Evaluating the Wage-Pension Trade-Off in a Dynamic Model of Search and Savings

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Abstract

In the last thirty years there has been a considerable change in the way people save for retirement. The once traditional defined benefit plans are steadily being replaced by now dominant defined contribution plans. This shift in the type of pensions offered to the worker, can potentially affect job choice and savings decisions. By examining the trade-off that workers face when choosing between compensation in the form of wages versus pension contributions, this paper explores the behavioral effects resulting from changes in the workers' environment and the potential impact of policy interventions. I formulate a lifecycle model, in which individuals search for jobs, consume, save on their own and through their employer-provided pension, so as to maximize utility. Job offers are wage-pension packages, consisting of a total compensation component , and a pension plan, which could be defined benefit, defined contribution or neither. Each worker's job acceptance and rejection decisions is the result of interplay between his preferences and the set of different incentives and risks that each pension plan comes with. The structural model is simulated and will later be estimated using the Panel Study of Income Dynamics (PSID). The estimates of the structural parameters can be used to address a set of policy question such as: How does the shift in pension type coverage impact wage growth and job-to-job transitions? Which compensation packages lead to higher retirement savings?

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1 Introduction

This paper explores the trade-off that workers face when choosing between compensation in the form of wages versus pension contributions. This question has particular policy relevance in the light of the ongoing debate on issues regarding the adequacy of retirement income and ways to encourage workers to save more. Since the end of WWII employer-provided pensions have been an important part of workers' compensation and constituted a significant portion of retirees' wealth. For today's retirees, the employer-provided pension benefits are the second largest component of retirement income. They are surpassed only by social security benefits, and are twice as big as other forms of personal savings. Recognizing that pensions come as a part of the job offer package and understanding how individuals value compensation in the form of wages versus pension contributions will provide valuable insight into how individuals save for retirement and how policy reform can affect savings behavior.

During the last twenty years pension coverage in the US has remained stagnant at around 50 percent of the working population. The type of coverage, however, has changed dramatically. Pensions have shifted from traditional defined benefit plans, which provide retirees with a set benefit based on their wage history, to 401(k)-style plans (also called defined contribution plans), in which benefits are determined by the accumulated amount in a worker's account.[1] One of the main policy concerns with the now popular defined contribution plans, is the fact that most of the responsibility for retirement saving has been shifted from the employer to the worker. Workers must make decisions on how much to contribute, where to invest the money, whether to cash out when changing jobs, and how to manage their accumulated funds upon retirement. While, in theory, defined contribution plans have the potential to provide substantial retirement income, in practice most participants have only modest account balances.[2] The concern has been that these plans expose workers to too much retirement income volatility and they do not provide adequate saving incentives.

[1] For example, in 1980, of all workers with pension coverage, only 40 percent had a 401(k)-style plan and 83 percent had a defined benefit plan. In 2004, those numbers changed to 89 percent and 39 percent respectively, Munnell and Perun (2006)

[2] Munnell and Perun (2006)

Moreover, pension coverage does not occur randomly in the population. Worker's income has been shown as a major factor in determining who gets covered. Pension coverage is much more extensive for high-income households -around 85 percent of the households in the top two quintiles of the income distribution have pensions, compared to mere 28 percent for the bottom quintile.[3] The financing strain on Social Security caused by the retiring Baby Boom cohort, combined with the lack of pension coverage for many low income households, and the shifts in the type of plans, raises serious concerns about future retirees' wellbeing. If we want to understand how individuals save for retirement and how policy can affect their behavior, it is important to understand the trade-off that they face when choosing between wages and pensions and when choosing between jobs offering different pension plans.

The existing literature on the trade-off between wages and pension benefits is quite limited. A survey paper by Gustman et al. (1994) reports that most of the studies that address this question, estimate the compensating differential for pensions by using a hedonic pension-earnings equation. The hedonic analysis views employees and employers as choosing jointly the employment characteristics so as to maximize their objective functions. Typically such models have an earnings measure as a dependent variable and pension accruals or promised benefits on the right-hand side. Moreover, the existing empirical evidence is very inconclusive. Schiller and Weiss (1980), Ehrenberg (1980), and Smith (1981) and later Montgomery et al. (1992) as well as Gunderson et al. (1992) all find some evidence for the existence of a trade-off. On the contrary, Gustman and Steinmeier (1986) find no trade-off, but rather a significantly positive relationship.

Usually those studies fail to account properly for the endogeneity of pensions and the results suggest more of a correlation rather than causation. Treating pensions as exogenous fails to recognize that workers' job search behavior determines simultaneously their wage and benefits. Not all employers offer pensions, and those that do, offer one at most two pension plans.[4] So, the worker cannot decide whether and what pension plan to have; he can only choose his employer and enroll in the plan that employer offers. In that respect, any job

[3]Munnell and Perun (2006)
[4]Decressin et al. (2005)

acceptance decision in intertwined with the saving for retirement decision. When deciding between job offers, the worker has to take into account how that particular offer will affect his consumption today versus his ability to save for the future. This alone will make us expect that individuals at different points in their lifecycle would prefer different types of compensation packages. Differences in workers preferences like risk-aversion will also impact the job decision. In the end, the positive correlation in the data could simply be due to the fact that more productive workers are the ones who also care more about the future. They consciously choose to work for employers who offer pensions and thus end up both with a high wage and high retirement income. Obviously pensions cannot be treated as being exogenous.

This paper suggests a new approach for addressing the question by taking into account that the wage vs. pension benefit decision is crucially related to the job search and saving for retirement decisions. In this respect, the current paper is related to two quite different strands of literature in labor economics. On one hand, by modeling the job offers as compensation packages, this paper fits within the literature on search models that include job characteristic other than the offered wage. Some of the recent work in this fields includes Bloemen (2006), in which jobs are characterized by a combination of a wage and number of working hours, and Dey and Flinn (2005) who focus on wage and employer-provided health insurance. Similarly to those papers, the current study models the job search decision in a structural dynamic framework following workers job decisions over time. The current paper is the first one to consider job offers as wage-pension packages and thus contributes to this literature from a methodological stand point.

On the other hand, this paper is related to the growing literature that studies retirement behavior, where dynamic discrete choice models have already seen numerous applications. Many authors have found the dynamic modeling approach to be better at explaining consumption and employment behavior of older workers-Gustman and Steinmeier (1986), M.D.Hurd (1990), J.F.Quinn and Burkhauser (1990). This paper uses a similar theoretical framework to build a better intuition of how the saving for retirement decision interrelates with job choice and employment decisions of workers of all ages. The main advantage of such a framework is the ability to measure the effect of changes in uncertain future circumstances, allowing for an inter-temporal evaluation of opportunities.

Last but not least, this project studies workers' behavior when choosing between jobs offering defined benefit vs those offering defined contribution plans. As such it falls among the broader literature on the benefits and risks of DB and DC plans. Many studies in this area have already documented a link between between increased job mobility and the growing popularity of DC plans. Other papers have made comparisons of simulated retirement wealth under DB and DC - Poterba et al. (2006), Samwick and Skinner (2006), Schrager (2006). This paper contributes to both of these questions by estimating a behavioral and performing counterfactual experiments on how the shift from DB to DC plans affects the job-to-job transitions and the distribution of retirement wealth.

To capture all of the above mentioned features of workers' behavior, I construct a lifecycle model in which individuals search for jobs, consume, save on their own and through their employer-provided pension, so as to maximize utility. Job offers are wage-pension packages, consisting of a total compensation component, related to worker's productivity and a pension plan (Defined Benefit, Defined Contribution or none). The decision of a worker to accept or reject a job offer is the result of an interplay between his preferences and the set of incentives and risks that each pension plan comes with. Although from the employer's point of view the two pension plans and the no plan option will be considered exactly equivalent, the worker will not necessarily be indifferent between the three. More specifically, given the same total compensation, a DB plan will be more attractive to a worker who is highly risk-averse. The reason is that DB plans offer a certain replacement rate upon retirement, while under a DC plan the worker will have to face financial risk on his pension assets. On the other hand, DC plans will be preferred by more mobile agents. Because of the backloaded nature of the benefit accrual in DB plans, a job change poses a significant risk to retirement saving, especially later in life. This is absent in DC plans whose benefits accrue more evenly throughout the work life, making them easily portable across employers.

Liquidity constraints and pension tax benefits will also influence the decision between a job with a DB, DC or no plan at all. While well paid individuals might prefer a DB plan, reaping the benefits of a secure future income and high tax savings, less wealthy workers might prefer the ability to make early withdrawals from their DC plan, or even to enjoy their total labor compensation today. In addition, differences in expected future offers and career paths based on unobserved worker differences (like innate ability) will also influence the wage-pension decision.

When all of the above factors are considered, it is not obvious how to answer questions like: How will the shift from DB to DC plans impact wage growth and job-to-job transitions? How will changes in the tax treatment of pension contributions and benefits impact the distribution of retirement wealth or which compensation packages lead to higher retirement savings? Estimating the lifecycle model to match the observed trends in the data will allow me to uncover the underlying structural parameters and perform counterfactual experiments to address such questions.

The remainder of this paper is organized as follows. Section 2 presents the dynamic job search model and its solution. The main features of the data and simulations based on the theoretical model are presented in section 3. Section 4 discusses the estimation procedure. Section 5 concludes.

2 Model

Time is discrete and indexed by t. There's a finite horizon, T. The individual is in the work force from period 1 to period R-1 and is retired from period R to period T. Death is certain at period T+1. All shocks dated period t are realized at the beginning of the period, before period t choices are made.

Employment. The employment choice variable is j_t which takes a value of 1 if the agent chooses to be employed in period t and 0 if he chooses non-employment. δ is the probability of job separation if employed. Each period, a new job offer arrives with probability ϕ^e if the worker is currently employed, and with probability ϕ^u if currently unemployed. If the agent receives such an offer, then in addition to choosing between employment and non-employment, he chooses wether to remain on the current job or take the new offer. The job choice variable is k_t , where $k_t = 0$ if the individual chooses to remain on the current job, and $k_t = 1$ if he accepts the new offer. Let $L_{1t} = 1$if the individual is laid off at the beginning of period t, and $L_{1t} = 0$ otherwise. Let $L_{2t} = 1$if an offer is received in period t and $L_{2t} = 0$ otherwise.

Utility: Individuals maximize the expected present value of utility, subject to budget constraint, while incorporating expectations about future job offers and layoffs. Period t instantaneous utility , if alive, is specified by the following function:

$$u(C_t, j_t, j_{t-1}, k_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} e^{\gamma_1 j_t} + \gamma_2 (1 - j_{t-1}) j_t + \gamma_3 k_t + \varepsilon_{jkt}$$

where ε_{jkt} is an iid (across periods) shock to utility for employment choice j and job choice k. ($\varepsilon_{jkt} \backsim N(0, \sigma_\varepsilon^2)$); the term $\gamma_1 j_t$ captures the disutility of work; $\gamma_2(1 - j_{t-1})j_t$ - the cost of reentering employment and $\gamma_3 k_t$ - the cost of switching employers.

Job offer: wage and pension: Each job offer is characterized by the total compensation component y and a pension component λ. Each job offer is assumed to arrive randomly from a joint total compensation-pension offer distribution, denoted by $f(y, \lambda)$.

$$f(y, \lambda) = f(y) * q_l, 0 < y < \infty, \; q_l = pr(\lambda = \lambda_l) \text{ where } l \; = 1, 2, 3$$

The pension is modeled with a discrete distribution where $l = 1$ indicates no pension, $l = 2$ indicates Defined Benefit type of pension and $l = 3$ indicates Defined Contribution type of pension. More specifically, there are three types of employers- those who offer no pension plans ($\lambda = \lambda_1$), those who offer DB plans ($\lambda = \lambda_2$) and those who offer DC plans ($\lambda = \lambda_3$).

Total labor compensation and worker heterogeneity: The log of the total compensation offer is $\ln y_t = \mu_{th} + \eta_t$,where η_t is an iid mean-zero normal shock with variance σ_η^2 and μ_{th}, varies with age and worker productivity, where $h = 1, 2$ low and high productivity worker respectively. This is done in order to accommodate rising wages with age and also different wage paths depending on ability (e.g. flatter wage profiles for low productivity workers and steeper for high productivity workers). When employed, the agent experiences wage growth due to specific human capital accumulation. His current wage $w_t(w_0, z_t)$ depends on the initial wage draw w_0 and the number of periods he has been working for the same employer z_t. I assume the following functional form: $w_t(w_0, z_t) = w_0 exp(\alpha_1 z_t + \alpha_2 z_t^2 + \zeta_t)$, where ζ_t is an iid mean-zero normal shock with variance σ_ζ^2.

During his working life the agent receives transfers b when unemployed, which include nonlabor income, like family transfers, plus unemployment compensation net of out-of-pocket search costs.

Mortality: $\pi_{(t|t-1)}$ is the probability of being alive at the beginning of period t given that the worker was alive in period t-1.

Firm's indifference: Although firms' behavior is not modeled, there's one important identifying assumption coming from the firm's side, namely that the firms are indiffirentbetween offering a DB plan, a DC plan or no plan at all.[5] Assume further that firms can finance DB or DC schemes at zero administrative costs, so the only costs the firms face are contributions to the plans.The firm completely finances the DB pension and contributes to the DC participant's pension account.

Definded Benefit plans*:*In a DB scheme the employer pays the worker an annuity upon retirement and until his death that is a product of the generosity factor α , the employee's wage during his last year of employment, and the number of years the worker has been at the firm. Denote by $\widetilde{t}$ the period when the worker leaves the firm

$$Pension_{DB_t} = \alpha w_{\widetilde{t}} z_{\widetilde{t}} \tag{1}$$

When the firm hires a worker at time t, it contributes to a fund that will finance future benefit payments. When calculating the expected benefit stream the firm estimates the expected number of years the worker is expected to be with the firm, his expected salary when he leaves, and how long the worker will be retired; assuming he survives to retirement. The present discounted value of benefits that will be paid each year after retirement, R, until death T, is:

$$EPDV_t^{benefits} = \pi_{R|t}\, E \frac{\sum_{i=R}^{T} \pi_{i+1|i} \left(\frac{\alpha E(w_{\widetilde{t}}) E(z_{\widetilde{t}})}{\prod_{s=R}^{i}(1+r_s)} \right)}{\prod_{l=t}^{R}(1+r_l)} \tag{2}$$

[5] Similar to the approach taken by Schrager (2006)

It is assumed that once the worker dies the firm does not pay any more benefits. It is as if the worker is single, because most DB plans offer survivor benefits.

Defined Contribution plans: In the DC plan the firm contributes a fixed fraction of the employee's wage , ρ_t, into his retirement account when he begins work and in each period until the worker leaves the firm (ρ_t is set at the time of the offer and remains constant for the duration of the job). Period t's contribution to the account is $\rho_t w_t$. The expected present discounted value of DC contributions (benefits) is:

$$EPDV_t^{benefits} = E\sum_{i=t}^{\widetilde{t}} \frac{\rho w_i}{\prod_{s=t}^{i}(1+r_s)} \tag{3}$$

For a firm making an offer at period the indifference assumption imposes the following 3 conditions:

1. $2 = 3$
2. Total compensation$^{\text{under DB}}$ =Total compensation$^{\text{no plan}}$
3. Total compensation$^{\text{under DC}}$ =Total compensation$^{\text{no plan}}$

Let's denote by $w_t^{NO\ PLAN}$, w_t^{DB} and w_t^{DC} period t's wage under No pensions plan, DB plan and DC plan respectively. The wage under no plan is just the draw from the distribution of total compensation. $w_t^{NO\ PLAN} = y_t$. For a given $w_t^{NO\ PLAN}$, α and a known wage growth function, a firm offering DB plan will set w_t^{DB}, so as to satisfy condition 2. What is left is to determine the contribution rate and the wage for a firm offering DC plan.

$$\begin{aligned}
\text{Total compensation}^{\text{under DC}} &= EPDV_t^{wages} + EPDV_t^{benefits} \\
\text{Total compensation}^{\text{under DC}} &= E\sum_{i=t}^{\widetilde{t}} \frac{w_i^{DC}}{\prod_{s=t}^{i}(1+r_s)} + E\sum_{i=t}^{\widetilde{t}} \frac{\rho w_i^{DC}}{\prod_{s=t}^{i}(1+r_s)} \\
&= E\sum_{i=t}^{\widetilde{t}} \frac{(1+\rho) w_i^{DC}}{\prod_{s=t}^{i}(1+r_s)}
\end{aligned}$$

which leads to the conclusion that in order for condition 3 to hold, the following has to be true: $w_t^{NO\ PLAN} = (1+\rho)w_t^{DC}$, which however is satisfied by multiple combinations of w_t^{DC}

and ρ. To pinpoint a unique solution, use condition 1 and for a given $w_t^{NO\ PLAN}$ and α ,solve for ρ.

Social Security: Worker starts receiving Social Security benefit in retirement period R. The Social Security Retirement benefit is $B_t = B(AIME_t, t)$, where $AIME_t$ is average indexed monthly earnings of the worker. During the agent's working life, average earnings (AIME) evolve according to the rule:

$AIME_t = (AIME_{t-1}(t-1) + j_t\{W_t, W_m\})/t$

where W_m is the maximum amount of taxable Social Security earnings. Beginning at the retirement, $AIME_t = AIME_{t-1}$

Assets and returns: Besides having a pension account(s), the worker is able to save also on his own. A_t denotes financial assets at the beginning of period t, r_t is the rate of return on assets. The same interest rate applies to all types of assets - worker's and firm's , meaning that the worker and the firm have access to the same asset market. The return factor, $R_t = 1 + r_t$ is determined by $\ln R_t = \ln \overline{R} + \xi_t$, where ξ_t is an iid normal shock $\xi_t \sim N(0, \sigma_\xi^2)$.[6] Returns are defined to include capital gains, which means that R can be less than one, corresponding to a capital loss. I also assume that there is a liquidity constraint, so that $A_t \geq 0$. The liquidity constraint prevents agents from borrowing against uncertain future income.

Withdrawals from DC account before retirement are subject to a penalty of θ, while withdrawals form DB account are not permitted before reaching retirement.

Law of motion for DC assets: $DC_{t+1} = (DC_t + \rho_t w_t - \omega_t) * R_t$, where ω_t indicates how much is withdrawn from the DC account in period t.

Taxes: Progressive taxation. $\tau_t = \tau(j_t w_t, B_t, A_t r_t, (DC_t + \rho_t w_t - \frac{DC_{t+1}}{R_t}))$. Pensions are taxed at withdrawal.

No bequest motive*:* Retirees live off their assets and pensions and consume everything before death.

The agent's goal in this model is to choose j_t , k_t, ω_t and $C_t, t = 1, \ldots T$ so as to maximize

[6] As in Blau (2004)

the expected present discounted value of lifetime utility, with a discount factor of β.

$$\max_{j_t,k_t,\omega_t,C_t} E\sum_{t=1}^{T}\pi_{t|t-1}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}+\gamma_1 j_t+\gamma_2(1-j_{t-1})j_t+\gamma_3 j_{t-1}j_t k_t+\varepsilon_{jkt}\right)$$

subject to

$$C_t=(1-\tau_t)w_t+A_t(1+(1-\tau_t)r_t)-A_{t+1}+(1-\theta)(1-\tau_t)(DC_t+\rho_t w_t-\frac{DC_{t+1}}{R_t})\text{ for }t\leq R$$

subject to

$$C_t=b_t+(1-\tau_t)Pension_{DB_t}+A_t(1+(1-\tau_t)r_t)-A_{t+1}+DC_t(1+(1-\tau_t)r_t)-DC_{t+1}\text{ for }t>R$$

$$A_{T+1}=0, DC_{T+1}=0$$

Every time the worker changes jobs he can only start a new DB pension plan. Thus, $Pension_{DB_t}=\sum_{i=1}^{f}\alpha w_{\widetilde{t}_{jobi}}z_{\widetilde{t}_{jobi}}$ and $\sum_{i=1}^{f}z_{\widetilde{t}_{jobi}}\leq R$. A worker will hold a total of f jobs with DB plans in his lifetime and job i lasts for $z_{\widetilde{t}_{jobi}}$ years.

Vesting: At this time, both DB and DC plans vest immediately.

2.1 Solving the model

To solve the model I use dynamic programming, which is a standard tool for solving dynamic models. In the DP approach, a multi-period optimization problem under uncertainlty is reformulated as a two-period problem: today and the future. The assumption is that individuals choose their behavior so as to maximize the Expected Present Discounted Value (EPDV) of remaining lifetime utility. This assumption implies that future decisions will be made optimally given the information available in the future. The DP approach allows us to focus on today's decision accounting for its future consequences in a simple way.

The *value function* is defined as the *maximized* EPDV of remaining lifetime utility, where the maximization is with respect to the current-period decision variables, and the expectation is with respect to the distribution of future random variables. DP allows me to write the maximized EPDV of remaining lifetime utiliy as the maximized value of {current utility + the maximized value of remaining lifetime utility next period conditional on today's choices}.[7]

Let S_t represent the set of state variables that characterize the agent's state(information) at the beginning of period t, after realization of the shocks.

$$S_t = (A_t, DB_{t,}DC_t, AIME_t, j_{t-1}, w_{t-1}, \lambda_{t-1}, L_{1t}, L_{2t}, \eta_t, \xi_t, \zeta_t, \varepsilon_{00t}, \varepsilon_{01t}, \varepsilon_{10t}, \varepsilon_{11t})$$

Also define $\overline{S_t}$ as the state space known as of the end of t-1, before period t shocks are realized and ϵ_t as the vector of continuous random variables.

$$\overline{S_t} = (A_{t-1}, DB_{t-1,}DC_{t-1}, AIME_{t-1}, j_{t-1}, w_{t-1}, \lambda_{t-1},), \quad \epsilon_t = (\eta_t, \xi_t, \zeta_t, \varepsilon_{00t}, \varepsilon_{01t}, \varepsilon_{10t}, \varepsilon_{11t})$$

By DB_t I denote the total annual DB pension benefit already earned. In other words, if the worker is to stop working from time t onward, what will his DB pension benefit of all jobs (previous and current) be. Let $u_{jk\omega C_t}(S_t)$ be the period-t choice-specific reward function, characterizing net period-t utility from a given choice j_t, k_t, ω_t, C_t conditional on the state space, and with the budget constraint substituted in. Let $F_{t+1}(S_{t+1}|j_t, k_t, \omega_t, C_t, S_t)$ denote the cumulative distribution function of next period state variables S_{t+1} given S_t, j_t, k_t,ω_t, and C_t.

The value function in period t is the Expected Present Discounted Value (EPDV) of remaining lifetime utility from entering period t with state variables S_t, including the vector of shocks ϵ_t and making optimal choices from t through the end of life.

$$\begin{aligned}
V_t(S_t) &= \max_{j_t,k_t,\omega_t,C_t} \{u_{jk\omega C_t}(S_t) + \beta E_t[V_{t+1}(S_{t+1})|j_t, k_t, \omega_t, C_t, S_t]\} \\
&= \max_{j_t,k_t,\omega_t,C_t} \{u_{jk\omega C_t}(S_t) + \beta E_t \left[\max_{j_{t+1},k_{t+1},\omega_{t+1},C_{t+1},} V_{jk\omega Ct+1}(S_{t+1})|j_t, k_t, \omega_t, C_t, S_t\ \right]\} \\
&= \max_{j_t,k_t,C_t} \{u_{jk\omega C_t}(S_t) + \beta \int \max_{j_{t+1},k_{t+1},\omega_{t+1},C_{t+1}} V_{jk\omega Ct+1}(S_{t+1}) dF_{t+1}(S_{t+1}|j_t, k_t, \omega_t, C_t, S_t)\ \}
\end{aligned}$$

where $V_{jk\omega Ct+1}(S_{t+1}|j_t, k_t, \omega_t, C_t S_t)$ is the choice specific value function in t+1, given S_t

[7] The solution approach follows closely David Blau. 2004. "Retirement and Consumption in a Life Cycle Model" under revision for Journal of Labor Economics

and the decisions made in t. The expression $E_t \underset{j_{t+1},k_{t+1},\omega_{t+1},C_{t+1},}{[\max} V_{jk\omega Ct+1}(S_{t+1})|j_t, k_t, \omega_t, C_t, S_t\ ]$ on the second line is known as the "Emax". The third line just follows the definition of mathematical expectation. As we see, the object that's being maximized is the sum of the preiod t net utility plus the expected discounted value of the period t+1 value function given a period t choice. Under uncertainty, one will in general want to optimize sequentially, period by period, as new information is revealed. Writing the problem this way, guarantees that each agent makes today's choice optimally accounting for the distribution of its future consequences. In addition, if we know the Emax function, then it would be straightforward to compute $V_t(S_t)$, since it would be a standard optimization problem. This suggests a solution strategy where we solve the model backwards, starting at period T. Then when we get to period t, we will have already computed and stored the entire function $E_t[\max V_{jk\omega Ct+1}(S_{t+1})|j_t, k_t, \omega_t, C_t, S_t\ ]$, so it will be available. Solving the model will mean computing the value function, or more precisely the Emax, for every period and every point in the state space. I do this numerically by backward induction starting at period T and using Monte Carlo integration as suggested by Keane and Wolpin (1994). This approach provides the full solution of the model which is necessary in order to conduct policy experiments. Details on the solution are provided in the appendix.

3 Data

This study uses the Panel Study of Income Dynamics (PSID) as the primary source of data. The PSID is a longitudinal survey conducted by the Survey Research Center at the Institute for Social Research at the University of Michigan. Beginning in 1968, the study followed the same set of households, emphasizing the dynamic aspects of their economic and demographic behavior. As of 2005, the sample size consists of almost 8000 households. An important reason that makes the PSID the preferred dataset for the purposes of this study, in comparison to other datasets that also provide wage and pension data, is the fact that the PSID allows me to study the wage-pension tradeoff for workers at different stages of their lifecycle. Unlike other datasets like the Health and Retirement Study where the majority of the individuals are in the later phases of their working lives, the PSID enables me to track the job market and savings decisions of individuals of all ages. Although in the past, the pension data from the PSID has been somewhat limited, in 1999 a new section was added to the core questionnaire that introduced detailed questions on pension participation, eligibility, type and number of plans, percent worker and employer contribution, account balance, vesting. In addition, the PSID contains detailed information on other kind of variables that are likely to enter the estimation, such as socio-demographic characteristics, education, health, employment status, income, wealth, etc. So far the 1999, 2001, 2003 and 2005 waves with detailed pension sections have been released, with the 2007 wave forthcoming.

3.1 Descriptive Statistics and Simulations

Before discussing estimation, it will be useful to simulate the model. This not only allows the researcher to check for errors, but also to examine the characteristics of the behavior generated by a given set of parameters. Moreover, by determining the sensitivity of the results to alternative parameter values, we can get a sense of how robust the findings are. Simulation can be used to calibrate the model, where the parameters are chosen such as to try to "match some moments" of the data.

Initially, the primitive parameters of the model developed in this paper will be assumed to be independent of observable individual characteristics. Laters, this will be relaxed. For the time being, I attempted to define a sample that is relatively homogeneous with respect to a number of demographic characteristics. In particular, I include only white males between the ages of 25 and 54 with at least a high school education. In addition, any individual who reports attendance in school, self-employment, military service, or participation in any government welfare program (i.e., AFDC, WIC, or Food Stamps) over the sample period is excluded. Individuals who have missing data at some point during the panel have also been excluded. Although the format of the PSID data makes the task of defining job changes somewhat difficult[8] , in other respects the survey information is well-suited to the requirements

[8] The interviews are done every couple of years.

of this analysis since it follows individuals for up to six years and includes data on both wages,employment and type of pension plans held during the observation period.

Table 1 contains some descriptive statistics from the sample.[9] All working individuals in 1999 are tracked over the waves till they make their first job transition- either to employment or unemployment. Wee see that a majority of employment spells with pensions end with a transition to a job with pension again. On the contrary, the majority of employment spells without pensions end with a transition to a job without pension. Perhaps the most striking feature of the data is the difference between the average salaries of jobs conditional on providing pensions. Jobs with pensions have a mean salary which is almost twice as high as the average salary on jobs without pensions. Also, transitioning from a job with pension to one without pension, on average, results in a lower salary than if the second job was a pension job as well.We also see that jobs with pensions tend to last somewhat longer than jobs without pensions .

Another feature of the data that is interesting to note is the difference in initial salaries for the various transitions out of pensions.In particular, while the mean initial salary for all jobs with pension is above $46,000, individuals who subsequently move into a job without pension are earning $44,599 initially, on average. In addition, the mean salary in the subsequent no pension job is $48,047, well above the mean salary for nopension jobs accepted coming out of a nopension job as well ($31,611).

[9] Only the 1999,2001 and 2003 waves are considered. Included are only individuals who were employed in 1999.

Table 1:Descriptive Statistics

Characteristics of Jobs with Pensions (866 observations)				
Type of Transition	Number	Spell Duration	Initial Salary	Accepted Salary
Right-censored	434	—–	46,059 (62,087)	—–
To a job with pension	218	1.67 (1.2)	49,246 (80,718)	57,089 (87,280)
To a job without pension	96	1.77 (1.3)	44,599 (38,743)	48,047 (60,602)
To unemployment	118	2.23 (1.2)	41,892 (30,076)	——
Characteristics of Jobs without Pensions (539 observations)				
Type of Transition	Number	Spell Duration	Initial Salary	Accepted Salary
Right-censored	270	——	25,082 (30,124)	——
To a job with pension	85	1.44 (1.1)	25,376 (23,809)	36,879 (33,854)
To a job without pension	110	1.61 (1.1)	23,795 (28,894)	31,611 (39,097)
To unemployment	74	2.1 (1.2)	26,840 (37,906)	——
*standard errors in brackets				

Table 2 presents descriptive statistics derived from simulating a simplified version of the full model.[10] On the face of it, the model seems to be relatively consistent with the trends observed in the sample descriptive statistics. Even in the simplified version, we observe that workers with pension who transition to a non-pension job end up with a lower salary that

[10]The simplified version assumes discrete distribution for y (with 2 points of support), no unemployment, no involuntary separations, flat tax rate, equal contribution rates on DB and DC plans- DB plan accrues evenly throughout the worklife, asset return is constant and equals return on both DB and DC plans.3000 simulated histories; R=5, T=6, y_{min}=26900, y_{max}=50500, $\beta = 0.97, \sigma = 2, \theta = 0.1, \tau = 0.18, contributionrate = 0.03, prob(noplan) = 0.3, prob(DC) = 0.35$

those who transition to another pension job.We also see the similar trend that workers in jobs with pensions who subsequently move to jobs without pension have an initial salary lower than the mean salary for pension job. Also jobs with pensions indeed tend to last longer.

Table 2: Model Simulations

Characteristics of Jobs with Pensions (2100 observations)				
Type of Transition	**Number**	**Spell Duration**	**Initial Salary**	**Accepted Salary**
Right-censored	902	——	43,503 (9.774)	——
To a job with pension	838	2,2 (1.1)	33,386 (10,672)	46,745 (6,805)
To a job without pension	360	2.3 (1.1)	31,943 (9,998)	45,321 (9,780)
Characteristics of Jobs without Pensions (539 observations)				
Type of Transition	**Number**	**Spell Duration**	**Initial Salary**	**Accepted Salary**
Right-censored	306	——	43,250 (10,905)	——
To a job with pension	514	2.1 (1.1)	37,506 (11,750)	45,021 (8,670)
To a job without pension	80	1.9 (0.9)	26,900 (0)	50,500 (0)
*standard errors in brackets				

These results are very preliminary and depend heavily on the chosen parameter values. Nevertheless, they signal for the potential of the model for explaining workers behavior. In the work to follow, the goal will be to recover the primitive parameters of the model from the observed data.

4 Estimation Approach

There are two problems that arise in estimating the behavioral model. First, since PSID samples people of all ages, the decisions that we observe for the majority of the people, do not start at the beginning of their lifecycles but at some later period and are thus conditioned on state variables that arise from prior unobserved decisions. To the extent that those "initial" conditions are not exogenous, e.g., if there is unobserved heterogeneity in preferences or constraints, direct estimation will lead to bias. Second, some of the state variables are missing every other year, which is due to the biennial nature of the PSID. The "intial" conditions problem can be solved by assuming that the probabilities of the unobserved heterogeneity types can be represented by parametric functions of the intial state variables. If the shocks to preferences, wages, etc. are serially independent, the initial state variables will be exogenous given type. The second problem however will be harder to solve with a likelihood-based estimation approach. It would require integrating over the distribution of the missing state variables. Because the missing observations problem affects elements of the state space that take on many values (e.g. assets are treated as continuous), this approach poses a huge computational burden.

I therefore pursue a non-likelihood-based estimation strategy, efficient method of moments,henceforth EMM, which is a type of indirect inference(see Gallant and Tauchen (1996), Gourieroux and Monfort (1996), Smith (1993)). The basic idea is to fit simulated data obtained from the behavioral model to an auxiliary statistical model that can easily be estimated and that provides a complete enough statistical description of the data to be able to identify the behavioral parameters. Following van der Klaauw and Wolpin (2007), I use a combination of approximate decision rules (that link endogenous outcomes of the model and elements of the state space) and modified structural relationships (such as the wage equations).

More specifically, using actual data, y_A , I estimate a set of M_A auxiliary statistical relationships with parameters θ_A. By construction, at the maximum likelihood estimates $\widehat{\theta}_A$, the scores of the likelihood function(L_j for $j = 1, ..., M_A$) are zero. That is, $\frac{\partial L_j}{\partial \theta_{A,j}} = 0$ where $\theta_{A,j}$ is the vector of model j's parameters. Denoting θ_B the parameters of the behavioral model, the idea of EMM is to choose parameters that generate simulated data ($y_B(\theta_B)$) that make the score functions as close to zero as possible. This is accomplished by minimizing the weighted squared deviations of the score functions evaluated at the simulated data. The EMM estimator of θ_B is thus

$$\widehat{\theta}_B = \arg\min_{\theta_B} \frac{\partial L}{\partial \theta_A}(y_B(\theta_B); \widehat{\theta}_A)\Lambda \frac{\partial L}{\partial \theta'_A}(y_B(\theta_B); \widehat{\theta}_A) \ ,$$

where Λ is a weighting matrix and $\frac{\partial L}{\partial \theta_A}(y_B(\theta_B)$ is a vector collecting the scores of the likelihood functions across auxiliary models. When $M_A = 1$, the optimal weighting matrix is the inverse Hessian and has a limiting normal distribution. For tractability, I estimate M_A auxiliary models separately and choose as a weighting matrix a block diagonal matrix Λ^* such that each block is a consistent estimate of the inverse Hessian of the corresponding auxiliary model evaluated at the actual data. The estimator of is consistent when the number of simulated observations grows proportionately with the number of actual observations as the latter goes to infinity.

4.1 The Auxiliary Statistical Models

The solution of the optimization problem of section 2 is a set of decision rules in which the optimal choice made in any decision period is a function of the state space in that period. Parametric approximations to these decision rules will serve as one class of auxiliary models to be used in the estimation. Following van der Klaauw and Wolpin (2007), to keep these approximations parsimonious (as to preserve precision in the parameter estimates), I do not include all the state variables as suggested by the theory, and for that reason it is best to think of them as "restricted" approximate decision rules. A second set of auxiliary models comprises quasi-structural relationships related to the wage equation.

The specific type of parametric approximation adopted depends on whether the choice and state variables are discrete or continuous. The following list consists of auxiliary models to be used in estimation:

1. Logits of unemployment vs employment: variables included in the specification - age,

race, education, tenure at current employer, experience, lagged net assets, lagged AIME, lagged DC balance.

2. Multinomial logits of work in job without pension, job with DB, job with DC pension: variables included in specification - age, education, lagged employment status, tenure, experience, AIME, lagged DC balance.

3. Multinomial logits of transitions b/n job without pension to jobs with DB or DC, and vice versa: variables included - age, tenure, experience, lagged net worth, lagged DC balance.

4. Regression of net assets : variables included- double lagged net assets, age, lagged employment status, lagged pension status, lagged DC balance.

5. Regression of log(accepted) wages: variables included - age, education, lagged log wage, tenure, experience, AIME, DB/DC/no pension dummies.

5 Conclusion

This paper studies the trade-off that workers face when choosing between compensation in the form of wages versus pension contributions. This is a particularly important question, given the recent shifts in the type of pension coverage and the financial strain on Social Security. From a policy stand point, if we want to understand how people save for retirement and how policy can affect their behavior, we need to understand the value that workers place on the wage and pension components of their compensation packages.

This paper suggests a new approach for addressing this question, which focuses on the idea that the wage vs. pension benefit decision is crucially related to the job search and saving for retirement decisions. I formulate, solve and simulate a dynamic discrete choice model in which people search for jobs, consume, save on their own and through their employer so as to maximize utility. What makes this framework different from the classical job search model is the fact that the job offer is a wage-pension package. The decision of a worker to accept or reject a job offer is the result of an interplay between his preferences and the set of incentives and risks associated with the offered pension plan.

Future work includes estimation of the model using the suggested estimation approach. The estimates of the structural parameters will be used to address a set of policy question such as: How does the shift in pension type coverage impact wage growth and job-to-job transitions? What will be the effect of changes in the social security benefit formula or the tax treatment of pensions? Which compensation packages lead to higher retirement savings?

References

Blau, D. M. (2004). Retirement and consumption in a life cycle model. *under revision for Journal of Labor Economics*.

Bloemen, H. G. (2006). Job search, hours restrictions and desired hours of work. *Serie Research Memoranda 0038, Free University Amsterdam.*.

Decressin, A., J. Lane, K. McCue, and M. Stinson (2005). Employer-provided benefit plans, workforce composition and firm outcomes. *U.S. Census Bureau, LEHD Program, Technical Paper No. TP-2005-01*.

Dey, M. S. and C. J. Flinn (2005). An equilibrium model of health insurance provision and wage determination. *Econometrica 73(2)*, 571–627.

Ehrenberg, R. G. (1980, July). Retirement system characteristics and compensating wage differentials in the public sector. *Indistrial and Labor Relations Review 33*(4).

Gallant, R. A. and G. Tauchen (1996). Which moments to match? *Econometric Theory*.

Gourieroux, C. and A. Monfort (1996). Simulation-based econometric methods. *Oxford University Press*.

Gunderson, M., D. Hyatt, and J. E. Pesando (1992, October). Wage-pension trade-offs in collective agreements. *Industrial and Labor Relations Review 46*(1).

Gustman, A. L., O. S. Mitchell, and T. L. Steinmeier (1994, April). The role of pensions in the labor market:a survey of the literature. *Indistrial and Labor Relations Review 47*(3).

Gustman, A. L. and T. L. Steinmeier (1986). A structural retirement model. *Econometrica 54*, 555–584.

J.F.Quinn and R. Burkhauser (1990). Work and retirement. *Handbook of Ageing and the social Sciences,*, Binstock RH, George LK(eds).Academic Press, San Diego.

Keane, M. P. and K. I. Wolpin (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte carlo evidence. *The Review of Economics and Statistics 76*, 648–672.

M.D.Hurd (1990). Research on the elderly :economic status, retirement and consumption and saving. *Journal of Economic Literature 28*, 565–637.

Montgomery, E., K. Shaw, and M. E. Benedict (1992, February). Pensions and wages: An hedonic price theory approach. *International Economic Review 33*(1).

Munnell, A. H. and P. Perun (2006). An update on private pensions. *Center for Retirement Research, Boston College,Issue in Brief 50*.

Poterba, J. M., S. F. Venti, and D. A. Wise (2006). Defined contribution plans, defined benefit plans, and the accumulation of retirement wealth.

Samwick, A. A. and J. Skinner (2006). How will 401(k) pension plans affect retirement income? *American Economic Review 94*.

Schiller, B. R. and R. D. Weiss (1980, November). Pensions and wages: A test for equilizing differences. *The Review of Economics and Statistics 62*(4), 529–538.

Schrager, A. (2006). A life-cycle analysis of the decline of defined benefit plans and job tenure. *Job Market Paper, Columbia University*.

Smith, A. A. (1993). Estimating nonlinear time-series models using simulated vector auto regressions. *Journal of Applied Econometrics*.

Smith, R. S. (1981, August). Compensating differentials for pensions and underfunding in the public sector. *The Review of Economics and Statistics 63*(3), 463–468.

van der Klaauw, W. and K. I. Wolpin (2007). Social security and the retirement and savings behavior of low income households. *revised and resubmitted to Journal of Econometrics*.

6 Appendix

Model Solution

In period T there's no future, just present. So, the value function if alive in period T is:

$$V_T(S_T) = \max_{j_T,k_T,\omega_T,C_T} V_{jk\omega CT}(S_T) = \max_{j_T,k_T,\omega_T,C_T} u_{jk\omega C_T}(S_T)$$

where

$$u_{jk\omega C_T}(S_T) = \frac{C_T^{1-\sigma}-1}{1-\sigma} e^{\gamma_1 j_T} + \gamma_2(1-j_{T-1})j_T + \gamma_3 k_T + \varepsilon_{jkT}$$

This is a one period (static) optimization problem, where the agent just chooses consumption-work is not an option during retirement.

When we move back to period T-1, we want to compute

$$\begin{aligned} V_{T-1}(S_{T-1}) &= \max_{j_{T-1},k_{T-1},\omega_{T-1},C_{T-1}} \{u_{jk\omega C_{T-1}}(S_t) + \\ &\quad +\beta E_{T-1}[V_T(S_T)|j_T-1, k_{T-1}, \omega_{T-1}, C_{T-1}, S_{T-1}]\} \text{ or} \\ V_{T-1}(S_{T-1}) &= \max_{j_{T-1},k_{T-1},\omega_{T-1},C_{T-1}} \{u_{jk\omega C_{T-1}}(S_{T-1}) + \\ &\quad +\beta E_{T-1} [\max_{j_T,k_T,\omega_T,C_T} V_{jk\omega CT}(S_T)|j_{T-1}, k_{T-1}, \omega_{T-1}, C_{T-1}, S_{T-1}\ ]\} \end{aligned}$$

So, we want the Emax not just the max of V_T, since at T-1 we do not yet know the realizations of period T random variables. Moreover, Emax is a function $j_{T-1}, k_{T-1}, \omega_{T-1}, C_{T-1}$ and S_{T-1}, so we need to compute the Emax for all possible values of that S_{T-1} could take on. The agent could arrive in period T-1 with many different combinations of the state variables. We have to account for all possibilities, because we do not know, neither does he, which combination he will realize.

The state space at the beginning of T-1 before the realization of the T-1 shocks is: $\overline{S_{T-1}} = (A_{T-1}, DB_{T-1,}DC_{T-1}, AIME_{T-1}, j_{T-1}, w_{T-1}, \lambda_{T-1})$. One complication comes from the fact that $A_{T-2}, DB_{T-2,}DC_{T-2}$ and $AIME_{T-2}$ are continuous. But we could easily discretize these into grids of values. The finer the grid, the better the approximation.

Still, for each point in the state space, we need to compute

$$E_{T-1}\ [\max_{j_T,k_T,\omega_T,C_T} V_{jk\omega CT}(S_T)|j_{T-1},k_{T-1},C_{T-1},S_{T-1}\ ]\} =$$
$$= \int \max_{j_T,k_T,\omega_T,C_T} V_{jk\omega CT}(S_T)dF_T(S_T|j_{T-1},k_{T-1},\omega_{T-1},C_{T-1},S_{T-1})\ \}$$

Notice also, that the situation entering period T depends on work status in period T-1 (j_{T-1}).Let's write out separately the equations for $j_{T-1}=0$ and $j_{T-1}=1$. Let $E_{T-1}(Max|j_{T-1})$ denote EMax conditional on the value of j_{T-1}.

$$\begin{aligned} E_{T-1}(Max|j_{T-1} &= 0) = \int [\pi_{T|T-1}\{\phi^u \max_{j_T\in(0,1),\omega_T,C_T} u_{j0\omega C_T}(S_T) + (1-\phi^u)\max_{\omega_T,C_T} u_{00\omega_T,C_T}(S_T)]dF_T^{'}(\epsilon_T) \\ E_{T-1}(Max|j_{T-1} &= 1) = \int [\pi_{T|T-1}\{\delta[\phi^u \max_{j_T\in(0,1),\omega_T,C_T} u_{j0\omega C_T}(S_T) + \\ &+(1-\phi^u)\max_{\omega_T,C_T} u_{00\omega C_T}(S_T) + (1-\delta)[\phi^e \max_{j_T\in(0,1),k_T\in(0,1),\omega_T,C_T} u_{jk\omega C_T}(S_T) \\ &+(1-\phi^e)\max_{j_T\in(0,1),\omega_T,C_T} u_{j0\omega C_T}(S_T)]\}dF_T^{'}(\epsilon_T) \end{aligned}$$

where $dF_T^{'}(\epsilon_T)$ is the CDF of the continuous random variables $\eta_T, \xi_T, \zeta_T, \varepsilon_{jkT}$ which is independent of the state space in T-1, since these are all iid by assumption.

This integral is not easy to compute even with iid disturbances, because of the max function and because some disturbances enter nonlinearly. It is possible to achieve an analytical solution, but there's just one functional form for the disturbances that allows that - Type I Extreme Value. This might be a reasonable assumption for the utility disturbances, but not necessarily for the wages or asset returns. Instead, I will use a much more general approach as described in Keane and Wolpin (1994). They suggest using Monte Carlo integration to approximate the Emax function, which gives the researcher a lot of flexibility in the choice of functional form for the disturbances. The idea is to simply compute the integral numerically, averaging over a sufficiently large number of random draws from the joint distribution of the disturbances.

Given the distributional assumption for the disturbances ($\eta_t \sim N(0,\sigma_\eta^2), \xi_t \sim N(0,\sigma_\xi^2), \zeta_t \sim N(0,\sigma_\zeta^2), \varepsilon_{jkt} \sim N(0,\sigma_\varepsilon^2)$), take N draws from the joint distribution $F_T^{'}(\epsilon_T)$ of the period T random variables. Let $\widetilde{\eta}_T^r, \widetilde{\xi}_T^r, \widetilde{\zeta}_T^r, \widetilde{\varepsilon}_{jkT}^r$ represent the rth draws. Then,

$$\begin{aligned}
\widetilde{u}_{jk\omega C_T}^r(\widetilde{S}_T^r) &= \frac{C_T^{1-\sigma}-1}{1-\sigma}e^{\gamma_1 j_T} + \gamma_2(1-j_{T-1})j_T + \gamma_3 k_T + \widetilde{\varepsilon}_{jkT}^r \\
\ln \widetilde{y}_T^r &= \mu_{Th} + \widetilde{\eta}_T^r \\
\widetilde{w}_T^r(\omega, z_T) &= \omega exp(\alpha_1 z_T + \alpha_2 z_T^2 + \widetilde{\zeta}_T^r) \\
ln\widetilde{R}_T^r &= \ln \overline{R} + \widetilde{\xi}_T^r
\end{aligned}$$

and let $\widetilde{S}_T^r$ represent the vector of period T state variables evaluated at the rth draws.

Finally,

$$E_{T-1}(Max|j_{T-1} = 0) \approx \frac{1}{N}\sum_{r=1}^{N}[\pi_{T|T-1}\{\phi^u \max_{j_T\in(0,1),\omega_T,C_T} \widetilde{u}_{j0\omega C_T}^r(\widetilde{S}_T^r) + (1-\phi^u)\max_{\omega_T,C_T}\widetilde{u}_{00\omega C_T}^r(\widetilde{S}_T^r)] \quad (4)$$

$$\begin{aligned}
E_{T-1}(Max|j_{T-1} = 1) \approx& \frac{1}{N}\sum_{r=1}^{N}[\pi_{T|T-1}\{\delta[\phi^u \max_{j_T\in(0,1),\omega_T,C_T} \widetilde{u}_{j0\omega C_T}^r(\widetilde{S}_T^r) + \quad (5)\\
&+(1-\phi^u)\max_{\omega_T,C_T}\widetilde{u}_{00\omega C_T}^r(\widetilde{S}_T^r)] + \\
&(1-\delta)[\phi^e \max_{j_T\in(0,1),k_T\in(0,1),\omega_T,C_T} \widetilde{u}_{jk\omega C_T}^r(\widetilde{S}_T^r) + (1-\phi^e)\max_{j_T\in(0,1),\omega_T,C_T}\widetilde{u}_{j0\omega C_T}^r(\widetilde{S}_T^r)]\}
\end{aligned}$$

As N becomes large, the approximation converges to the true value of the expectation. Notice, that inside the summations, the max functions are easy to compute - they require just finding the maximal $\widetilde{u}_{jkC_T}^r(\widetilde{S}_T^r)$ depending on the available choice set. The two expressions above must be computed for all points in the state space that could be reached at the end of period T-1, given the possible state space at the beginning of T-1, and the feasible choices in T-1. This allows us to now calculate the value function if alive in period T-1.

Going back in period T-2:

$$V_{T-2}(S_{T-2}) = \max_{j_{T-2},k_{T-2},\omega_{T-2},C_{T-2}} V_{jk\omega CT-2}(S_{T-2}) =$$

$$= \max_{j_{T-2},k_{T-2},\omega_{T-2},C_{T-2}}\{u_{jk\omega C_{T-2}}(S_{T-2}) + \beta E_{T-2}[V_{T-1}(S_{T-1})|j_{T-2},k_{T-2},\omega_{T-2},C_{T-2},S_{T-2}]\}$$

where the expected value value of V_{T-1} is then approximated by equations 4 and 5 lagged one period. Repeat back to period 1. The solution approach is identical.