Pro-cyclical Unemployment Benefits? Optimal Policy in an Equilibrium Business Cycle Model

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Abstract

We study the optimal provision of unemployment insurance (UI) over the business cycle. We use an equilibrium Pissarides search and matching model with aggregate shocks to labor productivity, augmented to incorporate risk-averse workers, endogenous worker search effort decisions, and unemployment benefit expiration. Both the vacancy creation decisions of firms and the search effort decisions of workers respond to changes in UI policy. Our calibrated model is consistent with aggregate US data on labor market volatility, as well as with micro studies on the responsiveness of unemployment duration to benefit generosity. We use the model to characterize the optimal UI policy, allowing both the benefit level and benefit duration to depend on the history of past aggregate shocks. We find that, all else equal, the optimal benefit is decreasing in current productivity and decreasing in current unemployment. Optimal benefits are therefore lowest when current productivity is high and current unemployment is high. The optimal path of benefits reacts non-monotonically to a productivity shock. Following a drop in productivity, benefits initially rise in order to provide short-run relief to the unemployed and stabilize wages, but then fall significantly below their pre-recession level, in order to speed up the subsequent recovery. Under the optimal policy, the path of benefits is pro-cyclical overall. As compared to the existing US UI system, the optimal history-dependent benefits smooth cyclical fluctuations in unemployment and deliver substantial welfare gains.

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1 Introduction

The unemployment insurance (UI) system in the US, as well as in other economies, contains provisions for extending unemployment benefits in response to adverse economic conditions. The optimality of such extensions is the subject of an ongoing debate, which is particularly relevant in the context of the recent recession. On one hand, unemployment benefits provide insurance to workers against heightened unemployment risk. On the other hand, they may distort worker search decisions as well as firms' hiring decisions, possibly exacerbating the negative effects of an adverse economic shock. In this paper, we use a general equilibrium search model to characterize optimal UI policy over the business cycle.

We study UI provision in a Pissarides model with risk-averse workers and aggregate shocks to labor productivity. The advantage of a general equilibrium model is that it enables us to capture the effects of policy changes on both firms' vacancy creation and worker search behavior. Wages are determined by Nash bargaining and therefore respond to both aggregate productivity and the UI policy. The vacancy posting decisions of firms respond to the UI policy because changes in the worker outside option affect wages, and thus the returns to posting a vacancy. Worker search effort decisions respond to the UI policy for two reasons: first, benefits directly affect the value of being unemployed; second, benefits affect the aggregate job-finding rate, and therefore the returns to search, through their effect on vacancy posting. Our general equilibrium approach acknowledges that the fluctuations in the returns to search are themselves endogenous and, in particular, respond to changes in policy.

We consider the optimal policy choice of a benevolent, utilitarian government that can choose both the level of unemployment benefits and the rate at which they expire. The government can change the benefit level and duration in response to aggregate shocks and to run deficits in some states of nature, as long as it balances its budget on average. We solve for the optimal state-contingent UI policy and find that it prescribes for benefits to rise immediately following a drop in productivity. Subsequently, however, it prescribes a persistent decline in benefits below their pre-recession level. The response of benefits to a negative shock is thus non-monotonic. We find that the optimal path of benefits is pro-cyclical overall.

The features of the optimal benefit policy can be explained as follows. Higher and longer-lasting benefits translate into a lower job-finding rate through both lower returns to posting a vacancy and lower returns to search effort. Immediately after a negative productivity shock hits, the social returns to job creation are low, so the government is more concerned with providing short-term relief for the unemployed and slowing the decline of wages than with inducing high job finding. It therefore raises the generosity of benefits temporarily, triggering a decrease in both vacancy creation and worker search effort. Subsequently, since the shock is mean-reverting, the government expects an economic recovery and would like to stimulate job finding, which requires lowering benefits.

Our paper contributes to the literature on the design of optimal UI policy in response to aggregate economic conditions. While a huge literature (see, for example, Baily (1978), Shavell and Weiss (1979), and Hopenhayn and Nicolini (1997) for seminal contributions) has analyzed the insurance-incentives trade-off involved in optimal UI provision, most of this literature has bypassed the optimal response of benefits to aggregate shocks. Recently, several studies (Kiley (2003) , Sanchez (2008), Andersen and Svarer (2010, 2011), Kroft and Notowidigdo (2010), Landais, Michaillat, and Saez (2010)) have examined the optimal design of a state-contingent policy. The focus of this emerging literature is the notion that the moral hazard distortion resulting from unemployment insurance depends on the underlying state of the economy. In particular, an argument can be made for countercyclical unemployment benefits if unemployment benefits distort job search incentives less in recessions than in booms. Our paper reassesses the desirability of such state-contingent policies in a general equilibrium framework.[1] Our result that the optimal benefit path is pro-cyclical is new to this literature.

Our paper is not the first to analyze the design of optimal unemployment insurance in equilibrium search models. A number of studies, such as Fredriksson and Holmlund (2001), Coles and Masters (2006), and Lehmann and van der Linden (2007), study optimal UI design in models with endogenous

[1] Andersen and Svarer (2010) and Landais, Michaillat, and Saez (2010) also consider models with endogenously determined vacancy creation but assume rigid wages, implicitly assuming that changes in UI benefits leave wages unaffected.

job creation and wage bargaining. The contribution of our paper is to introduce aggregate productivity shocks into such optimal policy analysis and to quantitatively characterize the optimal policy.

Finally, our paper differs from the above literature in allowing the government to optimally choose both the level of unemployment benefits and their duration. While the policy debate on the desirability of extending benefits in recessions has focused largely on the duration of benefits, the literature has instead considered how the level of benefits should vary. To our knowledge, our paper is the first to incorporate both policy dimensions in the context of optimal UI provision over the business cycle.

The paper is organized as follows. We present the model in section 2. Section 3 describes the optimal policy. We describe how we calibrate the model to US data in section 4. We report our results in section 5. Finally, we conclude in section 6.

2 Model Description

2.1 Economic Environment

We consider an infinite-horizon discrete-time model. The economy is populated by a unit measure of workers and a larger continuum of firms.

Agents. In any given period, a worker can be either employed (matched with a firm) or unemployed. Workers are risk-averse expected utility maximizers and have expected lifetime utility

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(x_t\right) - c\left(s_t\right) \right],$$

where $\mathbb{E}_0$ is the period-0 expectation operator, $\beta \in (0,1)$ is the discount factor, x_t denotes consumption in period t, and s_t denotes search effort exerted in period t if unemployed. Only unemployed workers can supply search effort: there is no on-the-job search. The within-period utility of consumption $u : \mathbb{R}_+ \to \mathbb{R}$ is twice differentiable, strictly increasing, strictly concave, and satisfies $u'(0) = \infty$. The cost of search effort for unemployed workers $c : [0,1] \to \mathbb{R}$ is twice differentiable, strictly increasing, strictly convex, and satisfies $c'(0) = 0$, $c'(1) = \infty$. An unemployed worker produces h units of the consumption good via home production. There do not exist private insurance markets and workers cannot save or borrow.

Firms are risk-neutral and maximize profits. Workers and firms have the same discount factor β. A firm can be either matched to a worker or vacant. A firm posting a vacancy incurs a flow cost k.

Production. The economy is subject to aggregate shocks to labor productivity. Specifically, a matched worker-firm pair produces output z_t, where z_t is stochastic. We assume that $\ln z_t$ follows an AR(1) process

$$\ln z_t = \rho \ln z_{t-1} + \sigma_\varepsilon \varepsilon_t,$$

where $0 \leq \rho < 1$, $\sigma_\varepsilon > 0$, and ε_t are independent and identically distributed standard normal random variables. We will write $z^t = \{z_0, z_1, ..., z_t\}$ to denote the history of shocks up to period t.

Matching. Job creation occurs through a matching function. The number of new matches in period t equals

$$M\left(S_t\left(1 - L_{t-1}\right), v_t\right),$$

where $1 - L_{t-1}$ is the unemployment level in period $t-1$, S_t is the average search effort exerted by unemployed workers in period t, and v_t is the measure of vacancies posted in period t. The quantity $\mathcal{N}_t = S_t\left(1 - L_{t-1}\right)$ represents the measure of efficiency units of worker search.

The matching function M exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and has the property that the number of new matches cannot exceed the number of potential matches: $M\left(\mathcal{N}, v\right) \leq \min\{\mathcal{N}, v\}\ \forall \mathcal{N}, v$. We define

$$\theta_t = \frac{v_t}{S_t\left(1 - L_{t-1}\right)}$$

to be the market tightness in period t. We define the functions

$$f\left(\theta\right) = \frac{M\left(S\left(1-L\right), v\right)}{S\left(1-L\right)} = M\left(1, \theta\right) \quad \text{and}$$
$$q\left(\theta\right) = \frac{M\left(S\left(1-L\right), v\right)}{v} = M\left(\frac{1}{\theta}, 1\right)$$

where $f(\theta)$ is the job-finding probability per efficiency unit of search and $q(\theta)$ is the probability of filling a vacancy. By the assumptions on M made above, the function $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ. For an individual worker exerting search effort s, the probability of finding a job is $sf(\theta)$. When workers choose the amount of search effort s, they take as given the aggregate job-finding probability $f(\theta)$.

Existing matches are exogenously destroyed with a constant job separation probability δ. Thus, any of the L_{t-1} workers employed in period $t-1$ has a probability δ of becoming unemployed.

2.2 Government Policy

The US UI system is financed by payroll taxes on firms and is administered at the state level. However, under the provisions of the Social Security Act, each state can borrow from a federal unemployment insurance trust fund, provided it meets certain federal requirements. Motivated by these features of the UI system, we assume that the government in the model economy can insure against aggregate shocks by buying and selling claims contingent on the aggregate state and is required to balance its budget only in expectation. Further, we assume that the price of a claim to one unit of consumption in state z_{t+1} after a history z^t is equal to the probability of z_{t+1} conditional on z^t; this would be the case, e.g., in the presence of a large number of out-of state risk-neutral investors with the same discount factor.

Government policies are restricted to take the following form. The government levies a constant lump sum tax τ on firm profits and uses its tax revenues to finance unemployment benefits. Only a subset of unemployed workers may be eligible for benefits; the government is allowed to choose both the level of benefits and the rate at which they expire. We assume stochastic benefit expiration. A benefit policy at time t thus consists of a pair (b_t, e_t), where $b_t \geq 0$ is the level of benefits provided to those workers who are eligible at time t, and $e_t \in [0, 1]$ is the probability that an individual eligible worker becomes ineligible the following period. The eligibility status of a worker evolves as follows. A worker employed in period t is automatically eligible for benefits in case of job separation. An unemployed worker eligible for benefits in period t becomes ineligible the following period with probability e_t, and an ineligible worker does not regain eligibility until he finds a job. All eligible workers receive the same benefits b_t; ineligible workers receive no unemployment benefits, but instead receive an exogenously given welfare payment p.

We allow the benefit policy to depend on the entire history of past aggregate shocks; thus the policy $b_t = b_t\left(z^t\right), e_t = e_t\left(z^t\right)$ must be measurable with respect to z^t. Benefits are constrained to be non-negative: the government cannot tax home production.

2.3 Timing

The government commits to a policy $(\tau, b_t(\cdot), e_t(\cdot))$ once and for all before the period-0 shock realizes. Within each period t, the timing is as follows.

1. The economy enters period t with a level of employment L_{t-1}. Of the $1 - L_{t-1}$ unemployed workers, a measure $D_{t-1} \leq 1 - L_{t-1}$ are eligible for benefits, i.e. will receive benefits in period t if they do not find a job.

2. The aggregate shock z_t then realizes. Firms observe the aggregate shock and decide how many vacancies to post, at cost k per vacancy. At the same time, workers choose their search effort s_t at the cost of $c(s_t)$. Letting S_t^E and S_t^I be the search effort exerted by an eligible unemployed worker and an ineligible unemployed worker, respectively, the aggregate search effort is then equal to $S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})$, and the market tightness is therefore equal to

$$\theta_t = \frac{v_t}{S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})} \tag{1}$$

3. $f(\theta)\left(S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})\right)$ unemployed workers find jobs. At the same time, a fraction δ of the existing L_{t-1} matches are exogenously destroyed.

4. All the workers who are now employed produce z_t and receive a bargained wage w_t (below we describe wage determination in detail). Workers who (i) were employed and lost a job, or (ii) were eligible unemployed workers and did not find a job, consume home production plus unemploy-

ment benefits, $h + b_t$ and lose their eligibility for the next period with probability e_t. Ineligible unemployed workers who have not found a job consume home production plus public assistance, $h + p$, and remain ineligible for the following period.

This determines the law of motion for employment

$$\begin{aligned} L_t\left(z^t\right) &= (1-\delta) L_{t-1}\left(z^{t-1}\right) \\ &\quad + f\left(\theta_t\left(z^t\right)\right)\left[S_t^E\left(z^t\right) D_{t-1}\left(z^{t-1}\right) + S_t^I\left(z^t\right)\left(1 - L_{t-1}\left(z^{t-1}\right) - D_{t-1}\left(z^{t-1}\right)\right)\right] \end{aligned} \tag{2}$$

and the law of motion for the measure of eligible unemployed workers:

$$D_t\left(z^t\right) = \left(1 - e_t\left(z^t\right)\right)\left[\delta L_{t-1}\left(z^{t-1}\right) + \left(1 - s_t\left(z^t\right) f\left(\theta_t\left(z^t\right)\right)\right) D_{t-1}\left(z^{t-1}\right)\right] \tag{3}$$

Thus, the measure of workers receiving benefits in period t is $\delta L_{t-1} + (1 - s_t f(\theta_t)) D_{t-1} = \frac{D_t}{1-e_t}$.

Since we assume that the government has access to financial markets in which a full set of state-contingent claims is traded, its budget constraint is a present-value budget constraint

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t\left(z^t\right)\tau - \left(\frac{D_t\left(z^t\right)}{1 - e_t\left(z^t\right)}\right) b_t\left(z^t\right) \right\} \geq 0 \tag{4}$$

2.4 Worker Value Functions

A worker entering period t employed retains his job with probability $1 - \delta$ and loses it with probability δ. If he retains his job, he consumes his wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he loses his job, he consumes his home production plus benefits, $h + b_t\left(z^t\right)$ and proceeds as unemployed to period $t+1$. With probability $1 - e_t$ he then retains his eligibility for benefits in period $t+1$, and with probability e_t he loses his eligibility. Denote by $W_t\left(z^t\right)$ the value after a history z^t for a worker who enters period t employed.

A worker entering period t unemployed and eligible for benefits chooses search effort s_t^E and suffers the disutility $c\left(s_t^E\right)$. He finds a job with probability $s_t^E f\left(\theta_t\left(z^t\right)\right)$ and remains unemployed with the complementary probability. If he finds a job, he earns the wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he remains unemployed, he consumes his home production plus benefits, $h+b_t\left(z^t\right)$, and proceeds as unemployed to the next period. With probability $1-e_t$ he retains his eligibility for benefits in period $t+1$, and with probability e_t he loses his eligibility. Denote by $U_t^E\left(z^t\right)$ the value after a history z^t for a worker who enters period t as eligible unemployed.

Finally, a worker entering period t unemployed and ineligible for benefits chooses search effort s_t^I and suffers the disutility $c\left(s_t^I\right)$. He finds a job with probability $s_t^I f\left(\theta_t\left(z^t\right)\right)$ and remains unemployed with the complementary probability. If he finds a job, he earns the wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he remains unemployed, he consumes his home production plus welfare payments, $h+p$, and proceeds as ineligible unemployed to the next period. Denote by $U_t^I\left(z^t\right)$ the value after a history z^t for a worker who enters period t as ineligible unemployed.

The Bellman equations for the three types of workers are then:

$$\begin{aligned}
W_t\left(z^t\right) =& (1-\delta)\left[u\left(w_t\left(z^t\right)\right)+\beta\mathbb{E}W_{t+1}\left(z^{t+1}\right)\right] \\
&+\delta\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\right)\mathbb{E}U_{t+1}^E\left(z^{t+1}\right)+\beta e_t\mathbb{E}U_{t+1}^I\left(z^{t+1}\right)\right] && (5)\\
U_t^E\left(z^t\right) =& \max_{s_t^E} -c\left(s_t^E\right)+s_t^E f\left(\theta_t\left(z^t\right)\right)\left[u\left(w_t\left(z^t\right)\right)+\beta\mathbb{E}W_{t+1}\left(z^{t+1}\right)\right] \\
&+\left(1-s_t^E f\left(\theta_t\left(z^t\right)\right)\right)\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\left(z^t\right)\right)\mathbb{E}U_{t+1}^E\left(z^{t+1}\right)+\beta e_t\mathbb{E}U_{t+1}^I\left(z^{t+1}\right)\right] && (6)\\
U_t^I\left(z^t\right) =& \max_{s_t^I} -c\left(s_t^I\right)+s_t^I f\left(\theta_t\left(z^t\right)\right)\left[u\left(w_t\left(z^t\right)\right)+\beta\mathbb{E}W_{t+1}\left(z^{t+1}\right)\right] \\
&+\left(1-s_t^I f\left(\theta_t\left(z^t\right)\right)\right)\left[u\left(h+p\right)+\beta\mathbb{E}U_{t+1}^I\left(z^{t+1}\right)\right] && (7)
\end{aligned}$$

It will be useful to define the worker's surplus from being employed. The surplus utility from being employed, as compared to eligible unemployed, in period t is

$$\Delta_t\left(z^t\right)=\left[u\left(w_t\left(z^t\right)\right)+\beta\mathbb{E}_t W_{t+1}\left(z^{t+1}\right)\right]-\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\right)\mathbb{E}U_{t+1}^E\left(z^{t+1}\right)+\beta e_t\mathbb{E}U_{t+1}^I\left(z^{t+1}\right)\right] \tag{8}$$

Similarly, we define the surplus utility from being employed as compared to being unemployed and ineligible for benefits:

$$\Xi_t\left(z^t\right) = \left[u\left(w_t\left(z^t\right)\right) + \beta\mathbb{E}_t W_{t+1}\left(z^{t+1}\right)\right] - \left[u\left(h+p\right)\right) + \beta\mathbb{E}U^I_{t+1}\left(z^{t+1}\right)\right] \tag{9}$$

2.5 Firm Value Functions

A matched firm retains its worker with probability $1-\delta$. In this case, the firm receives the output net of wages and taxes, $z_t - w_t\left(z^t\right) - \tau$, and then proceeds into the next period as a matched firm. If the firm loses its worker, it gains nothing in the current period and proceeds into the next period unmatched. A firm that posts a vacancy incurs a flow cost k and finds a worker with probability $q\left(\theta_t\left(z^t\right)\right)$. If the firm finds a worker, it gets flow profits $z_t - w_t\left(z^t\right) - \tau$ and proceeds into the next period as a matched firm. Otherwise, it proceeds unmatched into the next period.

Denote by $J_t\left(z^t\right)$ the value of a firm that enters period t matched to a worker, and denote by $V_t\left(z^t\right)$ the value of an unmatched firm posting a vacancy. These value functions satisfy the following Bellman equations:

$$J_t\left(z^t\right) = (1-\delta)\left[z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right] + \delta\beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{10}$$

$$V_t\left(z^t\right) = -k + q\left(\theta_t\left(z^t\right)\right)\left[z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right] + \left(1 - q\left(\theta_t\left(z^t\right)\right)\right)\beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{11}$$

The firm's surplus from employing a worker in period t is denoted

$$\Gamma_t\left(z^t\right) = z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right) - \beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{12}$$

2.6 Wage Bargaining

We assume that wages are determined according to Nash bargaining: the wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus. Further, the worker's outside option is being unemployed and eligible for benefits, since he becomes eligible upon locating an employer and retains eligibility if negotiations with the employer break down. The worker-firm pair therefore chooses the wage $w_t\left(z^t\right)$ to maximize

$$\Delta_t\left(z^t\right)^{\xi}\Gamma_t\left(z^t\right)^{1-\xi}, \tag{13}$$

where $\xi \in (0,1)$ is the worker's bargaining weight.

2.7 Equilibrium Given Policy

In this section, we define the equilibrium of the model, taking as given a government policy $(\tau, b_t(\cdot), e_t(\cdot))$ and characterize it.

2.7.1 Equilibrium Definition

Taking as given an initial condition (z_{-1}, L_{-1}), we define an equilibrium given policy:

Definition 1 *Given a policy* $(\tau, b_t(\cdot), e_t(\cdot))$ *and an initial condition* (z_{-1}, L_{-1}) *an equilibrium is a sequence of* z^t*-measurable functions for wages* $w_t\left(z^t\right)$*, search effort* $S_t^E\left(z^t\right)$*,* $S_t^I\left(z^t\right)$*, market tightness* $\theta_t\left(z^t\right)$*, employment* $L_t\left(z^t\right)$*, measures of eligible workers* $D_t\left(z^t\right)$*, and value functions*

$$\left\{W_t\left(z^t\right), U_t^E\left(z^t\right), U_t^I\left(z^t\right), J_t\left(z^t\right), V_t\left(z^t\right), \Delta_t\left(z^t\right), \Xi_t\left(z^t\right), \Gamma_t\left(z^t\right)\right\}$$

such that:

1. *The value functions satisfy the worker and firm Bellman equations (5), (6), (7), (8), (9), (10), (11), (12)*

2. *Optimal search: The search effort* S_t^E *solves the maximization problem in (6) for* s_t^E*, and the search effort* S_t^I *solves the maximization problem in (7) for* s_t^I

3. *Free entry: The value* $V_t\left(z^t\right)$ *of a vacant firm is zero for all* z^t

4. *Nash bargaining: The wage maximizes equation (13)*

5. *Law of motion for employment and eligibility status: Employment and the measure of eligible unemployed workers satisfy (2) , (3)*

6. *Budget balance: Tax revenue and benefits satisfy (4)*

2.7.2 Characterization of Equilibrium

We characterize the equilibrium given policy via a system of equations that involves allocations only, and does not involve the value functions. This will be helpful in computing the optimal policy.

Lemma 1 *Fix an initial condition and a policy $(\tau, b_t(\cdot), e_t(\cdot))$. Suppose that the sequence*

$$\Upsilon_t(z^t) = \{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t),$$
$$W_t(z^t), U_t^E(z^t), U_t^I(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Xi_t(z^t), \Gamma_t(z^t)\}$$

is an equilibrium. Then the sequences $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$ satisfy

1. *The laws of motion (2) , (3)*

2. *The budget equation (4)*

3. *Modified worker Bellman equations*

$$\frac{c'(S_t^E)}{f(\theta_t)} = u(w_t) - u(h+b_t) + (1-e_t)\beta\mathbb{E}_t\left(c(S_{t+1}^E) + (1-\delta-S_{t+1}^E f(\theta_{t+1}))\frac{c'(S_{t+1}^E)}{f(\theta_{t+1})}\right)$$
$$+ e_t\beta\mathbb{E}_t\left(c(S_{t+1}^I) + (1-S_{t+1}^I f(\theta_{t+1}))\frac{c'(S_{t+1}^I)}{f(\theta_{t+1})} - \delta\frac{c'(S_{t+1}^I)}{f(\theta_{t+1})}\right) \tag{14}$$

$$\frac{c'(S_t^I)}{f(\theta_t)} = u(w_t) - u(h+p) + \beta\mathbb{E}_t\left(c(S_{t+1}^I) + (1-S_{t+1}^I f(\theta_{t+1}))\frac{c'(S_{t+1}^I)}{f(\theta_t)} - \delta\frac{c'(S_{t+1}^E)}{f(\theta_{t+1})}\right) \tag{15}$$

4. *Modified firm Bellman equation*

$$\frac{k}{q(\theta_t(z^t))} = z_t - w_t(z^t) - \tau + \beta(1-\delta)\mathbb{E}_t\frac{k}{q(\theta_{t+1}(z^{t+1}))} \tag{16}$$

5. *Nash bargaining condition*

$$\xi u'(w_t(z^t))\,k\theta_t(z^t) = (1-\xi)\,c'(S_t^E(z^t)) \tag{17}$$

Conversely, if $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$ *satisfy (2)-(4) and (14)-(17), then there exist value functions such that* $\Upsilon_t(z^t)$ *is an equilibrium.*

Proof. First, observe that the necessary first-order conditions for optimal search effort are

$$\Delta_t = \frac{c'\left(S_t^E\right)}{f(\theta_t)} \tag{18}$$

$$\Xi_t = \frac{c'\left(S_t^I\right)}{f(\theta_t)} \tag{19}$$

Next, taking the differences of the workers' value functions from equations (5), (6), (7), we have

$$\begin{aligned} W_t(z^t) - U_t^E(z^t) &= c\left(S_t^E(z^t)\right) + \left(1 - \delta - S_t^E(z^t) f\left(\theta_t(z^t)\right)\right) \Delta_t(z^t) \\ &= c\left(S_t^E(z^t)\right) + \left(1 - \delta - S_t^E(z^t) f\left(\theta_t(z^t)\right)\right) \frac{c'\left(S_t^E\right)}{f(\theta_t)} \end{aligned} \tag{20}$$

$$\begin{aligned} W_t(z^t) - U_t^I(z^t) &= c\left(S_t^I(z^t)\right) + \left(1 - S_t^I(z^t) f\left(\theta_t(z^t)\right)\right) \Xi_t(z^t) - \delta \Delta_t(z^t) \\ &= c\left(S_t^I(z^t)\right) + \left(1 - S_t^I(z^t) f\left(\theta_t(z^t)\right)\right) \frac{c'\left(S_t^I(z^t)\right)}{f\left(\theta_t(z^t)\right)} - \delta \frac{c'\left(S_t^E(z^t)\right)}{f\left(\theta_t(z^t)\right)} \end{aligned} \tag{21}$$

Next, we rearrange the expressions for worker surpluses (8), (9) to get

$$\begin{aligned} \Delta_t(z^t) =& u\left(w_t(z^t)\right) - u\left(h + b_t(z^t)\right) \\ &+ \beta\left(1 - e_t(z^t)\right) \mathbb{E}_t\left(W_{t+1}(z^{t+1}) - U_{t+1}^E(z^{t+1})\right) + \beta e_t(z^t) \mathbb{E}_t\left(W_{t+1}(z^{t+1}) - U_{t+1}^I(z^{t+1})\right) \end{aligned} \tag{22}$$

$$\Xi_t(z^t) = u\left(w_t(z^t)\right) - u(h + p) + \beta \mathbb{E}_t\left(W_{t+1}(z^{t+1}) - U_{t+1}^I(z^{t+1})\right) \tag{23}$$

Now, substituting (18) and (20) into the left and right hand sides of (22) gives (14); similarly, substituting (19) and (21) into the left and right hand sides of (23) gives (15).

Next, we derive the law of motion for the firm's surplus from hiring. By the free-entry condition, the value $V_t\left(z^t\right)$ of a firm posting a vacancy must be zero. Equations (10) and (11) then simplify to:

$$J_t\left(z^t\right) = (1-\delta)\left[z_t - w_t\left(z^t\right) - \tau + \beta \mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right] \tag{24}$$

$$0 = -k + q\left(\theta_t\left(z^t\right)\right)\left[z_t - w_t\left(z^t\right) - \tau + \beta \mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right]; \tag{25}$$

which together imply

$$J_t\left(z^t\right) = (1-\delta)\frac{k}{q\left(\theta_t\left(z^t\right)\right)} \tag{26}$$

$$\Gamma_t\left(z^t\right) = \frac{k}{q\left(\theta_t\left(z^t\right)\right)} \tag{27}$$

Equations (24) and (26) imply that $\Gamma_t\left(z^t\right)$ follows the law of motion $\Gamma_t\left(z^t\right) = z_t - w_t\left(z^t\right) - \tau + \beta(1-\delta)\mathbb{E}_t\Gamma_{t+1}\left(z^{t+1}\right)$, which, by (27), is precisely (16).

Finally, the first-order condition with respect to $w_t\left(z^t\right)$ for the Nash bargaining problem (13) is

$$\xi u'\left(w_t\left(z^t\right)\right)\Gamma_t\left(z^t\right) = (1-\xi)\Delta_t\left(z^t\right) \tag{28}$$

Substituting (27) and (18) into (28) and using the fact that $f(\theta) = \theta q(\theta)$ yields (17).

The converse of the result holds since the value functions can be recovered via the corresponding Bellman equations. ■

The conditions (14)-(17) are straightforward to interpret. Equations (14) and (15) state that the marginal cost of increasing the job finding probability for the eligible and ineligible workers, respectively, equals the marginal benefit. The marginal cost (left-hand side of each equation) of increasing the job finding probability is the marginal disutility of search for that worker weighted by the aggregate job finding rate. The marginal benefit (right-hand side of each equation) equals the current consumption gain from becoming employed plus the benefit of economizing on search costs in the future. Equation (16) gives a similar optimality condition for firms: it equates the marginal cost of creating a vacancy, weighted by the probability of filling that vacancy, to the benefit of employing a worker. Finally, (17) is a restatement of the first-order condition of the bargaining problem. It will be clear in section 3 that the conditions (14)-(17) will play the role of incentive constraints in the optimal policy problem, analogous to incentive constraints in principal-agent models of unemployment insurance, e.g. Hopenhayn and Nicolini (1997).

3 Optimal Policy

We assume that the government is utilitarian: it chooses a policy to maximize the period-0 expected value of worker utility, taking the equilibrium conditions as constraints.

Definition 2 *A policy $\tau, b_t\left(z^t\right), e_t\left(z^t\right)$ is feasible if there exists a sequence of z^t-measurable functions $\{w_t\left(z^t\right), S_t^E\left(z^t\right), S_t^I\left(z^t\right), \theta_t\left(z^t\right), L_t\left(z^t\right), D_t\left(z^t\right)\}$ such that (2), (3), (14)-(17) hold for all z^t, and the government budget constraint (4) is satisfied.*

Definition 3 *The optimal policy is a policy $\tau, b_t\left(z^t\right), e_t\left(z^t\right)$ that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t\left(z^t\right) u\left(w_t\left(z^t\right)\right) + \left(\frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+b_t\left(z^t\right)\right) + \left(1 - L_t\left(z^t\right) - \frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+p\right) \\ -D_{t-1}\left(z^{t-1}\right) c\left(S_t^E\left(z^t\right)\right) - \left(1 - L_{t-1}\left(z^{t-1}\right) - D_{t-1}\left(z^t - 1\right)\right) c\left(S_t^I\left(z^t\right)\right) \end{array} \right\} \tag{29}$$

over the set of all feasible policies.

The government's problem can be written as one of choosing a policy $\tau, b_t\left(z^t\right), e_t\left(z^t\right)$ together with functions $\{w_t\left(z^t\right), S_t^E\left(z^t\right), S_t^I\left(z^t\right), \theta_t\left(z^t\right), L_t\left(z^t\right), D_t\left(z^t\right)\}$ to maximize (29) subject to (2), (3), (14)-(17) holding for all z^t, and subject to the government budget constraint (4). We find the optimal policy by solving the system of necessary first-order conditions for this problem. The period-t solution will naturally be state-dependent: in particular, it will depend on the current productivity z_t, as well as the current unemployment level $1 - L_{t-1}$, and current measure of benefit-eligible workers D_{t-1} with which the economy has entered period t. However, in general the triple $(z_t, 1 - L_{t-1}, D_{t-1})$ is not a sufficient state variable for pinning down the optimal policy, which may depend on the entire past history of aggregate shocks. In the appendix, we show that the optimal period t solution is a function of $(z_t, 1 - L_{t-1}, D_{t-1})$ as well as $(e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$, where e_{t-1} is the previous period's benefit expiration rate and $\mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$ are Lagrange multipliers on the constraints (14),(15),(16), respectively, in the maximization problem (29). The tuple $(z_t, 1 - L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$ captures the dependence of the optimal b_t, e_t on the history z^t. The fact that the z_t, $1 - L_{t-1}$ and D_{t-1} are not sufficient reflects the fact that the optimal policy is time-inconsistent: for example, the optimal benefits after two different histories of shocks may differ even though the two histories result in the same current productivity and the same current unemployment level. Intuitively, the government might want to induce firms to post vacancies - and workers to search - by promising low unemployment benefits, but has an ex post incentive to provide higher benefits, so as to smooth worker consumption, after employment outcomes have realized. Including the variables $e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$ as state variables in the optimal policy captures exactly this trade-off. Note that we assume throughout the paper that the government can fully commit to its policy. In the appendix we explain the method used to solve for the optimal policy.

4 Calibration

We calibrate the model to verify that it captures salient features of the US labor market, and is thus a useful one for studying optimal policy design. Unlike previous versions of the Pissarides model calibrated in the literature, e.g. Shimer (2005) and Hagedorn and Manovskii (2008), our model incorporates endogenous search intensity choices and stochastic benefit expiration. Moreover, the market tightness in our model is not equal to the vacancy-unemployment ratio; rather, it is the object defined in (1), which we do not directly observe in the data. Our calibration strategy will be correspondingly modified relative to the previous literature. As explained below, we will calibrate the model to ensure that it is consistent both with aggregate US labor market data and with results from micro studies on the responsiveness of unemployment duration to benefit generosity.

We normalize mean productivity to one. We assume a benefit scheme that mimics the benefit extension provisions currently in place within the US policy. The standard benefit duration is 26 weeks; local and federal employment conditions trigger automatic 20-week and 33-week extensions. In the model we assume that $e_t = 1/59$ when productivity is below two standard deviations below the mean, $e_t = 1/46$ when productivity is between one and two standard deviations below the mean, and $e_t = 1/26$ otherwise. We pick the tax rate $\tau = 0.023$ so that the government balances its budget if the unemployment rate is 5.5%.

We assume log utility: $u(x) = \ln x$. We will also consider a constant relative risk aversion utility specification and investigate the robustness of our results to different risk aversion parameters. For the cost of search, we assume the functional form

$$c(s) = \frac{A}{1+\psi}\left[(1-s)^{-(1+\psi)} - 1\right] - As \tag{30}$$

This functional form satisfies all the assumptions made on the search cost function; in particular, it implies that the optimal search effort will always be between 0 and 1 for any $A > 0$.

For the matching function, we follow den Haan, Ramey, and Watson (2000) and pick

$$M(\mathcal{N}, v) = \frac{\mathcal{N} v}{\left[\mathcal{N}^{\chi} + v^{\chi}\right]^{1/\chi}}$$

This matching technology satisfies all the assumptions made earlier, in particular the assumption that the implied job-finding rate is always less than one. We have:

$$f(\theta) = \frac{\theta}{(1+\theta^{\chi})^{1/\chi}}$$

$$q(\theta) = \frac{1}{(1+\theta^{\chi})^{1/\chi}}$$

The model period is taken to be 1 week. We set the discount factor $\beta = 0.99^{1/12}$, implying a yearly discount rate of 4%. Following Shimer (2005), labor productivity z_t is taken to mean real output per person in the non-farm business sector. This measure of productivity is taken from the data constructed by the BLS and the parameters for the shock process are estimated, at the weekly level, to be $\rho = 0.9895$ and $\sigma_{\varepsilon} = 0.0034$. The job separation parameter δ is set to 0.0081 to match the average weekly job separation rate.[2] We set $k = 0.58$ following Hagedorn and Manovskii (2008), who estimate of the costs of vacancy creation to be 58% of weekly labor productivity.

This leaves five parameters to be calibrated: (1) the value h of home production; (2) the worker bargaining weight ξ; (3) the matching function parameter χ; (4) the level coefficient of the search cost function A; and (5) the curvature parameter of the search cost function ψ. We jointly calibrate these five parameters to simultaneously match five data targets: (1) the average vacancy-unemployment ratio; (2) the standard deviation of vacancy-unemployment ratio; (3) the average weekly job-finding rate; (4) the average duration of unemployment; and (5) the elasticity of unemployment duration with respect to benefits. The first four of these targets are directly measured in the data. For the elasticity of unemployment duration with respect to benefits, $\mathcal{E}_{d,b}$, we use micro estimates reported by Meyer (1990) and target an elasticity of 0.9. Intuitively, given the first three parameters, the average unemployment duration and its elasticity with respect to benefits identify the parameters A and ψ, since these parameters govern the distortions in search behavior induced by benefits. Table 1 below reports the calibrated parameters. Our calibrated model is also consistent with non-targeted observations in the data: for example, the elasticity of unemployment duration with respect to the potential duration of benefits is 0.167 in the model, consistent with the estimates reported in Moffitt (1985) and close to other estimates in the literature.

Table 1: Internally Calibrated Parameters

	Parameter	Value	Target	Data	Model
h	Home production	0.580	Mean $v/(1-L)$	0.634	0.634
ξ	Bargaining power	0.114	St. dev of $\ln(v/(1-L))$	0.259	0.259
χ	Matching parameter	0.492	Mean job finding rate	0.139	0.139
A	Disutility of search	0.0015	Unemployment duration	13.2	13.2
ψ	Search cost curvature	3.786	$\mathcal{E}_{d,b}$	0.9	0.9

[2] See Hagedorn and Manovskii (2008) on how to obtain the weekly estimates for the job finding rate and the job separation rate from monthly data.

5 Results

In order to illustrate the mechanism behind the optimal policy, in Figure 1 we plot the optimal benefit policy function $b_t\left(z, 1-L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1}\right)$ as a function of current z and last period's $1-L$ only, keeping D_{t-1}, e_{t-1}, μ_{t-1}, ν_{t-1} and γ_{t-1} fixed at their average values. The optimal benefit level is decreasing in current productivity z and decreasing in unemployment $1-L$. The intuition for this result is that the optimal benefit is lower in states of the world when the marginal social benefit of job creation is higher, because lower benefits are used to encourage search effort by workers and vacancy creation by firms. The marginal social benefit of job creation is higher when z is higher, since the output of an additional worker-firm pair is then higher. The marginal social benefit is also higher when current employment is lower. As a consequence, optimal benefits are lowest, all else equal, when current productivity is high and current employment is low, i.e. at the beginning of an economic *recovery*. Figure 2 illustrates the same result for the optimal duration of benefits: optimal benefit duration is lowest at times of high productivity and high unemployment. This shape of the policy function also implies that during a recession, there are two opposing forces at work - low productivity and high unemployment - which give opposite prescriptions for the response of optimal benefits. This gives an ambiguous prediction for the overall cyclicality of benefit levels and benefit duration.

In order to understand the overall behavior of the optimal policy, in Figures 3 and 4 we analyze the response of the economy to a negative productivity shock under the optimal policy and compare it to the response under the current policy. In Figure 3 we plot the response of the optimal policy when productivity drops by 1% after a long sequence of productivity held at 1. The optimal benefit level initially jumps up, but then falls for about two quarters following the shock, and slowly reverts to its pre-shock level. The same is true of optimal benefit duration. Unemployment rises in response to the drop in productivity and continues rising for about one quarter before it starts to return to its pre-shock level. Note that the rise in unemployment is significantly lower than under the current benefit policy.

In Figure 4 we plot the response of other key labor market variables. As compared to the current benefit policy, the optimal policy results in a faster recovery of the vacancy-unemployment ratio, the search intensity of unemployed workers eligible for benefits, and the job finding rate. Wages also fall more gradually under the optimal policy than they do under the current policy.

The intuition for this optimal policy response is that the government would like to provide immediate insurance against the negative shock and, expecting future productivity to rise, would like to induce a recovery in vacancy creation and search effort. Thus, benefit generosity responds positively to the initial drop in productivity but negatively to the subsequent rise in unemployment, precisely as implied by Figures 1 and 2. The initial rise in benefits smooths the fall in wages. The subsequent benefit decline, as well as the increase in the rate of benefit expiration, ameliorates the rise in unemployment. The government optimally uses a combination of both available policy instruments - benefit level and benefit duration - to achieve this effect.

We next investigate how the economy behaves over time under the optimal policy. To this end, we simulated the model both under the current benefit policy and under the optimal policy. Table 2 reports the summary statistics, under the optimal policy, for the behavior of unemployment benefit levels b and potential benefit duration $1/e$. Benefits are higher and expire faster under the optimal policy than under the current policy. The optimal tax rate under the optimal policy is $\tau = 0.018$, lower than under the current policy.

The key observation is that, over a long period of time, the correlation of optimal benefits with productivity is positive: both benefit levels and potential benfit duration are pro-cyclical in the long run and, in particular, negatively correlated with the unemployment rate. Moreover, this result is not driven by any balanced budget requirement, since we allow the government to run deficits in recessions.

Our results are qualitatively robust to modifications of the model parameters. In particular, we have computed the optimal policy under smaller values of home production, larger values of worker bargaining power, and larger values of risk aversion. These alternative calibrations of the model retain the shape of the optimal policy functions a well as the feature that the optimal benefit schedule is pro-cyclical.

Tables 3 and 4 report the moments of key labor market variables when the model is simulated under the current policy and the optimal policy, respectively. As compared to the optimal policy, the optimal policy results in lower average unemployment and lower unemployment volatility. These results corroborate our earlier intuition that the benefit policy serves to smooth the cyclical fluctuations in unemployment.

Finally, we compute the expected welfare gain from switching from the current policy to the optimal policy. We find that implementing the optimal policy results in a significant welfare gain: 0.67% as measured in consumption equivalent variation terms.

6 Conclusion

We analyzed the design of an optimal UI system in the presence of aggregate shocks in an equilibrium search and matching model. Optimal benefits respond non-monotonically to productivity shocks: while raising benefit generosity may be optimal at the onset of a recession, it becomes suboptimal as the recession progresses and inducing a recovery is desirable. We find that optimal benefits are pro-cyclical overall, counter to previous results in the literature. Furthermore, we find that the optimal benefit policy, in addition to providing insurance to workers, results in the smoothing of unemployment over the business cycle.

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A Solving for the Optimal Policy

The government is maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t\left(z^t\right) u\left(w_t\left(z^t\right)\right) + \left(\frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+b_t\left(z^t\right)\right) + \left(1 - L_t\left(z^t\right) - \frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+p\right) \\ -D_{t-1}\left(z^{t-1}\right) c\left(S_t^E\left(z^t\right)\right) - \left(1 - L_{t-1}\left(z^{t-1}\right) - D_{t-1}\left(z^t - 1\right)\right) c\left(S_t^I\left(z^t\right)\right) \end{array} \right\} \tag{31}$$

subject to the conditions (2), (3), (14). (15),(16),(17) holding for all z^t, and subject to the government budget constraint (4).

Let $\pi\left(z^t\right)$ be the probability of history $z^t = \{z_0, z_1, ..., z_t\}$ given the initial condition z_{-1}. Denote by η the Lagrange multiplier on (4), and denote the Lagrange multipliers on (2), (3), (14). (15),(16),(17) by

$$\beta^t \pi\left(z^t\right) \lambda_t\left(z^t\right), \beta^t \pi\left(z^t\right) \alpha_t\left(z^t\right), \beta^t \pi\left(z^t\right) \mu_t\left(z^t\right), \beta^t \pi\left(z^t\right) \nu_t\left(z^t\right), \beta^t \pi\left(z^t\right) \gamma_t\left(z^t\right), \beta^t \pi\left(z^t\right) \phi_t\left(z^t\right),$$

respectively. In what follows, we suppress the dependence on z^t for notational simplicity. The first order necessary conditions with respect to $b_t, e_t, w_t, S_t^E, S_t^I, L_t, D_t, \theta_t$, respectively, are:

$$\left(D_t - \left(1 - e_t\right) \mu_t\right) u'\left(h + b_t\right) = \eta D_t \tag{32}$$

$$D_t\left[u\left(h+b_t\right) - u\left(h+p\right) - \eta b_t - \alpha_t\right] = \mu_t\left(1 - e_t\right)\left[u\left(h+b_t\right) - u\left(h+p\right) - \frac{c'\left(S_t^I\right) - c'\left(S_t^E\right)}{f\left(\theta_t\right)}\right] \tag{33}$$

$$\gamma_t = \left(L_t + \mu_t + \nu_t\right) u'\left(w_t\right) - \phi_t \xi u''\left(w_t\right) k \theta_t \tag{34}$$

$$\begin{aligned} \phi_t\left(\xi - 1\right) c''\left(S_t^E\right) =& D_{t-1}\left[\left(\lambda_t - \alpha_t\right) f\left(\theta_t\right) - c'\left(S_t^E\right)\right] \\ &+ \frac{c''\left(S_t^E\right)}{f\left(\theta_t\right)}\left[\mu_{t-1}\left(\left(1 - e_{t-1}\right)\left(1 - S_t^I f\left(\theta_t\right)\right) - \delta\right) - \mu_t - \delta \nu_{t-1}\right] \end{aligned} \tag{35}$$

$$\left(1 - L_{t-1} - D_{t-1}\right)\left[c' - \lambda_t f\left(\theta_t\right)\left(S_t^I\right)\right] = \frac{c''\left(S_t^I\right)}{f\left(\theta_t\right)}\left[\left(\mu_{t-1} e_{t-1} + \nu_{t-1}\right)\left(1 - S_t^I f\left(\theta_t\right)\right) - \nu_t\right] \tag{36}$$

$$\lambda_t = u\left(w_t\right) - u\left(h+p\right) + \eta \tau + \beta \mathbb{E}_t\left\{c\left(S_{t+1}^I\right) + \lambda_{t+1}\left(1 - \delta - S_{t+1}^I f\left(\theta_{t+1}\right)\right) + \alpha_{t+1} \delta\right\} \tag{37}$$

$$\begin{aligned} \alpha_t =& u\left(h+b_t\right) - u\left(h+p\right) - \eta b_t \\ &+ \beta\left(1 - e_t\right) \mathbb{E}_t\left\{c\left(S_{t+1}^I\right) - c\left(S_{t+1}^E\right) + \lambda_{t+1} f\left(\theta_{t+1}\right)\left(S_{t+1}^E - S_{t+1}^I\right) + \alpha_{t+1}\left(1 - S_{t+1}^E f\left(\theta_{t+1}\right)\right)\right\} \end{aligned} \tag{38}$$

$$\phi_t \xi u'(w_t) k - f'(\theta_t) \left\{ \lambda_t \left[S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1}) \right] - \alpha_t S_t^E D_{t-1} \right\} - \left[\gamma_t - (1-\delta) \gamma_{t-1} \right] \frac{k q'(\theta_t)}{(q(\theta_t))^2}$$
$$= \left[\mu_t - \mu_{t-1} (1 - e_{t-1} - \delta) + \nu_{t-1} \delta \right] \frac{c'(S_t^E) f'(\theta_t)}{(f(\theta_t))^2} + \left[\nu_t - \nu_{t-1} - \mu_{t-1} e_{t-1} \right] \frac{c'(S_t^I) f'(\theta_t)}{(f(\theta_t))^2} \tag{39}$$

The first-order necessary condition for the optimal tax rate τ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \eta L_t(z^t) - \gamma_t(z^t) \} = 0 \tag{40}$$

To find the optimal policy given η and τ, we solve the above system of difference equations (32)-(39) and (2), (3), (14). (15),(16),(17) for the optimal policy vector

$$\Omega(z^t) = \{ b_t(z^t), e_t(z^t), w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t),$$
$$\lambda_t(z^t), \alpha_t(z^t), \mu_t(z^t), \nu_t(z^t), \gamma_t(z^t), \phi_t(z^t) \}$$

We then pick η and τ so that (4) and (40) are satisfied.

Observe that the only period-$t-1$ variables that enter the period-t first-order conditions are

$$L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1},$$

and no variables from periods prior to $t-1$ enter the period-t first-order conditions. This implies that $(z_t, L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$ is a sufficient state variable for the history of shocks z^t up to and including period t. Specifically, fixing η, τ, let

$$\Psi : (z, L_-, D_-, e_-, \mu_-, \nu_- \gamma_-) \mapsto \left(b, e, w, S^E, S^I, L, D, \theta, \lambda, \alpha, \mu, \nu, \gamma, \phi \right)$$

be a function that satisfies

$$(D - (1-e)\mu) u'(h+b) = \eta D \tag{41}$$

$$D \left[u(h+b) - u(h+p) - \eta b - \alpha \right] = \mu (1-e) \left[u(h+b) - u(h+p) - \frac{c'(S^I) - c'(S^E)}{f(\theta)} \right] \tag{42}$$

$$\gamma = (L + \mu + \nu) u'(w) - \phi \xi u''(w) k \theta \tag{43}$$

$$\phi (\xi - 1) c''(S^E) = D_- \left[(\lambda - \alpha) f(\theta) - c'(S^E) \right]$$
$$+ \frac{c''(S^E)}{f(\theta)} \left[\mu_- \left((1 - e_-) \left(1 - S^I f(\theta) \right) - \delta \right) - \mu - \delta \nu_- \right] \tag{44}$$

$$(1-L_{-}-D_{-})\left[c'-\lambda f(\theta)\left(S^{I}\right)\right]=\frac{c''\left(S^{I}\right)}{f(\theta)}\left[(\mu_{-}e_{-}+\nu_{-})\left(1-S^{I}f(\theta)\right)-\nu\right] \tag{45}$$

$$\lambda=u(w)-u(h+p)+\eta\tau+\beta\mathbb{E}\left\{c\left(S_{+}^{I}\right)+\lambda_{+}\left(1-\delta-S_{+}^{I}f(\theta_{+})\right)+\alpha_{+}\delta\right\} \tag{46}$$

$$\begin{aligned}\alpha&=u(h+b)-u(h+p)-\eta b\\&+\beta(1-e)\mathbb{E}\left\{c\left(S_{+}^{I}\right)-c\left(S_{+}^{E}\right)+\lambda_{+}f(\theta_{+})\left(S_{+}^{E}-S_{+}^{I}\right)+\alpha_{+}\left(1-S_{+}^{E}f(\theta_{+})\right)\right\}\end{aligned} \tag{47}$$

$$\begin{aligned}&\phi\xi u'(w)k-f'(\theta)\left\{\lambda\left[S^{E}D_{-}+S^{I}(1-L_{-}-D_{-})\right]-\alpha S^{E}D_{-}\right\}-\left[\gamma-(1-\delta)\gamma_{-}\right]\frac{kq'(\theta)}{(q(\theta))^{2}}\\&=\left[\mu-\mu_{-}(1-e_{-}-\delta)+\nu_{-}\delta\right]\frac{c'\left(S^{E}\right)f'(\theta)}{(f(\theta))^{2}}+\left[\nu-\nu_{-}-\mu_{-}e_{-}\right]\frac{c'\left(S^{I}\right)f'(\theta)}{(f(\theta))^{2}}\end{aligned} \tag{48}$$

as well as

$$\begin{aligned}L&=(1-\delta)L_{-}+f(\theta)\left[S^{E}D_{-}+S^{I}(1-L_{-}-D_{-})\right]\\D&=(1-e)\left[\delta L_{-}+(1-sf(\theta))D_{-}\right]\end{aligned} \tag{49}$$

$$\begin{aligned}\frac{c'\left(S^{E}\right)}{f(\theta)}=&u(w)-u(h+b)+(1-e)\beta\mathbb{E}\left(c\left(S_{+}^{E}\right)+\left(1-\delta-S_{+}^{E}f(\theta_{+})\right)\frac{c'\left(S_{+}^{E}\right)}{f(\theta_{+})}\right)\\&+e\beta\mathbb{E}\left(c\left(S_{+}^{I}\right)+\left(1-S_{+}^{I}f(\theta_{+})\right)\frac{c'\left(S_{+}^{I}\right)}{f(\theta_{+})}-\delta\frac{c'\left(S_{+}^{I}\right)}{f(\theta_{+})}\right)\end{aligned} \tag{50}$$

$$\frac{c'\left(S^{I}\right)}{f(\theta)}=u(w)-u(h+p)+\beta\mathbb{E}\left(c\left(S_{+}^{I}\right)+\left(1-S_{+}^{I}f(\theta_{+})\right)\frac{c'\left(S_{+}^{I}\right)}{f(\theta)}-\delta\frac{c'\left(S_{+}^{E}\right)}{f(\theta_{+})}\right) \tag{51}$$

$$\frac{k}{q(\theta)}=z-w-\tau+\beta(1-\delta)\mathbb{E}\frac{k}{q(\theta_{+})} \tag{52}$$

$$\xi u'(w)k\theta=(1-\xi)c'\left(S^{E}\right) \tag{53}$$

Then the sequence defined by

$$\Omega\left(z^{t}\right)=\Psi\left(z_{t},L_{t-1}\left(z^{t-1}\right),D_{t-1}\left(z^{t-1}\right),e_{t-1}\left(z^{t-1}\right),\mu_{t-1}\left(z^{t-1}\right),\nu_{t-1}\left(z^{t-1}\right),\gamma_{t-1}\left(z^{t-1}\right)\right)$$

satisfies the system (32)-(39) and (2), (3), (14). (15),(16),(17).

To find the optimal policy given η, we therefore solve the system of functional equations (41)-(53). The details of the computation are in a supplementary appendix, available by request.

B Tables and Figures

Figure 1: Optimal policy: benefit level

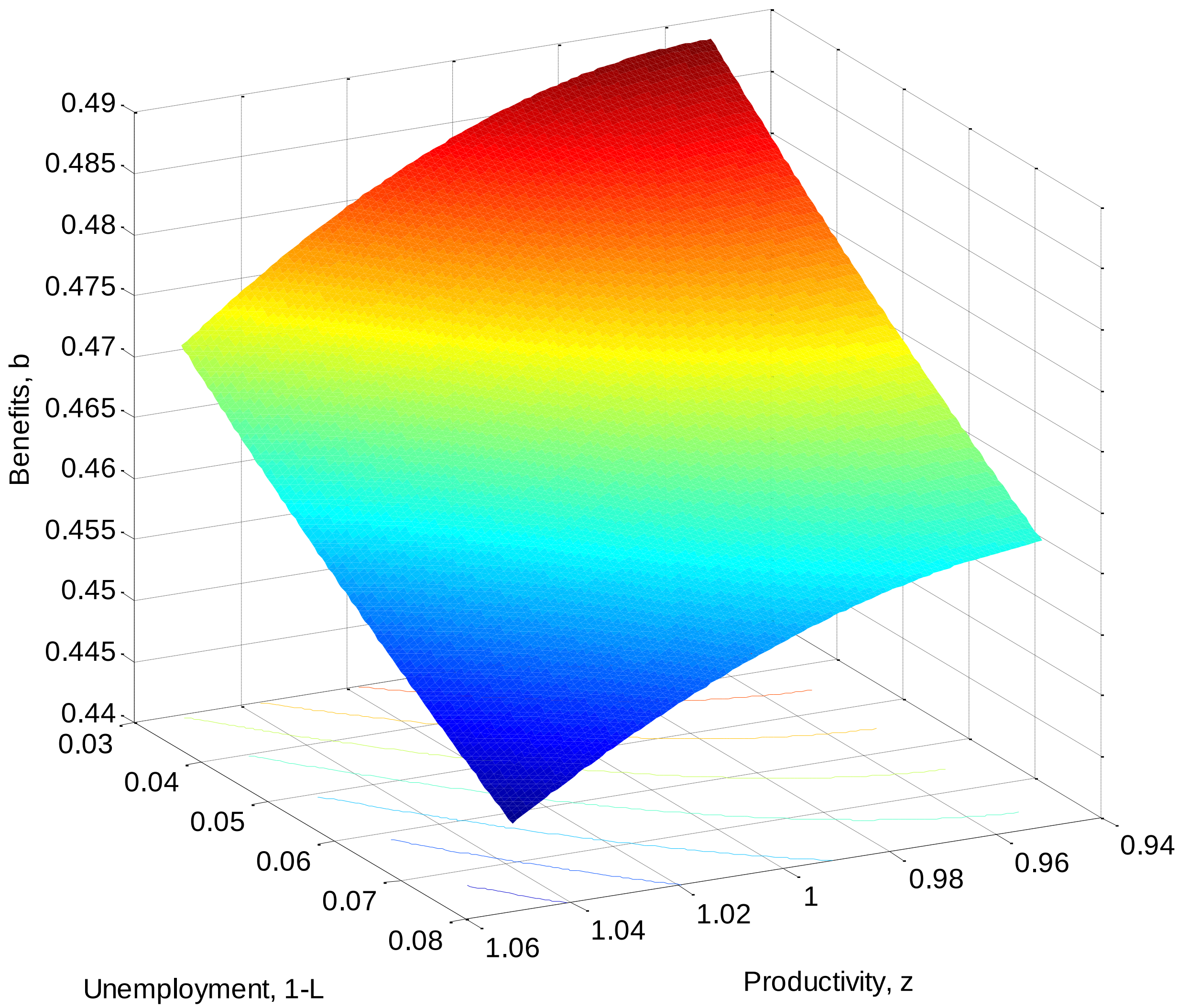

Figure 2: Optimal policy: benefit duration

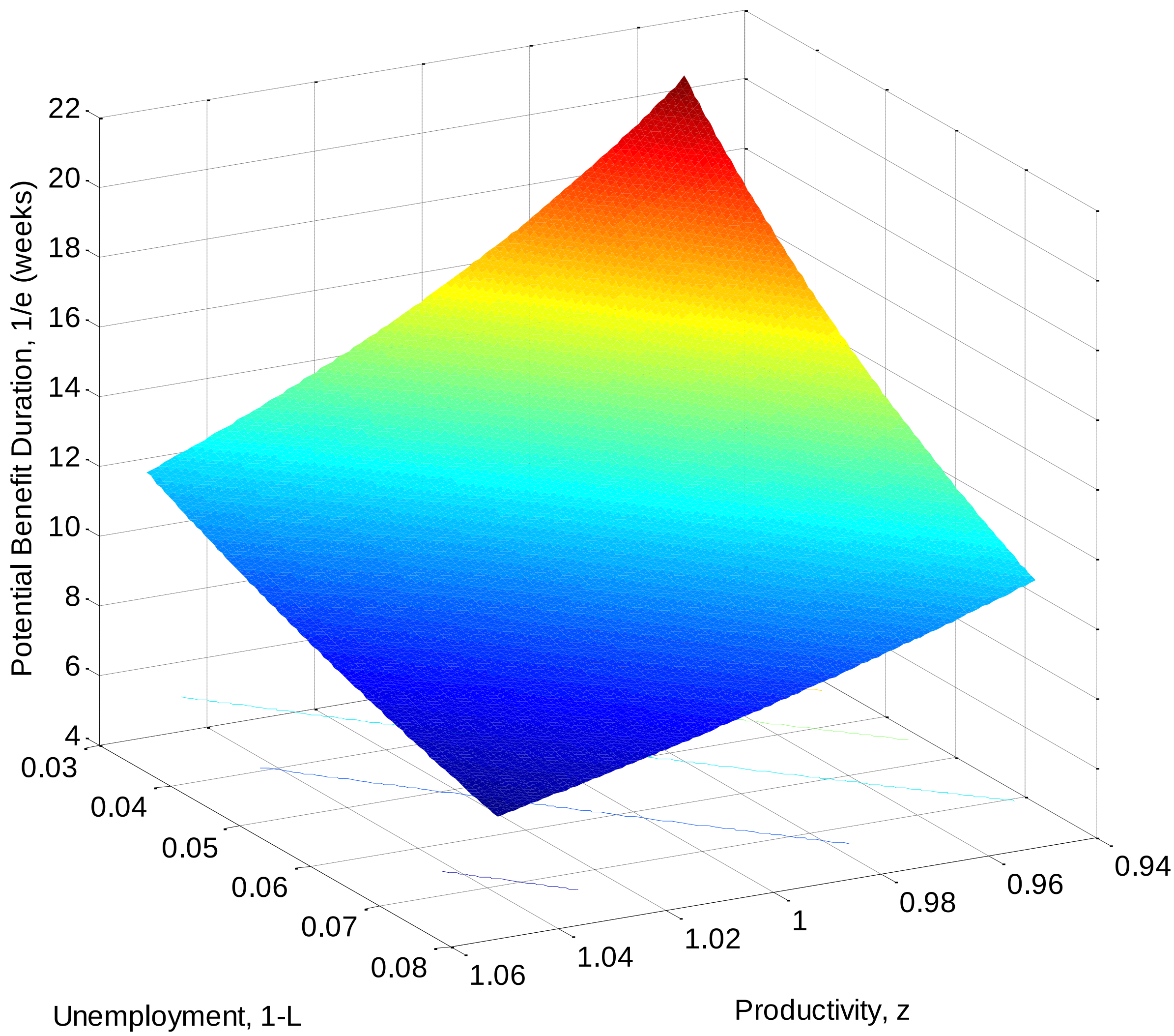

Table 2: Optimal benefit behavior

	Benefit level b	Potential duration $1/e$
Mean	0.472	12.5
Standard deviation	0.010	0.059
Correlation with z	0.758	0.520
Correlation with $1-L$	-0.420	-0.136
Correlation with b	1	0.950

Table 3: Model statistics simulated under the current US policy

		z	$1-L$	$v/(1-L)$	$\hat{f}$	w	S^E	S^I
Mean		1	0.059	0.634	0.139	0.954	0.505	0.655
Standard Deviation		0.013	0.128	0.259	0.150	0.010	0.040	0.003
	z	1	-0.849	0.907	0.945	0.883	0.873	0.943
	$1-L$	-	1	-0.902	-0.723	-0.908	-0.916	-0.891
	$v/(1-L)$	-	-	1	0.775	0.996	0.987	0.959
Correlation	$\hat{f}$	-	-	-	1	0.742	0.731	0.861
Matrix	w	-	-	-	-	1	0.997	0.958
	S^E	-	-	-	-	-	1	0.960
	S^I	-	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. $\hat{f}$ denotes the weekly job finding rate.

Table 4: Model statistics simulated under the optimal US policy

		z	$1-L$	$v/(1-L)$	$\hat{f}$	w	S^E	S^I
Mean		1	0.049	0.742	0.157	0.956	0.520	0.655
Standard Deviation		0.13	0.027	0.062	0.032	0.011	0.008	0.003
	z	1	-0.877	0.814	0.775	0.917	0.743	0.995
	$1-L$	-	1	-0.934	-0.920	-0.656	-0.904	-0.847
	$v/(1-L)$	-	-	1	0.998	0.515	0.993	0.768
Correlation	$\hat{f}$	-	-	-	1	0.459	0.999	0.726
Matrix	w	-	-	-	-	1	0.416	0.942
	S^E	-	-	-	-	-	1	0.692
	S^I	-	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. $\hat{f}$ denotes the weekly job finding rate.

Figure 3: Responses to 1% drop in productivity

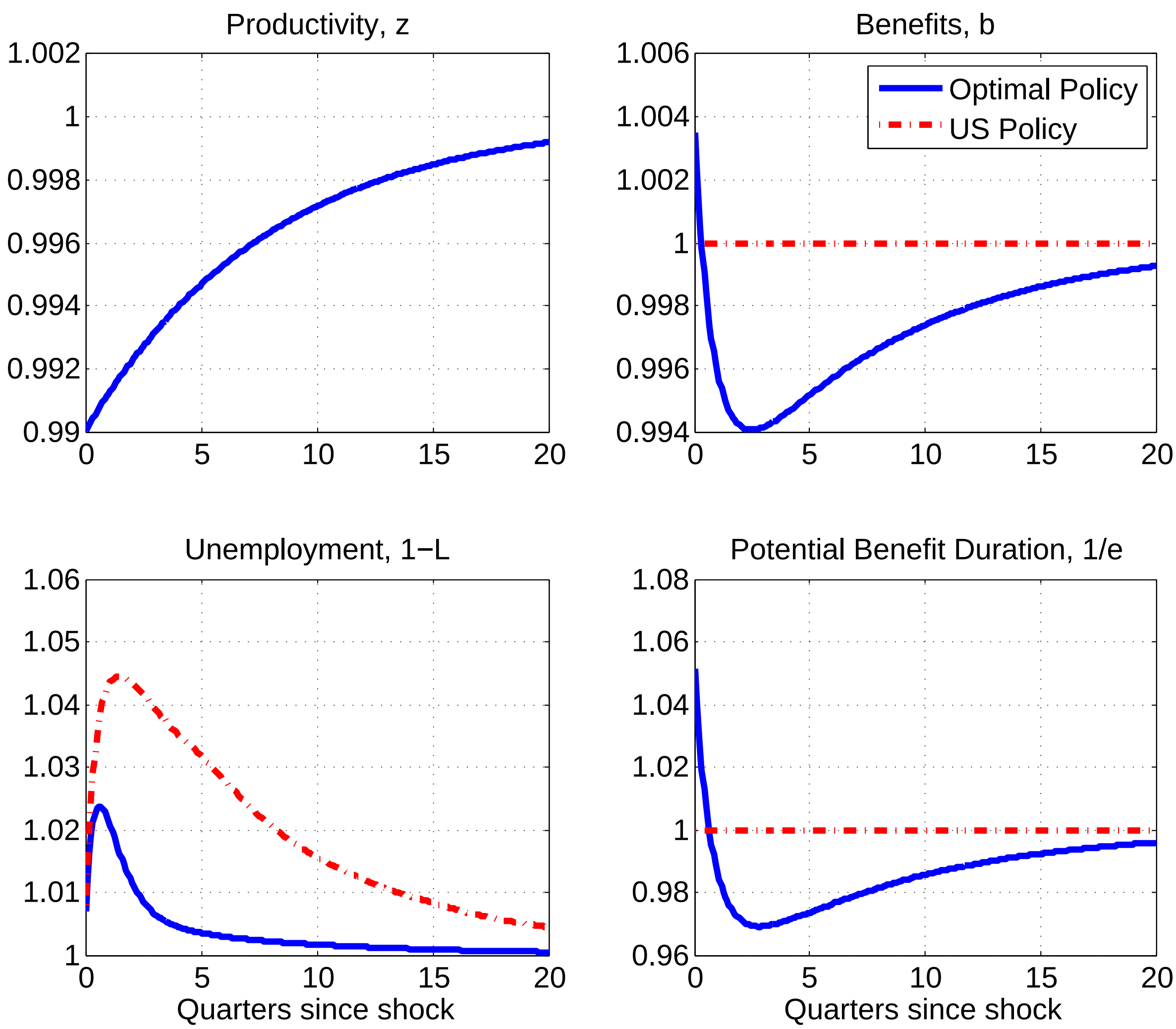

Figure 4: Responses to 1% drop in productivity