Cyclical and welfare effects of public sector unions in a Real-Business-Cycle model

Aleksandar Vasilev*

August 23, 2012

Abstract

Motivated by the highly-unionized public sectors, the high public shares in total employment, and the public sector wage premia observed in Europe, this paper examines the importance of public sector unions for macroeconomic theory. Following Fernandez-de-Cordoba *et al.* (2009), a public sector union is incorporated in a real-business-cycle (RBC) setup with valuable government consumption and productive public investment. The model generates cyclical behavior in hours and wages that is consistent with data behavior in an economy with highly-unionized public sector, namely Germany during 1970-2007 period. The main findings are: (i) the model with public sector union performs reasonably well *vis-a-vis* data; (ii) overall, the union model is a significant improvement over a similar model with exogenous public employment, namely Finn (1998); (iii) endogenously-determined public wage and hours add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. Additionally, the union model requires larger changes in tax rates to achieve a pre-specified increase in tax revenue, as compared to Finn's model with exogenous public sector hours. Ignoring the positive co-movement between public and private hours leads to a significant underestimation of the welfare effect of fiscal regime changes.

Keywords: fiscal policy, public wages, public employment, labor market, collective bargaining

JEL Classification: C68, E62, J45, J51

*The author is a PhD candidate at the University of Glasgow Business School, Department of Economics, Main Building G12 8QQ. Tel: +44 (0)141 330 2276. E-mail: a.vasilev.1@research.gla.ac.uk.

Contents

Contents

List of Figures

List of Tables

1 Introduction

The behavior of the labor input is very important for output fluctuations, as Cooley and Prescott (1995) and Kydland (1995) have pointed out. In particular, changes in hours account for two-thirds of the movement in US output per person over the business cycle. Despite this, real business cycle (RBC) theory has been predominantly focused on the private sector and largely ignored the dynamic general-equilibrium effects of public sector labor choice. This paper adds to earlier research by distinguishing between the two types of hours and argues that the presence of the public sector labor market in European economies generates significant interaction with the private sector labor and capital markets. If public sector labor choice is ignored, then important effects on cyclical fluctuations, as well as welfare, due to fiscal regime changes, will be missed.

Furthermore, several stylized facts suggest that this labor market is driven by non-competitive arrangements: As reported in Table 1 below, the public sectors in the major European Union (EU) member states are highly unionized, and significantly more than the respective private sectors. Even though the unionization rates in each sector were calculated in Visser (2003) for the EU countries for just one year only, the wide gap in union density indicates that the two labor markets operate under different setting. On their own, high unionization rates do not necessarily translate into strong unions, but the significance of unions in Europe can be inferred from the generally high coordination, centralization and especially the extensive coverage rate. Therefore, collective bargaining agreements are often used to set public wage rates and employment levels in European economies.

Central governments in EU countries are the biggest employer on a national level, with high public share in total employment, as documented in Table 1 for the largest EU economies. The large share of public employees in total employment in itself could constitute a source of union power, and could explain the positive public sector wage premia over the private wage, which are observed in most post-WWII European economies over the period 1970-2008. The Wage Dynamics Network (WDN) 2010 Final Report[1] also emphasizes that wage bargaining institutions are an important determinant of the wage dynamics and wage struc-

[1]"WDN is a research network consisting of economists from the European Central Bank (ECB) and the national central banks (NCBs) of the EU countries, which aims at studying in depth the features and sources of wage and labor cost dynamics and their implications for monetary policy in the euro area."(ECB 2011)

Table 1: Labor market facts 1970-2008

Country	Private sector union density	Public sector union density	Coverage rate (2000)	Average publ./priv. compensation	Average publ./priv. employment
Euro Area (2001)	26	N/A	78	1.22	0.22
France (1993)	4	25	95	1.00	0.32
Germany (1997)	22	56	73	1.20	0.17
Italy (1997)	36	43	82	1.30	0.26
Spain (1997)	15	32	80	1.60	0.16
UK (2003)	18	59	36	1.08	0.27
US (2010)	7	35	15	1.08	0.16

Sources: BLS (2011), OECD (2011), Visser (2003)

ture in the EU countries, and the major reason for the existence of the public wage premium.[2]

Additionally, Forni *et al.* (2009) and Gomes (2010) show that the compensation of public employees in OECD countries takes 60% of total government expenditure. Furthermore, Lane (2003) shows that the public wage bill in OECD countries is pro-cyclical, as opposed to government purchases, which are acyclical. Next, empirical work from Lamo, Perez and Schuknecht (2007, 2008) concludes that pro-cyclical discretionary fiscal policy can have important effects on the economy through the unions. In particular, unions act as organized groups that constantly press for an expansion in the the government wage bill. Therefore, the presence of interest groups in the public sector imposes a significant constraint on the use of fiscal policy in Europe as a tool for economic stabilization, and thus accentuates cyclical fluctuations.

This paper uses the RBC framework to study the cyclical properties of European public sector labor markets. The benchmark RBC model has established itself as a useful environment to study aggregate fluctuations in developed economies. In addition, the baseline

[2]Other reasons for the existence of a public sector wage premium, as documented in Ehrenberg and Schwartz (1986) can be due to skill and experience differences: on average, public employees are older and have higher qualification. In addition, females and employees belonging to a minority group receive higher labor compensation compared to the remuneration package for similar duties in the private sector.

RBC model performance improves significantly when extended to capture specific features of the economy of interest. Some examples include: distortionary taxation (McGrattan 1994), government spending (Christiano and Eichenbaum 1992), and productive public investment (Baxter and King 1993). Most of the extensions to the benchmark RBC model, which allow for public employment, however, model public sector labor market variables predominantly as exogenous, *e.g.* Finn (1998), Cavallo (2005), and Linnemann (2009). Those models feature a representative household and two sectors - public and private - where labor hours can be supplied. A serious shortcoming in these models is that wage rates in the economy are identical, with public hours approximated by a stationary stochastic process. These models, despite being an improvement over earlier vintages of RBC models, produce a good match *vis-a-vis* data along the public sector labor market dimension, *e.g.* public hours volatility, mostly by construction. The absence of an internal propagation mechanism for public employment is a serious limitation in this class of models, especially when the research focus falls on the interactions between the two labor markets and their relation to the business cycle.

There are few RBC models with endogenous public sector wages and employment. Ardagna (2007), for example, departs from the representative-agent assumption. Total population is split into capitalists and workers, with workers being either employed in the private or public sector, or unemployed. In addition, both sectors are unionized, and public sector wage is different from the private sector wage rate. Public wage and employment are obtained from the government's maximization problem, where the government profit function is augmented with a term capturing equity considerations. However, a major limitation of Ardagna's (2007) setup is that it assigns each household to a sector and by default excludes further labor reallocation, which is the focus in this paper.

Additionally, RBC models that incorporate endogenously-determined wage and hours in the public sector, and also reflect the importance of public sector unionization for the business cycles in EU countries, are even fewer: Fernandez-de-Cordoba *et al.* (2009, 2012) are the first to develop a Dynamic Stochastic General Equilibrium (DSGE) model with public and private wages being determined in different environments. The private sector wage is determined within a competitive market framework, while the public wage is a optimal solution to the union's optimization problem. In addition, the impulse response analysis in Fernandez-de-Cordoba *et al.* (2009, 2012) generates pro-cyclical public wage and hours.

Another important finding is the positive co-movement between the two wage rates, and public and private hours. These are all robust patterns have been observed previously in the empirical work of Lamo, Perez and Schuknecht (2007, 2008) and Perez and Sanchez (2010).

The model in this paper combines two ingredients used in earlier research to address new aspects of the economy and produce new results: it adopts the public sector union maximization problem from Fernandez-de-Cordoba *et al.* (2009, 2012) and incorporates it into a RBC model with richer tax structure and fiscal policy instruments, *i.e.* Finn (1998). Thus, the individual quantitative effect of union optimization can be assessed relative to Finn's (1998) setup with exogenous public hours and a single, competitive wage rate. In addition, the fiscal policy instruments will be the shares of government consumption and investment in output, which allows the government to react to output. The presence of a union in the public sector will crowd out the other types of the government spending at the expense of the public sector wage bill, an effect not present in Fernandez-de-Cordoba *et al.* (2009, 2012). Additionally, in contrast to Fernandez-de-Cordoba *et al.* (2009, 2012), who model public employment as output-enhancing, public employment in this paper is a wasteful expenditure from a productive point of view. This modeling choice is used to reflect the view that the public sector bureaucracy's direct contribution to the national product in the economy is rather small. Moreover, the setup is consistent with Blanchflower (1991), who suggests that some governments use 'safe' public sector jobs as a tool to fight unemployment and generate votes for re-election. Lastly, government's completely wasteful public wage bill spending is expected to amplify fluctuations in hours, as there will be no direct substitutability/complementarity between private and public hours. In other words, the allocative efficiency will decrease significantly, as a wasteful hour spent working in the public sector receives a higher return relative to a productive hour of work in the private sector.

The analysis in this paper is done at the country level, as taxation and government spending decisions are still to a great extent country-specific for individual EU member states. Furthermore, based on their extensive compilation of case studies, Ebbinghaus and Visser (2000) and Visser (2003) conclude that international unionism is weak, *i.e.* labor unions' influence in Europe tends to be constrained to the respective country's borders. This approach differs from Fernandez-de-Cordoba *et al.* (2009, 2012), who analyze the Euro Area as a whole. Germany is the preferred choice for calibration in this paper, as it is the classical example for a large EU economy. Some of the features of the German economy include strong public sector unions, and a large and growing gap between public and private sector unionization, as reported in *The Economist* (2011). Additionally, Germany has a similar to the EU average public sector wage premium and public/private employment ratio.

The study in this paper takes a much wider scope relative to Finn (1998) and at the same time is complementary to Fernandez-de-Cordoba *et al.* (2009, 2012). It includes a complete evaluation of an RBC model with optimizing public sector union, following the widely-accepted methodology in the RBC literature. The model here matches the cyclical fluctuations in the public and private sector labor markets. Additionally, it also compares well against the empirical autocorrelation and cross-correlation functions generated from an unrestricted Vector Auto Regression (VAR). While all these features are important for understanding the aggregate fluctuations, these aspects were not addressed in the earlier studies on the dynamic general equilibrium effects of public sector unions. Lastly, endogenously-determined public wage and hours will be shown to add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. The union model requires larger changes in tax rates to achieve a pre-specified increase in tax revenue, as compared to Finn's model with exogenous public sector hours. Thus, endogenous public hours are quantitatively important for fiscal policy evaluation. Ignoring the interaction between hours and wages leads to a significant underestimation of the welfare effect of tax regime changes.

The paper is organized as follows: Section 2 presents the model setup in the context of the relevant literature. Section 3 and 4 lay out the system of equations describing the decentralized competitive equilibrium and the model steady-state, respectively. Section 5 explains the data used and model calibration. Section 6 solves for the steady-state, and section 7 approximates the model using log-linearization. Section 8 presents the model solution procedure, discusses the effects of different shocks and the impulse responses of variables across model. Section 9 simulates the competing models and evaluates their properties for the calibrations performed for Germany; it also computes the long-run welfare costs of exogenous tax regime changes, both across models and across countries. Section 10 concludes.

2 Model setup

2.1 Description of the model:

The model builds upon Finn (1998). There is a representative household, as well as a representative firm. The household owns the private physical capital and labor, which it supplies to the firm. Hours supplied in the public sector are decided via a collective agreement between a union and the government. The perfectly-competitive firm produces output using labor, private and public capital. The government uses tax revenues from consumption expenditure, labor and capital income to finance: (1) government consumption (which is valued by the representative household), (2) government investment (public capital generates mild increasing returns to scale in the aggregate output production function), (3) government transfers, and (4) public wage bill. The wage rate and hours supplied in the public sector are determined by a utility-maximizing public sector union, as in Fernandez-de-Cordoba *et al.* (2009), subject to the government period budget constraint.

2.2 Households

There is an infinitely-lived representative household in the model economy, and no population growth. The household maximizes the following expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h, G_t^c, N_t^h), \tag{2.2.1}$$

where E_0 is the expectation operator as of period 0; C_t^h, G_t^c and N_t^h are household's consumption, per household consumption of government services, and hours worked by the household at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. Following Finn (1998), the CRRA form for utility is:

$$U(C_t^h, G_t, N_t^h) = \frac{[(C_t^h + \omega G_t^c)^{\psi}(1 - N_t^h)^{(1-\psi)}]^{(1-\alpha)} - 1}{1 - \alpha}, \tag{2.2.2}$$

where ($\alpha > 1$). The parameter ψ is the weight of consumption in utility, $0 < \psi < 1$, and $0 < 1-\psi < 1$ is the weight in the utility function that the household puts on leisure. Government consumption is a substitute to private consumption, and the degree of substitutability is measured by ω, where $0 \leq \omega \leq 1$.

The household has an endowment of one unit of time in each period t, which is split between work, N_t^h and leisure, L_t^h, so that

$$N_t^h + L_t^h = 1. \tag{2.2.3}$$

The household can supply hours of work in the public sector, N_t^{gh}, or in the private one, N_t^{ph}, with $N_t^h = N_t^{ph} + N_t^{gh}$. The wage rates per hour of work in private and public sector are denoted by w_t^p and w_t^g, respectively. The household chooses N_t^{ph} only; public hours will be endogenously chosen by the government, so N_t^{gh} will be taken by the household as given, as in Fernandez-de-Cordoba *et al.* (2012).

The representative household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to private capital, and K_t^{ph} denotes private capital stock in the beginning of period t. As in Finn (1998), the household receives capital depreciation allowance in the amount of $\tau^k \delta^p K_t^{ph}$, where τ^k is the capital income tax rate and $0 < \delta^p < 1$ is the depreciation rate of private physical capital. In other words, capital income taxes are levied net of depreciation as in Prescott (2002, 2004) and in line with the methodology used in Mendoza, Razin, and Tesar (1994).

Finally, the household owns all firms in the economy, and receives all profit (Π_t^h) in the form of dividends. Household's budget constraint is

$$(1+\tau^c)C_t^h + I_t^h \leq (1-\tau^l)[w_t^p N_t^{ph} + w_t^g N_t^{gh}] + (1-\tau^k) r_t K_t^{ph} + \tau^k \delta^p K_t^{ph} + G_t^t + \Pi_t^h, \tag{2.2.4}$$

where τ^c, τ^l are the proportional tax rates on consumption and labor income, respectively, and G_t^t is the per household transfer from the government.

Household's private physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1-\delta^p) K_t^{ph}. \tag{2.2.5}$$

The representative household acts competitively by taking prices $\{w_t^p, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau^c, \tau^l, \tau^k\}$, policy variables $\{w_t^g, N_t^{gh}, G_t^c, G_t^i, G_t^t\}_{t=0}^{\infty}$ as given, and chooses allocations $\{C_t^h$, $N_t^{ph}, I_t^h, K_{t+1}^{ph}\}_{t=0}^{\infty}$ to maximize Equation (2.2.1) subject to Equations (2.2.2)-(2.2.5), and initial condition for private physical capital, K_0^{hp}.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for private physical capital, are as follows[3]

$$C_t\text{: } \left[(C_t^h + \omega G_t^c)^{\psi}(1-N_t^h)^{(1-\psi)}\right]^{-\alpha} \psi (C_t^h + \omega G_t^c)^{\psi-1}(1-N_t^h)^{(1-\psi)} = \Lambda_t(1+\tau^c) \quad (2.2.6)$$

$$N_t^p\text{: } \left[(C_t^h + \omega G_t^c)^{\psi}(1-N_t^h)^{(1-\psi)}\right]^{-\alpha} (1-\psi)\left[\frac{C_t^h + \omega G_t^c}{1-N_t^h}\right]^{\psi} = \Lambda_t(1-\tau^l)w_t^p \quad (2.2.7)$$

$$K_{t+1}^p\text{: } \beta E_t \Lambda_{t+1}\left[(1-\tau^k)r_{t+1} + \tau^k \delta^p + (1-\delta^p)\right] = \Lambda_t \quad (2.2.8)$$

$$\text{TVC: } \lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^p = 0, \quad (2.2.9)$$

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Private hours are chosen so that the disutility of an hour work in the private sector at the margin equals the after-tax return to labor. Next, the Euler equation describes the optimal capital accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two neighboring periods. The last expression is the TVC, imposed to ensure that the value of the private physical capital that remains at the end of the optimization horizon is zero. This boundary condition guarantees that the model equilibrium is well-defined by ruling out explosive solution paths.

2.3 Firms

Following Finn (1998), there is a representative private firm in the model economy as well. It produces a homogeneous final product using a production function that requires private and public physical capital, K_t^p, K_t^g respectively, and labor hours N_t^p. The production function is as follows

$$Y_t = A_t (N_t^p)^{\theta}(K_t^p)^{1-\theta}(K_t^g)^{\nu}, \quad (2.3.1)$$

where A_t measures the total factor productivity in period t; $0 < \theta, (1-\theta) < 1$ are the productivity of labor and private physical capital, respectively. Parameter $\nu \geq 0$ measures the degree of increasing returns to scale (IRS) that public capital has on output.

[3]Detailed derivations in Appendix 11.1.2.

The representative firm acts competitively by taking prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and policy variables $\{\tau^c, \tau_k, \tau_l, w_t^g, N_t^g, G_t^c, G_t^i, G_t^t, K_{t+1}^g\}_{t=0}^{\infty}$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize firm's static aggregate profit,

$$\Pi_t = A_t(N_t^p)^{\theta}(K_t^p)^{1-\theta}(K_t^g)^{\nu} - r_t K_t^p - w_t^p N_t^p. \tag{2.3.2}$$

In equilibrium, profit is zero. In addition, labor and capital receive their marginal products, i.e[4]

$$w_t^p = \theta \frac{Y_t}{N_t^p}, \tag{2.3.3}$$

$$r_t = (1-\theta)\frac{Y_t}{K_t^p}. \tag{2.3.4}$$

2.4 Government budget constraint

Government purchases goods, G_t^c, invests in public capital G_t^i, distributes transfers G_t^t, hires labor N_t^g and sets the public sector wage rate w_t^g. Public capital evolves according to the following law of motion

$$K_{t+1}^g = G_t^i + (1-\delta^g)K_t^g, \tag{2.4.1}$$

where $0 < \delta^g < 1$ is the linear depreciation rate on government physical capital.

Total government expenditure, $G_t^c + G_t^i + w_t^g N_t^g + G_t^t$, is financed by levying proportional taxes on consumption, capital and labor income. Thus, the government budget constraint is

$$G_t^c + G_t^i + w_t^g N_t^g + G_t^t = \tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t^p + \tau^l \left[w_t^p N_t^p + w_t^g N_t^g \right]. \tag{2.4.2}$$

Government takes market prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and allocations $\{N_t^p, K_t^p\}$ as given.

The following six policy instruments, $\{\tau^c, \tau^k, \tau^l, \frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}, \frac{G_t^t}{Y_t}\}$, will be exogenously set. In particular, shares of government consumption and investment in output, rather than the levels of the fiscal variables, will follow stochastic processes. Thus, public consumption and investment will respond to both exogenous shocks and output. (K_{t+1}^g will be exogenously determined as well, subject to the initial condition K_0^g and the law of motion for G_t^i.) Gov-

[4]Detailed derivations in Appendix 11.1.1

ernment transfers-to-output ratio $G^{ty} \equiv \frac{G_t^t}{Y_t}$ will be fixed,[5] but the level of public transfers will vary with output (*i.e.* $G_t^t = G^{ty} Y_t$). All three tax rates $\{\tau^c, \tau^k, \tau^l\}$ will be kept constant. Finally, the pair $\{N_t^g, w_t^g\}$ will be determined as an optimal solution from a collective bargaining problem between the government and a public sector union, which is described in the next subsection.

2.5 Government sector union objective function

In contrast to Finn's (1998) model, which features a single wage rate w_t and exogenous public employment, modeled as an AR(1) process, in this paper the two variables will be obtained as optimal choices from an explicit objective function maximization, as in Fernandez-de-Cordoba *et. al* (2009):[6]

$$\max_{w_t^g, N_t^g} \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{1/\rho}, \tag{2.5.1}$$

where $\eta > 0$ is the relative weight put on wages, and ρ is the parameter determining the constant elasticity of substitution between wages and hours, $\frac{1}{1-\rho}$. Hence, the pair $\{N_t^g, w_t^g\}$ solves (2.5.1) s.t (2.4.1)-(2.4.2) and the processes for the other policy instruments.[7]

In the union literature, Doiron (1992) uses equivalent representation to model union preferences over wages and employment between a private sector union and a firm.[8] Furthermore, the representation used in this paper can be traced back to Oswald *et al.* (1984) and Alogoskoufis and Manning (1988). Recent studies by Demekas and Kontolemis (2000), and Forni and Giordano (2003) also use social welfare functions where public sector wage and employment appear as separate arguments. Indeed, all those studies ignore the process of

[5]The fixed government transfers/output ratio is to be interpreted as an "implied" one, as it will be set so that the model matches the long-run wage and employment ratios, as it will be shown in the following sections. In this sense, it bears little correspondence to the average ratio in data.

[6]The difference in this paper is that instead of having two separate weights, ϕ on wages, and $(1 - \phi)$ on employment, only one relative weight will be used.

[7]The public sector union should be taken as an aggregation of smaller unions who operate on federal and state/local levels, who maximize the same objective function over local government period budget constraint. The coalition of workers is large on regional level, thus able to influence the public sector wage rate. Still, local union is small relative to the size of the economy, hence w^p is taken as given. Still, both wage rates will be determined within the system, so there will be some feedback effect from public to private wage.

[8]The equivalence is shown in Appendix 11.1.3.

bargaining, and thus the objective function used here is not micro-founded as well.[9] Alternatively, deriving the reduced-form utility function using union aggregation over individual worker preferences is still an open question in the literature. Oswald (1982) shows in a simple static framework that if the union is utilitarian, *i.e.*, maximizes the expected utility of a representative worker, and if members are risk-averse, then there exists a well-behaved union utility function defined over both wage rate and employment. Since those conditions are assumed to hold in this paper, Oswald's (1982) result is one way to rationalize the *ad hoc* union utility function used here. Additionally, a CES union utility function, which is concave and increasing in wage and employment, has proven to be a successful modeling choice in econometric studies.[10] In contrast, simple models such as wage bill maximization ($w_t^g N_t^g$) and rent maximization ($[w_t^g - w_t^a]N^g$, where w_t^a denotes the alternative wage at time t) have been rejected in many studies.[11] In what is to follow, it will be shown that the CES union maximization function is empirically relevant, and thus a useful modeling device, similar to the household's utility function and the aggregate production function, which could help generate several new and interesting results.

The interaction between the public sector union and the government is as follows: the wage bill in the public sector, modeled as a residual spending item that balances the budget constraint in every period, is distributed between wages and hours according to the union utility function (2.5.1) specified above.[12] Additionally, government period budget constraint serves the role of a labor demand function, which will be subject to shocks, resulting from innovations to total factor productivity and the fiscal shares. The balanced budget assumption is thus important in the model setup. Since wage bill is a residual, if wage rate is increased, then hours need to be decreased. Now the problem in the public sector is a standard representation used in union literature, where a labor union maximizes utility, constrained by a

[9]Despite researchers' claims that this representation is consistent with Nash bargaining, such statements are incorrect. The author is not aware of any studies that explicitly show how the union objective function can be obtained from a Nash bargaining procedure.

[10]Papers applying this approach are Akerlof (1969), Atherton (1973), Calmfors (1981), Carruth and Oswald (1981), Cartter (1959), Corden (1981), Hersoug (1978), Fellner (1949), Mulvey (1978), Oswald (1979), Rees (1977), and Warren-Boulton (1977), among others.

[11]A partial list of such studies is Brown and Ashenfelter (1986), Card (1986), Carruth and Oswald (1983), Eberts and Stone (1986), Farber (1978), MaCurdy and Pencavel (1986), and Martinello (1989).

[12]The modeling choice is also consistent with Tullock's (1974) hypothesis, which states that bureaucrats first exert effort to increase their number; once staff is expanded, the bureaucrats will then use their newly-increased power to negotiate higher wages.

stochastic labor demand curve. In addition to producing endogenous public wage and public hours, this optimization problem generates a public sector wage that features a positive premium over the private sector one. Therefore, at least part of this premium can be justified by the gains from unionization in the public sector. In equilibrium, a positive linear relation exists between the public wage rate and public sector hours, which is obtained from the marginal rate of substitution between the two:[13]

$$N_t^g = \eta^{\frac{1}{\rho}} w_t^g. \tag{2.5.2}$$

There are several interpretations for Eq. (2.5.2): first, it can be recognized as a standard neoclassical labor supply curve. Hence, this model can be viewed as one emphasizing the relative importance of supply-side factors, *i.e.* unions, in the economy. Second, and more important, such a relationship is called a "contract curve" in the union literature. In particular, this curve defines the set of allocations $\{w_t^g, N_t^g\}$, generated as an outcome of the collective bargaining between the government and the union. Since union optimizes over both the public wage and hours, the outcome is efficient. The solution pair is at the intersection point of the contract curve, and the labor demand curve (government budget constraint).

Next, Eq. (2.5.2) is plugged back into (2.4.2) to obtain a solution for the public sector wage:

$$w_t^g = \eta^{-\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^t - G_t^i}{1 - \tau^l} \right]^{\frac{1}{2}}. \tag{2.5.3}$$

Optimal public hours are obtained by substituting (18) into (17) to obtain

$$N_t^g = \eta^{\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^t - G_t^i}{1 - \tau^l} \right]^{\frac{1}{2}}. \tag{2.5.4}$$

Both public sector wage and hours will be negatively related to government consumption and investment, and positively related to tax revenue from consumption, capital income and private sector labor income. Public hours and the wage rate are directly affected by fiscal policy variables: a decrease in government consumption, for example, will have a direct positive effect on both public hours and wages, and thus on the household's income. Such effect are empirically observed in Lano, Perez, and Schuknecht (2008). In the model, the crowding out effect of government spending will generate important differences from earlier

[13]Detailed derivations in Appendix 12.1.4

literature. This makes it relevant for the analysis of the impulse responses to fiscal shares shocks and for the long-run welfare effects of fiscal policy. These effects will be discussed at length in the following sections.

2.6 Stochastic processes for the policy variables

The exogenous stochastic variables are the total factor productivity A_t, and the policy instruments $\frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$, where $\frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$ denote the shares of government consumption and government investment in output, respectively. Then assume that $A_t, \frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$ follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1-\rho_a)\ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a, \tag{2.6.1}$$

where $A_0 = A > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in the total factor productivity process.

The stochastic process for the government consumption/output share $\{\frac{G_t^c}{Y_t}\}$ is

$$\ln\left(\frac{G_{t+1}^c}{Y_{t+1}}\right) = (1-\rho_c)\ln\left(\frac{G_0^c}{Y_0}\right) + \rho_c \ln\left(\frac{G_t^c}{Y_t}\right) + \epsilon_{t+1}^c, \tag{2.6.2}$$

or

$$\ln G_{t+1}^{cy} = (1-\rho_c)\ln G_0^{cy} + \rho_c \ln G_t^{cy} + \epsilon_{t+1}^c, \tag{2.6.3}$$

where $G_{t+1}^{cy} = \frac{G_{t+1}^c}{Y_{t+1}}$, and $\frac{G_0^c}{Y_0} > 0$ is the steady-state public consumption/output ratio, $0 < \rho^c < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^c \sim iidN(0, \sigma_c^2)$ are random shocks to government consumption/output share. Hence, the innovations ϵ_t^c represent unexpected changes in government consumption/output share.

The stochastic process followed by the government investment/output share $\{\frac{G_t^i}{Y_t}\}$ is

$$\ln\left(\frac{G_{t+1}^i}{Y_{t+1}}\right) = (1-\rho_i)\ln\left(\frac{G_0^i}{Y_0}\right) + \rho_i \ln\left(\frac{G_t^i}{Y_t}\right) + \epsilon_{t+1}^i, \tag{2.6.4}$$

or

$$\ln G_{t+1}^{iy} = (1-\rho_i)\ln G_0^{iy} + \rho_i \ln G_t^{iy} + \epsilon_{t+1}^i, \tag{2.6.5}$$

where $G^{iy}_{t+1} = \frac{G^i_{t+1}}{Y_{t+1}}$, and $\frac{G^i_0}{Y_0} > 0$ is the steady-state public investment/output ratio, $0 < \rho^i < 1$ is the first-order autoregressive persistence parameter and $\epsilon^i_t \sim iidN(0, \sigma^2_i)$ are random shocks to government investment/output share. Hence, the innovations ϵ^i_t represent unexpected changes in government investment/output share.

Additionally, in Finn (1998), public hours will follow AR(1) process as well:

$$\ln N^g_{t+1} = (1 - \rho_n) \ln N^g_0 + \rho_n \ln N^g_t + \epsilon^n_{t+1}, \tag{2.6.6}$$

where $N^g_0 = N^g > 0$ is the steady-state public employment, $0 < \rho^n < 1$ is the first-order autoregressive persistence parameter and $\epsilon^n_t \sim iidN(0, \sigma^2_n)$ are random shocks to government employment. Hence, the innovations ϵ^n_t represent unexpected changes in government employment.

2.7 Decentralized competitive equilibrium

Given the fixed value of government transfers/output ratio G^{ty}, the exogenous processes followed by $\{A_t, G^{cy}_t, G^{iy}_t\}^{\infty}_{t=0}$ and initial conditions for the state variables $\{A_0, G^{cy}_0, G^{iy}_0, K^{ph}_0, K^g_0\}$, a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C^h_t, N^{ph}_t, N^{gh}_t, I^{ph}_t, K^{ph}_{t+1}, K^g_{t+1}\}$ $\forall h$, prices $\{r_t, w^p_t, w^g_t\}^{\infty}_{t=0}$ and the tax rates $\{\tau^c, \tau^l, \tau^k\}$ such that (i) the representative household maximizes utility; (ii) the stand-in firm maximizes profit every period; (iii) government objective function is maximized s.t the government budget constraint being satisfied in each time period; (iv) all markets clear.

3 Per capita stationary DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, i.e $Z^h_t = Z_t = z_t$. The system of equations that describes the DCE is as follows:

$$y_t = a_t (k^p_t)^{1-\theta} (n^p_t)^{\theta} (k^g_t)^{\nu} \tag{3.0.1}$$

$$y_t = c_t + g^c_t + g^i_t + k^p_{t+1} - (1 - \delta^p) k^p_t \tag{3.0.2}$$

$$\psi (c_t + \omega g^c_t)^{\psi(1-\alpha)-1} (1 - n^p_t - n^g_t)^{(1-\alpha)(1-\psi)} = (1 + \tau^c) \lambda_t \tag{3.0.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left[1-\delta^p+(1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p\right] \tag{3.0.4}$$

$$(1-\psi)(c_t+\omega g^c_t) = \psi(1-n^p_t-n^g_t)\frac{(1-\tau^l)}{(1+\tau^c)}\theta\frac{y_t}{n^p_t} \tag{3.0.5}$$

$$k^p_{t+1} = i_t+(1-\delta^p)k^p_t \tag{3.0.6}$$

$$k^g_{t+1} = g^i_t+(1-\delta^g)k^g_t \tag{3.0.7}$$

$$g^i_t = g^{iy}_t y_t \tag{3.0.8}$$

$$g^c_t = g^{cy}_t y_t \tag{3.0.9}$$

$$g^t_t = g^{ty} y_t \tag{3.0.10}$$

$$w^p_t = \theta\frac{y_t}{n^p_t} \tag{3.0.11}$$

$$r_t = (1-\theta)\frac{y_t}{k^p_t} \tag{3.0.12}$$

$$w^g_t = \eta^{-\frac{1}{2\rho}}\left[\frac{\tau^c c_t+\tau^k r_t k^p_t-\tau^k\delta^p k^p_t+\tau^l w^p_t n^p_t-g^c_t-g^t_t-g^i_t}{1-\tau^l}\right]^{\frac{1}{2}} \tag{3.0.13}$$

$$n^g_t = \eta^{\frac{1}{\rho}}w^g_t. \tag{3.0.14}$$

Therefore, the DCE is summarized by Equations (3.0.1)-(3.0.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, g^c_t, g^i_t, g^t_t, k^p_t, k^g_t, n^p_t, n^g_t, \lambda_t, w^g_t, w^p_t, r_t\}^{\infty}_{t=0}$ given the paths of technology $\{a_t\}$, the fixed level of implied government transfers/output ratio $\{g^{ty}\}$, $\forall t$, and the exogenously set stationary government spending/output and government investment/output ratio processes, $\{g^{cy}_t, g^{iy}_t\}^{\infty}_{t=0}$, whose motion was determined in the previous subsection.[14]

[14]Note that Eq. (3.0.13)-(3.0.14) imply the government budget constraint.

4 Steady-state system

In steady-state, there is no uncertainty, and $z_{t+1} = z_t = z$. Thus, remove expectations operators and time subscripts to obtain

$$y = a(k^p)^{1-\theta}(n^p)^{\theta}(k^g)^{\nu} \tag{4.0.15}$$

$$y = c + g^c + g^i + \delta^p k^p \tag{4.0.16}$$

$$\psi(c + \omega g^c)^{\psi(1-\alpha)-1}(1 - n^p - n^g)^{(1-\alpha)(1-\psi)} = (1 + \tau^c)\lambda \tag{4.0.17}$$

$$1 = \beta\left[1 - \delta^p + (1 - \tau^k)(1 - \theta)\frac{y}{k^p} + \tau^k \delta^p\right] \tag{4.0.18}$$

$$(1 - \psi)(c + \omega g^c) = \psi(1 - n^p - n^g)\frac{(1 - \tau^l)}{(1 + \tau^c)}\theta\frac{y}{n^p} \tag{4.0.19}$$

$$i = \delta^p k^p \tag{4.0.20}$$

$$g^i = \delta^g k^g \tag{4.0.21}$$

$$g^i = g^{iy} y \tag{4.0.22}$$

$$g^c = g^{cy} y \tag{4.0.23}$$

$$g^t = \ ^{ty} y \tag{4.0.24}$$

$$w^p = \theta\frac{y}{n^p} \tag{4.0.25}$$

$$r = (1 - \theta)\frac{y}{k^p} \tag{4.0.26}$$

$$n^g = \eta^{\frac{1}{\rho}} w^g \tag{4.0.27}$$

$$w^g = \eta^{-\frac{1}{2\rho}}\left[\frac{\tau^c c + \tau^k(r - \delta^p)k^p + \tau^l w^p n^p - g^c - g^t - g^i}{1 - \tau^l}\right]^{\frac{1}{2}} \tag{4.0.28}$$

5 Data and model calibration

Both the model in this paper and Finn (1998) are calibrated for German data at annual frequency. The paper follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate certain features of the reference economy.

5.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007, while the sub-period 1970-91 covers West Germany only. For Germany, data on real output per capita, household consumption per capita, gross fixed capital formation per capita, as well as government consumption and population was taken from the World Development Indicators (WDI) database. OECD statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Public transfers ratio were calculated from the *CES-Ifo* DICE Database (2011). Public and private investment and capital stock series were obtained from EU Klems database (2009). German average annual real public compensation per employee was estimated by dividing real government wage bill (OECD 2011) by the number of public employees. Due to data limitations on the average hours worked in each sector, employment statistics will be used. To make empirical variables comparable with model variables, employment series in Germany were normalized by total population (obtained from WDI).

5.2 Calibrating model parameters to German data

In German data, the average public/private employment ratio over the period 1970-2007 is 0.17, and the average wage ratio in data equals 1.20. The weight put on public wages, η, as well as government transfers/output ratio g^{ty} will be set so that the steady-state wage and employment ratios in the model match the corresponding data averages.The curvature parameter of the union's CES maximization function, was set to a standard value, $\rho = -1$, as in Fernandez-de-Cordoba et al. (2010).[15] The average effective tax rates in EU countries

[15]A robustness check on the curvature parameter was performed with $\rho = [-5, -4, -3, -2, -0.5]$, which did not produce any significant difference in the results obtained, as parameter η adjusted accordingly.

were obtained from McDaniel's (2009) dataset. McDaniel's approach was preferred to the one used by Mendoza et al. (1984) and the subsequent updates due to the more careful treatment of property and import taxes. Over the period studied, German economy is characterized by a low average capital income tax rate, $\tau^k = 0.16$, and a relatively high labor income tax rate, $\tau^l = 0.409$. The labor share, $\theta = 0.71$, was computed as the average ratio of compensation of employees in total output.[16] Private and public capital depreciation rates, $\delta^p = 0.082$ and $\delta^g = 0.037$, respectively, were approximated from the EU Klems Database as the average ratio of gross fixed capital formation in constant 1995 prices and and the corresponding value of fixed capital stock in constant 1995 prices over the 1970-2007.[17] The discount rate $\beta = 0.973$ was calibrated from the steady-state Euler equation (41). The parameter describing the curvature of the household's utility function was again set to $\alpha = 2$. The weight on consumption, $\psi = 0.296$, was set equal to the average steady-state total hours of work in data as a share of total hours available. The weight put on government consumption in the utility function, $\omega = 0.099$, was calibrated using (42) and data averages. The public capital share in the production function, $\nu = 0.0233$, equals the average public investment/output ratio in German data. Persistence and innovation volatility of the stochastic processes, as well as the AR(1) process for public employment in Finn (1998), were estimated using OLS. Total factor productivity parameters, $\rho^a = 0.943$ and $\sigma^a = 0.013$, were estimated using the logged and linearly detrended Solow residual series, obtained from the model's aggregate production function and data. Table 2 on the next page summarizes the model parameters for Germany.

[16] Alternatively, capital share, $1-\theta$, can be obtained as the mean ratio of gross private capital compensation in output from EU Klems.

[17] To impute public sector capital stocks and investment, the series for education, public administration, social security and health sectors were used.

Table 2: Model Parameters

Param.	Value	Definition	Source
β	0.973	Discount factor	Calibrated
θ	0.710	Labor income share	Data average
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on government capital	Data average
α	2.000	Curvature parameter of the utility function	Set
ψ	0.296	Weight on consumption in utility	Set
ν	0.023	Degree of increasing returns to scale of public capital	Set
ρ	-1.000	Curvature parameter of the union's maximization function	Set
ω	0.099	Weight on government services in household's consumption	Calibrated
τ^c	0.148	Effective tax rate on consumption	Data average
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
ρ^c	0.976	AR(1) parameter government consumption/output ratio	Estimated
ρ^i	0.853	AR(1) parameter government investment/output ratio	Estimated
ρ^n	0.915	AR(1) parameter government employment (Finn's model)	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated
σ_c	0.016	SD of government consumption/output share innovation	Estimated
σ_i	0.023	SD of government investment/output share innovation	Estimated
σ_n	0.016	SD of government employment innovation (Finn's model)	Estimated

6 Solving for the steady-state

Once model parameters were obtained, the unique steady-state of the system was computed numerically for the Germany-calibrated model. Results are reported in Table 3 below.

Table 3: Data averages and long-run solution

	Description	GE Data	Finn GE	Union GE
c/y	Consumption-to-output ratio	0.590	0.576	0.576
i/y	Investment-to-output ratio	0.210	0.212	0.212
g^c/y	Gov't consumption-to-output ratio	0.189	0.189	0.189
g^i/y	Gov't investment-to-output ratio	0.023	0.023	0.023
g^t/y	Gov't transfers-to-output ratio	0.170	0.047	0.047
k^p/y	Private capital-to-output ratio	2.350	2.350	2.350
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
$w^p n^p/y$	Priv. labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill-to-output ratio	0.130	0.146	0.146
rk^p/y	Capital share in output	0.290	0.290	0.290
n^g/n^p	Public-private employment ratio	0.170	0.170	0.170
w^g/w^p	Public-private employment ratio	1.200	1.200	1.200
n^p	Private sector employment	0.253	0.210	0.211
n^g	Public sector employment	0.043	0.043	0.036
η	Relative weight on public wage rate	-	31.63	N/A
$\tilde{r}$	After-tax net return to capital	0.036	0.028	0.028

Note that the public transfers share, g^{ty}, and the relative weight attached to public wages, η are set so that the wage and hours ratios match the corresponding data averages.[18] In addition, the steady-state values for hours in data are approximated by splitting the average hours, expressed as a share of total available hours of work, according to the average hours

[18]In this model, the implied η cannot be interpreted directly, but should rather be regarded as containing a scaling factor, as n^g and w^g differ in magnitude (due to the normalization of the time endowment to unity). Therefore, once this is accounted for, i.e. when η is normalized by w^g/n^g, the "corrected" parameter, $\bar{\eta}$, equals 0.998 for Germany. In other words, wage rate and hours are equally-weighted in the generalized Stone-Geary union utility function, as typically assumed in the trade union literature.

ratio.[19] In Finn (1998), public hours are set to match the corresponding data average.

Overall, the long-run solutions of both models are good approximations to the data averages. The steady-state real after-tax real interest rate, net of depreciation, delivered by the two models, *i.e.* $\tilde{r} = (1-\tau^k)(r-\delta^p)$, is close to the average real interest rate on 10-year bonds, which is taken as a proxy for the return to private physical capital in the model. Both models capture the public wage bill share of GDP in Germany. Furthermore, public sector labor income is a significant share relative to capital in Germany as well.

Across models, several important differences can be noted: in steady-state, Finn (1998) produces a slightly higher level of total hours and lower public sector wages, compared to the model in this paper. That is due to the additional constraint imposed in the union model on the steady-state public-private hours ratio. In addition, the model with collective bargaining produces larger a steady-state public sector labor income/output ratio. Next, model dynamics out of the steady-state is investigated in the following section.

7 The Log-linearized system of equations

Since there is no closed-form general solution for the model in this paper, a typical approach followed in the RBC literature is to log-linearizing the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The log-linearized system of model equations is as below[20] ($\hat{rw}_t$ and $\hat{rl}_t$ denote the log-deviations in the wage and hours ratios, respectively):

$$k^p\hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t - g^c\hat{g}^c_t - g^i\hat{g}^i_t + (1-\delta^p)k^p\hat{k}^p_t \quad (7.0.1)$$

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta\hat{n}^p_t + \nu\hat{k}^g_t \quad (7.0.2)$$

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g}\hat{c}_t + \frac{\omega g(\psi-1-\alpha\psi)}{c+\omega g}\hat{g}^c_t - (1-\alpha)(1-\psi)\frac{n^p}{1-n}\hat{n}^p - \hat{\lambda}_t = 0 \quad (7.0.3)$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \quad (7.0.4)$$

[19] In this way hours/employment data averages are made comparable in magnitude with the corresponding theoretical variables in the union model.

[20] Detailed derivations in Appendix 11.2.1-12.2.19

$$\frac{c}{c+\omega g}\hat{c}_t + \frac{\omega g}{c+\omega g}\hat{g}_t + (1+\frac{n^p}{1-n})\hat{n}^p - \hat{y}_t \quad = \quad 0 \tag{7.0.5}$$

$$\hat{k}^p_{t+1} \quad = \quad \delta^p \hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{7.0.6}$$

$$\hat{k}^g_{t+1} \quad = \quad \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{7.0.7}$$

$$\hat{a}_{t+1} \quad = \quad \rho^a \hat{a}_t + \epsilon^a_{t+1} \tag{7.0.8}$$

$$\hat{g}^{cy}_{t+1} \quad = \quad \rho_c \hat{g}^{cy}_t + \epsilon^c_{t+1} \tag{7.0.9}$$

$$\hat{g}^{iy}_{t+1} \quad = \quad \rho_i \hat{g}^{iy}_t + \epsilon^i_{t+1} \tag{7.0.10}$$

$$\hat{g}^c_t = \hat{g}^{cy}_t + \hat{y}_t \tag{7.0.11}$$

$$\hat{g}^i_t = \hat{g}^{iy}_t + \hat{y}_t \tag{7.0.12}$$

$$\hat{w}^g_t = \frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\hat{c}_t + \frac{(1/2)[\tau^k(1-\theta)+\tau^l\theta]y}{(1-\tau^l)w^g n^g}\hat{y}_t$$
$$-\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\hat{k}_t - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\hat{g}^c_t - \frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\hat{g}^i_t - \frac{(1/2)g^t}{(1-\tau^l)w^g n^g}\hat{g}^t_t \tag{7.0.13}$$

$$\hat{n}^g_t \quad = \quad \hat{w}^g_t \tag{7.0.14}$$

$$\hat{w}^p_t \quad = \quad \hat{y}_t - \hat{n}^p_t \tag{7.0.15}$$

$$\hat{r}_t \quad = \quad \hat{y}_t - \hat{k}^p_t \tag{7.0.16}$$

$$\hat{n}_t \quad = \quad \frac{n^p}{(n^p+n^g)}\hat{n}^p_t + \frac{n^g}{(n^p+n^g)}\hat{n}^g_t \tag{7.0.17}$$

$$r\hat{w}_t \quad = \quad \hat{w}^g_t - \hat{w}^p_t \tag{7.0.18}$$

$$\hat{rl}_t \quad = \quad \hat{n}^g_t - \hat{n}^p_t \tag{7.0.19}$$

8 Model solution and impulse responses

The linearized DCE system can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988):

$$\mathbf{A}E_{t+1}\mathbf{\hat{x}_t} \;=\; \mathbf{B}\mathbf{\hat{x}_t} + \mathbf{C}\epsilon_t \tag{8.0.20}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\epsilon_\mathbf{t}$ is a matrix of innovations, and $\mathbf{\hat{x}_t}$ is the stacked vector of state (also called 'predetermined') variables, $\mathbf{\hat{s}_t} = \begin{bmatrix} \hat{a}_t & \hat{g}_t^{cy} & \hat{g}_t^{iy} & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\mathbf{\hat{z}_t} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^c & \hat{g}_t^i & \hat{g}_t^t & r\hat{w}_t & r\hat{l}_t \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model.

Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to either productivity, or government consumption. The effects of total factor productivity (TFP) and fiscal shocks to the government consumption and investment shares in a model with public sector union are different compared to Finn (1998), especially when the behavior of labor market variables and the labor reallocation is taken under close scrutiny.

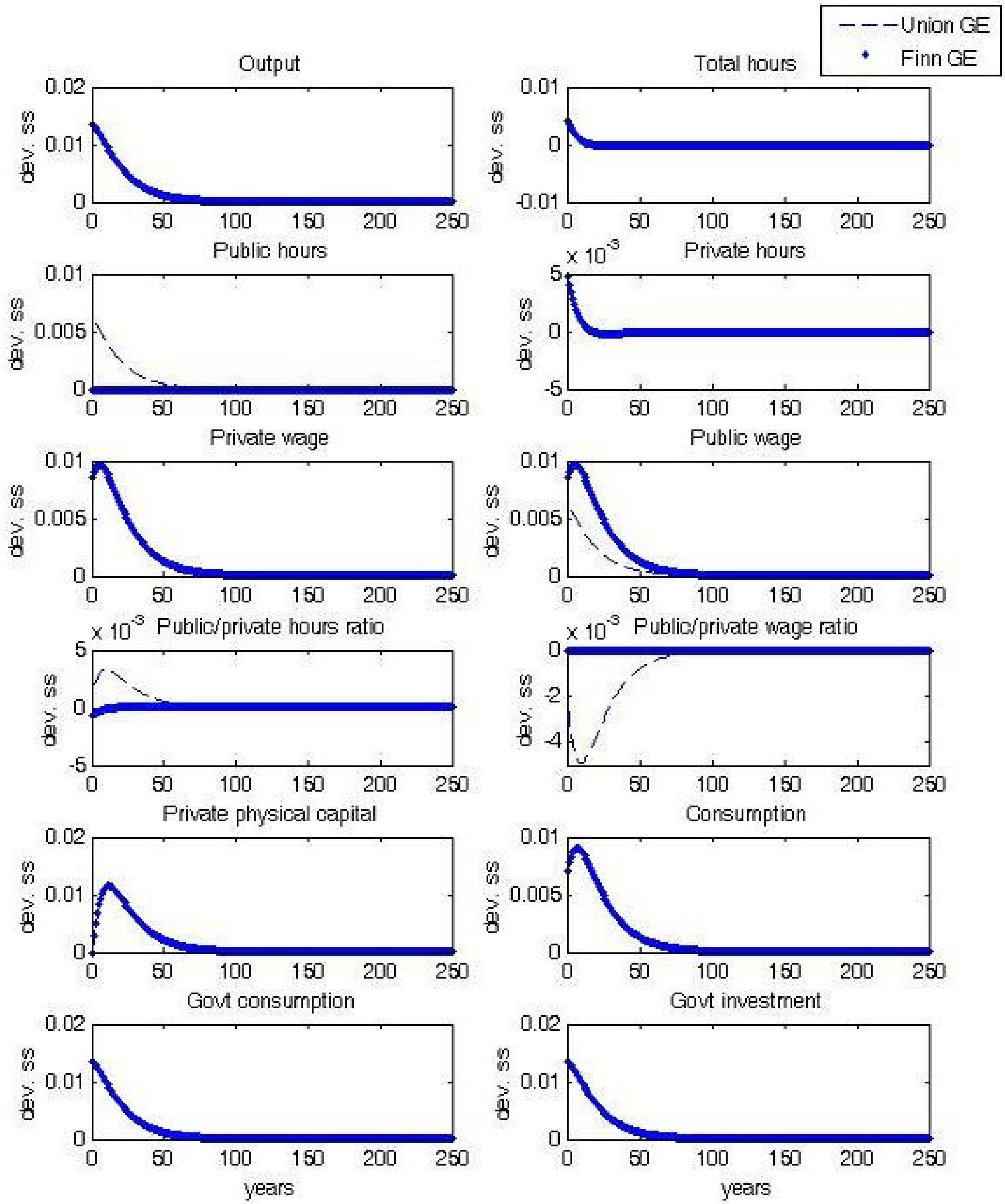
8.1 The Effect of a positive productivity shock

Figure 1 shows the impact of a 1% surprise TFP innovation on the economy with public sector union and Finn's setup. The impulse responses are expressed in log-deviation from the variables' original steady-states in the model economy calibrated to annual German data. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as investment. From the rules for the government spending, investment and transfers in levels, a higher output translates into higher level of expenditure in each of the three categories. In turn, there is also a feedback effect from government investment to output through the public capital, which comes with a one-period lag. This indirect effect is quite small. Meanwhile, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not pictured) and the private wage rate increase. The household responds to the price signals and supplies more hours in the private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private consumption and leisure. In terms of the labor-leisure trade-off, the income effect (”work more”) produced by the increase in the private wage dominates the substitution effect (”work less”). Furthermore, the increase of private hours expands output even further, thus both output and government spending categories increase more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

An observational equivalence is noted in the responses of most of the model variables across the two models. Public sector labor dynamics, however, is quite distinct: In Finn (1998), public hours stay fixed at their steady-state, and public wage transition is identical to the private wage one. In the model with collective bargaining, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. Higher income and consumption lead to larger tax revenue. The growth in government revenue exceeds the increase in the fiscal spending instruments, so the additional funds available for the wage bill, lead to an expansion in both public sector wage and hours. The effect on total hours in Germany is very small.[21] In addition, the model with collective bargaining in this paper generates an interesting dynamics in the wage and hours ratio, which is not present in Finn (1998). The two wage rates, as well as the two types of hours move together, making the model consistent with the empirical evidence presented in Lamo, Perez and Schuknecht (2007, 2008).

[21] Still, the increase in hours is much larger in magnitude compared to the responses reported in Fernandez-de-Cordoba *et al.* (2009, 2010).

Figure 1: Impulse Responses to a positive 1% productivity shock in Germany

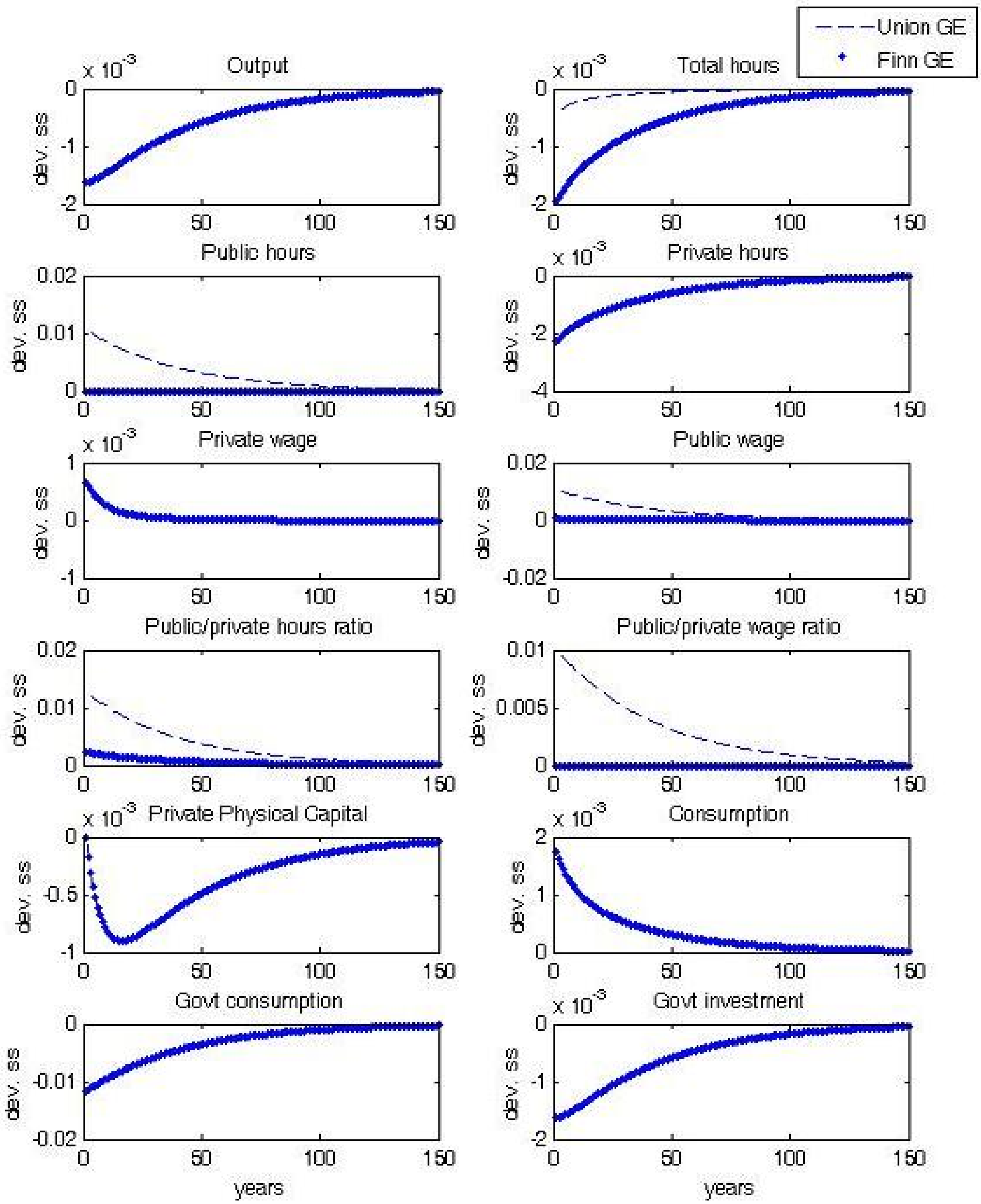


Overall, the endogenous public sector hours model shows an important difference in the composition of household's labor income with the public sector share increasing at the expense of private sector labor income. At aggregate level, however, this distributional effect washes away, as output and consumption dynamics are identical across models. Another important observation to make is that the TFP shocks, being the main driving force in the union model, induce pro-cyclical behavior in public wage and hours. In the German model economy, the shock effects are smaller and variables reach their peak response much quicker. This means the impulse effect dies out much faster but the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks in the labor markets, and particularly on the wage- and hours ratios.

8.2 The effect of a negative government consumption share shock

The second scenario is an exogenous restrictive fiscal policy, which is an unexpected decrease in the government consumption/output ratio. The impulse response functions for this scenario are reported in Figure 2. The results are similar to those obtained from a standard RBC model. The plots show that a negative government consumption shock partially crowds-in private consumption, as public consumption is only an imperfect substitute for private consumption from the household's point of view. This creates a significant positive welfare effect in the model economy as the decrease in the government consumption ratio frees additional resources that could be directed to private use. The increase in consumption at the expense of a drop in investment, triggers a decrease in private sector hours through the marginal rate of substitution between consumption and leisure. In other words, the increase in consumption, resulting from the positive wealth effect, decreases the need to supply labor, so the household enjoys more leisure. The decrease in labor input leads to a fall in output, and an increase in the private wage. Since government expenditure categories follow output, public consumption, investment, and government transfers (not presented) fall as well. Over time, all variables return to their old-steady states.

Figure 2: Impulse Responses to a negative 1% government consumption/output share shock in Germany



Those common responses are typical in the RBC literature but in the presence of a union in the public sector, the fall in labor supply leads to a lower tax revenue, while the increase in consumption increases the tax revenue. The other spending categories decrease as well, thus leaving more funds available for the public sector wage bill. The effect on public hours is very pronounced, when total hours responses are compared across models. Furthermore, the model with public sector union generates a realistic labor reallocation from private to public sector meaning that in times of fiscal restraint, government jobs become more attractive. In a model with exogenous public employment, public sector hours stay fixed at their steady-state value and do not respond to fiscal shocks. The effect of a decrease in the government consumption/output ratio in Finn (1998) leads to a significant underestimation in total hours. Additionally, the model with public sector union could again address the relative labor income share evolution, which is the product of the public-private wage and employment ratios. The results in this subsection differ from the ones in Fernandez-de-Cordoba *et al.* (2009, 2010) in important ways: The negative shock to the fiscal instruments creates a substitution effect and leads to the crowding-in of the public wage bill. In other words, even under a regime of fiscal tightening, public employment and the public wage are increased, *i.e.* shocks to the government consumption share make public wage and hours behave counter-cyclically.

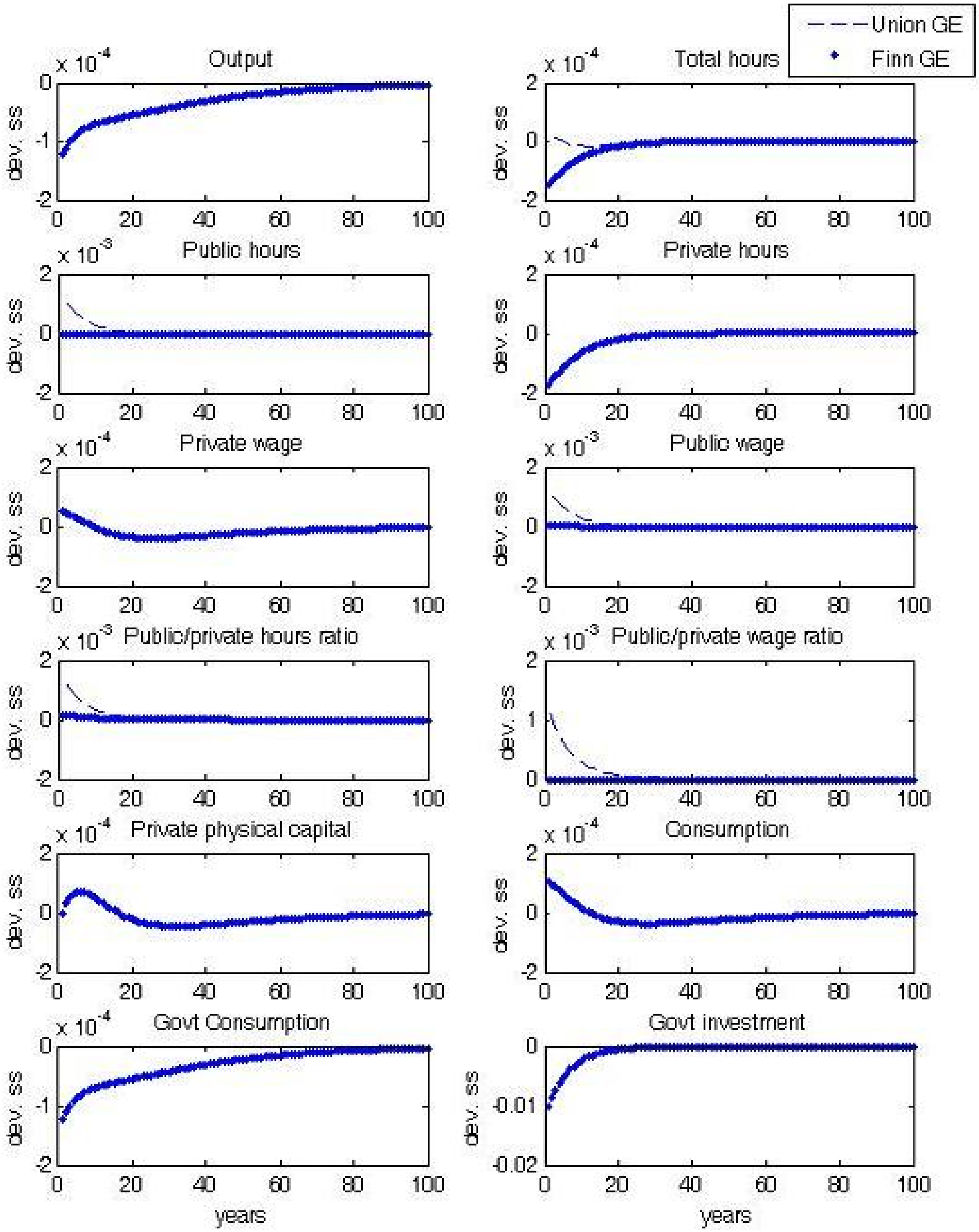
8.3 The Effect of a negative government investment share shock:

This experiment simulates the effect of a surprise negative innovation in the government investment/output ratio. The impulse response functions are reported in Figure 3. This scenario is very relevant in times of crisis, as public investment projects are small relative to the GDP, thus usually the first ones to be cut. The decrease in the government investment share has a direct negative effect due to the decrease in the public physical capital input in the aggregate production function. The magnitude of the shock effect depends on the degree of IRS, captured by the parameter ν. Public investment falls both because of the fall in the public investment/output ratio, and the fall of output itself. Following the output fall, public consumption and government transfers also fall. On aggregate level, there is a positive welfare effect: output falls less compared to the fall in government consumption and investment. Therefore, the extra resources available now in the economy are used for private consumption and investment. Private physical capital increases, but the effect is short-lived as the marginal product of capital decreases fast, and capital even falls below the steady-state level along the transition path. Meanwhile, the positive wealth effect leads to a fall in the private sector hours supplied by the household, meaning that private wage increases; The subsequent transition behavior of the private sector wage is determined by the private physical capital dynamics. In the long run, all variables return to their old steady-state values.

The model with public sector union generates the expected additional positive effect on the public wage bill. As total tax revenue increases, and other spending items decrease, the additional revenue is allocated to raise private wage and hours. The total contemporaneous effect on hours changes from negative in Finn (1998) to slightly positive, with the overall impact on model variables being very small and short-lived. The model with collective bargaining, however, produces important transition in the wage and hours ratio and is present for almost 20 periods. In addition, the shocks to public investment share add to the counter-cyclical behavior of public hours and wage rate.

To investigate fully the forces that operate within the model and to study in detail the dynamic interaction among model variables, a complete simulation of the model is performed in the next subsection.

Figure 3: Impulse Responses to a negative 1% government investment/output share in Germany



9 Model simulation, goodness-of-fit, and the welfare effect of tax reforms

Using the model solutions, shock series were added to produce simulated data series. The length of the draws for the series of innovations is 138, and the simulation is replicated 1000 times. Natural logarithms are taken, and then all series are run through the Hodrick-Prescott filter with a smoothing parameter equal to 100. The first 100 observations are then excluded to decrease any dependence on the initial realizations of the innovations. Average standard deviation of each variable and its correlation of output of are estimated across the 1000 replications.[22] The large number of replications implemented is to average out sampling error across simulations, before comparing model moments to the ones obtained from data.

9.1 Relative second moments evaluation

This section compares the theoretical second moments of the simulated data series with their empirical counterparts, with special attention paid to the behavior of public sector hours and wages. Table 4 on the next page summarizes the empirical and simulated business cycle statistics for the two models calibrated for Germany.

In the German data, relative consumption volatility exceeds one, as the available series does not provide a breakdown into consumption of non-durables and consumption of durables.[23] Durable products behave like investment, and vary much more than non-durables, while model consumption corresponds to non-durable consumption. Since a major force in all the three models is consumption smoothing, as dictated by the Euler equation, both models under-predict consumption volatility and investment variability. Across models, private sector employment and private wage also vary less compared to data. Total employment in German data varies less than either private or public employment due to smaller variation in the number of self-employed individuals. It is evident from Table 5 that the model with public sector union underestimates public wage volatility, but matches public employment quite well. Finn's model captures the volatility of public employment due to the fact that it

[22] As an additional model check, the autocorrelation (ACFs) and the cross-correlation functions (CCFs) are also generated, and compared it to the ones computed from German data. The results of this computational experiment are presented in Appendix 11.3.

[23] Another possible reason could be the presence of strong habits in consumption.

Table 4: Business Cycle Statistics Germany, 1970-2007

	GE Data	Finn (1998)	Public Sector Union
$\sigma(y)$	0.0154	0.0165	0.0165
$\sigma(c)/\sigma(y)$	1.11	0.56 [0.49,0.62]	0.56 [0.49,0.62]
$\sigma(i)/\sigma(y)$	3.57	2.30 [2.24,2.36]	2.30 [2.24,2.36]
$\sigma(n^p)/\sigma(y)$	1.05	0.45 [0.40,0.50]	0.45 [0.40,0.49]
$\sigma(n^g)/\sigma(y)$	1.06	0.91 [0.69,1.13]	1.27 [0.98,1.56]
$\sigma(n)/\sigma(y)$	0.73	0.38 [0.33,0.43]	0.39 [0.38,0.40]
$\sigma(w^p)/\sigma(y)$	1.16	0.63 [0.59,0.68]	0.63 [0.59,0.68]
$\sigma(w^g)/\sigma(y)$	3.50	0.63 [0.59,0.68]	1.19 [0.92,1.47]
$corr(c,y)$	0.80	0.85 [0.79,0.92]	0.85 [0.79,0.92]
$corr(i,y)$	0.85	0.99 [0.98,0.99]	0.99 [0.98,0.99]
$corr(n^p,y)$	0.60	0.89 [0.84,0.93]	0.89 [0.85,0.94]
$corr(n^g,y)$	0.11	-0.05 [-0.29,0.20]	0.19 [0.04,0.43]
$corr(n,y)$	0.60	0.84 [0.78,0.91]	0.97 [0.97,0.98]
$corr(w^p,y)$	0.60	0.95 [0.92,0.97]	0.94 [0.93,0.97]
$corr(w^g,y)$	0.35	0.95 [0.92,0.97]	0.19 [0.04,0.43]
$corr(n,n^p)$	0.92	0.90 [0.86,0.95]	0.88 [0.79,0.92]
$corr(n,n^g)$	0.43	0.28 [0.06, 0.51]	0.27 [0.05,0.49]
$corr(n^p,n^g)$	0.12	-0.15 [-0.38,0.08]	-0.21 [-0.44,0.02]
$corr(n^p,w^p)$	0.21	0.70 [0.59,0.81]	0.71 [0.61,0.81]
$corr(n^g,w^g)$	-0.38	0.03 [-0.22,0.28]	1.00 [1.00,1.00]
$corr(n^g,w^p)$	0.20	0.03 [-0.22,0.28]	0.45 [0.26,0.64]
$corr(n^p,w^g)$	0.34	0.70 [0.59,0.81]	-0.21 [-0.44,0.02]
$corr(w^p,w^g)$	0.48	1.00 [1.00,1.00]	0.45 [0.26,0.65]

is modeled as an exogenous stochastic process to mimic public hours time series behavior.

Both models capture relatively well the high contemporaneous correlations of main variables with output. Moreover, public sector variables are also pro-cyclical, but not as much as the models predict: Finn (1998) even predicts that public employment is countercyclical. Nevertheless, the model with the public union captures quite well the co-movement between labor market variables, as well as their contemporaneous correlations with output, compared to the alternative. The German data, as well as the model with public sector union, provide some support to the "private sector wage-leader" hypothesis. In other words, there is some evidence that public sector wage follows the one in the private sector but only moderately so. The dimension where the union model fails, however, is the correlation between public sector hours and wages: in German data, it is negative, while the union model predicts a perfect positive linear relationship. The reason is that the empirical correlation can be interpreted as showing the net effect of supply and demand factors, while the model models concentrates exclusively on the supply-side forces. It is plausible that due to the population aging, demand for public employees will be high as well, especially in healthcare, social security and senior care. The empirical correlation between wages also well-captured by the model with collective bargaining. In other words, empirical public sector wage follows to a much lesser degree the private sector wage. A failure of the model with public sector union is the predicted negative correlation between the two types of hours. To a certain extent, this is an artifact of the way fiscal instruments were specified. The prediction of the model along this dimension greatly improves if government consumption and investment follow AR(1) processes in levels, and thus do not react to output. Furthermore, it is a well-known fact (*e.g.* Prescott 1986, Hansen 1992) that the RBC model does not capture private sector labor market dynamics very well.

Overall, the model with the public sector union captures the labor market dynamics in Germany, addressing dimensions that were ignored in earlier RBC models. Thus, an optimizing union in the public sector proves to be an important ingredient in RBC models when studying European labor markets with strong public sector unions. To assess the welfare cost of fiscal policy in the presence of public sector union, several fiscal experiments are performed in the following subsection.

9.2 Welfare evaluation of fiscal regime changes

The goal of this section is to quantify the importance of endogenously-determined public sector hours for fiscal policy, relative to Finn's setup with exogenously-fixed public hours. Additionally, the explicit welfare analysis complements earlier studies in Finn (1998) and Fernandez-de-Cordoba *et al.* (2009, 2012). To understand the adjustment mechanisms after an exogenous change in fiscal policy, each tax rate in the two models is varied over the $[0, 1]$ interval. Since all three tax rates were exogenously-specified, Schmitt-Grohe and Uribe

(1997) show that for a wide class of RBC models, and plausible values for model parameters, a unique long-run solution exists. When tax rates are plotted against tax revenues, Laffer curves (Laffer 1974, Schmidt-Grohe and Uribe 1997) appear: in both Finn and public sector union model, an inverted U-shape relationship is observed between labor and capital income tax rates and total tax revenues. Thus, there are pairs of tax rates that generate the same level of tax revenue.[24] In general, increasing tax rates could lead to either an increase or a decrease in total tax revenue, depending on which side of the Laffer curve the economy is situated. For the German model economy, however, both setups place Germany on the left side of the labor and capital tax Laffer curve, as seen in Fig. 4-5. Furthermore, a change in a tax rate affects the tax receipts from other tax bases as well, by influencing steady-state allocations and prices. Therefore, to gain an additional insight of the effect of fiscal policy in the steady state, total tax revenue is broken down into individual tax revenues corresponding to the tax bases, and plotted as a function of each individual tax rate in Fig. 4-6, for both the public union model and Finn.

The shape of the capital tax Laffer curve, for example, presents an interesting case: an increase in τ^k leads to a negligible marginal increase in total tax revenue, since total tax revenue is essentially flat in the $\tau^k \in [0, 0.5]$ range, and for $\tau^k \in [0.5, 1]$ total revenue is negatively related to capital income tax rate.[25] The German economy features a low rate of capital income taxation, $\tau^k = 0.16$, thus the economy is situated safely away from the downward sloping segment of the Laffer curve. The reason for the flat Laffer curve is clearly seen from the breakdown in individual tax revenues as a function of capital income tax rate: All increases in capital income tax revenue are offset by corresponding decreases in labor income and consumption tax revenue. Since τ^c and τ^l are held fixed while τ^k is varied, the fall in labor income and consumption tax revenue is entirely driven by the shrinking tax bases. Across models, union framework features only slightly higher capital income and consumption revenue, and lower labor income tax revenue for each τ^k, as compared to Finn's setup.

On the other hand, labor income tax rate places Germany much closer to the peak of the

[24]Sensitivity analysis of the effect of model parameters on the shape of the Laffer curves is performed in Appendix 11.4.

[25]Uhlig and Trabandt (2010) find a similarly-shaped capital tax Laffer curve in an RBC model without public employment, calibrated to the EU-15 data.

labor tax Laffer curve, but still far away from the downward-sloping segment. Thus, the government could increase tax revenue by increasing τ^l. The computed total tax revenue-maximizing τ^l is approximately 50% in the union model, and 55% in Finn. As demonstrated in Fig. 5, the difference in computed total tax revenue with respect to labor income tax in the union model and Finn is due to the difference in the steady-state public and private hours, as well as the wage rates in the two models: Finn's model, featuring a single wage rate and fixed public employment, generates both a higher total tax revenue and a higher labor income tax revenue Laffer curve, as compared to the union model.

Lastly, for the consumption tax rate, no Laffer curve is observed: within a realistic range,

Figure 4: Capital tax Laffer curve

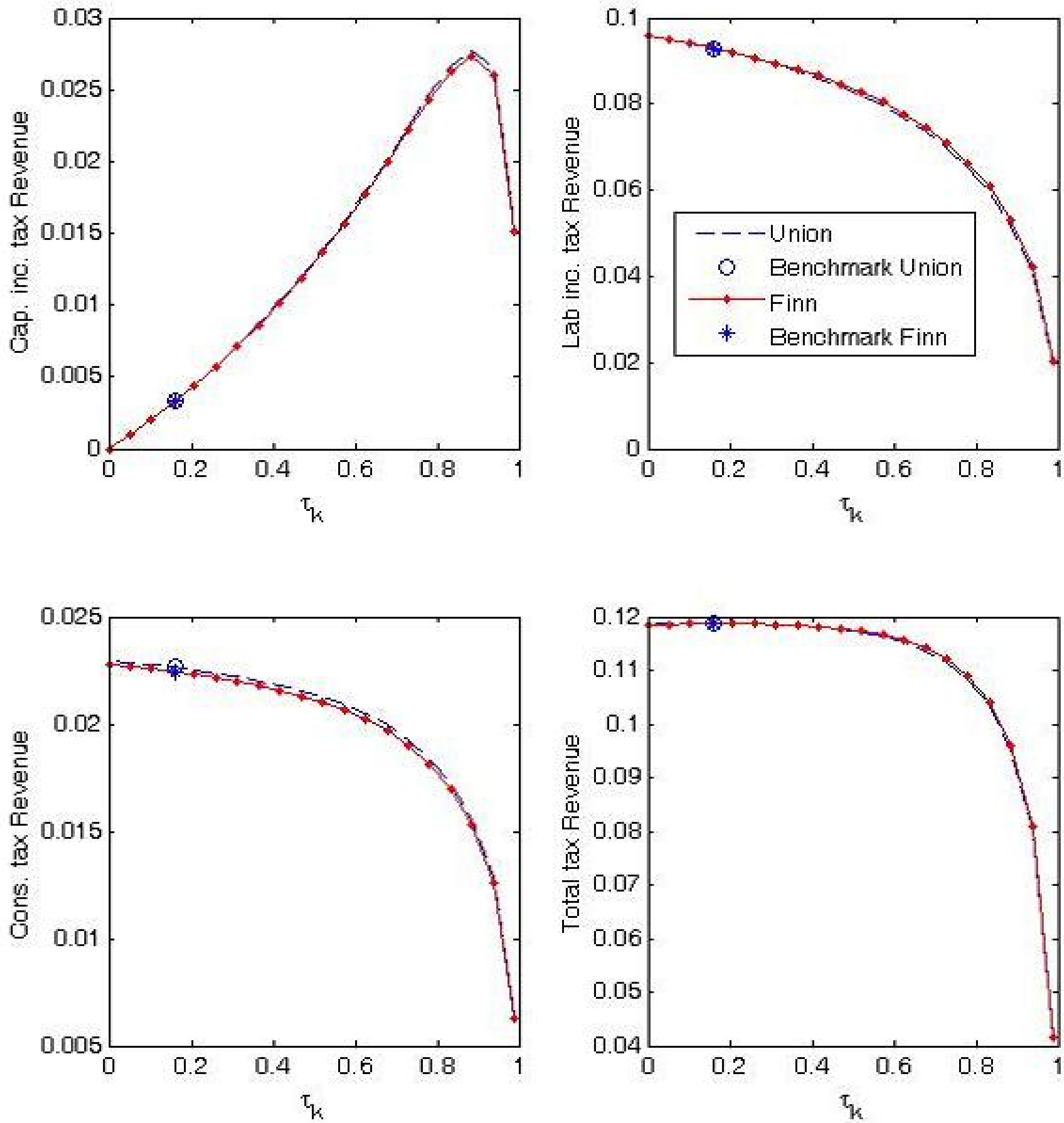


Figure 5: Labor tax Laffer curve

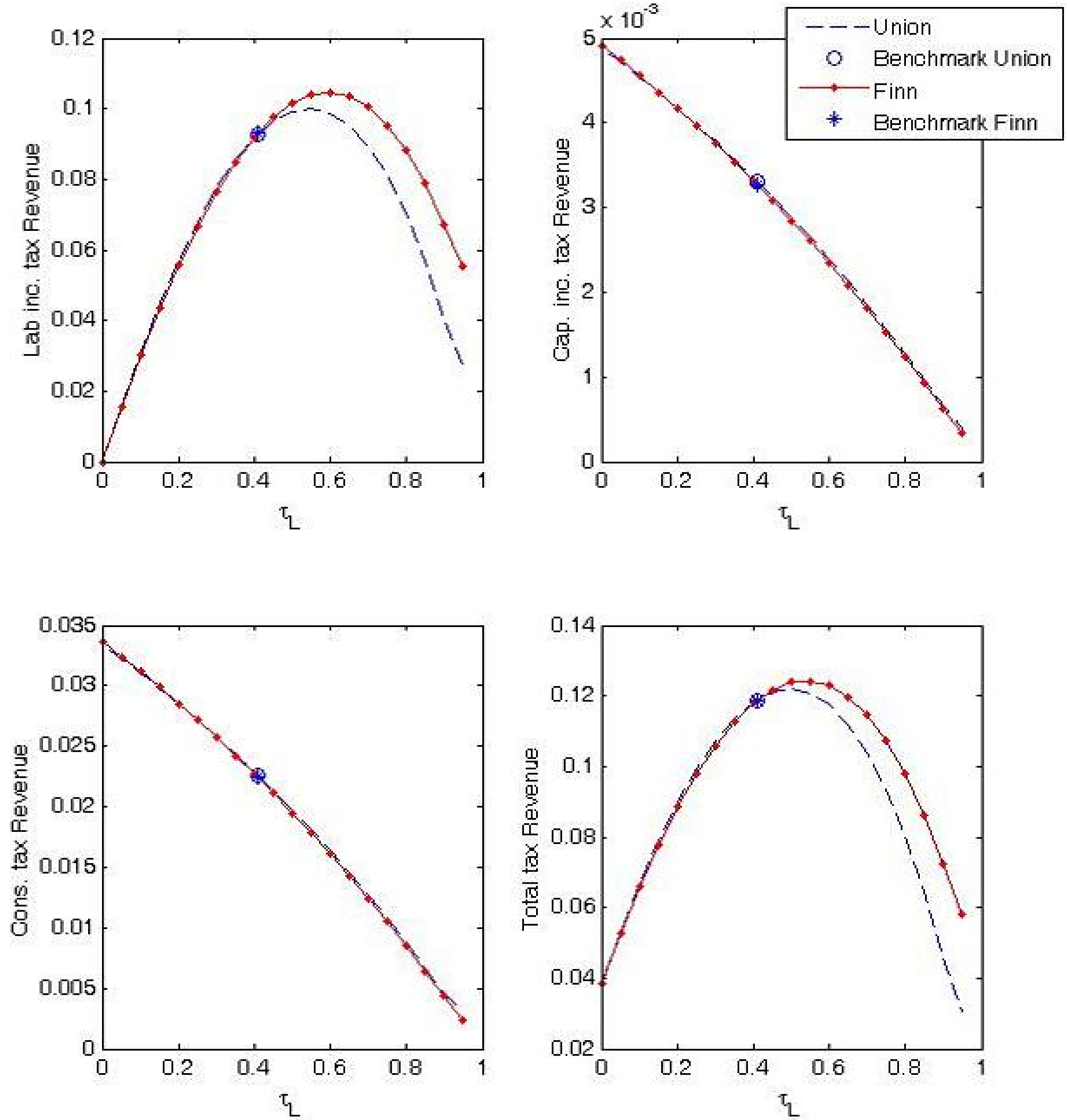


Figure 6: Consumption tax Laffer curve

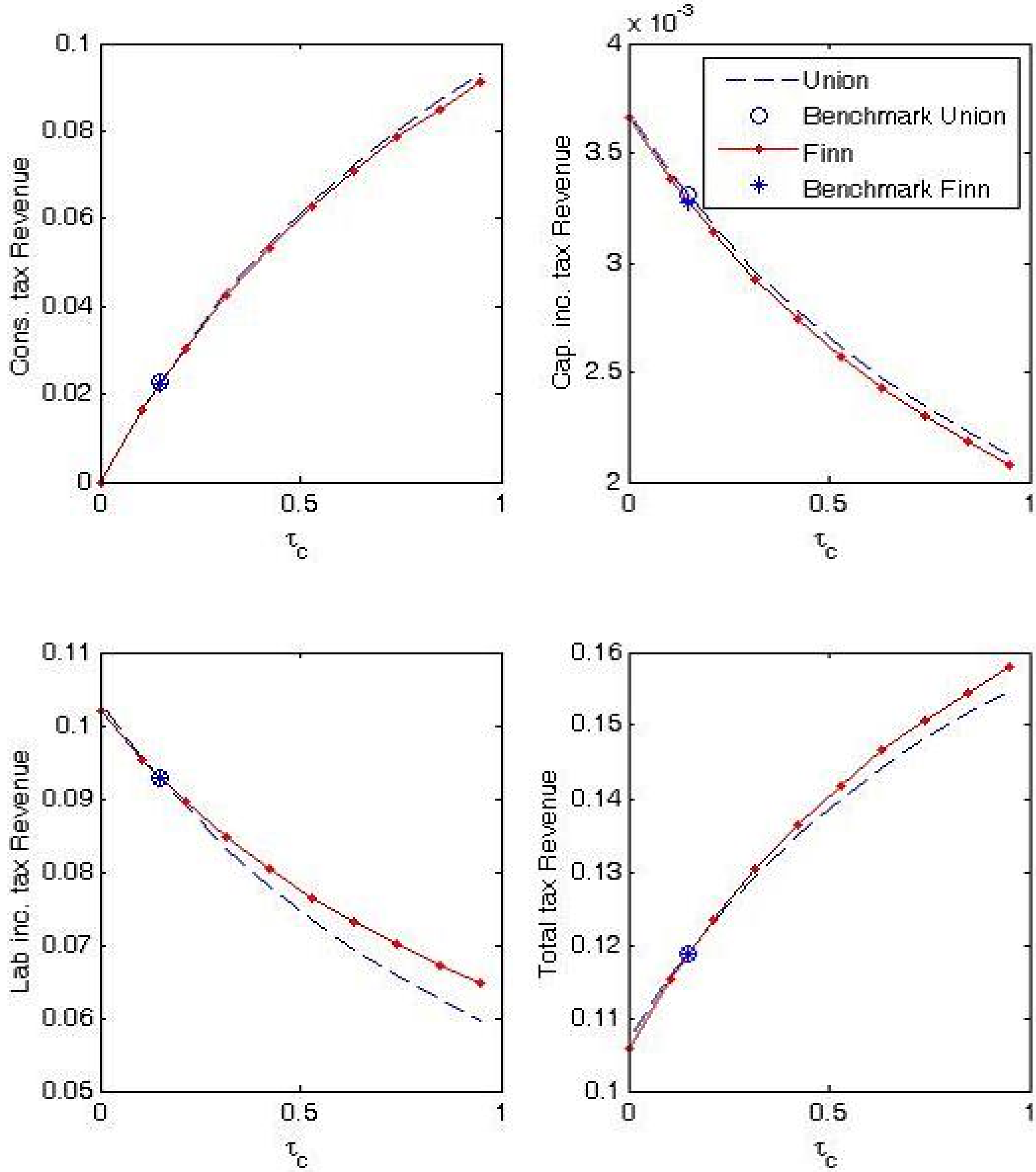


Fig. 6 shows no negative relationship between τ^c and tax revenue. The reason for that is as follows: In the model parameterizations $\alpha > 1$, thus the income effect dominates the substitution effect: when τ^c increases, labor supply and capital stock increase while consumption falls.[26] As argued in Trabandt and Uhlig (2010), a consumption tax Laffer curve arises if $\alpha < 1$, so that after an increase in τ^c, the substitution effect dominates the income effect and hours and capital stock fall together with consumption. In the union model, public employment falls as well, driven by the fall in tax revenue. In the borderline case, when $\alpha = 1$ (log-utility), the two effects offset one another. Again, no consumption tax Laffer curve occurs.

Across models, the exogenous public hours in Finn produce a slightly flatter total tax revenue curve as a function of τ^c. In particular, the important difference across the setups is a steeper labor income tax revenue curve in the union model vs. a flatter labor income tax revenue curve in Finn's model. The slope of the labor tax revenue curve is determined by the elasticity of hours with respect to changes in the tax rate. In both models, a higher τ^c decreases the labor wedge, $(1 - \tau^l)/(1 + \tau^c)$. However, the response in hours is larger in the case of the union model, which features endogenous public sector hours, as compared to Finn's setup, where n^g is held.

After characterizing and comparing the shapes of the Laffer curves in both models, this section proceeeds to welfare-evaluate the effects of different tax regimes. This is achieved through several normalized fiscal policy experiments. In all of the experiments a combination of tax rate changes will be specified so that total tax revenue is kept constant.[27] The general usefulness of this approach is that it separates tax and spending issues. In the framework considered in this paper, however, public sector labor income appears on both sides of the government budget constraint. In addition, the substitutability/complementarity of the capital and labor input in the Cobb-Douglas production function, the substitutability between consumption and labor, as well as the substitutability between consumption and investment implies that changes in a single tax rate will affect the tax revenue generated from the other two tax bases.

[26] Note that the increase in private hours and capital, driven by the increase in consumption tax rate does not translate in an increase in the corresponding tax revenue category. In addition, a higher τ^c leads to lower steady-state consumption, but a higher consumption revenue.

[27] As it will be seen in the next section, not all such combinations will be feasible.

Following Lucas (1987), the approach taken is to compute the compensatory variation in consumption.[28] In other words, this section calculates the percentage of compensating consumption, ζ, that is to be given to the household to make it indifferent between the two regimes. The initial regime for Germany is as described in Section 2, with the calibration and steady state solution presented in Section 4. The value of ζ is calculated for all restrictive fiscal policy scenarios, where a positive (negative) value indicates a welfare gain (loss).

Three different policies will be examined: a 1% increase in capital income, labor income, as well as consumption tax rate will be considered. In order to keep total tax revenue constant, whenever a tax rate increases, one of the other two tax rates will be allowed to adjust, holding all other model parameters fixed.[29]

9.2.1 Revenue-neutral increase in capital income tax rate

This subsection discusses the steady-state effect of a 1% increase in τ^k, with results presented in Table 5 on the next page. Higher capital income tax rate enters the Euler equation and thus

Table 5: Welfare gains/costs of 1% increase in τ^k in Germany

Model	τ^l fixed, τ^c adjusts	τ^c fixed, τ^l adjusts
	$\tau^c = 0.4033 \uparrow (25.52\%)$	$\tau^c = 0.1481$
	$\tau^l = 0.4085$	$\tau^l = 0.5535 \uparrow (14.50\%)$
Union	$\zeta = -0.2093$	$\zeta = -0.2425$
	$\tau^c = 0.3657 \uparrow (21.76\%)$	$\tau^c = 0.1481$
	$\tau^l = 0.3596$	$\tau^l = 0.5415 \uparrow (13.30\%)$
Finn	$\zeta = -0.1430$	$\zeta = -0.1745$

decreases the steady state private capital-to-output ratio. Since total revenue with respect to τ^k is relatively flat in both models, the increase in capital income tax essentially does not change total revenue. Variations in labor income tax rate, or consumption tax rate, however, are very distortionary, as they operate through the marginal rate of substitution. A higher labor, or a higher consumption tax rate, lower private hours. From the complementarity

[28]Detailed derivations are shown in Appendix 11.5.

[29]For example, η and g^{ty} in the union model, and g^{ty} in Finn, are held fixed at the values obtained in the original steady-state computation.

of hours and capital in the production function, capital stock falls as well. Lower levels of labor and capital inputs shrink output, which in turn decreases consumption. This change in steady-state allocation requires additional adjustment in the varying tax rate (τ^l or τ^c) to preserve revenue neutrality. The computational experiment performed shows that in either case, the adjusting tax rate have to change significantly to satisfy the revenue neutrality restriction. Across models, consumption tax is the less distortive instrument. Additionally, the computed welfare cost is higher in the union model by 6.63 % (6.8 % when τ^l varies) due to the endogenous response of public hours, which requires significantly larger tax rate increases in the union model.

9.2.2 Revenue-neutral increase in labor income tax rate

In this case, an increase in τ^l affects the marginal rate of substitution (MRS) between steady-state hours and consumption. As in the previous subsection, the analysis is split in two sub-cases, with results summarized in Table 6 on the next page. When the consumption

Table 6: Welfare gains/costs of 1% increase in τ^l in Germany

Model	τ^k fixed, τ^c adjusts	τ^c fixed, τ^k adjusts
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^c = 0.3862 \uparrow (23.81\%)$	$\tau^c = 0.1481$
Union	$\zeta = -0.2105$	$\zeta = N/A$
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^c = 0.35 \uparrow (20.19\%)$	$\tau^c = 0.1481$
Finn	$\zeta = -0.1444$	$\zeta = N/A$

tax rate is the adjusting rate, a 23.81% increase in τ^c is required in the union model. Again, Finn's setup generates much smaller welfare cost as compared to the union model, as the setup with exogenous public sector hours requires consumption tax rate to increase by 17% to preserve the initial level of tax revenues.[30] In both models, the increase in the consumption tax rate relative to the increase in the labor income tax rate is larger. Therefore, the labor wedge, $(1-\tau^l)/(1+\tau^c)$, decreases in both cases, which leads to an increase in private hours. Since hours and private physical capital are complements in the production function, the increase in labor input raises the marginal product of private capital, hence real interest rate

[30] Note that the higher fall in a tax rate results in a lower level of distortions in the economy.

will increase as well. The higher return to capital encourages investment, thus steady-state private capital stock expands. Following the expansion in capital input, output increases as well. In turn, higher output leads to higher consumption. The increase in consumption, however, is dominated by the increase in hours, so long-run welfare decreases relative to the one obtained in the initial steady-state. In addition, in the union model, there is an important feedback effect, which further increases welfare cost. This effect works to increase public hours, as a result of the higher tax revenue. In effect, endogenously-determined public hours add to the allocative distortions in the union model. Public hours enter the MRS condition, and thus necessitate a much larger adjustment in the union economy, as compared to Finn's framework. The presence of endogenously-determined public hours and wages adds 6.6% to the computed welfare loss.

In the second sub-case, when capital income tax rate varies in response to the increase in labor income tax, no reasonable level of τ^k (i.e. $\tau^k \in [-1, 1]$) exists that satisfies the revenue neutrality restriction. This is a straightforward consequence of the relatively flat Laffer curve with respect to the capital income tax rate, as demonstrated in the section on capital tax Laffer curve. Additionally, in both models the share of capital income tax revenue is less than 3%, which is very small when compared to consumption tax revenue share (22%) and labor income tax revenue share (75%). Thus, capital income tax rate is not a suitable instrument for fiscal adjustment, due to its limited ability to affect total tax revenue.

9.2.3 Revenue-neutral increase in consumption tax rate

The increase in τ^c affects the marginal rate of substitution between steady-state hours and consumption as well, so the effect on allocations is qualitatively similar to the one described in the previous section. In the first sub-case of this scenario (Table 7 below), when labor income tax rate changes to preserve the tax revenue, the labor income tax rate needs to increase by 12.73% and 16.96% in Finn and the union model, respectively.

This upward change in the labor income tax rateis significantly larger than the increase in consumption tax rate. The resulting decrease in the effective labor wedge, $(1-\tau^l)/(1+\tau^c)$, affects labor supply and consumption decisions: the household responds to the dominating income effect and supplies more hours in the private sector. Next, the higher level of labor input in the production function raises both output and the interest rate. The higher return to private physical capital leads to an increase in investment, which adds to the capital

Table 7: Welfare gains/costs of 1% increase in τ^c in Germany

Model	τ^k fixed, τ^l adjusts	τ^l fixed, τ^k adjusts
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^l = 0.5781 \uparrow (16.96\%)$	$\tau^l = 0.4085$
Union	$\zeta = -0.2404$	$\zeta = N/A$
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^l = 0.5358 \uparrow (12.73\%)$	$\tau^l = 0.4085$
Finn	$\zeta = -0.1724$	$\zeta = N/A$

stock and expands output. The positive wealth effect then translates into an increase in consumption. However, the higher consumption is offset by the increase in hours, so welfare decreases. Additionally, the increase in hours is higher in the union model, driven by the endogenously-determined public hours, which positively co-move with private hours. Thus the required increases in labor income tax rates produce nearly 6.8% larger welfare losses in the union model, a result attributed to the endogenously-determined public hours.

The case when τ^k is the adjusting tax rate unravels exactly as the case when τ^l increased by 1% and τ^k was the adjusting tax rate. Intuitively, both an increase in τ^c and τ^l decrease the effective labor wedge, thus the resulting adjustments through τ^k are qualitatively smilar. Again, there is no feasible capital income tax rate that preserves revenue neutrality.

Overall, the experiments performed in this section uncovered some important limitations of Finn's model with exogenous public hours. The presence of endogenously-determined public sector hours and wage rate was shown to generate important interactions, which add to the distortionary effect of taxes. If ignored, the long-run welfare cost of revenue-neutral tax increase policies could be significantly underestimated. To strengthen the results obtained so far, a robustness check in next subsection will consider tax reform scenarios that depart from the revenue neutrality restriction.

9.2.4 Non-revenue-neutral tax rate increases

In contrast to revenue-neutral policy experiments, this section quantifies the welfare effect of a contractionary fiscal regime, when the increases in one tax rate are not offset by a change in another tax rate. The exogenously-specified common objective in both models is to increase total tax revenue by 10 % and 5 %, respectively, by allowing a single tax rate to vary, while keeping all other parameters fixed at their initial steady-state values.

Table 8: Welfare effect of 10%-tax-revenue increase

Model	τ^k	τ^l	τ^c
Union	N/A	0.6405 $\uparrow$(+23.20%)	0.5033 $\uparrow$(+35.52%)
ζ	N/A	-0.3432	-0.2406
Finn	N/A	0.6032 $\uparrow$(+19.47%)	0.4990 $\uparrow$(+35.09%)
ζ	N/A	-0.2332	-0.1787

As in the revenue neutral experiments, there is no feasible capital tax rate, which can satisfy the objective, a result which follows directly from the flatter Laffer curve with respect to τ^k. Next, if labor taxes are the instrument used to achieve the targeted revenue revenue, the required increase in τ^l in Finn, is almost 4.3% smaller compared to the union model. This outcome is due to the exogenously-fixed public sector hours in Finn: the distortions caused by an increase in τ^l, and thus in the effective labor wedge, which appears in the MRS condition are smaller. In addition, in both models, the new level of τ^l places the German economy on the downward-sloping segment of labor tax Laffer curve. [31]

As shown in Table 8, across both models, changing τ^c is the cheaper option to raise additional tax revenue, measured in terms of the welfare cost incurred. Additionally, the required change in consumption tax rates to achieve 10 % increase in total revenue, is approximately 9 % larger in the union model. The two models produce significant differences in terms of the magnitude of the tax rate changes required to achieve a pre-specified tax revenue increase. When public hours are considered to be endogenously-determined in the model, the tax rates increase by a significantly larger amount. This is a new result in the literature, with important policy implications.

For the 5% tax-revenue-increase objective scenario, the results reported in Table 9 below are qualitatively very similar to the outcomes in the 10% revenue increase scenario. Again, there is no feasible capital income tax rate to achieve the new objective. However, when τ^l is the instrument used to increase total tax revenue, the two models generate different

[31] The computed revenue-maximizing τ^l is 50% in the union model, and 55% in Finn.

Table 9: Welfare effect of 5%-tax-revenue increase

Model	τ^k	τ^l	τ^c
Union	N/A	0.6065 $\uparrow$(+19.80%)	0.5033 $\uparrow$(+35.52%)
ζ	N/A	-0.2988	-0.2406
Finn	N/A	0.5728 $\uparrow$(+16.43%)	0.4300 $\uparrow$(+28.19%)
ζ	N/A	-0.2026	-0.1600

predictions: after the resulting increase in labor tax rate, the German economy is again situated on the slippery slope of the respective Laffer curve in the union model, while Finn places the economy on its upward segment. In terms of welfare loss, consumption tax rate is again the preferred instrument to achieve the 5% total tax revenue increase. Finally, the new tax rate levels, as well as the welfare costs are higher in the union model, due to the additional allocative distortion caused by the endogenous adjustment of public hours.

10 Summary and Conclusions

Motivated by the highly-unionized public sectors, the high public shares in total employment, and public sector wage premia observed in most post-WWII European economies, this paper examined the role of public sector unions in a DSGE framework. A strong union, operating in a largely non-market sector was shown to be relevant for business cycle fluctuations, and when evaluating the welfare effects of fiscal policy. Following Fernandez-de-Cordoba *et al.* (2009), an optimizing public sector union was incorporated in a real business cycle model with valuable government consumption and productive public investment. The RBC model generated cyclical behavior in hours and wages that is consistent with data behavior in an economy with highly-unionized public sector, Germany during 1970-2007 period. The main findings are: (i) the model with collective bargaining performs reasonably well *vis-a-vis* data; (ii) overall, the model with collective bargaining in the public sector is an improvement over a similar model with exogenous public employment, namely Finn (1998); (iii) endogenously-determined public wage and hours add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. In addition, the endogeneity of public hours in union model generates larger changes in tax rates to achieve a pre-specified increase in tax revenue and produce significantly higher welfare losses, as compared to Finn's model with exogenous public sector hours. Thus, endogenous public hours are quantitatively important model ingredient when evaluating fiscal policy. In particular, ignoring the positive co-movement between public and private wage and hours leads to a significant underestimation of the welfare effect of tax regime changes.

There are some limitations of the model setup: the dynamics of public hours and wage in the model is identical, which constraints the ability of the model to match well the behavior in the two variables simultaneously. Furthermore, the theoretical framework ignores household's increased demand for labor-intensive programs such as healthcare, education and social security, which would require additional employment in the public sector. More realistically, public sector unions and government usually bargain over nominal wage increases, and against redundancies. They do not negotiate hours and the level of the real wage directly. Before engaging in negotiations, unions also take into consideration many macroeconomic indicators. Labor productivity in the private sector and the private wage, are often used as a leverage in the negotiations over the public wage. The simple union objective used in this paper ignores other possible demands by unions, such as job security, work conditions, government pensions, other non-monetary benefits, etc. Indeed, some of those factors can be incorporated in the union utility function and thus extend the basic model. Nevertheless, the importance of public sector unions is evident even from the reduced-form representation used in this paper. In addition, this paper suggests that the model with public sector unions could produce potentially useful insights regarding optimal taxation. The potentially interesting issue of public sector union power in the context of a Ramsey problem of setting tax rates in an optimal way is left for future research.

Given the overall reasonable performance of the model with public sector union, the organizational structure of public sector labor market deserves further and deeper investigation as well. Von Mises (1944), Parkinson (1957) and Tullock (1974) suggest that bureaucracy itself has been one of the most important factors affecting economic activity, mainly through the development and implementations of different legislative procedures, rules and regulations. In particular, civil servants are usually insulated from market forces in both the input and output markets: many government positions do not have a close equivalent in the private sector, and there is no direct way to measure performance. A closer analysis of bureaucrats' behavior, and their effect on aggregate fluctuations in European economies would be a logical extension to the work in this paper. This also complements earlier research on rent-seeking and the quality of institutions, as in Angelopoulos *et al.* (2009, 2012).

References

[1] Akerlof, G. (1969). "Relative wages and the rate of inflation," *Quarterly Journal of Economics*, vol. 83, pp. 353-74.

[2] Alogoskoufis, G. and Manning A. (1988) "On the Persistence of Unemployment," *Economic Policy* 7: 427-469.

[3] Ardagna, Silvia. (2007). "Fiscal policy in unionized labor markets," *Journal of Economic Dynamics and Control*, 31, pp. 1498-1534.

[4] Ashenfelter, O.C. (1978) "Union relative wage effects: new evidence and a survey of their implications of wage inflation." In: *Economic Contributions to Public Policy*, eds. R. Stone and W. Peterson. London: Macmillan.

[5] Atherton, W. (1973). *The Theory of Union Bargaining Goals*. Princeton: Princeton University Press.

[6] Bain, G.S. and Elsheikh, F. (1976) *Union Growth and the Business Cycle: An Econometric Analysis*. Basil Blackwell: Oxford, UK.

[7] Baxter, M. and King, R. G. (1993) "Fiscal policy in general equilibrium," *American Economic Review* 83, pp. 315-334.

[8] Brown, J.N., and O. Ashenfelter. (1986). "Testing the Efficiency of Employment Contracts," *Journal of Political Economy*, 94(3), S40-S87.

[9] Bureau of Economic Analysis (2011), www.bea.gov.

[10] Bureau of Labor Statistics (2011). *Union Members Summary*. Available on-line at: www.bls.gov

[11] Calmfors, L. (1981). "Employment and resource allocation in a small open economy with a trade union determined wage." Mimeo, Institute for International Economic Studies, Stockholm.

[12] Canova, Fabio. (2007) *Methods for Applied Macroeconomic Research*. Princeton University Press: Princeton, NJ.

[13] Cartter, A. M. (1959). *Theory of Wages and Employment*. Homewood, Illinois: Irwin.

[14] Carruth, A.A. and Oswald, A.J. (1981). "The determination of union and non-union wage rates." *European Economic Review*, vol. 30, pp. 285-302.

[15] Corden, W.M. (1981). "Taxation, real wage rigidity and employment," *Economic Journal*, vol. 91, pp. 309-30.

[16] Cavallo, M. (2005) "Government Employment Expenditure and the Effects of Fiscal Policy Shocks", Working Paper No. 2005-16, *Federal Reserve Bank of San Francisco.*

[17] Christiano, Lawrence and M. Eichenbaum (1992) "Current real business cycle theories and aggregate labor market fluctuations," *American Economic Review* 82, 430-450.

[18] Cooley, T. and Prescott, E.C. (1995) "Economic Growth and Business Cycles." In: T. Cooley (ed.), *Frontiers of Business Cycle Research*, Princeton University Press: Princeton, NJ.

[19] Danthine, Jean-Pierre and John B. Donaldson (1995). "Non-Walrasian Economies." In: *Frontiers in Business Cycle Research*,ed. Thomas F. Cooley. Princeton University Press: Princeton, NJ.

[20] Demekas, Dimitri and Zenon G. Kontolemis (2000). "Government Employment and Wages and Labour Market Performance," *Oxford Bulletin of Economics and Statistics*, 62(3), 391-415.

[21] Dertouzos, James N., and John Pencavel. (1981) "Wage and Employment Determination under Trade Unionism: The International Typographical Union," *Journal of Political Economy*, 89(6):1162-81.

[22] Doiron, Denise J. (1992) "Bargaining Power and Wage-Employment Contracts in a Unionized Industry," *International Economic Review*, Vol.33, No.3, pp.583-606.

[23] Ebbingshaus, B., and Wissler, J. (2000) "Trade Unions in Western Europe since 1945," Basingstoke: Palgrave Macmillan.

[24] *The Economist* (2011) "(Government) workers of the world unite!". January 6.

[25] Ehrenberg, R.G. and J. Schwartz. (1986) "Public Sector Labor Markets." In: *Handbook of Labor Economics*, Vol. 2, eds.O. Ashenfelter and R. Layard. North-Holland: Oxford, UK.

[26] *EU Klems Database* (2009). Available on-line at: www.euklems.net

[27] Fellner, W. (1949). *Competition Among the Few*. New York: A.A. Knopf.

[28] Fernandez-de-Cordoba, G., Perez, J.J., and Torres, J.L. (2009) "Public and private sector wages interactions in a general equilibrium model", *ECB WP* 1099 (October).

[29] Fernandez-de-Cordoba, G., Perez, J.J., and Torres, J.L. (2012) "Public and private sector wages interactions in a general equilibrium model", *Public Choice*, Published online: 29 September 2010, DOI 10.1007/s11127-010-9705-7

[30] Finn, Mary G. (1998). "Cyclical Effects of Government's Employment and Goods Purchases." *International Economic Review*, Vol. 39, No. 3 (Aug.), pp. 635-657.

[31] Forni, Lorenzo, and Raffaela Giordano (2003). "Employment in the public sector," *CES Ifo Working Paper*, No. 1085.

[32] Gregory, A. W. and Smith, G. W. (1991). "Calibration as testing: Inference in a simulated macroeconomic models," *Journal of Business and Economic Statistics* 9(3), pp. 297-303.

[33] Hansen, Gary D. and Randall Wright. (1992) "The labor market in real business cycle theory," *Federal Reserve Bank of Minneapolis Quarterly Review*, Spring 1992, pp.2-12.

[34] Giavazzi, F. and Pagano, M. (1990). "Can Severe Fiscal Contractions be Expansionary? Tales of Two Small European Countries," in Blanchard, O. and Fischer, S. (eds.), *NBER Macroeconomics Annual*, MIT Press.

[35] Gomes, P. (2009). "Fiscal policy and the labour market: the effects of public sector employment and wages," Available at http://personal.lse.ac.uk/gomesp/1-Academia/General.htm.

[36] Hersoug, T. (1978). "Effects of expansionary policies in unionized economy," University of Oslo. Mimeo.

[37] Johnson, G.E. (1975). "Economic analysis of trade unionism," *American Economic Review*, Papers and Proceeedings, vol. 65, pp. 23-9.

[38] King, Robert G., Plosser, Charles I. and Sergio T. Rebello. (1988)"Production, growth, and business cycles I: The basic neoclassical model," *Journal of Monetary Economics*, 21(2), pp. 195-232.

[39] King R. and S. Rebelo (1999). "Resuscitating real business cycles." In: *Handbook of Macroeconomics*, vol. 1B, eds J. Taylor and M. Woodford, North Holland.

[40] Klein, Paul. (2000) "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control* 24, pp. 1405-1423.

[41] Klein, Paul. (2000) MATLAB Code for "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control* 24, pp. 1405-1423.

[42] Kydland, F. (1995) "Business cycles and aggregate labor market fluctuations." In: T. Cooley (ed.), *Frontiers of Business Cycle Research*, Princeton University Press: Princeton, NJ.

[43] Kydland, Finn E. and Prescott, E.C. (1982). "Time to build and aggregate fluctuations," *Econometrica* 50(6), Nov 1982, pp. 1345-1369.

[44] Kydland, Finn E. and Prescott, E.C (1990). "Business Cycles: Real facts and monetary myth," *Federal Reserve Bank of Minneapolis Quarterly Review* 14(2), pp.3-18.

[45] Laffer, A. (1981) "Government Exactions and Revenue Deficiencies," *Cato Journal*, Vol.1, No.1.

[46] Lane, P. (2003) "The Cyclical Behavior of Fiscal Policy: Evidence from the OECD", *Journal of Public Economics* 87, 26612675.

[47] Lamo, A., J. Perez and L. Schuknecht (2007). "The cyclicality of consumption, wages and employment in the public sector in the Euro Area," *ECB Working paper Series* 757, May.

[48] Lamo, A., J. Perez and L. Schuknecht (2008). "Public and Private Sector Wages: Comovement and Causality," *ECB WDN Working Paper Series* 963, November.

[49] Linnemann, Ludger (2009) "Macroeconomic Effects of Shocks to Public Employment," *Journal of Macroeconomics* 31(2), 252-267.

[50] Lucas, Robert (1987). *Models of Business Cycles*. Basil Blackwell Inc: NY, USA.

[51] McGrattan, Ellen R., 1994. "The macroeconomic effects of distortionary taxation," *Journal of Monetary Economics*, Elsevier, vol. 33(3), pp. 573-601.

[52] McDaniel, Cara (2007) "Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003." Available on-line at: http://www.caramcdaniel.com/researchpapers.

[53] McDaniel, Cara (2009) *Updated tax series OECD 1950-2009.* Available on-line at at: http://www.caramcdaniel.com/researchpapers.

[54] Mendoza, E.G., Razin A., and Tesar, L. (1994) "Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption." *Journal of Monetary Economics* 34 (December): 297-323.

[55] Mulvey, C. (1978). *The Economic Analysis of Trade Unions.* Oxford: Martin Robertson.

[56] OECD Statistical Database (2011). Available on-line at: stats.oecd.org.

[57] Oswald, Andrew J., Paul A. Grout and David T. Ulph (1984). "Uncertainty, unions and the theory of public sector labor markets," *Working paper* No.176, Industrial Relations Section, Princeton University.

[58] Oswald, A. J. (1979). "Wage determination in an economy with many unions," *Oxford Economic Papers*, vol. 31, pp. 369-85.

[59] Pencavel, J. H. (1977). "The distributional and efficiency effects of trade unions in Britain," *British Journal of Industrial Relations*, vol. 15, pp. 137-56.

[60] Parkinson, C. Northcote (1957). *Parkinson's Law, or the Pursuit of Progress.* John Murray: London, Great Britain.

[61] Prescott, E. (2002). "Prosperity and depression," *American Economic Review* 92, 1-15.

[62] Prescott, E. (2004). "Why do Americans work so much more than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review*

[63] Rodrick, D. (2000). "What drives Public Employment in Developing Countries?", *Review of Development Economics*, 4(3), 229-243.

[64] Rees, Albert (1977). *The Economics of Trade Unions*. The University of Chicago Press: Chicago, USA.

[65] Rose, R. (1985). *Public Employment in Western Nations*. Cambridge University Press: Cambridge, UK.

[66] Schmitt-Grohe, S. and Uribe, M. (1997). "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability," *The Journal of Political Economy*, Vol. 105, No.5, pp. 976-1000.

[67] Trabandt, M. (2006) "How far are we from the slippery slope? The Laffer curve revisited," in: *Essays in Macroeconomics*, PhD Dissertation, Humboldt University, Berlin.

[68] Tullock, G. (1974) "Dynamic Hypothesis on Bureaucracy," *Public Choice* 19(1), pp. 127-131.

[69] Visser, J. (2003). "Unions and Unionism around the world," in J. Addison and C. Schnabel (eds), *The International handbook of Trade Unions*, pp. 366-413. Edward Elgar: Cheltenham, UK.

[70] Visser, J. (2006). "Union membership statistics in 24 countries," *Monthly Labor Review*, Vol. 129, No. 1.

[71] von Mises, L. (1944). *Bureaucracy*. Yale University Press: New Haven, USA.

[72] Warren-Boulton, F. R. (1977). "Vertical control by labour unions," *American Economic Review*, vol. 67, pp. 309-22.

[73] WDN (2010). "Wage Dynamics in Europe: Final Report of the Wage Dynamics Network (WDN)," *European Central Bank: Frankfurt, Germany.*

[74] *World Development Indicators* (2011) Available on-line at: www.wdi.org

11 Technical Appendix

11.1 Optimality conditions

11.1.1 Firm's problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} - r_t \quad = \quad 0 \tag{11.1.1}$$

where $(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu}$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, i.e the interest rate.

$$r_t \quad = \quad (1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} \tag{11.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} = r_t K_t^p \quad \text{or} \quad (1-\theta)Y_t = r_t K_t^p \tag{11.1.3}$$

because

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} = A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} \quad = \quad (1-\theta)Y_t$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1}(K_t^g)^{\nu} - w_t^p = 0 \text{ or } w_t^p = \theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1}(K_t^g)^{\nu} \tag{11.1.4}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, i.e the hourly wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1}(K_t^g)^{\nu} = w_t^p N_t^p \quad \text{or} \quad \theta Y_t = w_t^p N_t^p \tag{11.1.5}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \ = \ Y_t - r_t K_t^p - w_t^p N_t^p = Y_t - (1-\theta)Y_t - \theta Y_t = 0 \tag{11.1.6}$$

Indeed, in equilibrium, economic profits are zero.

11.1.2 Consumer problem

Set up the Lagrangian

$$\begin{aligned}\mathcal{L}(C_t, K_{t+1}^p, N_t^p; \Lambda_t) = E_0 \sum_{t=0}^{\infty} \Bigg\{ & \frac{\left[(C_t + \omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{1-\alpha} - 1}{1-\alpha} + \\ & +\Lambda_t \Bigg[(1-\tau^l)(w_t^p N_t^p + w_t^g N_t^g) + (1-\tau^k) r_t K_t^p + \\ & +\tau^k \delta^p K_t^p - (1+\tau^c) C_t - K_{t+1}^p + (1-\delta) K_t^p \Bigg]\Bigg\}\end{aligned} \tag{11.1.7}$$

This is a concave programming problem, so the FOCs, together with the additional, boundary ("transversality") conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\begin{aligned}&\beta^t \Bigg\{ \frac{1-\alpha}{1-\alpha} \left[(C_t + \omega G_t^c)^{\psi}(1-N_t^h)^{(1-\psi)}\right]^{-\alpha} \times \\ &\psi (C_t + \omega G_t^c)^{\psi-1}(1-N_t^h)^{(1-\psi)} - \Lambda_t (1+\tau^c) \Bigg\} = 0\end{aligned} \tag{11.1.8}$$

Cancel the β^t and the $1-\alpha$ terms to obtain

$$\left[(C_t + \omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha} \psi (C_t + \omega G_t^c)^{\psi-1}(1-N_t)^{(1-\psi)} - \Lambda_t(1+\tau^c) = 0 \tag{11.1.9}$$

Move Λ_t to the right so that

$$\left[(C_t + \omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha} \psi (C_t + \omega G_t^c)^{\psi-1}(1-N_t)^{(1-\psi)} = \Lambda_t(1+\tau^c) \tag{11.1.10}$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K^p_{t+1} (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K^p_{t+1}} = 0$. That will result in the following expression

$$\beta^t \Bigg\{ -\Lambda_t + E_t \Lambda_{t+1} \Bigg[(1-\tau^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \Bigg] \Bigg\} \quad = \quad 0 \qquad (11.1.11)$$

Cancel the β^t term to obtain

$$-\Lambda_t + \beta E_t \Lambda_{t+1} \Bigg[(1-\tau^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \Bigg] = 0 \qquad (11.1.12)$$

Move Λ_t to the right so that

$$\beta E_t \Lambda_{t+1} \Bigg[(1-\tau^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \Bigg] = \Lambda_t \qquad (11.1.13)$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1} = (1-\theta) \frac{Y_{t+1}}{K^p_{t+1}}$$

$$\beta E_t \Lambda_{t+1} \Bigg[(1-\tau^k)(1-\theta) \frac{Y_{t+1}}{K^p_{t+1}} + \tau^k \delta^p + (1-\delta^p) \Bigg] = \Lambda_t \qquad (11.1.14)$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N^p_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^p_t} = 0$. That will result in the following expression

$$\begin{aligned} \beta^t \Bigg\{ \frac{1-\alpha}{1-\alpha} \Bigg[(C_t + \omega G^c_t)^{\psi} (1-N_t)^{(1-\psi)} \Bigg]^{-\alpha} \times \\ (1-\psi)(C_t + \omega G^c_t)^{\psi} (1-N_t)^{-\psi}(-1) + \Lambda_t (1-\tau^l) w^p_t \Bigg\} \quad &= \quad 0 \qquad (11.1.15) \end{aligned}$$

Cancel the β^t and the $1-\alpha$ terms to obtain

$$\Bigg[(C_t + \omega G^c_t)^{\psi} (1-N_t)^{(1-\psi)} \Bigg]^{-\alpha} (1-\psi) \Bigg[\frac{C_t + \omega G^c_t}{1-N_t} \Bigg]^{\psi} (-1) + \Lambda_t (1-\tau^l) w^p_t = 0 \quad (11.1.16)$$

Rearranging, one can obtain

$$\Bigg[(C_t + \omega G^c_t)^{\psi} (1-N_t)^{(1-\psi)} \Bigg]^{-\alpha} (1-\psi)(C_t + \omega G^c_t)^{\psi} (1-N_t)^{-\psi} = \Lambda_t (1-\tau^l) w^p_t \qquad (11.1.17)$$

Plug in the expression for w_t^h, that is,

$$w_t^p = \theta \frac{Y_t}{N_t^p} \tag{11.1.18}$$

into the equation above. Rearranging, one can obtain

$$\left[(C_t + \omega G_t^c)^{\psi}(1 - N_t)^{(1-\psi)}\right]^{-\alpha}(1 - \psi)(C_t + \omega G_t^c)^{\psi}(1 - N_t)^{-\psi} = \Lambda_t(1 - \tau^l)\theta \frac{Y_t}{N_t^p} \tag{11.1.19}$$

Transversality conditions need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^p \quad = \quad 0 \tag{11.1.20}$$

11.1.3 The Objective Function of a Public Sector Union: Derivation

This subsection shows that the objective function in the government sector is a generalized version of Stone-Geary monopoly union utility function used in Dertouzos and Pencavel (1981) and Brown and Ashenfelter (1986). The utility function is

$$V(w^g, N^g) = (w^g - \bar{w}^g)^{\phi}(N^g - \bar{N}^g)^{(1-\phi)}, \tag{11.1.21}$$

where ϕ and $1 - \phi$ are the weights attached to public wage and hours, respectively, and $\bar{w}^g$ and $\bar{N}^g$ denote subsistence wage rate and hours. Since there is no minimum wage in the model, $\bar{w}^g = 0$. Additionally, as public hours are assumed to be unproductive, it follows that $\bar{N}^g = 0$ as well. Therefore, the utility function simplifies to

$$V(w^g, N^g) = (w^g)^{\phi}(N^g)^{(1-\phi)}. \tag{11.1.22}$$

Doiron (1992) uses a generalized representation, which encompasses (2) as a special case when $\rho \to 0$.

$$\left[\phi(N^g)^{-\rho} + (1 - \phi)(w^g - \bar{w})^{-\rho}\right]^{-1/\rho}, \tag{11.1.23}$$

when $\bar{w} = 0$, the function simplifies to

$$\left[\phi(N^g)^{-\rho} + (1 - \phi)(w^g)^{-\rho}\right]^{-1/\rho}, \tag{11.1.24}$$

Union objective function used in the paper is very similar to Doiron's (1992) simplified version:

$$\left[(N^g)^{\rho} + \eta(w^g)^{\rho}\right]^{1/\rho}, \tag{11.1.25}$$

can be transformed to

$$\left[(N^g)^\rho + \frac{\phi}{(1-\phi)}(w^g)^\rho\right]^{1/\rho}, \tag{11.1.26}$$

Collecting terms under common denominator

$$\left[\frac{(1-\phi)}{(1-\phi)}(N^g)^\rho + \frac{\phi}{(1-\phi)}(w^g)^\rho\right]^{1/\rho}, \tag{11.1.27}$$

Factoring out the common term

$$\left[\frac{1}{1-\phi}\right]^{1/\rho}\left[(1-\phi)(N^g)^\rho + \phi(w^g)^\rho\right]^{1/\rho}, \tag{11.1.28}$$

Note that the constant term $\left[\frac{1}{1-\phi}\right]^{1/\rho} > 0$ can be ignored, as utility functions are invariant to positive affine transformations. After rearranging terms, the equivalent function

$$\tilde{V} = \left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]^{1/\rho}. \tag{11.1.29}$$

Take natural logarithms from both sides to obtain

$$\ln \tilde{V} = \frac{1}{\rho}\ln\left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]. \tag{11.1.30}$$

Take the limit $\rho \to 0$

$$\lim_{\rho\to 0}\ln\tilde{V} = \lim_{\rho\to 0}\frac{\ln\left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]}{\rho} \tag{11.1.31}$$

Apply L'Hopital's Rule on the R.H.S. to obtain

$$\lim_{\rho\to 0}\ln\tilde{V} = \lim_{\rho\to 0}\frac{\frac{\partial}{\partial\rho}\ln\left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]}{\frac{\partial\rho}{\partial\rho}} \tag{11.1.32}$$

Thus

$$\ln\tilde{V} = \lim_{\rho\to 0}\frac{\left[\phi(w_t^g)^\rho \ln w^g + (1-\phi)(N^g)^\rho \ln N^g\right] / \left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]}{1} \tag{11.1.33}$$

Simplify to obtain

$$\ln\tilde{V} = \frac{\lim_{\rho\to 0}\left[\phi(w_t^g)^\rho \ln w^g + (1-\phi)(N^g)^\rho \ln N^g\right]}{\lim_{\rho\to 0}\left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]} = \frac{\phi\ln w^g + (1-\phi)\ln N^g}{\phi + (1-\phi)} \tag{11.1.34}$$

Therefore,

$$\ln \tilde{V} = \phi \ln w^g + (1-\phi) \ln N^g. \tag{11.1.35}$$

Exponentiate both sides of the equation to obtain

$$e^{\ln \tilde{V}} = e^{\phi \ln w^g + (1-\phi) \ln N^g}. \tag{11.1.36}$$

Thus

$$\tilde{V} = e^{\ln (w^g)^{\phi} + \ln (N^g)^{(1-\phi)}}. \tag{11.1.37}$$

or

$$\tilde{V} = e^{\ln (w^g)^{\phi} (N^g)^{(1-\phi)}}. \tag{11.1.38}$$

Finally,

$$\tilde{V} = (w^g)^{\phi} (N^g)^{(1-\phi)} \tag{11.1.39}$$

Furthermore, government period budget constraint serves the role of a labor demand function. Additionally, the public sector demand curve will be subject to shock, resulting from innovations to the fiscal shares. The balanced budget assumption is thus important in the model setup. Since wage bill is a residual, if wage rate is increased, then hours need to be decreased. Additionally, government period budget constraint can be expressed in the form $N^g = N^g(w^g)$ as

$$N^g = \frac{\tau^l w^p N^p + \tau^k (r - \delta^p) K^p + \tau^c C - G^c - G^i - G^t}{(1-\tau^l) w^g} \tag{11.1.40}$$

Therefore, the problem in the government sector is reshaped in the standard formulation in the union literature:

$$\max_{w^g, N^g} V(w^g, N^g) \quad \text{s.t.} \quad N^g = N^g(w^g) \tag{11.1.41}$$

Since union optimizes over both the public wage and hours, the outcome is efficient. The solution pair is on the contract curve (obtained from FOCs), at the intersection point with the labor demand curve (government budget constraint).

11.1.4 Public sector union optimization problem

The union solves the following problem:

$$\max_{w_t^g, N_t^g} \Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{1/\rho} \tag{11.1.42}$$

s.t

$$G_t^c + G_t^t + G_t^i + w_t^g N_t^g = \tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l [w_t^p N_t^p + w_t^g N_t^g] \tag{11.1.43}$$

Setup the Lagrangian

$$\mathcal{V}(w_t^g, N_t^g; \nu_t) = \max_{w_t^g, N_t^g} \Bigg\{ \Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{1/\rho} \tag{11.1.44}$$
$$-\nu_t \Bigg[G_t^c + G_t^t + G_t^i + w_t^g N_t^g - \tau^c C_t - \tau^k r_t K_t^p + \tau^k \delta^p K_t - \tau^l [w_t^p N_t^p + w_t^g N_t^g] \Bigg] \Bigg\}$$

Optimal public employment is obtained, when the derivative of the government Lagrangian is et to zero, i.e $\mathcal{V}_{N_t^g} = 0$

$$(1/\rho) \Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} \rho (N_t^g)^{\rho-1} - (1-\tau^l)\nu_t w_t^g = 0 \tag{11.1.45}$$

or, when ρ is canceled out and $(1-\tau^l)\nu_t w_t^g$ put to the right

$$\Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} (N_t^g)^{\rho-1} = (1-\tau^l)\nu_t w_t^g \tag{11.1.46}$$

Optimal public wage is obtained, when the derivative of the government Lagrandean is et to zero, i.e $\mathcal{V}_{w_t^g} = 0$

$$(1/\rho) \Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} \eta \rho (w_t^g)^{\rho-1} - (1-\tau^l)\nu_t N_t^g = 0 \tag{11.1.47}$$

or, when ρ is canceled out and$(1-\tau^l)\nu_t N_t^g$ term put to the right

$$\Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} \eta (w_t^g)^{\rho-1} = (1-\tau^l)\nu_t N_t^g \tag{11.1.48}$$

Divide (11.1.46) and (11.1.48) side by side to obtain

$$\frac{\Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} (N_t^g)^{\rho-1}}{\Bigg[(N_t^g)^\rho + \eta (w_t^g)^\rho \Bigg]^{(1/\rho)-1} \eta (w_t^g)^{\rho-1}} = \frac{(1-\tau^l)\nu_t w_t^g}{(1-\tau^l)\nu_t N_t^g} \tag{11.1.49}$$

Cancel out the common terms

$$\frac{(N_t^g)^{\rho-1}}{\eta(w_t^g)^{\rho-1}} = \frac{w_t^g}{N_t^g} \tag{11.1.50}$$

Now cross-multiply to obtain

$$\frac{(N_t^g)^{\rho}}{\eta} = (w_t^g)^{\rho} \tag{11.1.51}$$

Hence

$$w_t^g = \left(\frac{1}{\eta}\right)^{1/\rho} N_t^g \tag{11.1.52}$$

The wage bill expression, which is obtained after simple rearrangement of the government budget constraint, is as follows

$$w_t^g N_t^g = \frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^t - G_t^i}{1-\tau^l} \tag{11.1.53}$$

Use the wage bill equation and the relationship between public wage and employment in order to obtain

$$w_t^g = \eta^{-\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^t - G_t^i}{1-\tau^l}\right]^{\frac{1}{2}} \tag{11.1.54}$$

and

$$N_t^g = \eta^{\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^t - G_t^i}{1-\tau^l}\right]^{\frac{1}{2}} \tag{11.1.55}$$

11.2 Log-linearized model equations

11.2.1 Linearized market clearing

$$c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t = y_t \quad (11.2.1)$$

Take logs from both sides to obtain

$$\ln[c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t] = \ln(y_t) \quad (11.2.2)$$

Totally differentiate with respect to time

$$\frac{d\ln[c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t]}{dt} = d\ln(y_t) \quad (11.2.3)$$

$$[\frac{1}{c+g^c+g^i+\delta^p k^p}][\frac{dc_t}{dt}\frac{c}{c} + \frac{dg^c_t}{dt}\frac{g}{g} + \frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p} - (1-\delta^p)\frac{dk^p_t}{dt}\frac{k^p}{k^p}] = \frac{dy_t}{dt}\frac{1}{y} \quad (11.2.4)$$

Define $\hat{z} = \frac{dz_t}{dt}\frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y}[\hat{c}_t c + \hat{g}^c_t g^c + \hat{g}^i_t g^i + \hat{k}^p_{t+1}k^p - (1-\delta^p)\hat{k}^p_t k^p] = \hat{y}_t \quad (11.2.5)$$

$$\hat{c}_t c + \hat{g}^c_t g^c + \hat{g}^i_t g^i + \hat{k}^p_{t+1}k^p - (1-\delta^p)\hat{k}^p_t k^p = y\hat{y}_t \quad (11.2.6)$$

$$k^p\hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t - g^c\hat{g}^c_t - g^i\hat{g}^i_t + (1-\delta^p)k^p\hat{k}^p_t \quad (11.2.7)$$

11.2.2 Linearized production function

$$y_t = a_t(k^p_t)^{1-\theta}(n^p_t)^\theta(k^g_t)^\nu \quad (11.2.8)$$

Take natural logs from both sides to obtain

$$\ln y_t = \ln a_t + (1-\theta)\ln k^p_t + \theta\ln n^p_t + \nu\ln k^g_t \quad (11.2.9)$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln y_t}{dt} = \frac{d\ln a_t}{dt} + (1-\theta)\frac{d\ln k^p_t}{dt} + \theta\frac{d\ln n^p_t}{dt} + \nu\frac{d\ln k^g_t}{dt} \quad (11.2.10)$$

$$\frac{1}{y}\frac{dy_t}{dt} = \frac{1}{a}\frac{da_t}{dt} + \frac{1-\theta}{k^p}\frac{dk^p_t}{dt} + \frac{\theta}{n^p}\frac{dn^p_t}{dt} + \frac{\nu}{k^g}\frac{dk^g_t}{dt} \quad (11.2.11)$$

Pass to log-deviations to obtain

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta\hat{n}^p_t + \nu\hat{k}^g_t \quad (11.2.12)$$

11.2.3 Linearized FOC consumption

$$[(c_t+\omega g_t^c)^{\psi}(1-n_t)^{(1-\psi)}]^{-\alpha}\psi(c_t+\omega g_t^c)^{\psi-1}(1-n_t)^{(1-\psi)} = (1+\tau^c)\lambda_t \quad (11.2.13)$$

Simplify to obtain

$$\psi(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = (1+\tau^c)\lambda_t \quad (11.2.14)$$

Take natural logs from both sides to obtain

$$\ln\psi(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = \ln(1+\tau^c)+\ln\lambda_t \quad (11.2.15)$$

$$\ln(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = \ln(1+\tau^c)+\ln\lambda_t \quad (11.2.16)$$

$$(\psi-1-\alpha\psi)\ln(c_t+\omega g_t^c)+(1-\alpha)(1-\psi)\ln(1-n_t) = \ln(1+\tau^c)+\ln\lambda_t \quad (11.2.17)$$

Totally differentiate with respect to time to obtain

$$\begin{aligned}(\psi-1-\alpha\psi)\frac{d\ln(c_t+\omega g_t^c)}{dt}+(1-\alpha)(1-\psi)\frac{d\ln(1-n_t)}{dt} &= \\ = \frac{d\ln(1+\tau^c)}{dt}+\frac{d\ln\lambda_t}{dt}& \quad (11.2.18)\end{aligned}$$

$$(\psi-1-\alpha\psi)\frac{1}{c+\omega g^c}\left(\frac{dc_t}{dt}+\omega\frac{dg_t^c}{dt}\right)+(1-\alpha)(1-\psi)\frac{-1}{1-n}\frac{dn_t}{dt} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \quad (11.2.19)$$

$$\begin{aligned}\frac{(\psi-1-\alpha\psi)}{c+\omega g^c}\frac{dc_t}{dt}\frac{c}{c}+\frac{\omega(\psi-1-\alpha\psi)}{c+\omega g^c}\frac{dg_t^c}{dt}\frac{g^c}{g^c}+& \\ -(1-\alpha)(1-\psi)\frac{1}{1-n}\frac{dn_t}{dt}\frac{n}{n}=\frac{d\lambda_t}{dt}\frac{1}{\lambda}& \quad (11.2.20)\end{aligned}$$

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{g}_t^c-(1-\alpha)(1-\psi)\frac{n}{1-n}\hat{n} = \hat{\lambda}_t \quad (11.2.21)$$

Since

$$\hat{n} = \frac{n^p}{n^p+n^g}\hat{n}^p+\frac{n^g}{n^p+n^g}\hat{n}^g=\frac{n^p}{n}\hat{n}^p+\frac{n^g}{n}\hat{n}^g, \quad (11.2.22)$$

and consumers choose n^p only, pass to log-deviations to obtain

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c^c+\omega g}\hat{g}_t^c-(1-\alpha)(1-\psi)\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p = \hat{\lambda}_t \quad (11.2.23)$$

Since $n=n^p+n^g$, it follows that

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{g}_t^c-(1-\alpha)(1-\psi)\frac{n^p}{1-n}\hat{n}^p = 0 \quad (11.2.24)$$

11.2.4 Linearized no-arbitrage condition for capital

$$\lambda_t = \beta E_t \lambda_{t+1}[(1-\tau^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p)] \tag{11.2.25}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t = \beta E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k \delta^p + 1 - \delta^p)] \tag{11.2.26}$$

Take natural logs from both sides of the equation to obtain

$$\ln \lambda_t = \ln E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k \delta^p + 1 - \delta^p)] \tag{11.2.27}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \lambda_t}{dt} = \frac{d \ln E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k \delta^p + 1 - \delta^p)]}{dt} \tag{11.2.28}$$

$$\begin{aligned} \frac{1}{\lambda}\frac{d\lambda_t}{dt} = E_t \Bigg\{ & \frac{1}{\lambda((1-\tau^k)(1-\theta)\frac{y}{k^p} + 1 - \delta^p + \tau^k \delta^p} \times \\ & \Bigg[((1-\tau^k)(1-\theta)\frac{y}{k^p} + \tau^k \delta^p + 1 - \delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda} \\ & + \frac{\lambda(1-\tau^k)(1-\theta)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y} - \left[\frac{\lambda(1-\tau^k)(1-\theta)y}{(k^p)^2}\right]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\Bigg]\Bigg\} \end{aligned} \tag{11.2.29}$$

Pass to log-deviations to obtain

$$\begin{aligned} \hat{\lambda}_t = E_t \Bigg\{ & \hat{\lambda}_{t+1} + \Bigg[\frac{(1-\tau^k)(1-\theta)y}{((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k\delta^p + 1 - \delta^p)k^p}\hat{y}_{t+1} \\ & - \frac{(1-\tau^k)(1-\theta)y}{((1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k\delta^p + 1 - \delta^p)k^p}\hat{k}^p_{t+1}\Bigg]\Bigg\} \end{aligned} \tag{11.2.30}$$

Observe that

$$(1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k \delta^p + 1 - \delta^p = 1/\beta \tag{11.2.31}$$

Plug it into the equation to obtain

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{k}^p_{t+1}\right] \tag{11.2.32}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{11.2.33}$$

11.2.5 Linearized MRS

$$(1-\psi)(c_t+\omega g_t^c) \quad = \quad \psi(1-n_t)\frac{(1-\tau^l)}{(1+\tau^c)}\theta\frac{y_t}{n_t^p} \tag{11.2.34}$$

Take natural logs from both sides of the equation to obtain

$$\ln(1-\psi)(c_t+\omega g_t^c) \quad = \quad \ln\psi(1-n_t)\frac{(1-\tau^l)}{(1+\tau^c)}\theta\frac{y_t}{n_t^p} \tag{11.2.35}$$

$$\ln(c_t+\omega g_t^c) \quad = \quad \ln(1-n_t)+\ln y_t-\ln n_t^p \tag{11.2.36}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln(c_t+\omega g_t^c)}{dt} \quad = \quad \frac{d\ln(1-n_t)}{dt}+\frac{d\ln y_t}{dt}-\frac{d\ln n_t^p}{dt} \tag{11.2.37}$$

$$\frac{1}{c+\omega g^c}\left(\frac{dc_t}{dt}+\omega\frac{dg_t^c}{dt}\right) \quad = \quad -\frac{1}{1-n}\frac{dn_t}{dt}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{11.2.38}$$

$$\frac{1}{c+\omega g^c}\frac{dc_t}{dt}\frac{c}{c}+\frac{\omega}{c+\omega g^c}\frac{dg_t^c}{dt}\frac{g^c}{g^c} \quad = \quad -\frac{1}{1-n}\frac{dn_t}{dt}\frac{n}{n}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{11.2.39}$$

$$\frac{c}{c+\omega g^c}\frac{dc_t}{dt}\frac{1}{c}+\frac{\omega g^c}{c+\omega g^c}\frac{dg_t^c}{dt}\frac{1}{g^c} \quad = \quad -\frac{n}{1-n}\frac{dn_t}{dt}\frac{1}{n}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{11.2.40}$$

Pass to log-deviations to obtain

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g}\hat{g}_t^c \quad = \quad -\frac{n}{1-n}\hat{n}+\hat{y}_t-\hat{n}_t^p \tag{11.2.41}$$

Since

$$\hat{n}=\frac{n^p}{n^p+n^g}\hat{n}^p+\frac{n^g}{n^p+n^g}\hat{n}^g, \tag{11.2.42}$$

and noting that consumers are only choosing n^p, then

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g^c}\hat{g}_t^c \quad = \quad -\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p+\hat{y}_t-\hat{n}_t^p \tag{11.2.43}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g^c}\hat{g}_t^c \quad = \quad -\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p+\hat{y}_t-\hat{n}_t^p \tag{11.2.44}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g^c}\hat{g}_t^c \quad = \quad -\left(1+\frac{n}{1-n}\frac{n^p}{n^p+n^g}\right)\hat{n}^p+\hat{y}_t \tag{11.2.45}$$

Since $n=n^p+n^g$, it follows that

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g^c}\hat{g}_t^c \quad = \quad -\left(1+\frac{n^p}{1-n}\right)\hat{n}^p+\hat{y}_t \tag{11.2.46}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c}{c+\omega g^c}\hat{g}_t^c+\left(1+\frac{n^p}{1-n}\right)\hat{n}^p-\hat{y}_t \quad = \quad 0 \tag{11.2.47}$$

11.2.6 Linearized private capital accumulation

$$k_{t+1}^p \quad = \quad i_t + (1-\delta^p)k_t^p \tag{11.2.48}$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^p \quad = \quad \ln(i_t + (1-\delta^p)k_t^p) \tag{11.2.49}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}^p}{dt} \quad = \quad \frac{1}{i+(1-\delta^p)k^p}\frac{d(i_t+(1-\delta^p)k_t^p)}{dt} \tag{11.2.50}$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that} \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \text{ Then} \tag{11.2.51}$$

$$\frac{dk_{t+1}^p}{dt}\frac{1}{k^p} \quad = \quad \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i+(1-\delta^p)k_t^p}\frac{dk_t^p}{dt}\frac{k^p}{k^p} \tag{11.2.52}$$

Pass to log-deviations to obtain

$$\hat{k}_{t+1}^p \quad = \quad \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}_t^p \tag{11.2.53}$$

$$\hat{k}_{t+1}^p \quad = \quad \delta^p\hat{i}_t + (1-\delta^p)\hat{k}_t^p \tag{11.2.54}$$

11.2.7 Linearized government capital accumulation

$$k_{t+1}^g \quad = \quad g_t^i + (1-\delta^g)k_t^g \tag{11.2.55}$$

Take natural logs from both sides to obtain

$$\ln k_{t+1}^g \quad = \quad \ln(g_t^i + (1-\delta^g)k_t^g) \tag{11.2.56}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}^g}{dt} \quad = \quad \frac{1}{g^i+(1-\delta^g)k^g}\frac{d(g_t^i+(1-\delta^g)k_t^g)}{dt} \tag{11.2.57}$$

Observe that since

$$g^i = \delta^g k^g, \tag{11.2.58}$$

it follows that

$$g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \tag{11.2.59}$$

Hence,

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{x+(1-\delta^g)}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{11.2.60}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^g)k^g}{k^g}\hat{k}^g_t \tag{11.2.61}$$

Cancel out the k^g terms to obtain

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{11.2.62}$$

11.2.8 Public wage rate rule

$$w^g_t = \eta^{-\frac{1}{2\rho}}\left[\frac{\tau^c c_t + \tau^k r_t k^p_t - \tau^k \delta^p k^p_t + \tau^l w^p_t n^p_t - g^c_t - g^t_t - g^i_t}{1-\tau^l}\right]^{\frac{1}{2}} \tag{11.2.63}$$

Take logs from both sides to obtain

$$\begin{aligned}\ln w^g_t = -\frac{1}{2\rho}\ln\eta - \frac{1}{2}\ln(1-\tau^l) + \\ \frac{1}{2}\ln\left\{\tau^c c_t + \tau^k r_t k^p_t - \tau^k \delta^p k^p_t + \tau^l w^p_t n^p_t - g^c_t - g^t_t - g^i_t\right\}\end{aligned} \tag{11.2.64}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln w^g_t}{dt} = \frac{1}{2}\frac{d}{dt}\ln\left\{\tau^c c_t + \tau^k r_t k^p_t - \tau^k \delta^p k^p_t + \tau^l w^p_t n^p_t - g^c_t - g^t_t - g^i_t\right\} \tag{11.2.65}$$

Observe that

$$\begin{aligned}\tau^k r_t k^p_t - \tau^k \delta^p k_t + \tau^l w^p_t n^p_t = \tau^k(1-\theta)y_t + \tau^l\theta y_t - \tau^k\delta^p k^p_t = \\ = \left[\tau^k(1-\theta) + \tau^l\theta\right]y_t - \tau^k\delta^p k^p_t\end{aligned} \tag{11.2.66}$$

Also

$$(1-\tau^l)w^g n^g = \tau^c c + [\tau^k(1-\theta)+\tau^l\theta]y - \tau^k\delta^p k^p - g^c - g^i - g^t_t \tag{11.2.67}$$

Thus

$$\frac{dw_t^g}{dt}\frac{1}{w^g} = \frac{1}{2}\frac{1}{(1-\tau^l)w^g n^g}\left\{\tau^c\frac{dc_t}{dt} + [\tau^k(1-\theta)+\tau^l\theta]\frac{dy_t}{dt} - \tau^k\delta^p\frac{dk_t^p}{dt} - \frac{dg_t^c}{dt} - \frac{dg_t^i}{dt} - \frac{dg_t^t}{dt}\right\} \tag{11.2.68}$$

$$\frac{dw_t^g}{dt}\frac{1}{w^g} = \frac{1}{2}\frac{1}{(1-\tau^l)w^g n^g}\times$$
$$\left\{\tau^c\frac{dc_t}{dt}\frac{c}{c} + \left[\tau^k(1-\theta)+\tau^l\theta\right]\frac{dy_t}{dt}\frac{y}{y} - \tau^k\delta^p\frac{dk_t^p}{dt}\frac{k^p}{k^p} - \frac{dg_t^c}{dt}\frac{g^c}{g^c} - \frac{dg_t^i}{dt}\frac{g^i}{g^i} - \frac{dg_t^t}{dt}\frac{g^t}{g^t}\right\} \tag{11.2.69}$$

$$\begin{aligned}\frac{dw_t^g}{dt}\frac{1}{w^g} = {} & \frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\frac{dc_t}{dt}\frac{1}{c} + \frac{(1/2)\left[\tau^k(1-\theta)+\tau^l\theta\right]y}{(1-\tau^l)w^g n^g}\frac{dy_t}{dt}\frac{1}{y}\\ & -\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\frac{dk_t^p}{dt}\frac{1}{k^p} - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\frac{dg_t^c}{dt}\frac{1}{g^c}\\ & -\frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\frac{dg_t^i}{dt}\frac{1}{g^i} - \frac{(1/2)g^t}{(1-\tau^l)w^g n^g}\frac{dg_t^t}{dt}\frac{1}{g^t}\end{aligned} \tag{11.2.70}$$

Pass to log-deviations to obtain

$$\begin{aligned}\hat{w}_t^g = {} & \frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\hat{c}_t + \frac{(1/2)\left[\tau^k(1-\theta)+\tau^l\theta\right]y}{(1-\tau^l)w^g n^g}\hat{y}_t\\ & -\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\hat{k}_t - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\hat{g}_t^c - \frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\hat{g}_t^i - \frac{(1/2)g^t}{(1-\tau^l)w^g n^g}\hat{g}_t^t\end{aligned} \tag{11.2.71}$$

11.2.9 Public hours/employment rule

$$n_t^g = \eta^{\frac{1}{\rho}} w_t^g \tag{11.2.72}$$

Take logs from both sides to obtain

$$\ln n_t^g = \frac{1}{\rho}\ln\eta + \ln w_t^g \tag{11.2.73}$$

Totally differentiate both sides to obtain

$$\frac{d\ln n_t^g}{dt} = \frac{d\ln w_t^g}{dt} \tag{11.2.74}$$

$$\frac{dn_t^g}{dt}\frac{1}{n^g} = \frac{dw_t^g}{dt}\frac{1}{w^g} \tag{11.2.75}$$

Pass to log-deviations to obtain

$$\hat{n}_t^g = \hat{w}_t^g \tag{11.2.76}$$

11.2.10 Total hours/employment

$$n_t = n_t^g + n_t^p \tag{11.2.77}$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \tag{11.2.78}$$

Totally differentiate to obtain

$$\frac{d\ln n_t}{dt} = \frac{d\ln(n_t^g + n_t^p)}{dt} \tag{11.2.79}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{11.2.80}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{11.2.81}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{11.2.82}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{11.2.83}$$

11.2.11 Linearized private wage rate

$$w_t^p = \theta\frac{y_t}{n_t^p} \tag{11.2.84}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln\theta + \ln y_t - \ln n_t^p \tag{11.2.85}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln w_t^p}{dt} = \frac{d\ln\theta}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln n_t^p}{dt} \tag{11.2.86}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{11.2.87}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{11.2.88}$$

11.2.12 Linearized real interest rate

$$r_t = \theta \frac{y_t}{k_t^p} \tag{11.2.89}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln \theta + \ln y_t - \ln k_t^p \tag{11.2.90}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln r_t}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln k_t^p}{dt} \tag{11.2.91}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{11.2.92}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{11.2.93}$$

11.2.13 Public/private wage ratio

$$rw_t \quad = \quad w_t^g / w_t^p \tag{11.2.94}$$

Take logs from both sides of the equation

$$\ln rw_t \quad = \quad \ln w_t^g - \ln w_t^p \tag{11.2.95}$$

Totally differentiate to obtain

$$\frac{d \ln rw_t}{dt} \quad = \quad \frac{d \ln w_t^g}{dt} - \frac{d \ln w_t^p}{dt} \tag{11.2.96}$$

$$\frac{drw_t}{dt}\frac{1}{rw} \quad = \quad \frac{dw_t^g}{dt}\frac{1}{w^g} - \frac{dw_t^p}{dt}\frac{1}{w^p} \tag{11.2.97}$$

Pass to log-deviations to obtain

$$r\hat{w}_t \quad = \quad \hat{w}_t^g - \hat{w}_t^p \tag{11.2.98}$$

11.2.14 Public/private hours/employment ratio

$$rl_t = n_t^g / n_t^p \tag{11.2.99}$$

Take logs from both sides of the equation

$$\ln rl_t = \ln n_t^g - \ln n_t^p \tag{11.2.100}$$

Totally differentiate to obtain

$$\frac{d \ln rl_t}{dt} = \frac{d \ln n_t^g}{dt} - \frac{d \ln n_t^p}{dt} \tag{11.2.101}$$

$$\frac{drl_t}{dt}\frac{1}{rl} = \frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{11.2.102}$$

Pass to log-deviations to obtain

$$\hat{rl}_t = \hat{n}_t^g - \hat{n}_t^p \tag{11.2.103}$$

11.2.15 Linearized technology shock process

$$\ln a_{t+1} = \rho_a \ln a_t + \epsilon_{t+1}^a \tag{11.2.104}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln a_{t+1}}{dt} = \rho_a \frac{d \ln a_t}{dt} + \frac{d\epsilon_{t+1}^a}{dt} \tag{11.2.105}$$

$$\frac{da_{t+1}}{dt} = \rho_a \frac{da_t}{dt} + \epsilon_{t+1}^a \tag{11.2.106}$$

where for $t = 1$ $\frac{d\epsilon_{t+1}^a}{dt} \approx \ln(e^{\epsilon_{t+1}^a}/e^{\epsilon^a}) = \epsilon_{t+1}^a - \epsilon^a = \epsilon_{t+1}^a$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \epsilon_{t+1}^a \tag{11.2.107}$$

11.2.16 Linearized stochastic process for government consumption/output share

$$\ln g_{t+1}^{cy} \;=\; (1-\rho^g)\ln g^{cy} + \rho^g \ln g_t^{cy} + \epsilon_{t+1}^c \tag{11.2.108}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_{t+1}^{cy}}{dt} \;=\; (1-\rho^g)\frac{d\ln g^{cy}}{dt} + \rho^g \frac{d\ln g_t^{cy}}{dt} + \frac{d\epsilon_{t+1}^c}{dt} \tag{11.2.109}$$

$$\frac{dg_{t+1}^{cy}}{dt} \;=\; \rho_g \frac{dg_t^{cy}}{dt} + \epsilon_{t+1}^c \tag{11.2.110}$$

where for $t=1$ $\frac{d\epsilon_{t+1}^c}{dt} \approx \ln(e^{\epsilon_{t+1}^c}/e^{\epsilon^c}) = \epsilon_{t+1}^c - \epsilon^c = \epsilon_{t+1}^c$ since $\epsilon^c = 0$. Pass to log-deviations to obtain

$$\hat{g}_{t+1}^{cy} \;=\; \rho_g \hat{g}_t^{cy} + \epsilon_{t+1}^c \tag{11.2.111}$$

11.2.17 Linearized level of government consumption

$$g_t^c = g_t^{cy} y_t \tag{11.2.112}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^c = \ln g_t^{cy} + \ln y_t \tag{11.2.113}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_t^c}{dt} = \frac{d\ln g_t^{cy}}{dt} + \frac{d\ln y_t}{dt} \tag{11.2.114}$$

$$\frac{dg_t^c}{dt}\frac{1}{g^c} = \frac{dg_t^{cy}}{dt}\frac{1}{g^c} + \frac{dy_t}{dt}\frac{1}{y} \tag{11.2.115}$$

Pass to log-deviations to obtain

$$\hat{g}_t^c = \hat{g}_t^{cy} + \hat{y}_t \tag{11.2.116}$$

11.2.18 Linearized stochastic process for the government investment/output ratio

$$\ln g_{t+1}^{iy} = (1-\rho^i)\ln g^{iy} + \rho^i \ln g_t^{iy} + \epsilon_{t+1}^i \tag{11.2.117}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_{t+1}^{iy}}{dt} = (1-\rho^i)\frac{d\ln g^{iy}}{dt} + \rho^i \frac{d\ln g_t^{iy}}{dt} + \frac{d\epsilon_{t+1}^i}{dt} \tag{11.2.118}$$

$$\frac{dg_{t+1}^{iy}}{dt} = \rho_g \frac{dg_t^{iy}}{dt} + \epsilon_{t+1}^i \tag{11.2.119}$$

where for $t = 1$ $\frac{d\epsilon_{t+1}^i}{dt} \approx \ln(e^{\epsilon_{t+1}^i}/e^{\epsilon^i}) = \epsilon_{t+1}^i - \epsilon^i = \epsilon_{t+1}^i$ since $\epsilon^i = 0$. Pass to log-deviations to obtain

$$\hat{g}_{t+1}^{iy} = \rho_i \hat{g}_t^{iy} + \epsilon_{t+1}^i \tag{11.2.120}$$

11.2.19 Linearized level of government investment

$$g_t^i = g_t^{iy} y_t \tag{11.2.121}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g_t^{iy} + \ln y_t \tag{11.2.122}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_t^i}{dt} = \frac{d\ln g_t^{iy}}{dt} + \frac{d\ln y_t}{dt} \tag{11.2.123}$$

$$\frac{dg_t^i}{dt}\frac{1}{g^i} = \frac{dg_t^{iy}}{dt}\frac{1}{g^i} + \frac{dy_t}{dt}\frac{1}{y} \tag{11.2.124}$$

Pass to log-deviations to obtain

$$\hat{g}_t^i = \hat{g}_t^{iy} + \hat{y}_t \tag{11.2.125}$$

11.2.20 Linearized level of government transfers

$$g_t^t = g^{ty} y_t \tag{11.2.126}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^t = \ln g^{ty} + \ln y_t \tag{11.2.127}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^t}{dt} = \frac{d \ln g^{ty}}{dt} + \frac{d \ln y_t}{dt} \tag{11.2.128}$$

$$\frac{dg_t^t}{dt} \frac{1}{g^t} = \frac{dy_t}{dt} \frac{1}{y} \tag{11.2.129}$$

Pass to log-deviations to obtain

$$\hat{g}_t^t = \hat{y}_t \tag{11.2.130}$$

11.3 Auto- and cross-correlation functions

As an additional test of model fit, this appendix compares auto- and cross-correlation functions generated from the model with collective bargaining and Finn (1998) calibrated for Germany, with their empirical counterparts. The main emphasis in this subsection is on the ACFs and CCFs of labor market variables. In particular, close attention is paid to cyclical properties of public and private wage rates and hours. To establish 95% confidence intervals for the theoretical ACFs and CCFs, as in Gregory and Smith (1991), the simulated time series are used to obtain 1000 ACFs and CCFs. The mean ACFs and CCFs are computed by averaging across simulations, as well as the corresponding standard error across simulations. Those moments allow for the lower and upper bounds for the ACFs confidence intervals to be estimated. The empirical ACFs and CCFs are then plotted, together with the theoretical ones. If empirical ACFs lie within the confidence region, this means that the calibrated model fits data well.

Empirical ACFs and CCFs were generated from a Vector Auto-Regressive (VAR) process of order 1. Since ACFs and CCFs are robust to identifying restrictions (Canova (2007), Ch.7), the VAR(1) was left unrestricted. The figures on the following pages display empirical ACFs (solid line), together with the simulated average ACFs (dashed line) and the corresponding stochastic error bounds (dotted lines). This is done for the union model first , and then for the calibration using Finn's (1998) framework.

The model with the public sector union calibrated for Germany outperforms Finn (1998), especially in the prediction of the dynamic behavior of labor market variables. In terms of capturing the autocorrelation structure of the variables, the union model fits data quite well. One exception is the public sector wage: in data, it is highly autocorrelated, while the model generates low persistence. A possible explanation could be that the public union puts weight also on last year's public sector wage level, *i.e.* the union bargains over the public wage increase rate, and not just the wage level. Public and total hours are also borderline cases, as employment rates in data were used instead. In addition, the union model predicts

Figure 7: Theoretical and empirical ACFs for Germany: Union

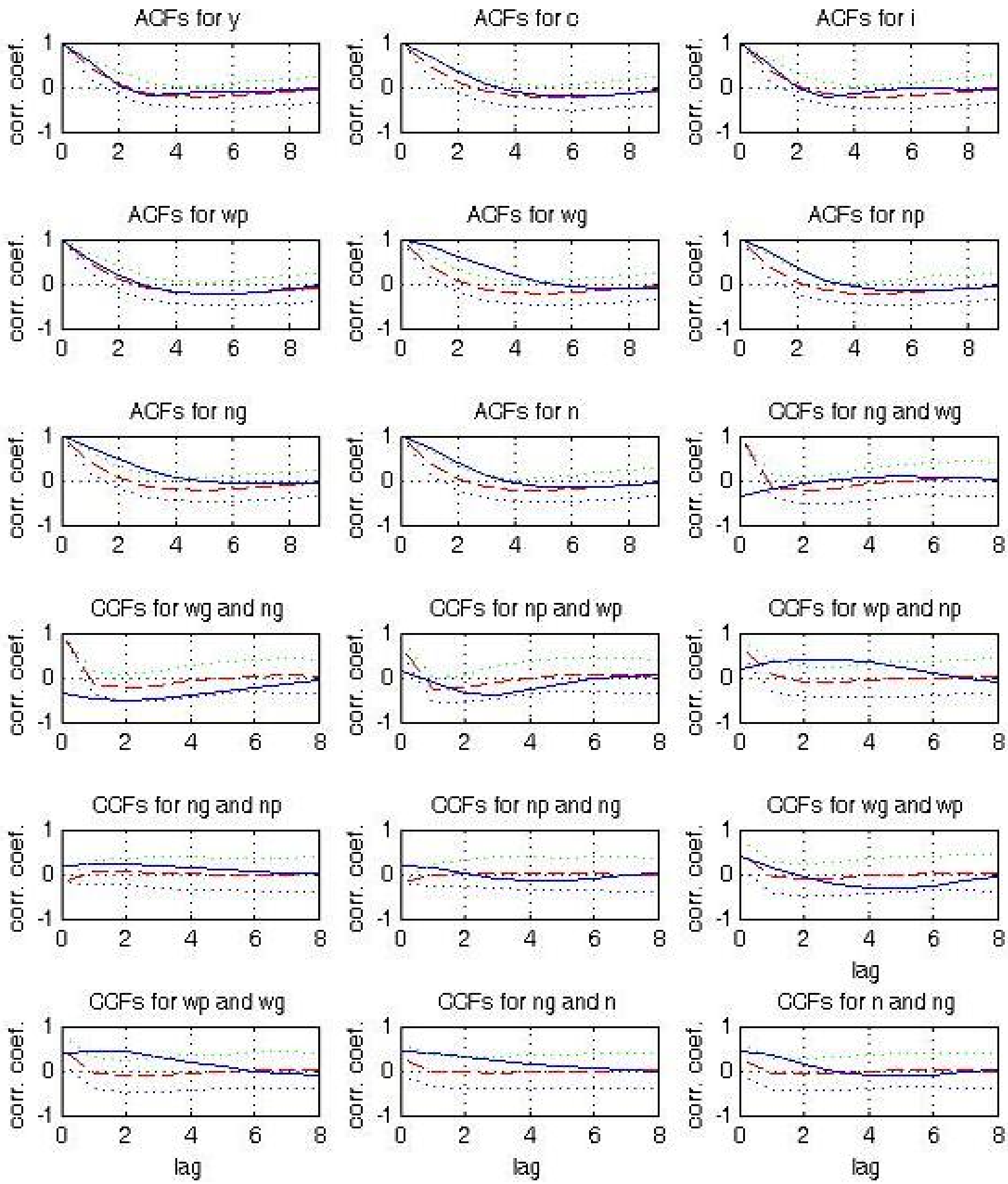


Figure 8: Theoretical and empirical ACFs for Germany: Union

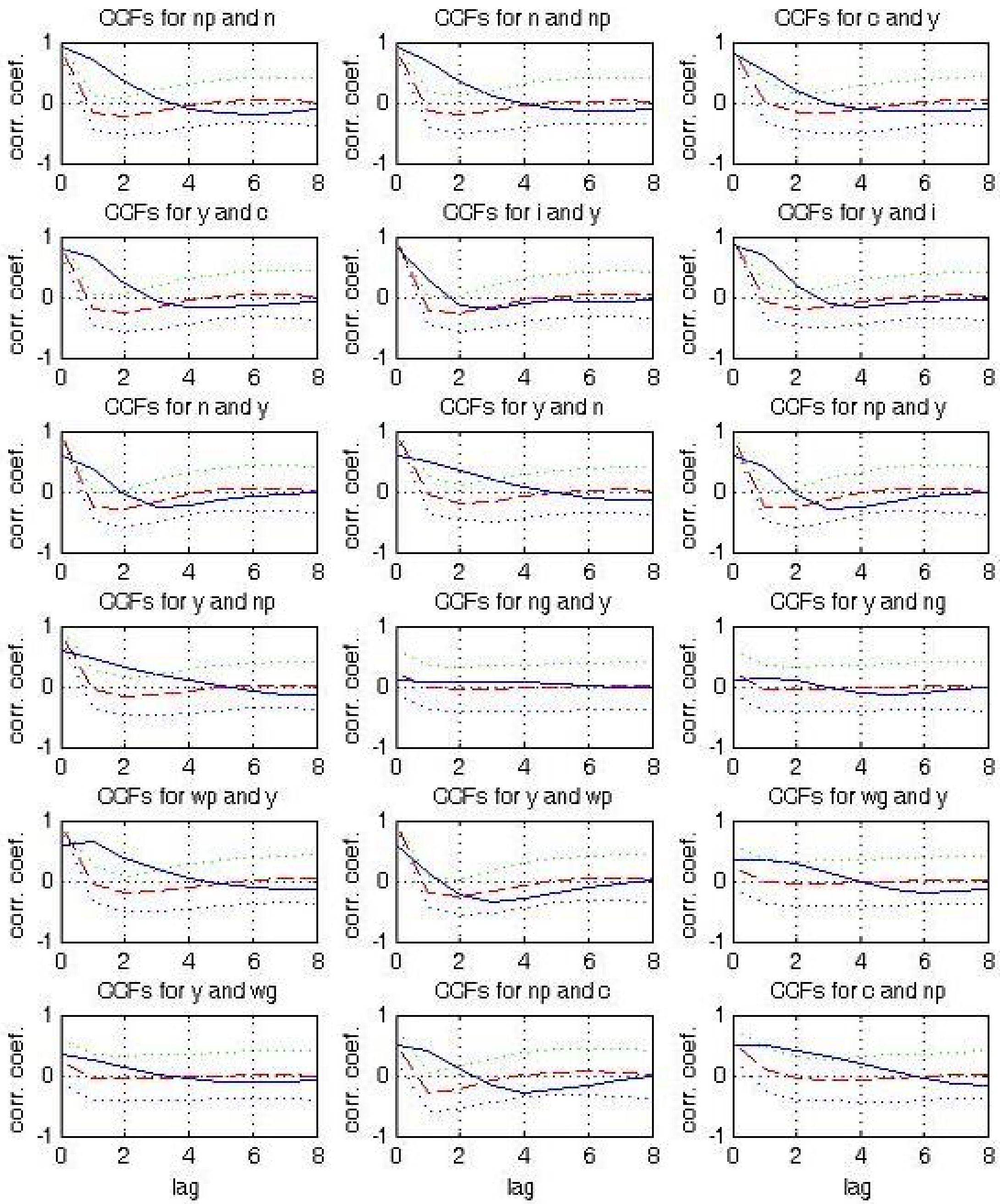


Figure 9: Theoretical and empirical ACFs for Germany: Union

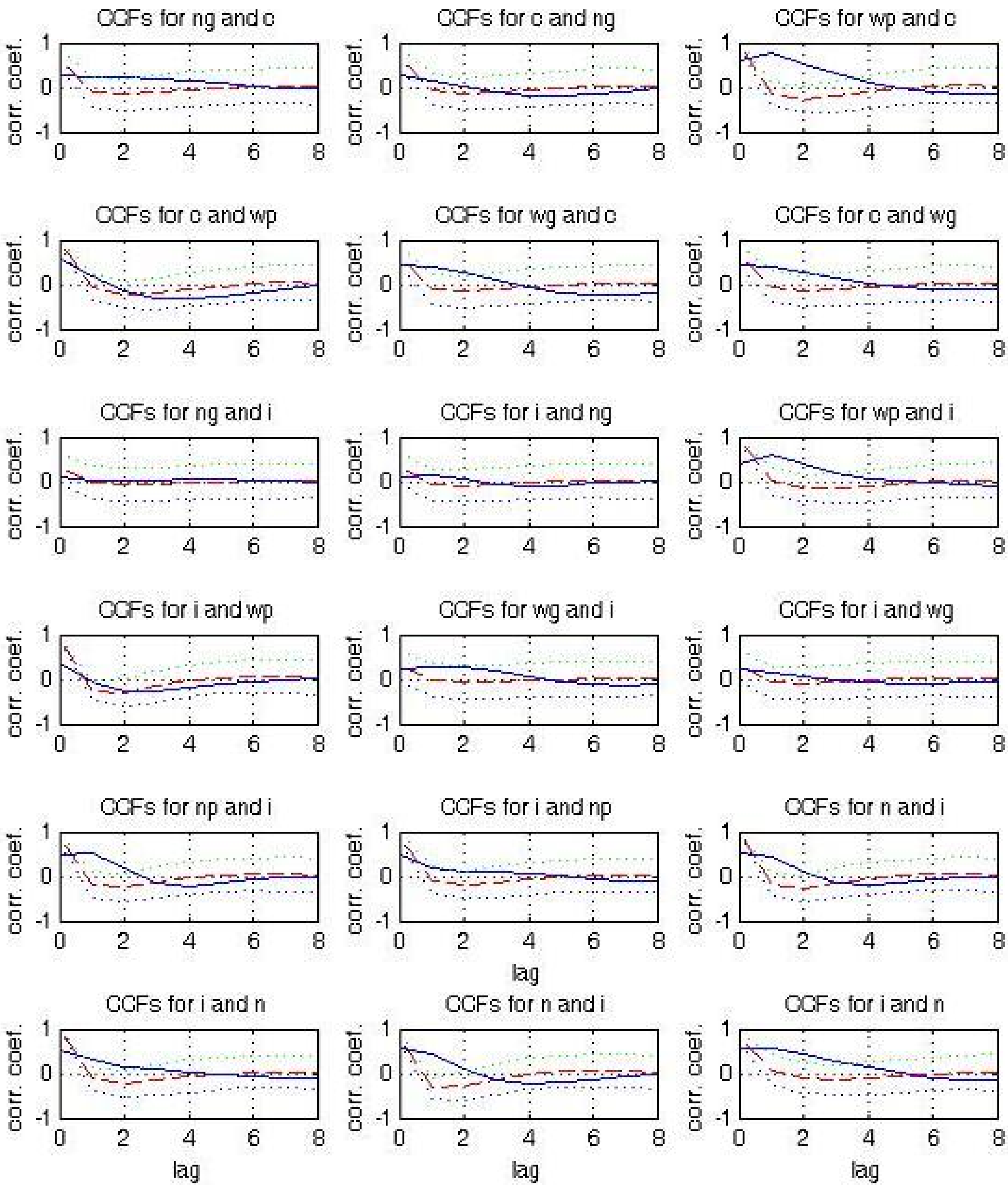


Figure 10: Theoretical and empirical ACFs for Germany: Finn

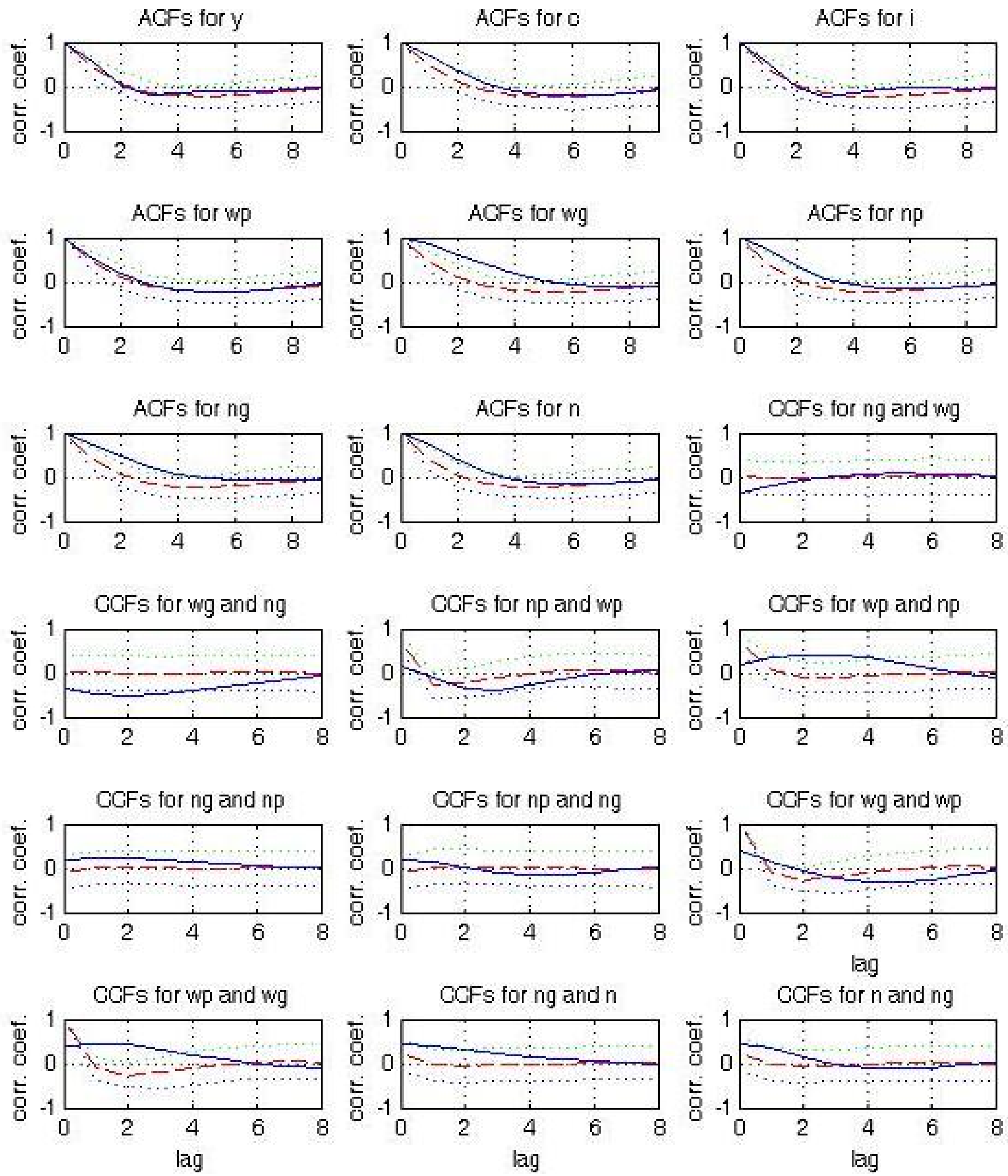


Figure 11: Theoretical and empirical ACFs for Germany: Finn

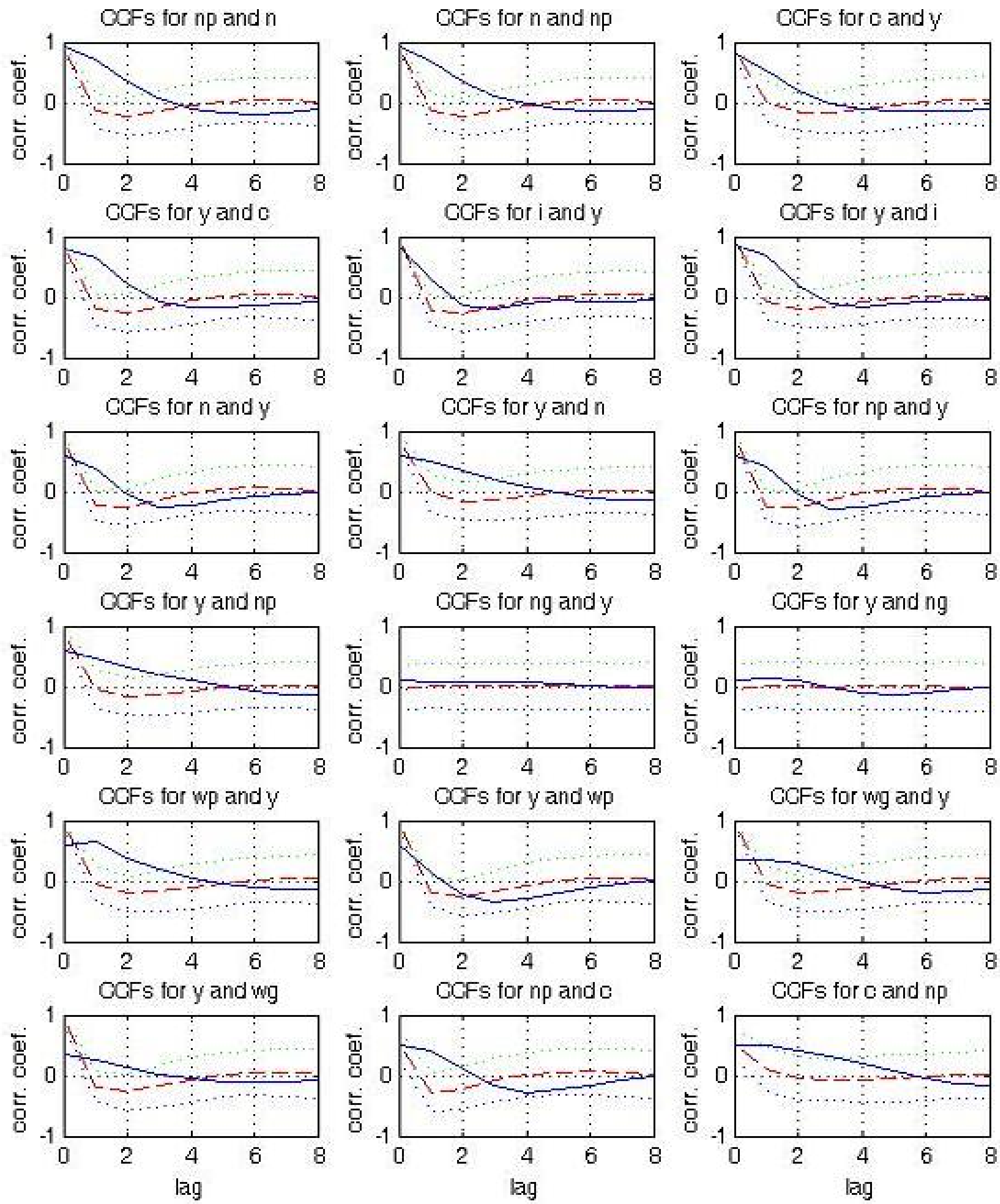
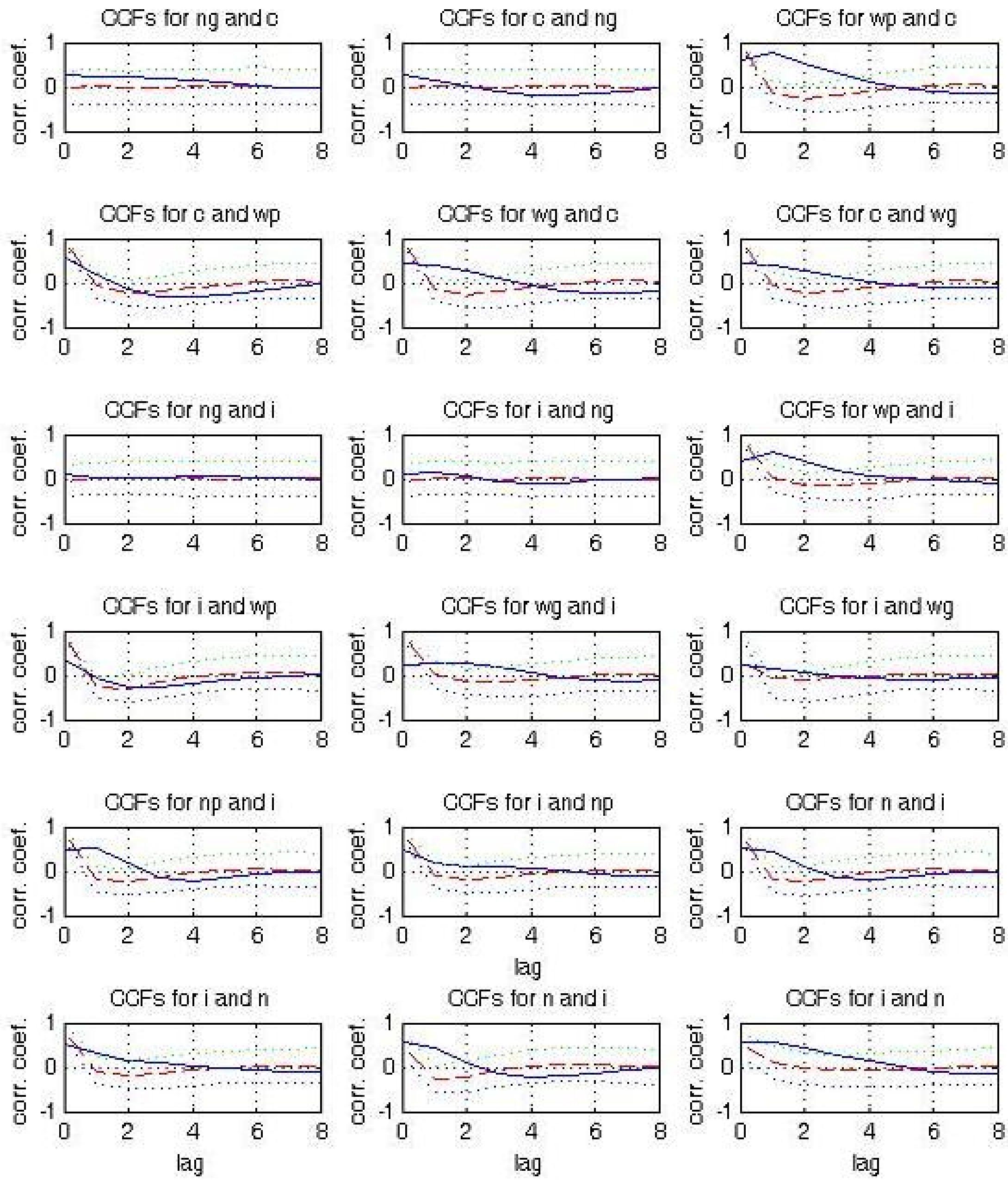


Figure 12: Theoretical and empirical ACFs for Germany: Finn



perfect positive contemporaneous correlation between public wages and hours, while in data, it is negative. Overall, the model with public sector union calibrated for Germany captures the dynamic co-movement of hours and wages with output, consumption and investment. In addition, union model is able to address and match some new dimensions such as the dynamic correlation of the two wage rates and the pair of hours worked.

11.4 Sensitivity analysis

To evaluate the effect of structural parameters on the shape of the Laffer curves, this section performs sensitivity analysis for different values of model parameters and how those affect tax revenues. The two parameters of interest are the curvature parameter of household's Cobb-Douglas utility function α, as well as the weight on composite consumption, ψ. Interestingly, as α is allowed to vary, steady-state revenues are essentially unchanged. Even an implausibly high value, $\alpha = 50$, does not produce any difference in steady state tax revenues. In both models considered in this paper, the preference parameter is not important for steady-state fiscal policy effect. This result is not surprising in the literature, as Trabandt and Uhlig (2010) obtain a very similar finding in their paper.[32]

In contrast, changes in the second parameter, ψ, yield significant differences. Both the capital and labor tax Laffer curves, and the responses of the other tax bases to capital and labor income tax rate are affected when ψ is allowed to vary.[33] Higher values of ψ shift up the Laffer curve and make it steeper, without significant change in its peak. The difference between Finn and the model with endogenous public employment becomes significant for implausibly high values of ψ, i.e. $\psi > 0.5$. (As explained in the calibration section, parameter $\psi = 0.296$, describing household's preference was calculated as the ratio of hours of work out of total potential hours in the model.) Intuitively, a higher ψ corresponds to a lower weight to leisure, $(1 - \psi)$, in the household's utility function. In other words, a higher ψ

[32] Parameter α is important for model dynamics, though.

[33] Consumption tax Laffer curve proves to be very sensitive to ψ parameter. In the majority of the cases it breaks down for values outside the benchmark value. This is also a typical result in the literature, e.g. Trabandt and Uhlig (2010).

Figure 13: Sensitivity analysis: Union

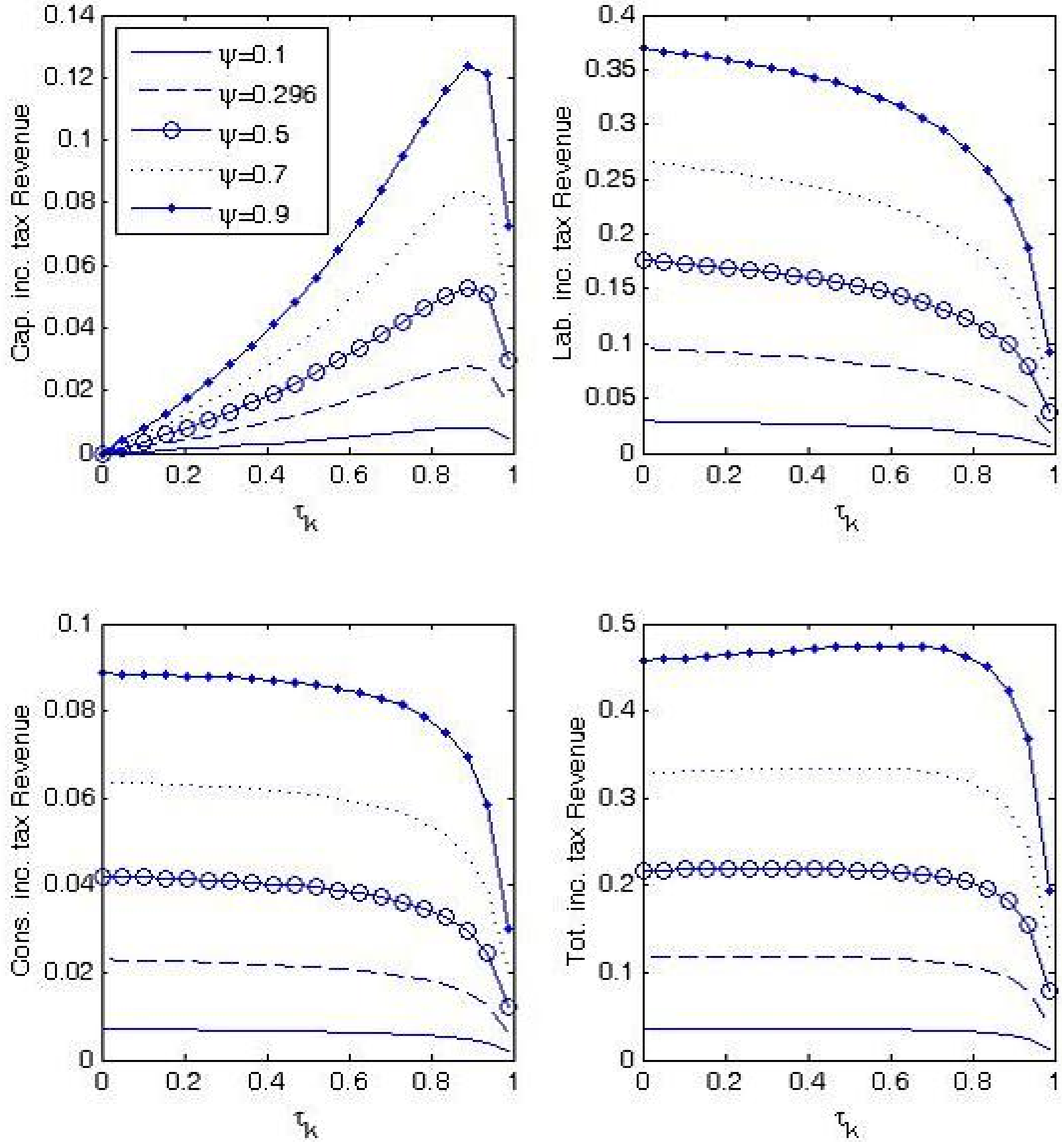


Figure 14: Sensitivity analysis: Union

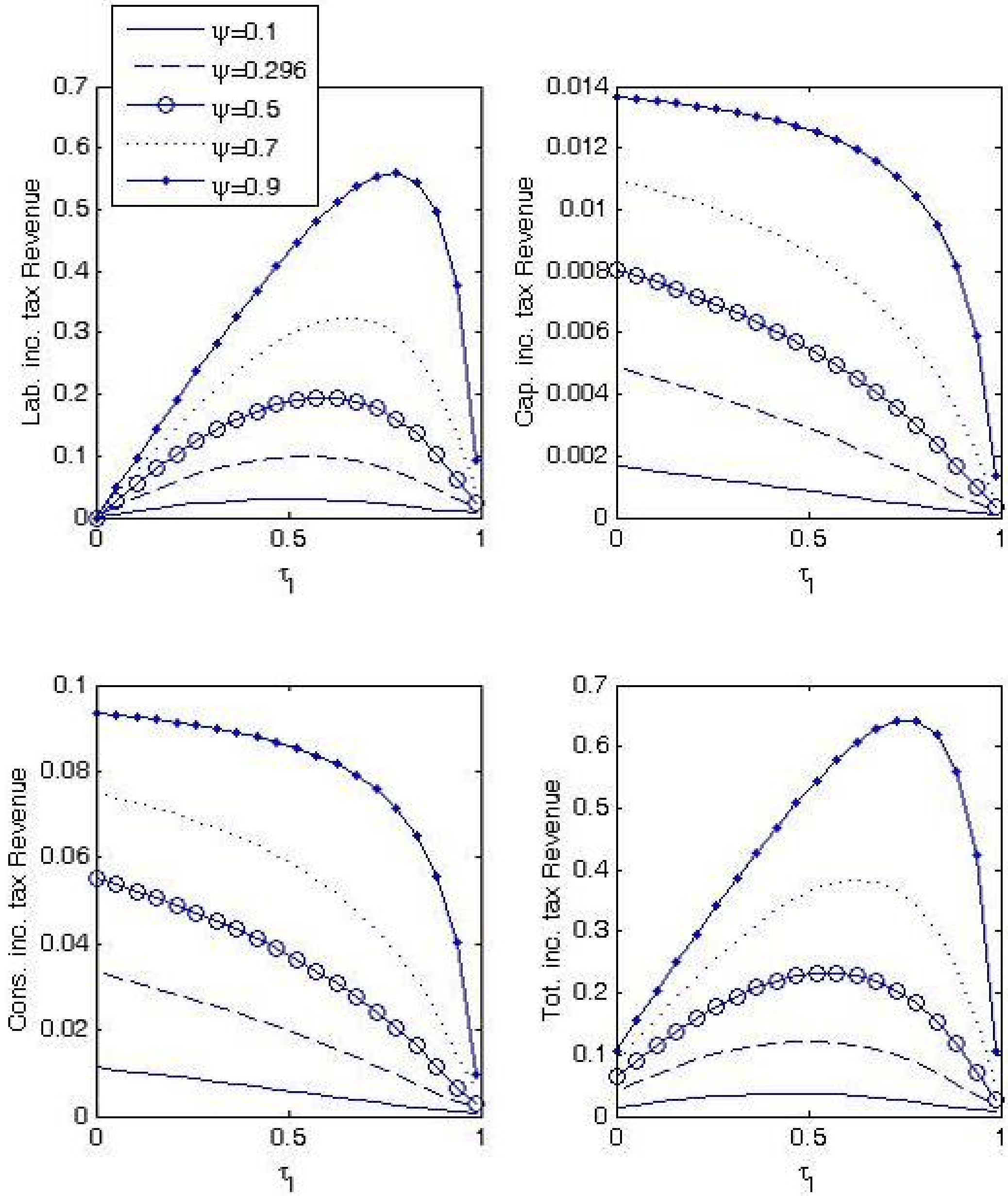


Figure 15: Sensitivity analysis: Finn

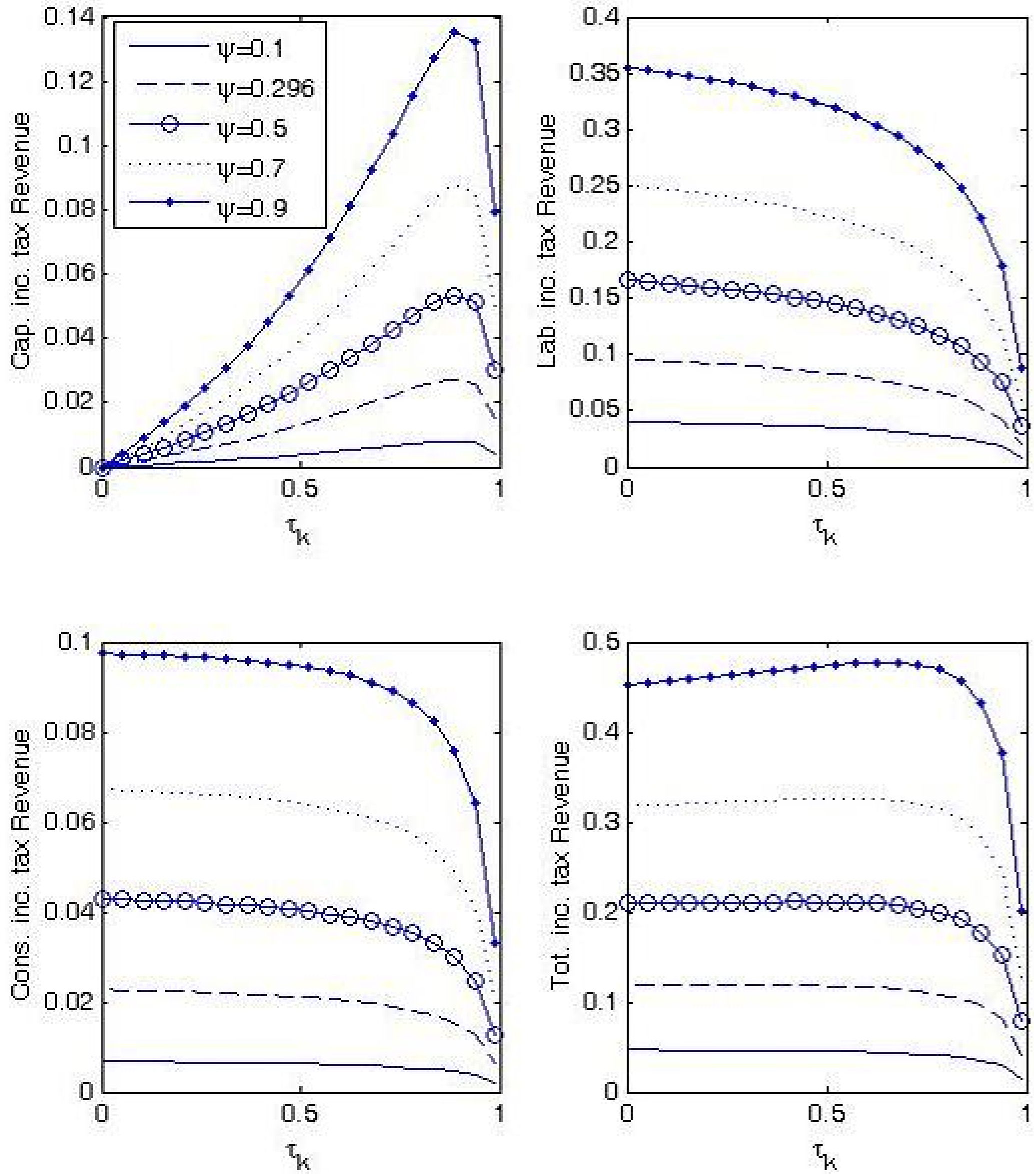
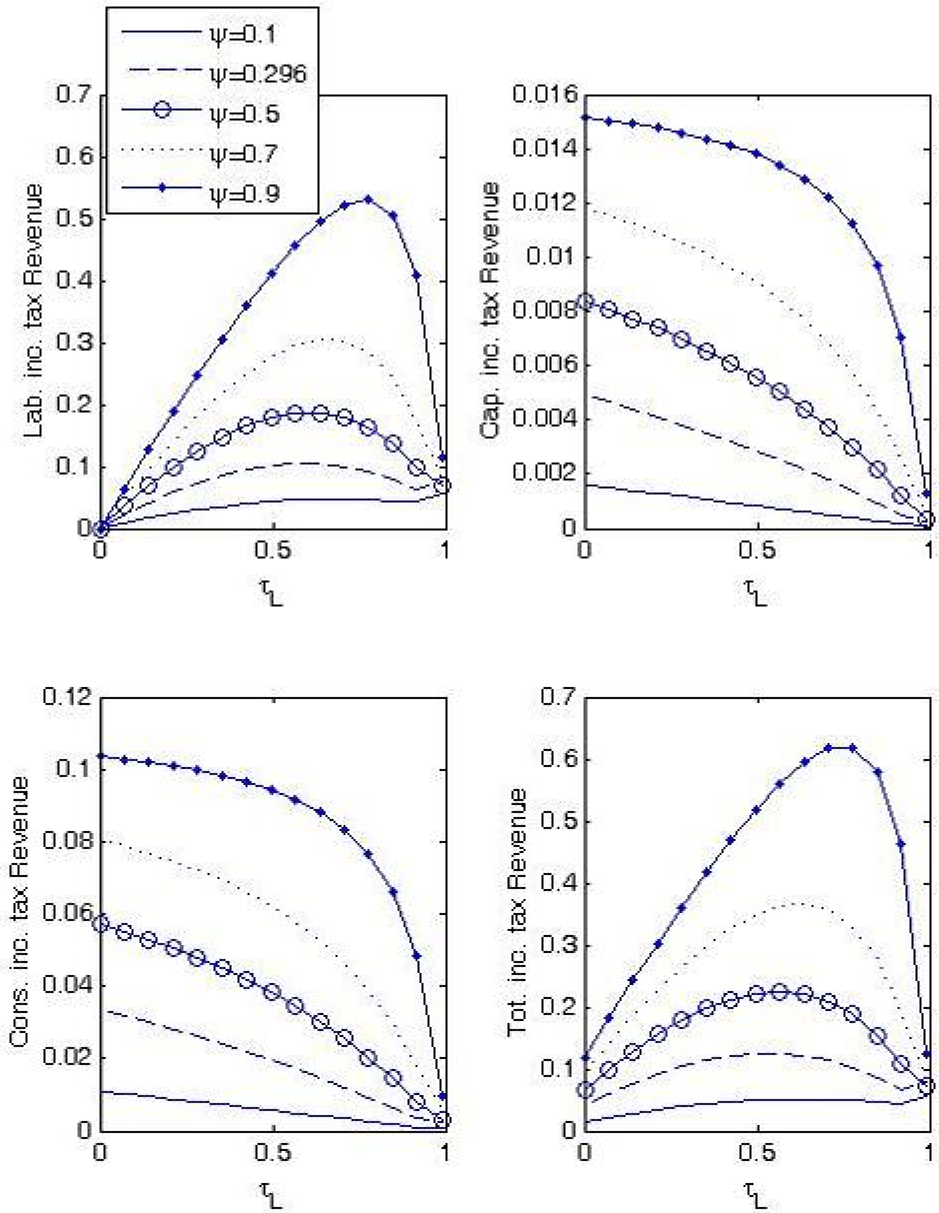


Figure 16: Sensitivity analysis ψ: Finn



decreases the elasticity of private labor supply. Intuitively, when labor tax rate increases, or equivalently, after tax private wage falls, private hours respond less, thus increasing labor income tax revenue, as well as total tax revenue.

The effect of higher ψ on capital tax Laffer curve is similar to ψ's effect on the labor tax Laffer curve above. When τ^k is allowed to vary, a higher weight attached to consumption in household's utility function, together with the optimality condition for the marginal rate of substitution between consumption and hours require private higher capital stock to finance private consumption. Therefore, a higher ψ shifts the capital tax Laffer curve upward as well.

11.5 Measuring conditional welfare

In steady state

$$u(c, g^c, 1-n) = \frac{[(c+\omega g^c)^\psi (1-n)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} \tag{11.5.1}$$

Let A and B denote two different regimes. The welfare gain, ζ, is the fraction of consumption that is needed to complement household's steady-state consumption in regime B so that the household is indifferent between the two regimes. Thus

$$\frac{[(c^A+\omega g^{c,A})^\psi (1-n^A)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} = \frac{[((1+\zeta)c^B+\omega g^{c,B})^\psi (1-n^B)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} \tag{11.5.2}$$

Multiply both sides by $(1-\alpha)$ to obtain

$$[(c^A+\omega g^{c,A})^\psi (1-n^A)^{(1-\psi)}]^{(1-\alpha)} - 1 = [((1+\zeta)c^B+\omega g^{c,B})^\psi (1-n^B)^{(1-\psi)}]^{(1-\alpha)} - 1 \tag{11.5.3}$$

Cancel -1 terms at both sides to obtain

$$[(c^A+\omega g^{c,A})^\psi (1-n^A)^{(1-\psi)}]^{(1-\alpha)} = [((1+\zeta)c^B+\omega g^{c,B})^\psi (1-n^B)^{(1-\psi)}]^{(1-\alpha)} \tag{11.5.4}$$

Raise both sides to the power $\frac{1}{1-\alpha}$ to obtain

$$(c^A+\omega g^{c,A})^\psi (1-n^A)^{(1-\psi)} = ((1+\zeta)c^B+\omega g^{c,B})^\psi (1-n^B)^{(1-\psi)} \tag{11.5.5}$$

Divide throughout by $(1-n^B)^{(1-\psi)}$ to obtain

$$((1+\zeta)c^B+\omega g^{c,B})^\psi = (c^A+\omega g^{c,A})^\psi \left(\frac{1-n^A}{1-n^B}\right)^{(1-\psi)}$$

Raise both sides to the power $1/\psi$ to obtain

$$(1+\zeta)c^B + \omega g^{c,B} = (c^A + \omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} \tag{11.5.6}$$

Move $\omega g^{c,B}$ term to the right to obtain

$$(1+\zeta)c^B \;=\; (c^A + \omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B} \tag{11.5.7}$$

Divide both sides by c^B to obtain

$$1+\zeta \;=\; \frac{1}{c^B}\left\{(c^A + \omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B}\right\} \tag{11.5.8}$$

Thus

$$\zeta \;=\; \frac{1}{c^B}\left\{(c^A + \omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B}\right\} - 1 \tag{11.5.9}$$

Note that if $\zeta > 0 (< 0)$, there is a welfare gain (loss) of moving from B to A. In this paper B is the initial scenario, while A will be the fiscal regime change.