled), the preferences generate a complementarity between assets and job productivity.

The implication of this condition is that high asset workers apply for high productivity jobs, they earn higher wages, they have higher unemployment, they consume more and they have higher expected utility. Likewise, high productivity firms post higher wages, they attract higher asset workers, they have higher expected profits and they fill vacancies faster.

3.2 Infinite Horizon

We now consider the stationary equilibrium allocation in the infinite horizon version of the model. The per period allocation problem in the labor market is very similar to the one analyzed for the two period model, with the exception of the continuation value. Making some simplifying assumptions, this allows us to derive a condition similar to the (**U**) condition, but now for the infinite horizon economy. Note that though that this is condition involves value functions, not primitives such as utilities and consumption bundles.

While we cannot derive an analytical solution in the infinite horizon economy generally, we can solve it under the assumptions that $\beta R = 1$ and when employed agents are infinitely lived ($\lambda = 0$ together with an exogenous inflow of new asset holders). The assumptions have been used extensively in the literature on unemployment insurance (see amongst others Acemoglu and Shimer (1999), Shimer and Werning (2008), Hopenhayn and Nicolini (1997) and Krusell, Mukoyama, and Sahin (2010) implies that assets levels when employed are invariant in equilibrium as employed workers consume a share of their assets exactly equal to the dividend.

Due to these simplifying assumptions, the employed worker problem turns into a stationary problem that is independent of $U(a)$. As a result, the employed worker's problem can be solved explicitly. The first-order condition of the employed worker is $u'(w + a - a') = \beta R E'(Ra')$. With $\beta R = 1$ and $\lambda = 0$ the solution is $a' = \frac{a}{R} = \beta a$ and we can explicitly write the value for employment:

$$E(a) = \frac{1}{1-\beta} u\left(w + (1-\beta)a\right).$$

We can then write the problem of the unemployed as:

$$U(a) = \max_{a',\theta} \left\{ u(a - a') + \beta \left[m \frac{1}{1-\beta} u\left(w' + (1-\beta)Ra'\right) + (1-m)U(Ra') \right] \right\}$$

subject to the firm's value:

$$\begin{aligned} V(y) &= \max_{w} \left\{ q\left(f(y) - w\right) + \beta(1-q)V(y) \right\} \\ &= \max_{w} \left\{ \frac{q}{1-\beta(1-q)} \left[f(y) - w \right] \right\}. \end{aligned}$$

Using the standard technique in directed search, and similar to what we did in the two-period model, we substitute the wage and rewrite the problem as

$$U(a, y, V) = \max_{a',\theta} \left\{ u(a - a') + \beta \left[m \frac{1}{1-\beta} u\left((1-\beta)Ra' + f(y) - V \frac{1-\beta(1-q)}{q} \right) + (1-m)U(Ra') \right] \right\}$$

We can now repeat the analysis in the proof of Proposition 1, to obtain the infinite horizon version of the result.

Proposition 4 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$\frac{E_a(a,w) - U_a(a)}{E(a,w) - U(a)} < \frac{E_{a,w}(a,w)}{E_w(a,w)} \tag{$\mathbf{U}_\infty$}$$

Proof. In Appendix. ■

The following result now follows immediately:

Corollary 2 *Under condition ($\mathbf{U}_\infty$) and for a given worker with assets a, the job productivity y decreases in the duration of unemployment.*

4 Steady State Experiments

We will now analyze the full model with an ergodic asset and firm productivity distribution but with non-stationary savings by individual workers while unemployed and employed. The objective is to study the impact of unemployment benefits. The key feature of the model is, as in the simplified versions in section 2, the sorting of unemployed workers into different productivity jobs, depending on their assets. We solve for the ergodic distribution of assets. Unemployed workers run down their assets while searching for a job. In the process, as their assets decrease, they apply for lower productivity jobs. When on the job, they face a probability of exogenous separation. Anticipating the possibility of unemployment, they accumulate assets while working. This gives rise to a pattern of individual asset fluctuations to endogenously insure against unemployment risk. Computationally, we derive the ergodic distribution of assets that results.

Our objective is to study the role of policy. As unemployment insurance changes, the incentives to save and accumulate assets change. Different asset holders have different preferences for insurance and therefore for benefits. We decompose the channels through which unemployment insurance affects the workers value of unemployment across the distribution. First however, we calibrate the model and report its basic properties.

4.1 Calibration

Benchmark Calibration. We set one period to be a quarter. The production function is $f(y) = y$. Following Krusell, Mukoyama, and Sahin (2010) the borrowing constraint is set at 0 and $\beta = 0.99$. We use the utility function $u(c) = \log(c)$. Following Shimer (2005), we set the flow value of the unemployed household production (benefit, b) to be 40% of the average wage at steady state. We thus set b to 60 which, in equilibrium, turns out to be about 40% of the average wage. The tax rate on wage income τ

is set to balance the budget, financing for the entire burden of unemployment benefits. That is, for a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int \omega(a) F_e(a) da. \tag{10}$$

The probability of separation (λ) is set to be 0.03 which implies an average job duration of 4 years. Following Menzio and Shi (2011), we pick the CES contact rate functions, $m(\theta) = \theta(1 + \theta^{\gamma})^{\frac{1}{\gamma}}$. To calibrate the elasticity of matching function (γ) we target the steady state unemployment rate to be 4.7%. We set the cost of posting a vacancy as 35% of the average productivity which is 50% of the lowest level of productivity and 25% of highest. The asset domain is $a \in \mathcal{A} = [0, 300]$ and the productivity domain is $y \in \mathcal{Y} = [100, 200]$. The measure of workers is equal to one and firms is equal to two. The distribution of firm productivities in the economy is uniform and fixed. However, only those firms with productivity $y \geq \underline{y}$ enter the market. Moreover, the distribution of *firms posting vacancies* is endogenous: in general, firms posting higher wages will attract longer queues and will therefore fill their vacancies faster. With a constant separation rate, there should therefore be fewer high productivity than low productivity firms posting vacancies (given a uniform distribution in the population). Integrating the measure of vacant firms $F_f(y)$ above the cutoff firm $\underline{y}$ gives us the measure vacancies posted. The ergodic distribution of asset holdings of the employed $F_e(a)$ and the unemployed $F_u(a)$ is endogenous.

Table 1 summarizes the value of the pre-calibrated parameters.

Parameter	Definition	Value
r	Interest rate	0.012
b	Flow value unemployment	40
k	cost of vacancy	50
λ	Probability of seperation	0.03
γ	elasticity of matching function	1.5
ϕ	matching efficiency	1

Table 1: Exogenous Parameters

Characterization of the Steady State. We first display some key model features and then discuss our experiments. Table 2 details the the summary statistics of the most important endogenous outcome at the benchmark steady state calibration.

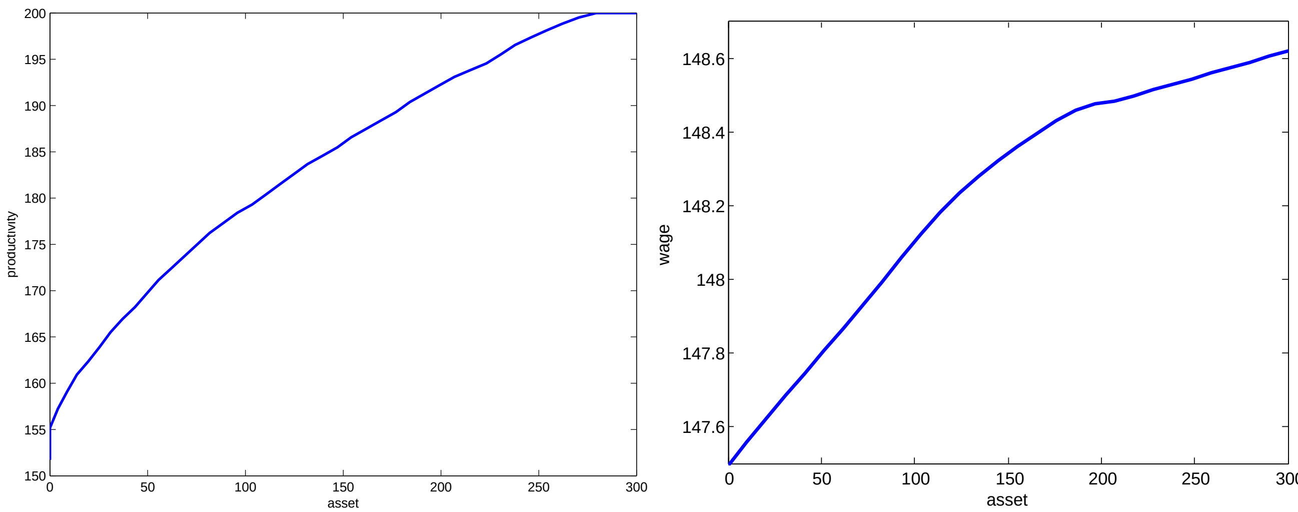
When there is positive assortative matching between workers' asset holdings and firms' productivity: in equilibrium workers with higher level of asset are matched with more productive firms.[9] Figure 1a

[9]The theoretical results indicate that under log preferences there is indeed positive sorting. Because we cannot solve the general model analytically, we guess the allocation is positively assorted and verify that the match surplus along the equilibrium allocation is supermodular.

u	τ	average θ	average w
4.7%	0.025	1.11	148.22

Table 2: Endogenous Outcomes

shows the allocation of workers to firms in the labor market. The market clearing condition implies that all workers are allocated to submarkets while firms below a productivity threshold are staying out of the market. This threshold is obviously sensitive to different parameterizations of the model. In particular, below we will study the impact of a change in unemployment benefits on the threshold and therefore on job creation. A higher threshold means more firms stay out of the market and hence fewer jobs are created.



(a) Allocation of firms and workers in the labor market

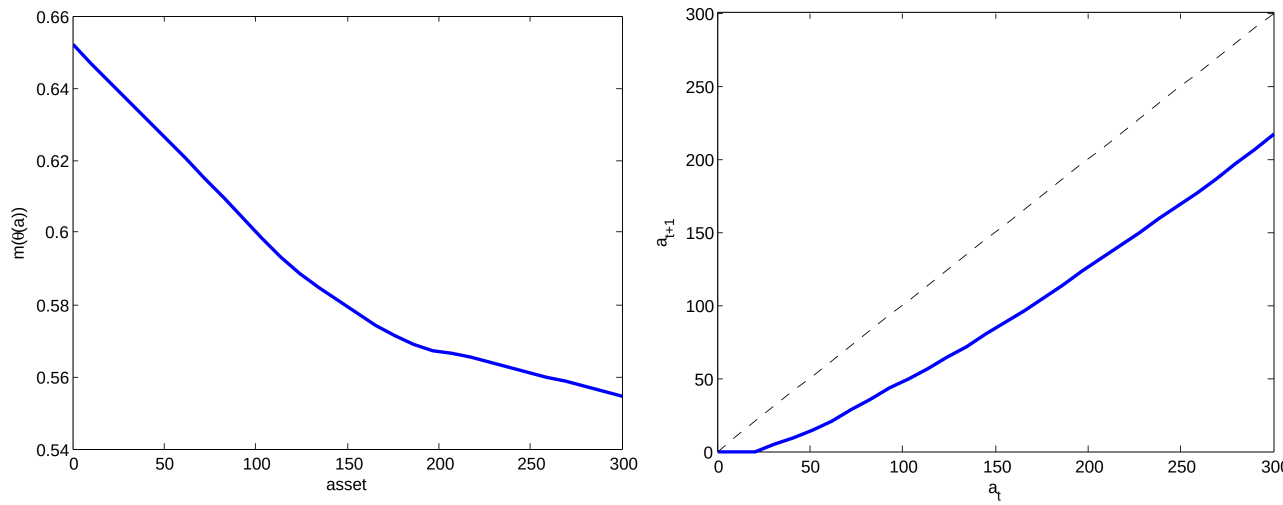
(b) Wages by Asset Level at Hiring

Figure 1: Equilibrium Allocation and Wages.

Figure 1b depicts equilibrium wages for different asset levels. Firms with more productive jobs post higher wages. This increases the vacancy to unemployment ratio θ and allows them to fill the vacancy with higher probability. Workers with more assets apply for the high wage jobs because they are able to insure better against unemployment. Their assets allow them to maintain a higher level of consumption.

Quantitatively, the key aspect is the role that assets play in the productivity of *equally skilled* workers. Workers with higher assets apply for jobs with a substantially higher productivity than those with low assets (200 versus 150, Figure 1a). The reason why they are able to get those better jobs is that they are taking a substantial amount longer than those with low asset holdings. As shown in Figure 2a, the matching probability decreases from 65% for the low asset unemployed workers to 55% for those with high asset levels. For the workers with low level of asset, unemployment is a worse option.

Therefore, if they do not find a job this period and deplete their asset stock further, next period they prefer to apply for a lower paid jobs where they can get them with a considerably higher probability. This dependence of the job search decision on assets is absent in the random search model: with random search, the probability of finding a job is the same for all workers regardless of their asset holding.[10]



(a) Job Finding Probability $m(\theta(a))$

(b) Savings Decision a'

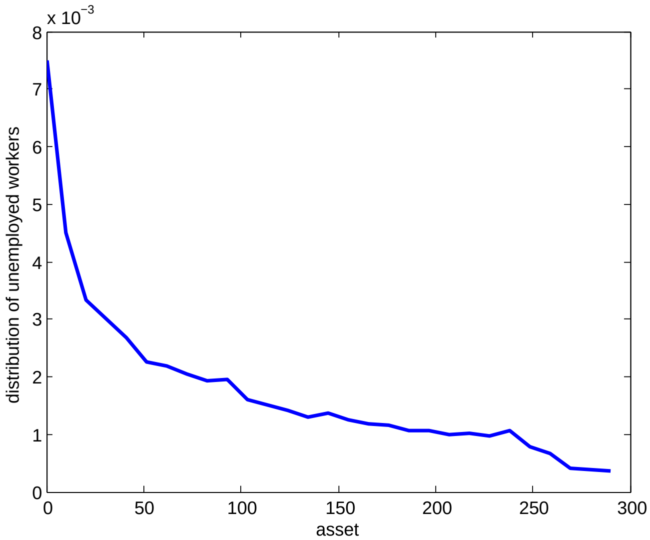
Figure 2: Policy functions of the unemployed

To maintain the same level of utility, the high asset holders consume more of their assets (Figure 2b). Interestingly, the dynamic nature of the problem now implies a time variant job choice decision. A worker who fails to become employed sees her assets gradually deplete ($a_{t+1} < a_t$). But the optimal search decision dictates application to less productive, lower wage jobs when assets are lower. As a result, over the duration of unemployment, workers will apply for less productive, lower wage jobs as assets deplete.

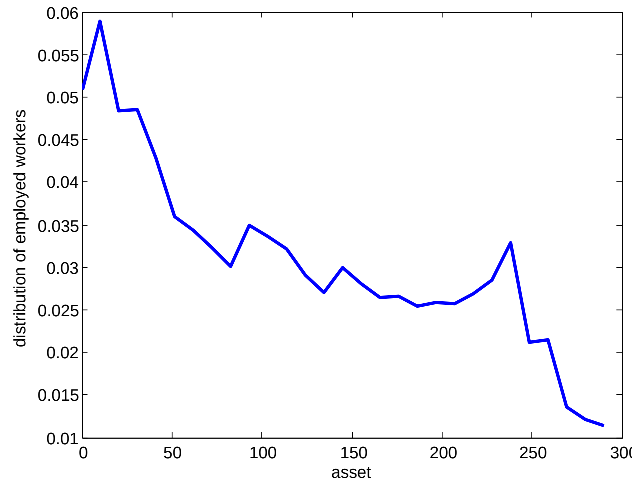
Of course, even though we analyze the steady state of the economy and derive the ergodic distributions, there is mobility at the individual level of workers. In particular, the unemployed who start off with high assets and who do not find a job deplete their assets and with time they change the type of jobs they apply for from high wage, high productivity to low wage, low productivity jobs. Instead, while employed, workers anticipate the possibility of unemployment and accumulate assets. Figure **??** shows a simulated path of a worker.

The depletion of asset during the unemployment together with different job finding rates for workers

[10]The endogenous matching probability explains why the wage function is increasing whereas it is mostly flat in the random search model (see Krusell, Mukoyama, and Sahin (2010)). The slope of the wage function is nonetheless small, because the sorting acts mostly through the matching probability. This is in part because the value of the unemployed exhibits little variation (as opposed to the value of the jobs), which can be attributed to two causes. First, the unemployed have no fixed level of assets. The value function thus is the expected value over future realizations, including while employed. Higher assets of course of course means higher utility (draws are independent), but assets can deplete fast if a worker does not find a job immediately. Second, workers have log preferences whereas firms are risk neutral.



(a) Asset Distribution Unemployed Workers



(b) Asset Distribution Employed Workers (integrated over wages)

Figure 3: Ergodic Asset Distributions (Densities)

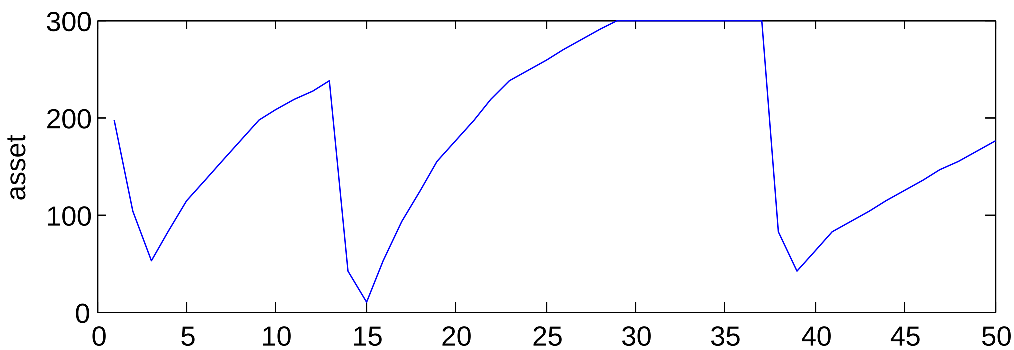


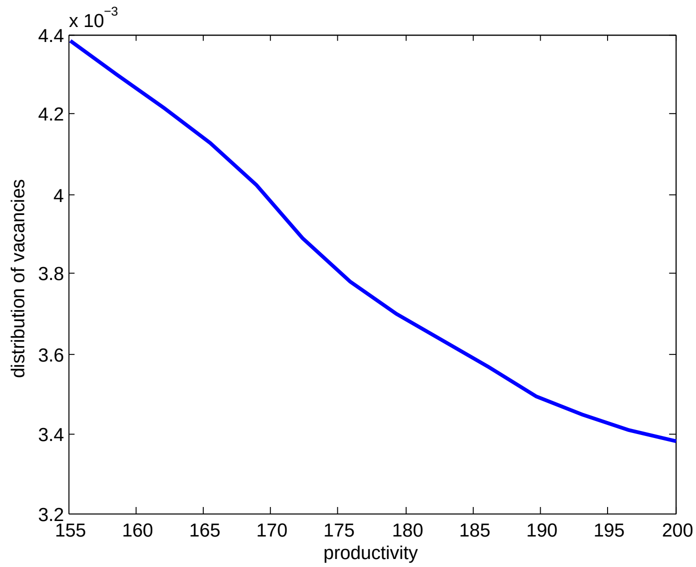
Figure 4: Simulated sample path of assets.

results in an ergodic distribution of unemployed worker assets with fat left tail (Figure 3). Even though the probability of job finding is significantly lower for high asset holders, in order to smooth consumption during unemployment, they reduce their stock of assets by dissaving and gradually stepping down the asset ladder.

Also on the firm side, we observe a fatter left tail for the stationary distribution of firms posting vacancies compared to the distribution of firms in the population, which is uniform (Figure 5). High productivity firms have a higher option value of filling a vacancy, so they increase the probability of filling the vacancy by offering higher wages to the unemployed. Therefore more productive firms leave the pool of searching firms faster than less productive ones. As a result, in steady state there are fewer high productive firms searching.

In addition to the endogeneity of the vacancy distribution, also marginal firm is endogenous. In

Figure 5: Distribution of firms

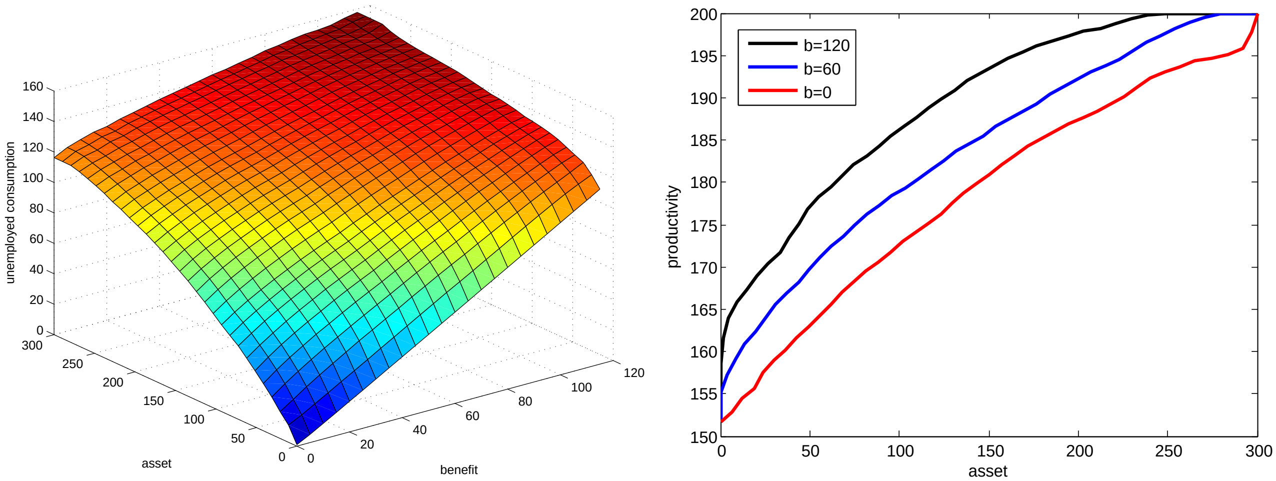


the benchmark calibration with firms on uniform on $[100, 200]$, only firms with productivity above 155 find it profitable to enter. This cutoff is thus a measure of job creation. Below, we investigate how job creation amongst other aspects is affected by unemployment benefits.

4.2 Unemployment Insurance and Value of Unemployment

We now study the impact of benefits on value of unemployment. In the absence of complete markets to insure the employment risk, we ask how changes in the government mandated unemployment insurance policy that is financed with income taxes affects value of unemployment. The direct impact of UI is that it allows higher benefits will allow workers to smooth consumption. However, UI will also affect value of unemployment through various general equilibrium channels. In the first place, higher unemployment insurance reduces the firm's share of the match surplus. With a higher outside option, workers command a higher wage. This reduces job creation as only firms with higher productivity enter the market to post vacancies. This mechanism is similar to the one pointed out in Krusell, Mukoyama, and Sahin (2010) with random search.

Specifically to our framework, direct insurance against unemployment not only affect the entry decision of firms by changing the entry threshold, but also it affects the distribution of unemployed workers by influencing their saving decisions. Guaranteed higher unemployment benefits, workers will save less, both while employed and while unemployed. More importantly, benefits also affects the workers' job search behavior. Since the workers with different asset levels direct their search to firms of different productivity, higher benefits will increase the unemployment rate (i.e., the matching probability) as well as the productivity of jobs that workers apply to. With less necessity to use their own assets for self insurance because of higher benefits, workers will increasingly direct their search towards high productivity jobs that pay higher wages at the expense of lower matching probabilities.



(a) Consumption of unemployed workers.

(b) Allocation for different levels of benefits.

Figure 6: Consumption and equilibrium allocation for different levels of benefits.

The general equilibrium effect unemployment benefits are made explicit in the following series of figures where in the benchmark economy we vary the benefits b between 0 and 120. First, consider the impact on consumption of the unemployed (Figure 6a). For all asset holders, equilibrium consumption of the unemployed increases in benefits. The effect however is much more pronounced for the low asset holders. In fact, those with close to zero assets nearly exclusively consume the entire benefits. For the high asset holders, benefits have a much more moderate impact on consumption.

Figure 6b illustrates the impact of benefits on the job search behavior. When benefits are higher, all workers direct their search to more productive, high paying jobs. As a result, for all asset levels, the allocation of assets to productivities shifts upwards as benefits increase. Benefits thus increase the productivity of the job, but it also increases the competition for jobs thus decreasing the job finding probability. This decrease is much more pronounced for the low asset holders. Benefits induces them to compete for higher productivity jobs, but while there is a big drop in the job finding probability as b increases, due to the general equilibrium effect there is only a minor increase in the allocation due to increased competition for high productivity jobs. This is most evident for the high asset holders. They apply for the high productivity jobs at any level of benefits, but they see their job finding probability drop from nearly 70% to less than 40% as benefits increase from 0 to 120. Interestingly, at the highest level of benefits, the job search behavior in terms of matching probability is very similar (both low and high asset holders find a job with probability between 35 and 40%). Yet, there continues to be a big difference in the productivity of the jobs they obtain in equilibrium.

Not surprisingly then, the impact of increased benefits is an increase in aggregate unemployment (Figure 8a). . The unemployment rate nearly doubles from 4 to 7.3% as benefits increase from 0 to 120. At the same time, the firms entering the market to open vacancies raises only modestly, from a

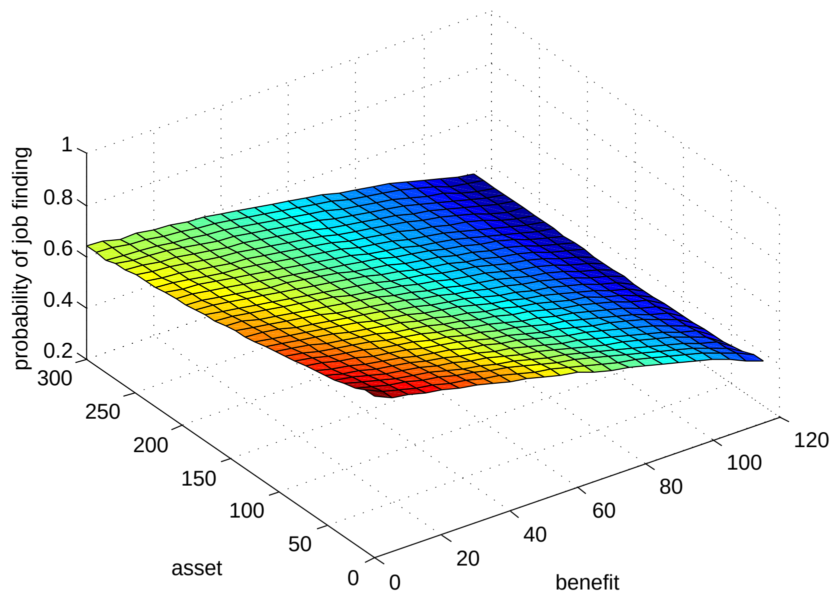
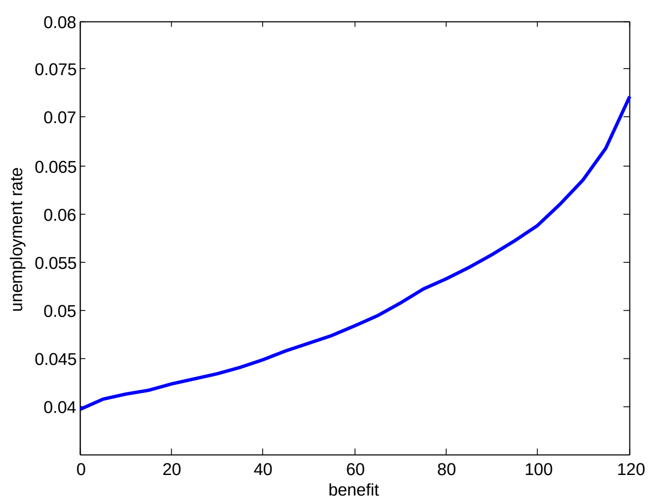
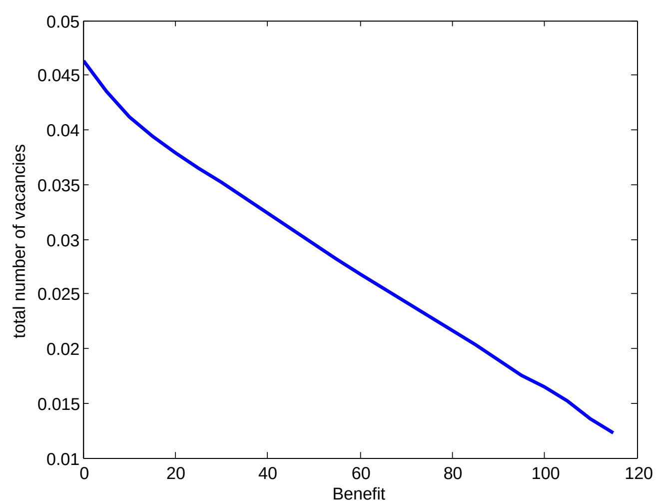


Figure 7: The job finding probability as a function of assets and unemployment benefit.

cutoff of 152 to 162. At first sight the effect on job creation is modest.



(a) Unemployment rate

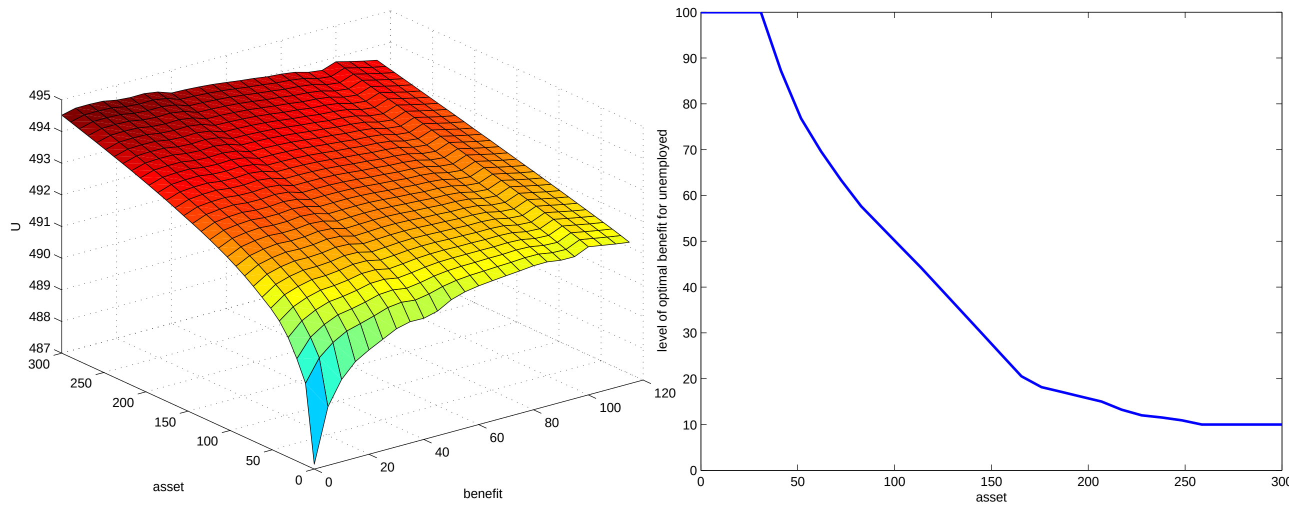


(b) Total number of vacancies

Figure 8: Equilibrium unemployment and vacancy creation for different levels of benefits.

Different benefit levels clearly pull value of unemployment in opposing directions: search for better jobs, more consumption smoothing and a more favorable asset distribution, but lower vacancy creation and lower job finding rates. To evaluate the net effect on value of unemployment, we evaluate the option value of unemployment as a function of assets and benefits. This is illustrated in Figure 9a.[11] For low asset holders, the value of unemployment increases as benefits increase. For high asset holders,

[11]Given log preferences, the variation in utility is nominally small, even if assets and benefits drop to zero.



(a) Value of unemployment as function of asset, benefits

(b) Optimal UI as function of assets

Figure 9: Value of Unemployment.

the opposite is true. Except for very low levels (benefits around 10), the value of unemployment is decreasing in benefits. While not immediately obvious from the three dimensional figure, the value of unemployment attains maximum at an interior benefit (except at the lowest assets) for any asset level. That maximum is depicted in Figure 9b. Low asset holders prefer maximal benefits, while preferred benefits are decreasing for those with more asset holdings. Overall, this suggest that for high asset holders, the allocation and probability of job finding effect dominates the consumption smoothing effect. These workers have already high level of asset an increase in UI does not affect their marginal utility of consumption much while it considerably reduces their probability of job finding. In contrast, low asset holders have high marginal utility of consumption. As mentioned, the highest asset holders prefer some benefits because life is so dire without any assets or benefits, and even the high asset holders have a positive probability of reaching that outcome.

To calculate the optimal benefit level for the economy, we integrate the preferred benefits in Figure 9b over the ergodic distribution of asset holdings of the unemployed.

5 Conclusion

We have analyzed the role of endogenous job search decisions and the distribution of assets in an economy with a precautionary savings motive. We have shown that the job search decision is an important source of self insurance for those with low assets levels, i.e. there is a precautionary job finding motive. To analyze this, we solve a model with directed search and consumption smoothing where workers with high asset holdings sort into more productive jobs. Because their asset holdings allow them to smooth consumption, they can afford to face a substantially lower job finding probability. For a calibration to the US economy, we find that the job finding rate for those with high assets is 18%

lower. This is consistent with independent findings that the poor without liquid assets find jobs faster.

In this framework, we analyze the effect of an income tax financde UI policy on value of unemployment. Not only is there the usual conflict of interest between the unemployed who receive the benefits and those with a job who pay for it, now there is a conflict between the unemployed with assets and those without. Both receive benefits, but the rich can rely on their savings for insurance, while poor workers prefers higher benefits.

It is clear that the optimal benefit scheme should be contingent on assets. With assets private information, this is however a complicated dynamic mechanism design problem in infinite horizon, with a continuum of agents and with time varying strategies. Unfortunately, we have not been able to solve that problem.

Appendix

Derivatives of U and a'

$$
\begin{aligned}
U_y &= \beta m u'(c_e')f_y + U_{a'}a_y' + U_\theta\theta_y = \beta m u'(c_e')f_y \\
U_a &= u'(a-a') + U_{a'}a_a' + U_\theta\theta_a = u'(a-a') \\
U_V &= \beta m u'(c_e')\frac{-1}{\beta q} + U_{a'}a_V' + U_\theta\theta_V = q'(c_e')\frac{-1}{q} \\
U_{ay} &= -u''(a-a')a_y'(= \partial/\partial a U_y) \\
U_{aV} &= -u''(a-a')a_V'
\end{aligned}
$$

where $U_{a'} = 0$ and $U_\theta = 0$ from the envelope theorem.

Denote the maximand of U by $\phi(a',\theta) = u(a-a') + \beta\left[mu\left(c_e'\right) + (1-m)\,u(Ra')\right]$, i.e., the objective function that is maximized with respect to a', θ. We calculate the derivative of a' using the implicit function theorem. For the problem to have a maximum, we require that the Hessian of the maximand is positive $|\mathbf{H}| > 0$ (recall that $\phi_{\theta\theta}$ is assumed negative), where:

$$
|\mathbf{H}| = \begin{vmatrix} \phi_{a'a'} & \phi_{a'\theta} \\ \phi_{\theta a'} & \phi_{\theta\theta} \end{vmatrix}
$$

Applying the implicit function theorem,

$$
a_y' = \frac{\partial a'}{\partial y} = -\frac{\begin{vmatrix} \phi_{a'y} & \phi_{a'\theta} \\ \phi_{\theta y} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'y}\phi_{\theta\theta} - \phi_{\theta y}\phi_{a'\theta}}{|\mathbf{H}|} \quad \text{and} \quad a_V' = \frac{\partial a'}{\partial V} = -\frac{\begin{vmatrix} \phi_{a'V} & \phi_{a'\theta} \\ \phi_{\theta V} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'V}\phi_{\theta\theta} - \phi_{\theta V}\phi_{a'\theta}}{|\mathbf{H}|}
$$

Proof of Proposition 1

Proof. $U_{ay} > \frac{U_y}{U_V}U_{aV}$ provided (where the partial derivatives of U are derived in the Appendix):

$$
\begin{aligned}
-u''(a-a')a_y' &> \frac{\beta m u'(c_e')f_y}{\beta m u'(c_e')\frac{-1}{q}}(-u''(a-a')a_V') \\
a_y' &> -mf_y a_V'
\end{aligned}
$$

We obtain the expressions for a_y' and a_V' from the first order conditions (above). Then the condition for positive sorting of a on y becomes:

$$
\left(\phi_{a'y} + mf_y\phi_{a'V}\right)\phi_{\theta\theta} \quad < \quad \left(\phi_{\theta y} + mf_y\phi_{\theta V}\right)\phi_{a'\theta}
$$

Observe that from the first order conditions to the maximization problem, we obtain the cross partials on ϕ. First, note that $\phi_{a'y} = -f_y q \phi_{a'V} = f_y \beta R m u''(c_e')$ so that the LHS is zero. Then we derive the following:

$$\begin{aligned}
\phi_{\theta y} &= \beta m' u'(c_e') f_y + \beta u''(c_e') f_y \frac{\theta q' V}{q} \\
\phi_{\theta V} &= \beta m' u'(c_e') \frac{-1}{q} + \beta u'(c_e') \frac{\theta q'}{q} + \beta u''(c_e') \frac{-1}{q} \frac{\theta q' V}{q} \\
&= \frac{-1}{f_y q} \phi_{\theta y} + \beta u'(c_e') \frac{\theta q'}{q}
\end{aligned}$$

Therefore, the inequality can be written as:

$$0 < \beta u'(c_e') \theta q' f_y \phi_{a'\theta}$$

The condition for positive sorting of a on y is $\phi_{a'\theta} < 0$ is thus,

$$\beta R \left(m'[u'(c_e') - u'(Ra')] + u''(c_e') \frac{\theta q' V}{q} \right) < 0.$$

From the first order condition $\phi_\theta = 0$ we obtain:

$$\frac{\theta m' V}{q} = -m' \frac{u(c_e') - u(Ra')}{u'(c_e')}.$$

Substituting in the condition $\phi_{a'\theta} > 0$:

$$m'[u'(c_e') - u'(Ra')] - u''(c_e') m' \frac{u(c_e') - u(Ra')}{u'(c_e')} > 0,$$

or, noting that $m' < 0$,

$$u'(c_e')[u'(c_e') - u'(Ra')] < u''(c_e') \left[u(c_e') - u(Ra') \right].$$

or alternatively

$$\frac{u'(c_e') - u'(Ra')}{u(c_e') - u(Ra')} < \frac{u''(c_e')}{u'(c_e')}.$$

■

Proof of Proposition 2

Proof. We calculate the derivatives:

$$\begin{aligned} u'(c) &= \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \\ u''(c) &= -\alpha^2 \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-2} \end{aligned}$$

and condition (**U**) becomes (where $c = Ra'$):

$$\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} \left[\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} - \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \right] <$$
$$-\alpha^2 \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-2} \left[\frac{1-\gamma}{\gamma} \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma} - \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma} \right]$$

and after dividing by α^2 and by $\left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{2\gamma-2}$, which under our assumptions are both positive, this implies:

$$1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma} \right],$$

or

$$1 - x^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - x^{\gamma} \right] \quad \text{where} \quad x = \frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \in (0,1).$$

First consider $\gamma > 0$. After rearranging and multiplying by $\gamma x^{1-\gamma}$, which is positive for $\gamma > 0$:

$$\begin{aligned} x^{1-\gamma} - (\gamma + (1-\gamma)x) &< 0 \\ G(\gamma) - H(\gamma) &< 0. \end{aligned}$$

At $\gamma = 0$ and $\gamma = 1$ the expression is exactly zero, i.e., G and H cross at 0 and 1. Now, $G'(\gamma) = -x^{1-\gamma} \log x, H'(\gamma) = 1 - x$, and $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0, H''(\gamma) = 0$. Observe that $G(\gamma)$ is convex, $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0$, while $H(\gamma)$ is linear. As a result, for $\gamma \in (0,1)$ condition (**U**) holds with strict inequality. For $\gamma = 1$, (**U**) holds with equality and for $\gamma > 1$ it holds with opposite inequality.

Now consider $\gamma < 0$. Since we multiplied by $\gamma < 0$, condition (**U**) now implies that $G(\gamma) - H(\gamma) > 0$. Using the same logic, we establish that condition (**U**) holds for $\gamma < 0$.

This establishes that for a risk averse worker with HARA utility function, condition (**U**) holds strictly if and only if $\gamma < 1$, i.e., there is DARA. The condition holds with opposite inequality when there is IARA and $\gamma > 1$. ■

Proof of Corollary 1

Proof. All the cases can immediately be verified from Proposition 2, except for the case of CARA. There, $u'(c) = \alpha e^{-\alpha c}, u''(c) = -\alpha^2 e^{-\alpha c}$, so that condition (**U**) becomes:

$$\begin{aligned} \alpha e^{-\alpha c_e}\left(\alpha e^{-\alpha c_e} - \alpha e^{-\alpha c}\right) &\leq -\alpha^2 e^{-\alpha c_e}\left(1 - e^{-\alpha c_e} - 1 + e^{-\alpha c}\right) \\ e^{-\alpha c_e} - e^{-\alpha c} &\leq -\left(-e^{-\alpha c_e} + e^{-\alpha c}\right) \end{aligned}$$

which holds with equality. ■

Proof of Proposition 3

Proof. It is immediate that this condition is not satisfied when $u''' = 0$. To see this, observe that then $u'(c'_e) - u'(Ra') = wu''(c'_e)$ and the condition (**U**) can be written as $u'(c'_e)wu''(c'_e) < u''(c'_e)\left[u(c'_e) - u(Ra')\right]$, or $u'(c'_e)w > u(c'_e) - u(Ra')$. This condition only holds under convexity of u, and therefore is never satisfied for risk averse agents.

When $u''' < 0$, we have instead that $u'(c'_e) - u'(Ra') > wu''(c'_e)$, so the left hand side is even smaller, and again, condition (**U**) implies $u'(c'_e)w > u(c'_e) - u(Ra')$, which is not satisfied for risk averse agents.

Now consider $u''' > 0$. Then we can write the utility function and its derivative as

$$\begin{aligned} u(c) &= u(c'_e) + u'(c'_e)(c - c'_e) + \frac{u''(c'_e)}{2}(c - c'_e)^2 + \cdots \\ u'(c) &= u'(c'_e) + u''(c'_e)(c - c'_e) + \frac{u'''(c'_e)}{2}(c - c'_e)^2 + \cdots \end{aligned}$$

and therefore condition (**U**) becomes:

$$\begin{aligned} u'(c'_e)\left[u''(c'_e)(c'_e - c) - \frac{u'''(c'_e)}{2}(c'_e - c)^2 + \frac{u''''(c'_e)}{6}(c'_e - c)^3 - \cdots\right] < \\ u''(c'_e)\left[u'(c'_e)(c'_e - c) - \frac{u''(c'_e)}{2}(c'_e - c)^2 + \frac{u'''(c'_e)}{6}(c'_e - c)^3 - \cdots\right]. \end{aligned}$$

Canceling terms and dividing by $(c'_e - c)^2$, this condition implies that at least for small $c'_e - c = w$ implies

$$u'''(c'_e) > \frac{u''(c'_e)^2}{u'(c'_e)}.$$

This is equivalent to requiring that the coefficient of risk aversion $A(c) = -\frac{u''}{u'}$ is decreasing, i.e., $A' = -\frac{u'''u' - (^2u'')}{u'^2}$ or $u''' > \frac{(u'')^2}{u'} = -u''A(c)$. ■

A Counter Example

Example 1 *Let w be large enough and find a u-function with u'''' suitably chosen such that the condition is not satisfied. Let $u(c)$ be defined as:*

$$u(c) = u(c_e') + u'(c_e')(c - c_e') + \frac{u''(c_e')}{2}(c - c_e')^2 + \frac{u'''(c_e')}{6}(c - c_e')^3 + \frac{u''''(c_e')}{24}(c - c_e')^4.$$

Evaluating u at $c = Ra'$ and observing that $c_e' - Ra' = w$ we can then write

$$\begin{aligned} u(c_e') - u(Ra') &= u'(c_e')w - \frac{1}{2}u''(c_e')w^2 + \frac{1}{6}u'''(c_e')w^3 - \frac{1}{24}u''''(c_e')w^4 \\ u'(c_e') - u'(Ra') &= u''(c_e')w - \frac{1}{2}u'''(c_e')w^2 + \frac{1}{6}u''''(c_e')w^3. \end{aligned}$$

*Now we can write condition (**U**) as (where u denotes $u(c_e')$):*

$$\begin{aligned} u'\left[u''w - \frac{1}{2}u'''w^2 + \frac{1}{6}u''''w^3\right] &< u''\left[u'w - \frac{1}{2}u''w^2 + \frac{1}{6}u'''w^3 - \frac{1}{24}u''''w^4\right] \\ u'u''' &> u''^2 - \frac{1}{3}u''u'''w + \frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right] \end{aligned}$$

Observe that $u'u''' > u''^2$ is the standard condition for Decreasing Absolute Risk Aversion. But for any $u''' > 0$, however large, we can find a utility function with $\frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right]$ sufficiently large such that the inequality is not satisfied. For example, if $u' + \frac{1}{4}u''w > 0$ we can choose u'''' positive and large. Conversely, if $u' + \frac{1}{4}u''w < 0$ we can choose u'''' sufficiently negative such that the inequality does not hold.

Proof of Proposition 4

Proof. The solution to the (interior) maximization problem is $a'(a, y, V), \theta(a, y, V)$ and satisfies:

$$\begin{aligned} -u'(c_u) + \beta[qE_{a'}(a', w') + (1 - m)U_{a'}(a')] &= 0 \\ m'[E(a', w') - U(a')] + qE_{w'}(a', w')\frac{\partial w'}{\partial \theta} &= 0 \end{aligned}$$

Now we have monotone matching of a in y provided: $U_{ay} > \frac{U_y}{U_V} U_{aV}$.

$$\begin{aligned}
U_y &= m\beta E_{w'}(a',w').\frac{\partial w'}{\partial y} + U_{a'}a'_y + U_\theta \theta_y = m\beta E_{w'}(a',w') f_y \\
U_a &= u'(c) + U_{a'}a'_a + U_\theta \theta_a = u'(c_u) \\
U_V &= m\beta E_{w'}(a',w').\frac{\partial w'}{\partial V} + U_{a'}a'_V + U_\theta \theta_V = -m\beta E_w(a',w')\frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{q} \\
U_{ya} &= -u''(a-a')a'_y \\
U_{aV} &= -u''(a-a')a'_V
\end{aligned}$$

where $U_{a'} = 0$ and $U_\theta = 0$ from the envelope theorem. Then:

$$\begin{aligned}
U_{ay} &> \frac{U_y}{U_V} U_{aV} \\
-u''(a-a')a'_y &> \frac{m\beta E_w(a',w') f_y}{-m\beta E_w(a',w')\frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{q}}(-u''(a-a')a'_V) \\
a'_y &> -\frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y a'_V
\end{aligned}$$

Writing the Hessian $|\mathbf{H}| > 0$ as:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a'a'} & \phi_{a'\theta} \\ \phi_{\theta a'} & \phi_{\theta\theta} \end{vmatrix}$$

then

$$a'_y = \frac{\partial a'}{\partial y} = -\frac{\begin{vmatrix} \phi_{a'y} & \phi_{a'\theta} \\ \phi_{\theta y} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} \quad \text{and} \quad a'_V = \frac{\partial a'}{\partial V} = -\frac{\begin{vmatrix} \phi_{a'V} & \phi_{a'\theta} \\ \phi_{\theta V} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|}$$

$$\begin{aligned}
a'_y &> -\frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y a'_V \\
\phi_{a'y}\phi_{\theta\theta} - \phi_{\theta y}\phi_{a'\theta} &< -\frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y \left(\phi_{a'V}\phi_{\theta\theta} - \phi_{\theta V}\phi_{a'\theta}\right) \\
\left(\phi_{a'y} + \frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y \phi_{a'V}\right)\phi_{\theta\theta} &< \left(\phi_{\theta y} + \frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y \phi_{\theta V}\right)\phi_{a'\theta} \quad (11)
\end{aligned}$$

Observe that from the first order conditions to the (interior) maximization problem, we obtain the cross partials on ϕ. First, note that:

$$\phi_{a'y} = -\frac{q}{[1-\beta(1-\lambda)][1-\beta(1-q)]} f_y \phi_{a'V}$$

so that the LHS is zero. Then deriving the expression for $\phi_{\theta y}$ and $\phi_{\theta V}$: Note that: $m'[E(a',w') - U(a')] + qE_{w'}\frac{\delta w'}{\delta\theta}] = 0$ and this implies:

$$\begin{aligned}
\phi_{\theta y} &= q'E_{w'}(a',w')\frac{\partial w'}{\partial y} + qE_{w'w'}(a',w')\frac{\partial w'}{\partial y}\frac{(1-\beta)(1-\beta(1-\lambda))}{q^2}V \\
\phi_{\theta V} &= q'E_{w'}(a',w')\frac{\partial w'}{\partial V} + qE_{w'w'}(a',w')\frac{\partial w'}{\partial V}\frac{(1-\beta)(1-\beta(1-\lambda))}{q^2}V + qE_{w'}(a',w')\frac{(1-\beta)(1-\beta(1-\lambda))}{q^2}
\end{aligned}$$

the RHS reduces to:

$$\frac{m}{q(1-\beta(1-1-q))}f_y E_{w'}(a',w')\phi_{a'\theta}$$

Therefore, the inequality (11) is satisfied provided $\phi_{a'\theta} > 0$.

$$\begin{aligned}
\phi_{a'\lambda} &= \beta m'[E_{a'}(a',w) - U_{a'}(a')] + \beta m E_{a',w}(a',w')\frac{\partial w'}{\partial\theta} \\
&= \beta m'[E_{a'}(a',w') - U_{a'}(a')] + \beta m E_{a',w'}(a',w)\frac{-1}{mE_{w'}(a',w')}m'[E(a',w') - U(a')]
\end{aligned}$$

since from FOC for θ

$$\frac{\partial w'}{\partial\theta} = \frac{-\beta m'}{qE_{w'}(a',w')}[E(a',w') - U(a')]$$

Therefore $\phi_{a'\lambda} > 0$ provided

$$\frac{E_{a'}(a',w') - U_{a'}(a')}{E(a',w') - U(a')} < \frac{E_{a',w'}(a',w')}{E_{w'}(a',w')}$$

■

The Algorithm to Calculate the Equilibrium Allocation

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