Understanding the Role of Public Employment Services: Theory and Empirical Evidence*

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Abstract

Using the German Job Vacancy Survey we are able to show how the Public Employment Services (PES) affects the labor market. Almost half of all vacancies in Germany are registered with the PES. The PES is shown to distribute workers more evenly among vacancies compared to the private market. This reduction in coordination frictions is going along with lower wages paid by registered firms. Applicants send by the PES are on average less suited for the job than applicants attracted through the private market. We rationalize our empirical findings using a directed search model, where firms can decide whether to search via the PES or the private market. Lower coordination frictions in the PES induce less wage competition between firms enabling them to pay lower wages. The implicit cost of registering a vacancy is due to the less suited pool of applicants and explains why not all vacancies are registered with the PES despite the fact that its service is free of charge.

Keywords: Labor Search, Intermediation, Public Employment Services

JEL: J6

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The survey data used are confidential but not exclusive. Access to the data is provided by the Research Data Centre of the German Federal Employment Agency. For details, see fdz.iab.de. To facilitate replication the Stata do-files are available from the first author upon request.

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1 Introduction

In most industrialized countries Public Employment Services (hereafter, PES) provide job-broking services – help job seekers to obtain jobs and employers to fill vacancies. The market places provided by PESs are open to all job seekers and vacancies at zero cost. Still, the costless services provided by the PESs do not attract all of them. The existing empirical literature shows that between 35 to 55 percent of all job seekers, including employed workers, and up to 85 percent of unemployed job seekers use the PESs.[1] The evidence on the fraction of registered vacancies is scarce. Pollard et al. (2012) report in a survey for the Department of Work and Pensions that in the Unitied Kingdom 39 percent of all establishments used the Jobcentre Plus in 2011. We find that in the years 2005 to 2010 around 49.7 percent of all vacancies in Germany registered with the PES.

Despite the importance of the PES as a search channel very little is known about its impact on the labor market. The existing empirical literature has only studied the use of the PES and the effects of using the PES from the job seekers perspective. It showed that unemployment benefit recipients, low skilled workers, long-term unemployed and workers with few job opportunities are more likely to use a PES.[2] The empirical evidence for the effect on wages is mixed. Holzer (1988) reports for the US and Addison and Protugal (2002) for Portugal that workers, who searched through the PES, received lower wage offers (after controlling for worker characteristics). Osberg (1993) finds for Canada and Weber and Mahringer (2002) for Austria that the wage difference disappears after controlling for worker characteristics.

Many questions regarding the effect of the PES on the labor market still remain unanswered, first and foremost, the question whether the PES is able to reduce search frictions in the labor market, the core goal set by governments, which finance the PES, the question whether registered vacancies pay lower wages compared to unregistered vacancies, the question whether applicants send by the PES are on average less suited for the job than applicants attracted through the private market, and finally the question why are not all vacancies registered with the PES given that advances in technology make it possible for PESs to centralize job applications and to mitigate coordination frictions.

We use the German Job Vacancy Survey, a representative establishment data-set collected by the Federal Employment Agency in Germany, which contains numerous questions regarding the recruitment process, to investigate these questions. Most importantly we know how many applicants and how many suitable applicants a registered firm received from the PES and through other search channels. We first show that job- and firm-characteristics that are strongly correlated with the number of applicants attracted through the private market play no role in explaining the number of applicants send by the PES. This indicates that the PES distributes its job seekers independent of these characteristics. In a second step we compare the (residual) variance of the number of applicants send by the PES and

[1] See Holzer (1988) and Blau and Robins (1990) for the US, Osberg (1993) for Canada, Gregg and Wadsworth (1996) for the UK, Addison and Protugal (2002) for Portugal, and Weber and Mahringer (2002) for Austria.

[2] See Holzer (1988) and Blau and Robins (1990) for the US, Osberg (1993) for Canada and Gregg and Wadsworth (1996) for the UK.

attracted through the private market. The significantly lower variance in the number of applicants send by the PES compared to the number of applicants received through the private market shows that the PESs distribute workers more evenly among vacancies. This supports the hypothesis that the PESs reduce coordination frictions.

We address the question how affect the PES wages by looking at two pieces of information. First, we are able to show that registered firms pay 8 percent lower wages than not registered firms. Although we do not claim causality, we can exclude that this effect is driven by observable person characteristics like work experience and previous employment status or by observable job and firm characteristics including industry and occupation. We can also show that firms registered with the PES are more likely to report difficulties in the recruitment process, because their applicants demanded higher wages. This supports not only the direct evidence above that firms, which register their vacancy with the PES, offer lower wages than firms, which did not register their vacancy, it also implies that one and the same worker can receive multiple wage offers, which differ systematically depending on whether the vacancy is registered with the PES or not. Our evidence from the vacancies perspective is complementary to the inconclusive evidence of the existing literature, which used individual data on search channels used. It suggests that there is indeed a significant difference in the wages paid by registered and unregistered vacancies.

By comparing the fraction of suitable applicants send by the PES and attracted through the private market we can show that the fraction of suitable applicants at vacancies registered with the PES is about 10 percent lower than at vacancies that only use decentralized search channels. This evidence is complementary to the evidence provided in the existing literature, which shows that less suited workers are more likely to use PESs (see footnote 2). Our complement is important in order to understand the role of PESs in the labor market, since the evidence from the workers' side alone does not necessarily imply that registered vacancies receive on average a less suited pool of applicants. It could have been that PESs sort out less suited workers and thereby help registered firms to overcome information asymmetries. Instead, our empirical result suggests that this is not the case.

This last finding is also able to explain why not all vacancies register with the PES despite the fact that, it reduces coordination frictions, enables firms to pay lower wages. and provides its service is free of charge. It is the implicit cost associated with the less suited pool of applicants, which is able to explain why not all vacancies are registered with the PES.

To rationalize our empirical findings we construct a directed search model, where firms can decide whether to search via the PES or the private market. Our theoretical model extends the pioneering work on the role of PESs by Pissarides (1979). Like him we assume that all unemployed are registered at the PES, that searching in the decentralized market is costly for workers, and that firms can choose between two alternative methods of finding a worker: the search market and the PES.

There are two major differences between his and our model. First, workers are homogeneous in Pissarides (1979), while the key ingredient of our model is that workers differ in productivity, and

that firms have some technologies to screen applicants, imperfectly though, at the recruitment stage. Second, Pissarides (1979) assumes an exogenous and identical wage in both markets. In contrast, by adopting a directed search approach, we show that firms in the decentralized market choose to post higher wages than firms registered with the PES in order to attract workers despite the cost associated with searching in the private market. Since the value of searching in the private market is higher for high productivity workers, the endogenous wage differential allows firms in the private market to attract a better pool of applicants. In an extension, Pissarides (1979) considers the limiting case where search frictions are eliminated in the PES. He finds that in this case the private market collapses and all workers search via the PES. This is in contrast to our model. We show that the positive selection of workers in the search market ensures the existence of the decentralized search market even if the PES manages to match the short side of the market.

Our model is related to the literature of intermediation. Watanabe (2010, 2013) provide a directed search model of middlemen (e.g., retailers, wholesalers, trading entrepreneurs, dealers or brokers of services and durable goods and assets).[3] He demonstrates that backed by the capability of dealing with many agents at a time, middlemen find it optimal to provide customers with proximity or a lower likelihood of experiencing stockout, charging a higher price. That the PES in the present framework provides a coordinated transaction is similar to his middlemen's capability of pursuing large-scaled dealings. However, in contrast to the literature, the PES does not act as a private agent, who charges a premium for their service, which raises the question why not all agents use the middlemen. In a recent progress, Gautier, Hu and Watanabe (2015) offer a hybrid model of middlemen and market-makers (i.e., platform), and study the choice of the two alternative intermediation modes. Unlike in the present model, agents are homogeneous and the issue of differential composition of heterogeneous agents is not addressed.

Our framework is also related to the directed search literature with private information, which assumes that firms observe a signal before hiring a worker. Menzio (2007) considers wage-bargaining in a framework where firms use cheap talk to signal private information about the quality of a vacancy. Delacroix and Shi (2013) consider a directed search model with signaling, where sellers can distort their posted prices upward in order to signal high quality. In our case, firms receive a signal after screening workers, which allows them to reject workers with a low signal. We also assume that firms cannot condition their wages on the signal they receive (or demand an application fee). We therefore have a directed search model with incomplete contracts like Michelacci and Suarez (2006), who analyze under which conditions firms prefer wage posting over wage bargaining. The incomplete contracting assumption distinguishes our model also from the work by Guerrieri, Shimer, and Wright (2010), who consider an environment where firms can post complete contracts and who establish that firms can

[3] A seminal work in the literature of middlemen is Rubinstein and Wolinsky (1987). While most of the models in the literature would be viewed as describing general markets, there are some models (e.g., Yavas, 1994, Masters, 2007, Watanabe, 2010) that feature labor market intermediation or turnover behaviors. See Watanabe (2013) for the references of the recent contributions.

always use these contracts to separate types.[4] The novelty of our approach is that our workers hold not only private information about their type but they are also able to search in two markets simultaneously.

In the directed search literature, where workers simultaneously apply for multiple jobs, Albrecht, Gautier and Vroman (2006) show that firms engage in Bertrand competition, if their applicant receives two or more offers. Unlike in their setup, we assume wage commitment and show that a low wage can survive in equilibrium due to the coordinated allocation mechanism used by the PES. Galenianos and Kircher (2009) consider the case of commitment with homogeneous workers and show the existence of an equilibrium wage dispersion. In our model, workers can apply only to one firm in the decentralized market, just like in the standard directed search models, but there are some workers who use both the search market and the PES, and hence receive multiple offers, one from the search market and the other from the PES. In this setup we show that, while the search market has a unique wage, a wage differential exists between the search market and the PES. Our modeling choice reflects the institutional difference between the two market places in reality.

There are very few other papers that consider the role of PESs. Fugère et al. (2009) uses a structural partial search equilibrium model to investigate the hypothesis by Pissarides (1979) that more efficient PESs crowed out private search effort. Their partial equilibrium model considers exogenous, search-channel-specific wage-offer distributions and allows for an endogenous search intensity of workers. Using French data to structurally estimate the model they find that the exit rate from unemployment increases with the arrival rate of job contacts obtained by the PES. This is especially the case for low-skilled workers. Launov and Wälde (2014) use a structural model to analyze to which extend an increase in operating efficiency of PESs on the one hand and a reduction of unemployment benefits on the other are responsible for the decline in unemployment in Germany from 11.7% in 2005 to 7.8% in 2008.

Finally, Casella and Hanaki (2008) and Galenianos (2013) study firms' use of referrals by their own employees in addition to formal hiring channels.[5] Referred workers may be more suited for the job, because referred workers carry a more accurate productivity signal than workers contacted through a formal search channel. The search channels in our model do not differ in their signaling ability. They differ in the allocation mechanism used. In the search market firms can increase the probability to meet a worker by offering a higher wage, while firms' meeting probability at the PES is independent of the wages they offer. The novelty of our paper is to show that the higher degree of wage competition among firms in the search market compared to the PES leads to a positive selection of applicants.

The sequel of the paper is organized as follows. In Section 2, we use data from the German Job Vacancy Survey to show which factors determine the use of PESs as search channel, to show that PESs reduce coordination frictions, that registered vacancies pay lower wages, and that the applicants send

[4] In the competing auctions literature by e.g. McAfee (1993), Peters and Severinov (1997), Albrecht, Gautier, and Vroman (2012, 2014) sellers uses auctions to direct buyers with private information.

[5] Galenianos (2014) models the use of referrals by expanding firms in order to explain inter-industry variation in aggregate matching efficiency.

by the PESs are on average less suited. Section 3 presents the theoretical model of the labor market in the presence of the PES. Section 4 concludes. All proofs are collected in the Appendix.

2 Empirical Analysis

2.1 German Job Vacancy Survey

We use the German Job Vacancy Survey, a dataset collected by the Federal Employment Agency in German. The German Job Vacancy Survey is based on a representative sample of establishments, which is newly sampled each year. The yearly survey started in 1989 and was initially conducted to provide an estimate of the total number of vacancies in Germany relative to the number of vacancies registered with the PES.

The survey includes establishment level data on the number of employees, number of vacancies, hires and quits in the last 12 months, and information on the industry and region of the firm. The economic conditions of a firm can be proxied by binary indicator variables for "low sales", "financial constraints", and "not enough suitable employees". It also contains a number of questions concerning the last case of a successfully filled vacancy. In these questions firms were asked to provide information on the qualification and experience level required for the job, on job characteristics like permanent/temporary, full-/part-time, and weekend-work and on whether the vacancies were registered with the PES. We use the years 2005 to 2010, since the information on the number of applicants and the number of suitable applicants is only available since 2005. For the years 2005 to 2008 the data also contains information on the number of applicants and the number of suitable applicants, which were send by the PES. The data also contains information on whether a firm experienced difficulties in the recruitment process due to "high wage demands" of applicants, which we use to investigate the existence of a wage difference between registered and unregistered vacancies. Data on wages paid to newly hired workers is only available for the year 2014.[6]

2.2 Who registers with and hires through the PES?

In the years 2005 to 2010 53.7% of all vacancies in Germany were advertised in news papers, 49.7% have been registered with the PES (16.6% only at the agency, 11.9% only on the PES-online platform and 21.2% on both), and 46.8% of all vacancies were posted on the internet (excluding the PES-online-platform). The PES is thus the second most used search channel to contact workers. We start with investigating which types of firms register which type of vacancy with the PES. We denote those vacancies as registered that use the PES among other search channels. This definition also includes registrations on the online platform of the PES.

[6]This data is to date not publicly available. We are very grateful to Hermann Gartner from the IAB that he did these regressions for us.

Table 1: **PES used as search and hiring channel (deskriptive statistics)**

	Fraction of vacancies (in %) using PES as search channel	hiring channel
all vacancies	49.7	18.4
low qualification	59.2	19.6
medium qualification	52.2	22.0
high qualification	34.4	5.9
occupation specific experience	48.9	17.8
permanent	47.8	16.0
full-time	50.1	18.1
weekend-work	51.3	17.9
firm size (1 - 9)	55.7	25.6
firm size (10 - 19)	48.9	21.0
firm size (20 - 49)	48.6	20.2
firm size (50 - 199)	52.0	21.7
firm size (200 - 499)	49.2	15.7
firm size (500 +)	45.2	9.7
agriculture	60.7	27.1
food, textile, clothes and furniture	41.2	18.2
wood, paper and printing	45.5	15.3
chemicals and plastic products	40.7	14.5
basic metal and metal products	51.9	16.8
machinery, electric, motor vehicles	35.5	11.1
energy and mining	29.1	9.6
construction	56.5	25.8
wholesale and retail trade	44.3	19.6
hotels and restaurants	44.2	18.9
traffic and transportation	50.5	15.3
banking and insurance	31.8	8.7
business related services	56.0	20.2
other public and private services	49.3	20.4
health and social services	48.1	16.2
public administration	60.4	19.7

Source: German Job Vacancy Survey 2005-2010.
Weighted averages using sampling weights.

The use of the PES as a search channel is almost evenly distributed across all firm sizes. Only small firms with less than 10 employees, which register around 55.7% of their vacancies, are more likely and large firms with more than 500 employees are less likely to use the PES as a search channel. Also the fraction of small (large) firms that hire their worker through the PES is with 25.5% (9.7%)

substantially higher (smaller). The use of the PES as a search channel also differs across industries. The agricultural sector with 60.7% registered vacancies, the construction sector with 56.5% registered vacancies and business related services with 56.0% are on the upper end while the use of the PES as a search channel is far less widespread among the energy and mining industry (29.1%), the banking and insurance industry (31.8%), and in the machinery, electric, motor vehicles industry (35.5%).

As shown in the multivariate regression below the differences across industries are mainly driven by the kind of jobs that are posted. Jobs, which require no formal or very low qualifications, are with 59.2% more likely to be registered with the PES than jobs, which require a medium (52.2%) or a high qualification (34.4%). The fraction of registered jobs requiring occupation specific training (48.9%) or weekend-work (51.3%) and the fractions of full-time (50.1%) and permanent (47.8%) jobs that are registered with the PES do not differ much from the average. Table 1 shows all summary statistics.

Firms use on average three different search channels. Without having any prior on the effectiveness of a search channel one would expect that one-third of the 49.7% registered firms hire a worker through the PES. The fraction of firms that contacted the person, which they hired, through the PES is with 18.4% slightly higher than one-third of 49.7%. The effectiveness differs. The effectiveness is above average for jobs requiring a medium qualification (with a hiring rate of 22% for 52.2% using the PES) and jobs at firms with less than 20 employees (with a hiring rate of 25.6% and 21.0% for 55.7% and 48.9% using the PES). For jobs requiring high qualifications and jobs in firms with more than 500 employees the hiring rate is with 5.9% and 9.7% substantially lower than what one would have expected according to the respective 34.4% and 45.2% registration rate. The effectiveness also differs a lot across industries.

In Table 2 we show which firm and vacancy characteristics are associated with using the PES as search or hiring channel. Table 2 presents marginal effects based on multivariate probit regressions. One would expect that those firm and vacancy characteristics, which are highly correlated with a successful hiring through the PES, to be also highly correlated with the use of the PES as a search channel. This is true for jobs requiring high qualifications and weekend-work, permanent jobs and jobs at large firms. All these factors are negatively correlated with the use of the PES as hiring and search channel. Firms that are financially constrained are more likely to use the PES as hiring as well as search channel. Positively correlated with using the PES as search channel are full-time jobs and jobs at firms, which report "low sales" or "having not enough suitable employees". Full-time jobs and jobs at firms, which report "low sales", are, however, not more likely to hire a worker through the PES. A job posted by a firm, which reports "having not enough suitable employees", is even less likely to hire through the PES. Table 2 also shows that job and firm characteristics explain most differences in PES registration and hiring success among industries. Only the energy and mining industry and the banking and insurance industry are less likely to post vacancies at the PES although they are not less likely to hire a worker through the PES once they have registered their vacancy with the PES.

In columns (3) and (4) of Table 2 we present the correlations between job and firm characteristics and PES use as a search and hiring channel for a subsample, which only includes vacancies that

were successful in filling their vacancy before the intended starting data of the employment contract. Comparing columns (3) and (4) with the results of the full sample in columns (1) and (2) is reassuring. It shows that there is no systematic difference between vacancies that are successful within the expected period and vacancies that need longer.

Table 2: **PES used as search and hiring channel**

	Probit Regressions: Marginal Effects			
	full sample		restricted sample	
	PES used as		PES used as	
	search channel	hiring channel	search channel	hiring channel
	(1)	(2)	(3)	(4)
number of search channels		-0.3687***		-0.3888***
		(0.0117)		(0.0184)
number of search channels^2		0.0340***		0.0366***
		(0.0017)		(0.0028)
low qualification	0.0097	0.0226	0.0125	0.0413*
	(0.0150)	(0.0189)	(0.0195)	(0.0251)
high qualification	-0.0712***	-0.0571***	-0.0627***	-0.0314*
	(0.0107)	(0.0147)	(0.0135)	(0.0186)
occupation specific experience	0.0116	-0.0145	0.0027	-0.0164
	(0.0083)	(0.0107)	(0.0106)	(0.0139)
permanent	-0.0400***	-0.0718***	-0.0409***	-0.0739***
	(0.0086)	(0.0110)	(0.0110)	(0.0144)
full-time	0.0576***	0.0048	0.0537***	0.0025
	(0.0111)	(0.0140)	(0.0136)	(0.0174)
weekend-work	-0.0273**	-0.0436***	-0.0423***	-0.0496**
	(0.0121)	(0.0155)	(0.0151)	(0.0202)
firm size (10 - 19)	-0.0879***	-0.0305	-0.1192***	-0.0036
	(0.0167)	(0.0193)	(0.0216)	(0.0257)
firm size (20 - 49)	-0.1444***	-0.0314*	-0.1597***	-0.0262
	(0.0156)	(0.0188)	(0.0200)	(0.0248)
firm size (50 - 199)	-0.1504***	-0.0569***	-0.1684***	-0.0520**
	(0.0161)	(0.0192)	(0.0205)	(0.0251)
firm size (200 - 499)	-0.1354***	-0.0464**	-0.1379***	-0.0694**
	(0.0183)	(0.0233)	(0.0232)	(0.0300)
firm size (500 +)	-0.1326***	-0.0774***	-0.1326***	-0.0745**
	(0.0189)	(0.0246)	(0.0242)	(0.0313)
"financial constraints"	0.0530***	0.0527***	0.0616***	0.0352
	(0.0150)	(0.0189)	(0.0189)	(0.0226)
"low sales"	0.0278**	0.0286	0.0286*	0.0203
	(0.0122)	(0.0158)	(0.0158)	(0.0198)
"not enough suitable employees"	0.1636***	-0.0224	0.1261***	-0.0611**
	(0.0148)	(0.0178)	(0.0219)	(0.0270)
year-, region-, industry-FE	yes	yes	yes	yes
(year x region)-FE	yes	yes	yes	yes
Peudo-R^2	0.0480	0.2342	0.0494	0.2713
N	6,593	6,590	3,731	3,730

Source: German Job Vacancy Survey 2005-2010.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

Continued Table 2: **PES used as search and hiring channel**

	Probit Regressions: Marginal Effects			
	full sample		restricted sample	
	PES used as		PES used as	
	search channel	hiring channel	search channel	hiring channel
	(1)	(2)	(3)	(4)
food, textile, clothes and furniture	-0.0196	0.0409	-0.0523	0.0627
	(0.0327)	(0.0389)	(0.0441)	(0.0526)
wood, paper and printing	0.0008	-0.0555	-0.0309	-0.0485
	(0.0344)	(0.0407)	(0.0456)	(0.0538)
chemicals and plastic products	0.0112	0.0208	-0.0064	0.0279
	(0.0321)	(0.0378)	(0.0431)	(0.0500)
basic metal and metal products	0.0401	-0.0181	-0.0225	-0.0299
	(0.0321)	(0.0366)	(0.0440)	(0.0512)
machinery, electric, motor vehicles	0.0004	-0.0024	-0.0472	-0.0494
	(0.0308)	(0.0367)	(0.0422)	(0.0517)
energy and mining	-0.1479***	-0.0566	-0.1672***	-0.0795
	(0.0344)	(0.0450)	(0.0452)	(0.0590)
construction	0.0566*	0.0400	0.0009	0.0353
	(0.0335)	(0.0384)	(0.0463)	(0.0548)
wholesale and retail trade	0.0089	0.0028	-0.0234	-0.0015
	(0.0323)	(0.0385)	(0.0436)	(0.0520)
hotels and restaurants	0.0597*	-0.0025	0.0692	-0.0267
	(0.0321)	(0.0373)	(0.0429)	(0.0506)
traffic and transportation	-0.0382	-0.0399	-0.0528	-0.0554
	(0.0325)	(0.0396)	(0.0435)	(0.0547)
banking and insurance	-0.1825***	-0.0330	-0.1972***	-0.0277
	(0.0339)	(0.0448)	(0.0442)	(0.0573)
business related services	-0.0178	-0.0014	-0.0238	-0.0269
	(0.0306)	(0.0361)	(0.0414)	(0.0492)
other public and private services	-0.0147	0.0084	-0.0344	0.0018
	(0.0286)	(0.0331)	(0.0382)	(0.0438)
health and social services	0.0143	-0.0017	-0.0074	-0.0235
	(0.0284)	(0.0326)	(0.0378)	(0.0434)
public administration	0.0449	-0.0018	0.0214	0.0134
	(0.0292)	(0.0338)	(0.0386)	(0.0444)
year-, region-, industry-FE	yes	yes	yes	yes
(year x region)-FE	yes	yes	yes	yes
Pseudo-R^2	0.0480	0.2342	0.0494	0.2713
N	6,593	6,590	3,731	3,730

Source: German Job Vacancy Survey 2005-2010.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

2.3 The PES reduces coordination frictions

The aim of the PES is to reduce coordination frictions in the labor market. To evaluate whether the PES is able to achieve this goal, we investigate how the PES distributes job applicants among registered vacancies and compare it to the distribution of job applicants coming from the private market. To control for firm fixed effects, we use the information on the number of applicants that a registered

vacancy received from the PES and the private market. The German Job Vacancy Survey provides this information for the years 2005 to 2008.

Differences in the number of applicants from the PES and the private market

On average a registered vacancy received 20.6 applications, 7.2 of those were send by the PES and 13.4 came through other search channels. An unregistered vacancy received on average 21.2 applicants from the private market. Thus, using the PES as a search channel helps registered firms to attract on average roughly the same number of applicants that unregistered vacancies received.

Table 3: **Number of applicants through the PES and the private market**

	OLS-Regressions			
	full sample		restricted sample	
	Number of applicants (log) through		Number of applicants (log) through	
	PES	private market	PES	private market
	(1)	(2)	(3)	(4)
low qualification	-0.0052	-0.1780**	-0.0112	-0.2194**
	(0.0647)	(0.0728)	(0.0895)	(0.0976)
high qualification	-0.1333***	0.2400***	-0.1185**	0.1875**
	(0.0399)	(0.0605)	(0.0512)	(0.0796)
occupation specific experience	-0.0605**	0.0485	-0.0946**	0.0756
	(0.0296)	(0.0414)	(0.0407)	(0.0569)
permanent	0.0077	0.2338***	0.0223	0.2386***
	(0.0304)	(0.0422)	(0.0415)	(0.0574)
full-time	0.0266	0.1043*	0.0812	0.1231*
	(0.0417)	(0.0563)	(0.0541)	(0.0738)
weekend-work	-0.0136	-0.0559	0.0286	-0.0254
	(0.0419)	(0.0601)	(0.0574)	(0.0851)
firm size (log)	0.0066	0.1240***	0.0300*	0.1042***
	(0.0113)	(0.0156)	(0.0156)	(0.0214)
"financial constraints"	0.0159	-0.0180	-0.0107	-0.0029
	(0.0519)	(0.0715)	(0.0689)	(0.1004)
"low sales"	-0.0349	-0.0436	0.0086	-0.1122
	(0.0396)	(0.0541)	(0.0557)	(0.0785)
"not enough suitable employees"	-0.0458	-0.1450**	-0.0667	-0.1696*
	(0.0494)	(0.0621)	(0.0812)	(0.0997)
year-, region-, industry-FE	yes	yes	yes	yes
(year x region)-FE	yes	yes	yes	yes
R^2	0.1289	0.2227	0.1747	0.2602
N	2,761	2,761	1,609	1,609

Source: German Job Vacancy Survey 2005-2008.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

If we compare the factors that explain the number of applications that a registered vacancy received through the PES and the private market, we see large differences. These are shown in Table 3, which presents OLS regressions on the (log) number of applicants send by the PES in columns (1) and (3)

and the number of applications received by the same vacancies through the private market in columns (2) and (4). Columns (1) and (2) present the results for the full sample. Columns (3) and (4) present the results for the restricted sample, which only includes vacancies that were successful in finding an applicant before the initial starting date of the employment contract. The restricted sample is most likely not to suffer from reverse causality, i.e., that firms which were unsuccessful in attracting applicants until the intended starting date decide to register their vacancy with the PES later in the hiring process. We prefer the results based on the restricted sample.

While jobs requiring low qualifications receive less applicants from the private market, no such effect can be found for applicants send by the PES. A similar phenomena can be observed if the offered job is permanent or the firm offering the job is large. Both are positively correlated with the number of applicants in the private market, but the respective coefficients for the number of applicants send by the PES are insignificant and close to zero. These cases, where job or firm characteristics are insignificant in explaining the number of applicants send from the PES but significant in explaining the number of applicants coming from the private market, suggest that the PES distributes its applicants evenly among vacancies irrespective of these characteristics. The significant negative correlations between jobs requiring high qualifications or occupation specific experience and the number of applicants send through the PES is most like driven by the shortage of the supply of the respective workers in the PES.[7] All these correlations indicate that the PES helps to reduce coordination frictions by allocating applicants evenly among registered vacancies as long as the supply of the respective workers in the PES permits it.

Differences in the variance of the number of applicants through the PES and the private market

If it is true that the PES coordinates applicants, we should also see that the number of applicants send by the PES to registered firms are more evenly distributed among vacancies than the number of applicants that a firm receives through the private market. In other words, if the PES reduces coordination frictions we should see that the variance of the number of applicants send by the PES is smaller than the variance of the number of applicants that apply through the decentralized market. To calculate the respective variances, we use again the information on the number of applicants that a registered vacancy received through the PES and compare it to the number of applicants that the same vacancy receives through other search channels.

The upper part of Table 4 shows the total and the residual variances of the (log) number of applicants for the full sample and the bottom part of Table 4 shows the respective results for the restricted sample. The residual variances of the (log) number of applicants are calculated based on the respective regressions in Table 3. Columns (1) and (2) show the respective variances of the (log) number of applicants. Column (3) presents the F-statistic of the variance-ratio-test, which equals the ratio of the

[7]See Holzer (1988) and Blau and Robins (1990) for the US, Osberg (1993) for Canada and Gregg and Wadsworth (1996) for the UK.

variances in column (1) to column (2).

Table 4: **Differences in the variances of the number of applicants between PES and private market**

	Variance of the number of applicants (log)		Variance-Ratio-Test
	private market	PES	F-statistic = ratio
	(1)	(2)	(3)
full sample			
total variance	1.2329	0.5836	F(4013, 4013) = 2.1126***
residual variance	0.0064	0.0039	F(2760, 2760) = 1.6598***
restricted sample			
total variance	1.2946	0.5841	F(2236, 2236) = 2.2166***
residual variance	0.0126	0.0081	F(1606, 1607) = 1.5521***

Source: German Job Vacancy Survey 2005-2008.
Variance-Ratio-Text: H_0: Ratio=1 *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.
Residual variances of the (log) number of applicants are based on the respective regressions in Table 3.

Table 5: **Differences in the variances of the number of applicants between PES and private market for different subgroups**

	total variance F-statistic = ratio (1)	residual variance F-statistic = ratio (2)
low qualification	F(141, 141) = 1.1578	F(103, 103) = 2.0734***
medium qualification	F(1615, 1615) = 2.2309***	F(1208, 1209) = 1.5567***
high qualification	F(418, 418) = 2.4574***	F(293, 293) = 1.4113***
permanent	F(1085, 1085) = 2.2581***	F(760, 761) = 1.3322***
temporary	F(1124, 1124) = 2.1740***	F(845,845) = 1.7386***
full-time	F(1754, 1754) = 2.1924***	F(1263, 1264) = 1.5097***
part-time	F(463, 463) = 2.3080***	F(342, 342) = 1.7419***
firm size (1 - 19)	F(658, 658) = 1.8319***	F(455, 456) = 1.3847***
firm size (20 - 49)	F(498, 498) = 2.3771***	F(353, 353) = 2.0919***
firm size (50 - 199)	F(552, 552) = 2.2741***	F(400, 400) = 1.3537***
firm size (200 - 499)	F(289,289) = 2.2903***	F(222, 222) = 1.4852***
firm size (500+)	F(235, 235) = 2.0246***	F(172, 172) = 1.6410***

Source: German Job Vacancy Survey 2005-2008, restricted sample.
Variance-Ratio-Test: H_0: Ratio=1 *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.
Residual variances of the (log) number of applicants are based on column (3) and (4) in Table 3.

The total variance of the number of applicants that a registered firm receives from the private market is more than twice the total variance of the number of applicants a registered firm receives from the PES. This holds for the full and the restricted sample. The respective ratio for the residual variances is roughly one and a half. The ratio of the variances of column (1) to column (2) (F-statistic) is statistically different from one with a confidence interval of 99% for the total and the residual variances and the full and the restricted sample. These results show that the PES is indeed distributing job seekers more evenly than the private market.

The result that the PES distributes its unemployed more evenly than the private market also holds for vacancies with different skill requirement, for permanent and temporary jobs, for full- and part-time jobs and for jobs posted by firms of different sizes. The respective F-statistics based on the total and the residual variances of the (log) number of applicants received from the the private market and the PES are presented in Table 5. They are based on the restricted sample.

2.4 Wage differences between registered and unregistered jobs

Wages paid by registered compared to unregistered jobs

The evidence that the PES reduces coordination frictions, i.e., distributes its unemployed more evenly among registered vacancies than the private market, suggests that the probability to receive an applicant from the PES should be less dependent on the offered wage than the probability to receive an applicant via the private market. Thus, firms in the private market should offer higher wages than firms registered with the PES. To investigate this hypothesis we use the German Job Vacancy Survey in 2014, which – for the first time – collected the information on the wages paid to workers hired for the last filled vacancy.

A first look at the descriptive statistics confirms the hypothesis. A worker hired for a job, which was registered with the PES, is on average paid 12.48 Euros per hour. This is 1.36 Euro less than the 13.84 Euros per hour, which workers are paid by for jobs in the private market. This difference could of course be driven by worker-, vacancy-, and firm-level characteristics.

The German Job Vacancy Survey provides information on the age, the gender and the previous employment status of the hired worker. The information on age and the job's qualification requirement allows us to calculate the potential labor market experience of the hired worker.To do so we use job qualification requirements to approximate the years of schooling. Variables that control for vacancy characteristics include the required qualification and experience level, whether it is a permanent or temporary and full-time or part-time job. We also include whether the job requires regular weekend-work. On the firm level we control for the number of employees (log) as well as the economic condition of a firm as captured by the binary indicator variables "low sales", "financial constraints", and "not enough suitable employees". Admittedly, there might be unobservable characteristics, which influence a firms' decision to register a vacancy and which are correlated with the wages offered. If the unobserved

characteristic varies on the industrial or regional level, then this is controlled for by including industry- and region-fixed-effects.

Table 6: **Wage difference between registered and unregistered jobs**

	OLS-Regressions: Wages (log)		OLS-Regressions: Wages (log)	
	full sample		restricted sample	
	(1)	(2)	(3)	(4)
PES search channel	-0.0485***	-0.0434***	-0.0561***	-0.0530***
	(0.0069)	(0.0075)	(0.0108)	(0.0120)
PES hiring channel		-0.0181*		-0.0109
		(0.0108)		(0.0177)
previously unemployed	-0.0645***	-0.0624***	-0.0683***	-0.0670***
	(0.0077)	(0.0079)	(0.0124)	(0.0127)
worker's experience	0.0134***	0.0134***	0.0143***	0.0143***
	(0.0012)	(0.0012)	(0.0019)	(0.0019)
worker's experience2	-0.0265***	-0.0265***	-0.0277***	-0.0276***
	(0.0029)	(0.0029)	(0.0046)	(0.0046)
female	-0.0776***	-0.0774***	-0.0907***	-0.0906***
	(0.0079)	(0.0079)	(0.0119)	(0.0119)
low qualification	-0.1837***	-0.1839***	-0.1851***	-0.1852***
	(0.0108)	(0.0108)	(0.0174)	(0.0174)
high qualification	0.3567***	0.3566***	0.3516***	0.3516***
	(0.0109)	(0.0109)	(0.0163)	(0.0163)
occupation specific experience	0.1058***	0.1056***	0.1062***	0.1061***
	(0.0080)	(0.0080)	(0.0118)	(0.0118)
permanent	0.0722***	0.0720***	0.0772***	0.0772***
	(0.0071)	(0.0071)	(0.0109)	(0.0109)
full-time	0.2832***	0.2830***	0.3095***	0.3095***
	(0.0170)	(0.0170)	(0.0251)	(0.0251)
weekend-work	-0.0566***	-0.0572***	-0.0482***	-0.0486***
	(0.0100)	(0.0101)	(0.0155)	(0.0156)
unionized firm	0.0642***	0.0637***	0.0654***	0.0652***
	(0.0076)	(0.0076)	(0.0116)	(0.0117)
firm size (log)	0.0385***	0.0382***	0.0395***	0.0392***
	(0.0031)	(0.0031)	(0.0047)	(0.0047)
"financial constraints"	-0.0352*	-0.0349*	-0.0362	-0.0357
	(0.0184)	(0.0184)	(0.0300)	(0.0300)
"low sales"	0.0070	0.0071	-0.0042	-0.0042
	(0.0122)	(0.0122)	(0.0193)	(0.0193)
"not enough suitable employees"	-0.0053	-0.0058	0.0006	0.0002
	(0.0094)	(0.0095)	(0.0166)	(0.0167)
region-, industry-FE	yes	yes	yes	yes
R^2	0.5810	0.5811	0.5986	0.5987
N	6,940	6,940	3,432	3,432

Source: German Job Vacancy Survey 2014.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

Table 6 presents in columns (1) and (2) the OLS-regression results on (log) wages for the full sample and in columns (3) and (4) for the restricted sample, where we exclude all those observations where firms agreed on an applicant after the intended starting date of the employment contract. The restricted

sample should therefore rule out reverse causality, i.e., that firms, which were not successful in filling the vacancy before the intended starting date of the employment contract, adjusted their wage offer and registered their vacancy with the PES. We regard the results in columns (3) and (4) as being more reliable.

The results show that wages paid by firms that registered their vacancy at the PES are 4% to 5% lower than wages paid by firms that did not register comparable vacancies. Note, that we controlled for the previous labor market status of the hired worker, i.e., the result cannot be driven by the effect that registered vacancies are more likely to hire unemployed workers, who have a lower outside option, than unregistered vacancies.

Let us focus on our preferred specification in columns (3) and (4). Comparing columns (3) with (4) shows that the effect is robust to including an indicator variable for cases where the worker is hired through the PES. The fact that the coefficient on the "PES hiring channel" indicator variable is statistically insignificant suggests that wage offers do not depend on whether the worker was hired through the PES or not. What matters is whether the firm registered its vacancy with the PES, i.e., used the PES as a search channels.

Table 7: **Wage difference between registered and unregistered jobs for different subgroups**

	OLS-Coefficient for PES search channel		OLS-Coefficient for PES hiring channel	
	(1)	(2)	(3)	(4)
low qualification	-0.0781	(0.0805)	0.0804	(0.0969)
medium qualification	-0.0510***	(0.0141)	-0.0096	(0.0212)
high qualification	-0.0311	(0.0474)	-0.0771	(0.0640)
permanent	-0.0607***	(0.0161)	0.0117	(0.0248)
temporary	-0.0434**	(0.0219)	-0.0135	(0.0305)
full-time	-0.0527***	(0.0116)	-0.0354**	(0.0171)
part-time	-0.2308*	(0.1204)	0.3037	(0.1936)
firm size (1 - 19)	-0.0415	(0.0283)	0.0286	(0.0424)
firm size (20 - 49)	-0.0605**	(0.0277)	-0.0196	(0.0323)
firm size (50 - 199)	-0.0582**	(0.0288)	-0.0392	(0.0428)
firm size (200 - 499)	0.0671	(0.1257)	-0.0862	(0.2576)
firm size (500+)	-0.1512	(0.1281)	-0.1429	(0.1947)

Source: German Job Vacancy Survey 2014, restricted sample.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

To investigate the effect for different subgroups Table 7 presents the OLS-coefficients of the binary variables that indicate whether the vacancy was registered with the PES (PES search channel) and whether the hired worker was contacted through the PES (PES hiring channel) of wage regressions

for the respective subgroups. The results suggest that the wage difference between registered and unregistered vacancies is driven by full-time jobs requiring medium qualifications, and by jobs offered by firms with 20 to 200 employees. Table 7 shows the results for the restricted sample.

Differences in experiencing difficulties due to "high wage demands"

The German Job Vacancy Survey also contains information on whether a firm had difficulties in filling the vacancy because its applicants demanded higher wages. This information can shed light not only into the question on whether registered firms offer lower wages than unregistered firms, it also provides information on whether a worker receives different wage offers from registered firms compared to unregistered firms.

Table 8: **Recruiting difficulties due to "high wage demands"**

	Recruiting difficulties due to "high wage demands" Probit Regressions: Marginal Effects			
	full sample		restricted sample	
	(1)	(2)	(3)	(4)
PES search channel	0.0315***	0.0349***	0.0254***	0.0289***
	(0.0032)	(0.0035)	(0.0034)	(0.0038)
PES hiring channel		-0.0094**		-0.0101**
		(0.0041)		(0.0046)
low qualification	0.0220***	0.0222***	0.0124**	0.0128**
	(0.0049)	(0.0049)	(0.0054)	(0.0054)
high qualification	0.0061	0.0056	0.0028	0.0023
	(0.0041)	(0.0041)	(0.0044)	(0.0044)
occupation specific experience	0.0084***	0.0083***	0.0054*	0.0053
	(0.0031)	(0.0031)	(0.0033)	(0.0033)
permanent	0.0069**	0.0064**	0.0051	0.0046
	(0.0032)	(0.0032)	(0.0034)	(0.0034)
full-time	0.0142***	0.0142***	0.0093**	0.0092**
	(0.0043)	(0.0043)	(0.0044)	(0.0044)
weekend-work	0.0103**	0.0098**	0.0110**	0.0104**
	(0.0042)	(0.0042)	(0.0045)	(0.0045)
firm size (log)	-0.0028**	-0.0031***	-0.0042***	-0.0044***
	(0.0011)	(0.0012)	(0.0012)	(0.0012)
"financial constraints"	0.0267***	0.0270***	0.0184***	0.0189***
	(0.0051)	(0.0051)	(0.0056)	(0.0055)
"low sales"	0.0219***	0.0219***	0.0107**	0.0108**
	(0.0039)	(0.0039)	(0.0044)	(0.0044)
"not enough suitable employees"	0.0569***	0.0561***	0.0470***	0.0463***
	(0.0042)	(0.0042)	(0.0049)	(0.0049)
year-, region-, industry-FE	yes	yes	yes	yes
(year x region)-FE	yes	yes	yes	yes
Pseudo-R^2	0.1009	0.1014	0.1116	0.1128
N	21,603	21,603	12,347	12,347

Source: German Job Vacancy Survey 2005-2010.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

Given the evidence above that registered vacancies pay lower wages, we expect that they are also more likely to experience difficulties in the hiring process due to higher wage demands by applicants. Registered vacancies experience such difficulties in 7.7% of all cases, while unregistered vacancies experience such difficulties only in 3.9% of all cases.

We use the same job and firm characteristics as control variables as in Table 6. We do not include individual characteristics of the hired worker, since the question underlying the indicator variable "high wage demands" explicitly addresses the experience with all applicants.

The results in Table 8 indicate a strong positive correlation between registering a vacancy with the PES and experiencing difficulties in the recruitment process due to high wage demands by applicants. Thus, having registered the vacancy with the PES (and posted a respective wage) is associated with an increase in the probability to experience difficulties due to higher wage demands by 3.2 to 3.5 percentage points. The effect is reduced slightly if we exclude all observations where firms agreed on an applicant after the intended starting date of the employment contract (restricted sample in columns (3) and (4)). Firms which hire a worker, who was first contacted through the PES, report that they were less likely to experience difficulties due to higher wage demands. Thus, hiring a worker through the PES decreases the respective probability by around one percentage point.

Table 9: **Recruiting difficulties due to "high wage demands" for different subgroups**

	Recruiting difficulties due to "high wage demands"			
	Marginal effects (probit)		Marginal effects (probit)t	
	PES search channel		PES hiring channel	
	(1)	(2)	(3)	(4)
low qualification	0.0673***	(0.0216)	-0.0418*	(0.0250)
medium qualification	0.0273***	(0.0050)	-0.0083	(0.0067)
high qualification	0.0307***	(0.0094)	-0.0366***	(0.0114)
permanent	0.0353***	(0.0067)	-0.0073	(0.0099)
temporary	0.0224***	(0.0057)	-0.0153**	(0.0067)
full-time	0.0299***	(0.0050)	-0.0142**	(0.0066)
part-time	0.0272***	(0.0087)	-0.0091	(0.0113)
firm size (1 - 9)	0.1041***	(0.0232)	-0.0435*	(0.0263)
firm size (10 - 19)	0.0108	(0.0113)	0.0093	(0.0148)
firm size (20 - 49)	0.0227***	(0.0086)	-0.0178*	(0.0101)
firm size (50 - 199)	0.0270***	(0.0092)	-0.0144	(0.0116)
firm size (200 - 499)	0.0255**	(0.0103)	-0.0146	(0.0144)
firm size (500+)	0.0016	(0.0096)	0.0003	(0.0122)

Source: German Job Vacancy Survey 2005-2010, restricted sample.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

The same pattern is preserved if we divide the sample into different subgroups according to the

required qualification level, permanent/temporary, full-/part-time, weekend-work required as well as firm size. Tables 9 shows these results for the restricted sample.

Our evidence supports the hypothesis that firms, which register their vacancy with the PES, offer lower wages than firms, which did not register their vacancy. It also suggests residual wage dispersion, which differs systematically depending on whether the vacancy is registered with the PES or not.

2.5 Does the PES help to reduce information frictions?

In this section we investigate to which extend the PES is able to reduce information frictions by sending only suitable applicants to registered vacancies. Comparing the share of suitable applicants among all applicants at registered vacancies (41.4%) with the share of suitable applicants at unregistered vacancies (45.3%) we see that using the PES as a search channel reduces the share of suitable applicants by around 3.9 percentage points.

In Table 10 we investigate this correlation further and control for observable job- and firm-characteristics by including the (log) number of employees, binary indicator variables for the economic condition of the establishment like "low sales", "financial constraints", and "not enough suitable employees", indicator variables for the qualification and occupation specific experience required for the job, indicator variables for full-time and temporary jobs and for jobs requiring weekend-work. In addition we include year-, industry- and region-fixed effects. To control for regional time-varying effects we interact the year- and region-dummy variables.

Table 10 provides in columns (1) and (2) the OLS estimates for the share of suitable applicants. The dependent variable in columns (3) to (4) is the logarithm of the number of suitable applicants. To make these results comparable to columns (1) and (2) we include in these regressions the logarithm of the total number of applicants.

The share of suitable applicants is around 5.3 percentage points lower at vacancies registered with the PES compared to unregistered vacancies. This negative correlation between registering a vacancy at the PES and the number of suitable applicants is significant at a 1% level. In columns (2) we excluding those vacancies, where firms agreed on the candidate after the earliest intended starting date of the employment contract. The estimate in column (2) does not change significantly. The difference between registered and unregistered vacancies is round 1.5 percentage point lower if the fraction of suitable applicants is replaced by the number of suitable applicants (column (3) and (4)).

Unobserved heterogeneity between registered and unregistered vacancies might bias the correlation between the use of PES as a search channel and the fraction of suitable applicants found above. We use the information on the number of total and suitable applicants and the number of total and suitable applicants send by the PES for the registered firms in the years 2005 and 2008. This allows us to calculate the counterfactual, i.e., the number of (suitable) applicants that a registered firm would have received had it not posted its vacancy at the PES. Table 11 presents the respective difference results

Table 10: **Share of suitable applicants**

	OLS-Regressions			
	Share of suitable applicants		number of suitable applicants (log)	
	full sample	restricted sample	full sample	restricted sample
	(1)	(2)	(3)	(4)
PES search channel	-0.0529***	-0.0552***	-0.0377***	-0.0410***
	(0.0040)	(0.0051)	(0.0065)	(0.0083)
number of applicants			0.5105***	0.5257***
			(0.0041)	(0.0051)
low qualification	0.0363***	0.0414***	0.0254**	0.0287*
	(0.0080)	(0.0103)	(0.0121)	(0.0154)
high qualification	-0.0136***	-0.0120*	-0.0198**	-0.0168
	(0.0052)	(0.0066)	(0.0087)	(0.0108)
occupation specific experience	-0.0438***	-0.0425***	-0.0672***	-0.0677***
	(0.0041)	(0.0052)	(0.0065)	(0.0083)
permanent	-0.0419***	-0.0390***	-0.0328***	-0.0319***
	(0.0042)	(0.0053)	(0.0068)	(0.0086)
full-time	-0.0398***	-0.0350***	-0.0431***	-0.0392***
	(0.0056)	(0.0069)	(0.0087)	(0.0106)
weekend-work	0.0193***	0.0082	0.0117	0.0076
	(0.0060)	(0.0074)	(0.0091)	(0.0114)
firm size (log)	-0.0058***	-0.0052***	0.0274***	0.0270***
	(0.0014)	(0.0018)	(0.0024)	(0.0030)
"financial constraints"	0.0138*	0.0194*	0.0073	0.0098
	(0.0076)	(0.0100)	(0.0122)	(0.0163)
"low sales"	0.0127**	0.0132*	0.0005	-0.0017
	(0.0055)	(0.0073)	(0.0086)	(0.0114)
"not enough suitable employees"	-0.0497***	-0.0509***	-0.1308***	-0.1426***
	(0.0068)	(0.0100)	(0.0108)	(0.0159)
firm-, job-characteristics	yes	yes	yes	yes
year-, region-, industry-FE	yes	yes	yes	yes
(year x region)-FE	yes	yes	yes	yes
R^2	0.0893	0.0910	0.6145	0.6274
N	21,925	13,731	21,666	13,634

Source: German Job Vacancy Survey 2005-2010.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

for the full and the restricted sample.

The results are very similar to those obtained in Table 10 columns (3) and (4), where we used the (log) number of suitable applicants as dependent variable. The results show that using the PES as a search channel decreases the fraction of suitable applicants by 4.5 percentage points for the full sample and by 3.3 percentage points for the restricted sample.

Table 12 displays the same estimates for different subgroups. It shows that the difference in the fraction of suitable applicants is driven by vacancies, which require a low or medium qualification. The difference is no longer statistically significant for vacancies which require a high qualification

Table 11: **Fraction of suitable applicants**

	Fraction of suitable applicants			
	total (incl. PES)	private market (without PES)	difference	N
full sample	0.3372	0.3826	-0.0454***	3,270
	(0.0042)	(0.0058)	(0.0045)	
restricted sample	0.3588	0.3919	-0.0331***	1,867
	(0.0055)	(0.0077)	(0.0059)	

Source: German Job Vacancy Survey 2005-2008.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

level. The differences in the fraction of suitable applicants is very similar if we compare permanent versus temporary jobs or full-time versus part-time jobs. The difference in the fraction of suitable applicants decreases with the number of employees and becomes insignificant for firms with more than 200 employees.

Table 12: **Differences in the fraction of suitable workers for different subgroups**

	Differences in the fraction of suitable workers			
	full sample		restricted sample	
	(1)	(2)	(3)	(4)
low qualification	0.0544***	(0.0188)	0.0501*	(0.0262)
medium qualification	0.0521***	(0.0054)	0.0391***	(0.0071)
high qualification	0.0140	(0.0092)	0.0062	(0.0120)
permanent	0.0586***	(0.0063)	0.0567***	(0.0085)
temporary	0.0327***	(0.0067)	0.0108	(0.0084)
full-time	0.0492***	(0.0051)	0.0370***	(0.0069)
part-time	0.0297***	(0.0104)	0.0199	(0.0121)
firm size (1 - 9)	0.0797***	(0.0155)	0.0676***	(0.0205)
firm size (10 - 19)	0.0661***	(0.0115)	0.0405**	(0.0165)
firm size (20 - 49)	0.0562***	(0.0100)	0.0482***	(0.0136)
firm size (50 - 199)	0.0385***	(0.0084)	0.0358***	(0.0107)
firm size (200 - 499)	0.0193*	(0.0113)	0.0093	(0.0137)
firm size (500+)	-0.0117	(0.0093)	-0.0274**	(0.0115)

Source: German Job Vacancy Survey 2005-2008.
Robust standard errors in brackets. *** indicates $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

The result that the fraction of suitable applicants at vacancies registered with the PES is about 4.5 percentage points lower than at vacancies that only use decentralized search channels is complementary to the empirical evidence on worker self-selection into the PES based on worker-level data. Using a US household survey Blau and Robins (1990) show that unemployment insurance and welfare recipients are more likely to use the PES. Using the Canadian Labor Force Survey Osberg (1993) emphasizes

the importance to control for sample selection into PES use. Using the British Labour Force Survey Gregg and Wadsworth (1996) find that PES use is highest among less skilled and long-term unemployed workers. Holzer (1988) uses the Youth Cohort of the National Longitudinal Survey to analyze the self-selection of workers and concludes that PESs are primarily used by individuals with few job opportunities.

Our analysis is the first to provide evidence from firms' perspective that the same type of worker self-selection can also be found among applicants send by the PES. This is important, because the worker-based evidence in the literature mentioned above does not necessarily imply our result. To see this note that it could be that the PES can detect unsuitable workers and prevents them from applying. This would imply that registered firms receive more applications from suitable workers. The fact that our results show the opposite, i.e., that registered firms receive more applications from less suitable applicants, implies that the PES is not able/willing to screen workers efficiently enough to ensure that the applicants they allocate are on average as suitable as the workers that apply through the private market.

3 Theoretical part

We rationalize our empirical results by building a theoretical model, which allows firms to decide whether they want to register their vacancy with the PES or search in the private market.

3.1 The model

We consider an economy with a unit mass of unemployed workers and a mass $v \in (0, \infty)$ of firms. Each firm has one job vacancy that needs to be filled, and each worker wishes to find a job. There are two types of workers. A fraction m of them are *productive workers*, who can produce an output normalized to 1, and the rest are *unproductive workers*, who produce 0. The worker's type is private knowledge. Firms possess screening technologies to hire workers. But their technologies are not perfect. We assume that screening can fail with probability $\delta \in (0, 1)$. So firms can detect an unproductive worker only with probability $1 - \delta$. To simplify the model we assume that interviewing costs are such that firms interview only one worker.

There are two channels through which matching between firms and unemployed workers can occur. One is a Public Employment Agency (hereafter, PES), where all unemployed workers are registered in order to collect unemployment benefit (normalized to zero). We model the job-brokering service provided by the PES as follows. All job applications by registered workers are coordinated so that workers and firms are brought together on a one by one basis. Denote by $a \in (0, 1]$ the maximum number of matching pairs the PES can propose. a is a technological parameter and represents the efficiency of the PES. The other channel is a search market, which may be referred to as a decentralized

or private market. Here, unlike in the PES, search is costly for workers and job applications are not coordinated. Workers have to incur an individual specific search cost represented by c drawn from a uniform distribution with support $[0,1]$. The parameter c is uncorrelated with the worker's type. The firms' cost required to post a vacancy is normalized to zero for both markets. How workers search without coordination will be specified below.

The economy lasts only one period and has the following stages. In the first stage, firms decide whether to post their vacancy in the search market or the PES. Once firms are registered, the PES selects randomly $\min\{v\rho, a\}$ workers and suggests each of them to match with one of the registered firms. In the second stage, all firms post simultaneously a wage at which they are willing to hire a worker. The wage posted in the search market is denoted by w, and the wage posted in the PES by w_a. Having observed those wages, workers decide whether or not to enter the search market in the third stage. Once in the search market, workers must choose to which firm to send an application. Assuming that each worker can send only one application and that workers cannot coordinate their actions over which firm to apply, we investigate a symmetric equilibrium where all workers use the identical application strategy for any configuration of the announced wages. This is the standard notion of directed search equilibria, see e.g., Peters (1991, 2001), Moen (1997), Acemoglu and Shimer (1999), Shi (2001), Burdett, Shi and Wright (2001) and Guerrieri, Shimer and Wright (2010). Finally, given those applications in the search market or an assigned worker in the PES, firms select and screen one of their applicants and make a job offer in the final stage. Those workers, who receive multiple offers, can select the highest wage. Once employed, productive workers produce and matched workers and firms receive their payoffs. Unmatched workers and firms get zero payoff.

3.2 Equilibrium definition

In what follows, we construct a search market equilibrium which has the following characteristics. A fraction $\rho \in [0,1)$ of firms registered with the PES post a wage $w_a = 0$ and a fraction $1-\rho$ of firms in the search market post a wage $w \in (0,1)$. All workers are registered in the PES and accept the wage $w_a = 0$ if it is the only offer they have received. A productive (an unproductive) worker enters a search market if and only if his search cost c is no greater than a reservation value c^m (c^u). Given the search behaviors of workers, each individual firm in the directed search market is characterized by a queue of applicants, denoted by x. The number of applicants $n = 1, 2, 3, ...$ each individual firm receives is a random variable and follows from a Poisson distribution with density $\text{Prob}[n = \tilde{n}] = \left(x^{\tilde{n}} e^{-x}\right) / (\tilde{n}!)$. The expected queue length x satisfies,

$$x = \frac{mc^m + (1-m)c^u}{v(1-\rho)}, \tag{1}$$

where the numerator equals the total number of workers in the search market, mc^m productive and $(1-m)c^u$ unproductive workers, while the denominator equals the total number of vacancies in the search market. Each productive worker, who searches in the private market, expects to be hired with proba-

bility $\eta(x) = (1 - e^{-x})/x$ in the search market, whereas each unproductive worker, who searches in the private market, expects to be hired with probability $\delta\eta(x)$. The matching probability in the PES equals $\min\{v\rho, a\}$ for productive and $\delta \min\{v\rho, a\}$ for unproductive workers. Each individual firm expects to employ a productive worker (and can produce output 1) with probability $x\eta(x)m/(m + (1-m)\delta)$ in the search market and with probability $\min\{1, a/(v\rho)\}m(1 - c^m\eta(x))$ in the PES. The respectively probabilities to hire an unproductive worker are given by $x\eta(x)(1-m)\delta/(m + (1-m)\delta)$ and $\min\{1, a/(v\rho)\}(1-m)\delta(1-c^u\eta(x))$. In the following we show that workers and firms have no incentive to deviate from the proposed search market equilibrium.

An equilibrium without active search market, i.e., $\rho = 1$, will be characterized when we describe the first-stage entry decision.

3.3 Existence and characterization

Workers' search decision: Assuming for the moment the existence of an equilibrium, we first describe workers' search decision. In any equilibrium where U^m (U^u) is the expected value of search for a productive (an unproductive) worker, the participation decision is described by a reservation value for the search cost, i.e.,

$$\begin{aligned} c^m &= U^m, \\ c^u &= U^u, \end{aligned}$$

respectively. Since a productive (an unproductive) worker with search cost c searches if and only if $c \leq c^m$ ($c \leq c^u$), the threshold values c^m and c^u determine the fraction of productive and unproductive workers that choose to search in the decentralized market.

Given the participation decision, we now describe workers' application decision in the search market. For that purpose, consider a situation in which a firm in the search market deviates to a wage $w' > 0$, given that all other firms post w with associated queue length x. Note that this deviation is a measure zero event for the entire market. Let x' be the expected queue of workers at this firm. Then, we must have,

$$U^m = \eta(x')w', \tag{2}$$

where $\eta(x') = \sum_{i=0}^{\infty} \frac{x'^i e^{-x'}}{i!} \frac{1}{i+1} = \left(1 - e^{-x'}\right)/x'$ is the probability that a productive worker is employed if he applies to this firm. To derive this probability note that if there are $i = 0, 1, 2, ...$ other applications to this firm, which happens with probability $\left(x'^i e^{-x'}\right)/(i!)$, then a given worker's application is selected with probability $1/(i+1)$. Similarly, for unproductive workers, it is,

$$U^u = \delta\eta(x')w',$$

where, the employment probability for an unmotivated worker is given by $\delta\eta(x')$, since the firm's screening succeeds and detects an unproductive worker only with probability $1 - \delta$. Observe that

$\delta U^m = U^u$ and so $\delta c^m = c^u$. Equation (2) defines an implicit equation that determines $x' = x\left(w'|U^m\right)$ as a strictly increasing function of the wage w' given the market value U^m.

Firms' wage offers: Given the search behaviors of workers described above, the next step is to characterize the equilibrium wages. Given $w_a = 0$ in the PES, we first derive an equilibrium wage in the search market. In any equilibrium where U^m (U^u) is the value of a(n) (un)productive worker, the optimal wage of a firm, denoted by $w(U^m)$, satisfies,

$$w(U^m) = \arg\max_{w'} x(w'|U^m)\eta(x(w'|U^m))\frac{m(1-w') - (1-m)\delta^2 w'}{m + (1-m)\delta}.$$

Here, the firm with a wage w' and a queue x' expects to receive at least one application with probability $1 - \text{Prob.}(n = 0) = 1 - e^{-x'} = x(w'|U^m)\eta(x(w'|U^m))$. Note that there are mc^m productive and $(1-m)c^u = (1-m)\delta c^m$ unproductive workers in the search market, thus in total $[m + (1-m)\delta]c^m$ workers. Hence, a randomly selected applicant is a productive worker with probability $m/\left(m + (1-m)\delta\right)$, in which case the firm's payoff is $1 - w'$, and is an unproductive worker with probability $\left((1-m)\delta\right)/\left(m + (1-m)\delta\right)$. In the latter case, if the firm detects successfully the worker's type, which is possible with probability $1 - \delta$, he does not hire this worker, yielding zero payoff. If the screening fails, which occurs with probability δ, then the firm employs this worker, who produces nothing, and the firm's payoff is $-w'$.

Substituting out w' using equation (2), the objective function of a firm, denoted by $\Pi_s(x')$, can be written as,

$$\Pi_s(x') = \frac{m(x'\eta(x') - x'U^m) - (1-m)\delta^2 x'U^m}{m + (1-m)\delta},$$

where $x' = x(w'|U^m)$ satisfies equation (2). The first-order condition is,

$$\frac{\partial \Pi_s(x')}{\partial x'} = \frac{m(e^{-x'} - U^m) - (1-m)\delta^2 U^m}{m + (1-m)\delta} = 0.$$

The second order condition can be easily verified. Rearranging this condition using equation (2) one can obtain,

$$w(U^m) = \frac{m}{m + (1-m)\delta^2}\frac{x'e^{-x'}}{1 - e^{-x'}}.$$

In a directed search equilibrium, workers must be indifferent between any of the individual firms. This leads to,

$$w = \frac{m}{m+(1-m)\delta^2}\frac{xe^{-x}}{1-e^{-x}}, \tag{3}$$

$$U^m = \frac{m}{m+(1-m)\delta^2}e^{-x}. \tag{4}$$

Hence, we have shown that given that $w_a = 0$, the equilibrium wage in the search market $w > 0$ is given by equation (3).

Given $w > 0$ in the search market, we show next that the equilibrium wage in the PES is given by the reservation wage $w_a = 0$. Given that a proportion $\rho \in [0, 1)$ of firms are in the PES, the wage $w_a = 0$ in the PES yields an equilibrium profit,

$$\Pi_a(x) = \min\{\frac{a}{v\rho}, 1\} m(1 - c^m \eta(x)), \tag{5}$$

where, given the probability of being allocated a worker $\min\{a/(v\rho), 1\}$, $m(1 - c^m\eta(x))$ represents the number of productive workers, who do not receive a job offer in search market and are wiling to accept $w_a = 0$.

The PES matches registered workers and firms using its job-brokering mechanism. This allocation is independent of the wages offered by registered firms. The fact that registered firms cannot increase the PES-internal matching probability $\min\{a/(v\rho), 1\}$ by offering a higher wage implies that registered firms will never compete among themselves. They will only compete with firms in the decentralized market. This is the reason why a wage offer $w'_a \in (0, w)$ cannot be profitable since such a deviation implies a mere increase in the wage cost without improving the probability of hiring a productive worker. If a deviating firm posts $w'_a \geq w$, then it can hire an assigned productive worker, irrespective of whether the worker gets another offer in search market. Hence, the best deviation $w'_a = w$ yields the profit,

$$\Pi'_a = \min\{\frac{a}{v\rho}, 1\} [m(1 - w) - (1 - m)\delta w].$$

We show in the Appendix that deviating and paying the private market wage $w'_a = w$ is not profitable, i.e., that $\Pi_a(x) > \Pi'_a$ for any $x \in (0, \infty)$. The reason, why the increase in the hiring probability associated with offering the private market wage $w'_a = w$ is not able to compensate the for the higher wage cost, is that the average productivity of applicants at the PES is lower than the average productivity of applicants in the decentralized market. It follows that registered firms offer only the reservation wage $w_a = 0$, because the absence of PES-internal wage competition due to the job-brokering mechanism at the PES does not force them to offer higher wages. Thus, $w_a = 0$ is the unique equilibrium wage in the PES.

Firms' market choice: In the first stage, firms decide whether to enter the PES or the search market for hiring a worker. Firms will choose the market that offers the highest expected profit. Thereby the equilibrium condition is given by,

$$\rho = \begin{cases} 0 & \text{if } \Pi_a(x) < \Pi_s(x), \\ (0,1) & \text{if } \Pi_a(x) = \Pi_s(x), \\ 1 & \text{if } \Pi_a > \Pi_s, \end{cases}$$

where the equilibrium queue length in the search market $x = x(\rho)$ is given in equation (1) for $\rho \in [0, 1)$. Given this equilibrium queue length, the equilibrium wages $w > 0$, in equation (3), and $w_a = 0$, and the

equilibrium search values of workers $U^m \geq 0$, in equation (4), and $U^u = \delta U^m$, the equilibrium profit in the PES, $\Pi_a(x)$, is given by equation (5) and the equilibrium profit in the search market by,

$$\Pi_s(x) = \frac{m}{m + (1-m)\delta} \left(1 - e^{-x} - xe^{-x}\right). \tag{6}$$

If $\rho = 1$, all firms are in the PES. By offering the equilibrium wage $w_a = 0$ they earn $\Pi_a = \min\{\frac{a}{v}, 1\}m$. Since no jobs are posted in the private market, only workers with zero search costs will visit the search market. The proportion of productive (unproductive) workers in the search market is given by m $(1-m)$, since the density of productive and unproductive workers with zero search costs are the same. If a firm deviates from $\rho = 1$ and enters the search market with a wage $w' = \epsilon > 0$, then the firm meets with a worker for sure, and makes profits, $\Pi_s = m\,(1-\epsilon)$. This deviation is profitable if and only if $\Pi_s > \Pi_a$, or if and only if $a/v < 1$ for an arbitrary small ϵ. Hence, an equilibrium without active search market, i.e., with $\rho = 1$, can only exist if and only if $a/v \geq 1$, i.e., if the PES is efficient enough.

Before we state our Theorem let us first define,

$$v^* \equiv \frac{m + (1-m)\,\delta}{m + (1-m)\,\delta^2} \frac{m}{x^*} e^{-x^*}, \tag{7}$$

where x^* is a unique solution to $\Pi_a(x^*) = \Pi_s(x^*)$. We now summarize the main result of our analysis on labor market equilibria with the PES.

Theorem 1 *A search market equilibrium with an active PES, $\rho \in (0,1)$, exists, if and only if $v > v^*$, and a search market equilibrium with an inactive PES, $\rho = 0$, exists if and only if $v \leq v^*$. This equilibrium is unique if and only if $v > a$. If $v \leq a$ a pure PES equilibrium, $\rho = 1$, also exists.*

Our theory estabishes that firms find it optimal to post higher wages in a decentralized search market in order to obtain a better selection of workers. Thus, firms induce workers to search in a costly decentralized market in order to reduce the information friction that is associated with a coordinated market like the PES. The tradeoff between the search market and the PES can be seen by looking at the benefits and costs of both markets. The benefit of using the search market is that it attracts a better selection of workers, i.e., the share of productive workers among all applicants is higher in the decentralized search market than in the PES, i.e., for $\rho \in [0,1)$,

$$\frac{m}{m + (1-m)\delta} > m. \tag{8}$$

The benefit of having a better pool of applicants has to be weighted against the higher wage cost $w > w_a = 0$, which firms have to pay to workers in order to induce them to engage in costly search in the decentralized market. The wage cost in the decentralized market is lower, when the number of firms v is lower so that the search market is less tight and less competitive. Hence, Theorem 1 shows that the search market is used exclusively, $\rho = 0$, when the number of vacancies v is low, and the search market coexists with the PES, $\rho \in (0,1)$, when the number of vacancies v is high.

4 Conclusion

Using the German Job Vacancy Survey we are able to show how the Public Employment Services (PES) affects the labor market. Almost half of all vacancies in Germany are registered with the PES. The PES is shown to distribute workers more evenly among vacancies compared to the private market. This reduction in coordination frictions is going along with lower wages paid by registered firms. Applicants send by the PES are on average less suited for the job than applicants attracted through the private market. We rationalize our empirical findings using a directed search model, where firms can decide whether to search via the PES or the private market. Lower coordination frictions in the PES induce less wage competition between firms enabling them to pay lower wages. The implicit cost of registering a vacancy is due to the less suited pool of applicants and explains why not all vacancies are registered with the PES despite the fact that its service is free of charge.

An interesting topic for future research would be to assess the effect of labor market reforms, e.g. the so-called Hartz Reform in Germany, since part of the reform package (Hartz III) aimed at restructuring the Public Employment Services. One issue would be to study whether the crowding out effect studied by Pissarides (1979) has been sustained – more efficient PESs crowd out private search effort. To evaluate this effect in the context of the Hartz Reform, it would be necessary to extend the model and data to incorporate other parts of the reform – creating new types of employment opportunities (Hartz I), introducing additional wage subsidies (Hartz II), and cutting the unemployment benefits for the long-term unemployed (Hartz IV). We believe our framework will best fit to study this and other related issues, like the effect on the wage inequality, of such a reform policy.

Appendix

Proof of $\Pi_a(x) > \Pi'_a$

Using (3) and (4) simplifies the inequality in question to $\Pi_a(x) > \Pi'_a \iff m(1-c^m\eta(x)) > m(1-w) - (1-m)\delta w$

$$\begin{aligned} &\iff \quad m\left(1 - \frac{m}{m+(1-m)\,\delta^2}\frac{e^{-x}(1-e^{-x})}{x}\right) > m - (m+(1-m)\delta)\frac{m}{m+(1-m)\delta^2}\frac{xe^{-x}}{1-e^{-x}} \\ &\iff \quad (m+(1-m)\delta)\frac{x}{1-e^{-x}} - m\frac{1-e^{-x}}{x} > 0, \end{aligned}$$

where the last inequality follows from $\frac{x}{1-e^{-x}} > \frac{1-e^{-x}}{x} \iff 1 > \eta(x)^2$. ■

Proof of Theorem 1

Define $\Gamma \equiv \frac{1}{m}\{\Pi_s(x) - \Pi_a(x)\}$ for $x \in [0,\infty)$, where by (1) and (4), $x = x(\rho)$ is determined by

$$\frac{e^{-x}}{x} = \frac{m+(1-m)\delta^2}{m+(1-m)\delta}\frac{v(1-\rho)}{m}. \tag{9}$$

This expression shows that $x(\rho)$ is strictly increasing in $\rho \in [0,1)$ and satisfies $x(0) \equiv \underline{x} \in (0,\infty)$ and $x(\rho) \to \infty$ as $\rho \to 1$.

In what follows, we use the implicit function Γ to show the existence and uniqueness of an equilibrium $\rho \in [0,1)$. There are two possible cases. Suppose in equilibrium $a > v\rho$. This implies $\rho \in [0, \bar{\rho})$ where $\bar{\rho} \equiv \min\{\frac{a}{v}, 1\}$. Then, $\Gamma = \Gamma(x)$ where

$$\Gamma(x) = \frac{1 - e^{-x} - xe^{-x}}{m + (1-m)\delta} - 1 + \frac{m}{m + (1-m)\delta^2} \frac{e^{-x}(1 - e^{-x})}{x}. \tag{10}$$

Observe that: $\Gamma(0) = -1 + \frac{m}{m+(1-m)\delta^2} < 0$; $\Gamma(x) \to \frac{1}{m+(1-m)\delta} - 1 > 0$ as $x \to \infty$. Hence, since $\Gamma(x)$ is continuous in $x \in [0, \infty)$, there exists an $x^* \in (0, \infty)$ that satisfies $\Gamma(x^*) = 0$. Observe further that

$$\begin{aligned}
&\frac{d\Gamma(x)}{dx} \mid_{x=x^*} \\
&= \frac{xe^{-x}}{m + (1-m)\delta} - \frac{m}{m + (1-m)\delta^2} \frac{e^{-x}}{x^2} \left[(x+1)(1 - e^{-x}) - xe^{-x}\right] \mid_{x=x^*} \\
&= \frac{x^* e^{-x^*}}{1 - e^{-x^*} - x^* e^{-x^*}} \\
&- \frac{m}{m + (1-m)\delta^2} \frac{e^{-x^*}}{1 - e^{-x^*} - x^* e^{-x^*}} \left[(1 - \frac{1 - e^{-x^*}}{x^*})e^{-x^*} + (\frac{1 - e^{-x^*}}{x^*} - e^{-x^*})(x^* + 1)\frac{1 - e^{-x^*}}{x^*}\right] \\
&> \frac{e^{-x^*}}{1 - e^{-x^*} - x^* e^{-x^*}} \left[x^* - (1 - \frac{1 - e^{-x^*}}{x^*})e^{-x^*} - (\frac{1 - e^{-x^*}}{x^*} - e^{-x^*})(x^* + 1)\frac{1 - e^{-x^*}}{x^*}\right] \\
&= \frac{e^{-x^*}}{1 - e^{-x^*} - x^* e^{-x^*}} \left[\frac{(x^* + 1 - e^{-x^*})^2}{x^*} - \frac{(1 - e^{-x^*} - x^* e^{-x^*})^2}{x^{*2}}\right] \\
&> 0.
\end{aligned}$$

In the above, we use $\Gamma(x^*) = 0 \Leftrightarrow \frac{x^* e^{-x^*}}{m+(1-m)\delta} = \frac{x^* e^{-x^*}}{1 - e^{-x^*} - x^* e^{-x^*}} \left[1 - \frac{m}{m+(1-m)\delta^2} \frac{e^{-x^*}(1-e^{-x^*})}{x^*}\right]$ in the second equality, and $x + 1 - e^{-x} > \frac{1-e^{-x}}{x} - e^{-x}$ and $x + 1 - e^{-x} > 1 - e^{-x} - xe^{-x}$ in the last inequality. Since $\Gamma(0) < 0 < \Gamma(\infty)$, $\frac{d\Gamma}{dx} > 0$ at $x = x^*$ implies $x^* \in (0, \infty)$ is unique (that is, $\Gamma(x)$ curve cannot cross the line $\Gamma(x) = 0$ more than once).

Finally, notice that the $x^* \in (0, \infty)$ satisfying $\Gamma(x^*) = 0$ determined above does not depend on v, whereas $\underline{x}$ $(\equiv x(0))$ determined by (9) is strictly decreasing in v. Hence, we have $x^* > \underline{x} \Leftrightarrow v > v* \equiv \frac{m+(1-m)\delta}{m+(1-m)\delta^2} \frac{m}{x^*} e^{-x^*}$. On the other hand, denote by $\bar{x}$ the solution of $x = x(\rho)$ to (9) as $\rho \to \bar{\rho}$. If $\frac{a}{v} \geq 1$ then $\bar{\rho} = 1$ and $\bar{x} = \infty$, so $x^* < \bar{x}$. If $\frac{a}{v} < 1$ then $\bar{\rho} = \frac{a}{v} < 1$ and $\bar{x} < \infty$, so $x^* < \bar{x}$ if and only if $v < v^* + a$. The very last inequality follows, by (9), from

$$\frac{e^{-\bar{x}}}{\bar{x}} - \frac{e^{-x^*}}{x^*} = \frac{m + (1-m)\gamma^2}{(m + (1-m)\gamma)m}(v - a - v^*).$$

which implies

$$x^* < \bar{x} \iff \frac{e^{-\bar{x}}}{\bar{x}} < \frac{e^{-x^*}}{x^*} \iff v < v^* + a.$$

To sum up, there exists a unique $\rho \in (0, \bar{\rho})$ that satisfies $\frac{e^{-x^*}}{x^*} = \frac{m+(1-m)\delta^2}{m+(1-m)\delta} \frac{v(1-\rho)}{m}$ (and hence $\Pi_s(x^*) = \Pi_a(x^*)$) if and only if $v \in (v^*, v^* + a)$, and $\rho = 0$, satisfying $\Pi_s(x^*) > \Pi_a(x^*)$, if and only if $v \in (0, v^*)$

Suppose next in equilibrium $a \leq v\rho$. This implies $\rho \in [\bar{\rho}, 1)$, where $\bar{\rho} \equiv \min\{\frac{a}{v}, 1\}$, and is possible only when $\frac{a}{v} < 1$. Then, $\Gamma = \Gamma(\rho, x)$ where

$$\Gamma(\rho, x) = \frac{1 - e^{-x} - xe^{-x}}{m + (1-m)\delta} - \frac{a}{\rho v}\left[1 - \frac{m}{m + (1-m)\delta^2} \frac{e^{-x}(1 - e^{-x})}{x}\right], \tag{11}$$

where $x = x(\rho)$ is determined by (9) as before. Observe that:

$$\Gamma(\bar{\rho}, \bar{x}) = \frac{1 - e^{-\bar{x}} - \bar{x}e^{-\bar{x}}}{m + (1-m)\delta} - \left[1 - \frac{m}{m + (1-m)\delta^2} \frac{e^{-\bar{x}}(1 - e^{-\bar{x}})}{\bar{x}}\right] \leq 0,$$

if and only if $v \geq v^* + a$ (see above that $\Gamma(\bar{\rho}, \bar{x}) = \Gamma(\bar{x}) < 0$ for $v \geq v^* + a$); $\Gamma(\rho, x) \to \frac{1}{m+(1-m)\delta} - \frac{a}{v} > 0$ as $\rho \to 1$. Hence, since $\Gamma(\cdot)$ is continuous in $\rho \in [\bar{\rho}, 1)$, there exists an $\rho^* \in [\bar{\rho}, 1)$ that satisfies $\Gamma(\rho^*, x(\rho^*)) = 0$ if and only if $v \geq v^* + a$. Observe further that

$$\frac{d\Gamma(\cdot)}{d\rho} \mid_{\rho=\rho^*} = \frac{\partial \Gamma(\cdot)}{\partial \rho} + \frac{dx}{d\rho} \frac{\partial \Gamma(\cdot)}{\partial x} \mid_{\rho=\rho^*},$$

where $\frac{\partial \Gamma(\cdot)}{\partial \rho} = \frac{a}{\rho^2 v} \left[1 - \frac{m}{m+(1-m)\delta^2} \frac{e^{-x}(1-e^{-x})}{x} \right] > 0$, $\frac{dx}{d\rho} > 0$ (by (9)), and

$$\frac{\partial \Gamma(\cdot)}{\partial x} \mid_{\rho=\rho^*} > \frac{a}{\rho v} \frac{e^{-x}}{1 - e^{-x} - xe^{-x}} \left[\frac{(x + 1 - e^{-x})^2}{x} - \frac{(1 - e^{-x} - xe^{-x})^2}{x^2} \right] \mid_{\rho=\rho^*} > 0,$$

which follows from exactly the procedure developed above to show $\frac{d\Gamma(x)}{dx} \mid_{x=x^*} > 0$ in (10). Therefore, $\rho^* \in [\bar{\rho}, 1)$ that satisfies $\Gamma(\rho^*, x(\rho^*)) = 0$ is unique given $v \geq v^* + a$.

Combining with the previous result, we have shown that there exist a unique $\rho \in (0, 1)$ if and only if $v > v^*$, and $\rho = 0$ if and only if $v \leq v^*$. This completes the proof of Theorem 1. ■

References

[1] Acemoglu, D. and R. Shimer, (1999), Holdup and Efficiency with Search Frictions, *International Economic Review*, 47, 651-699.

[2] Addison, J.T. and P. Portugal, (2002), Job serach methods and outcomes, *Oxford Economic Papers*, 54, 505-533.

[3] Albrecht, J., Pieter A.G., and S. Vroman, (2006), Equilibrium Directed Search with Multiple Applications, *Review of Economic Studies*, 73, 869-891.

[4] Albrecht, J., Pieter A.G., and S. Vroman, (2012), A note on Peters and Severinov, Competition among sellers who offer auctions instead of prices, *Journal of Economic Theory*, 147, 389-392.

[5] Albrecht, J., Pieter A.G., and S. Vroman, (2014), Efficient Entry in Competing Auctions, *American Economic Review*, forthcoming.

[6] Blau, D.M., and P.K. Robins, (1990), Job search outcomes for the employed and unemployed, *Journal of Political Economy*, 98 (3), 637-655.

[7] Burdett, K., S. Shi and R. Wright, (2001), Pricing and Matching with Frictions, *Journal of Political Economy* 109, 1060-1085.

[8] Casella, A., and N. Hanaki, (2008), Information Channels in Labor Markets: One the Resilience of Referral Hiring, *Journal of Economic Behabior and Organization* 66, 492-513.

[9] Delacroix, A., and S. Shi (2013), Pricing and signaling with frictions, *Journal of Economic Theory*, 148, 1301-1332.

[10] Fougère, D., J. Pradel, and M. Roger, (2009), Does the public employment service affect search effort and outcomes?, *EuroPESn Economic Review* 53, 846-869.

[11] Galenianos, M., (2013), Learning About Match Quality and the Use of Referrals. *Review of Economic Dynamics* 16, 668-690.

[12] Galenianos, M., (2014), Hiring through Referrals. *Journal of Economic Theory* 152, 304-323.

[13] Galenianos, M. and P. Kircher, (2009), Directed Search with Multiple Job Applications, *Journal of Economic Theory*, 114, 445-471.

[14] Gautier, P., B. Hu, and M. Watanabe, (2015), Marketmaking Middlemen, *Tinbergen Institute Working paper*.

[15] Gregg, P., and J. Wadsworth, (1996), How effective are state employment agencies? Jobcentre use and job matching in Britain, *Oxford Bulletin of Economics and Statistics*, 58 (3), 442-467.

[16] Guerrieri, V., R. Shimer and R. Wright, (2010), Adverse Selection in Competitive Search Equilibrium, *Econometrica*, 78, 1823-1862.

[17] Holzer, H.J., (1988), Search method by use by unemployed youth, *Journal of Labor Economics*,6, 1-20.

[18] Launov, A., and K. Wälde (2014), The employment effects of reforming a public employment agency, *mimeo*.

[19] Masters, A. (2007), Middlemen in Search Equilibrium, *International Economic Review*, 48, 343-362.

[20] McAfee, R. P. (1993), Mechanism Design by Competing Sellers, *Econometrica*, 61, 1281-1312.

[21] Menzio, G. (2007), A Theory of Partially Directed Search, *Journal of Political Economy*, 115, 748-769.

[22] Michelacci, C., and J. Suarez (2006), Incomplete Wage Posting, *Journal of Political Economy*, 114, 1098-1123.

[23] Moen, E. R. , (1997), Competitive Search Equilibrium, *Journal of Political Economy*, 105, 385-411.

[24] Müller, A., M. Rieben, and M. Stops (2011), Einschaltungspotenzial fr den Arbeitgeber-Service der Bundesagentur für Arbeit: Ergebnisse aus der IAB-Erhebung des Gesamtwirtschaftlichen Stellenangebots, *IAB-Stellungnahme*, 10, 1-15.

[25] Osberg, L., (1993), Fishing in different pools: Job-search strategies and job finding success in Canada in the Early 1980s, *Journal of Labor Economics*, 11, 348-386.

[26] Peters, M., (1991), Ex Ante Pricing in Matching Games: Non Steady States, *Econometrica*, 59, 1425-1454.

[27] Peters, M., (2001), Limits of Exact Equilibria for Capacity Constrained Sellers with Costly Search, *Journal of Economic Theory*, 95, 139-168.

[28] Peters, M., and S. Severinov (1997), Competition among Sellers who Offer Auctions Instead of Prices, *Journal of Economic Theory*, 75, 141-179.

[29] Pissarides, C., (1979), Job Matching with State Employment Agencies and Random Search, *The Economic Journal*, 89, 818-833.

[30] Pollard, E., F. Behling, J. Hillage, and S. Seckesser, (2012), *Jobcentre Plus Employer Satisfaction and Experience Survey 2012*, Research Reort No 806, Department for Work and Pensions, UK.

[31] Rubinstein, A., and A. Wolinsky. (1987), Middlemen, *Quarterly Journal of Economics* 102, 581-593.

[32] Shi, S., Frictional Assignment I: Effciency, *Journal of Economic Theory*, 98, 232- 260.

[33] Watanabe, M., (2010), A Model of Merchants, *Journal of Economic Theory*, 145, 1865-1889.

[34] Watanabe, M., (2013), Middlemen: A Directed Search Equilibrium Approach, *Tinbergen Institute Working paper*.

[35] Weber, A., and H. Mahringer, (2002), Choice and success of job search methods, *Empirical Economics*, 35, 153-178.

[36] Wielgosz, J.B., and S. Carpenter (1987), The effectiveness of alternative methods of searching for jobs and finding them: An exploratory analysis of the data bearing upon the ways of coping with joblessness, *American Journal of Economics and Sociology*, 46 (2), 151-164.

[37] Yavas, A., (1994), Middlemen in Bilateral Search Markets, *Journal of Labor Economics* 12, No. 3, 406-429.